

# Cosmology from Large Scale full-shape analyses combining 2PCF and 3PCF

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ALMA MATER STUDIORUM  
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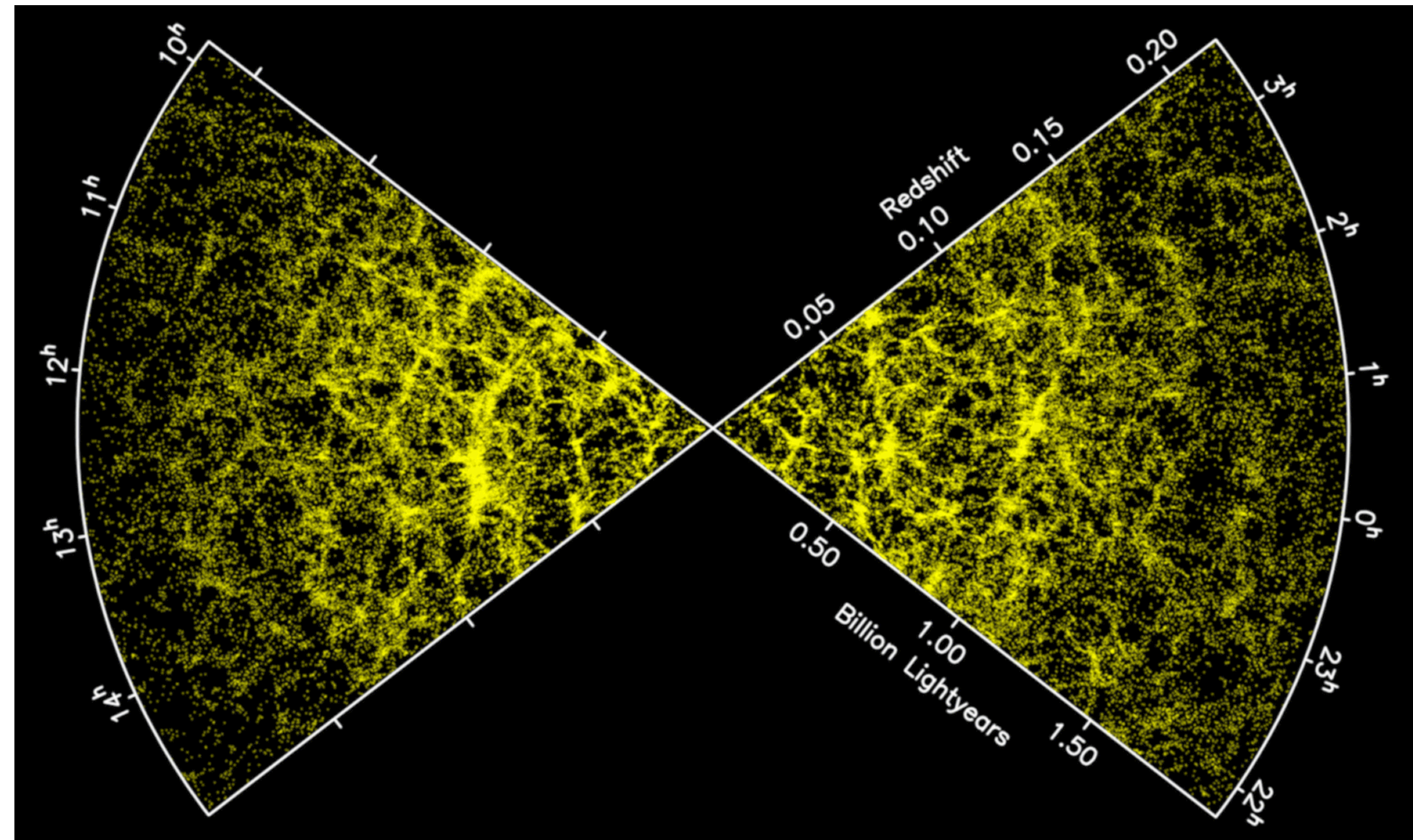
In collaboration with: M. Moresco, K. Nagainis, A. Labate, L. Cavazzini, B. Metcalf (UniBo); A. Veropalumbo, A. Farina, E. Branchini, I. Risso, B. Granett (INAF OABr, INFN-Genova, UniGE)



## Redshift surveys provide a description of the Large Scale Structure of the Universe

Large observational campaigns (such as SDSS, BOSS, DESI, Euclid, ...) aim to unveil the Dark Universe:

- Dark energy
- Dark matter
- Beyond  $\Lambda$ CDM (Inflation, General Relativity, Neutrinos, ...)

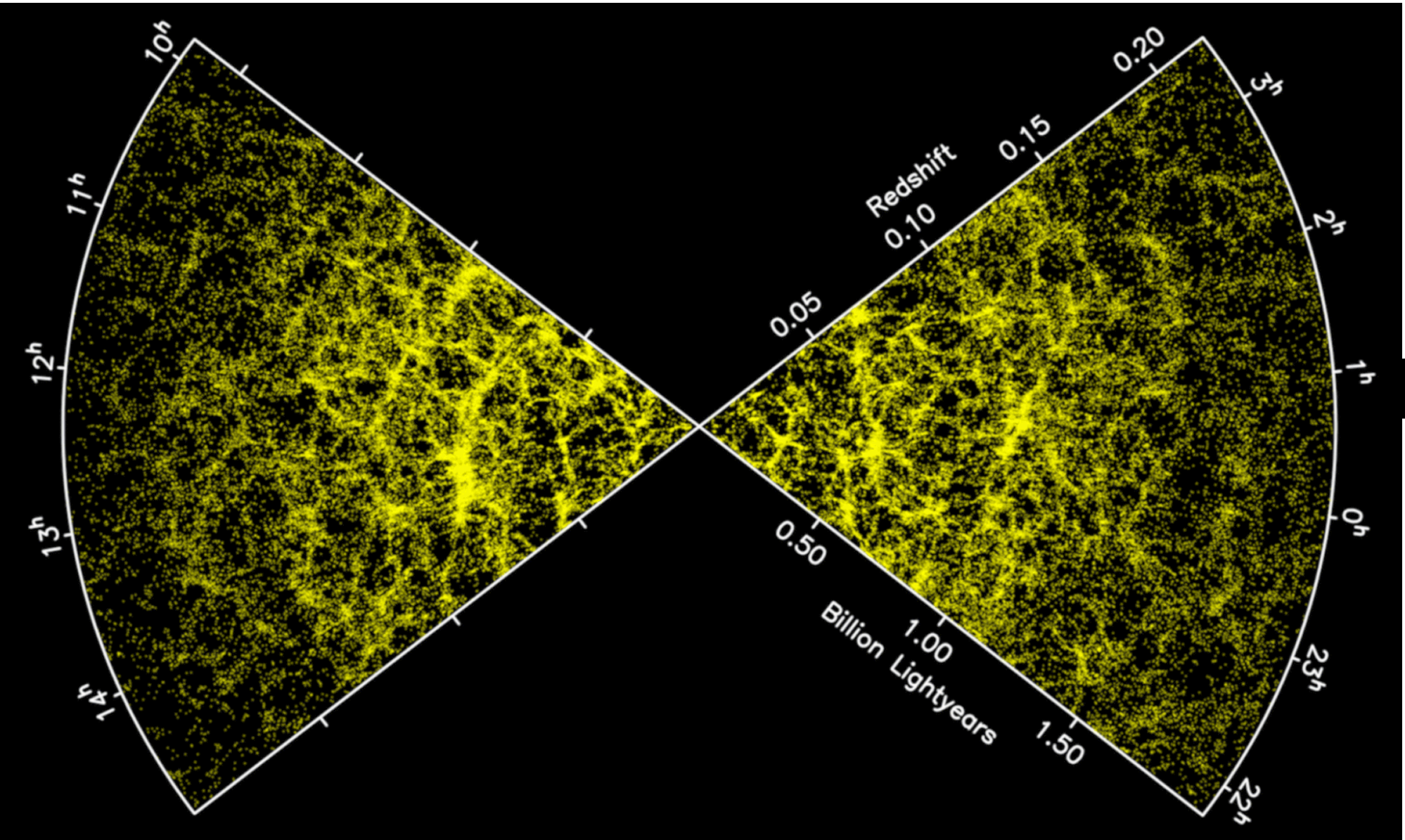


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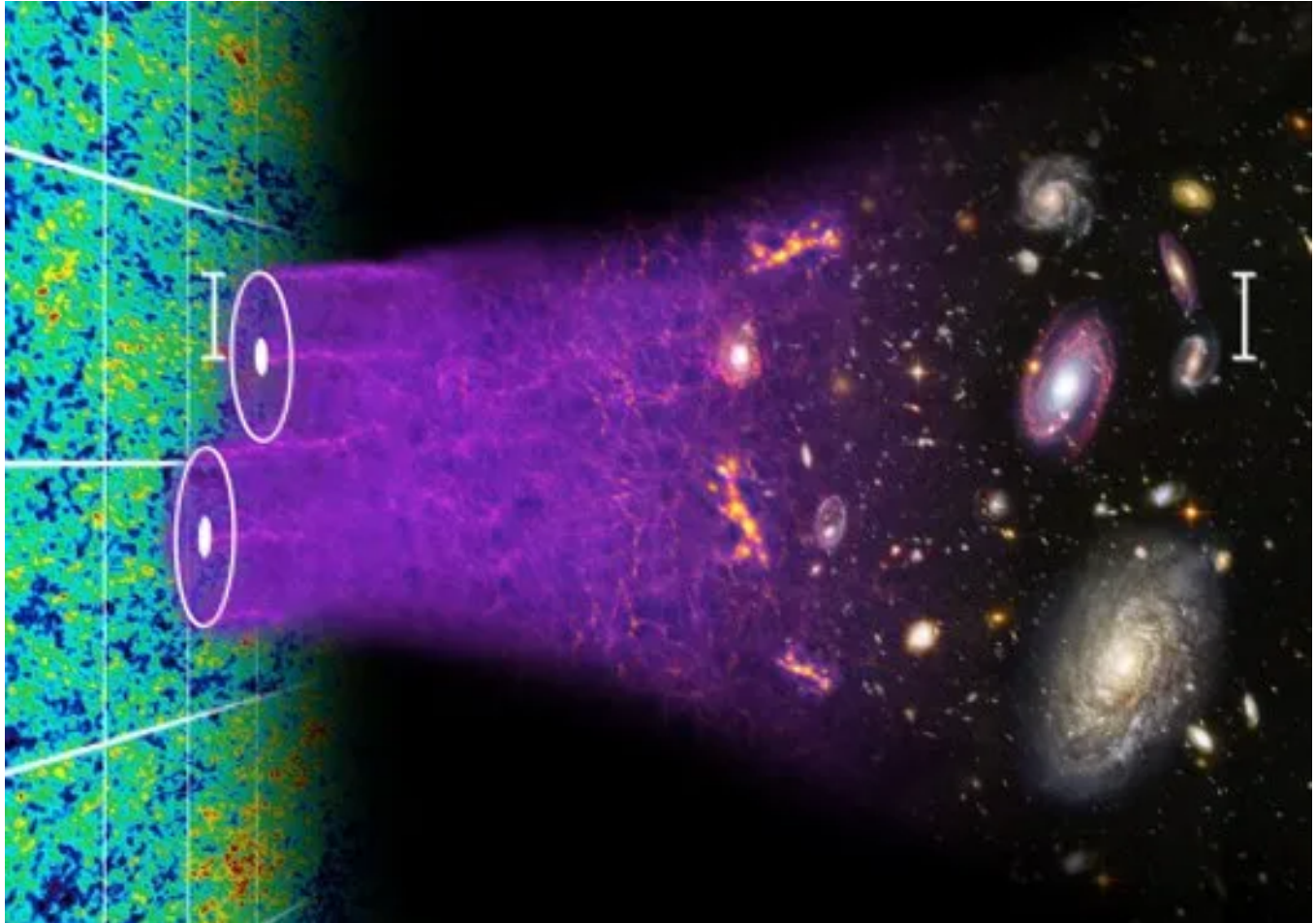
# Baryonic acoustic oscillations

How to extract cosmological information from redshift surveys?



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Baryonic acoustic oscillations after decoupling act as a **standard ruler** during the expansion history...



