Gaia and the Astrometric Sphere Reconstruction

The problem of the Global Astrometric Sphere Reconstruction in Gaia

Alberto Vecchiato[1]

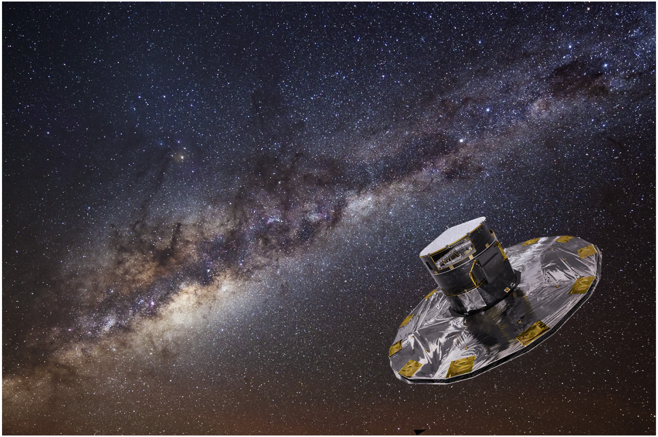
[1]INAF - Astrophysical Observatory of Torino

Rome - INAF, December 2, 2015

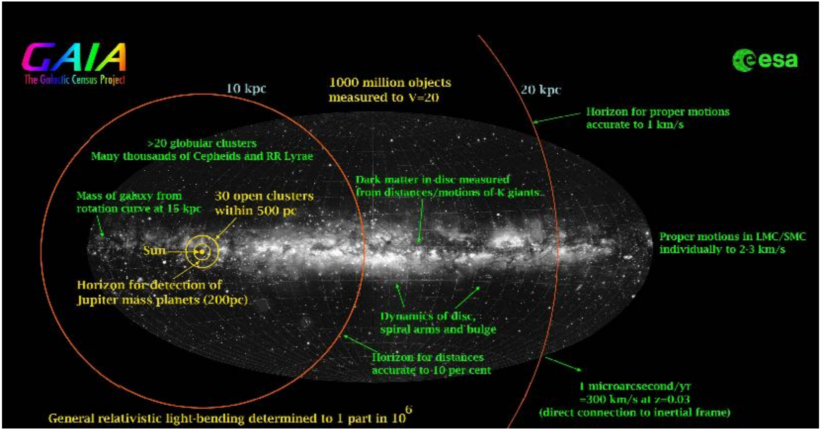
Outline

Outline

An ESA astrometric satellite

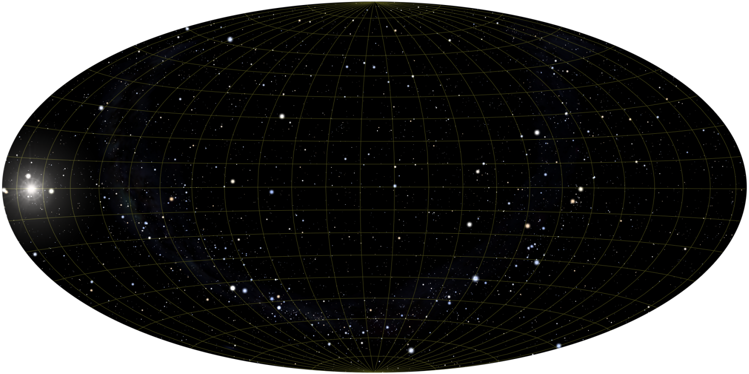


High-precision Milky Way mapping for science and astronomy



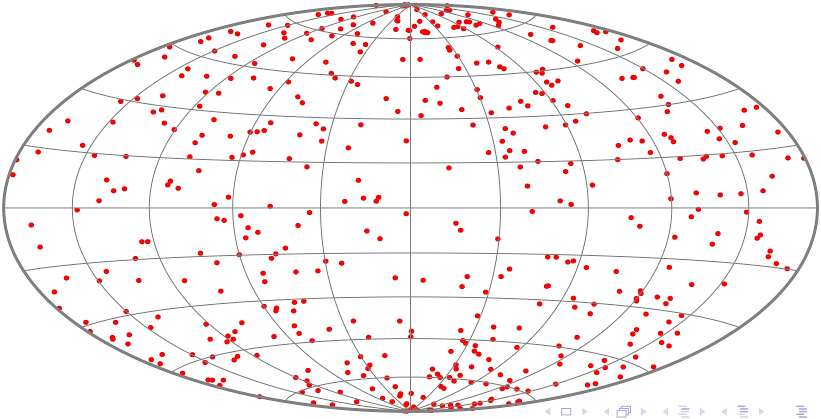
The reconstruction of the Global Astrometric Sphere

