Testing the asteroseismic estimates of stellar radii with surface brightness-colour relations and Gaia DR3 parallaxes

Pier Giorgio Prada Moroni

Dipartimento di Fisica “E. Fermi” – Università di Pisa

INFN – Sezione di Pisa

in collaboration with:

Matteo Dell’Omodarme

Giada Valle

Scilla Degl’Innocenti

Asteroseismic scaling relations

$$\frac{M}{M_\odot} \simeq \left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right)^{3} \left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-4} \left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{3/2}$$

$$\frac{R}{R_\odot} \simeq \left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right) \left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-2} \left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{1/2}$$

See e.g. Ulrich 1986; Kjeldsen & Bedding 1995; Stello et al. 2009; Kallinger et al. 2010; Chaplin & Miglio 2013

Testing asteroseismic scaling relations

Independent methods to estimate stellar radii:

- eclipsing binary stars *(Gaulme et al. 2016; Brogaard et al. 2016, 2018; Themessl et al. 2018)*
- Long-baseline interferometry *(Huber et al. 2012; Baines et al. 2014)*

These methods are robust, but they have been applied to a small number of stars

I will present a method that allows to increase the number of stars by a couple of order of magnitude (*Valle et al. 2024, 2025)*

Surface brightness-colour relation (SBCR)

- Surface brightness:

$$S_V = V_0 + 5 \log \theta,$$

V_0: apparent V magnitude corrected for extinction

θ: limb-darkened angular diameter

- Surface brightness-colour relation (SBCR)

$$S_{\rm V} \simeq \alpha + \beta (V - K)_{\rm O}$$

See e.g. Wesselink 1969; Barnes & Evans 1976

Stellar radius from SBCR and parallax

- limb-darkened angular diameter

$$\theta = 10^{0.2\,(S_V - V_0)}$$

- Once the distance *d* is calculated from the parallax, the radius *r* of the star can be estimated

$$r = 0.5\,d\,\theta$$

- Gaia DR3 provides accurate and precise parallaxes for over a billion of stars *(Gaia Collaboration 2021)*

Adopted SBCRs

- Pietrzynski et al. 2019 *(Nature, 567, 200):*

$$S_V^a = 1.330[(V-K)_0 - 2.405] + 5.869\,\mathrm{mag}$$

$2.0 \leq (V\text{-}K)_0 \leq 2.8$ mag

- Salsi et al. 2021 *(A&A, 652, A26):*

$$S_V^b = 1.22(V-K)_0 + 2.864\,\mathrm{mag}$$

$1.8 \leq (V\text{-}K)_0 \leq 3.9$ mag

See also Kervella et al. 2004; Di Benedetto 2005; Adams et al. 2018; Gallenne et al. 2018; Nardetto et al. 2023

Adopted asteroseismic catalogues

- APO-K2: 7500 RGB and RC stars *(Schnhut-Stasik et al. 2024)*

- Kepler: 16000 RGB and RC stars *(Yu et al. 2018)*

The K2 and Kepler datasets show differences *(Zinn et al. 2022; Lund et al. 2024; Schonhut-Stasik et al. 2024)*:

- Analysis pipelines
- Correction of systematics inherent to K2
- Target selection criteria
- Shorter duration of K2 light curves compared to Kepler

It's interesting to see whether differences will also emerge in the comparison with the radii obtained from the SBCRs.

APO-K2 sample

Data selection:

- [Fe/H]> -1 dex
- 4000 K < T_{eff} < 5300 K
- -0.1 dex < [α/Fe] <0.4 dex
- Log g [cm/s^2]: < 3.25
- $\Delta\pi/\pi \leq 0.1$
- $2.0 \leq (V-K)_0 \leq 2.8$ mag
- No binary flag
- $M \geq 0.75\ M_o$ for RGB stars

Final sample: 6420 stars

- 4202 RGB
- 2218 RC

Kepler sample

Data selection:

- [Fe/H]> -1 dex
- T_{eff} < 5500 K
- Log g [cm/s^2]: < 3.3
- $\Delta\pi/\pi \leq 0.1$
- $2.0 \leq (V-K)_0 \leq 2.8$ mag
- M ≥ 0.75 M_o for RGB stars

Final sample: 12492 stars

- 6034 RGB
- 6458 RC

Final samples

For each star in the two final samples:

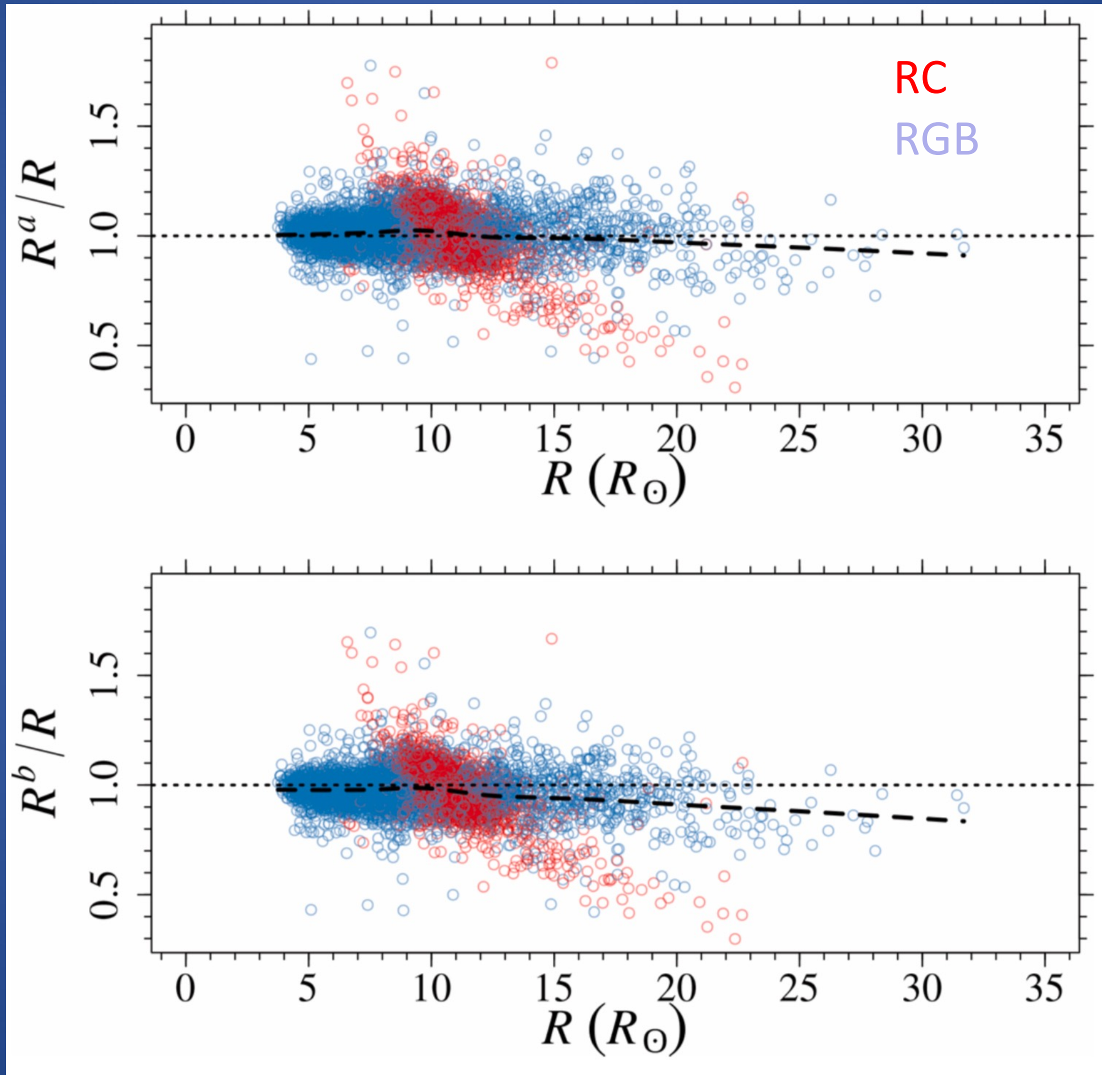
- we determined the linear radius from the Gaia parallax and the angular diameter θ derived from the SBCRs from Pietrzynski et al. 2019 (R^a) and Salsi et al. 2021 (R^b)
- we compared it with the linear radius R provided by the asteroseismic catalogues based on the scaling relation

