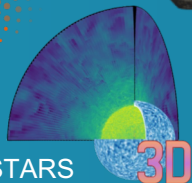


# Physics of Stellar Winds

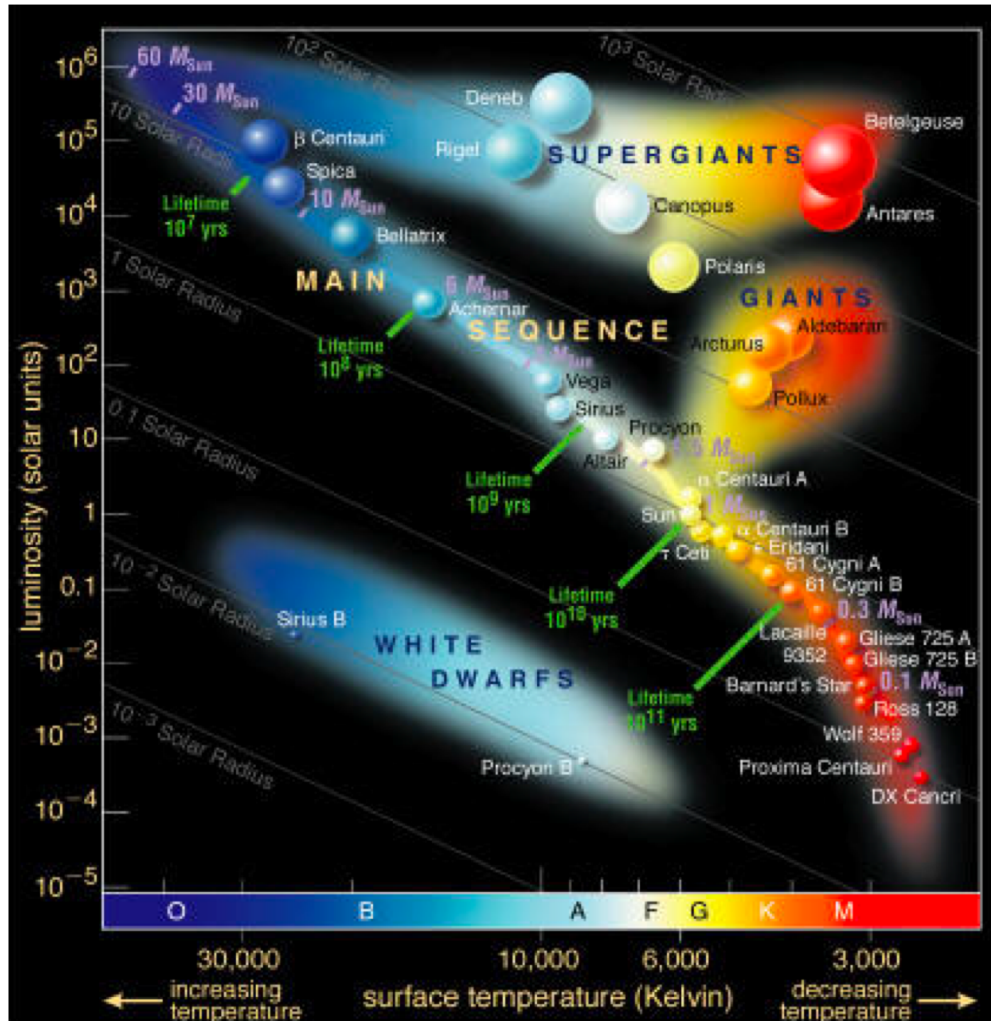
Jon Sundqvist

Tosca Workshop, Oct 30, 2024

erc



# Tour de force: what overcomes gravity?



Primary Agenda  
(upper HRD):

OB-stars

Too close to  
Eddington:  
WR, LBV-like

Cooler Side:  
RSGs / AGBs

But also small  
detour: Sun

# Governing Conservation Equations

$$\frac{\partial}{\partial t}(\text{Quantity}) + \nabla \cdot (\text{Flux}) = (\text{Source}) - (\text{Sink})$$

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0$$

$$\partial_t(\rho \vec{v}) + \nabla \cdot (\vec{v} \rho \vec{v} + p) = \vec{g}_{rad} \rho - \frac{GM}{r^2} \rho$$

$$\partial_t e + \nabla \cdot (e \vec{v} + p \vec{v}) = \vec{v} \cdot (\vec{g}_{rad} \rho + \vec{g} \rho) - \dot{q}$$

Conservation of mass, momentum, and energy for the gas – including dynamical effects of radiation (though neglecting e.g. magnetic fields)

# Parker's Stationary Isothermal Wind

- reference model for winds from Sun and similar stars



$$\frac{\partial v_r}{\partial r} = \frac{v_r}{r} \left( \frac{2c_i^2 - GM_*/r}{v_r^2 - c_i^2} \right)$$

**sonic point**  $v_r = c_i$  **must be at critical point**  $r_s = \frac{GM_*}{2c_i^2}$

⇒ dimensionless  $\bar{r} = r/r_0$ , Mach number  $M = \frac{v_r}{c_i}$

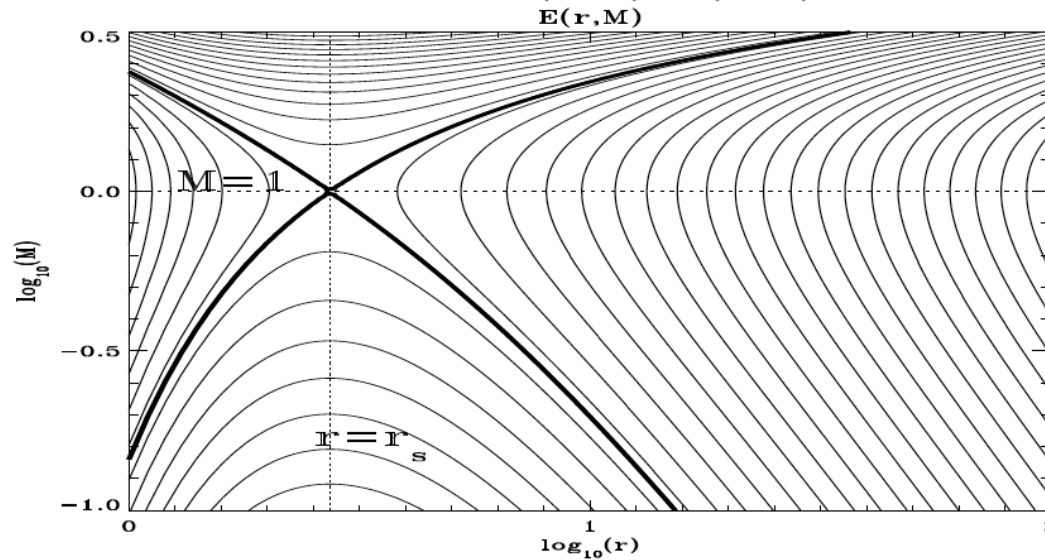
$$\frac{M^2}{2} + \ln \left( \frac{M_0}{\bar{r}^2 M} \right) - \frac{\bar{v}_{esc}^2}{2\bar{r}} = E/c_i^2 = \bar{E}$$

⇒ **'escape' speed**  $\bar{v}_{esc} = \sqrt{2GM_*/r_0}/c_i = 3.3015$  (Sun)

Gas sound speed high because of hot corona



plot contours of constant  $\bar{E}(\bar{r}, M)$  in  $(\bar{r}, M)$  plane



⇒ solutions correspond to contour lines

- 4 classes of ‘solutions’ (constant  $\bar{E}$  curves)
  - ⇒ double-valued (at given  $\bar{r}$ ) curves: unphysical
  - ⇒ purely supersonic  $M > 1$  solutions
  - ⇒ purely subsonic solutions: ‘stellar breeze’
  - ⇒ two transonic solutions: wind and accretion
- **Unique transonic wind solution**
  - ⇒ thermally driven transonic solar wind
  - ⇒ at sonic  $M = 1$  point  $r = r_s$

