



Swiss National  
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UNIVERSITÉ  
DE GENÈVE

# Orbital stability of circumbinary exoplanets orbiting double white dwarfs

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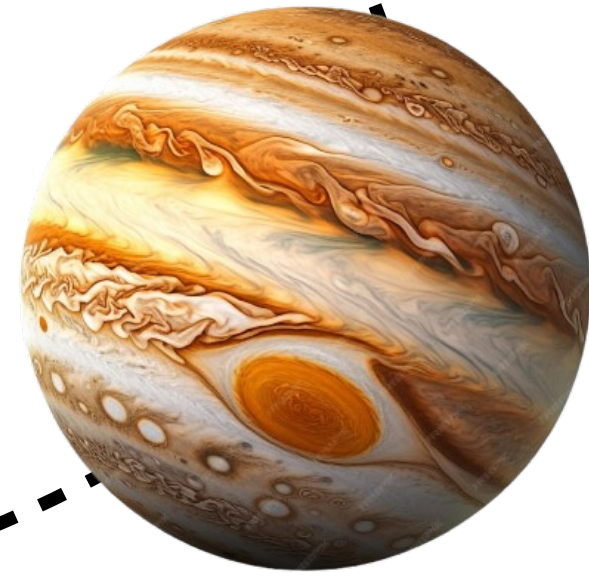
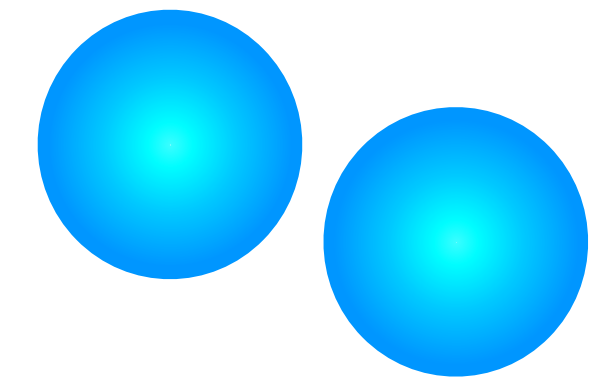
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Florence, CBP workshop  
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# Science case

**Double White Dwarf (DWD)**



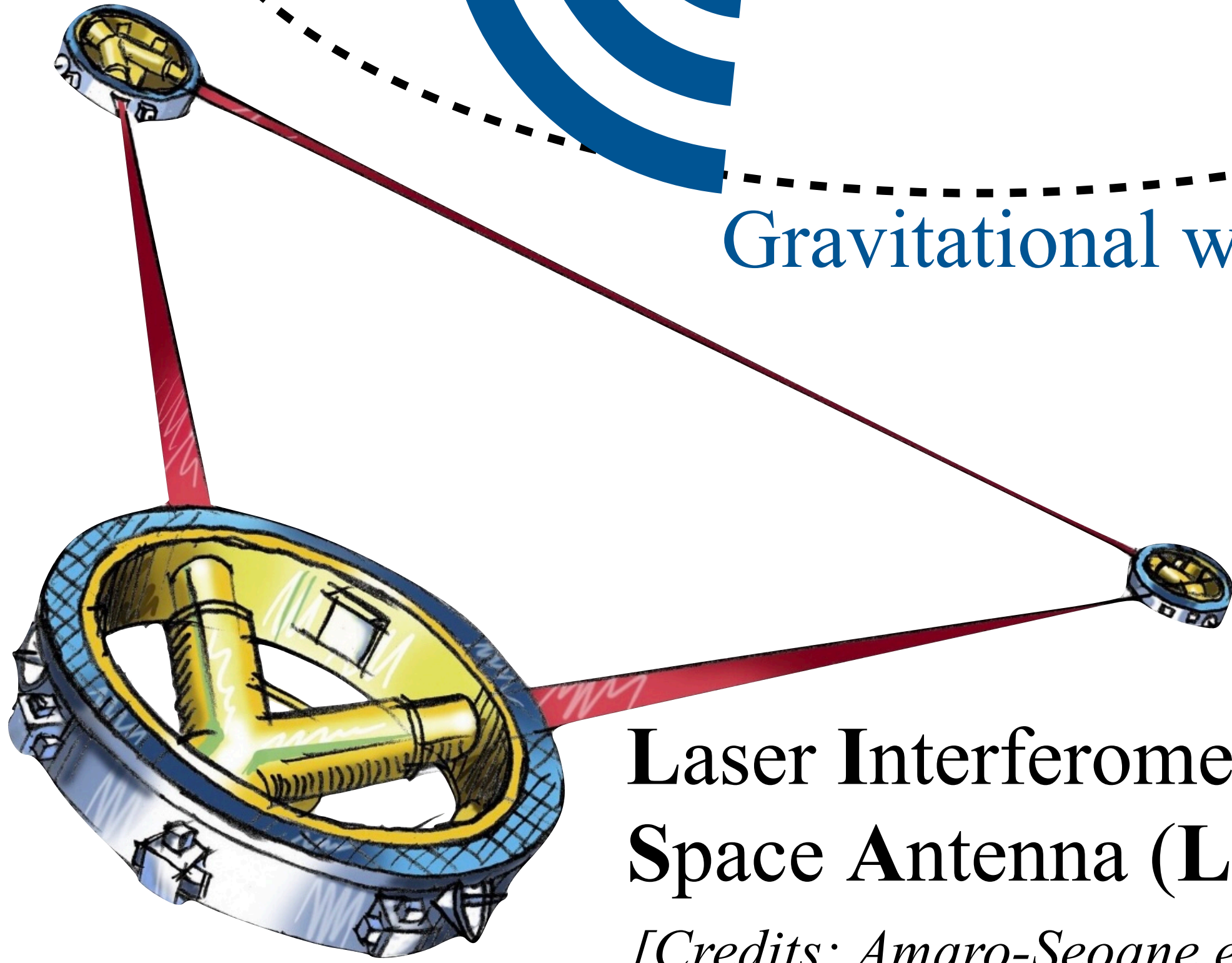
Can we form them ?  
**Yes !** [*Ledda et al. 2023*]



