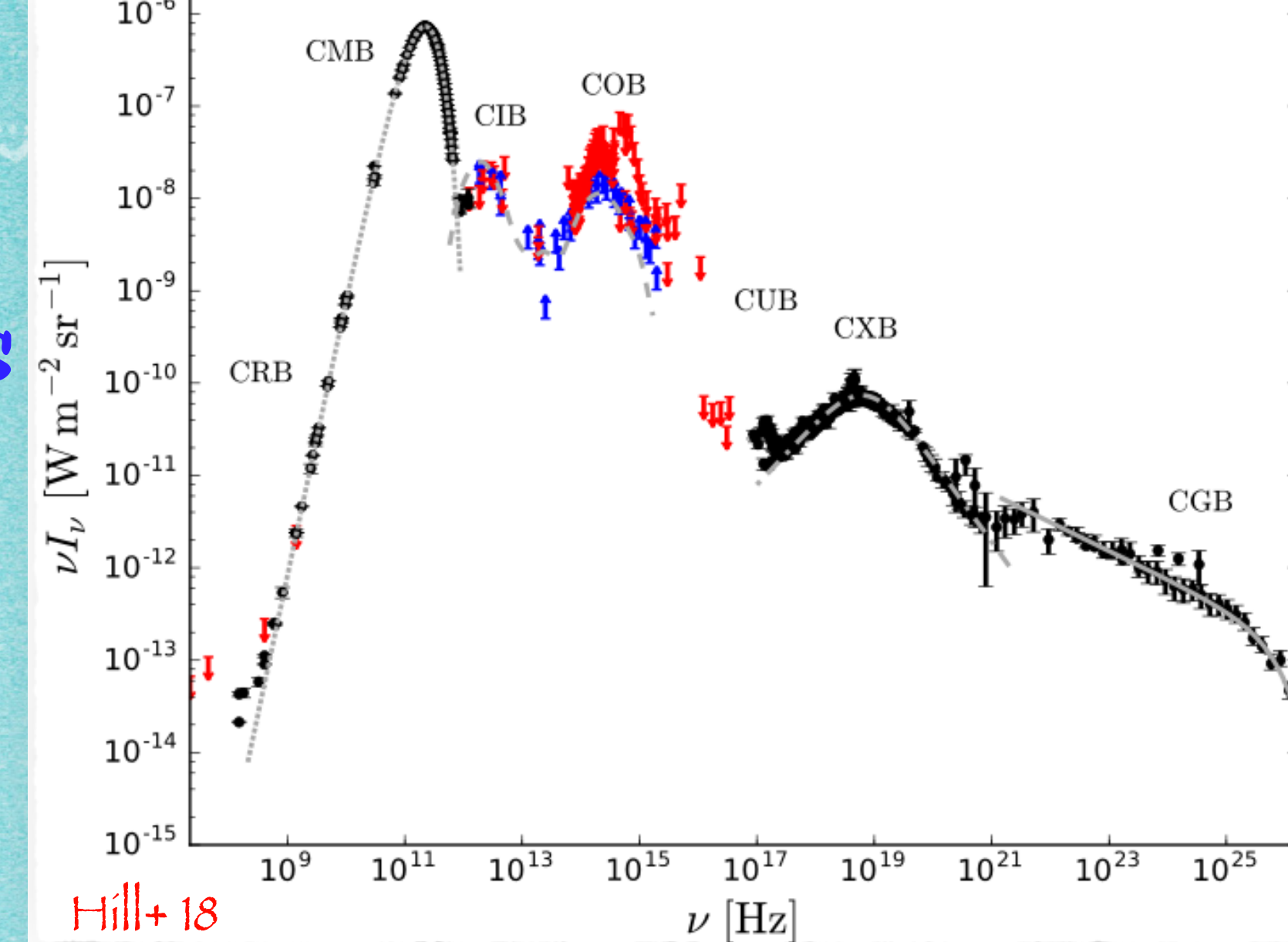


Constraining the frequency
spectrum of cosmic radio
background signals with dipole
analyses on sky areas
observed by SKA

Tiziana Trombetti & Carlo Burigana
INAF IRA Bologna

Best way to study the CB monopole frequency spectrum is to observe & map the whole sky with:

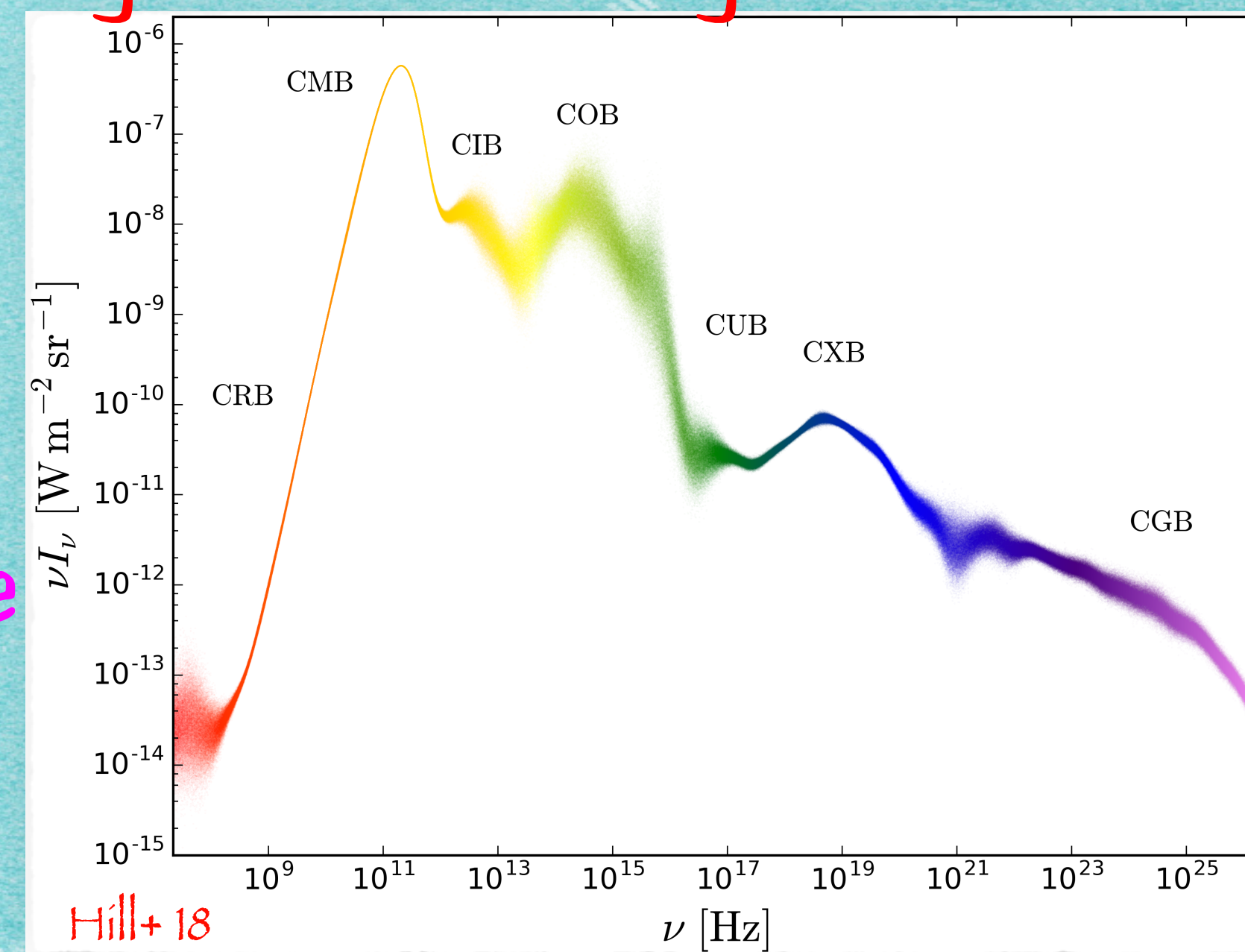
- ✓ precise absolute, relative and inter-frequency calibrations
- ✓ extreme sensitivity
- ✓ good resolution to identify & subtract discrete sources
- ✓ improve data analysis with a spectral resolution set according to expected spectral variations of the signal



Various experiments and observational projects dedicated to global radio signal BUT...

ABSOLUTE CALIBRATION is difficult... on-going & future microwave & far-IR projects (LiteBIRD) are inherently differential as well as radio interferometers \rightarrow best sensitivities & resolutions (LOFAR, SKA)

Why not take advantage of the possibility of measuring the CB spectrum by exploiting these intrinsically differential designs relying only on relative and interfrequency calibration?