# Parker’s Stationary Isothermal Wind

Predicted 1958,  
later observed

ApJ 128, 1958

DYNAMICS OF THE INTERPLANETARY GAS  
AND MAGNETIC FIELDS\*

E. N. PARKER

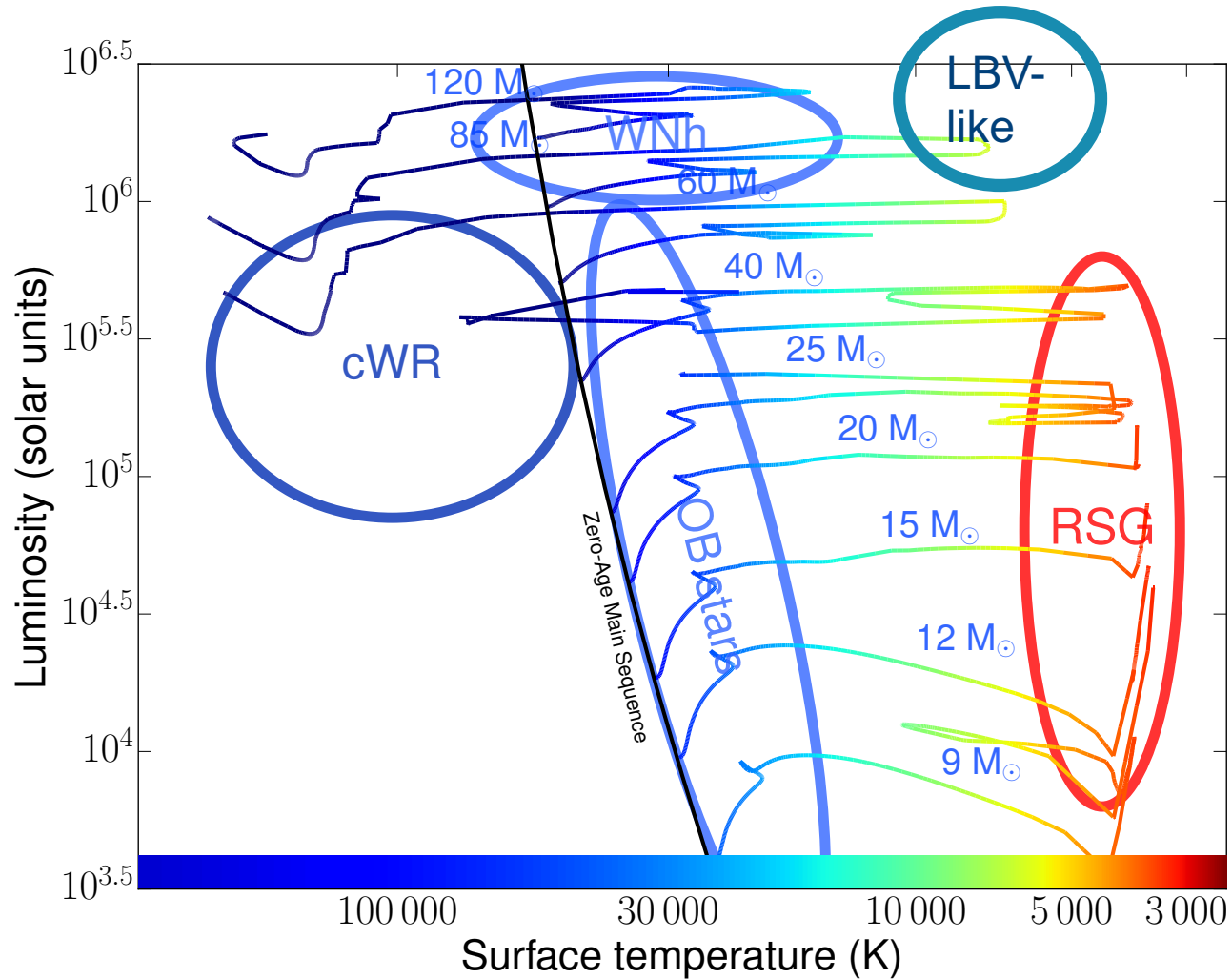
Enrico Fermi Institute for Nuclear Studies, University of Chicago  
Received January 2, 1958

ABSTRACT

We consider the dynamical consequences of Biermann’s suggestion that gas is often streaming outward in all directions from the sun with velocities of the order of 500–1500 km/sec. These velocities of 500 km/sec and more and the interplanetary densities of 500 ions/cm<sup>3</sup> (10<sup>11</sup> gm/sec mass loss from the sun) follow from the hydrodynamic equations for a 3 × 10<sup>6</sup> K solar corona. It is suggested that the outward-streaming gas draws out the lines of force of the solar magnetic fields so that near the sun the field is very nearly in a radial direction. Plasma instabilities are expected to result in the thick shell of disordered field (10<sup>3</sup> gauss) enclosing the inner solar system, whose presence has already been inferred from cosmic-ray observations.

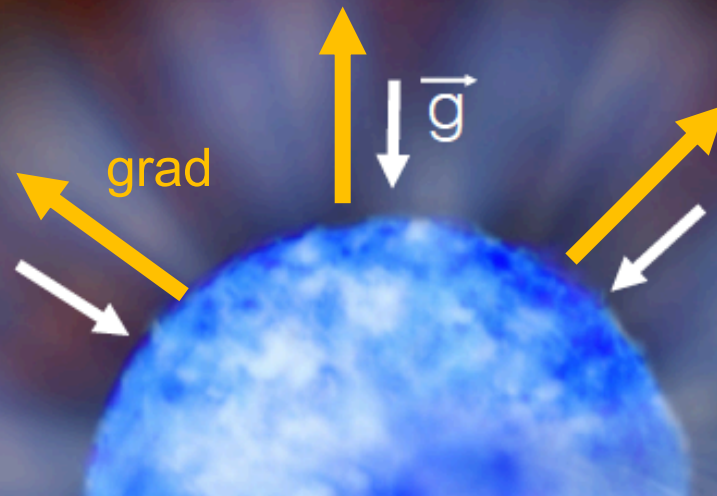


# Luminous Stars



# OR: How To Get Blown Away by Starlight

Dynamical effects of radiation



# The dynamical equations again

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0$$

$$\partial_t (\rho \vec{v}) + \nabla \cdot (\vec{v} \rho \vec{v} + p) = \vec{g}_{rad} \rho - \frac{GM}{r^2} \rho$$

$$\partial_t e + \nabla \cdot (e \vec{v} + p \vec{v}) = \vec{v} \cdot (\vec{g}_{rad} \rho + \vec{g} \rho) - \dot{q}$$

# Radiation force vs. gravity

Grey electron scattering minimum (“Thomson scattering”) benchmark value (‘classical Eddington limit’):

$$\vec{g}_{\text{rad}} = \frac{1}{c} \int \oint \kappa_{\mathbf{n},\nu} I_{\mathbf{n},\nu} \mathbf{n} d\Omega d\nu$$

$$g_{\text{rad}} = \kappa F / c$$

$$\Gamma = \frac{\kappa F / c}{g} = \frac{\kappa L}{GM4\pi c}$$

$$\Gamma_{e,\odot} \sim 2 \times 10^{-5} \quad L \sim M^3 \quad M/M_{\odot} \sim 50 \rightarrow \Gamma_e \sim 0.1 - 0.5$$



# Radiation force vs. gas pressure gradient

Equation of motion:

$$\frac{\partial v_r}{\partial r} = v_r \left( \frac{2c_i^2/r - GM_*/r^2(1 - \Gamma)}{v_r^2 - c_i^2} \right)$$

- For thermally driven solar wind (continuous coronal expansion)  
 $\Rightarrow T_{wind} \sim 2 \text{ MK} \rightarrow c_i \sim 165 \text{ km/s}$
- BUT luminous, massive stars do not have such a corona.

Therefore:

$$\Rightarrow T_{wind} \sim 40 \text{ kK} \rightarrow c_i \sim 25 \text{ km/s}$$

# Radiation force vs. gas pressure gradient

Equation of motion:

$$\frac{\partial v_r}{\partial r} = v_r \left( \frac{2c_i^2/r - GM_*/r^2(1 - \Gamma)}{v_r^2 - c_i^2} \right)$$

- For the Solar corona with stellar surface gravity  $g_\odot = GM_\odot/R_\odot^2 = 27500 \text{ g/cm}^2$ , we then have

$$\Rightarrow \frac{2c_i^2/R_\odot}{g_\odot} \sim 0.8$$

showing that the 'Parker term' ( $\propto 1/r$ ) will soon overcome gravity ( $\propto 1/r^2$ ) and start driving a thermal wind outflow.

# Radiation force vs. gas pressure gradient

Equation of motion:

$$\frac{\partial v_r}{\partial r} = v_r \left( \frac{2c_i^2/r - GM_*/r^2(1 - \Gamma)}{v_r^2 - c_i^2} \right)$$

- On the other hand, for a typical gravity at stellar surface  $g_* = GM_*/R_*^2 \sim 10^4 \text{ cm/s}^2$  and  $R_* = 10R_\odot$ , we have

$$\Rightarrow \frac{2c_i^2/R_*}{g_*} \sim 10^{-3}$$

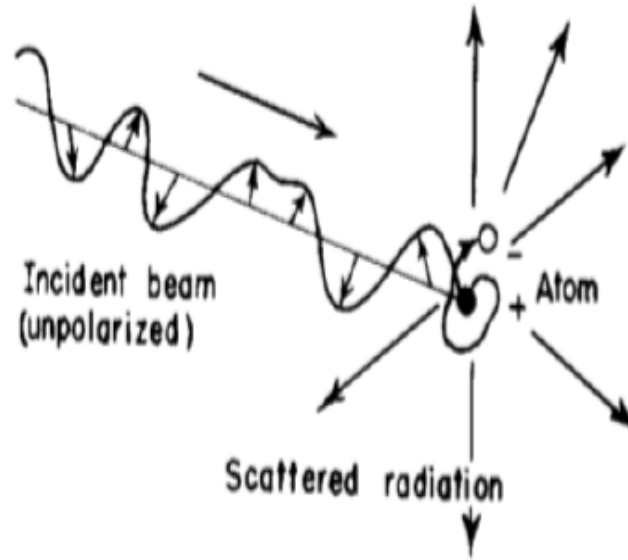
showing that for such luminous massive stars gas pressure effects are almost negligible close to the stellar surface. **Instead we overcome gravity by a strong radiation force.**

→ But what provides the little extra opacity (in addition to Thomson scattering) needed to push us above limit?