Gravitational wave signal

Can we detect them ?  
**Yes !** [*Tamanini & Danielski 2019*]

The gravitational wave signal is periodically modulated by the planet if  $0.2 M_J < M_p < 13 M_J$



**Laser Interferometer  
Space Antenna (LISA)**

[Credits: *Amaro-Seoane et al. 2017*]

# Science case

Double White Dwarf (DWD)

Can we form them ?  
**Yes !** [Ledda et al. 2023]

Can they survive during the evolutionary phase ?

[Nigioni A., Turrini D., Danielski C., Chambers J.E. under review]

[Danielski 2019]

The gravitational wave signal is periodically modulated by the planet if  $0.2 M_J < M_p < 13 M_J$

Laser Interferometer  
Space Antenna (LISA)

[Credits: Amaro-Seoane et al. 2017]

# N-body problems in P-type binaries

Short execution times

+

Small energy variation



Symplectic integrators

[Wisdom & Holman 1991]



Adaptation to the possibility of having close-encounters (Hybrid symplectic integrator)

[Chambers 1999]



Adaptation to presence of a secondary star

(P-type binary coordinates)

[Chambers et al. 2002]

Our systems' Hamiltonian in P-type binary coordinates

$$H = \begin{cases} \text{dominant term} & \longrightarrow H_{\text{Kep}} = H_{\text{P,Kep}} + H_{\text{B,Kep}} \\ + \\ \text{perturbation term} & \longrightarrow H_{\text{int}} = H_{\text{P,int}} + H_{\text{B,int}} \quad \& \quad H_{\text{jump}} \end{cases}$$

## Integration scheme

1. Advance  $H_{\text{P,int}}$  for  $\tau/2$ ;
2. Repeat the following  $N_{\text{bin}}$  times:
  - Advance  $H_{\text{B,int}}$  for  $\tau/(2N_{\text{bin}})$ ;
  - Advance  $H_{\text{B,Kep}}$  for  $\tau/(2N_{\text{bin}})$ ;
3. Advance  $H_{\text{jump}}$  for  $\tau/2$ ;
4. Advance  $H_{\text{P,Kep}}$  for  $\tau$ ;
5. Advance  $H_{\text{jump}}$  for  $\tau/2$ ;
6. Repeat the following  $N_{\text{bin}}$  times:
  - Advance  $H_{\text{B,int}}$  for  $\tau/(2N_{\text{bin}})$ ;
  - Advance  $H_{\text{B,Kep}}$  for  $\tau/(2N_{\text{bin}})$ ;
7. Advance  $H_{\text{P,int}}$  for  $\tau/2$ .