Peculiar motion of the observer naturally offers this possibility

The peculiar motion of an observer relative to an ideal reference frame at rest wrt the CB in a given frequency band produces boosting effects in the anisotropy patterns at low multipoles with frequency spectral behaviour related to the spectrum of the isotropic monopole emission

The largest effect is on the dipole ($\ell = 1$) mainly attributed to the solar system barycentre motion

The peculiar velocity effect can be evaluated on the whole sky using the complete description of the Compton-Getting effect (based on Lorentz invariance of photon distribution function $\eta(\nu)$)

The effect is given in terms of equivalent thermodynamic temperature, $T_{th}(\nu)$, defined as the temperature of the BB having the same $\eta(\nu)$ @ ν

$$T_{th}(\nu) = \frac{h\nu}{k} \frac{1}{\ln(1 + 1/\eta(\nu))}$$

Planck Collaboration 2020

where $\eta(\nu, \hat{n}, \beta) = \eta(\nu')$ with $\nu' = \nu(1 - \hat{n} \cdot \beta)/(1 - \beta^2)^{1/2}$

$$\beta \simeq 1.2336 \times 10^{-3} \simeq A_{dip} / T_0$$

CMB dipole direction in Galactic coord

$$l = 264.021^\circ, b = 48.253^\circ$$

$$v = (369.82 \pm 0.11) \text{ km/s}$$

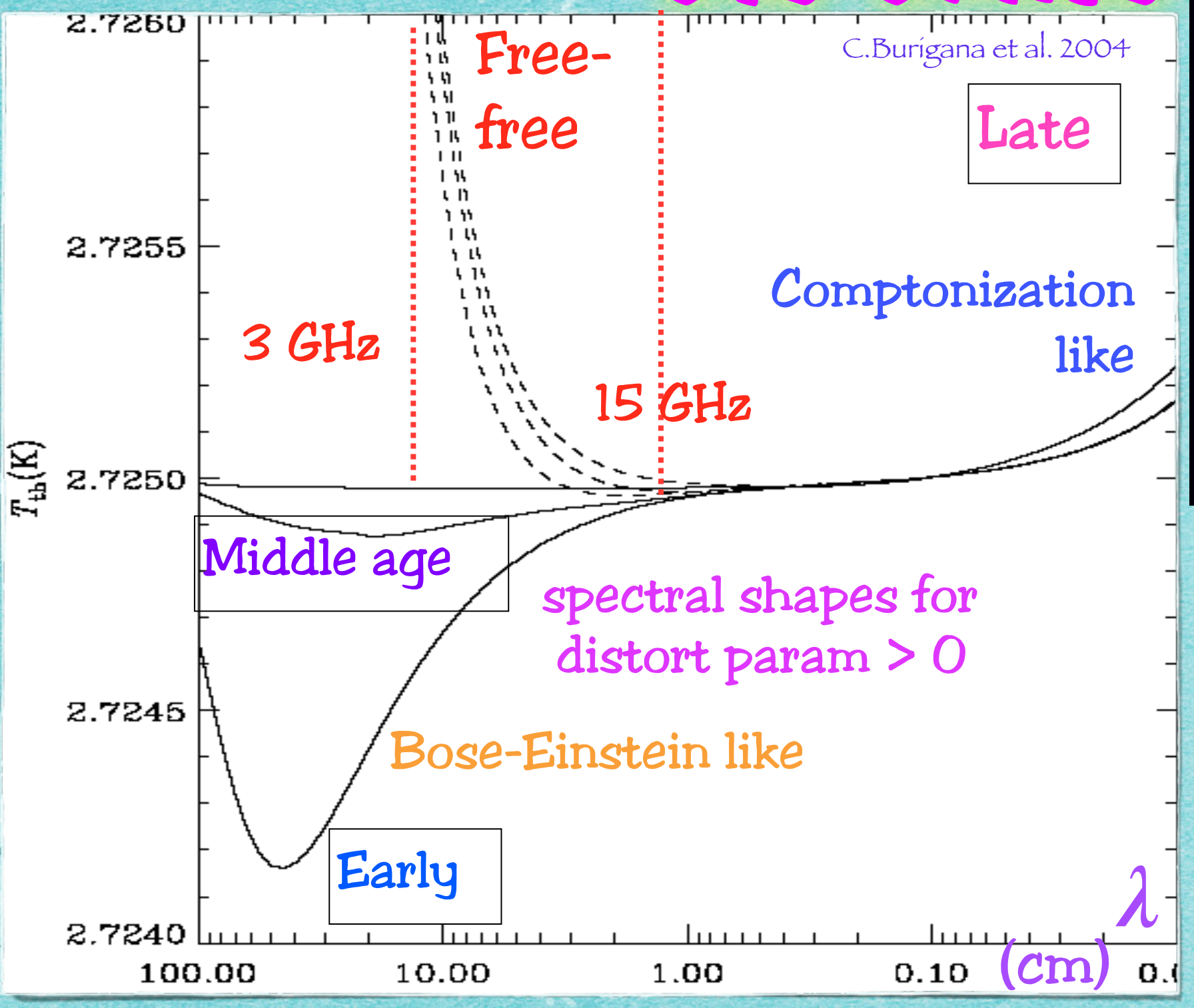
$\hat{n} \rightsquigarrow$ sky direction unit vector associated to polar

coordinates (colat) & (long) \rightarrow dipole vector: $\vec{\beta} = \vec{v}/c$

velocity of the Solar System barycentre wrt the CMB

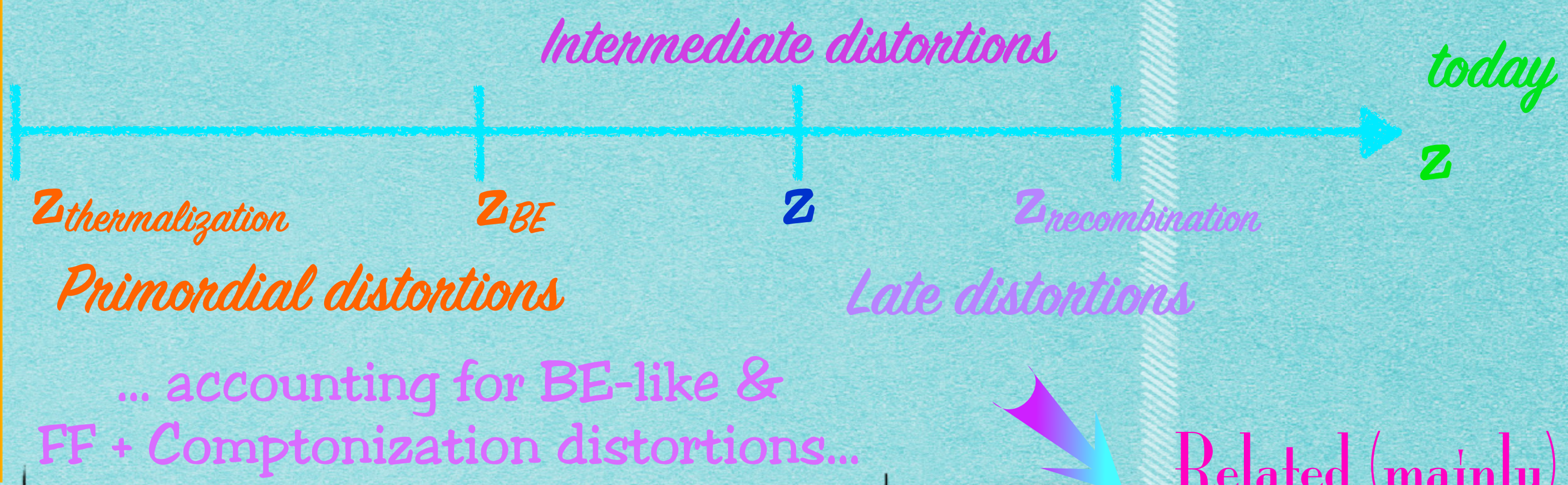
Zeldovich & Sunyaev 1969; Illarionov & Sunyaev 1974; Danese & de Zotti 1977; Burigana et al. 1991; Hu & Silk 1993