# The Enormous Resonance Effect of Line-Opacity

Classical Mechanics 101: Driven, Damped Classical Oscillator

Fig. from Feynman's notes in physics



$$\sigma_{\nu} \approx \sigma_{Th} \frac{\nu^2}{4(\nu - \nu_0)^2 + (\gamma/(2\pi))^2} \quad \sigma_{cl} = \int \sigma_{\nu} d\nu$$

# The Enormous Resonance Effect of Line-Opacity

Classical Mechanics 101: Driven, Damped Classical Oscillator

→ A GIGANTIC effect from the resonance Quality !

$$Q = \frac{\nu_0}{\gamma} = \frac{\sigma_{cl}}{\nu_0 \sigma_{Th}} \frac{1}{\pi^2} \approx 10^8 \frac{\lambda}{\lambda_{5000A}}$$

→ Cross section to opacity:

$$q = \frac{\kappa_L \rho}{\kappa_{Th} \rho \nu_0} = Q \frac{n_L}{n_e} f_{lu} \frac{1}{\pi^2}$$

$$Q \sim 10^8 \quad \frac{n_L}{n_e} \sim 10^{-4} \quad f_{lu} \sim 0.1$$

$$\rightarrow q \sim 10^3$$

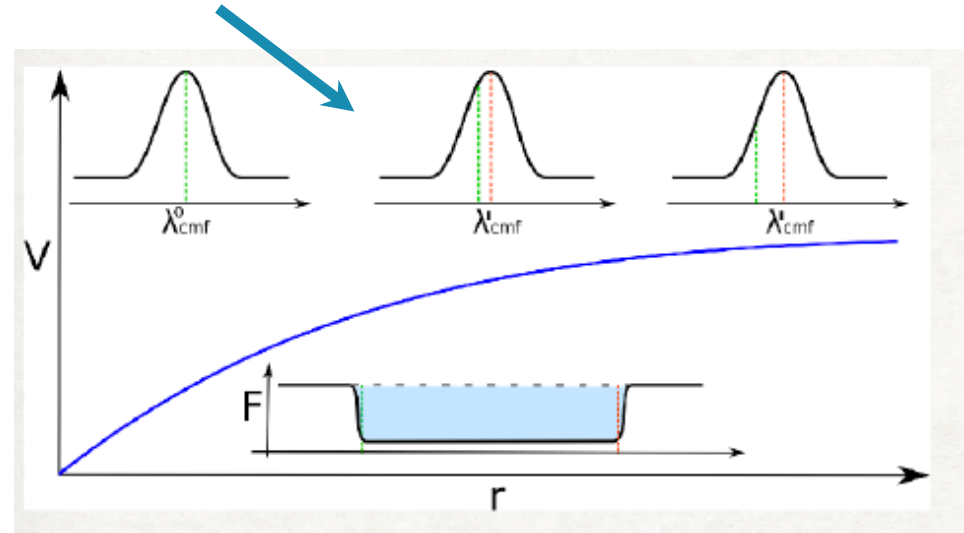
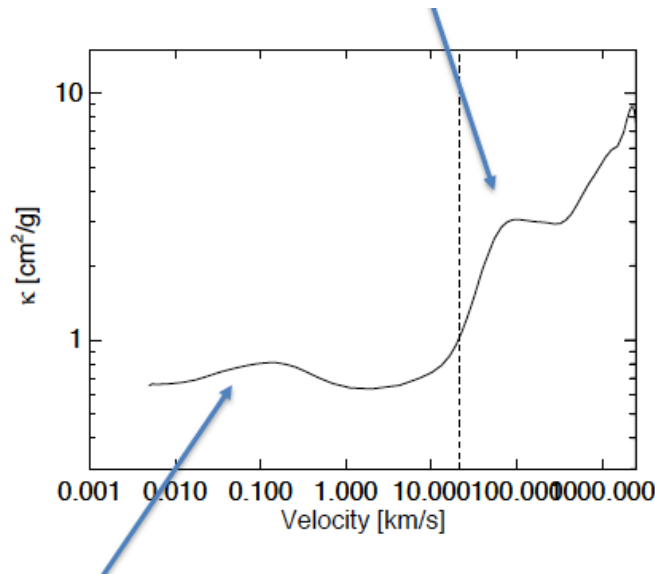
Meaning? Effect of line can be 1000 times that of e-scattering !

From Sundqvist, lecture-notes on radiative processes, partly based on formulation-idea by Gayley (1995)

# Line-Driving in Practice

## – Saturation and Doppler Shift

### Line-driving, from Doppler shift



### Rosseland-like

- No 'brute-force' formulation exists (only for 1D, steady and monotonic flows, e.g. talk by Sander)
- Long-term goal, general formalism, KU Leuven code-framework MPI-AMRVAC (CMPA, Keppens+), cf. Moens, Sundqvist+ (2022), Poniatowski, Sundqvist+ (2021, 2022), Debnath, Sundqvist+ (2024)
- Applications today all use various approximations





# Castor-Abbott-Klein (CAK) - reference model for line-driven winds from hot, luminous stars

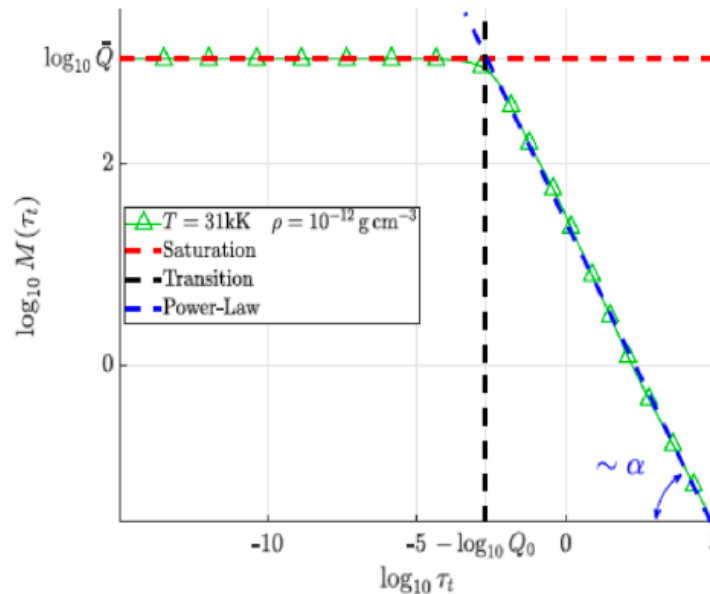
## The Upshot:

Surface regions around 10-100 kK, strong UV Flux. MANY, MANY lines available to tap from → STRONG line-force.

Avoid re-computing sum from, here, ~ 5 million lines; compute excitation-ionization balance, then tabulate fit-function for 'all' T, rho (similar to Rosseland means)

CAK 1975, Abbott 1980, Pauldrach+ (1986), Owocki+ (1988), Kudritzki+ 19189, Gayley (1995), Puls+ 2000, etc. .... Here modern reformulation and calculations by: Poniatowski, Sundqvist+ 2022, 'Munich' line data base from Pauldrach, Puls.

$$\frac{g_{line}^{tot}}{g_e} \equiv M(t) = \sum_i w_{\nu,i} q_i \left( \frac{1 - e^{-q_i t}}{q_i t} \right)$$



$$\alpha(\rho, T)$$

$$Q_0(\rho, T)$$

$$\bar{Q}(\rho, T)$$

$$M_{fit}(t) = \frac{\bar{Q}}{(1 - \alpha)} \frac{(1 + Q_0 t)^{1-\alpha} - 1}{Q_0 t}$$

# CAK as reference model for line-driven winds from hot, luminous stars

$$\frac{v_r^2 - c_i^2}{v_r} \frac{\partial v_r}{\partial r} = \frac{2c_i^2}{r} - \frac{GM_*}{r^2} (1 - \Gamma_e - A\Gamma_e \left( \frac{\partial v_r}{\partial r} / \rho \right)^\alpha) \quad \rightarrow$$

$$\dot{M}_{CAK} \approx \frac{L}{c^2} \frac{\alpha}{1 - \alpha} \left( \frac{\bar{Q}\Gamma_e}{1 - \Gamma_e} \right)^{1/\alpha - 1} \frac{Q_0}{\bar{Q}} \left( \frac{1}{1 + \alpha} \right)^{1/\alpha}$$

$$v(r) = v_\infty \left( 1 - \frac{R_*}{r} \right)^{\beta'}$$

for

$$\beta \approx 1/2 - 1 \text{ and } v_\infty \approx 2\sqrt{\alpha/(1 - \alpha)} \sqrt{\frac{2GM_*(1 - \Gamma_e)}{R_*}}$$

# Key Scalings of CAK-based models

$$\dot{M}_{CAK} \approx \frac{L}{c^2} \frac{\alpha}{1-\alpha} \left( \frac{\bar{Q}\Gamma_e}{1-\Gamma_e} \right)^{1/\alpha-1} \frac{Q_0}{\bar{Q}} \left( \frac{1}{1+\alpha} \right)^{1/\alpha}$$

Now re-inserting our typical values for a luminous O-star in the Milky Way,  $L \approx 8 \times 10^5 L_\odot$ ,  $\Gamma_e \approx 0.4$ , and  $v_{esc,eff} \approx 800$  km/s, we find  $\dot{M} \approx 2.0 \times 10^{-6} M_\odot/\text{yr}$  and  $v_\infty \approx 2200$  km/s for  $\bar{Q} \approx Q_0 \approx 2000$  and  $\alpha \approx 2/3$ .