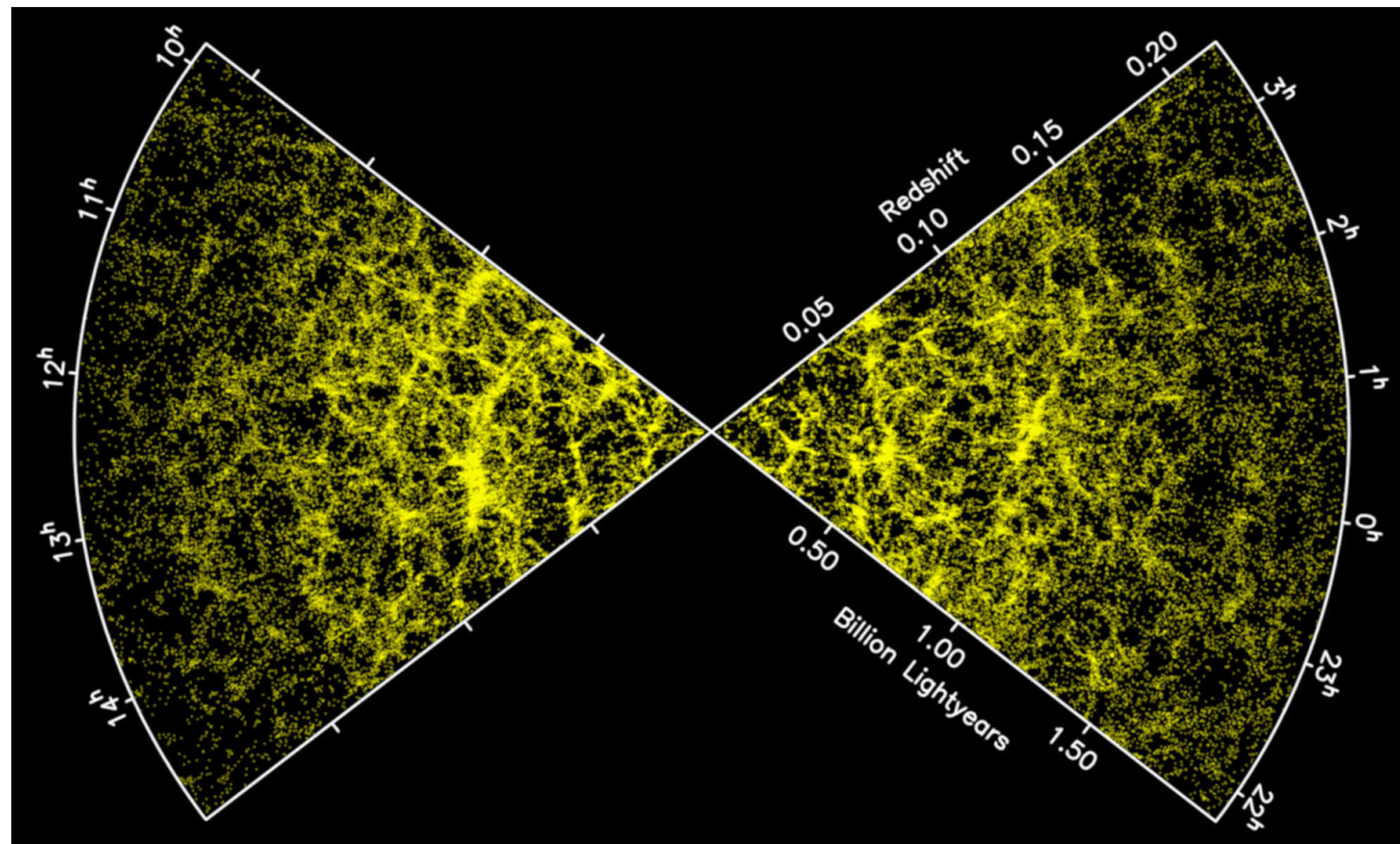
...like an expanding wave with an associated frequency



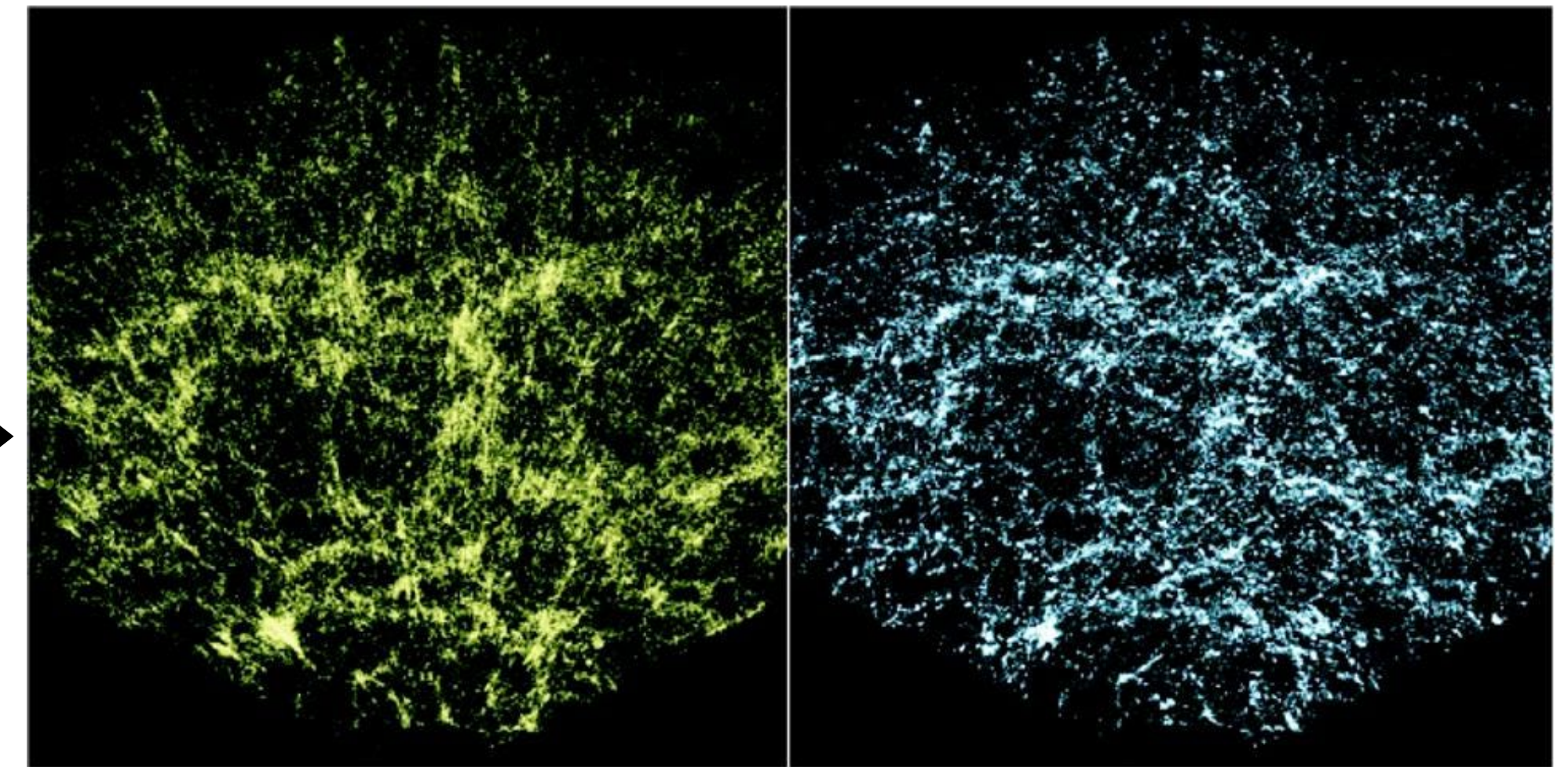


## How to extract cosmological information from redshift surveys?

When converting redshifts to physical distances, neglecting the effect of peculiar velocities gives rise to a distortion of clustering signal along the line of sight introducing observational **anisotropies**...



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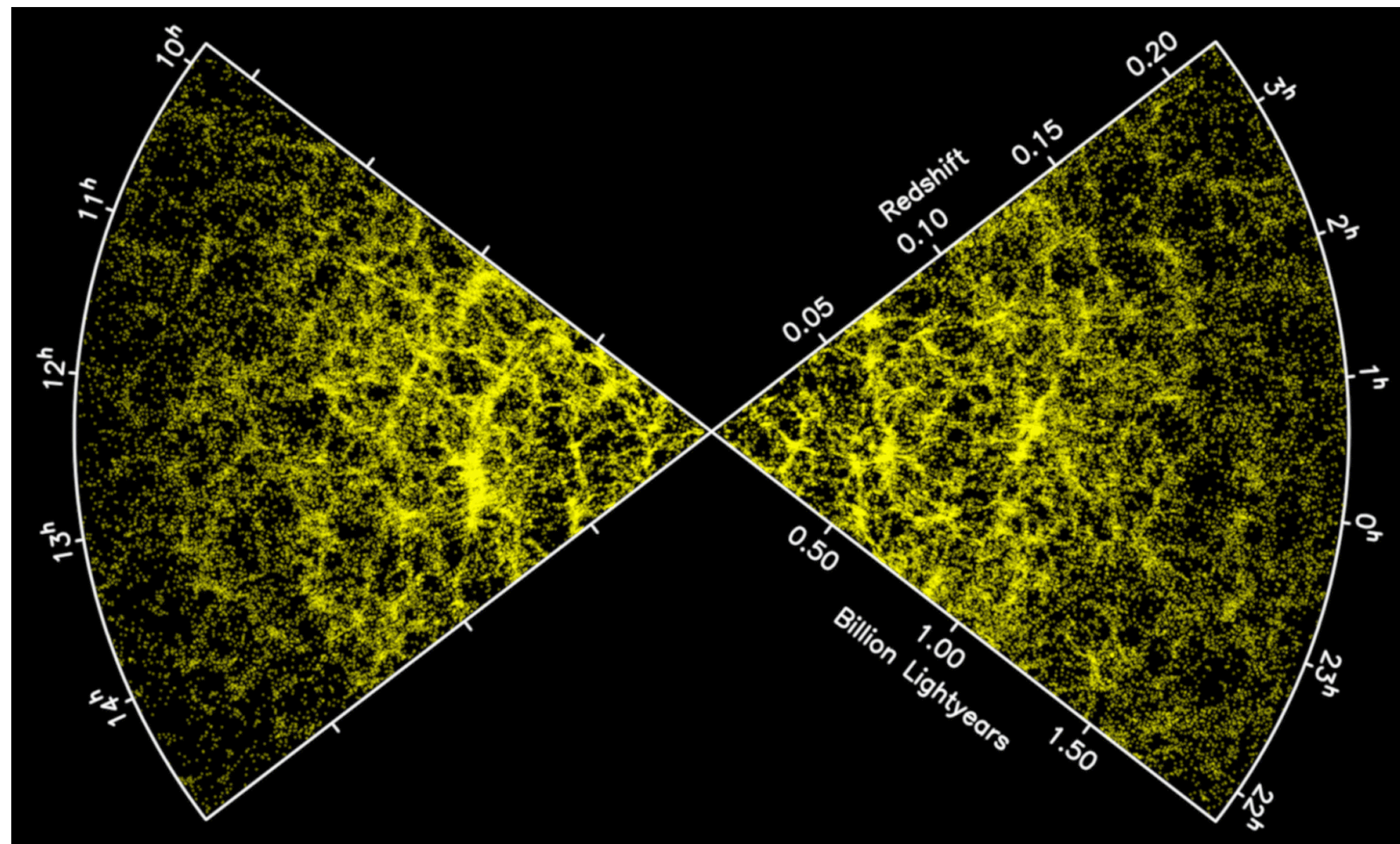


...degrading the quality of extraction of information from the BAO peak, but adding **extra information** depending on the **growth factor** of cosmic structures!



# Redshift space distortions

How to extract cosmological information from redshift surveys?