$$N_{\text{bin}} \simeq T_{\text{innermost planet}} / T_{\text{bin}}$$

# Initial conditions

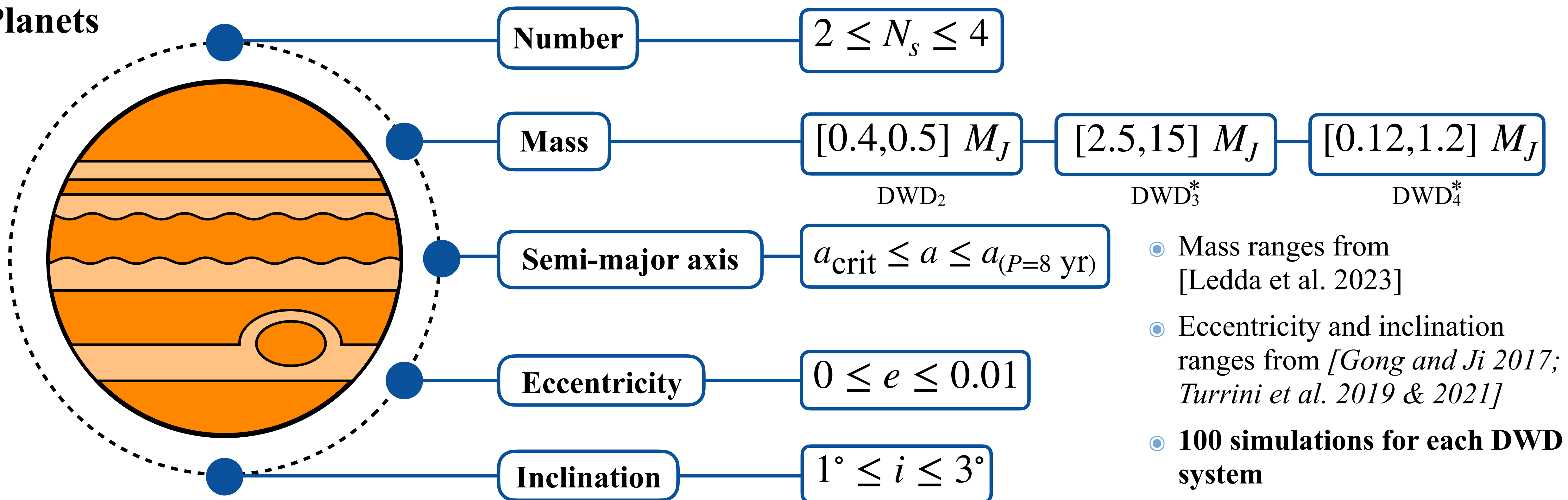
## Binary

system	$M_1 [M_\odot]$	$M_2 [M_\odot]$	$P_{\text{bin}} [\text{h}]$	$a_{\text{bin}} [\text{au}]$
DWD <sub>2</sub>	0.31	0.21	0.43	$1.078 \times 10^{-3}$
DWD <sub>3</sub> *	0.75	0.26	1.51	$3.106 \times 10^{-3}$
DWD <sub>4</sub> *	0.31	0.25	1.71	$2.773 \times 10^{-3}$

Eccentricity is zero [Ledda et al. 2023],  
inclination set to zero for simplicity,  
phase angles sampled randomly

The systems marked with the \* belong to the LISA DWD population presented in [Korol et al. 2019].

## Planets



# Metrics to analyse and compare planetary systems

## Normalized Angular Momentum Deficit

[Chambers 2001; Turrini et al. 2021]

$$\text{NAMD} = \frac{\text{AMD}}{\text{CAM}} = \frac{\sum_k m_k \sqrt{a_k} \left(1 - \sqrt{1 - e_k^2} \cos i_k\right)}{\sum_k m_k \sqrt{a_k}}$$

Threshold value:  $1.3 \times 10^{-3}$

Distinguish between dynamically COLD and HOT planetary system

## Orbital Spacing Statistics

$$S_s = \frac{6}{N-1} \left( \frac{a_{\max} - a_{\min}}{a_{\max} + a_{\min}} \right) \left( \frac{3M_{\text{bin}}}{2\bar{m}} \right)^{1/4} \quad \text{where } N > 1 \rightarrow S_s^f / S_s^s$$

## Fraction of total mass retained in the largest object

$$S_m = \frac{M_{\text{most massive}}}{\sum_i^N m_i} \rightarrow S_m^f / S_m^s$$

## Planetary centre of mass

$$\text{COM} = \frac{\sum_i^N m_i a_i}{\sum_i^N m_i} \rightarrow \Delta_{\text{COM}}$$