But line force parameters implicitly depend on stellar parameters and, in particular, stellar metallicity:

essentially,  $\bar{Q} \sim Z$ , which to first order gives  $\dot{M} \sim Z^{1/\alpha-1}$

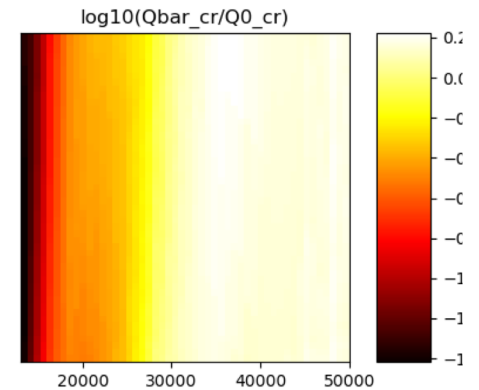
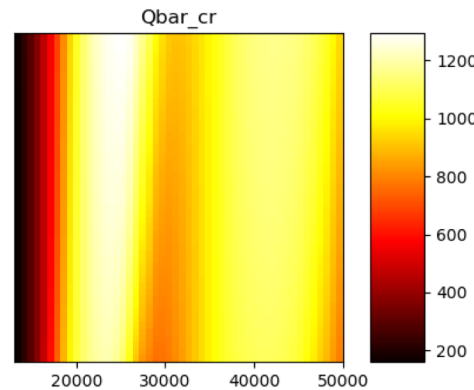
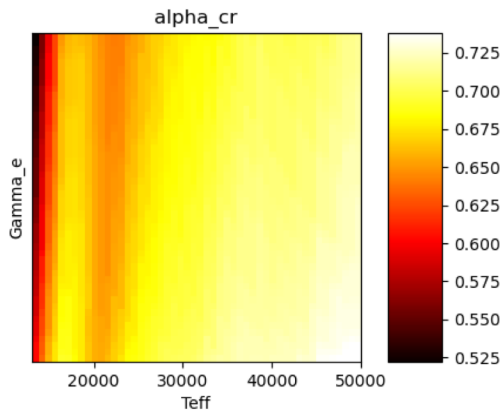
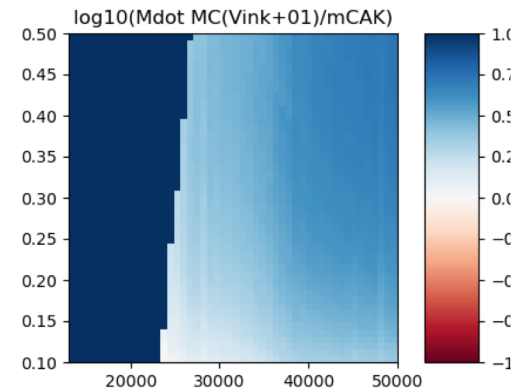
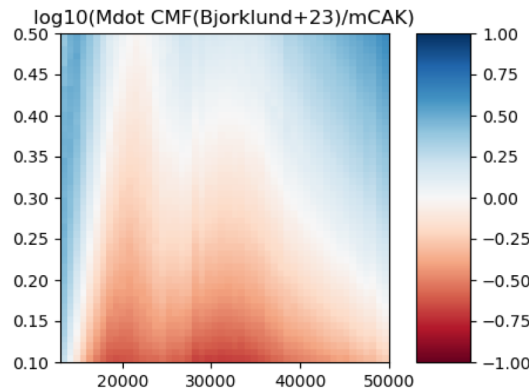
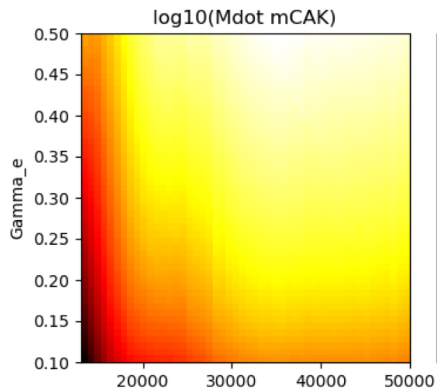
# Comparison of CAK-like analytic mass-loss rates to more elaborate ‘brute-force’ numerical model results