The reconstruction of the Global Astrometric Sphere



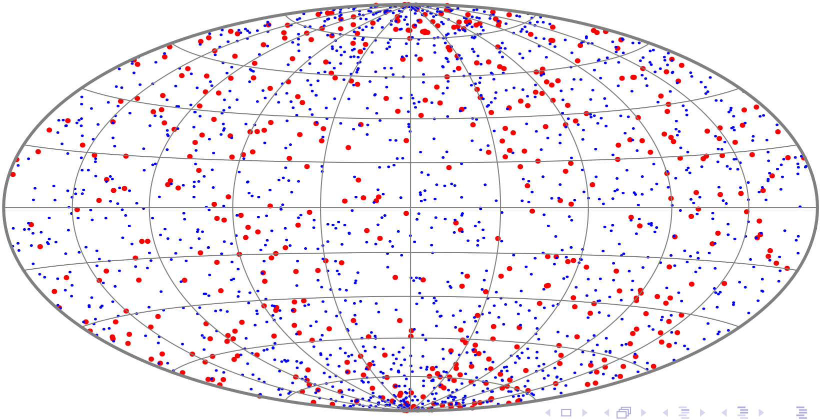
Primaries/Non-primaries

The Global Astrometric Sphere is first reconstructed with respect to a subset ($\sim 10^8$ out of $\sim 10^9$) of well-behaved stars called primaries.



Primaries/Non-primaries

The reference frame materialized by the primaries is used by other pipeline processes to include the other stars into the Gaia sphere.