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**Template fitting:** Alcock-Paczynski parameters (BAO) + growth factor (RSD)

**Full-shape:** direct sampling of the posterior distribution of cosmological parameters by fitting the shape dependence on cosmological parameters themselves



# Correlation functions - Two-Point Correlation Function (2PCF) or Power spectrum

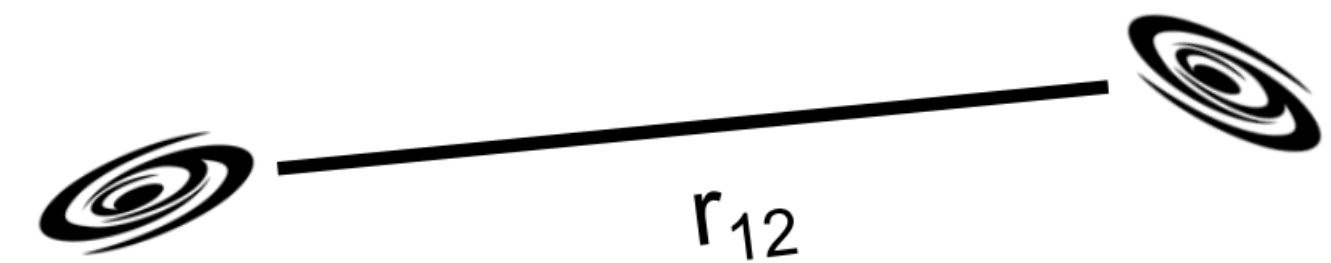
Probability of finding **pairs** of galaxies at a given  $r$  with respect to a random distribution:  $dP = n^2[1 + \xi(r)]dV_1dV_2$

2PCF estimator:  
*Landy & Szalay (1993)*

$$\xi(r) = \frac{DD - 2DR + RR}{RR}$$

*Peebles (1993)*

$$\xi(r) = \frac{DD}{RR} - 1$$



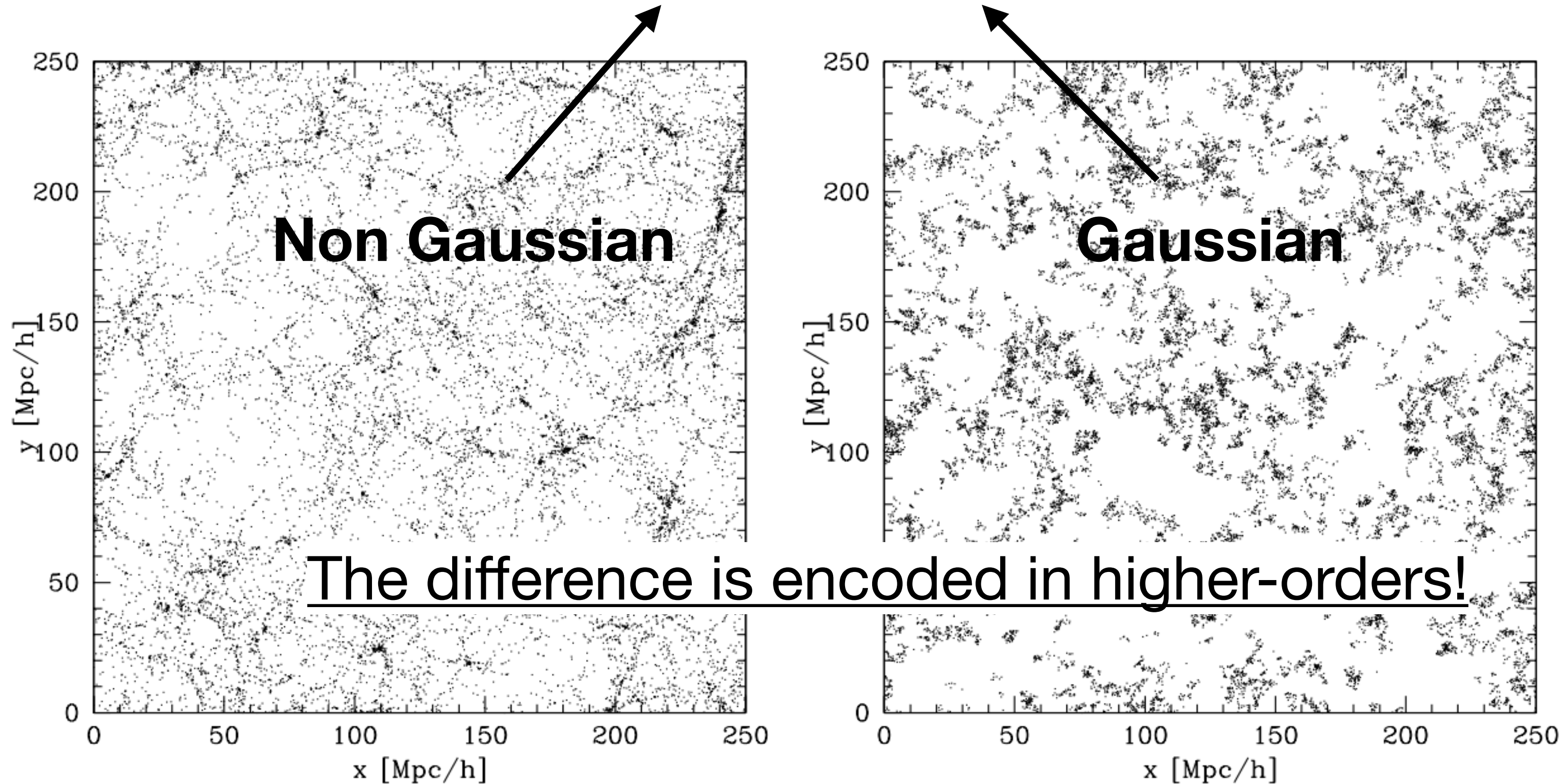
For a **Gaussian** Random Field, two-point statistics are enough to completely describe the distribution...

... but this is far from the large scale structure of the universe!



# Higher-order correlation functions

Same 2-point statistics



Slices of thickness 50  $\text{Mpc}/h$  of a mock galaxy distributions for SDSS (left) and a realisation of a Rayleigh-Lévy flight (right)

Sefusatti & Scoccimarro, 2005

## Non-Gaussianity:

- Nonlinear *gravitational* evolution
- Nonlinear relation between luminous tracers and dark matter perturbation (*galaxy biasing*)
- *Primordial* Non-Gaussianity



# Correlation function - Three-Point Correlation Function (3PCF) or Bispectrum

Probability of finding **triplets** of galaxies at a given *triangle* with respect to a random distribution:  $dP = n^3[1 + \xi(r_{12}) + \xi(r_{13}) + \xi(r_{23}) + \zeta(r_{12}, r_{13}, r_{23})]dV_1dV_2dV_3$

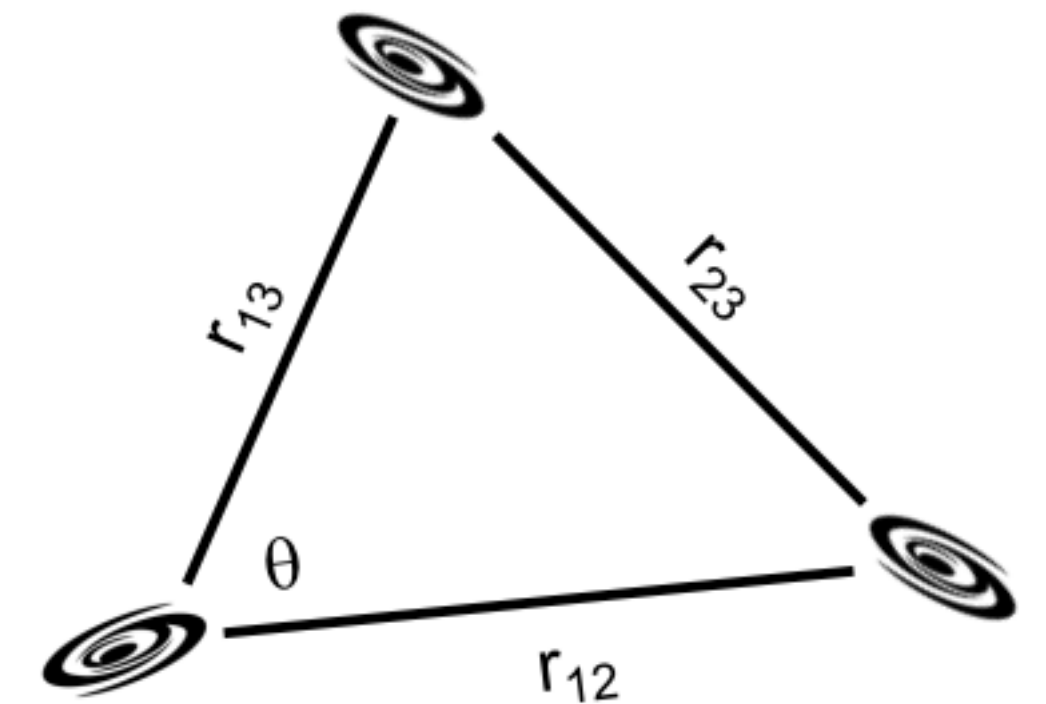
3PCF estimator:  $\zeta(r_{12}, r_{13}, r_{23}) = \frac{DDD - 3DDR + 3DRR - RRR}{RRR}$   
Szapudi & Szalay (1998)

## Spherical Harmonic Decomposition:

Slepian & Eisenstein (2015, 2017)

$$\zeta(r_{12}, r_{13}, r_{23}) = \sum_0^{\ell_{\max}} \zeta_{\ell}(r_{12}, r_{13}) \mathcal{L}(\hat{r}_{12} \cdot \hat{r}_{13})$$

$$\zeta_{\ell}(r_{12}, r_{13}) = \frac{DDD_{\ell} - 3DDR_{\ell} + 3DRR_{\ell} - RRR_{\ell}}{RRR_0}$$

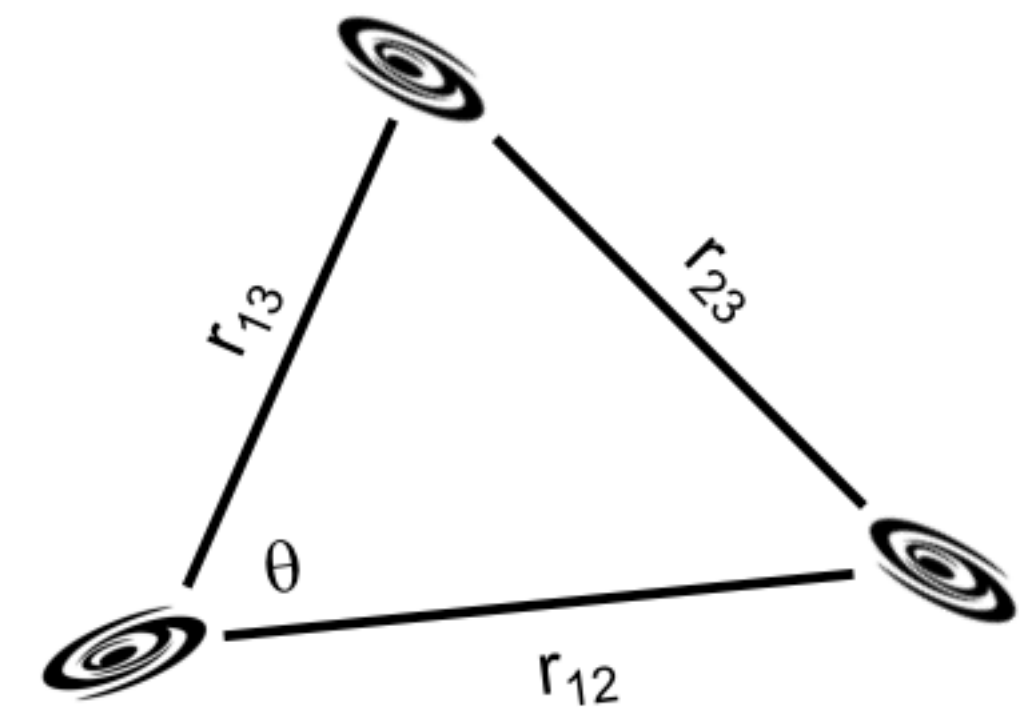
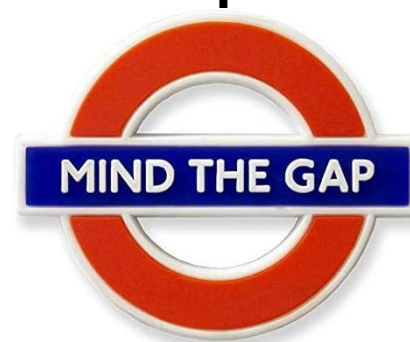




# Correlation function - 3PCF or Bispectrum

Probability of finding **triplets** of galaxies at a given *triangle* with respect to a random distribution:  $dP = n^3[1 + \xi(r_{12}) + \xi(r_{13}) + \xi(r_{23}) + \zeta(r_{12}, r_{13}, r_{23})]dV_1dV_2dV_3$