For  $\frac{Z}{Z_{\odot}} = 1$  and  $\frac{M}{M_{\odot}} = 40$

Modified CAK

CMF, Bjorklund, Sundqvist+ 2023

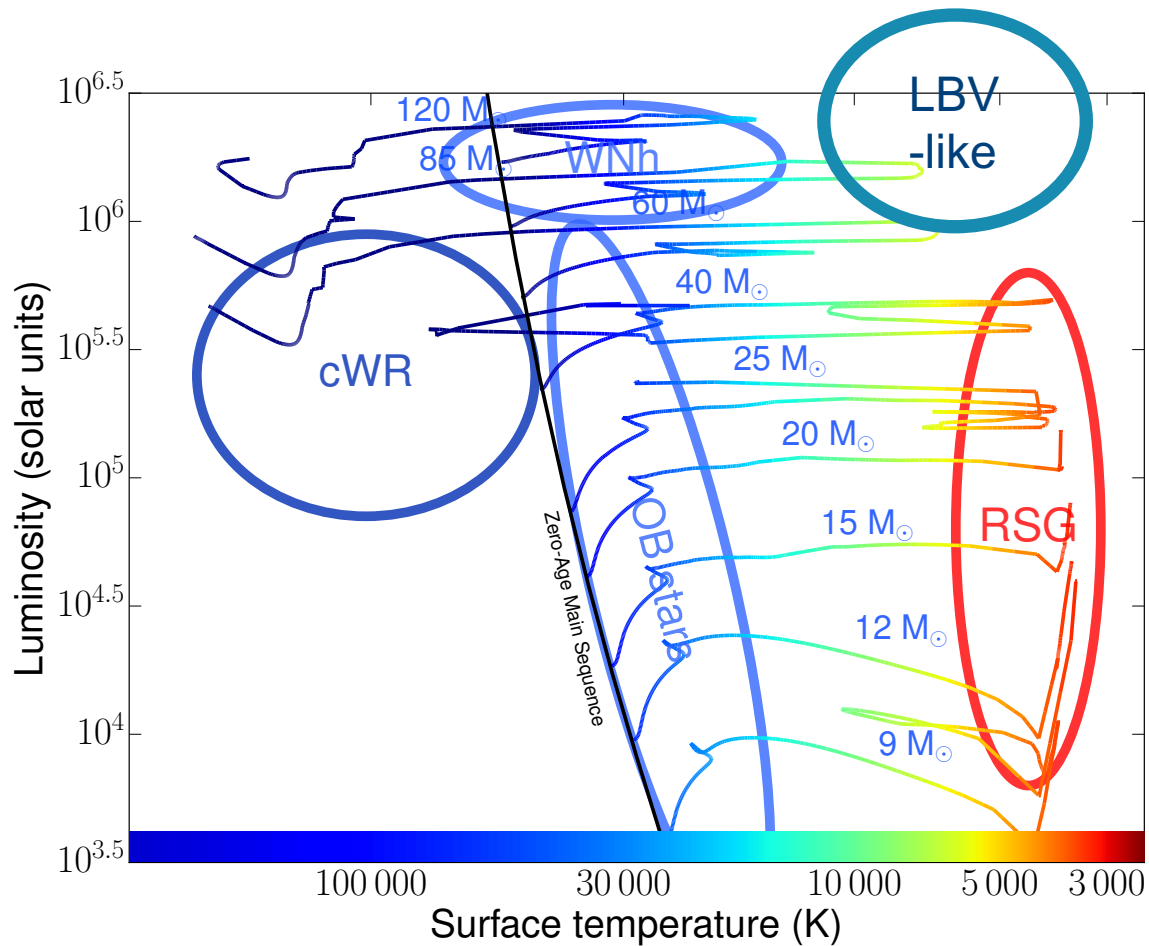
MC, Vink+ 2001



→ These are all ‘ready to go’ models / recipes for your favorite application

# (Too) Close to Eddington Limit ?

$$\Gamma_e \sim \frac{\kappa_e L}{M}$$

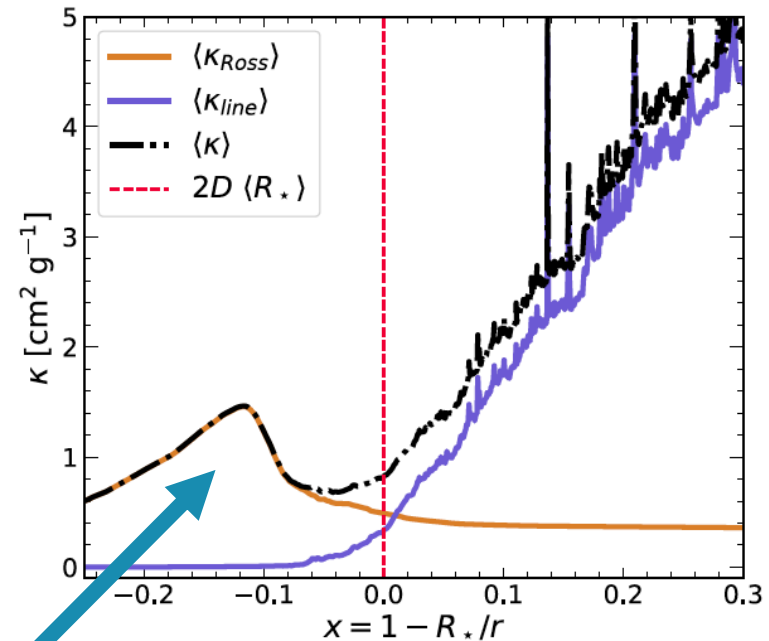


Standard CAK model ok for OB stars

# The Two Key Considerations For Stars Close to Classical Eddington Limit

1. Do you exceed local Eddington limit beneath surface?

$$\Gamma = \frac{\kappa F / c}{g} = \frac{\kappa L}{GM4\pi c}$$



'Iron opacity bump' in Rosseland mean,  
Around  $\sim 150$  kK



# The Two Key Considerations For Stars Close to Classical Eddington Limit

2. If yes on 1, instabilities will be induced. But can energy be efficiently transported by enthalpy (=convection), and so reduce radiative acceleration and retain a quasi-static envelope?

$$\Gamma = \frac{\kappa F / c}{g} = \frac{\kappa L}{GM4\pi c}$$

$$\rho v_r \left( \frac{5 P_g}{2 \rho} + 4 \frac{P}{\rho} + \frac{v^2}{2} - \frac{MG}{r} \right) + F = F_{\text{tot}}$$

$F_{\text{conv}}$ 
 $\frac{F_{\text{conv}}}{F}$

Energy conservation equation again

See also Owocki+ 2017

# The Two Key Considerations For Stars Close to Classical Eddington Limit

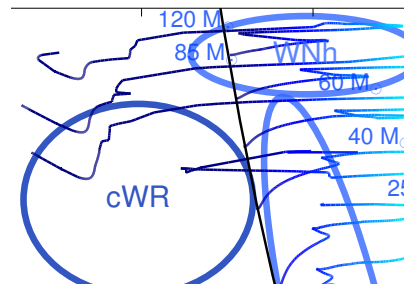
$$\frac{F_{\text{conv}}}{F} \sim \frac{v_s}{c} \left( \frac{T}{T_{\text{eff}}} \right)^4 \sim \frac{v_s}{c} \tau$$

At Fe opacity bump:  $T \sim 150\text{kK}$

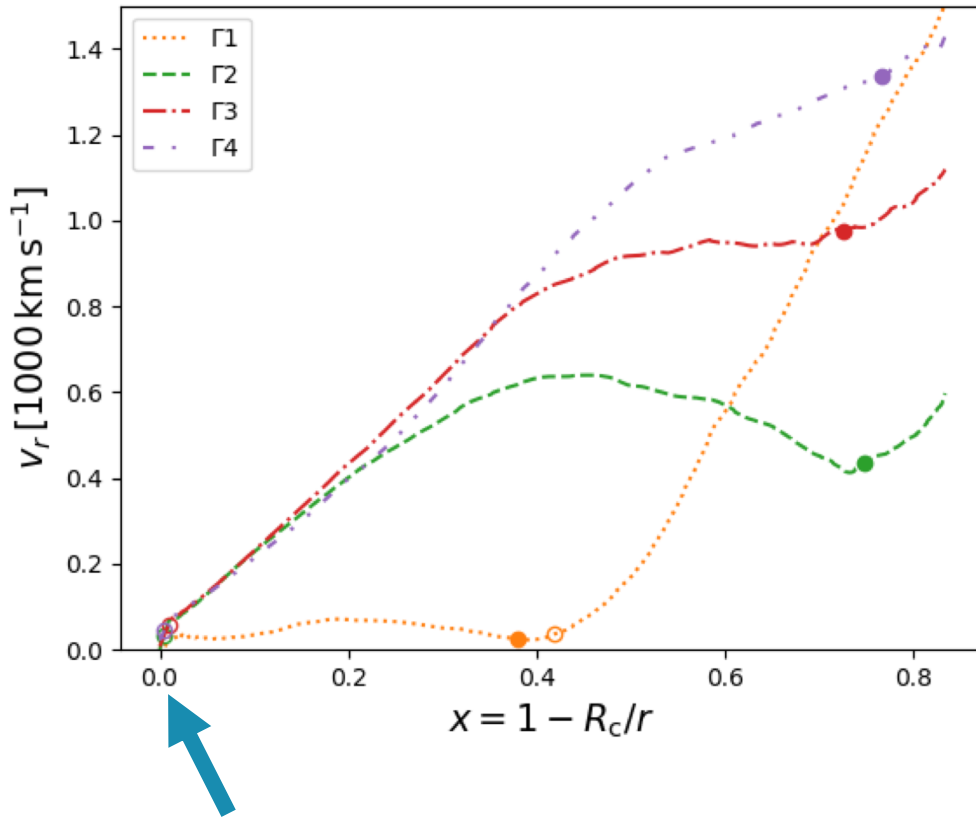
	100kK - WR		$\frac{F_{\text{conv}}}{F} \ll 1$
$T_{\text{eff}} \sim$	40kK - O	$\Rightarrow$	$\frac{F_{\text{conv}}}{F} < 1$
	10kK - LBV		$\frac{F_{\text{conv}}}{F} \gtrsim 1$

→ Wind launching. Q shifts: Can it be sustained?

# Hotter Side of HRD



Moenst+ 2022, Debnath+ 2024

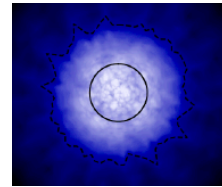


Evolved:

Transition from 'hot subdwarfs / stripped stars'

→ classical WR stars

Talk by Sander



Near MS:

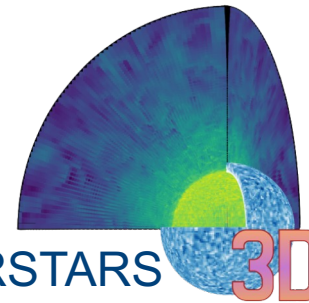
Transition from O-stars → WNH / VMS

Talk by Sabhahit

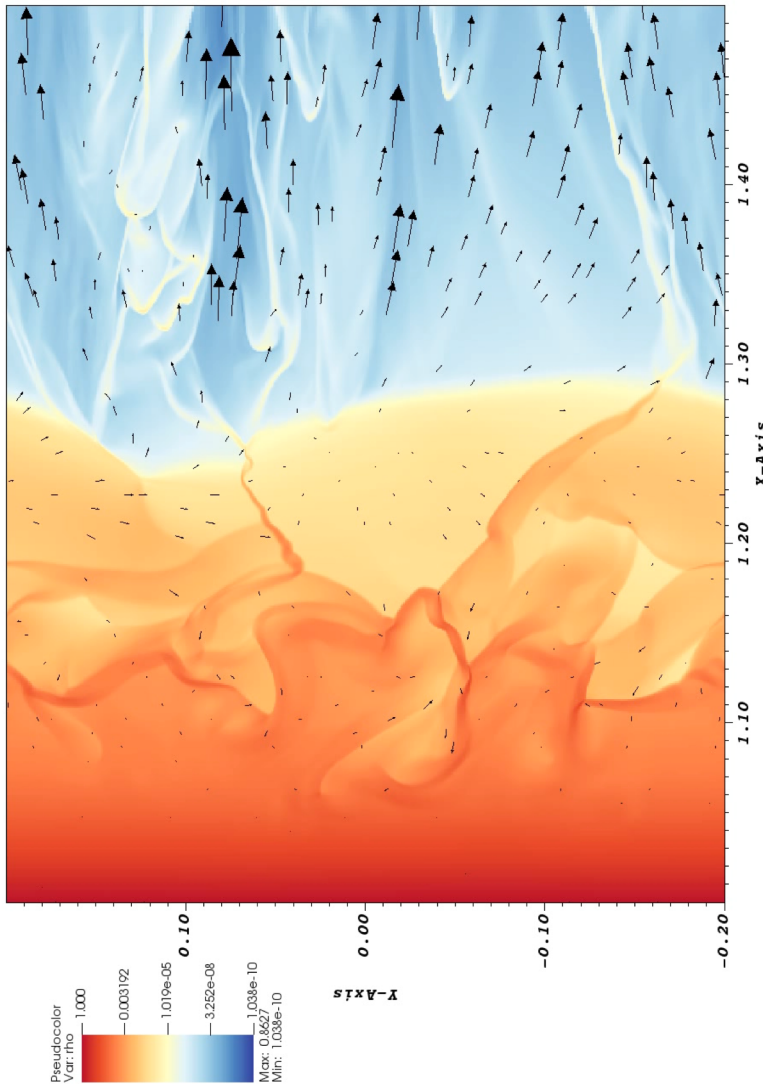
Lower boundary beneath iron-bump. → **At which  $\Gamma_e$  you will reach effective 'sub-surface' wind launching will also depend on  $Z$  and evolution state ( $T_{eff}$ , Metal, H/He content).**

