Basic statistics, and applications to X-ray spectral fitting

✓ Normal error (Gaussian) distribution

Most important in statistical analysis of data, describes the distribution of random observations for many experiments

✓ Poisson distribution

Generally appropriate for counting experiments related to random processes (e.g., radioactive decay of elementary particles)

✓ Statistical tests: χ^2 and F-test (more on these in the XSPEC tutorial)

Additional specific applications within XSPEC in the X-ray spectral analysis tutorial

All measurements should be provided with errors

- Measurement **X ± δX** (units of measure)

↓

Error associated with the measurement X

- Significant digits:

g (gravitational acceleration of an object in a vacuum near the Earth surface)=

=9.82±0.02385 m/s² ➔ 9.82±0.02 m/s²

Another example: v=100.2 ± 30 m/s ➔ 100±30 m/s

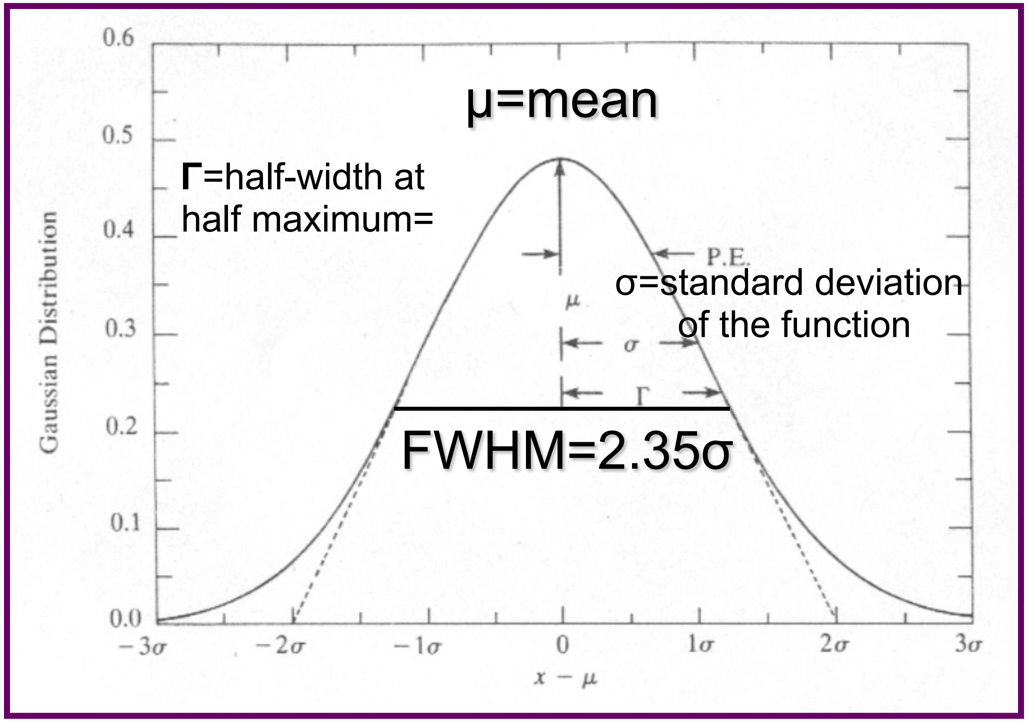
- Relative (fractionary) uncertainty: **δX/X**

The Gaussian (normal error) distribution. I

Averages of random variables (sufficiently large in number) independently drawn from independent distributions converge in distribution to the normal

Casual errors are above and below the "true" (**most "common"**) **value**

→ bell-shape distribution if systematic errors are negligible



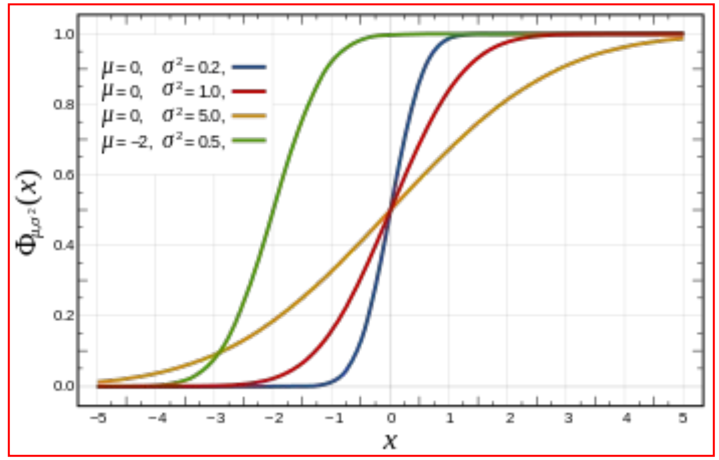
The Gaussian probability function. II

$$P(x;\ \mu,\ \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Probability Density Function
(centered on μ)