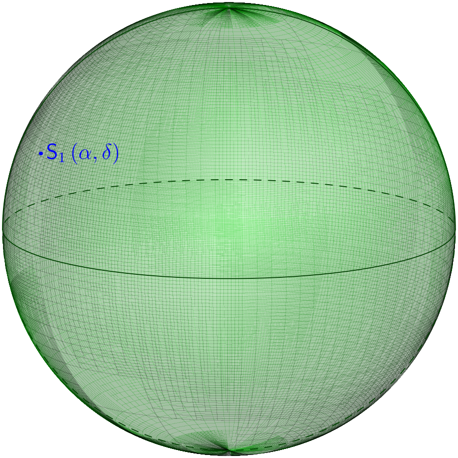


Outline

1. Gaia and GSR... What?
2. ... How?
3. ... When?
4. ... Where?
5. ... Why?
6. ... Who?

Principles of the sphere reconstruction

The ideal picture



Create a “geodetic” network of measurements

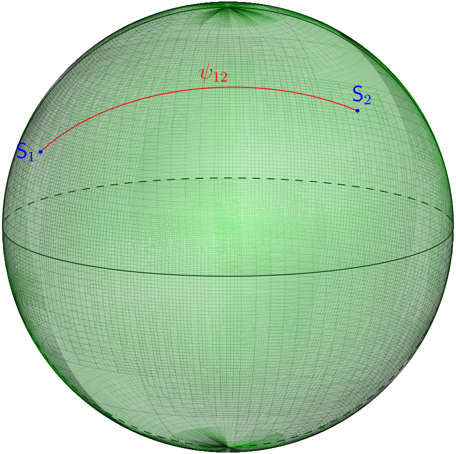
$$N_* = 1$$

$$N_{\text{unk}} = 2$$

$$N_{\text{arcs}} = 0$$

Principles of the sphere reconstruction

The ideal picture



Create a “geodetic” network of measurements

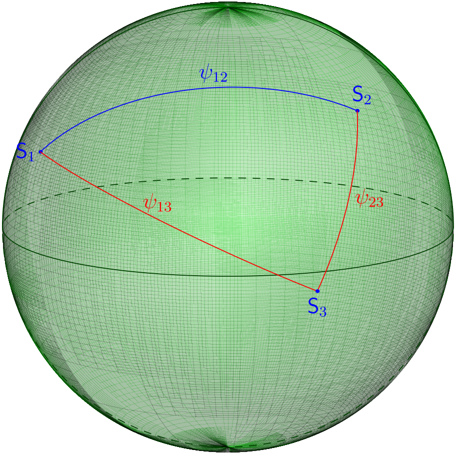
$$N_* = 2$$

$$N_{\text{unk}} = 4$$

$$N_{\text{arcs}} = 1$$

Principles of the sphere reconstruction

The ideal picture



Create a "geodetic" network of measurements

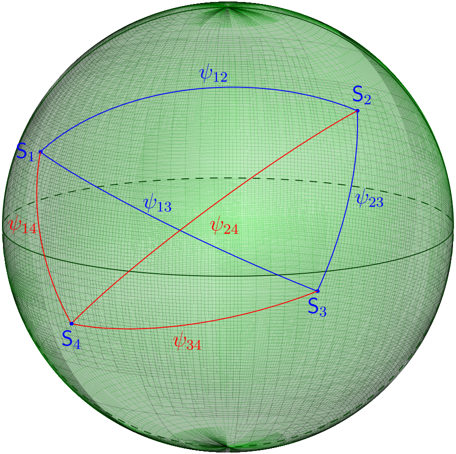
$$N_* = 3$$

$$N_{\rm unk} = 6$$

$$N_{\rm arcs} = 3$$

Principles of the sphere reconstruction

The ideal picture



Create a "geodetic" network of measurements

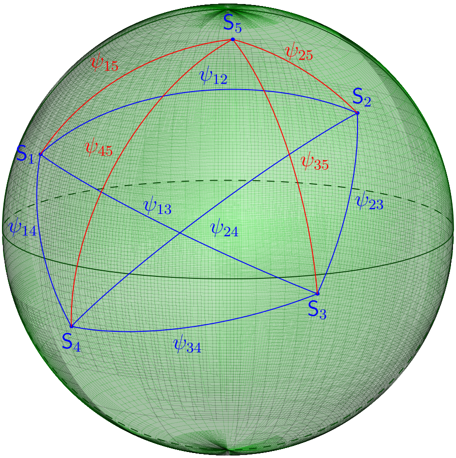
$$N_* = 4$$

$$N_{\rm unk} = 8$$

$$N_{\rm arcs} = 6$$

Principles of the sphere reconstruction

The ideal picture



Create a "geodetic" network of measurements

$$N_* = 5$$

$$N_{\rm unk} = 10$$

$$N_{\rm arcs} = 10$$

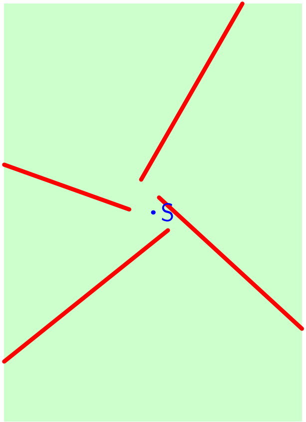
Network closed!
Solve an Equation System

Principles of the sphere reconstruction

The (almost) real picture

Observational errors ⇒

1. solution in the least-squares sense;
2. overdetermined system of equations.

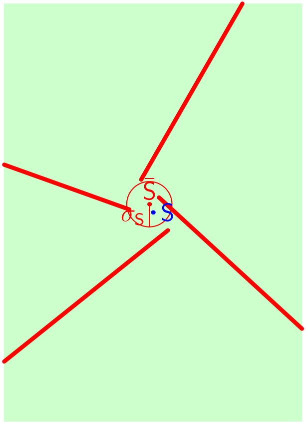


Principles of the sphere reconstruction

The (almost) real picture

Observational errors ⇒

1. solution in the least-squares sense;
2. overdetermined system of equations.



Principles of the sphere reconstruction

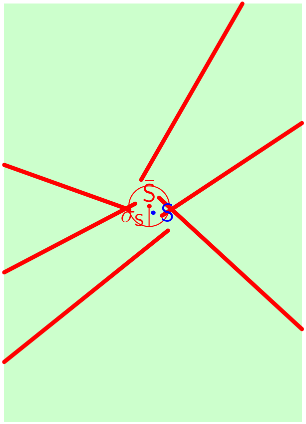
The (almost) real picture

Observational errors ⇒

1. solution in the least-squares sense;
2. overdetermined system of equations.