# Hotter Side of HRD

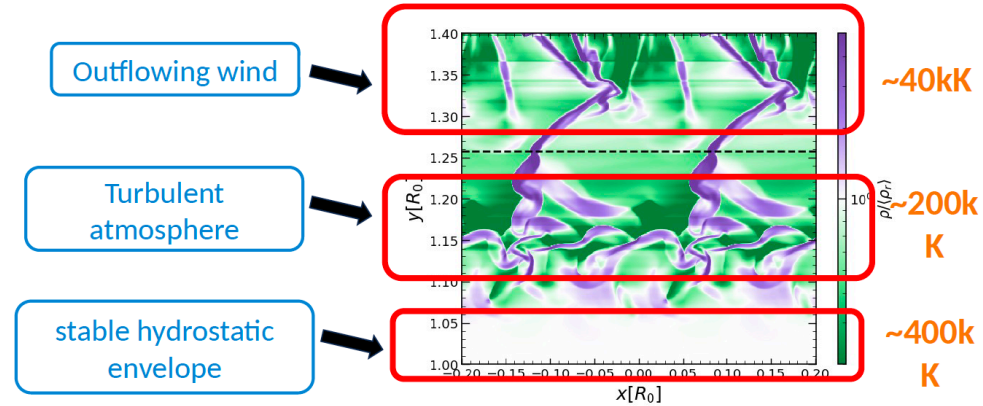
Sim by Nico Moens



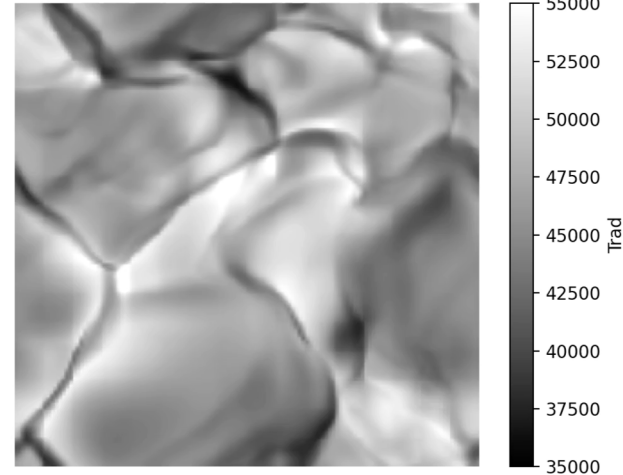
SUPERSTARS 3D



user: nicolasm  
Mon Oct 28 13:46:44 2024



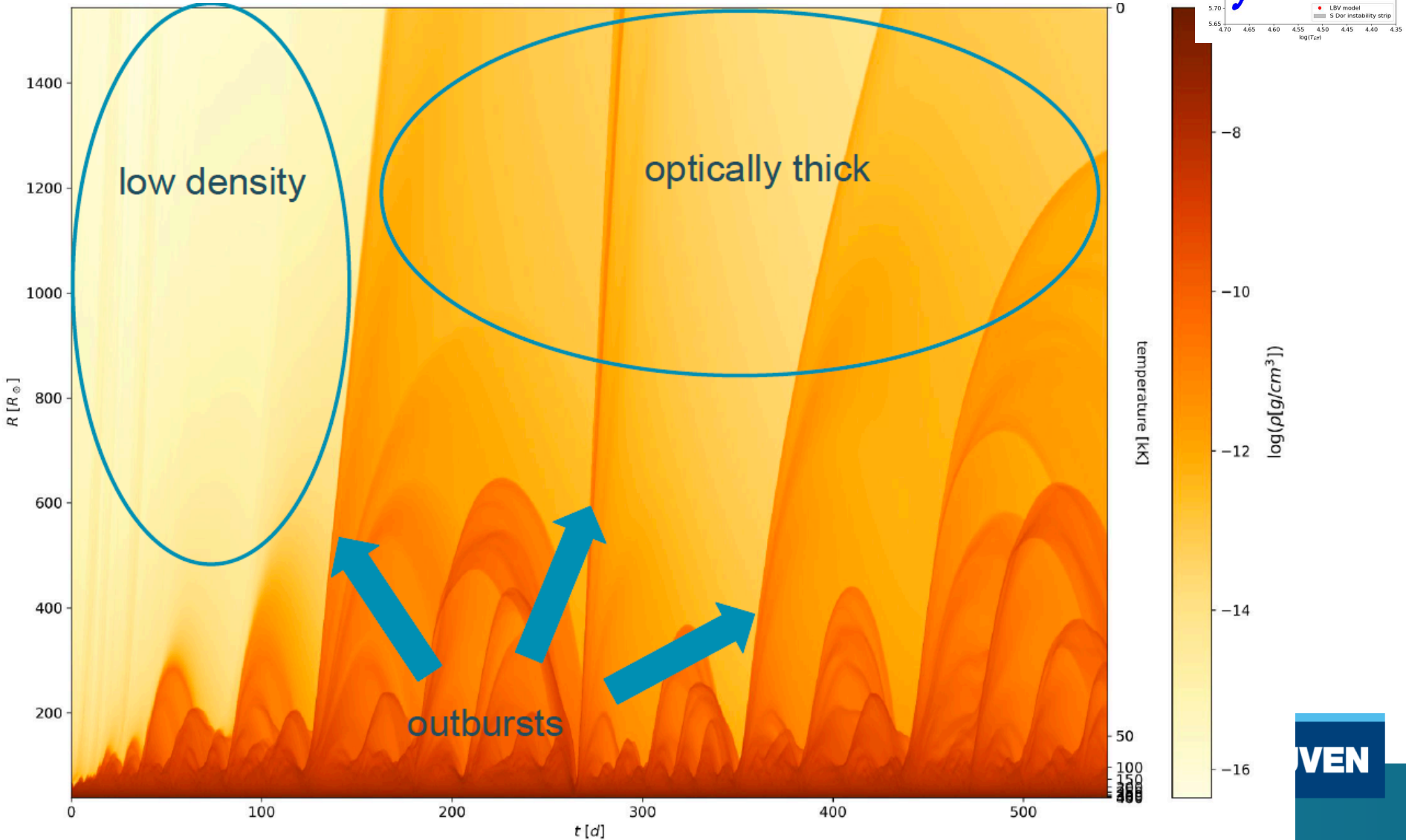
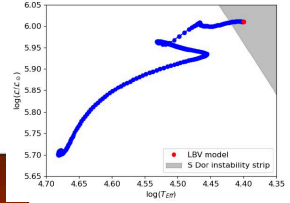
Emergent Continuum Intensity dt [sec] = 00.0