μ=mean (expectation) value
σ=standard deviation
σ^2=variance

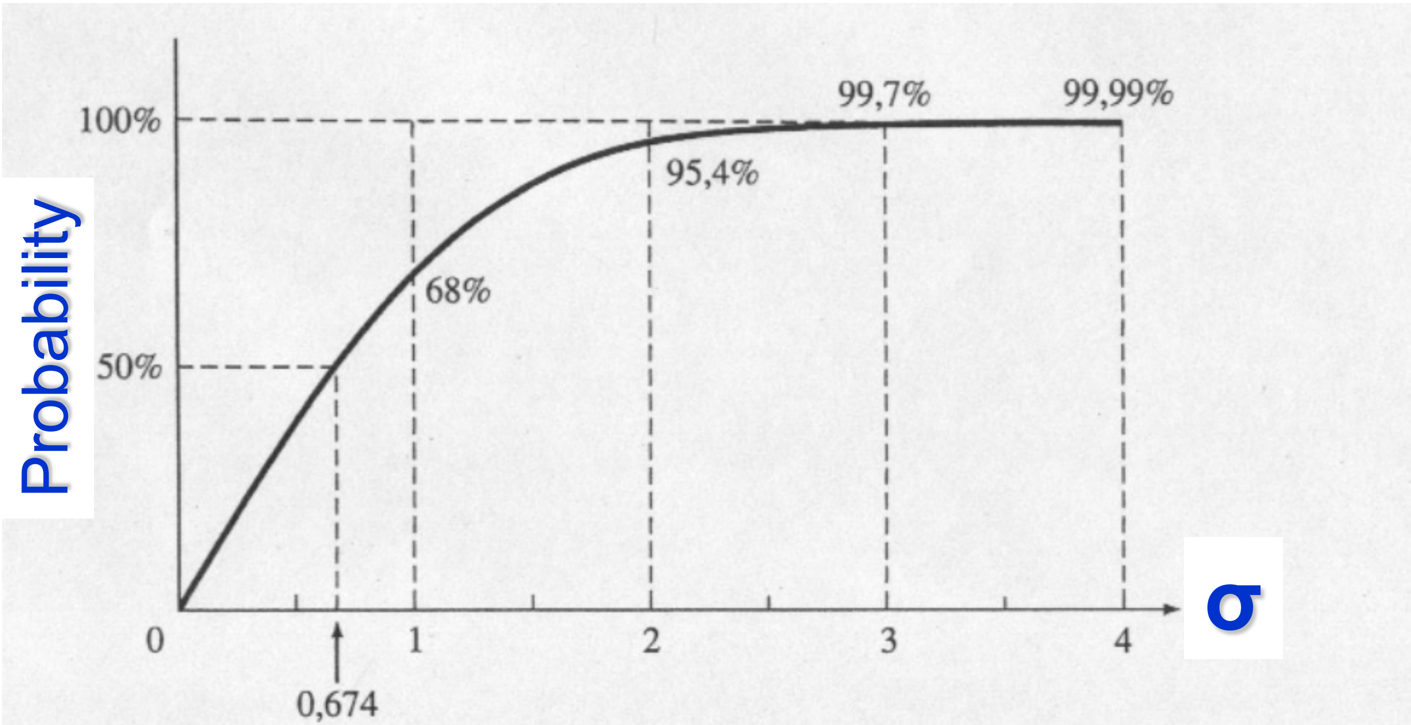
normalization factor, so that ∫f(x) dx=1



$$e^{-x^2/2\sigma^2}$$

function centered on 0

The Gaussian probability function. III



σ	0	0,25	0,5	0,75	1,0	1,25	1,5	1,75	2,0	2,5	3,0	3,5	4,0
P (%)	0	20	38	55	68	79	87	92	95,4	98,8	99,7	99,95	99,99

$$F(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{x} e^{-(x-\mu)^2/2\sigma^2} dx$$