BIG BANG



Blackbody Photosphere

CMB distortions @ different cosmic times



$$\eta^{FF+C} \simeq \eta_i + u \frac{x/\phi_i e^{x/\phi_i}}{(e^{x/\phi_i} - 1)^2} \left(\frac{x/\phi_i}{\tanh[x/(2\phi_i)]} - 4 \right) + \frac{y_B(x)}{x^3}$$

$$y_B(t, x) = \int_{t_h}^t (\phi - \phi_i) \phi^{-3/2} g_B(x, \phi) K_{0B} dt$$

$$= \int_z^{z_h} (\phi - \phi_i) \phi^{-3/2} g_B(x, \phi) K_{0B} t \exp \frac{dz}{1+z}$$

$$K_{0B}(z) = K_B(z) / \phi^{-7/2}$$

$$K_B(z) = \frac{8\pi}{3} \frac{e^6 h^2 n_e (n_H + 4n_{He})}{m (6\pi m k T_e)^{1/2} (k T_e)^3}$$

$$\sim 2.6 \times 10^{-25} \phi^{-7/2} (T_0 / 2.7 \text{ K})^{-7/2} (1+z)^{5/2} \hat{\Omega}_b^2 \chi_e^2 \text{ sec}^{-1}$$

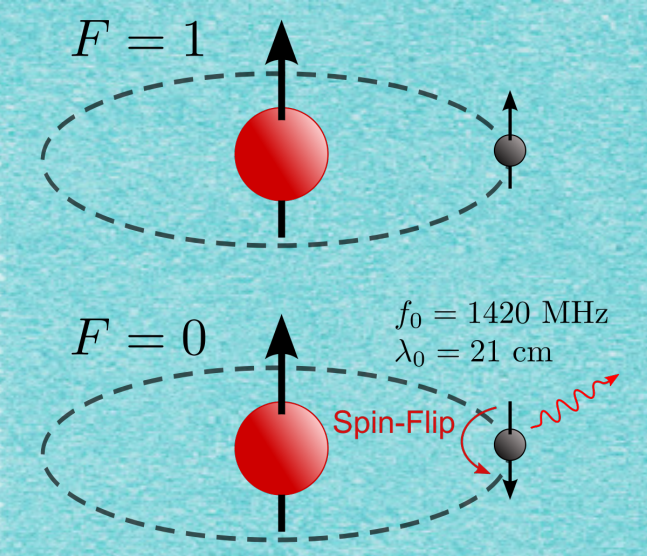
$$\eta_{BE} = \frac{1}{e^{x_e + \mu} - 1}$$

Quantify the fractional E exchanged in the plasma during the interaction

with μ function of x

$$\mu(x) \simeq \mu_0 e^{(-x_c/x)} \quad \mu_0 \sim 1.4 \Delta \epsilon / \epsilon_i \quad u \simeq (1/4) \Delta \epsilon / \epsilon_i$$

The 21 cm line corresponds to the spin-flip transition in the ground state of neutral hydrogen



In cosmology, this signal (usually in T_{ant}) is described as the offset of the 21 cm T_{bright} from T_{CMB} along the observed line of sight at ν

$$T_{\text{ant}}^{21\text{ cm}}(\nu) = \frac{T_S - T_{\text{CMB}}}{1+z} (1 - e^{-\tau_{\nu_0}})$$

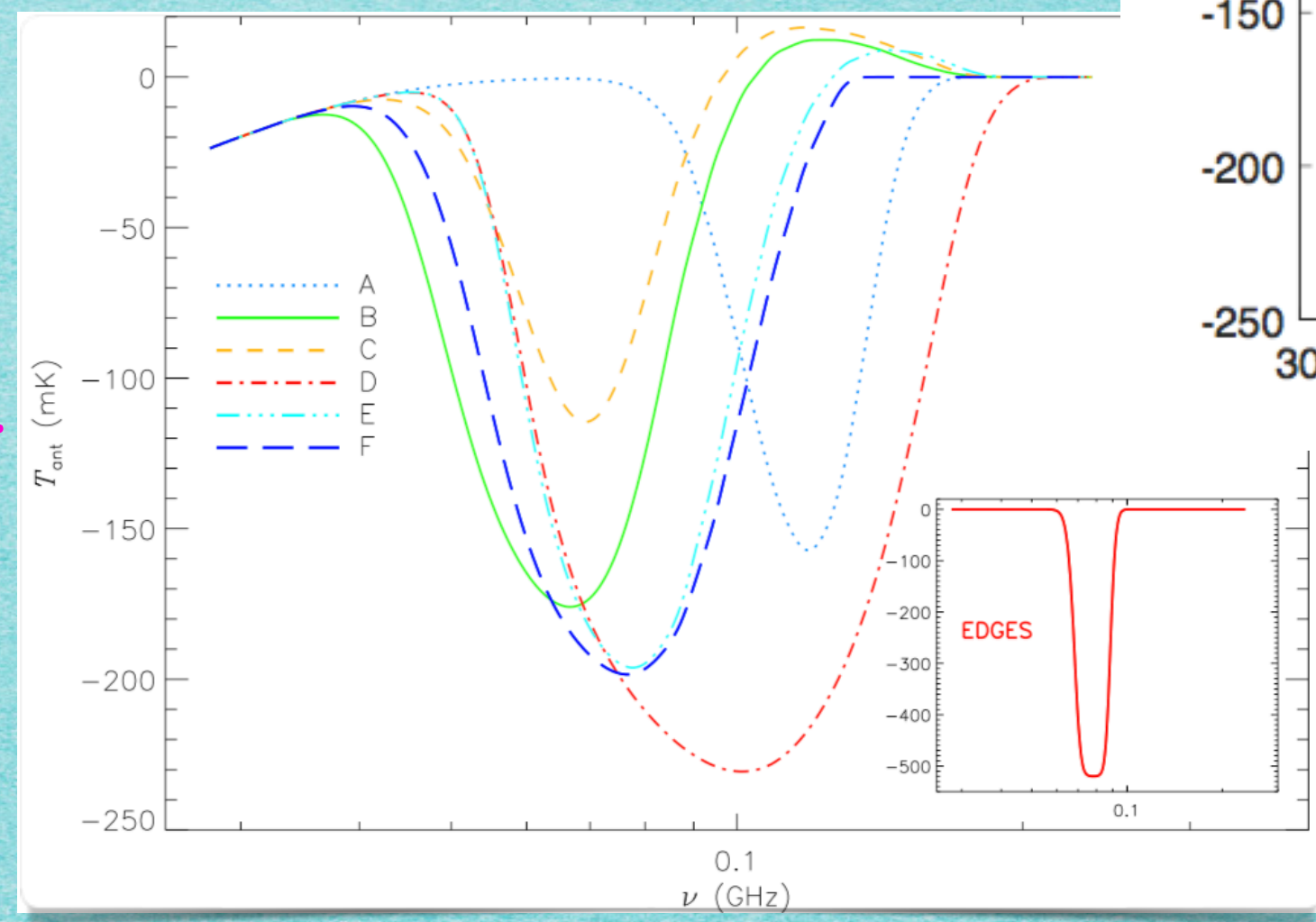
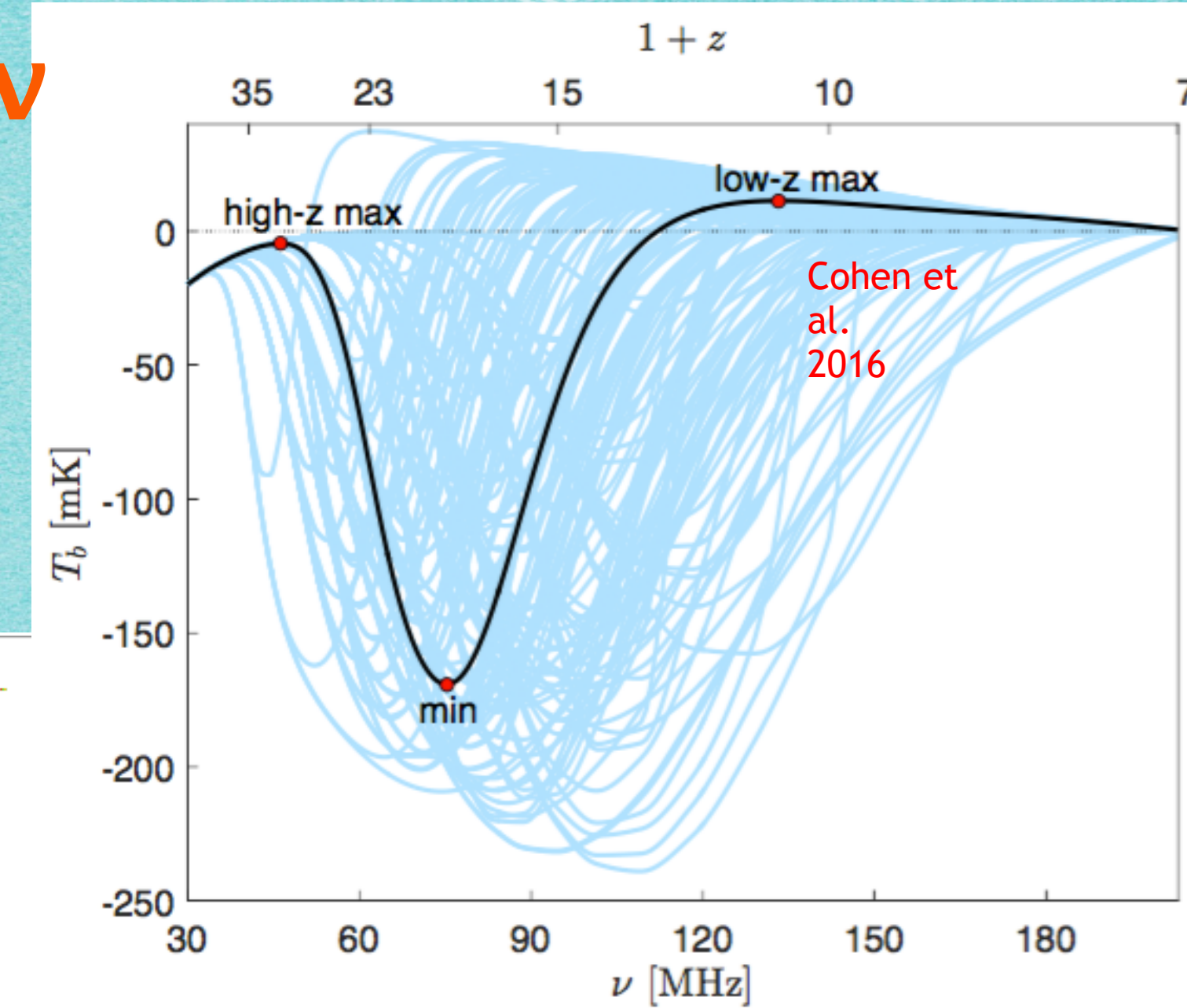
$$\approx 27 x_{\text{HI}} \left(1 - \frac{T_{\text{CMB}}}{T_S}\right) (1 + \delta_{nl}) \left(\frac{H(z)}{dv_r/dr + H(z)}\right) \frac{dv_r}{dr}$$