Results: APO-K2

Notable overall agreement

Pietrzynski SBCR

Salsi SBCR



1,2%

Standard deviation 10%

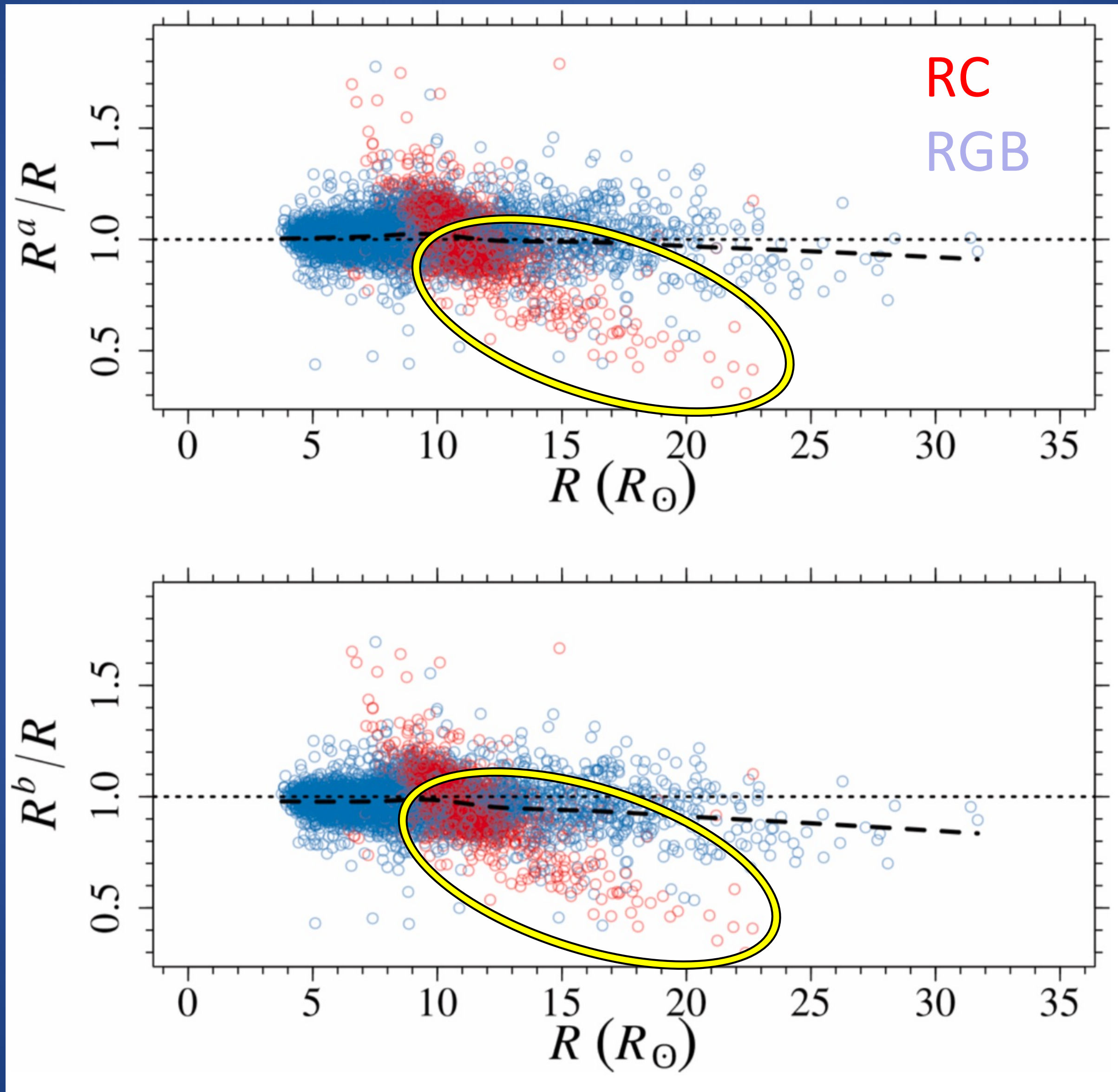
-2,5%

Valle et al. 2024 (A&A 690, A327)

Results: APO-K2

Pietrzynski SBCR

Salsi SBCR



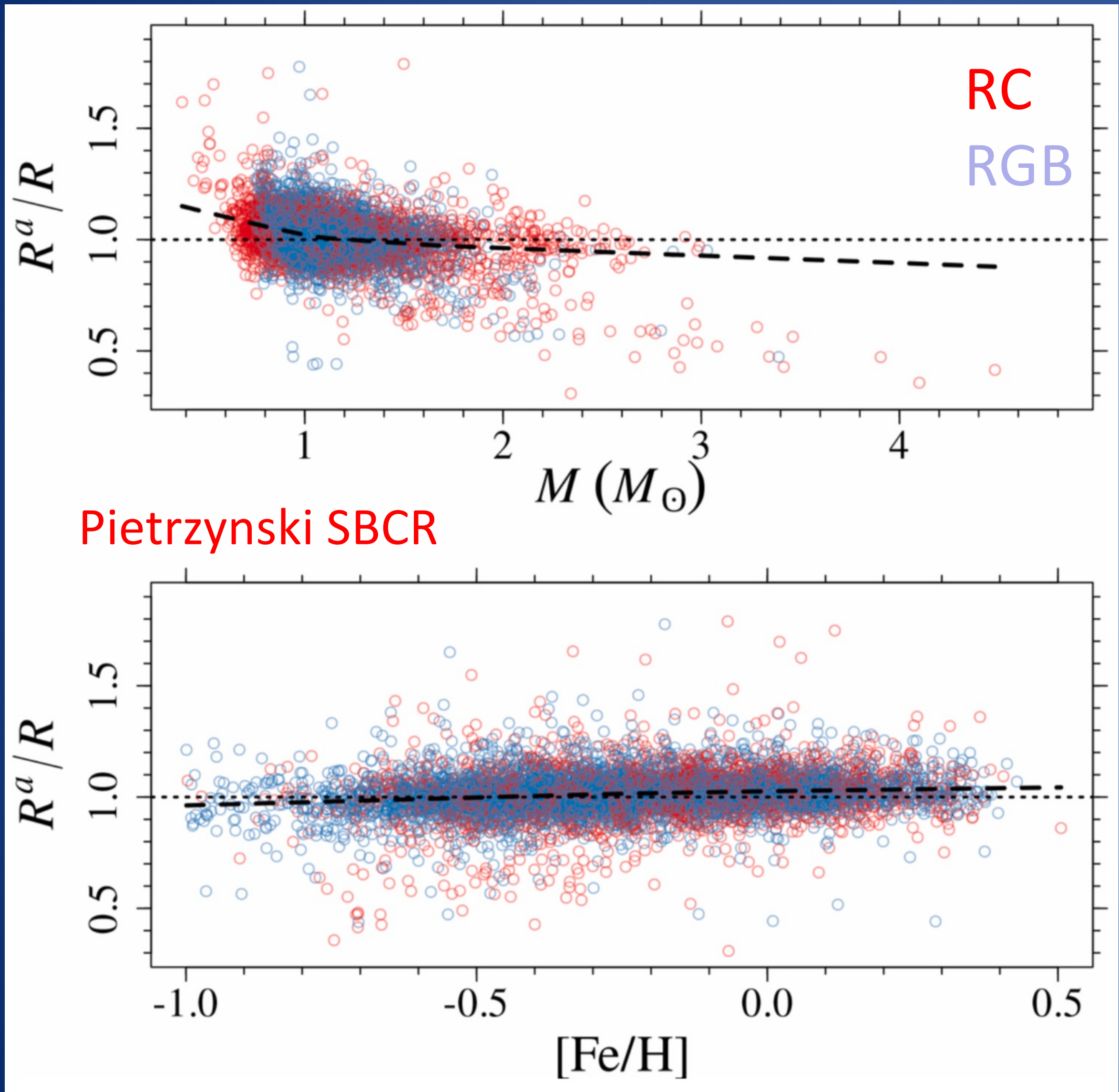
For R> 12 R_0

SBCR R << asteroseismic R

More pronounced for RC stars

Valle et al. 2024 (A&A 690, A327)

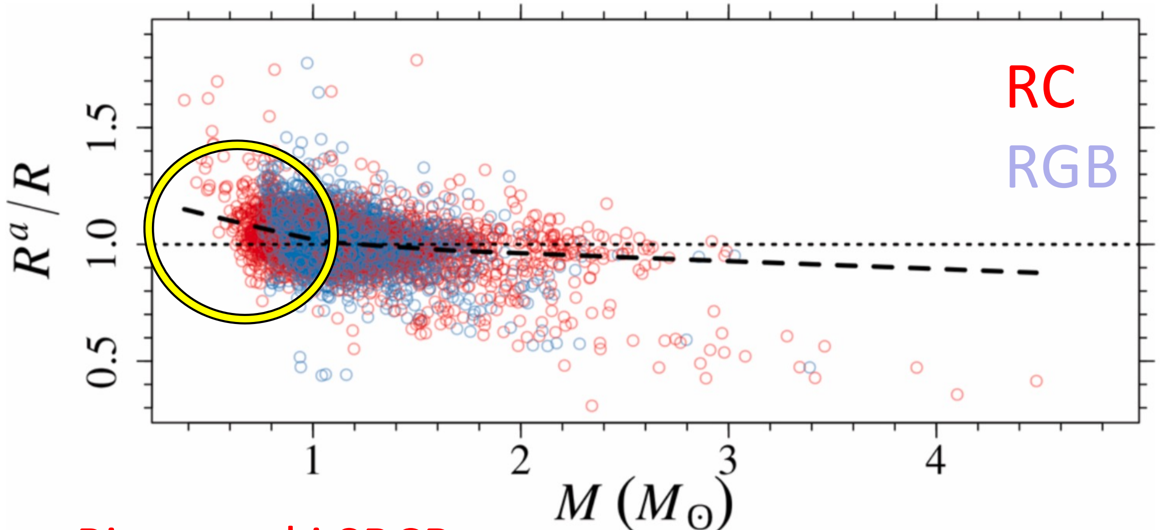
Results: APO-K2



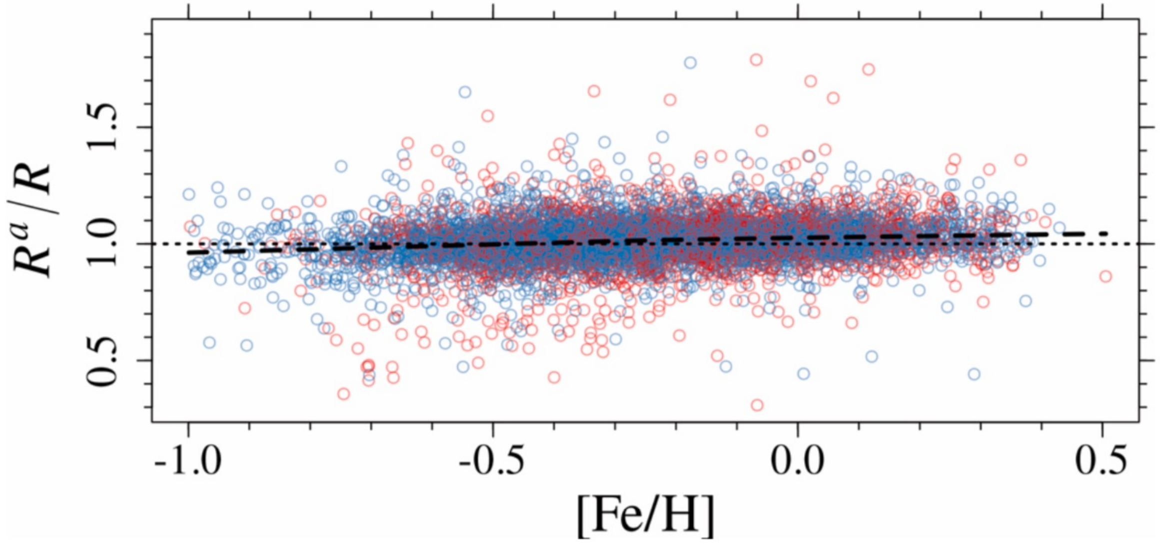
Metallicity dependence:
Weak positive trend 6% per dex