$$N_{\text{unk}} \sim N_* \simeq 10^8$$

$$N_{\text{obs}} \sim 10^2 N_{\text{unk}} \sim 10^{10}$$



Mathematical modeling: the Euclidean arc

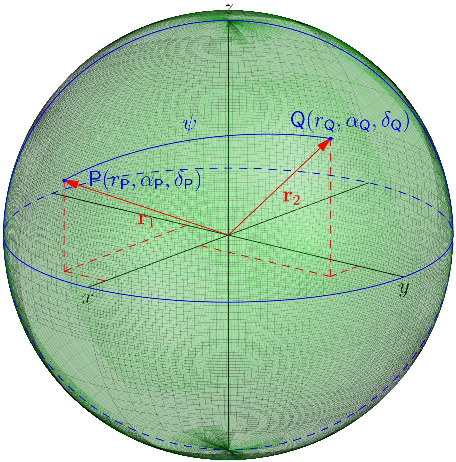
- The basic astrometric observable is an angle between two stars' directions

$$\cos\psi = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1|\,|\mathbf{r}_2|} \qquad (1)$$

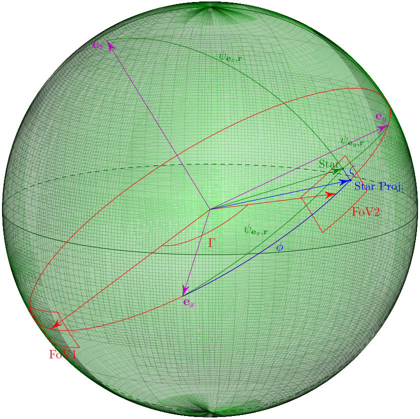
- It depends on the astrometric coordinates of the two stars:

$$\begin{aligned} \boldsymbol{r} &= \boldsymbol{r}\,(\alpha, \delta, \varpi) \\ &= \boldsymbol{r}\,(\alpha_0, \delta_0, \varpi, \mu_\alpha, \mu_\delta) \end{aligned}$$

- Stellar aberration enters in the definition of the satellite reference system



Mathematical modeling: the Euclidean abscissa



- The Gaia basic observable is the abscissa ϕ between the x axis and one viewing direction

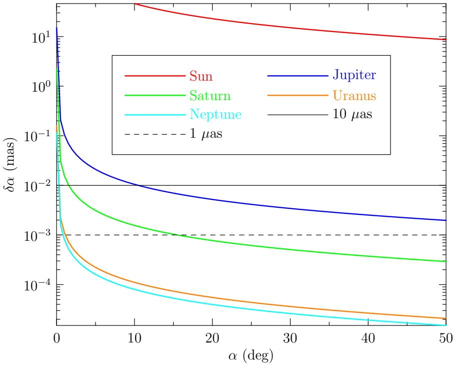
$$\cos\psi_{(\hat{a},\mathbf{r})} = \frac{\mathbf{e}_{\hat{a}}\cdot\mathbf{r}}{|\mathbf{r}|} \qquad (2)$$

$$\cos\phi = \frac{\cos\psi_{(\hat{x},\mathbf{r})}}{\sqrt{1-\cos^2\psi_{(\hat{z},\mathbf{r})}}} \qquad (3)$$

- Depends on the coordinates of one star (S) and on the satellite attitude (A) at the time of the observation
- The aberration enters in the same way as for the arcs

Mathematical modeling: enters General Relativity

Body	$\delta\alpha_M\,(\mu\mathrm{as})$	$\delta\alpha_Q\,(\mu\mathrm{as})$
Sun	1.75×10^6	~ 1
Mercury	83	
Venus	493	
Earth	574	0.6
Moon	26	
Mars	116	0.2
Jupiter	16270	240
Saturn	5780	95
Uranus	2080	8
Neptune	2533	10



The Linearized system of equations (I)

- The basic equations are highly non-linear

$$\cos\phi = \frac{\cos\psi_{(\hat{x},\mathbf{r})}}{\sqrt{1-\cos^2\psi_{(\hat{z},\mathbf{r})}}} = F\left(\boldsymbol{x}^{\mathrm{S}}, \boldsymbol{x}^{\mathrm{A}}, \boldsymbol{x}^{\mathrm{C}}, \boldsymbol{x}^{\mathrm{G}}\right)$$

- The Equation system is quite large ($\sim 10^{10} \times 10^{8}$)
- Solving a large system of non-linear equations is extremely complicated because of
  - the mathematical techniques involved
  - the computational power needed

The Linearized system of equations (II)