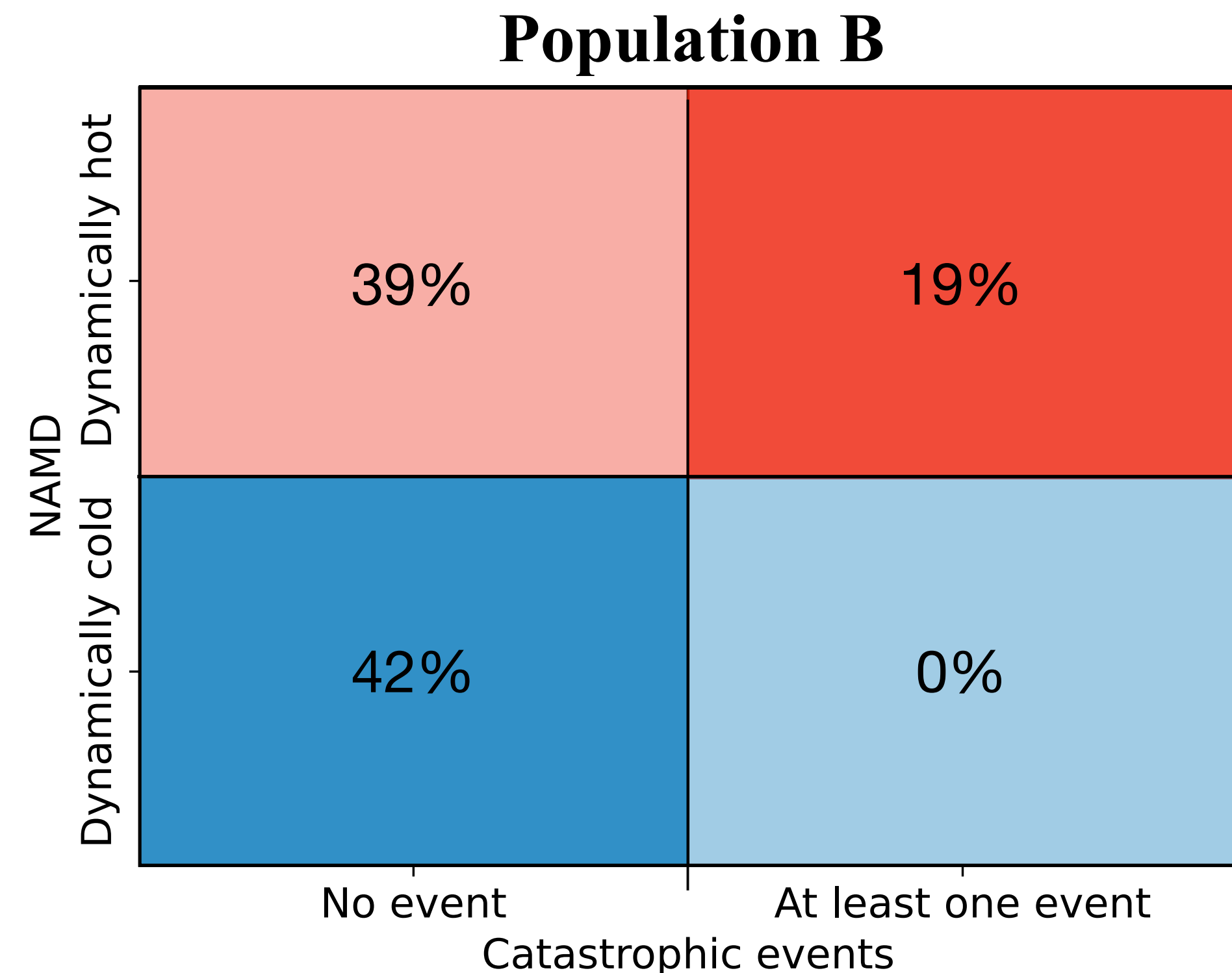
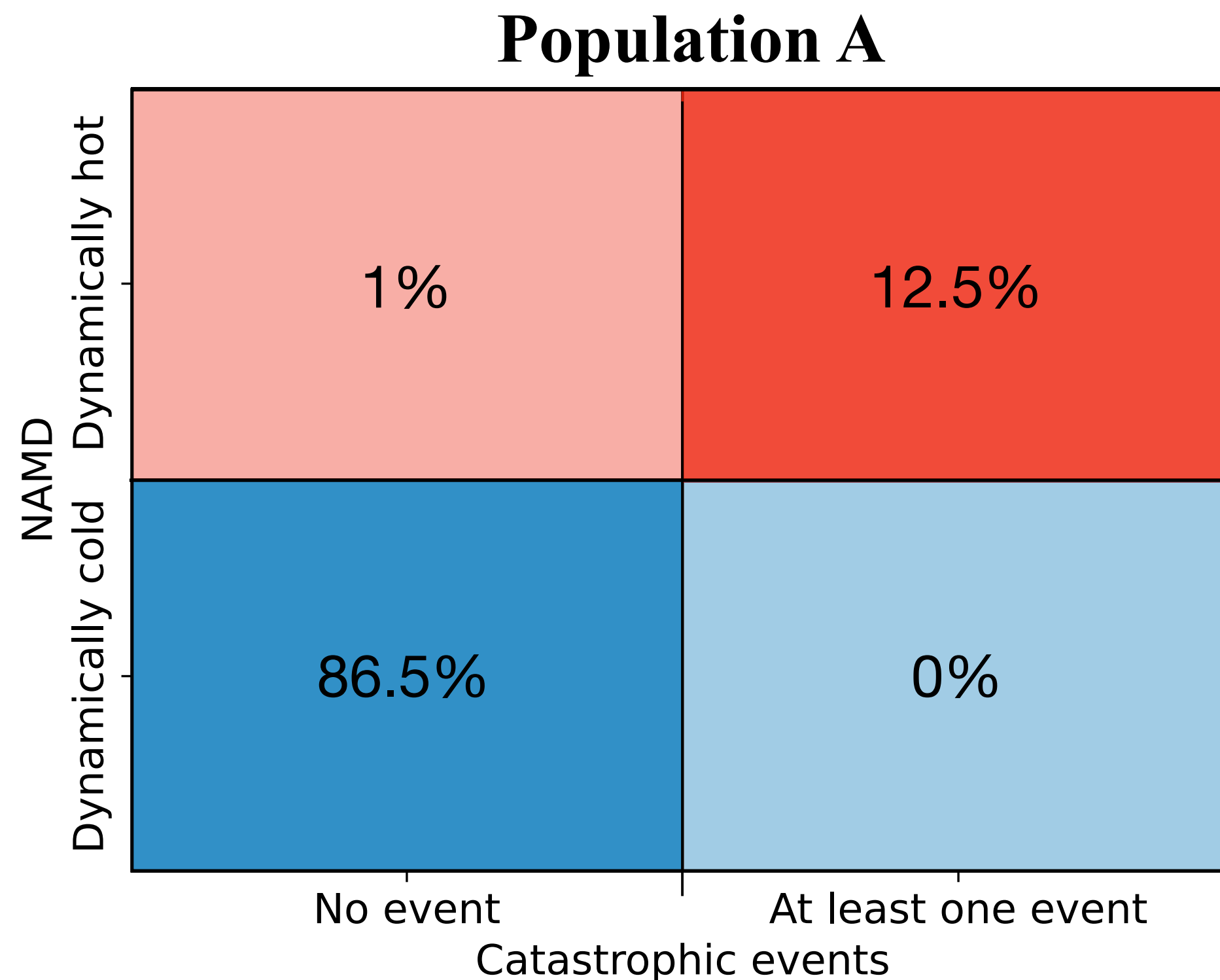
# Results (1/5)

We identify two populations:

- Population A:  $M_p^i \leq 1.2 M_J$  and orbit a binary with mass  $M_{\text{bin}} < 0.6 M_{\odot}$ . Corresponds to DWD<sub>2</sub> and DWD<sub>4</sub>\*
- Population B:  $M_p^i \geq 2.5 M_J$  and orbit a binary with mass  $M_{\text{bin}} \sim 1 M_{\odot}$ . Corresponds to DWD<sub>3</sub>\*

We define catastrophic events as:

- Single catastrophic events: ejections, planet-planet collisions, close approach to the binary
- Multiple catastrophic events: combination of the above



# Results (2/5)

Does the dynamical evolution depend on the initial multiplicity  $N_s$ ?