Valle et al. 2024 (A&A 690, A327)

Results: APO-K2



Pietrzynski SBCR



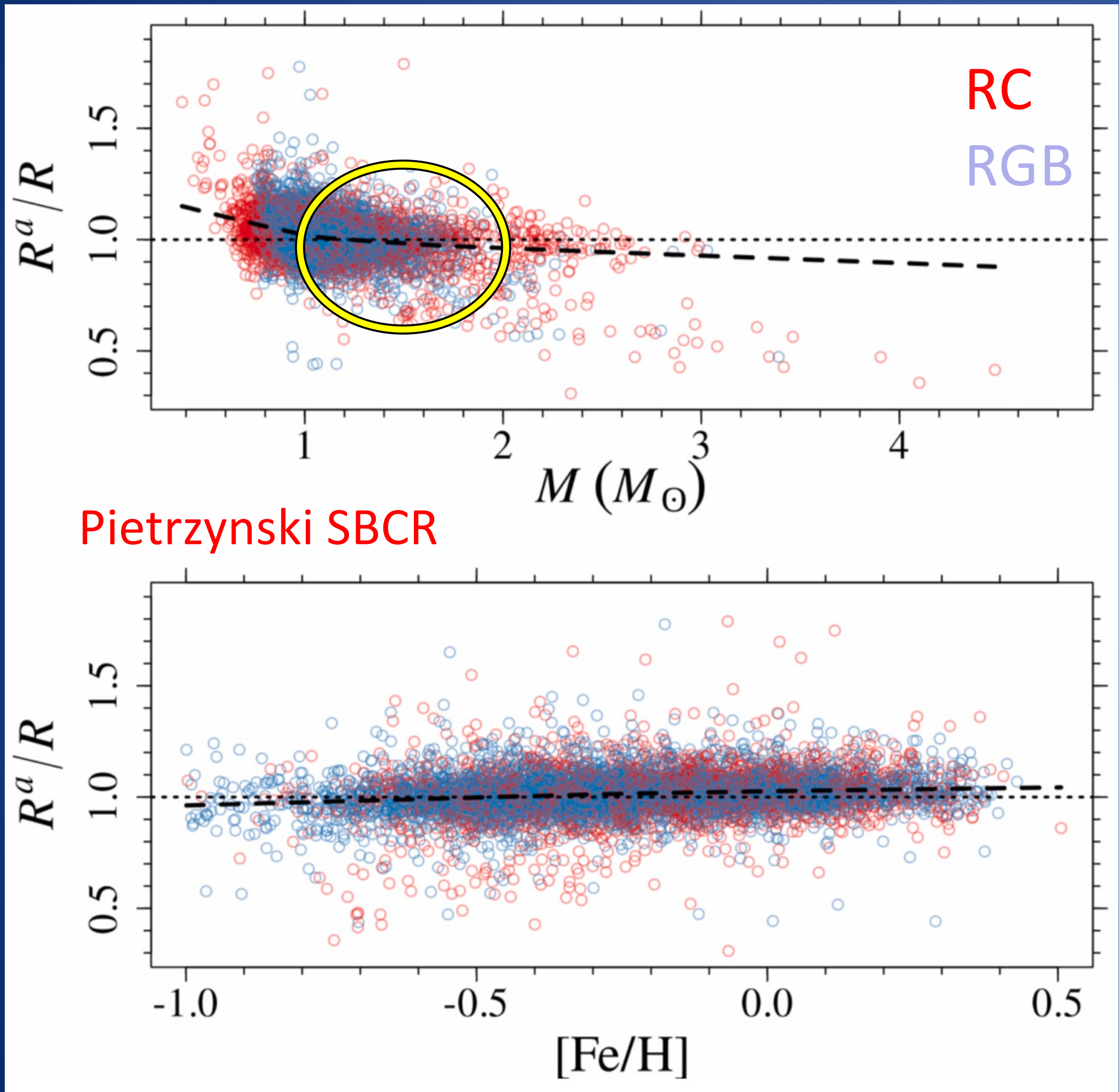
Mass dependence:

$M < 0.95 M_{\odot}$, $R^a > R$: 5.0% global
4.4% RGB
5.6% RC

Metallicity dependence:
Weak positive trend 6% per dex

Valle et al. 2024 (A&A 690, A327)

Results: APO-K2



Mass dependence:

$M < 0.95 M_o$, $R^a > R$: 5.0% global
4.4% RGB
5.6% RC

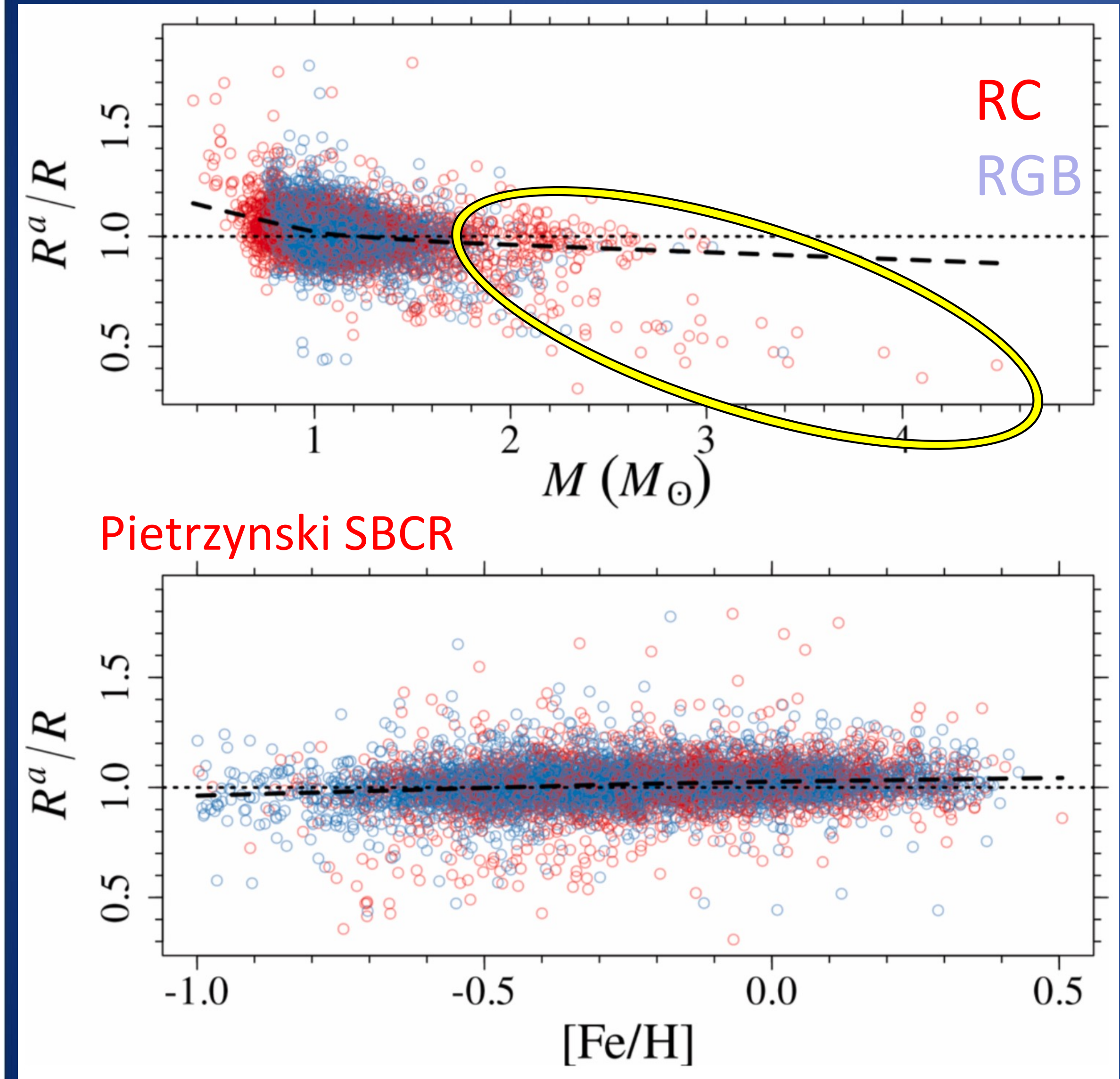
$1 M_o < M < 2 M_o$: discrepancy < 0,1%

Metallicity dependence:

Weak positive trend 6% per dex

Valle et al. 2024 (A&A 690, A327)

Results: APO-K2



Mass dependence:

$M < 0.95M_o$, $R^a > R$: 5.0% global
4.4% RGB
5.6% RC

$1M_o < M < 2M_o$: discrepancy < 0,1%

$M > 2M_o$ dichotomous trend for RC:

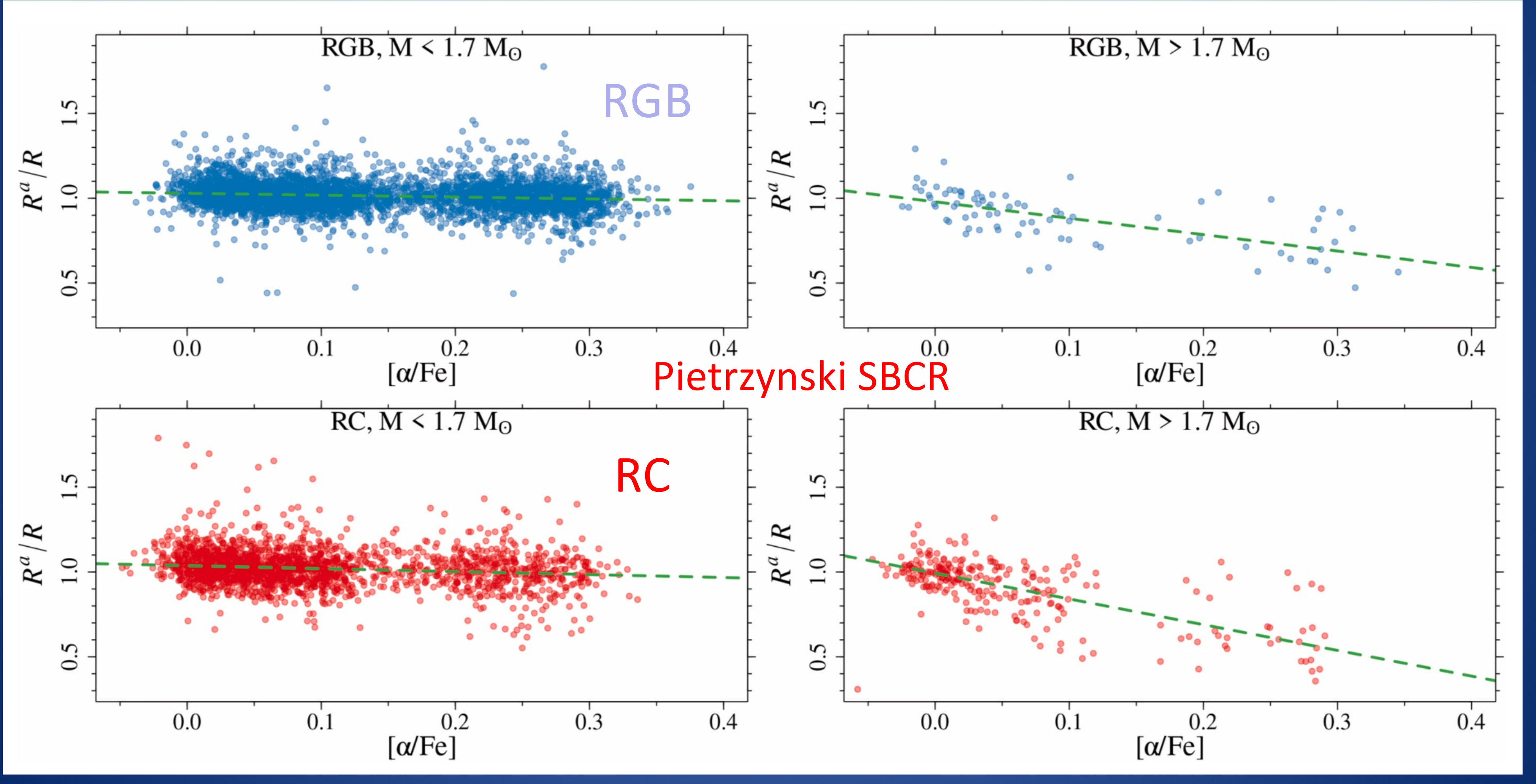
a) $R^a << R$
b) $R^a \approx R$

Metallicity dependence:
Weak positive trend 6% per dex

Valle et al. 2024 (A&A 690, A327)

Results: APO-K2

The dichotomy results from the different response to variations in [α/Fe] across different mass ranges in R estimates from asteroseismology and SBCR



Valle et al. 2024 (A&A 690, A327)

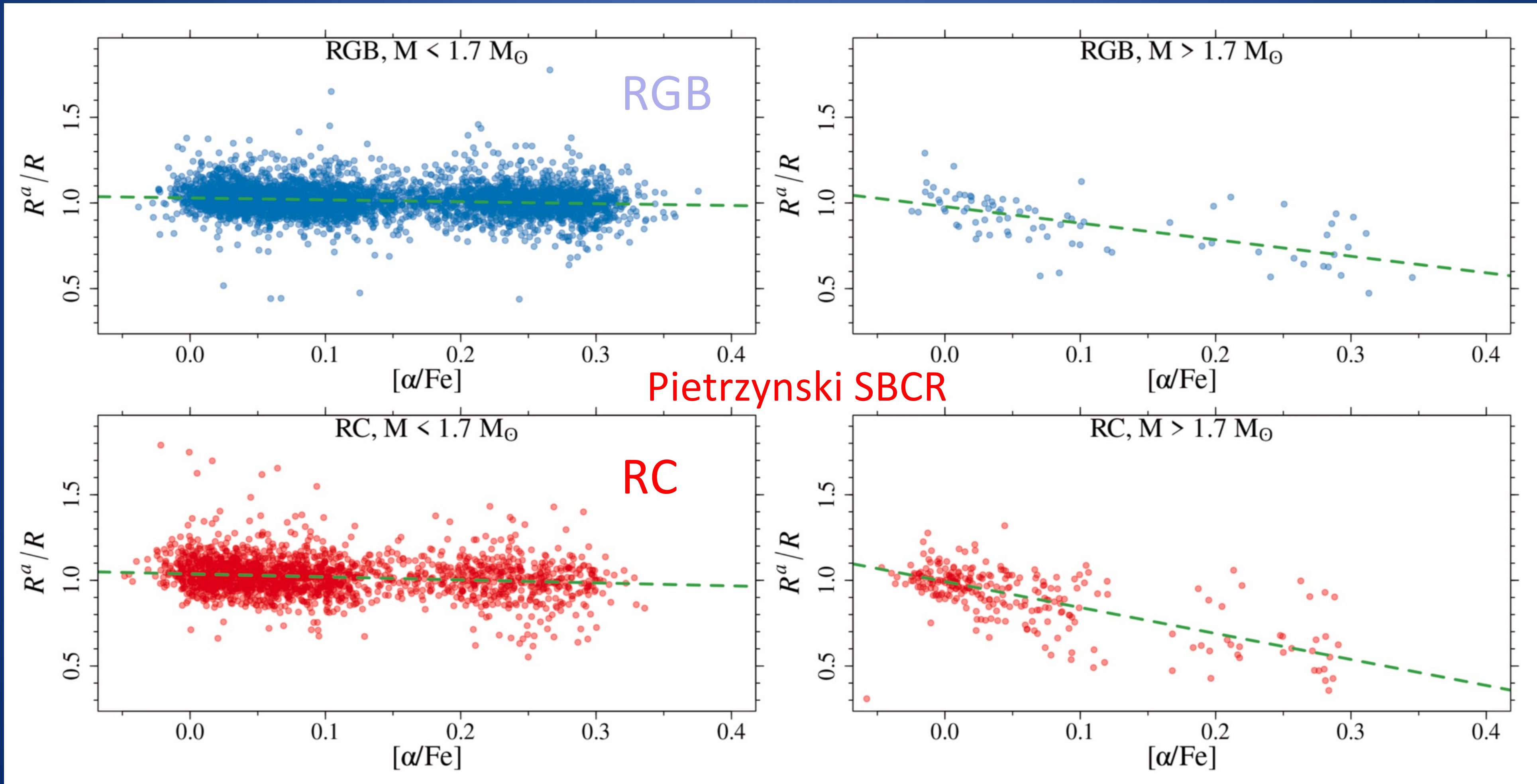
Results: APO-K2

M< 1,7 M$_o$:

- negligible trend with [α/Fe]

M> 1,7 M$_o$:

- significant trend with [α/Fe]
- steeper for RC stars

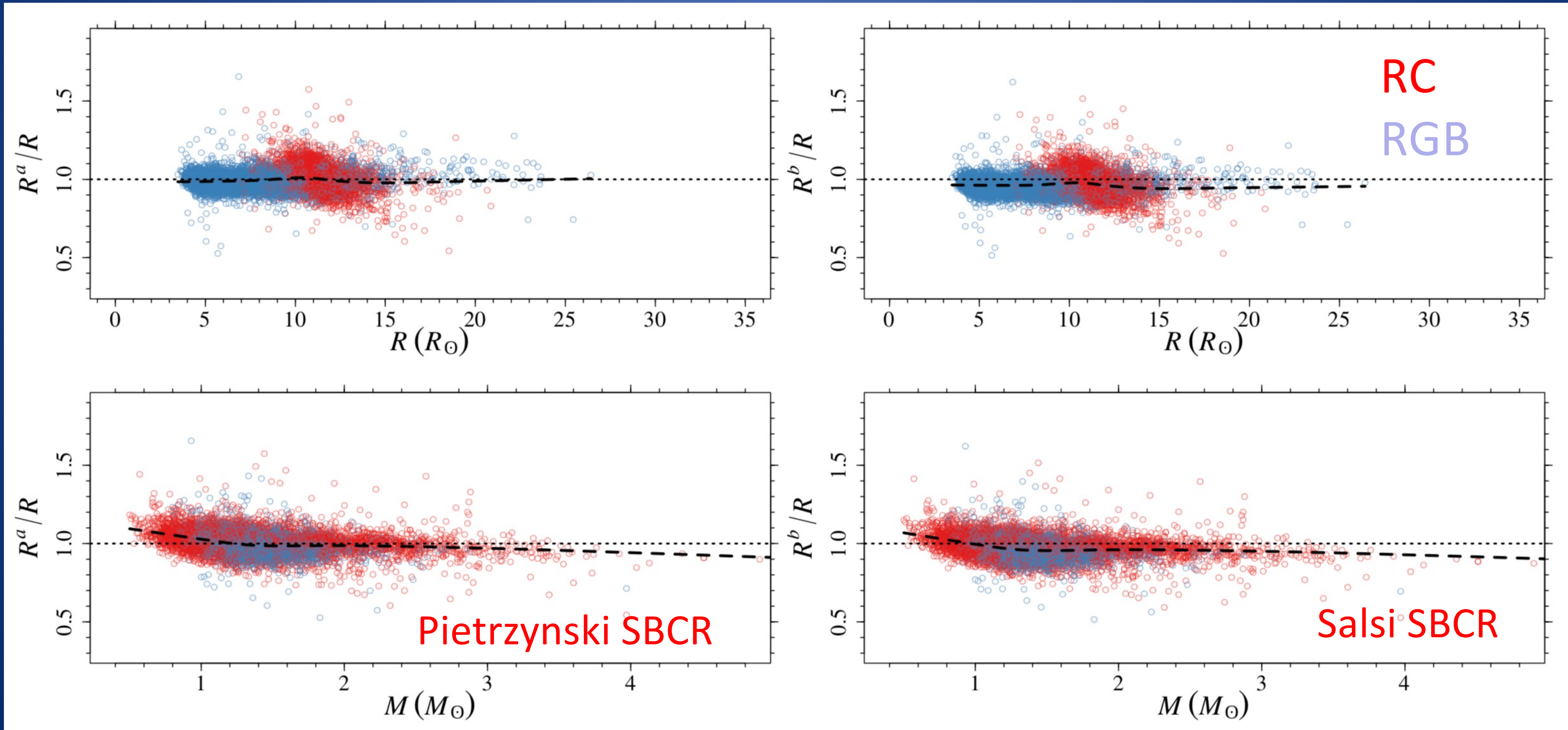


Valle et al. 2024 (A&A 690, A327)

Results: Kepler

Very good overall agreement

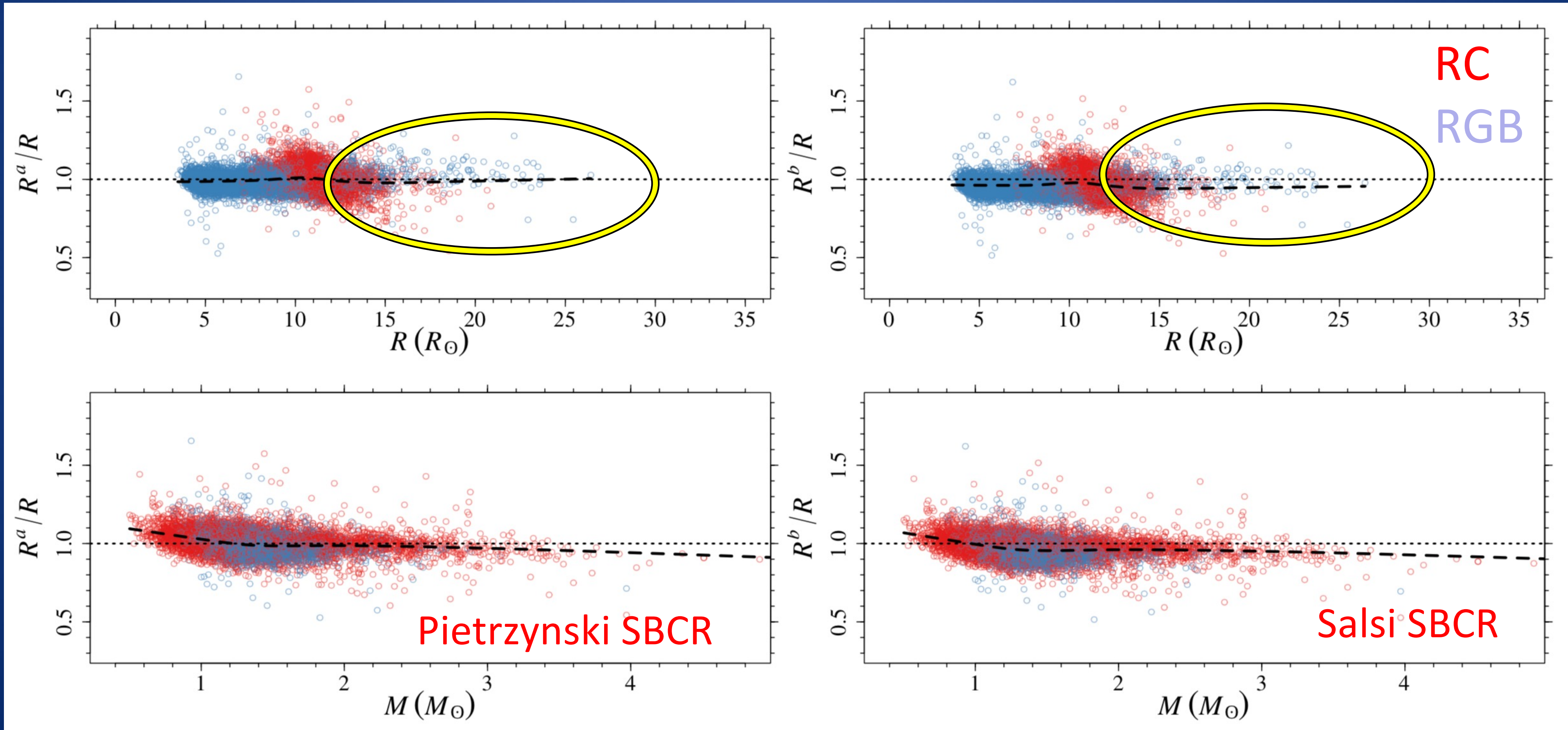
1.0% Standard deviation 7% 2.2%



Valle et al. 2025 (A&A 693, A159)

Results: Kepler

No systematic negative trend for R > 12 R_0

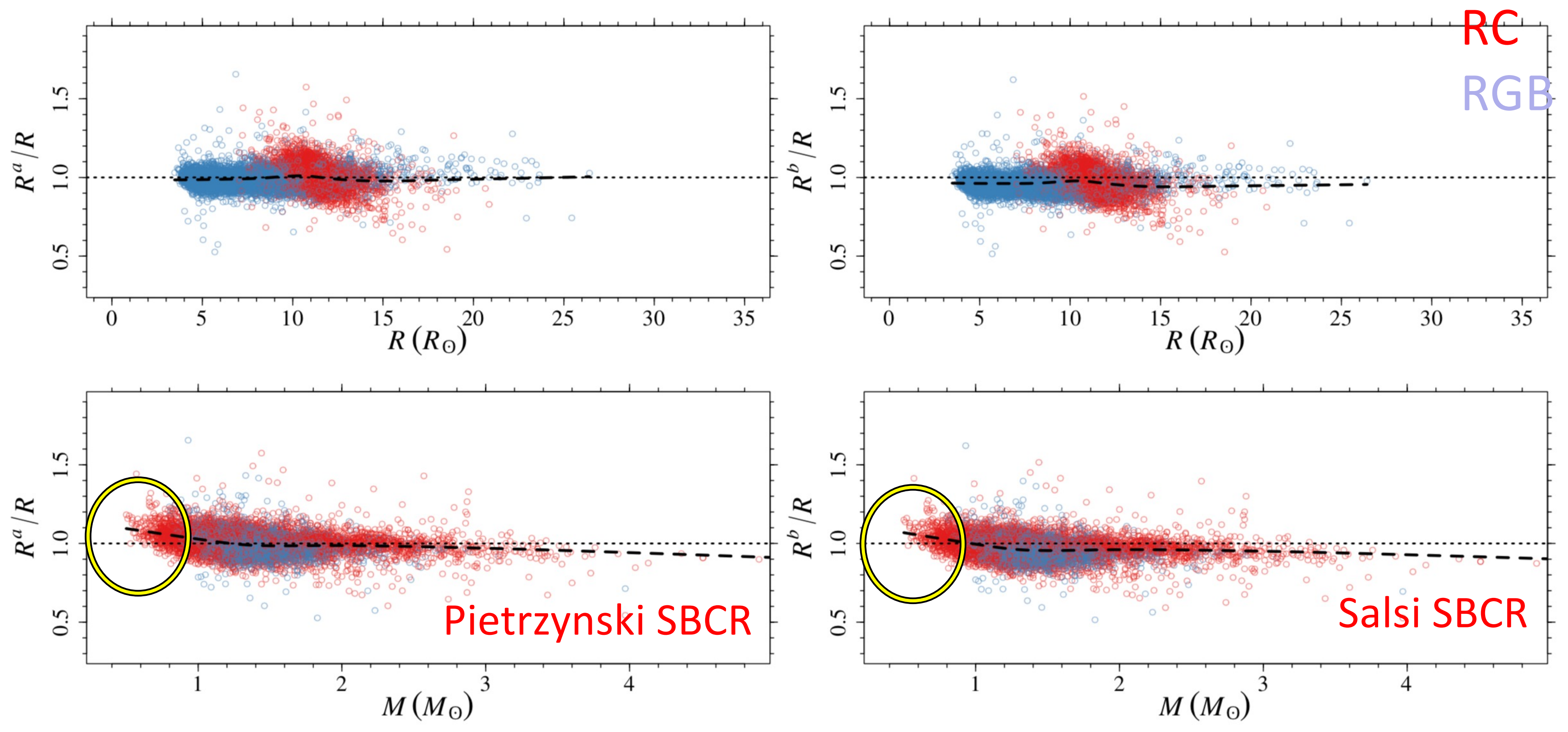


Valle et al. 2025 (A&A 693, A159)