Configuration space	Fourier space
<p><i>Estimator:</i></p> <ul style="list-style-type: none"> <li>Now feasible thanks to Spherical Harmonic Decomposition</li> <li><b>Survey footprint</b>: just random distribution needed</li> </ul> <p><i>Modelling (2PCF + 3PCF):</i></p> <ul style="list-style-type: none"> <li>Template fitting (BAO, BAO + RSD), computational intensive!</li> <li>No full-shape yet! <b>Gap</b> with Fourier space</li> </ul>	<p><i>Estimator:</i></p> <ul style="list-style-type: none"> <li><b>Survey footprint</b> requires window function to model mode coupling</li> </ul> <p><i>Modelling (P + B):</i></p> <ul style="list-style-type: none"> <li><b>Accessible modeling</b>: template fitting and full-shape (tree-level B)</li> </ul>





# Recap

- Extraction of **cosmological information** from redshift survey
  - Probes: BAO, RSD
  - Methods: Template fitting, Full-shape
- **Higher-order**: important as they add **constraining power**, configuration space data better deal with systematics due to the **footprint**, but both modelling and measuring have a **high computational cost**

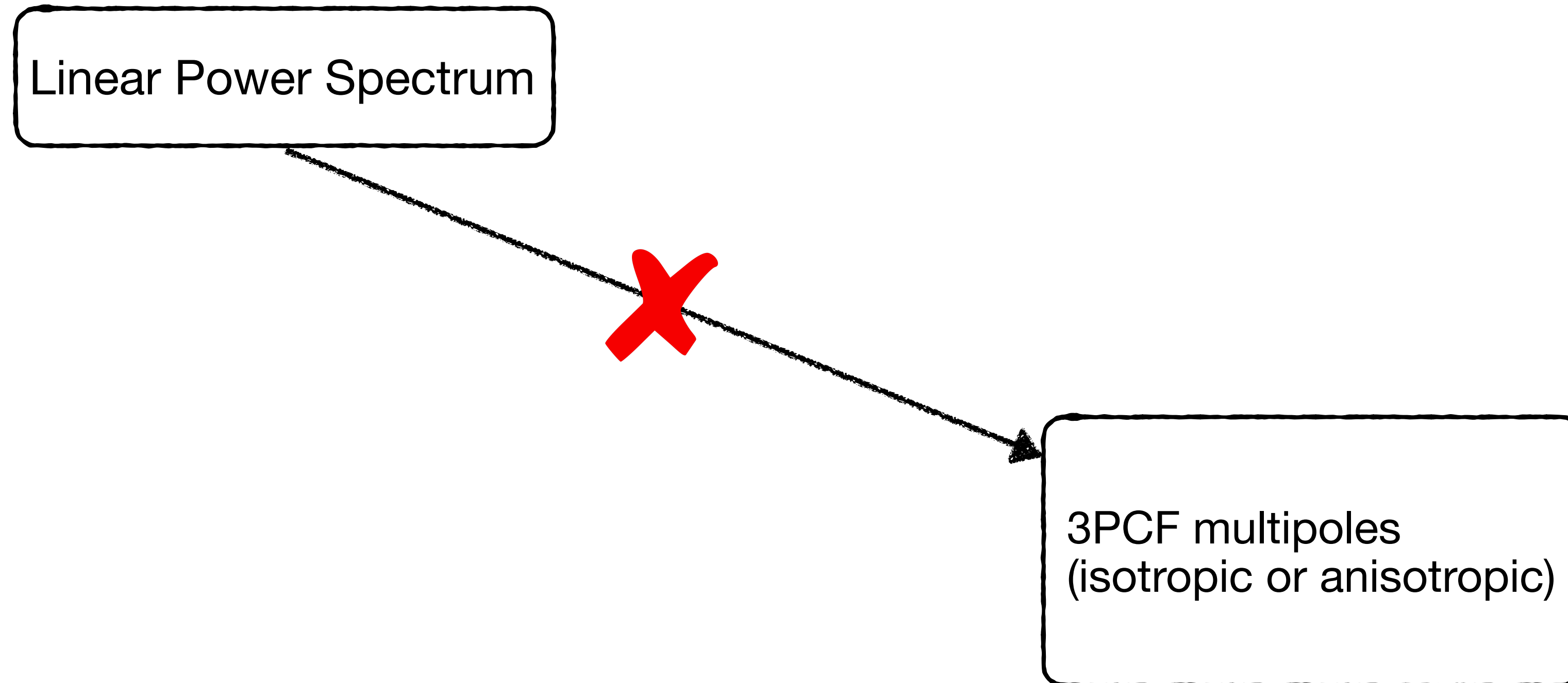


# How modelling the 3PCF?

3PCF multipoles  
(isotropic or anisotropic)

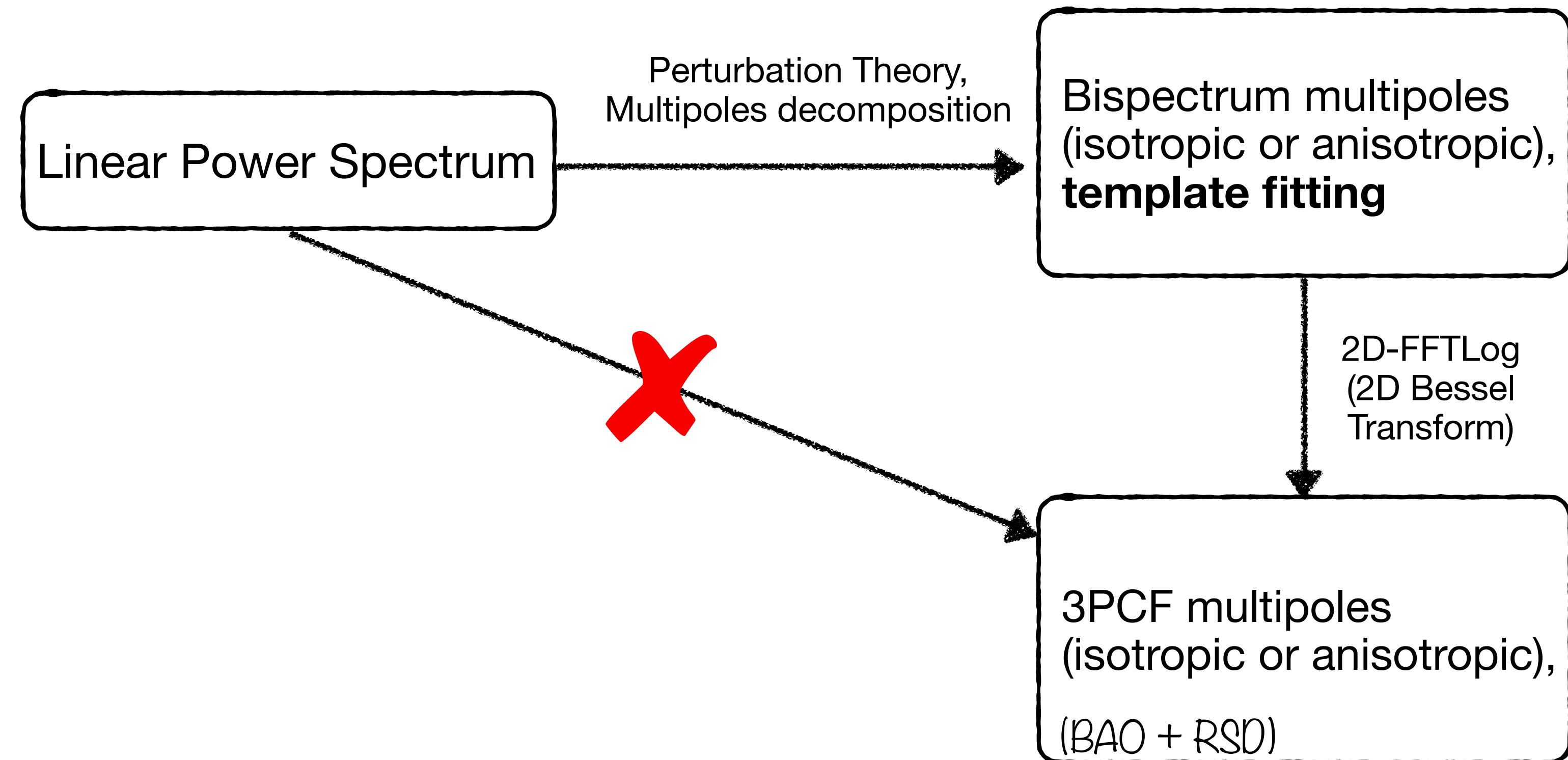


# How modelling the 3PCF?





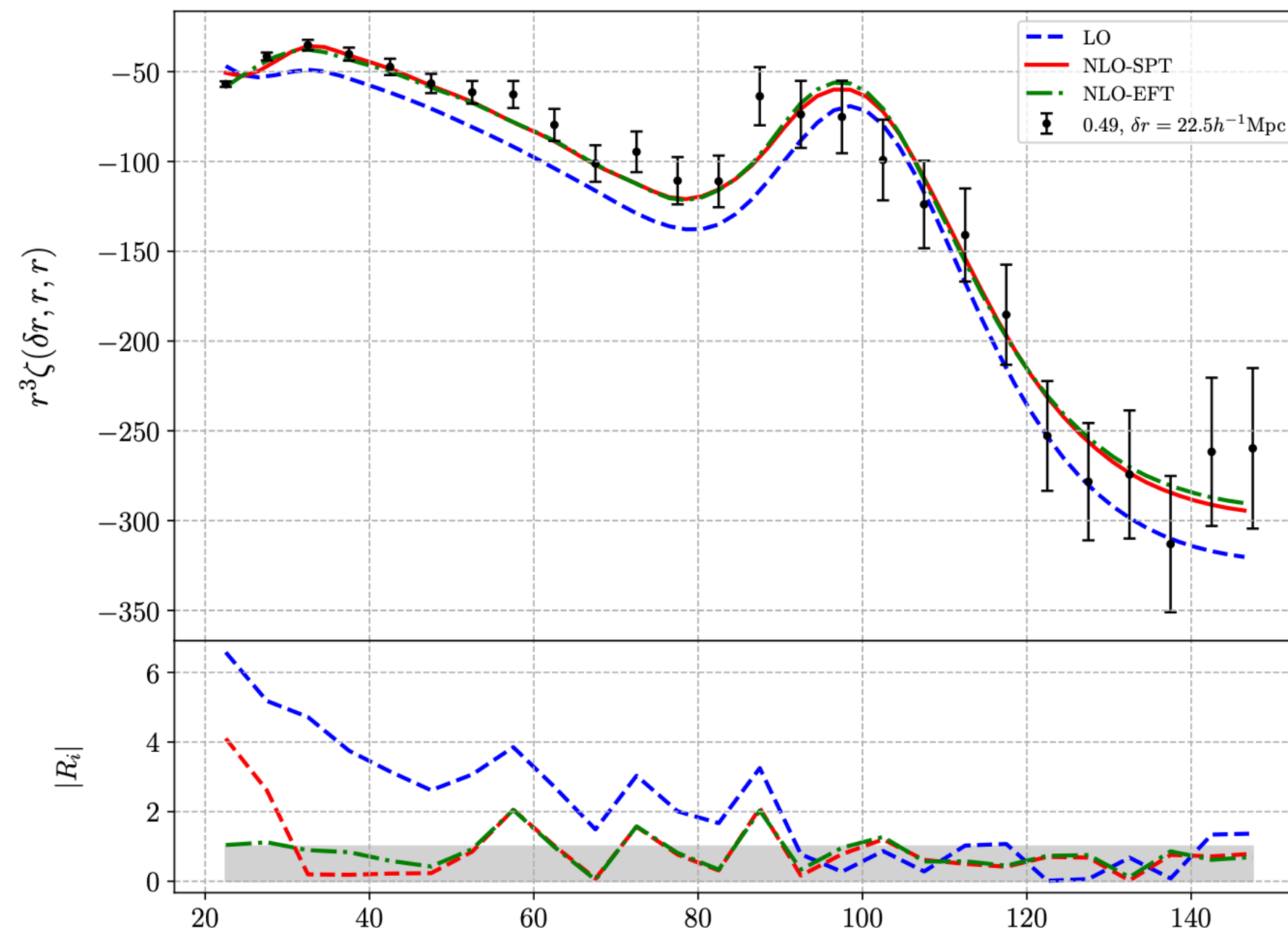
# How modelling the 3PCF?