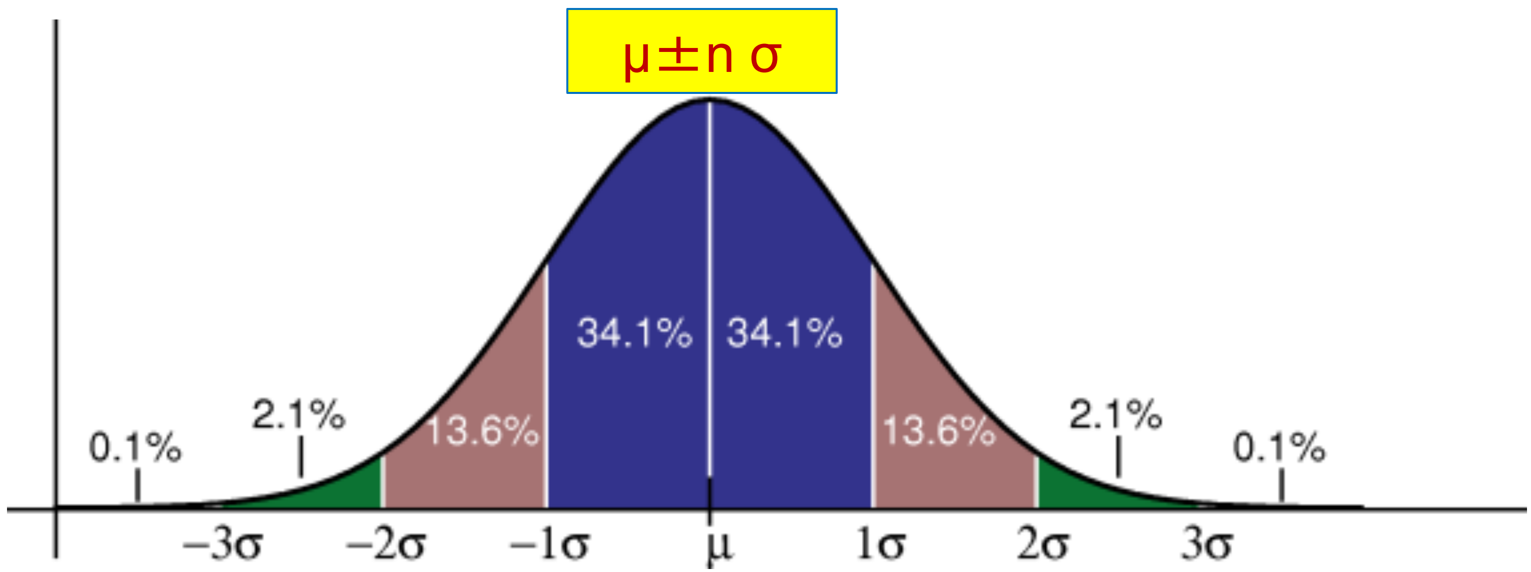
Cumulative Distribution Function

Chance to be outside the range

1σ	1/3
2σ	1/22
3σ	1/370
4σ	1/15787
5σ	1/1744277

Probability within +/- nσ

1σ	68.3%
2σ	95.45%
3σ	99.730%
4σ	99.99367%
5σ	99.999943%



$$F(x) = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{x} e^{-(x-\mu)^2/2\sigma^2}\,dx$$

Cumulative Distribution Function

Value±error at 1σ confidence level: if we make a measurement N times, in 68.3% of the times we obtain such value

Every measurement should be reported and considered along its own error

Percentage probability P within tσ: $P = \int_{X-t\sigma}^{X+t\sigma} G(x)\,dx$

Bevington textbook

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80
2.3	97.86	97.91	97.97	98.02	98.07	98.12	98.17	98.22	98.27	98.32
2.4	98.36	98.40	98.45	98.49	98.53	98.57	98.61	98.65	98.69	98.72
2.5	98.76	98.79	98.83	98.86	98.89	98.92	98.95	98.98	99.01	99.04
2.6	99.07	99.09	99.12	99.15	99.17	99.20	99.22	99.24	99.26	99.29
2.7	99.31	99.33	99.35	99.37	99.39	99.40	99.42	99.44	99.46	99.47
2.8	99.49	99.50	99.52	99.53	99.55	99.56	99.58	99.59	99.60	99.61
2.9	99.63	99.64	99.65	99.66	99.67	99.68	99.69	99.70	99.71	99.72
3.0	99.73									
3.5	99.95									
4.0	99.994									
4.5	99.9993									
5.0	99.99994									

3σ=99.73%: in 1000 experiments you can get results outside this ±3σ range only three times ($1000 \times 0.9973 = 997$ times within the 3σ range)

5σ=99.99994%: 6 cases out of 10^6

The Poisson distribution

Describes experimental results where events are counted and the uncertainty is not related to the measurement but reflects the **intrinsically casual behavior of the process** (e.g., radioactive decay of particles (Geiger counter), X-ray photons, etc.)