Results: Kepler

Mass dependence

M< 0.95M$_{o}$, R^{a}>R: 5.0% global
5.2% RGB
4.9% RC



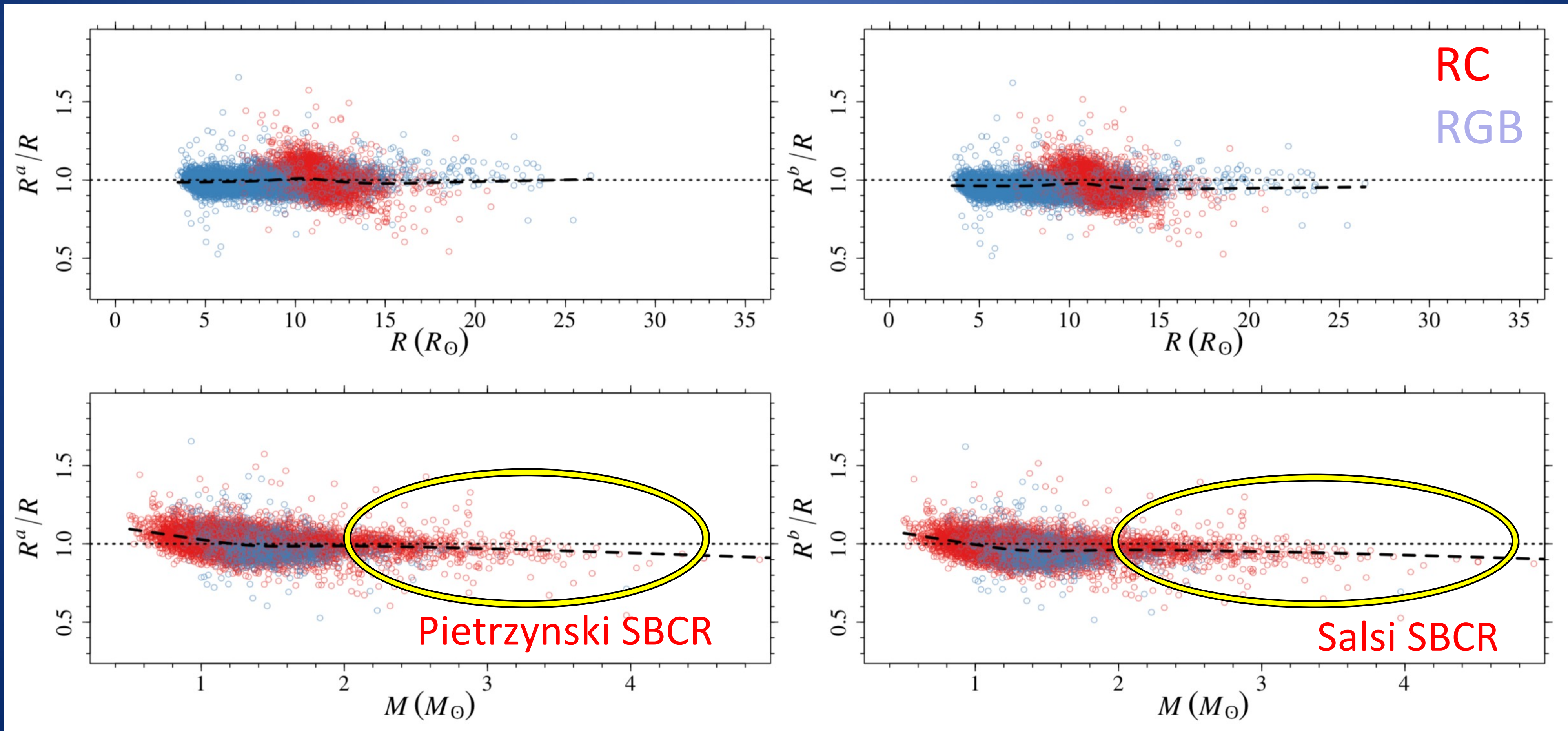
Valle et al. 2025 (A&A 693, A159)

Results: Kepler

Mass dependence

$M > 2M_o$: no dichotomous trend

no decreasing trend of R^a/R with M

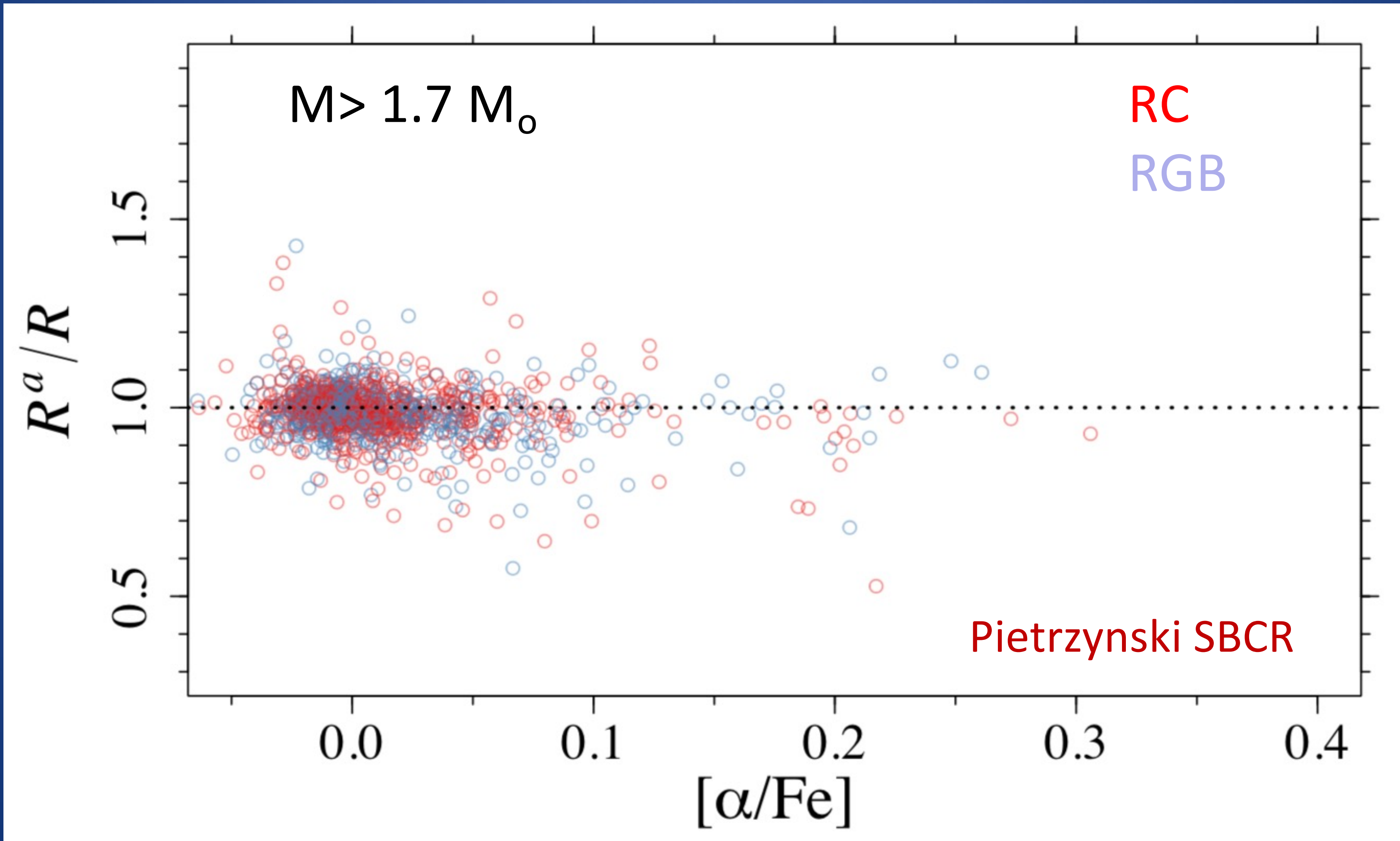


Valle et al. 2025 (A&A 693, A159)

Results: Kepler

no significant linear trend with [α/Fe]

no significant linear trend with [Fe/H]



Valle et al. 2025 (A&A 693, A159)

Conclusions

Very good agreement between SBCR+parallax and asteroseismic radii:

- median differences ~1-2.5%
- standard deviation ~7-10%

The technique for estimating radii from SBCR and parallaxes:

- reliable and accurate
- economical in terms of observation time, instrumentation, financial resources
- highly competitive and advantageous
- applicable to very large samples of single stars

Conclusions

Compared to the Kepler sample, the APO-K2 data set shows:

- greater dispersion
- suspicious and unexpected trends with [Fe/H], [α/Fe] and M