# How modelling the 3PCF?

A squeezed isosceles triangle BAO configuration in the small-scale regime:



One-loop (red and green) and tree-level 3PCF matter models compared with measurements from the DEMNUni N-Body simulation

Bispectrum multipoles  
(isotropic or anisotropic),  
**template fitting**

(2023) Reaching the small-scale regime

2D-FFTLog  
(2D Bessel Transform)

Some related works:

Umeh et al, 2020

Veropalumbo et al, 2022

**Guidi et al, 2023**

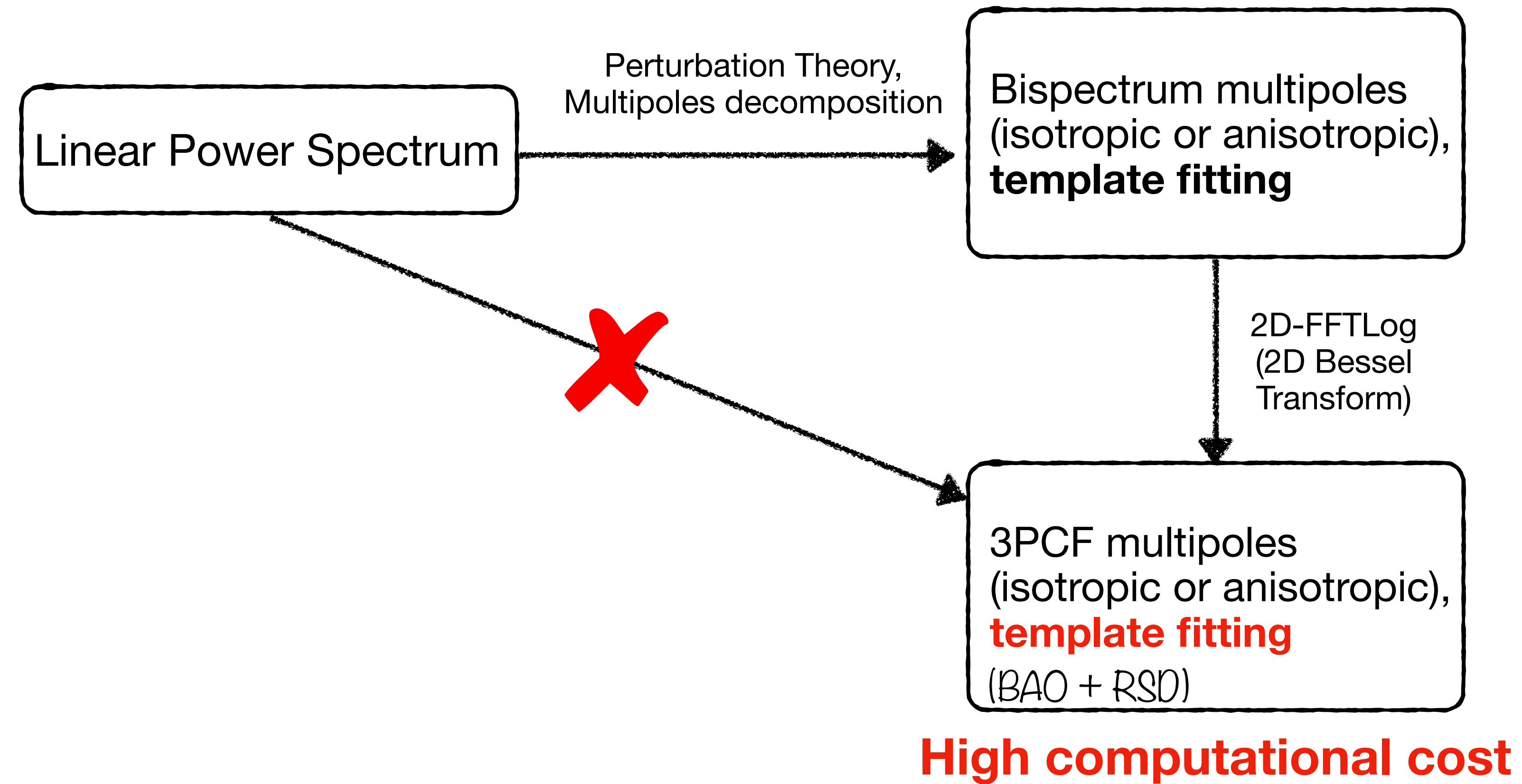
Farina et al, 2024

Pugno et al, 2024

3PCF multipoles  
(isotropic or anisotropic),  
(BAO + RSD)