$$\mathrm{P}(x) = e^{-\mu}\mu^{x}/x! \qquad (x=0,1,2,\ldots)$$

Probability of obtaining *x* events when μ events are expected
x=observed number of events in a time interval (frequency of events)

average number of events

$$\bar{x} = \sum_{x=0}^{\infty} xP(x) = \sum_{x=0}^{\infty} xe^{-\mu}\mu^{x}/x! = \mu$$

➔ μ=average number of expected events if the experiment is repeated many times

$$\sigma^2 = \langle (x-\mu)^2 \rangle =$$

expectation value of the square of the deviations

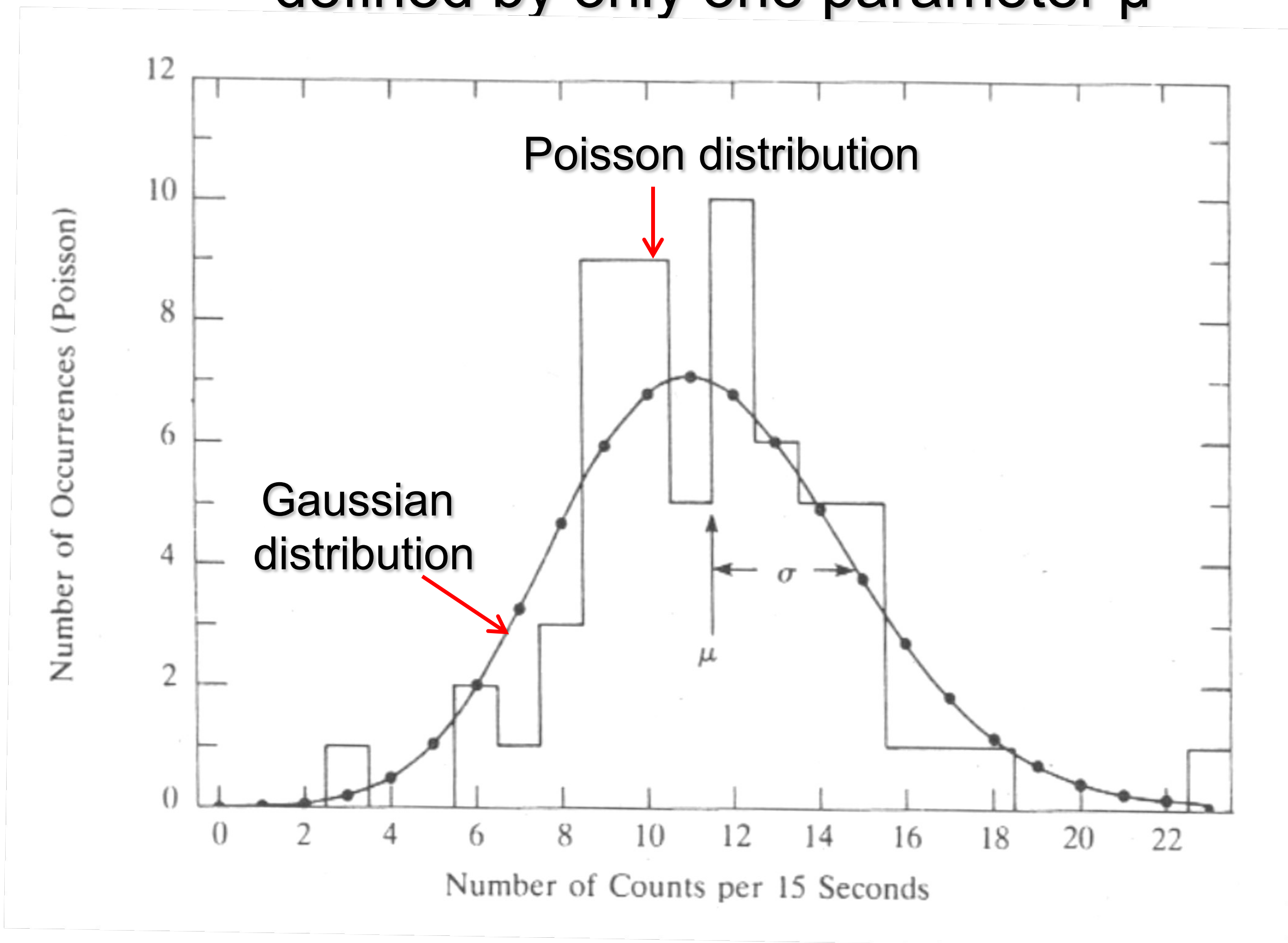
$$= \sum_{x=0}^{\infty} (x-\mu)^2 \frac{\mu^x}{x!} e^{-\mu} = \mu$$

the Poisson distribution with average counts=μ has standard deviation $\sqrt{\mu}$