$$\sqrt{\frac{1+z}{10} \frac{0.15}{\Omega_m h_{100}^2} \left(\frac{\Omega_b h_{100}^2}{0.023}\right)} \text{ mK,}$$

Furlanetto et al. 2006, Phys. Rep.

$\delta_{nl}(\vec{x}, z) \equiv \rho/\bar{\rho} - 1$ evolved (Eulerian) density contrast

dv_r/dr comoving gradient of the line of sight component of the comoving velocity



Assuming CMB assumed as a backlight

- > if $T_S < T_{\text{CMB}}$ the gas seen in absorption
- > if $T_S > T_{\text{CMB}}$ the gas seen in emission

Note: in principle T_{CMB} can be replaced by any possible background radiation

Wide envelope of possible predictions

✓ negative signals up to ~ 250mK
 ✓ positive signals up to ~ 50mK

peaking in a wide range of ν :
 ~50 and 150 MHz
 → $z \sim 30$ to 10

Important extragalactic (EG) background signal expected from the integrated contribution of discrete sources

Large fraction resolved through galactic surveys but a residual background (observationally diffuse) comes from faint sources below detection limits & depends on their number counts (very high compared to the fine reionization signatures)

Careful analysis by Gervasi+08 and prediction of the EG source radio background between 0.151 and 8.44 GHz, including different detection thresholds

Consider their best-fit single power-law model multiplied by 1.3 to account for a larger contribution likely ascribed to emerging SFG and radio-quiet AGN at fainter flux densities

LOFAR Boötes image support a flattening of differential number counts $N'(ν)$ @ 1.4 GHz below $\approx 100 \mu\text{Jy}$ (Prandoni+ 18) & @ 0.15 GHz below $\approx 1\text{mJy}$ (Retana-Montenegro+ 18)

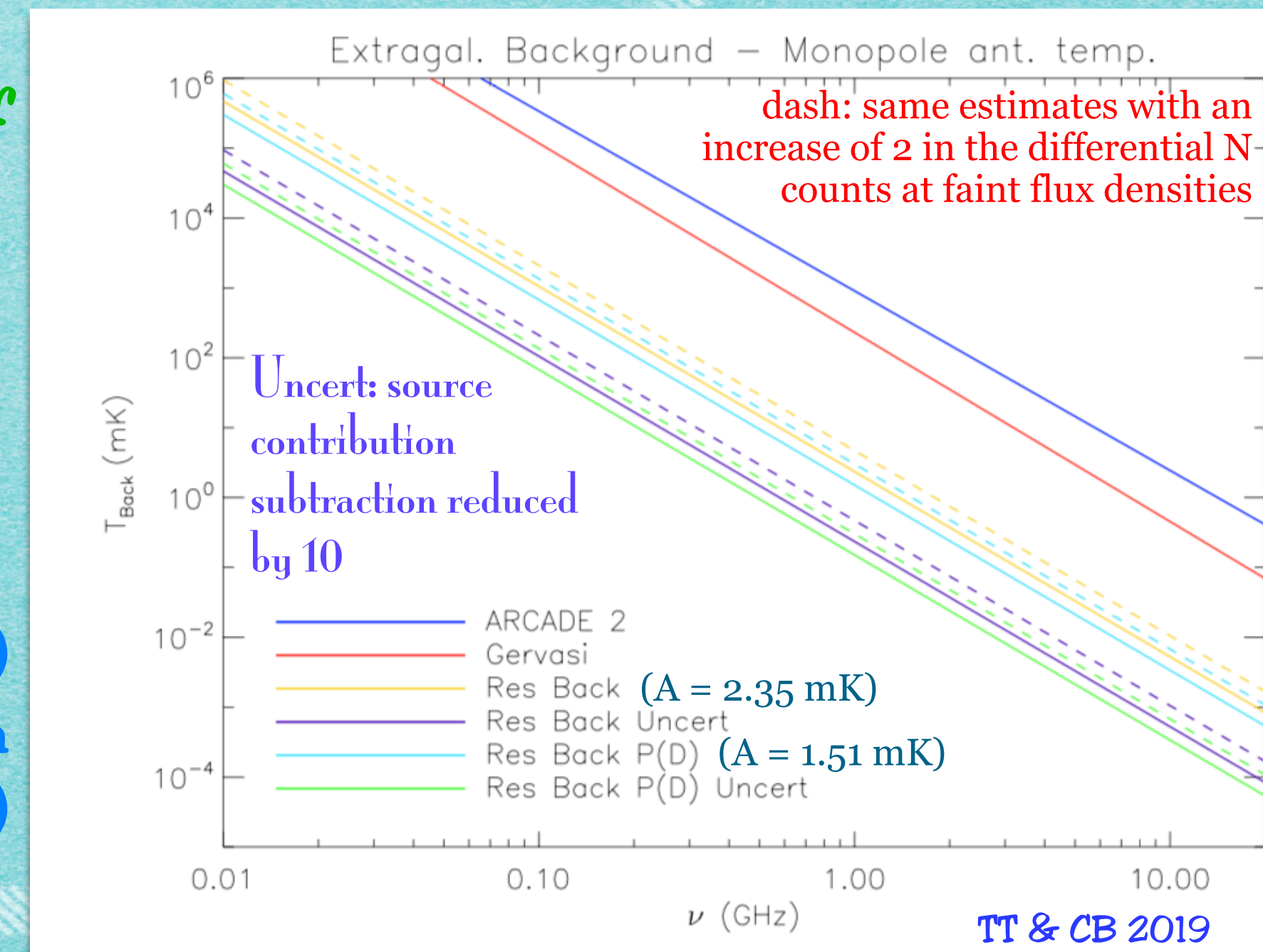
Suggest an increase in $N'(ν)$ of a factor of ~ 2 at the faint flux densities and a $\sim 30\%$ increase in the EG radio background