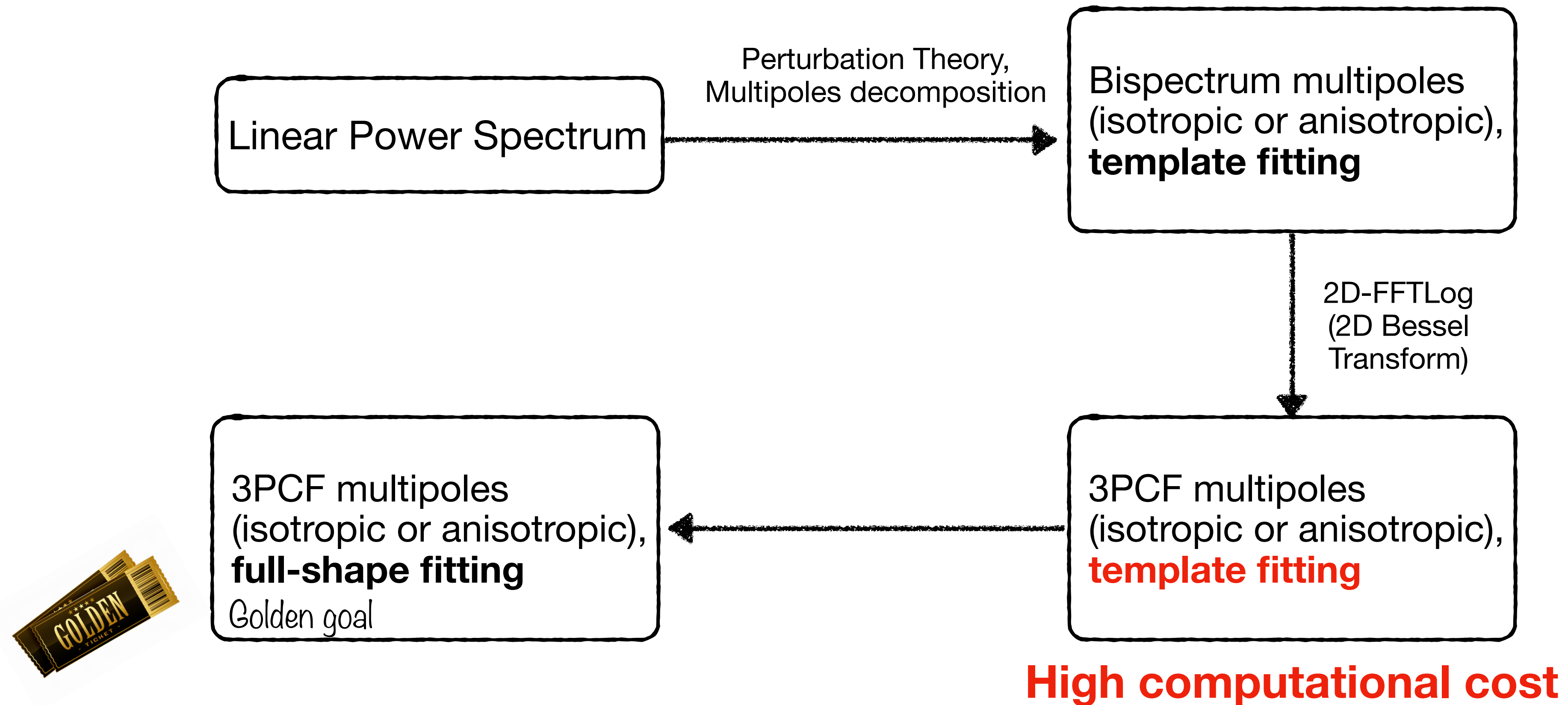


# How modelling the 3PCF?



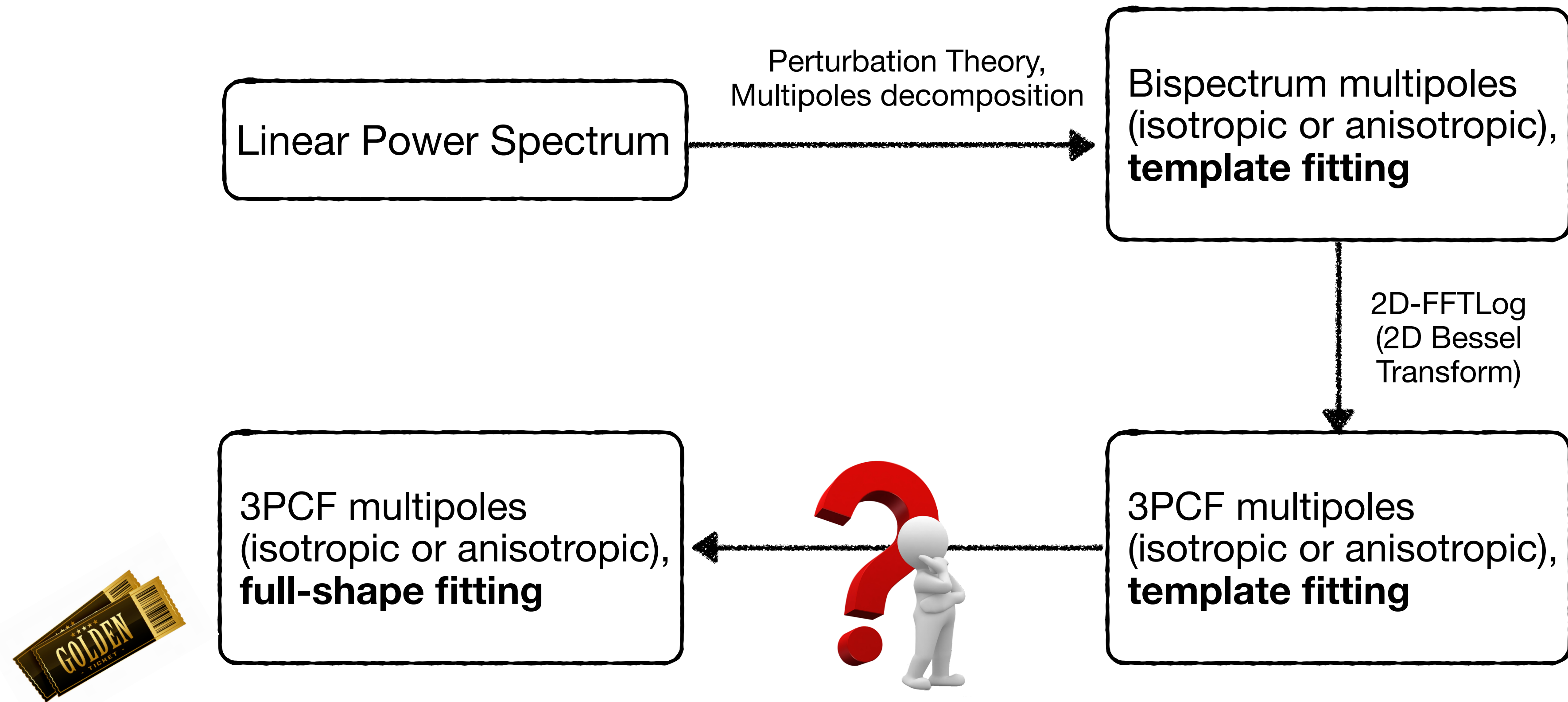


# How modelling the 3PCF?



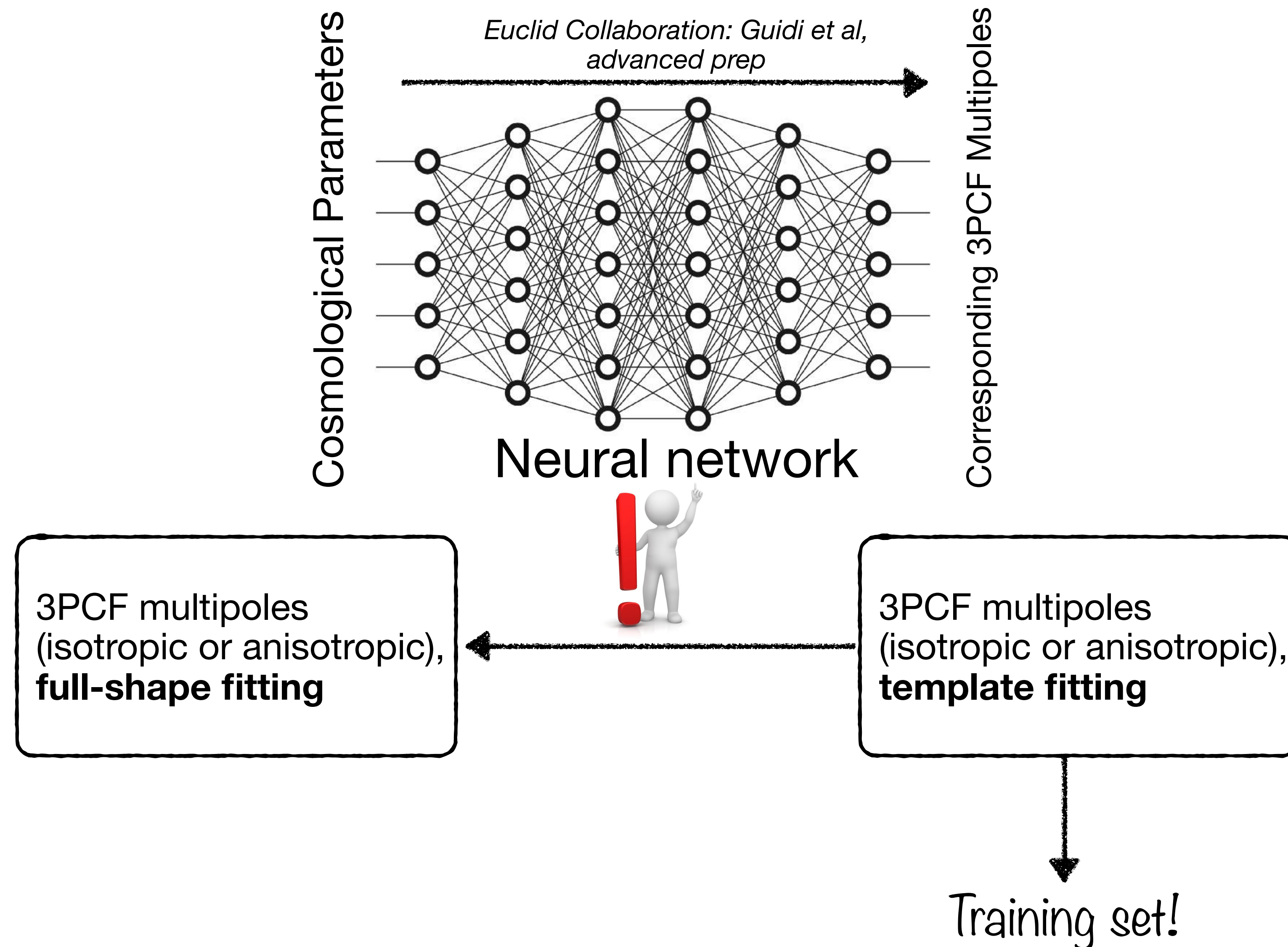


# How modelling the 3PCF?





# How modelling the 3PCF?





## First higher-order full-shape analysis in configuration space in the literature

### Euclid preparation. Full-shape modelling of 2-point and 3-point correlation functions in real space

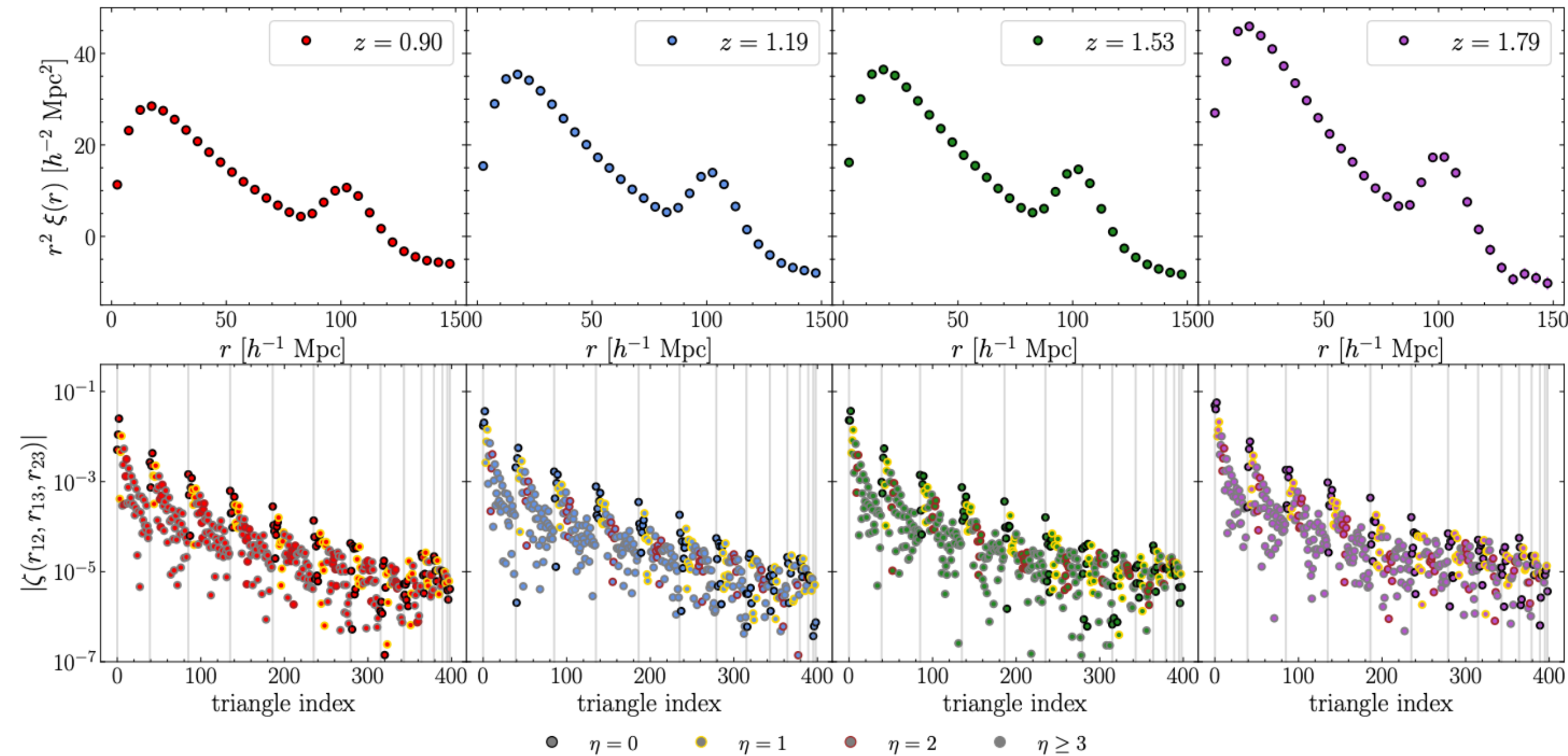
Euclid Collaboration: M. Guidi<sup>1,2</sup>, A. Veropalumbo<sup>3,4,5</sup>, A. Pugno<sup>6</sup>, M. Moresco<sup>7,2</sup>, E. Sefusatti<sup>8,9,10</sup>, C. Porciani<sup>6</sup>, E. Branchini<sup>5,4,3</sup>, M.-A. Breton<sup>11,12,13</sup>, B. Camacho Quevedo<sup>9,14,8,15,11</sup>, M. Crocce<sup>11,15</sup>, S. de la Torre<sup>16</sup>, V. Desjacques<sup>17</sup>, A. Eggemeier<sup>6</sup>, A. Farina<sup>5,3,4</sup>, M. Kärcher<sup>16,18,19</sup>, D. Linde<sup>20</sup>, M. Marinucci<sup>21,22</sup>, A. Moradinezhad Dizgah<sup>23</sup>, C. Moretti<sup>14,24,8,9,10</sup>, K. Pardede<sup>20</sup>, A. Pezzotta<sup>25,26</sup>, E. Sarpa<sup>14,24,10</sup>, A. Amara<sup>27</sup>, S. Andreon<sup>3</sup>, N. Auricchio<sup>2</sup>, C. Baccigalupi<sup>9,8,10,14</sup>, D. Bagot<sup>28</sup>, M. Baldi<sup>1,2,29</sup>, S. Bardelli<sup>2</sup>, P. Battaglia<sup>2</sup>, A. Biviano<sup>8,9</sup>, M. Brescia<sup>30,31</sup>, S. Camera<sup>32,33,34</sup>, G. Cañas-Herrera<sup>35,36,37</sup>, V. Capobianco<sup>34</sup>, C. Carbone<sup>38</sup>, V. F. Cardone<sup>39,40</sup>, J. Carretero<sup>41,42</sup>, M. Castellano<sup>39</sup>, G. Castignani<sup>2</sup>, S. Cavuoti<sup>31,43</sup>, K. C. Chambers<sup>44</sup>, A. Cimatti<sup>45</sup>, C. Colodro-Conde<sup>46</sup>, G. Congedo<sup>47</sup>, L. Conversi<sup>48,49</sup>, Y. Copin<sup>50</sup>, F. Courbin<sup>51,52</sup>, H. M. Courtois<sup>53</sup>, A. Da Silva<sup>54,55</sup>, H. Degaudenzi<sup>56</sup>, G. De Lucia<sup>8</sup>, H. Dole<sup>57</sup>, M. Douspis<sup>57</sup>, F. Dubath<sup>56</sup>, X. Dupac<sup>49</sup>, S. Dusini<sup>22</sup>, S. Escoffier<sup>58</sup>, M. Farina<sup>59</sup>, R. Farinelli<sup>2</sup>, F. Faustini<sup>39,60</sup>, S. Ferriol<sup>50</sup>, F. Finelli<sup>2,61</sup>, P. Fosalba<sup>15,11</sup>, S. Fotopoulou<sup>62</sup>, M. Frailis<sup>8</sup>, E. Franceschi<sup>2</sup>, M. Fumana<sup>38</sup>, S. Galeotta<sup>8</sup>, B. Gillis<sup>47</sup>, C. Giocoli<sup>2,29</sup>, J. Gracia-Carpio<sup>26</sup>, A. Grazian<sup>63</sup>, F. Grupp<sup>26,64</sup>, L. Guzzo<sup>19,3,65</sup>, S. V. H. Haugan<sup>66</sup>, W. Holmes<sup>67</sup>, F. Hormuth<sup>68</sup>, A. Hornstrup<sup>69,70</sup>, K. Jahnke<sup>71</sup>, M. Jhabvala<sup>72</sup>, B. Joachimi<sup>73</sup>, E. Keihänen<sup>74</sup>, S. Kermiche<sup>58</sup>, A. Kiessling<sup>67</sup>, B. Kubik<sup>50</sup>, M. Kümmel<sup>64</sup>, M. Kunz<sup>75</sup>, H. Kurki-Suonio<sup>76,77</sup>, A. M. C. Le Brun<sup>78</sup>, S. Ligori<sup>34</sup>, P. B. Lilje<sup>66</sup>, V. Lindholm<sup>76,77</sup>, I. Lloro<sup>79</sup>, G. Mainetti<sup>80</sup>, D. Maino<sup>19,38,65</sup>, E. Maiorano<sup>2</sup>, O. Mansutti<sup>8</sup>, S. Marcin<sup>81</sup>, O. Marggraf<sup>6</sup>, K. Markovic<sup>67</sup>, M. Martinelli<sup>39,40</sup>, N. Martinet<sup>16</sup>, F. Marulli<sup>7,2,29</sup>, R. Massey<sup>82</sup>, E. Medinaceli<sup>2</sup>, S. Mei<sup>83,84</sup>, M. Melchior<sup>85</sup>, Y. Mellier<sup>86,87</sup>, M. Meneghetti<sup>2,29</sup>, E. Merlin<sup>39</sup>, G. Meylan<sup>88</sup>, A. Mora<sup>89</sup>, B. Morin<sup>13</sup>, L. Moscardini<sup>7,2,29</sup>, E. Munari<sup>8,9</sup>, C. Neissner<sup>90,42</sup>, S.-M. Niemi<sup>35</sup>, C. Padilla<sup>90</sup>, S. Paltani<sup>56</sup>, F. Pasian<sup>8</sup>, K. Pedersen<sup>91</sup>, W. J. Percival<sup>92,93,94</sup>, V. Pettorino<sup>35</sup>, S. Pires<sup>13</sup>, G. Polenta<sup>60</sup>, M. Poncet<sup>28</sup>, L. A. Popa<sup>95</sup>, F. Raison<sup>26</sup>, R. Rebolo<sup>46,96,97</sup>, A. Renzi<sup>21,22</sup>, J. Rhodes<sup>67</sup>, G. Riccio<sup>31</sup>, E. Romelli<sup>8</sup>, M. Roncarelli<sup>2</sup>, R. Saglia<sup>64,26</sup>, Z. Sakr<sup>98,99,100</sup>, A. G. Sánchez<sup>26</sup>, D. Sapone<sup>101</sup>, B. Sartoris<sup>64,8</sup>, J. A. Schewtschenko<sup>47</sup>, P. Schneider<sup>6</sup>, T. Schrabback<sup>102</sup>, M. Scodeggio<sup>38</sup>, A. Secroun<sup>58</sup>, G. 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Di Ferdinando<sup>29</sup>, J. A. Escartin Vigo<sup>26</sup>, L. Gabarra<sup>117</sup>, J. Martín-Fleitas<sup>118</sup>, S. Matthew<sup>47</sup>, M. Maturi<sup>98,119</sup>, N. Mauri<sup>45,29</sup>, R. B. Metcalf<sup>7,2</sup>, A. A. Nucita<sup>120,121,122</sup>, M. Pöntinen<sup>76</sup>, I. Risso<sup>123</sup>, V. Scottez<sup>86,124</sup>, M. Sereno<sup>2,29</sup>, M. Tenti<sup>29</sup>, M. Viel<sup>9,8,14,10,24</sup>, M. Wiesmann<sup>66</sup>, Y. Akrami<sup>125,126</sup>, I. T. Andika<sup>127,128</sup>, S. Anselmi<sup>22,21,12</sup>, M. Archidiacono<sup>19,65</sup>, F. Atrio-Barandela<sup>129</sup>, A. Balaguera-Antolinez<sup>46,130</sup>, D. Bertacca<sup>21,63,22</sup>, M. Bethermin<sup>131</sup>, L. Blot<sup>132,78</sup>, H. Böhringer<sup>26,133,134</sup>, S. Borgani<sup>135,9,8,10,24</sup>, M. L. Brown<sup>136</sup>, S. Bruton<sup>137</sup>, A. Calabro<sup>39</sup>, F. Caro<sup>39</sup>, C. S. Carvalho<sup>105</sup>, T. Castro<sup>8,10,9,24</sup>, F. Cogato<sup>7,2</sup>, S. Conseil<sup>50</sup>, S. Contarini<sup>26</sup>, A. R. Cooray<sup>138</sup>, O. Cucciati<sup>2</sup>, S. Davini<sup>4</sup>, F. De Paolis<sup>120,121,122</sup>, G. Desprez<sup>110</sup>, A. Díaz-Sánchez<sup>139</sup>, J. J. Díaz<sup>46</sup>, S. Di Domizio<sup>5,4</sup>, J. M. Diego<sup>140</sup>, P. Dimauro<sup>39,141</sup>, A. Enia<sup>1,2</sup>, Y. Fang<sup>64</sup>, A. G. Ferrari<sup>29</sup>, P. G. Ferreira<sup>117</sup>, A. Finoguenov<sup>76</sup>, A. Franco<sup>121,120,122</sup>, K. Ganga<sup>83</sup>, J. García-Bellido<sup>125</sup>, T. Gasparetto<sup>8</sup>, V. Gautard<sup>142</sup>, E. Gaztanaga<sup>11,15,143</sup>, F. Giacomini<sup>29</sup>, F. Gianotti<sup>2</sup>, G. Gozalias<sup>144,76</sup>, C. M. Gutierrez<sup>145</sup>, C. Hernández-Monteagudo<sup>97,46</sup>, H. Hildebrandt<sup>146</sup>, J. Hjorth<sup>91</sup>, S. Joudaki<sup>41</sup>, J. J. E. Kajava<sup>147,148</sup>, Y. Kang<sup>56</sup>, V. Kansal<sup>149,150</sup>, D. Karagiannis<sup>112,151</sup>, K. Kiiveri<sup>74</sup>, C. C. Kirkpatrick<sup>74</sup>, S. Kruk<sup>49</sup>, M. Lattanzi<sup>113</sup>, L. Legrand<sup>152,153</sup>, M. Lembo<sup>87,113</sup>, F. Lepori<sup>154</sup>, G. Leroy<sup>155,82</sup>, G. F. Lesci<sup>7,2</sup>, J. Lesgourgues<sup>156</sup>, L. Leuzzi<sup>2</sup>, T. I. Liaudat<sup>157</sup>, A. Loureiro<sup>158,159</sup>, J. Macias-Perez<sup>160</sup>, G. Maggio<sup>8</sup>, M. Magliocchetti<sup>59</sup>, F. Mannucci<sup>161</sup>, R. Maoli<sup>162,39</sup>, C. J. A. P. Martins<sup>163,164</sup>, L. Maurin<sup>57</sup>, M. Miluzio<sup>49,165</sup>, P. Monaco<sup>135,8,10,9</sup>, G. Morgante<sup>2</sup>, S. Nadathur<sup>143</sup>, K. Naidoo<sup>143</sup>, A. Navarro-Alsina<sup>6</sup>, S. Nesseris<sup>125</sup>, L. Pagano<sup>112,113</sup>, F. Passalacqua<sup>21,22</sup>, K. Paterson<sup>71</sup>, L. Patrizii<sup>29</sup>, A. Pisani<sup>58</sup>, D. Potter<sup>154</sup>, S. Quai<sup>7,2</sup>, M. Radovich<sup>63</sup>, P. Reimberg<sup>86</sup>, P.-F. Rocci<sup>57</sup>, G. Rodighiero<sup>21,63</sup>, S. Sacquogna<sup>120,121,122</sup>, M. Sahlén<sup>166</sup>, D. B. Sanders<sup>44</sup>, A. Schneider<sup>154</sup>, D. Sciotti<sup>39,40</sup>, E. Sellentin<sup>167,37</sup>, L. C. Smith<sup>168</sup>, J. G. Sorce<sup>169,57</sup>, K. Tanidis<sup>117</sup>, C. Tao<sup>58</sup>, G. Testera<sup>4</sup>, R. Teyssier<sup>170</sup>, S. Tosi<sup>5,4,3</sup>, A. Troja<sup>21,22</sup>, M. Tucci<sup>56</sup>, C. Valieri<sup>29</sup>, A. Venhola<sup>171</sup>, D. Vergani<sup>2</sup>, F. Vernizzi<sup>172</sup>, G. Verza<sup>173</sup>, P. Vielzeuf<sup>58</sup>, and N. A. Walton<sup>168</sup>