Example: $N_{counts} \pm \sqrt{N}$

High μ: the Poisson distribution is approximated by the Gaussian distribution

defined by only one parameter μ



Statistical test: χ^2 – more on XSPEC tutorial

Test to compare the observed distribution of the results with that expected

$$\chi^2 = \sum_{k=1}^{n} \frac{(O_k - E_k)^2}{\sigma_k^2}$$

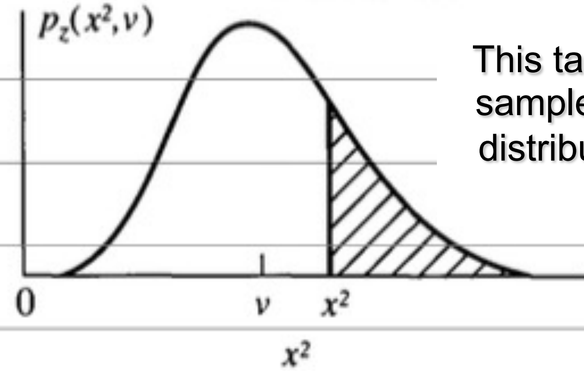
It provides a measure on how much the data differ from the expectations (model), taking into account the errors associated with the measurement (e.g., datapoints)

- O_k=observed values (e.g., spectral datapoints)
- E_k=expected values (model, i.e. predicted distribution)
- σ_k=error on the measured values (e.g., error on each spectral bin)
- k=number of datapoints (e.g., bins after rebinning)

$$\chi^2 / dof \approx 1$$

→ the observed and expected distributions are similar

dof=degrees of freedom = #datapoints − #free parameters



This table gives the probability that a random sample of data, when compared to its parent distribution, would yield values of X^2/v larger than the observed value

TABLE C.4 Reduced chi-squared distribution

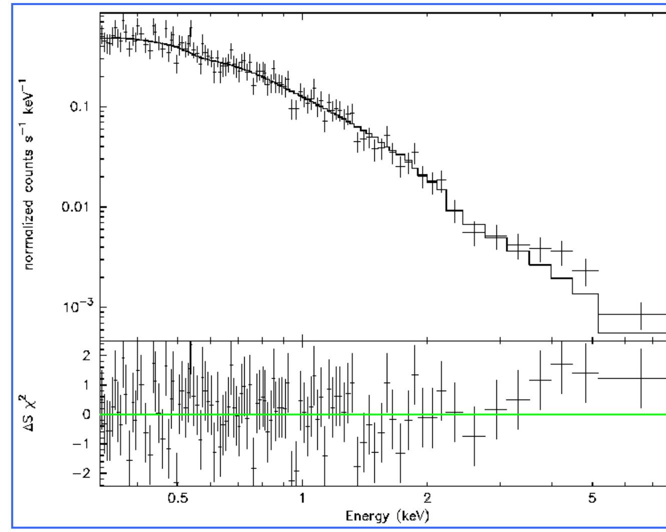
χ^2 distribution. Values of the reduced chi-square $\chi^2_\nu = \chi^2/\nu$ corresponding to the probability $P_\chi(\chi^2; \nu)$ of exceeding χ^2 versus the number of degrees of freedom ν