- $S_{\text{max}} = 50 \text{ nJy}$ (typical limits of ultra deep ref SKA continuum surveys planned)
- $S_{\text{max}} = 15 \text{ nJy}$ (number counts down to flux densities fainter than the threshold can be investigated through P(D) methods)

$$T_{\text{ant}}^{\text{Back}}(\nu) \simeq A (\nu/\text{GHz})^{-2.65}$$

$$\int_{S_{\text{min}}}^{S_{\text{max}}} S N'(ν) dS$$

P(D) confusion probability distribution



FORMALISM

observed signal map

$$T_{th}^{BB/dist}(\nu, \hat{n}, \beta) = \frac{xT_0}{\ln(1 + 1/(\eta(\nu, \hat{n}, \beta))^{BB/dist})} = \sum_{\ell=0}^{\ell_{max}} \sum_{m=-\ell}^{\ell} a_{\ell,m}(\nu, \beta) Y_{\ell,m}(\theta, \phi),$$

$$T_r = T_0(1 + z)$$

$$x = h\nu/(kT_r)$$

redshift invariant dimensionless frequency

CMB redshift dependent effective temperature

Expand the function $T_{th}^{BB/dist}(\nu, \hat{n}, \beta) = T_{th}^{BB/dist}(\nu, \theta, \phi, \beta)$ in spherical harmonics & adopt a reference system with the z axis parallel to the observer velocity

$Y_{\ell,m}(\theta, \phi)$ are the spherical harmonics related to the associated Legendre polynomials $P_m^\ell(\cos\theta)$ & $a_{\ell,m}(\nu, \beta)$ contain information on the background spectrum and the observer velocity

the isotropy of the background monopole implies that when $\vec{z} \parallel \beta \implies T_{th}^{BB/dist}$ depends only on colat. θ but not on long. $\phi \rightsquigarrow$ only the $a_{\ell,m}$ terms with $m=0$ do not vanish

Thus $Y_{\ell,m}(\theta, \phi) = \tilde{P}_\ell^m(\cos\theta) \implies$ renormalized associated Legendre polynomials

$$\tilde{P}_\ell^m(\cos\theta) = \sqrt{\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos\theta).$$

Method:

choose 7 directions, 7 linear equations from $l = 0$ up to $l = 6$

directions $\rightarrow N = l_{max} + 1$

Directions choice:

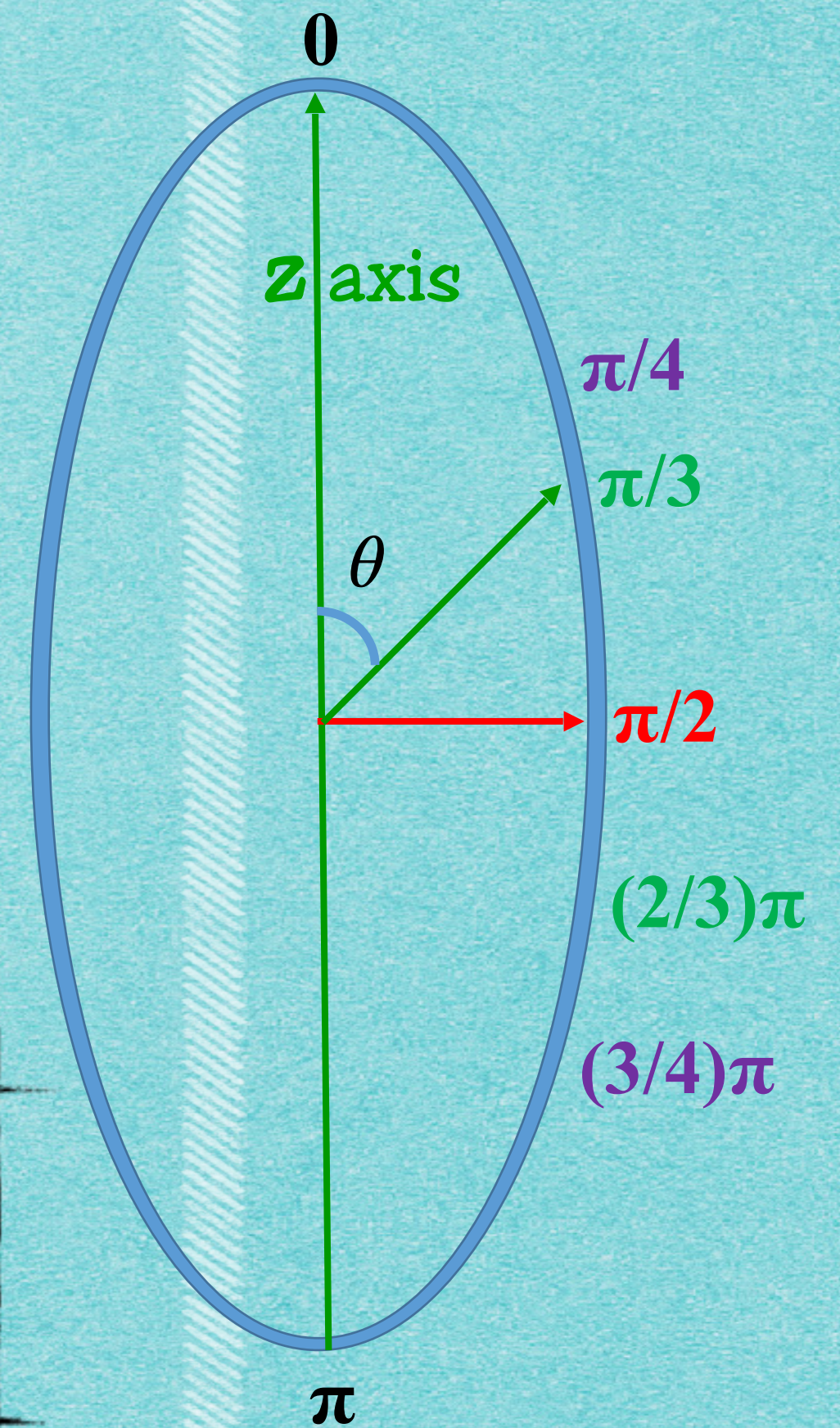
(anti)symmetric wrt $\pi/2$...
also to simplify algebra

$\nu = \nu_{obs}$
 $\nu' = \nu_{CMB}^{rest, frame}$

$$T_{th}^{BB/dist} = a_{0,0} \sqrt{\frac{1}{4\pi}} + a_{1,0} \sqrt{\frac{3}{4\pi}} \cos \theta + a_{2,0} \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) + a_{3,0} \sqrt{\frac{7}{4\pi}} \left(\frac{5}{2} \cos^3 \theta - \frac{3}{2} \cos \theta \right) + a_{4,0} \sqrt{\frac{9}{4\pi}} \left(\frac{35}{8} \cos^4 \theta - \frac{15}{4} \cos^2 \theta + \frac{3}{8} \right) + a_{5,0} \sqrt{\frac{11}{4\pi}} \left(\frac{63}{8} \cos^5 \theta - \frac{35}{4} \cos^3 \theta + \frac{15}{8} \cos \theta \right) + a_{6,0} \sqrt{\frac{13}{4\pi}} \left(\frac{231}{16} \cos^6 \theta - \frac{315}{16} \cos^4 \theta + \frac{105}{16} \cos^2 \theta - \frac{5}{16} \right).$$

& expanding $T_{th}^{BB/dist}$ in spherical harmonics

- * $Y_{0,0} = \sqrt{1/(4\pi)}$
- * for $l = 0$ & even l the Legendre $P_0^l(\cos \theta)$ are symmetric wrt $\theta = \pi/2$
- * for odd l they vanish at $\theta = \pi/2$ and are antisymmetric wrt $\theta = \pi/2$



$$\frac{1 - \beta}{(1 - \beta^2)^{1/2}} \leq \frac{\nu'}{\nu} \leq \frac{1 + \beta}{(1 - \beta^2)^{1/2}};$$

Symmetry of directions implies separation in 2 systems

- > for 0 and even multipoles
- > for odd multipoles

Among the possible choices satisfying these symmetry properties
select a set of colatitudes θ_i such that the values of $\cos(\theta_i)$ are
rational numbers or just involve $\sqrt{2}$ to simplify the algebra