# Cooler Side of HRD

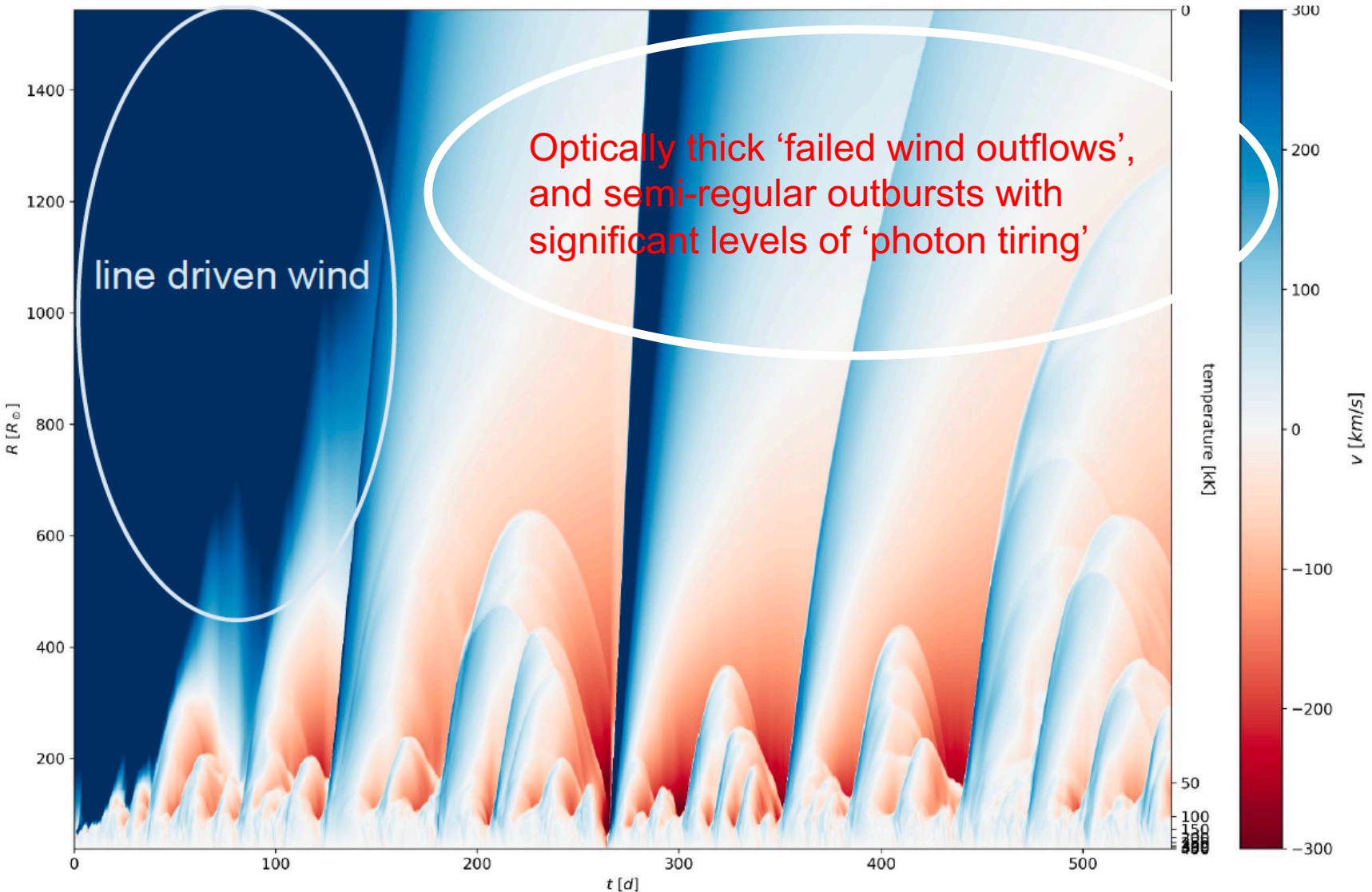
Sim by Pieter Schillemans

2D sim of Extreme Object Very Close to Eddington Limit, started from roughly (extreme) LBV / YHG positions. Illustrated as radial average Space-Time diagram





# Cooler Side of HRD



Feedback upon underlying star? Relevance? (e.g. eruptions, SN imposters, etc.)



# Cooler Side of HRD

Envelope very loosely bound. Current simulations very turbulent (also H-recombination), but majority of gas doesn't seem able to reach local escape speed (Freytag, Höffner+, Goldberg, Jiang+)

Observations indicate very large turbulent velocities:

## OBTAIN $V_{\text{TURB}}$ WITH OBSERVATIONS

Number	Name	Mass $M_{\odot}$	$T_{\text{eff}}$ K	Radius $R_{\odot}$	$\dot{M}_{\text{gas}}^a$ $10^{-7} M_{\odot} \text{ yr}^{-1}$	$v_{\text{turb, Obs}}$ km s $^{-1}$	$v_{\text{turb, Theory}}$ km s $^{-1}$
1	$\alpha$ Ori	15	3780	589	5.0	19	17
2	V466 Cas	12	3780	331	0.5	12	19
3	AD Per	12	3720	457	2.0	21	17
4	FZ Per	12	3920	324	1.75	16	20
5	BD+243902	15	4240	427	7.25	23	21
6	BI Cyg	20	3720	851	10.25	23	16
7	BC Cyg	20	3570	1230	8.0	22	13
8	RW Cyg	20	3920	676	8.25	20	19
9	SW Cep	9	3570	234	11.5	24	23
10	$\mu$ Cep	25	3750	1259	3.75	23	14
11	ST Cep	9	4200	174	6.25	23	26
12	TZ Cas	15	3670	646	9.5	17	17
13	Antares	12.7 <sup>b</sup>	3660 <sup>b</sup>	680 <sup>b</sup>	20.0 <sup>c</sup>	20 <sup>d</sup>	15

Kee, Sundqvist et al. (2021) using data from Josselin & Plez (2007) and Ohnaka et al. (2017)

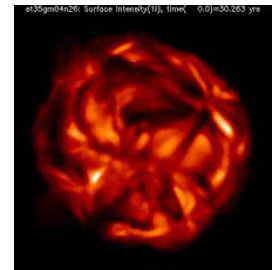
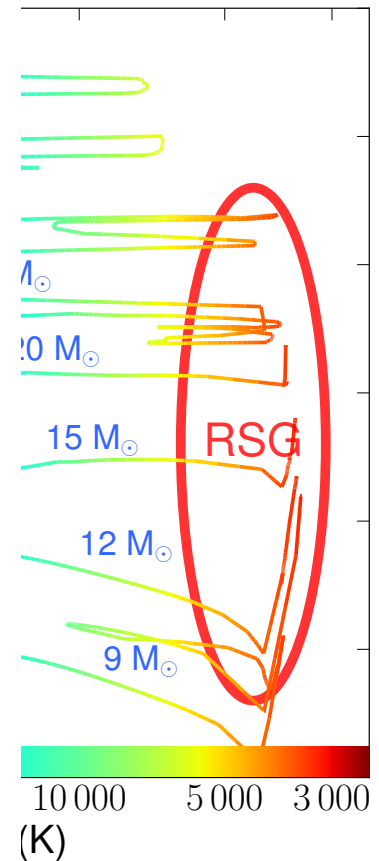


Image credit: Freytag

# Cooler Side of HRD

Modified Parker Wind Model including  $v_{turb}$  :

$$v \left( 1 - \frac{c_s^2 + v_{turb}^2}{v^2} \right) \frac{\partial v}{\partial r} = \frac{2 (c_s^2 + v_{turb}^2)}{r} - \frac{GM_* (1 - \Gamma)}{r^2}$$

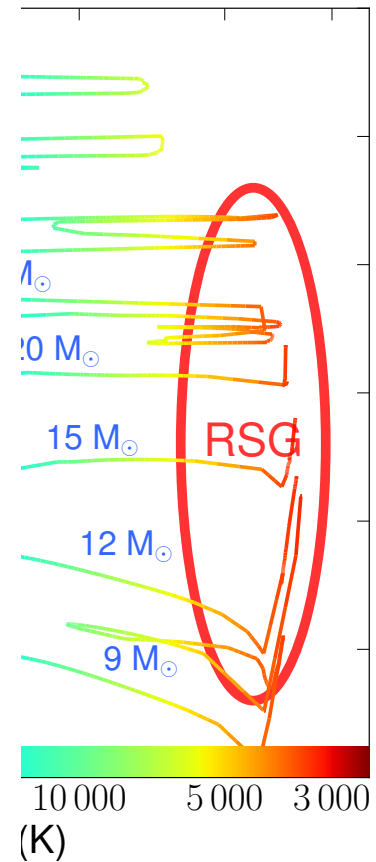
$$\frac{R_{p,mod}}{R_*} = \frac{1}{4} \frac{v_{esc,eff}^2}{a_{eff}^2}$$

For luminous RSG, only small 'extra push' required

$$\dot{M} = 4 \pi \rho(R_{p,mod}) \sqrt{c_s^2 + v_{turb}^2} R_{p,mod}^2$$

Connect to underlying hydrostatic photosphere:

$$\rho(R_{p,mod}) = \frac{4}{3} \frac{R_{p,mod}}{\kappa R_*^2} \frac{\exp \left[ -\frac{2R_{p,mod}}{R_*} + \frac{3}{2} \right]}{1 - \exp \left[ -\frac{2R_{p,mod}}{R_*} \right]}$$