v=dof=#datapoints − #free parameters

P — P=probability of exceeding χ_v^2

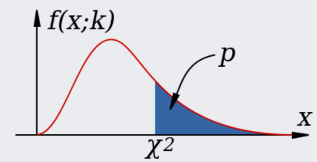
ν	0.99	0.98	0.95	0.90	0.80	0.70	0.60	0.50
1	0.00016	0.00063	0.00393	0.0158	0.0642	0.148	0.275	0.455
2	0.0100	0.0202	0.0515	0.105	0.223	0.357	0.511	0.693
3	0.0383	0.0617	0.117	0.195	0.335	0.475	0.623	0.789
4	0.0742	0.107	0.178	0.266	0.412	0.549	0.688	0.839
5	0.111	0.150	0.229	0.322	0.469	0.600	0.731	0.870
6	0.145	0.189	0.273	0.367	0.512	0.638	0.762	0.891
7	0.177	0.223	0.310	0.405	0.546	0.667	0.785	0.907
8	0.206	0.254	0.342	0.436	0.574	0.691	0.803	0.918
9	0.232	0.281	0.369	0.463	0.598	0.710	0.817	0.927
10	0.256	0.306	0.394	0.487	0.618	0.727	0.830	0.934
11	0.278	0.328	0.416	0.507	0.635	0.741	0.840	0.940
12	0.298	0.348	0.436	0.525	0.651	0.753	0.848	0.945
13	0.316	0.367	0.453	0.542	0.664	0.764	0.856	0.949
14	0.333	0.383	0.469	0.556	0.676	0.773	0.863	0.953
15	0.349	0.399	0.484	0.570	0.687	0.781	0.869	0.956

```
==========================================================================
Model phabs<1>*powerlaw<2> Source No.: 1   Active/On
Model Model Component  Parameter  Unit     Value
 par  comp
   1    1   phabs      nH         10^22    1.59000E-02  frozen
   2    2   powerlaw   PhoIndex            2.72811      +/-  0.0
   3    2   powerlaw   norm                1.51490E-04  +/-  0.0
__________________________________________________________________________

   Using energies from responses.

 Chi-Squared =          97.23 using 105 PHA bins.
 Reduced chi-squared =        0.9440 for    103 degrees of freedom
 Null hypothesis probability =   6.417127e-01
```



X^2=97.2, 103 d.o.f., Null-hypothesis probability (probability that the model reproduces the data, i.e., no difference between the two distributions)=**0.64** (6.4e-01 above)

Table of values of χ^2 in a Chi-Squared Distribution with *k* degrees of freedom such that *p* is the area between χ^2 and +∞



k=degrees of freedom

k	Probability Content, *p*, between χ^2 and +∞														
	0.995	**0.99**	**0.975**	**0.95**	**0.9**	**0.75**	**0.5**	**0.25**	**0.1**	**0.05**	**0.025**	**0.01**	**0.005**	**0.002**	**0.001**
1	3.927e-5	1.570e-4	9.820e-4	0.00393	0.0157	0.102	0.455	1.323	2.706	3.841	5.024	6.635	7.879	9.550	10.828
2	0.0100	0.0201	0.0506	0.103	0.211	0.575	1.386	2.773	4.605	5.991	7.378	9.210	10.597	12.429	13.816
3	0.0717	0.115	0.216	0.352	0.584	1.213	2.366	4.108	6.251	7.815	9.348	11.345	12.838	14.796	16.266
4	0.207	0.297	0.484	0.711	1.064	1.923	3.357	5.385	7.779	9.488	11.143	13.277	14.860	16.924	18.467
5	0.412	0.554	0.831	1.145	1.610	2.675	4.351	6.626	9.236	11.070	12.833	15.086	16.750	18.907	20.515
6	0.676	0.872	1.237	1.635	2.204	3.455	5.348	7.841	10.645	12.592	14.449	16.812	18.548	20.791	22.458
100	67.328	70.065	74.222	77.929	82.358	90.133	99.334	109.141	118.498	124.342	129.561	135.807	140.169	145.577	149.449
101	68.146	70.901	75.083	78.813	83.267	91.085	100.334	110.189	119.589	125.458	130.700	136.971	141.351	146.780	150.667
102	68.965	71.737	75.946	79.697	84.177	92.038	101.334	111.236	120.679	126.574	131.838	138.134	142.532	147.982	151.884
103	69.785	72.575	76.809	80.582	85.088	92.991	102.334	112.284	121.769	127.689	132.975	139.297	143.712	149.183	153.099
104	70.606	73.413	77.672	81.468	85.998	93.944	103.334	113.331	122.858	128.804	134.111	140.459	144.891	150.383	154.314

Statistical test: F-test – more on XSPEC tutorial

If two statistics following the χ^2 distribution have been determined, the ratio of the reduced chi-squares is distributed according to the F distribution

$$P_f(f;\upsilon_1,\upsilon_2) = \frac{\chi_1^2/\upsilon_1}{\chi_2^2/\upsilon_2}$$

$$\propto \Delta\chi^2/k$$

with k=number of additional terms (parameters)

Example: Use the F-test to evaluate the improvement to a spectral fit due to the assumption of a different model, with additional terms