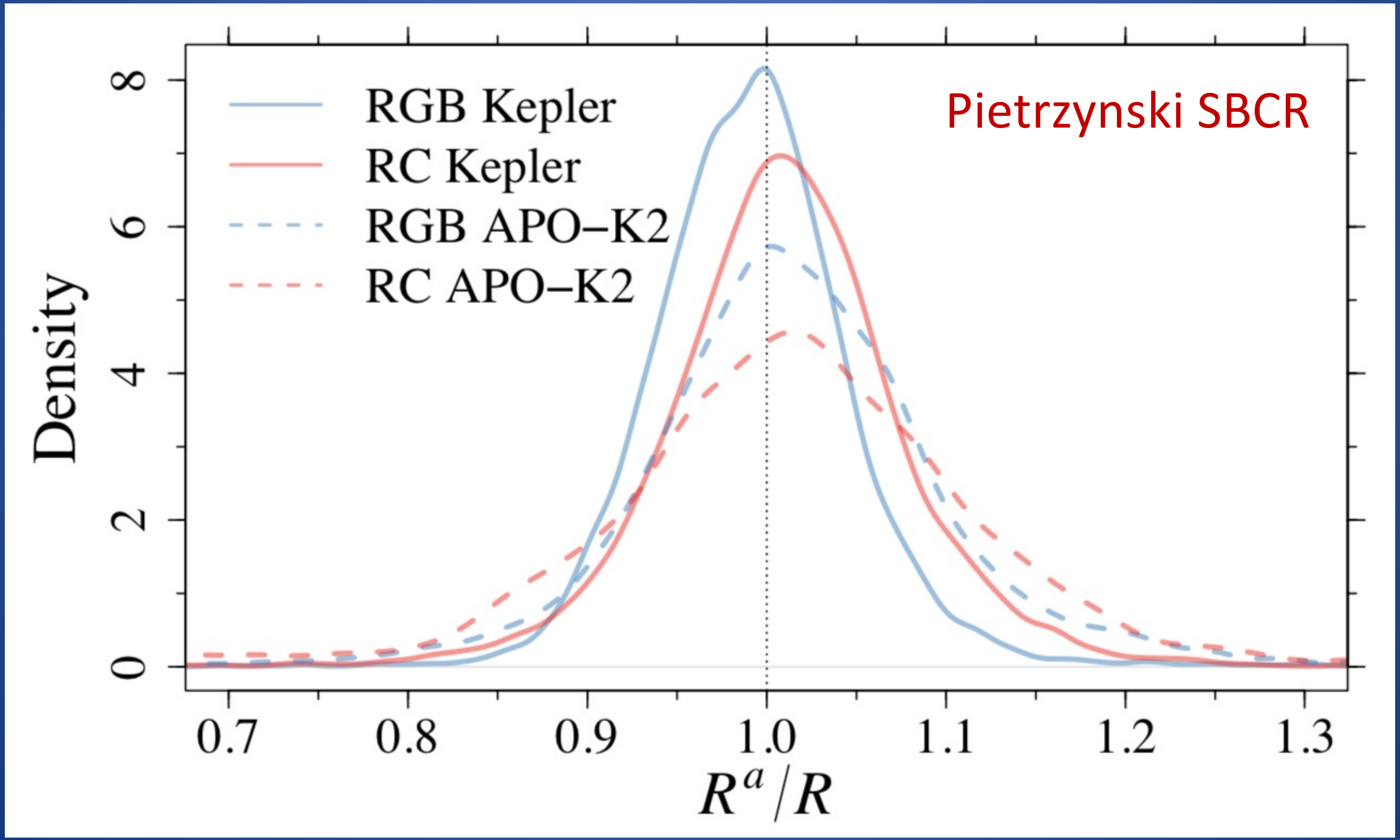
These differences:

- stem from differences in the asteroseismic data from K2 and Kepler
- might be artifact caused by biases in certain mass and metallicity regimes in K2

Thanks

Results: Kepler and APO-K2

Kernel density estimator of R^a/R for RGB and RC stars from the Kepler and APO-K2 datasets

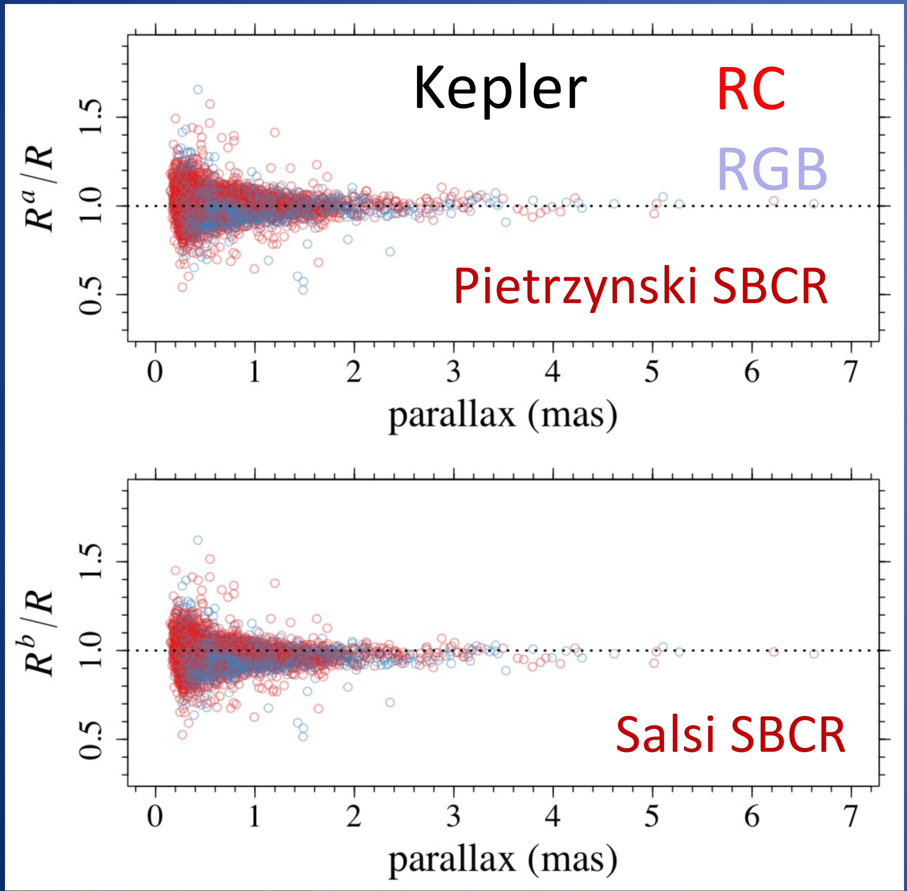


Valle et al. 2025 (A&A 693, A159)

Results

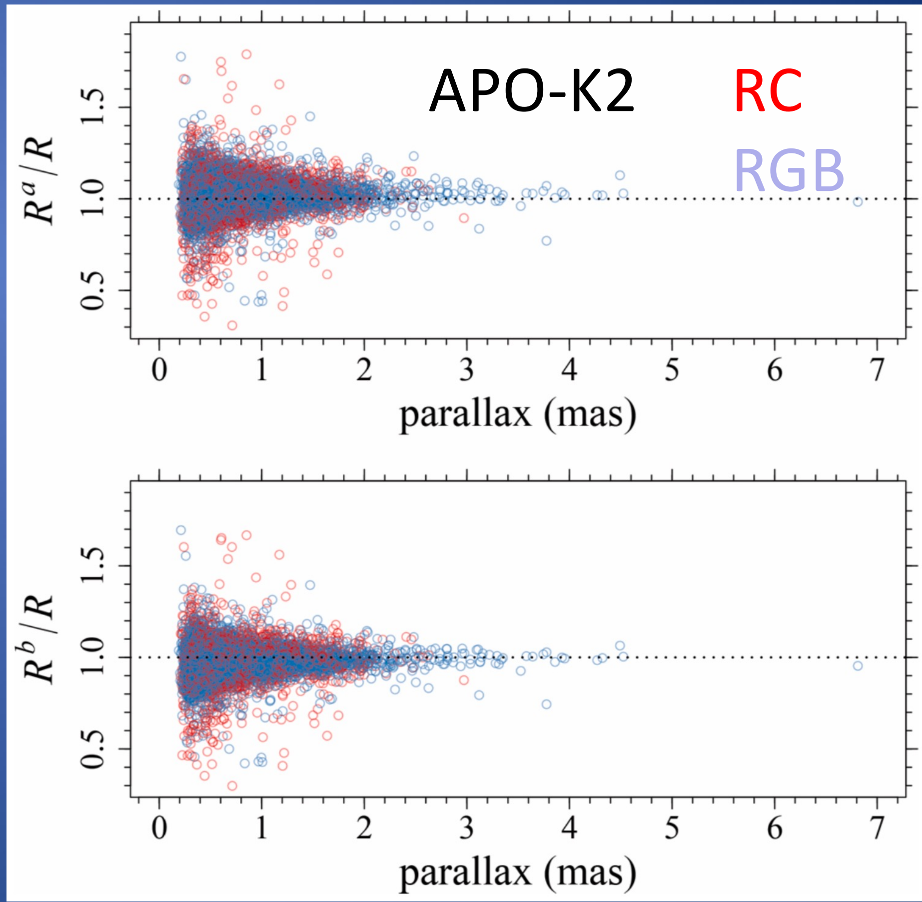
The agreement is better for nearby stars

π > 2.5 mas: standard deviation 3,7%



Valle et al. 2025 (A&A 693, A159)