- A first-order Taylor expansion around a convenient set $\mathbf{x}_0$ of starting values (catalog) of the unknown parameters $\mathbf{x} \equiv \left\{\mathbf{x}^{\mathrm{S}}, \mathbf{x}^{\mathrm{A}}, \mathbf{x}^{\mathrm{C}}, \mathbf{x}^{\mathrm{G}}\right\}$ linearizes the observation equations and the Equation system

$$-\sin\phi_{\mathrm{calc}}\,\delta\phi = \sum_{\mathrm{Source}} \frac{\partial F(\mathbf{x})}{\partial \mathbf{x}^{\mathrm{S}}}\bigg|_{\mathbf{x}_0} \delta\mathbf{x}^{\mathrm{S}} + \sum_{\mathrm{Attitude}} \frac{\partial F(\mathbf{x})}{\partial \mathbf{x}^{\mathrm{A}}}\bigg|_{\mathbf{x}_0} \delta\mathbf{x}^{\mathrm{A}} + \sum_{\mathrm{Cal}} \frac{\partial F(\mathbf{x})}{\partial \mathbf{x}^{\mathrm{C}}}\bigg|_{\mathbf{x}_0} \delta\mathbf{x}^{\mathrm{C}} + \sum_{\mathrm{Global}} \frac{\partial F(\mathbf{x})}{\partial \mathbf{x}^{\mathrm{G}}}\bigg|_{\mathbf{x}_0} \delta\mathbf{x}^{\mathrm{G}}$$

$$\delta\phi = \phi_{\mathrm{obs}} - \phi_{\mathrm{calc}}$$

$$\delta\mathbf{x} = \mathbf{x}_{\mathrm{true}} - \mathbf{x}_0$$

$$\phi_{\mathrm{calc}} = F(\mathbf{x}_0)$$

- The new unknowns are the corrections to the catalog values. Their estimation $\bar{\delta}\mathbf{x}$ gives

$$\mathbf{x}_{\mathrm{true}} \simeq \bar{\mathbf{x}} = \mathbf{x}_0 + \bar{\delta}\mathbf{x}$$

- The resulting $m \times n$ system of equations is:
    - sparse $\Rightarrow$ #of $(a_{ij} \neq 0) \ll m \times n$
    - overdetermined $\Rightarrow$ $n \ll m$

Solving the Equation System

The NON-feasibility of Direct methods

- Linear System of Equation: $\mathbf{b} = A\mathbf{x}$, sparse, overdetermined

$$\mathbf{x} = \left(A^T A\right)^{-1} A^T \mathbf{b}$$

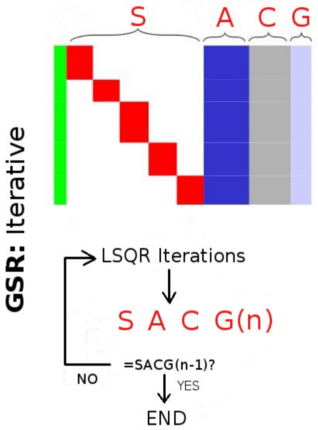
- Direct methods: needed operations$\propto N_{\mathrm{unk}}^3 \sim 2 \cdot 10^{26}$
- Most powerful supercomputer as of November 2015: Tianhe-2 (MilkyWay-2), National Super Computer Center in Guangzhou China, 33862.7 TFlop/s
- Time: ∼200 years!

Solving the Equation System
The GSR approach

- Linear System of Equation:
  $\mathbf{b} = A\mathbf{x}$, sparse, overdetermined

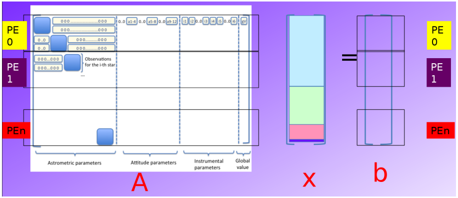
$$\mathbf{x} = \left(A^T A\right)^{-1} A^T \mathbf{b}$$

- GSR approach: iterative
  - complete system solved with an iterative algorithm (LSQR)
  - if needed, the process is repeated using the previous solution as starting values