➔ $\theta_i = 0, \pi/4, \pi/3, \pi/2, (2/3)\pi, (3/4)\pi, \pi$

Linear system to
be solved using
the methods of
elimination and
substitution

$$\begin{aligned}
 T_{\text{th}}^{\text{BB/dist}}(\theta = 0) &= \sqrt{\frac{1}{4\pi}}a_{0,0} + \sqrt{\frac{3}{4\pi}}a_{1,0} \\
 &+ \sqrt{\frac{5}{4\pi}}a_{2,0} + \sqrt{\frac{7}{4\pi}}a_{3,0} + \sqrt{\frac{9}{4\pi}}a_{4,0} \\
 &+ \sqrt{\frac{11}{4\pi}}a_{5,0} + \sqrt{\frac{13}{4\pi}}a_{6,0}, \\
 T_{\text{th}}^{\text{BB/dist}}(\theta = \pi/4) &= \sqrt{\frac{1}{4\pi}}a_{0,0} + \frac{\sqrt{2}}{2}\sqrt{\frac{3}{4\pi}}a_{1,0} \\
 &+ \frac{1}{4}\sqrt{\frac{5}{4\pi}}a_{2,0} - \frac{\sqrt{2}}{8}\sqrt{\frac{7}{4\pi}}a_{3,0} - \frac{13}{32}\sqrt{\frac{9}{4\pi}}a_{4,0} \\
 &- \frac{17\sqrt{2}}{64}\sqrt{\frac{11}{4\pi}}a_{5,0} - \frac{19}{128}\sqrt{\frac{13}{4\pi}}a_{6,0}, \\
 T_{\text{th}}^{\text{BB/dist}}(\theta = \pi/3) &= \sqrt{\frac{1}{4\pi}}a_{0,0} + \frac{1}{2}\sqrt{\frac{3}{4\pi}}a_{1,0} \\
 &- \frac{1}{8}\sqrt{\frac{5}{4\pi}}a_{2,0} - \frac{7}{16}\sqrt{\frac{7}{4\pi}}a_{3,0} - \frac{37}{128}\sqrt{\frac{9}{4\pi}}a_{4,0} \\
 &+ \frac{23}{256}\sqrt{\frac{11}{4\pi}}a_{5,0} + \frac{331}{1024}\sqrt{\frac{13}{4\pi}}a_{6,0}, \\
 T_{\text{th}}^{\text{BB/dist}}(\theta = \pi/2) &= \sqrt{\frac{1}{4\pi}}a_{0,0} + 0 \cdot a_{1,0} \\
 &- \frac{1}{2}\sqrt{\frac{5}{4\pi}}a_{2,0} + 0 \cdot a_{3,0} + \frac{3}{8}\sqrt{\frac{9}{4\pi}}a_{4,0}, \\
 &+ 0 \cdot a_{5,0} - \frac{5}{16}\sqrt{\frac{13}{4\pi}}a_{6,0}
 \end{aligned}$$

$$\begin{aligned}
 T_{\text{th}}^{\text{BB/dist}}(\theta = (2/3)\pi) &= \sqrt{\frac{1}{4\pi}}a_{0,0} - \frac{1}{2}\sqrt{\frac{3}{4\pi}}a_{1,0} \\
 &- \frac{1}{8}\sqrt{\frac{5}{4\pi}}a_{2,0} + \frac{7}{16}\sqrt{\frac{7}{4\pi}}a_{3,0} - \frac{37}{128}\sqrt{\frac{9}{4\pi}}a_{4,0} \\
 &- \frac{23}{256}\sqrt{\frac{11}{4\pi}}a_{5,0} + \frac{331}{1024}\sqrt{\frac{13}{4\pi}}a_{6,0}, \\
 T_{\text{th}}^{\text{BB/dist}}(\theta = (3/4)\pi) &= \sqrt{\frac{1}{4\pi}}a_{0,0} - \frac{\sqrt{2}}{2}\sqrt{\frac{3}{4\pi}}a_{1,0} \\
 &+ \frac{1}{4}\sqrt{\frac{5}{4\pi}}a_{2,0} + \frac{\sqrt{2}}{8}\sqrt{\frac{7}{4\pi}}a_{3,0} - \frac{13}{32}\sqrt{\frac{9}{4\pi}}a_{4,0} \\
 &+ \frac{17\sqrt{2}}{64}\sqrt{\frac{11}{4\pi}}a_{5,0} - \frac{19}{128}\sqrt{\frac{13}{4\pi}}a_{6,0}, \\
 T_{\text{th}}^{\text{BB/dist}}(\theta = \pi) &= \sqrt{\frac{1}{4\pi}}a_{0,0} - \sqrt{\frac{3}{4\pi}}a_{1,0} \\
 &+ \sqrt{\frac{5}{4\pi}}a_{2,0} - \sqrt{\frac{7}{4\pi}}a_{3,0} + \sqrt{\frac{9}{4\pi}}a_{4,0} \\
 &- \sqrt{\frac{11}{4\pi}}a_{5,0} + \sqrt{\frac{13}{4\pi}}a_{6,0}.
 \end{aligned}$$

T. Trombetti+ 2021

$$w_i = \cos \theta_i = 1, \sqrt{2}/2, 1/2, 0, -1/2, -\sqrt{2}/2, -1$$

For $\ell = 0$ and even multipoles

↳ sums of signals

$$a_{\ell,0} = A_\ell \sqrt{\frac{4\pi}{2\ell+1}} \left[d_{\ell,1} \left(T_{\text{th}}^{\text{BB/dist}}(w=1) + T_{\text{th}}^{\text{BB/dist}}(w=-1) \right) + d_{\ell,2} \left(T_{\text{th}}^{\text{BB/dist}}(w=\sqrt{2}/2) + T_{\text{th}}^{\text{BB/dist}}(w=-\sqrt{2}/2) \right) + d_{\ell,3} \left(T_{\text{th}}^{\text{BB/dist}}(w=1/2) + T_{\text{th}}^{\text{BB/dist}}(w=-1/2) \right) + d_{\ell,4} T_{\text{th}}^{\text{BB/dist}}(w=0) \right]$$

T. Trombetti+ 21
C. Burigana+ 24

neglects contributions
only from $\ell = 8, 10...$

Fully Explicit
Solution
(FES)

Simple & fast solution accurate up to $\ell = 6$

ℓ	A_ℓ	$d_{\ell,1}$	$d_{\ell,2}$	$d_{\ell,3}$	$d_{\ell,4}$
0	1/630	29	120	64	204
1	1/210	29	$60\sqrt{2}$	32	-
2	1/693	121	396	-352	-330
3	2/135	13	$15\sqrt{2}$	-56	-
4	8/385	9	-10	-16	34
5	32/189	1	$-3\sqrt{2}$	4	-
6	64/693	1	-6	8	-6

An expansion up to ℓ_{max} leads to neglect
the contributions from $\ell > \ell_{max}$

For odd multipoles → differences of signals

$$a_{\ell,0} = A_\ell \sqrt{\frac{4\pi}{2\ell+1}} \left[d_{\ell,1} \left(T_{\text{th}}^{\text{BB/dist}}(w=1) - T_{\text{th}}^{\text{BB/dist}}(w=-1) \right) + d_{\ell,2} \left(T_{\text{th}}^{\text{BB/dist}}(w=\sqrt{2}/2) - T_{\text{th}}^{\text{BB/dist}}(w=-\sqrt{2}/2) \right) + d_{\ell,3} \left(T_{\text{th}}^{\text{BB/dist}}(w=1/2) - T_{\text{th}}^{\text{BB/dist}}(w=-1/2) \right) \right]$$