(Affiliations can be found after the references)

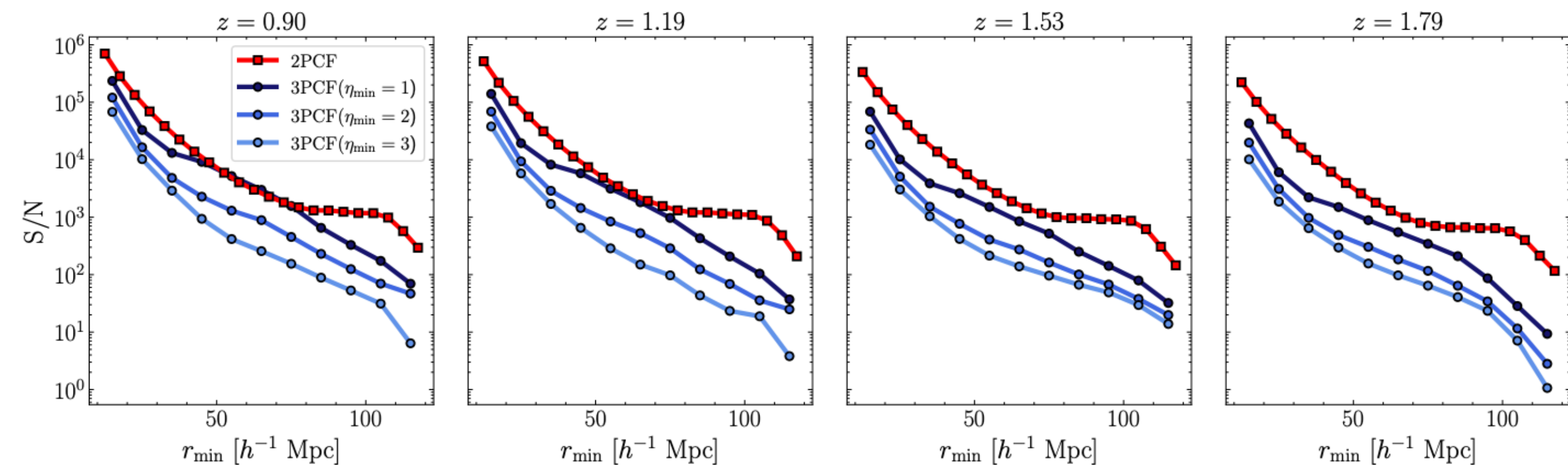
arXiv:2506.22257v1 [astro-ph.CO] 27 Jun 2025



- Four coming snapshots, 54 (Gpc/h)<sup>3</sup> coming volume
- **Real space** measurements
  - 2PCF
  - 3PCF: All triangle configurations, parametrised by  $\eta_{\min} = \frac{r_{13} - r_{12}}{\Delta}$ , where  $\Delta$  bin size
  - Cumulative SNR reveals how important is reaching the small-scale regime in the modeling



Top: 2PCF measurements at different four redshift snapshots. Bottom: as the top panel, but for the 3PCF

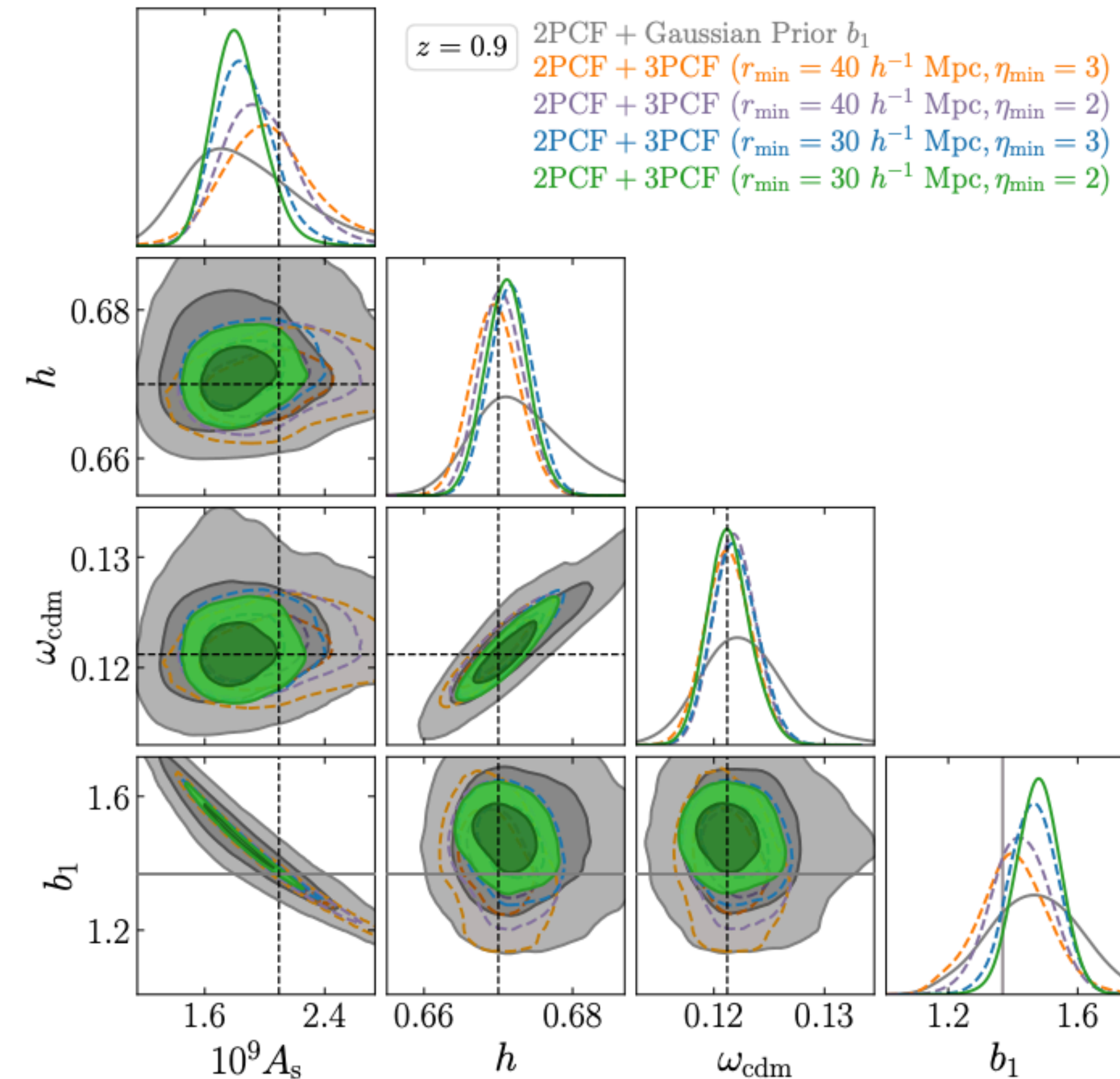


Cumulative SNR as a function of the minimum scale, for the 2PCF and the 3PCF (parametrised by different choices of  $\eta_{\min}$ )