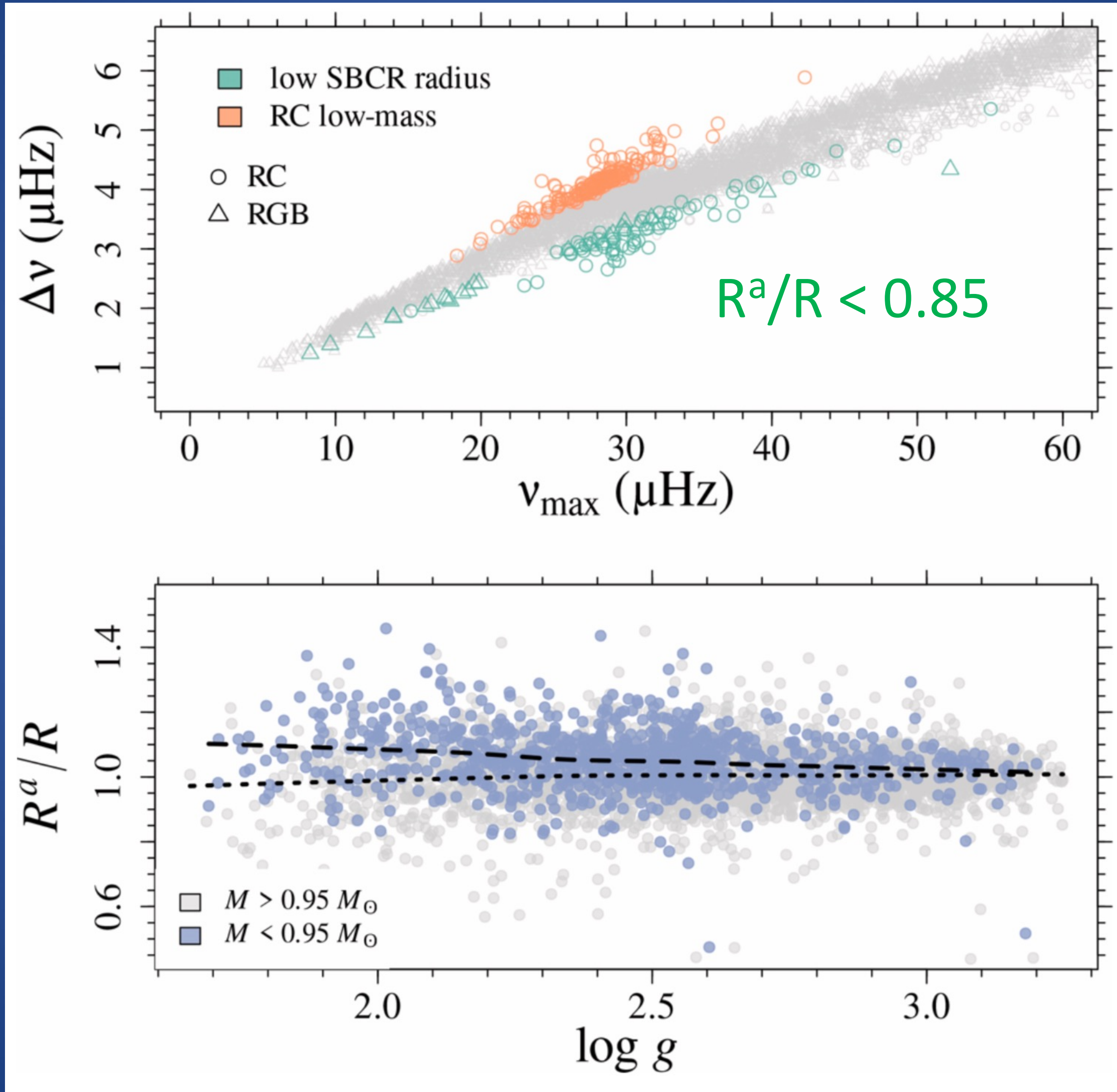
π > 2.5 mas: standard deviation 6%



Valle et al. 2024 (A&A 690, A327)

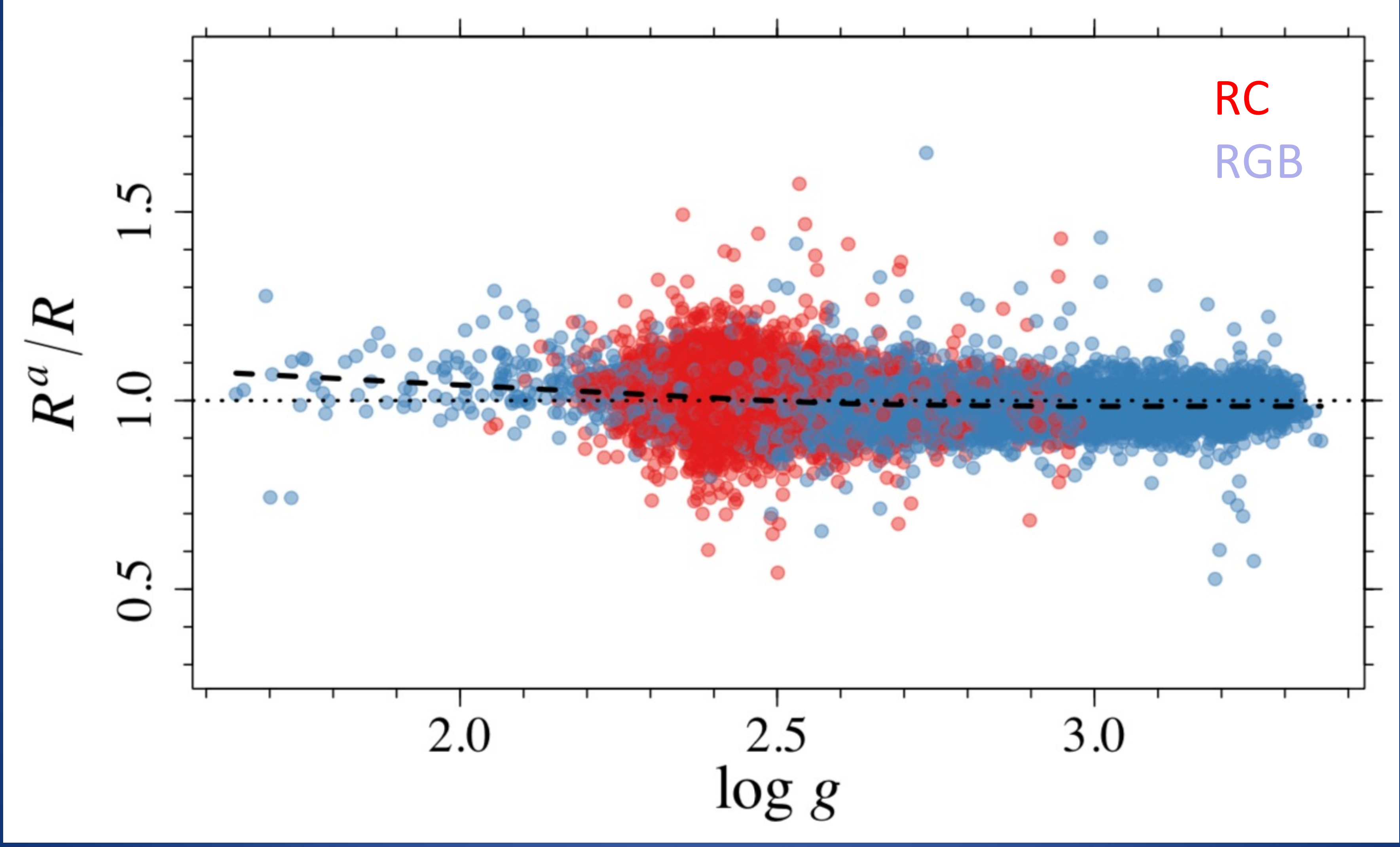
Results: APO-K2

Pietrzynski SBCR



Valle et al. 2024 (A&A 690, A327)

Results: Kepler



Pietrzynski SBCR

Valle et al. 2025 (A&A 693, A159)

Results: APO-K2

The dichotomy results from the different response to variations in [α/Fe] across different mass ranges of SBCR and asteroseismic radii estimates

Table 1. Robust regression fits of $R^{a/R}$ as a function of [α/Fe], according to the evolutionary phase and the stellar mass.

	Intercept	Slope
RGB, $M < 1.7\,M_\odot$	1.030 ± 0.002	-0.111 ± 0.012
RGB, $M > 1.7\,M_\odot$	0.979 ± 0.018	-0.967 ± 0.121
RC, $M < 1.7\,M_\odot$	1.037 ± 0.003	-0.174 ± 0.024
RC, $M > 1.7\,M_\odot$	0.993 ± 0.008	-1.517 ± 0.084

Valle et al. 2024 (A&A 690, A327)

Asteroseismic scaling relations

- RGB radii precision $\simeq$ 4% *(Pinsonneault et al. 2014; Martig et al. 2015; Valle et al. 2024)*

- The reliability of scaling relations when applied to RGB stars has been subject to debate *(Epstein et al. 2014; Gaulme et al. 2016; Viani et al. 2017; Brogaard et al. 2018; Buldgen et al. 2019)*

- Corrections have been proposed *(Sharma et al. 2016; Stello & Sharma 2022; Hekker 2020; Zinn et al. 2022)*

APO-K2 Catalogue (Schonhut-Stasik et al. 2024):

- 7500 RGB and red clump stars
- Spectroscopic data: APOGEE DR 17 (Abdurro'uf et al. 2022)
- Asteroseismic data: K2-GAP (Stello et al. 2015)
- Astrometric data: Gaia EDR3 (Brown et al. 2021)

APO-K2 sample

APO-K2 catalogue *(Schnhut-Stasik et al. 2024)* provides:

- Asteroseismic radius from scaling relations corrected according to Sharma et al. 2016
- K_s magnitude from 2MASS
- parallax from Gaia DR3, corrected according to the Gaia zero-point *(Lindegren et al. 2021)*

We cross-matched it with:

- the TESS Input Catalogue (TIC) v8.2 to obtain precise V magnitudes and E(B-V)

We computed the extinction coefficients: $A_V = 3.1\ E(B-V)$, $A_k = 0.114\ A_V$ *(Cardelli et al. 1989)*

Kepler sample

Kepler catalogue *(Yu et al. 2018)* provides:

- Asteroseismic radius from scaling relations corrected according to Sharma et al. 2016

We cross-matched it with:

- the TESS Input Catalogue (TIC) v8.2 to obtain precise V and K_s magnitudes and E(B-V)
- Gaia DR3, corrected according to the Gaia zero-point *(Lindegren et al. 2021),* to obtain parallax

We computed the extinction coefficients: A_V= 3.1 E(B-V), A_k= 0.114 A_V *(Cardelli et al. 1989)*

Surface brightness-colour relations

- Calibrated using very accurate angular diameters from long-baseline interferometry (*Kervella et al. 2004; Di Benedetto 2005; Adams et al. 2018; Gallenne et al. 2018; Salsi et al. 2021; Nardetto et al. 2023*)

- Comparisons among recent SBCRs show a very limited variability for late-type stars (*Pietrzynski et al. 2019; Salsi et al. 2022; Nardetto et al. 2023*)

- *Nardetto et al. 2023*: 19 SBCRs are in agreement better than 0.008 mag between $1.5 \leq (V\text{-}K)_0 \leq 2.5$ mag