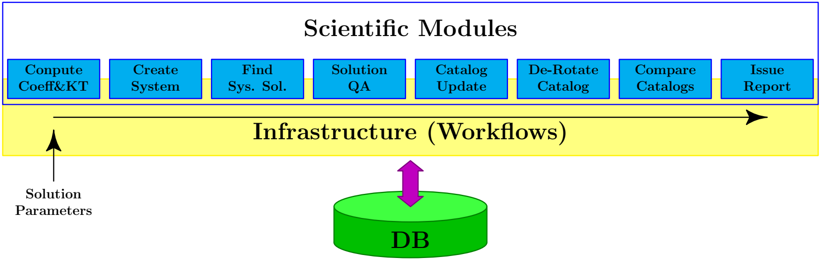


The need for HPC parallelization

- Contrary to the Block Iterative one, the Iterative approach needs “non-embarrassingly” parallel techniques
- This called for using:
  - C+MPI+OMP language for the Solver module
  - HPC-dedicated hardware



Structure of the GSR pipeline



Outline

GSR in the mission timeline

- GSR is a cyclic process (daily vs. cyclic processing)
- The milestones for the GSR processing are approximately the following (L means launch time):
  - December 2015 test processing first real data in validation mode
  - January 2015 starting of regular processing. First 12 months of operational data (approx. Cycles 00+01)
  - November 2015 first scientific Data Release
  - December 2015 starting processing of the approx. Cycles 00+01+02)
  - from #03 on cycles will last for 12 months. GSR processing will then proceed accordingly.

Outline

GSR @ CINECA

- The FERMI system will be the main facility to run the parallel AVU-GSR Solver module
- CINECA officially supports the Italian participation to the GAIA mission
- FERMI: IBM Blue Gene/Q FERMI, 10,240 Computing Nodes (CN) PowerA2, 1.6GHz, each with 16 cores. Totally: 163,840 computing cores
- Each CN has 16Gbyte of RAM (1 GB per core)
- GSR @ FERMI: up to 2,048 computing nodes will be used to compute the system solution

Outline

The importance of having an independent sphere reconstruction

- The Global Astrometric Sphere as a reference system determination (absolute measurements/parameters)
- Possible problems and pitfalls:
  - it is very difficult to pinpoint possible problems on absolute parameters
  - known correlations between different unknowns (e.g. ϖ vs. BAV, ϖ vs. γ)
  - estimation of variance-covariance matrix
- Perspectives:
  - *understanding* vs. *passive acceptance* of the sphere reconstruction results
  - alternative, more efficient methods to reduce the Astrometric Sphere can be conceived

The problem of the variance-covariance matrix

- An estimation of the errors on the system unknowns can be obtained by computing its variance-covariance matrix $S(\mathbf{x})$:

$$\begin{aligned} \mathbf{x} &= \left(A^T A\right)^{-1} A^T \mathbf{b} \\ A^{-g} &= \left(A^T A\right)^{-1} A^T \\ S(\mathbf{x}) &= A^{-g} \left(A^{-g}\right)^T \end{aligned}$$

- The evaluation of $S(\mathbf{x})$ has the same computational complexity of the finding of A^{-g}
- Iterative methods like the LSQR algorithm adopted by GSR can in principle solve the complete variance-covariance matrix
- Problem still relatively new in scientific literature

Scientific goals of the sphere reconstruction

- The challenge of *defining* and *solving* precise observations assembled in such a large system of equations is of huge scientific interest *per se*
  - calls for the determination of the best way to model the observations
  - helps to develop new perspectives on the reduction of global astrometric data
  - computationally intensive task (parallelization)
  - the problem of the variance-covariance matrix determination is still being investigated in the literature
- The determination of a full-sky "pseudo" inertial reference frame is a problem of fundamental physics
- An order of magnitude improvement of light deflection test for competing theories of Gravity in the PPN framework

Outline

1. Gaia and GSR... What?
2. ... How?
3. ... When?
4. ... Where?
5. ... Why?
6. ... Who?

The (enlarged) GSR team

- The development of GSR is an 8-year-long (up to now!) scientific effort that is involving the expertise of several people in many different research fields
  - **People:** Ummi Abbas, Ugo Becciani, Luca Bianchi, Beatrice Bucciarelli, Mariateresa Crosta, Mario G. Lattanzi, Roberto Morbidelli, Alberto Vecchiato (INAF) + Ruben De March, Rosario Messineo (ALTEC)
  - **Skills and expertise:** classical and relativistic astrometry, numerical algorithms, sparse systems of linear equations, catalog comparison, HPC parallelization, Java and C programming
  - **Scientific collaborations:** Stefano Bertone (ObsPM), Donato Bini (CNR & ICRA), Carlo Cavazzoni (CINECA), Fernando de Felice (UniPD, INAF)