neglects contributions only from $\ell = 7, 9...$

Assuming that contributions

scale as $\beta^{\ell \cdot p}$ with
 $p \sim 1$ & $\beta \simeq 10^{-3}$

then neglected

contributions are negligible
for current & future applications

Dipole sensitivity on patches

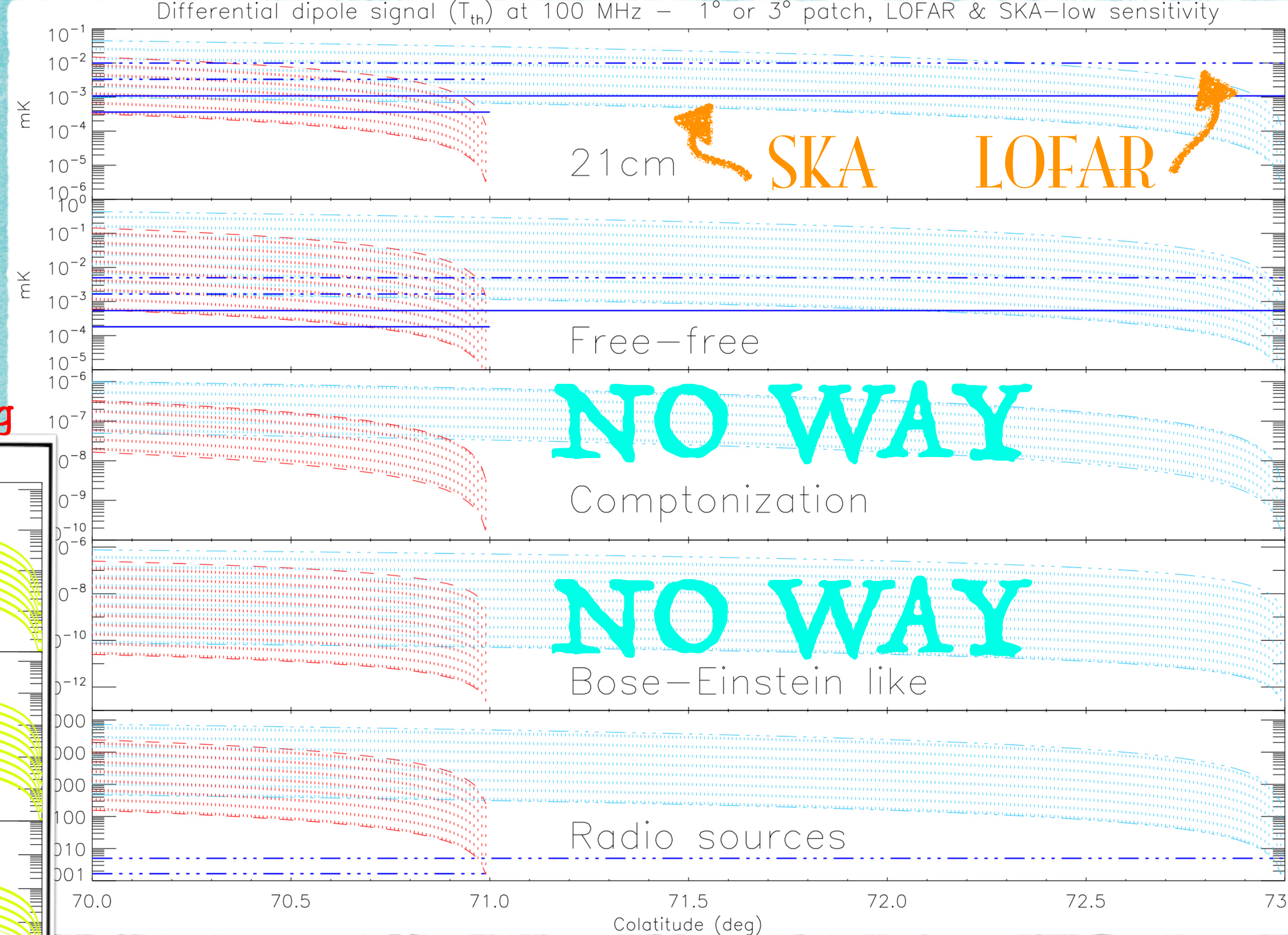
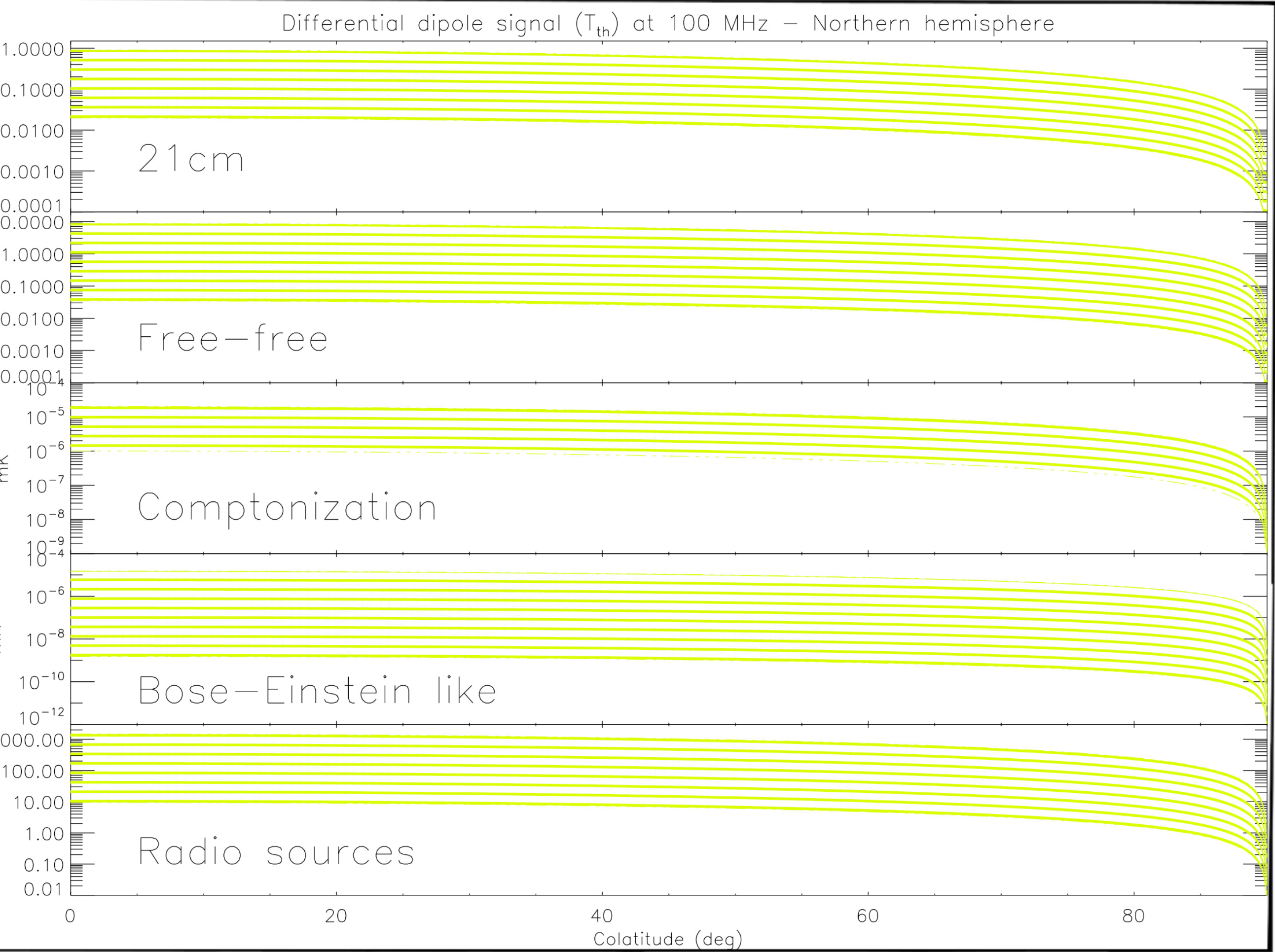
Sensitivity @ 100 kHz band