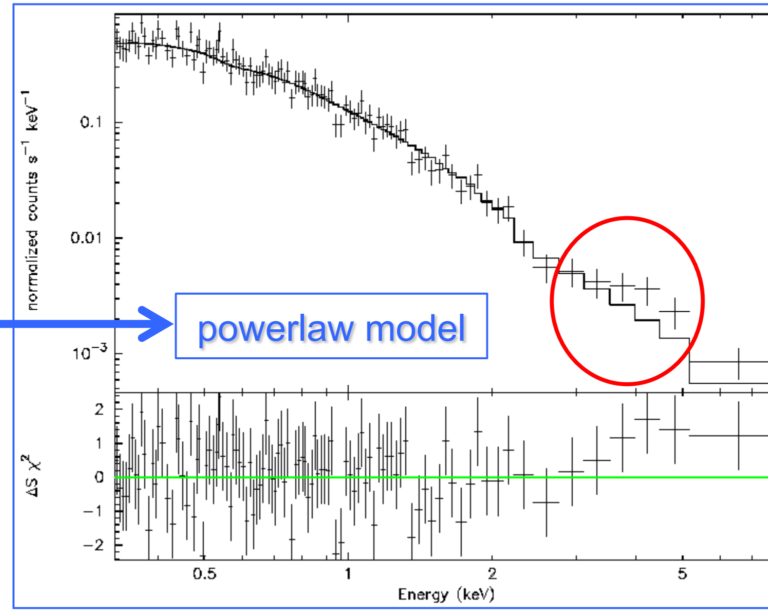
Conditions: (a) the simpler model is nested within the more complex model;
(b) the extra parameters have Gaussian distribution (not truncated by the parameter space boundaries)

➜ see the F-test tables for the corresponding probabilities (specific command in XSPEC)

An application of the F-test within XSPEC

```
========================================================================
Model phabs<1>*powerlaw<2> Source No.: 1   Active/On
Model Model Component  Parameter  Unit     Value
 par  comp
   1    1   phabs      nH         10^22    1.59000E-02  frozen
   2    2   powerlaw   PhoIndex            2.72811      +/-  0.0
   3    2   powerlaw   norm                1.51490E-04  +/-  0.0
________________________________________________________________________

   Using energies from responses.

 Chi-Squared =          97.23 using 105 PHA bins.
 Reduced chi-squared =       0.9440 for    103 degrees of freedom
 Null hypothesis probability =   6.417127e-01
```

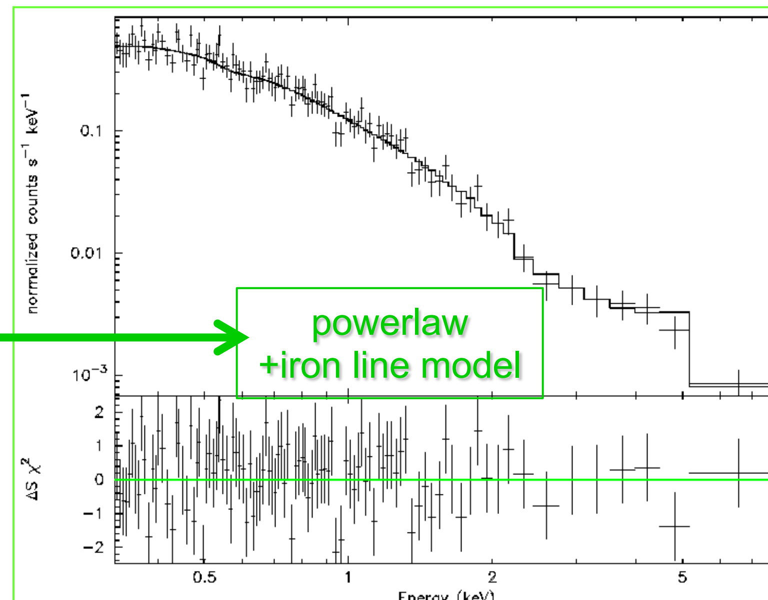
Model1




```
========================================================================
Model phabs<1>(laor<2> + powerlaw<3>) Source No.: 1   Active/On
Model Model Component  Parameter  Unit     Value
 par  comp
   1    1   phabs      nH         10^22    1.59000E-02  frozen
   2    2   laor       lineE      keV      5.23582      +/-  0.0
   3    2   laor       Index               3.00000      frozen
   4    2   laor       Rin(G)              1.23500      frozen
   5    2   laor       Rout(G)             400.000      frozen
   6    2   laor       Incl       deg      30.0000      frozen
   7    2   laor       norm                6.83065E-06  +/-  0.0
   8    3   powerlaw   PhoIndex            2.77137      +/-  0.0
   9    3   powerlaw   norm                1.48123E-04  +/-  0.0
________________________________________________________________________

   Using energies from response

 Chi-Squared =          90.84 using 105 PHA bins.
 Reduced chi-squared =       0.8994 for    101 degrees of freedom
 Null hypothesis probability =   7.557789e-01
 Current data and model not fit yet.
 Weighting method: standard
XSPEC12>ftest 90.84 101 97.2 103
 F statistic value = 3.53567 and probability 0.0327981
```