Population A			
	$N_s = 2$	$N_s = 3$	$N_s = 4$
NAMD $< 1.3 \times 10^{-3}$	100%	94%	58%
NAMD $> 1.3 \times 10^{-3}$	0%	6%	42%
Disrupted systems	0%	0%	0%

Population B			
	$N_s = 2$	$N_s = 3$	$N_s = 4$
NAMD $< 1.3 \times 10^{-3}$	79%	33%	13%
NAMD $> 1.3 \times 10^{-3}$	21%	61%	65%
Disrupted systems	0%	6%	22%

Population A			
	$N_s = 2$	$N_s = 3$	$N_s = 4$
No catastrophic events	100%	97%	59%
Ejections only	0%	2%	26%
Collisions <sub>p-p</sub> only	0%	1%	7.5%
Multiple catastrophic events	0%	0%	7.5%

Population B			
	$N_s = 2$	$N_s = 3$	$N_s = 4$
No catastrophic events	100%	81%	57%
Ejections only	0%	2%	13%
Collisions <sub>p-p</sub> only	0%	4%	4%
Multiple catastrophic events	0%	13%	26%

$N_s$  is the initial number of planets

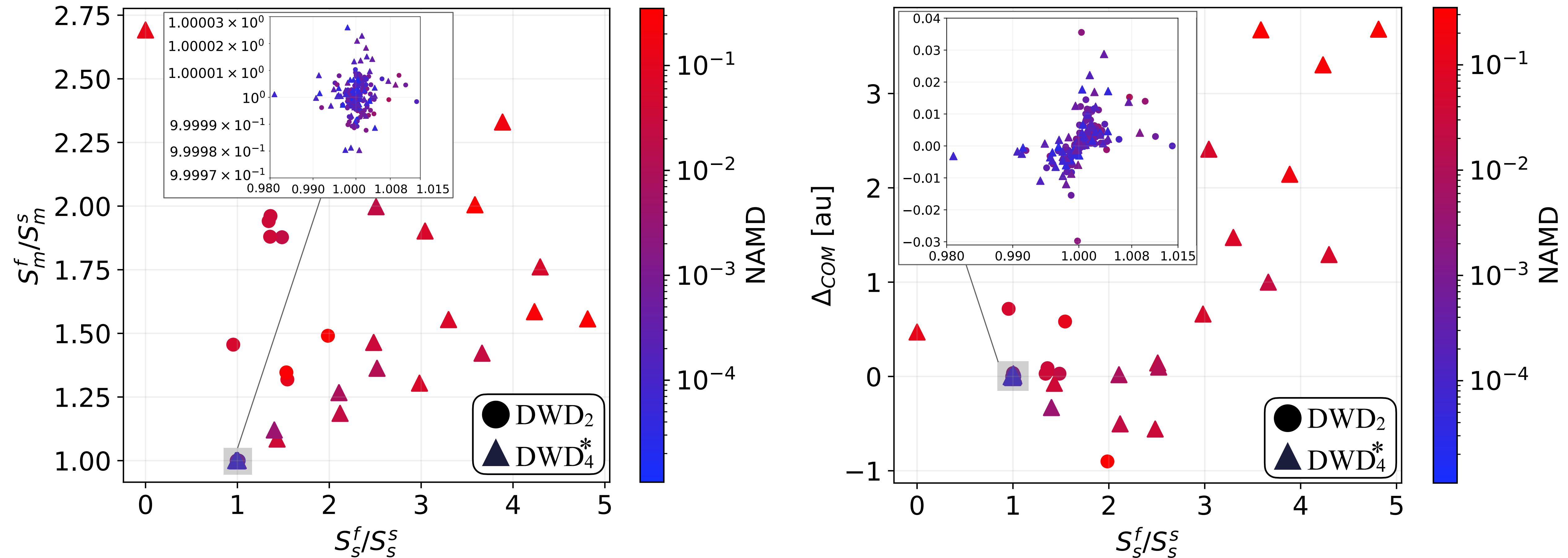
	$N_s = 2$	$N_s = 3$	$N_s = 4$
DWD <sub>2</sub>	32%	48%	20%
DWD <sub>3</sub> *	29%	48%	23%
DWD <sub>4</sub> *	31%	36%	33%

- Population A corresponds to DWD<sub>2</sub> and DWD<sub>4</sub>\*
- Population B corresponds to DWD<sub>3</sub>\*