Rescaled in 10 MHz (21cm) & 40 MHz bandwidth

24h of integration on 1° or 3° patch

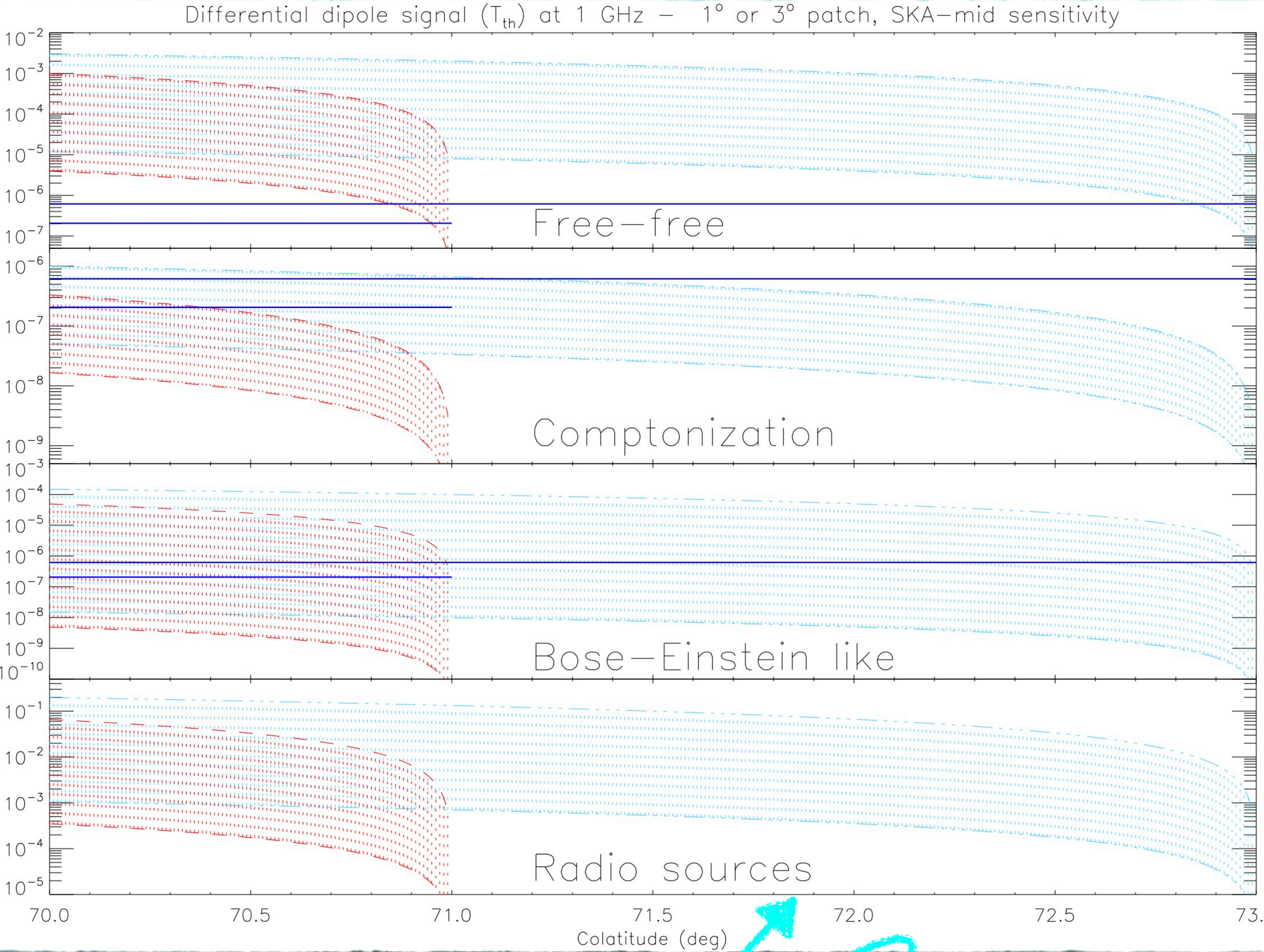
Rescaled on a 2 arcmin pixel size

Sensitivity levels: Dewdney+ 16 SKA1 System Baseline Design, SKA Org



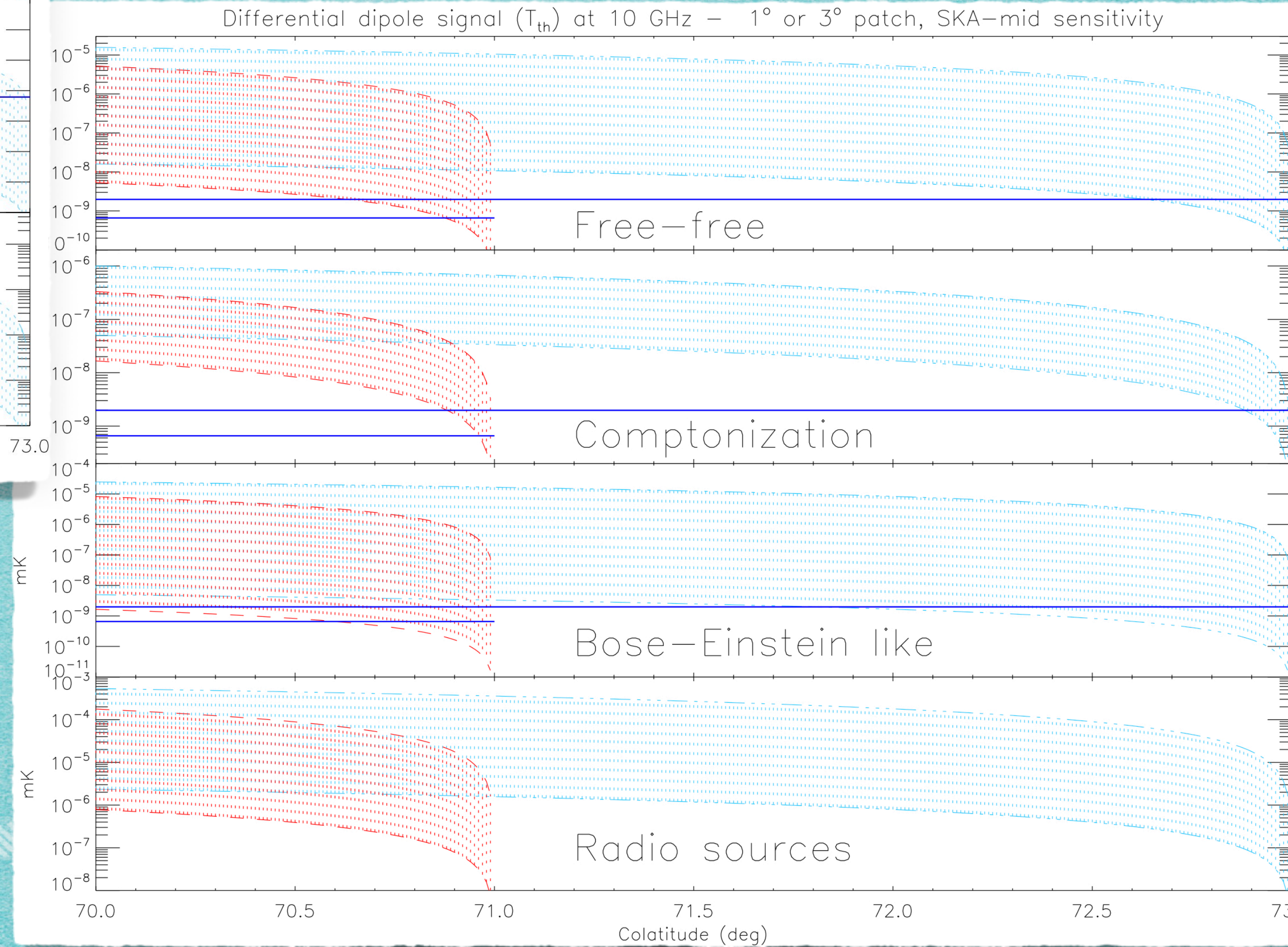
Frequency Range SKA-low (MHz): 50 - 350
 LOFAR (MHz): 30 - 220

Simple estimate: along a meridian with z-axis // to dipole direction signal variation @ colat θ from a dipole with amp ΔT in sky patch of linear size $\Delta\theta$ has amp $|\Delta T_{\Delta\theta}| \simeq \Delta T \cdot (\Delta\theta/90^\circ) \sin\theta$, $\sin\theta \simeq 1$ @ angles large from poles



Frequency Range SKA-mid (GHz): 0.35 - 14

Sensitivity @ 100 kHz band
Rescaled to 400 MHz (@ 1GHz)
& 4 GHz (@ 10GHz) bandwidth



several tens of mK could
identify EG background
from few μK to $\sim\text{mK}$ could
study the reionization imprints

ALWAYS

CONCLUSIONS

- ★ CB monopole ν spectrum \rightarrow key understand many physical, cosmological & astrophysical processes @ different cosmic epochs
- ★ Absolute measurements of the spectrum is standard & most accurate way
- ★ BUT precise absolute calibration is needed
- ★ Differential methods on precise inter- ν calibration are promising: by using low ℓ pattern (mainly dipole) could significantly improve current limits w/o precise absolute calibration measurements
- ★ Observations @ extreme sensitivity/resolution, differential method can be extended to sky patches \rightarrow interferometric observations \rightarrow SKA
- ★ SKA ν coverage jointly offered by low- & mid- is particularly suitable for 21cm redshifted line, radio background, FF signals & (depending on models) of the FF2Compton dominance transition
- ★ SKA resolution and sensitivity allow to detect point sources and to probe their number counts to very faint flux densities \rightarrow enable substantial subtractions of their contribution to radio background

CONCLUSIONS II

Subtracting sources with different detection thresholds would help to clarify to what extent the radio EG background could be ascribed to EG sources or if is of intrinsic cosmological or diffuse origin

Contribute to answer about level & origin of the radio EG background, still controversial
Collecting many patches would improve the statistical information & Galactic foreground subtraction

Despite an intense theoretical and experimental effort over the past decade EG radio background obs @ multiple $\nu < 10$ GHz are not understood in terms of known radiosources and may represent a sign of new physics (Caputo+ 23)