Model1+extra component

low F value ⇒low significance of the added component



F-test probability in XSPEC: probability of exceeding F (see tabulated values)

Fit (2) = Fit (1) + one component

xspec> ftest **χ^2** (best fit) **dof** (best fit) **χ^2** (previous fit) **dof** (previous fit)

xspec> ftest 90.8 101 97.2 103 → ftest=3.54 → prob=0.0328

$$F_t = \left(\frac{\chi^2(dof) - \chi^2(dof-k)}{dof-(dof-k)}\right)/(\chi^2(dof-k)/(dof-k)) =$$

$$=(\Delta\chi^2/k)/\chi^2_\nu$$

Ex: $\chi^2(103) = 97.23$

$\chi^2(101) = 90.84$

$\rightarrow \Delta\chi^2 = 6.39, k = 2 \rightarrow F_t = (6.39/2)/(90.84/101) = 3.55$

F_t follows the F distribution with v_1=k=Δ(dof) and v_2=dof-k(-1)

Search in the F-distribution tables for the probability of the null hypothesis (H_0) for v_1=2 and v_2~100

The significance of the improvement is given by **P=1-prob=1-0.032=96.8%** (i.e., not particularly significant)

Note of caution: F-test is an approximation (BUT quick); optimal solution would be running simulations (ses Protassov+2002)

You simulate N times (1000, 10000 trials) within XSPEC (command *fakeit*) data (source and background) of the same quality as that of your original data (including also response matrices ARF and RMF) and fit them with the same modeling without the line (e.g., a powerlaw); you then verify how many times your feature is found purely by chance

If you find it X times, the significance of the line =(1-X)/(number of trials)

Check the prob

Percentage probability P within tσ: $P = \int_{X-t\sigma}^{X+t\sigma} G(x)\,dx$

shaded region

X between –tσ and +tσ

(t means # in units of σ)

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80
2.3	97.86	97.91	97.97	98.02	98.07	98.12	98.17	98.22	98.27	98.32
2.4	98.36	98.40	98.45	98.49	98.53	98.57	98.61	98.65	98.69	98.72
2.5	98.76	98.79	98.83	98.86	98.89	98.92	98.95	98.98	99.01	99.04
2.6	99.07	99.09	99.12	99.15	99.17	99.20	99.22	99.24	99.26	99.29
2.7	99.31	99.33	99.35	99.37	99.39	99.40	99.42	99.44	99.46	99.47
2.8	99.49	99.50	99.52	99.53	99.55	99.56	99.58	99.59	99.60	99.61
2.9	99.63	99.64	99.65	99.66	99.67	99.68	99.69	99.70	99.71	99.72
3.0	99.73									
3.5	99.95									
4.0	99.994									
4.5	99.9993									
5.0	99.99994									

P=96.8% ➔≈2.1σ

Compute the significance of the improvement in terms of σ given P=0.0328, hence (1-P)=0.968

Gaussian probability table

$v_1=2$

$v_2=100$ (range 60−120)

F=3.15,3.07 at P(F)=0.05

TABLE 5 (Contd.)

$P(F) = 0.05$

f_2 \ f_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.09	251.14	252.20	253.25	254.32
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.84	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	[illegible]	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

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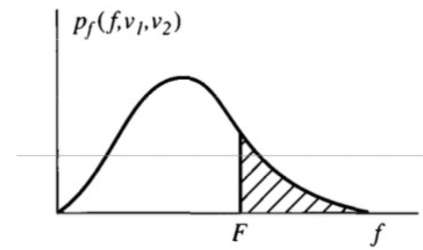
P(F)=0.05

F-test probability in XSPEC: probability of exceeding F (see tabulated values)



$F_{xspec}=3.54$

$v_1=2$

$v_2=100$ (range 60−120)

F=3.93,3.80 at P(F)=0.025

$P(F) = 0.025$

f_2 \ f_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63	976.71	984.87	993.10	997.25	1001.4	1005.6	1009.8	1014.0	1018.3
2	38.51	39.00	39.16	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.42	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.74	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.76	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.58	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	[illegible]	3.93	[illegible]	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	[illegible]	3.80	[illegible]	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

See notes at the beginning of the table.

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P(F)=0.025

F-test probability table

⇨ Probability intermediate between 0.05 and 0.025 (actually, **0.0323**)