# Results (3/5)

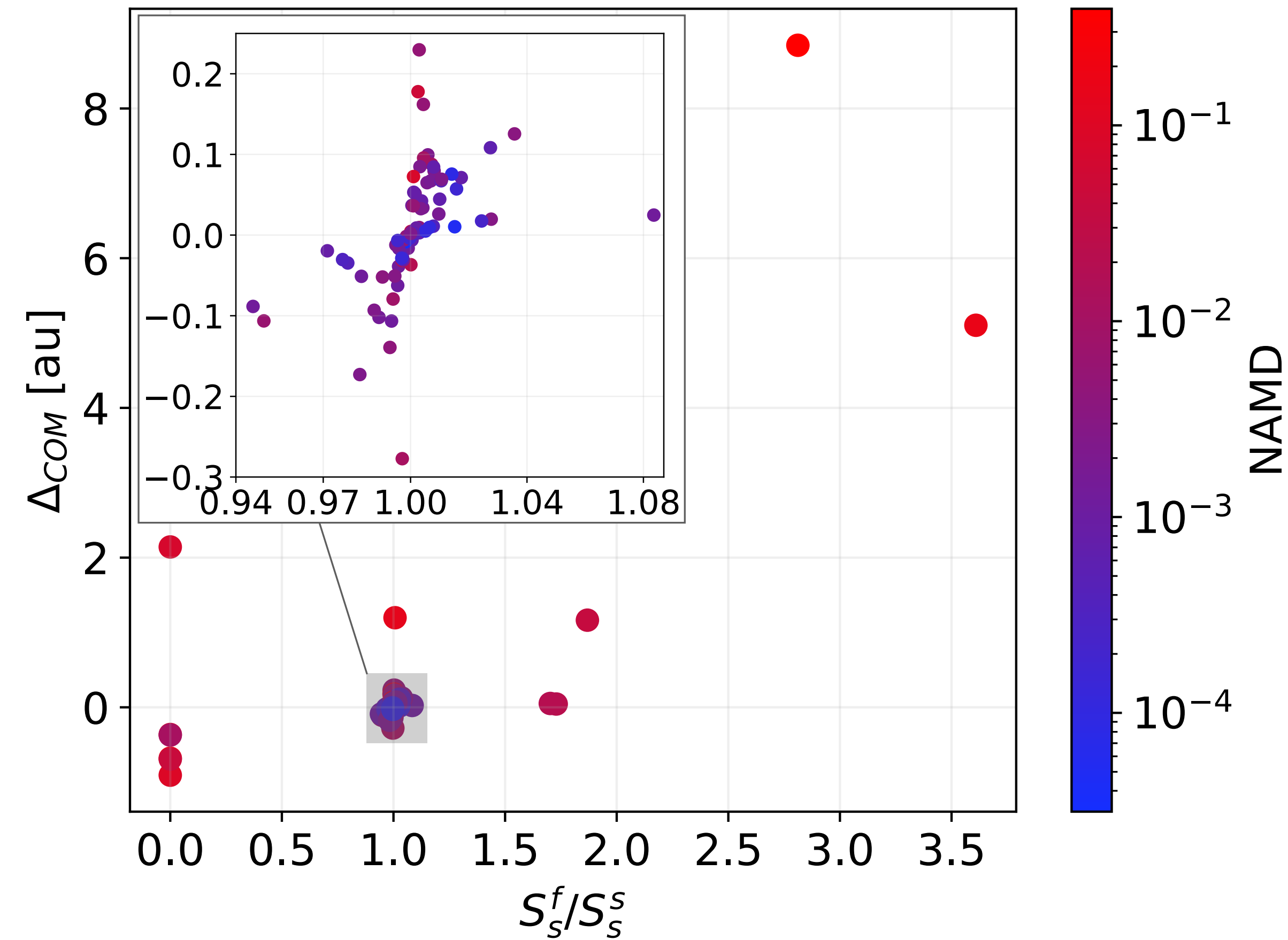
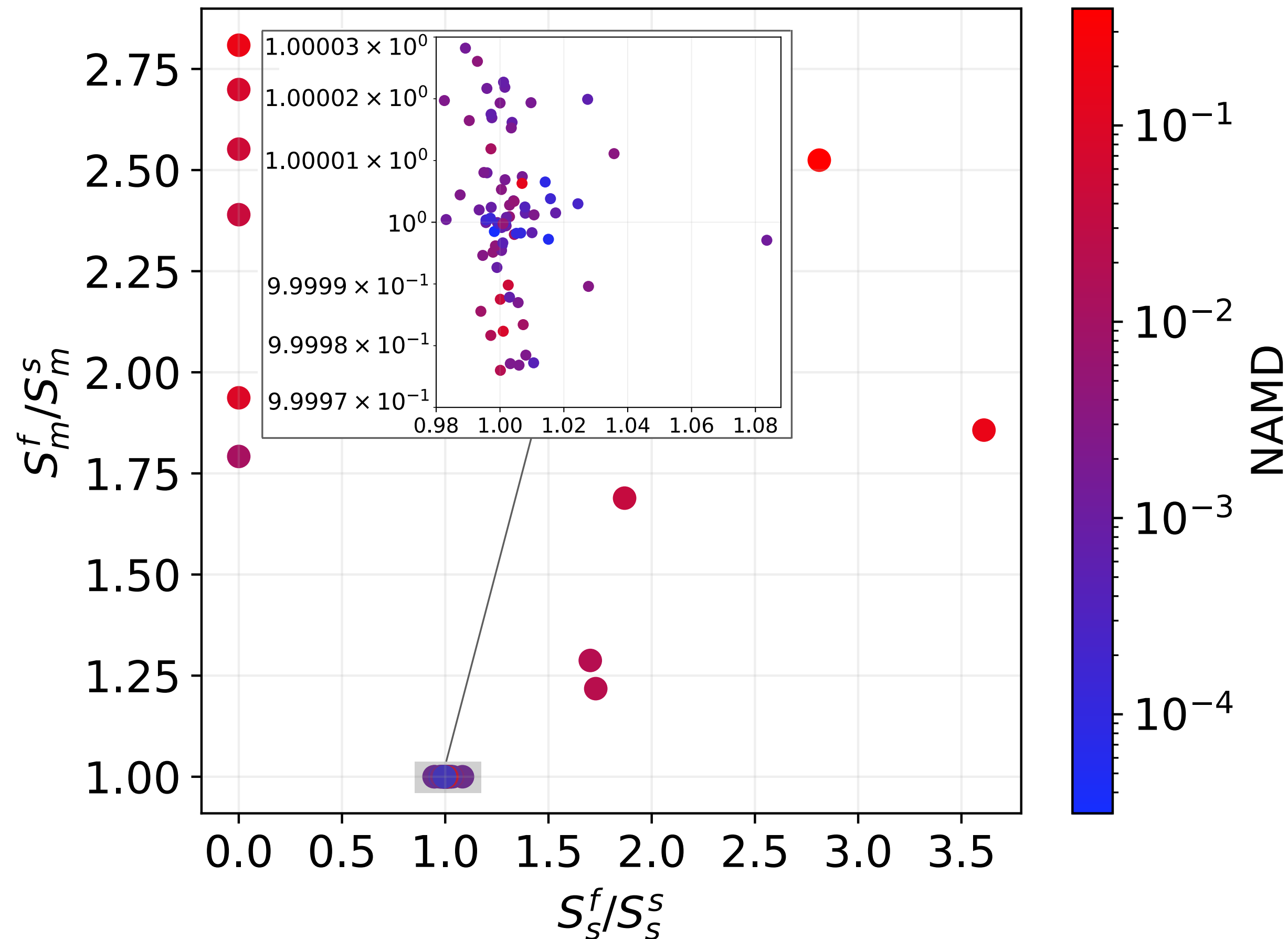
## Population A