## First higher-order full-shape analysis in configuration space in the literature

- Theoretical Gaussian Covariance
- 2PCF galaxy 1-loop model, 3PCF galaxy tree level. Parameter set:  $\{A_s, h, \omega_{\text{cdm}}\} + \{b_1, b_2, b_{\mathcal{E}_2}, b_{\Gamma_3}, c_0\}$
- 2PCF + 3PCF significantly improves the **constraining power** with respect to only 2PCF
- Full-shape **minimum scale** for constraining cosmological parameters consistent with template fitting and methodological studies (Veropalumbo et al, 2022)

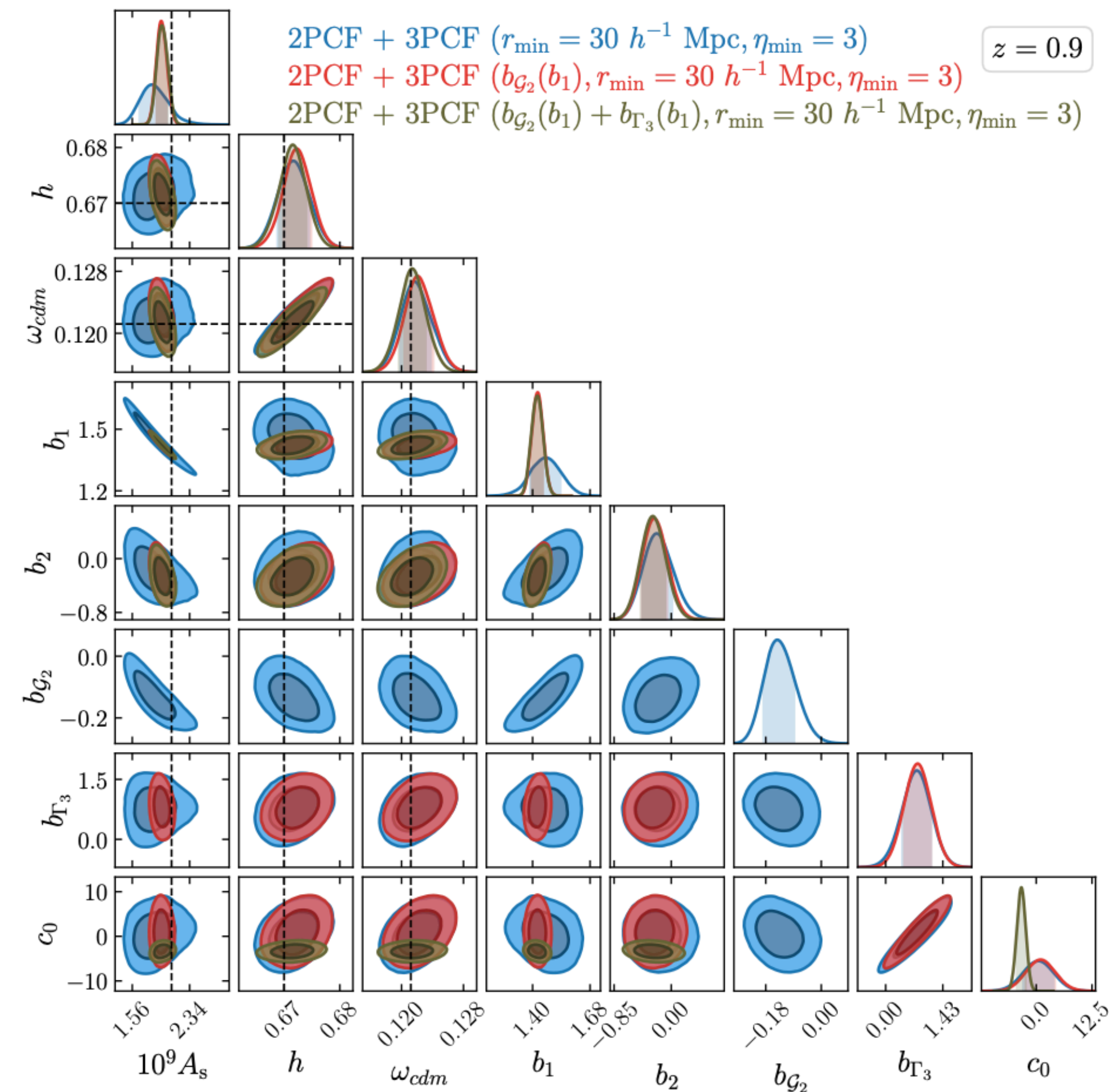


2D Posterior distribution of cosmological parameters + linear bias from a 2PCF + Gaussian prior on linear bias (grey) and a joint 2PCF + 3PCF analysis as a function of different  $r_{\text{min}}$  and  $\eta_{\text{min}}$  (different colors and line styles) at  $z = 0.9$



First higher-order full-shape analysis in configuration space in the literature

- Theoretical Gaussian Covariance
- 2PCF galaxy 1-loop model, 3PCF galaxy tree level. Parameter set:  $\{A_s, h, \omega_{\text{cdm}}\} + \{b_1, b_2, b_{\mathcal{G}_2}, b_{\Gamma_3}, c_0\}$
- **Bias relations help in constraining  $A_s$  and  $b_1$**
- Full-shape minimum scale for constraining cosmological parameters consistent with template fitting and methodological studies (Veropalumbo et al, 2022)



2D Posterior distribution of cosmological parameters + galaxy bias parameters from a joint 2PCF + 3PCF analysis as a function of different relations between bias parameters (different colors)

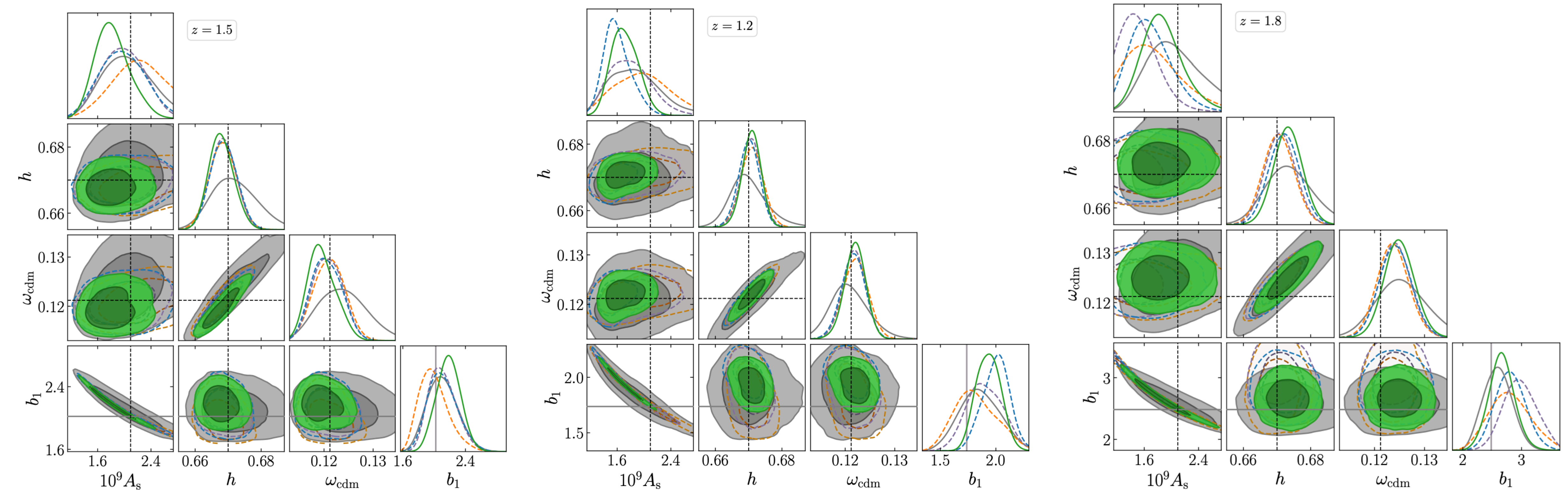


# Conclusions

- 3PCF as a powerful statistical tool combined with 2PCF, important to constrain physics responsible to non-Gaussianity in the density field
- **Unlocked** first full-shape 2PCF + 3PCF in real space in a simulated dataset. It maximise the extraction of cosmological information in a configuration space higher-order analysis and fills the gap with Fourier space modelling (power spectrum + bispectrum)
- Just the beginning, research line unfolding in many directions, some of them:
  - Exploring the full-shape information content of the **redshift space** (isotropic and anisotropic) 3PCF (Nagainis, *Guidi*, Moresco et al, in prep) —→ see Kristers' talk!
  - First analysis in redshift space with real data (BOSS, *Guidi* et al, in prep) + application to Stage IV upcoming datasets —→ (using of MEICorr code, see Alfonso's talk!)
  - Addressing beyond LCDM cosmologies, and the information content of the full-shape 3PCF. Role of massive neutrinos current ongoing (Labate, *Guidi*, et al in prep) —→ see Michele's talk!
- **Higher-order correlation** function will be fundament to **maximise the extraction of cosmological information**

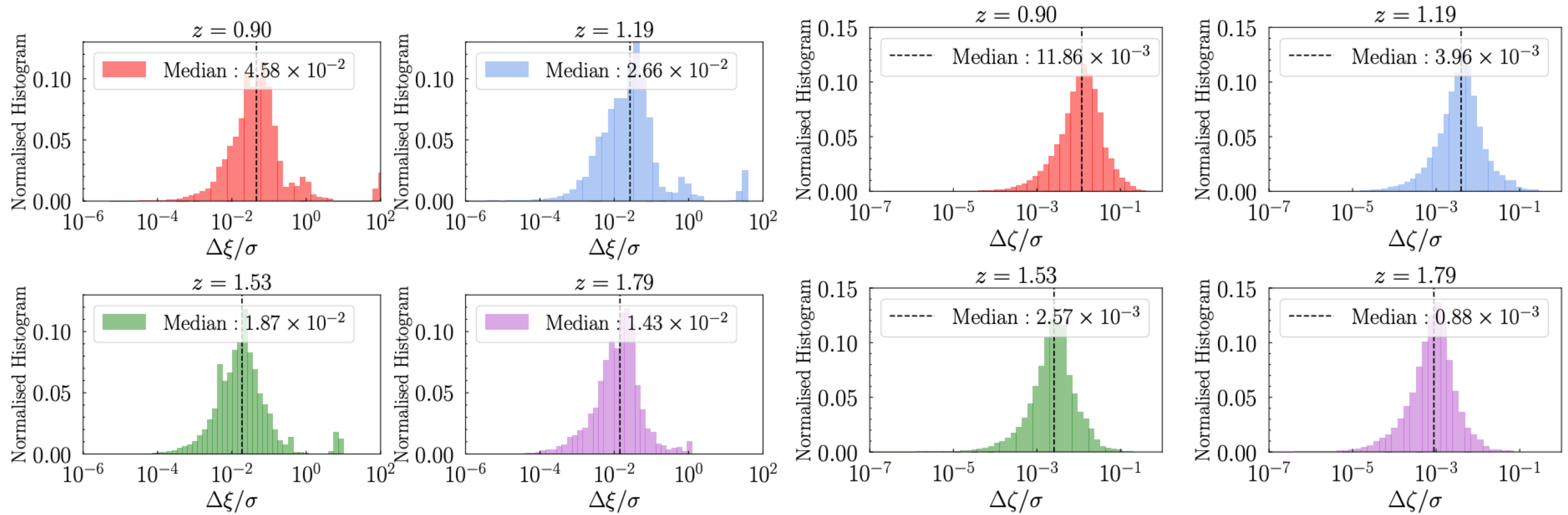


# Appendix: Full-shape modelling of 2PCF and 3PCF in real space



2D Posterior distribution of cosmological parameters + linear bias from a 2PCF + Gaussian prior on linear bias (grey) and a joint 2PCF + 3PCF analysis as a function of different  $r_{\text{min}}$  and  $\eta_{\text{min}}$  (different colors and linestyles). Different panels show different redshift snapshots





Normalised histogram of the ratio between the difference between emulated and exact modelling 2PCF (left), 3PCF (right) predictions and the corresponding uncertainty