- ⊙ Dynamically cold systems: no loss of planets,  $S_s^f/S_s^s \sim 1$ ,  $S_m^f/S_m^s \sim 1$ ,  $|\Delta_{\text{COM}}| < 0.04$
- ⊙ Dynamically hot systems: loss of planets,  $S_s^f/S_s^s > 1$ ,  $S_m^f/S_m^s > 1$ ,  $|\Delta_{\text{COM}}| > 0.04$

# Results (4/5)

## Population B



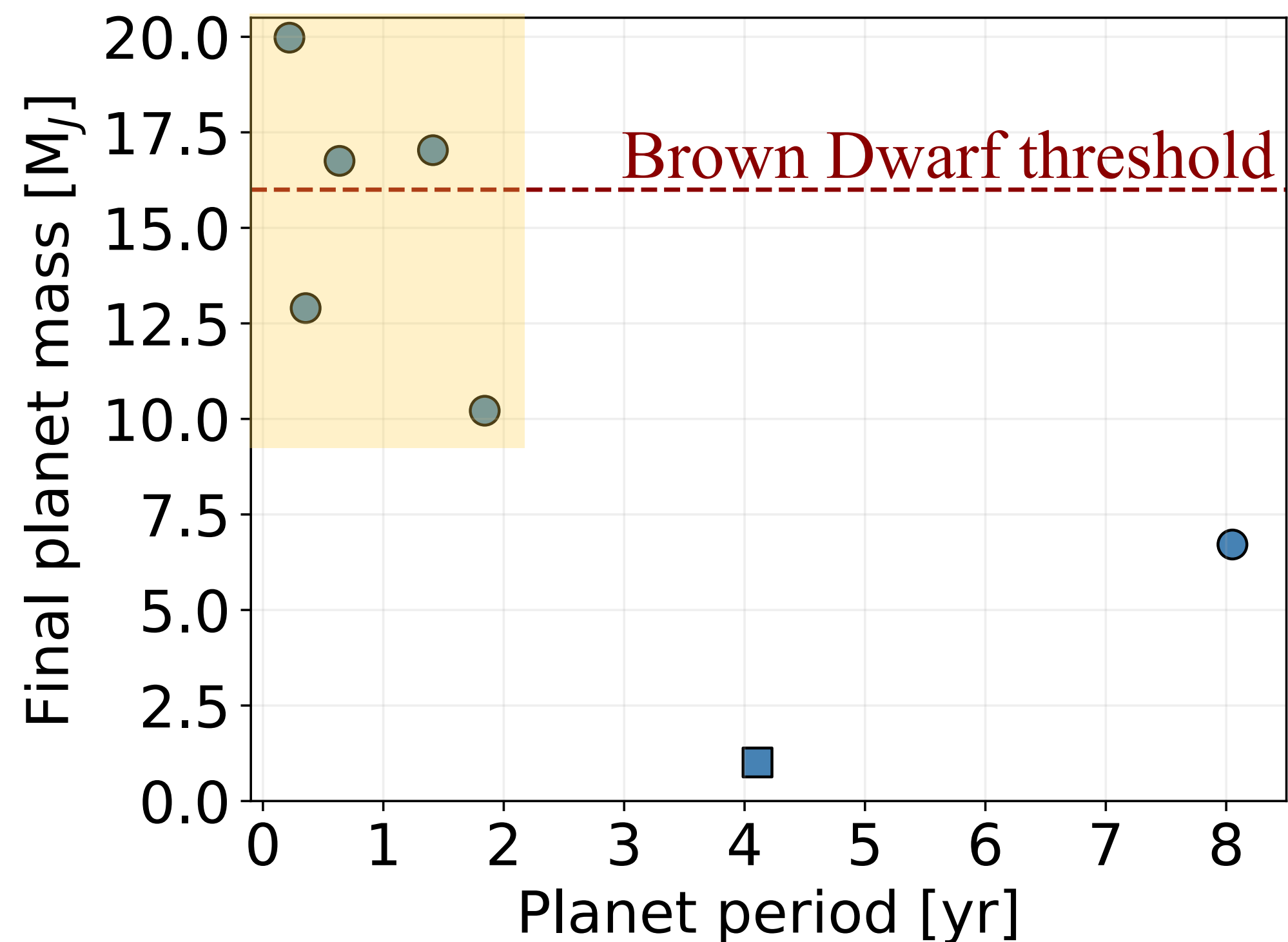
Same trends as population A but:

- More systems end up with one surviving planet (i.e.,  $S_s^f/S_s^s = 0$ )

- Larger  $|\Delta_{\text{COM}}|$
- Some dynamically hot systems keep their initial orbital configuration

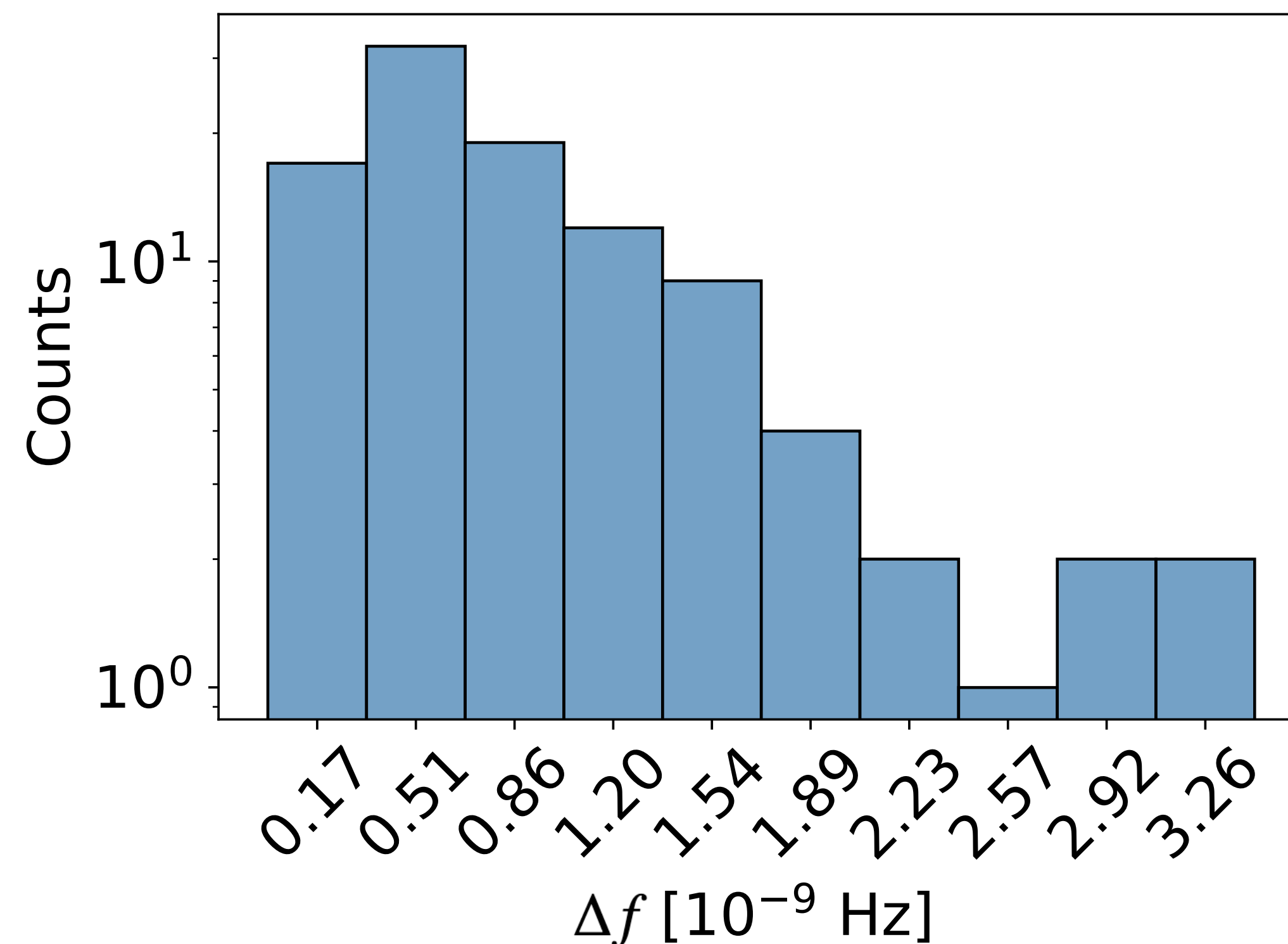
## Detectability with LISA ?

### One-planet population



Following the results by *Katz et al. (2022)* some of our single planet systems have the potential to be detected, pending though the distance of the systems, their sky-location, polarisation and inclination, which determine the GW signal-to-noise [*Robson et al. 2018*].

### DWD<sub>3</sub>\* multi-planet population gravitational wave frequency shift



LISA frequency resolution:

$\sim 7.92 \times 10^{-9}$  Hz for a 4 years mission

$\sim 3.96 \times 10^{-9}$  Hz for a 8 years mission

# Summary

- Multi-giant planet systems can survive around Double White Dwarf systems  
→ 97% of all our simulated systems have at least one surviving planet
- More massive binaries hosting more massive planets are more likely to go through unstable phases compared to the less massive counterpart
- Dynamically hot systems tend to loose planets and evolve towards less compact architectures and can experience large shifts of the planetary centre of mass (up to  $\sim 8$  au)
- Dynamically cold systems preserve their initial architecture
- Systems with initial higher multiplicity are more likely to undergo unstable phases and experience planet loss  
→ Creation of a one-planet population (3%) and most of these planets have the potential of being detected by LISA
- Our multi-planet systems are unlikely to be detectable by LISA