# Cooler Side of HRD

LEAVES US WITH  $V_{\text{TURB}}$   
AS A FREE PARAMETER

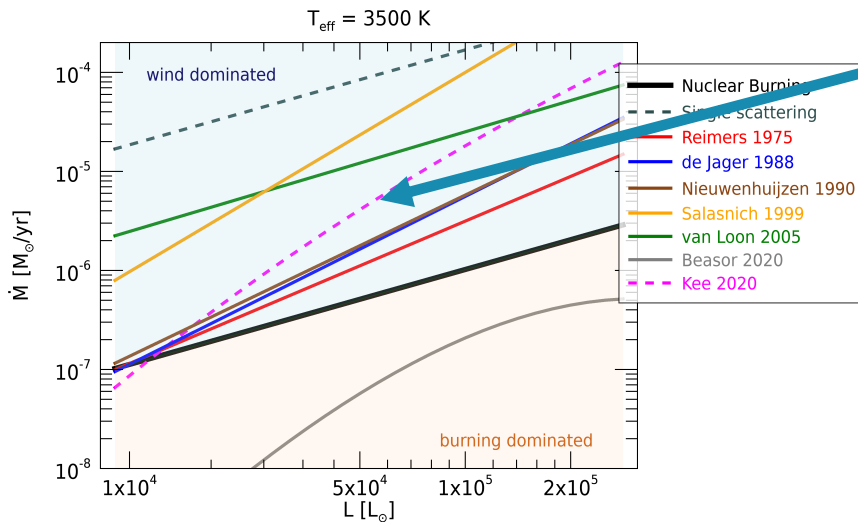
$$R_{p,\text{mod}} = \frac{GM_*(1-\Gamma)}{2c_{s,\text{eff}}^2}$$

$$\rho(R_{p,\text{mod}}) = \frac{4R_{p,\text{mod}}e^{3/2}}{3\kappa R_*^2(e^{2R_{p,\text{mod}}/R_*} - 1)}$$

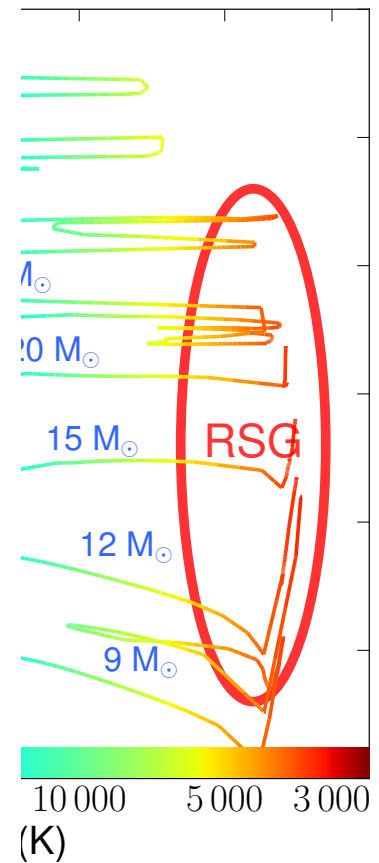
$$\dot{M} = 4\pi\rho(R_{p,\text{mod}})c_{s,\text{eff}}R_{p,\text{mod}}^2$$

$$\left(\frac{\dot{M}_{\text{num}}}{\dot{M}_{\text{an}}}\right) = \left(\frac{v_{\text{turb}}/(17\text{ km s}^{-1})}{v_{\text{esc}}(M_*, R_*)/(60\text{ km s}^{-1})}\right)^{1.30}$$

OBTAIN  $V_{\text{TURB}}$  WITH OBSERVATIONS



NOTE: Thus not complete theory



mean value  
 $v_{\text{turb}} = 18 \frac{\text{km}}{\text{s}}$   
 gets us straight into business..

Kee, Sundqvist+ 2021,  
 Sundqvist & Kee 2022  
 Plot from Decin 2020

# Cooler Side of HRD

Role of Dust:  
Also radiative acceleration

Not needed for wind launch?

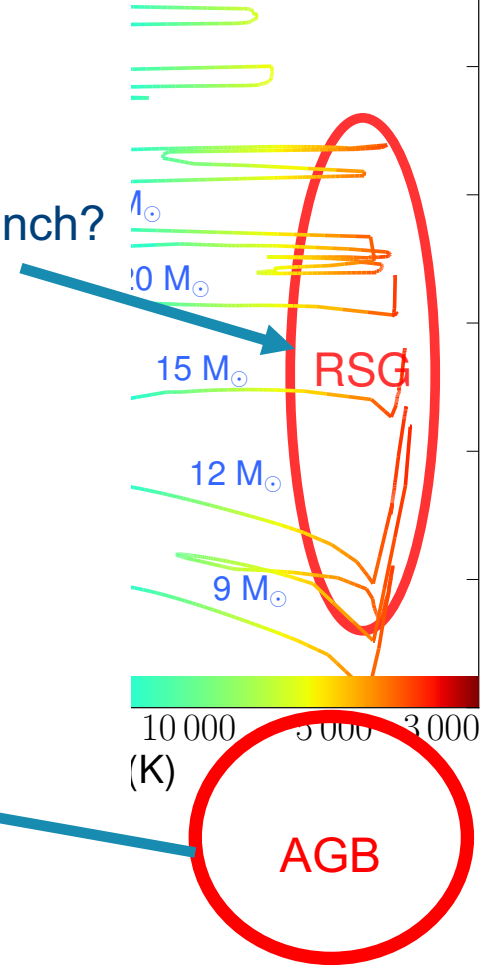
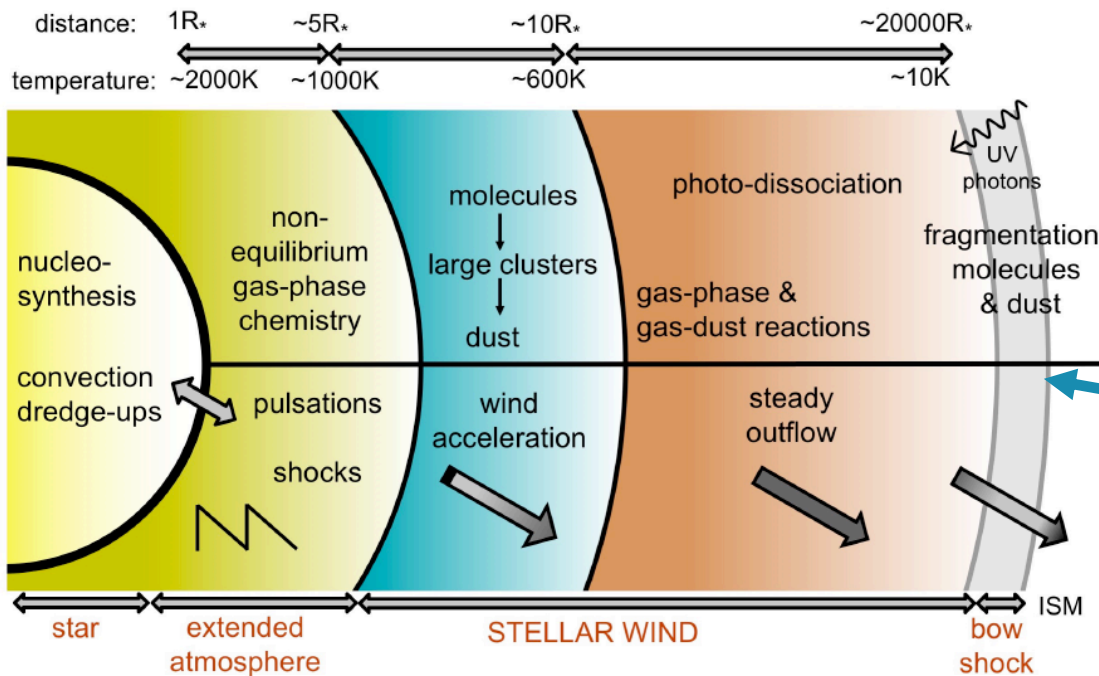
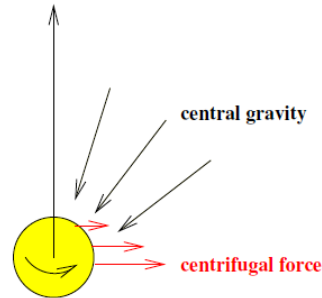


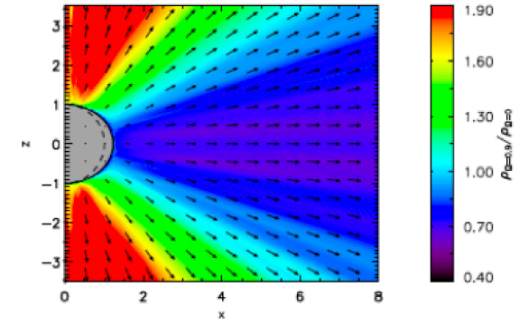
Fig. from review by Decin (2020)  
Full simulations carried out by Höffner, Freytag+

# Not Covered Here...

Fast Rotation:



Thermo-centrifugally bending  
of streamlines



Reversed effect,  
stronger above pole

## Instabilities, Clumps and Shocks, Magnetic Fields

(can lead to e.g. X-ray emission, see talk by Owocki)

## Wind Interactions in Binary Stars

(see talk by Pittard)

## Interactions on Larger Scales

(see talk by Mackey)

# Summary, in a nut shell:

## Radiative Force Key for Winds in Upper HRD

'Anti Gravity' character leads to fundamental scaling:  $v_{wind} \sim v_{esc}$

→ Fast Winds for OB-stars / WR-stars, Slow Winds for RSGs / AGBs

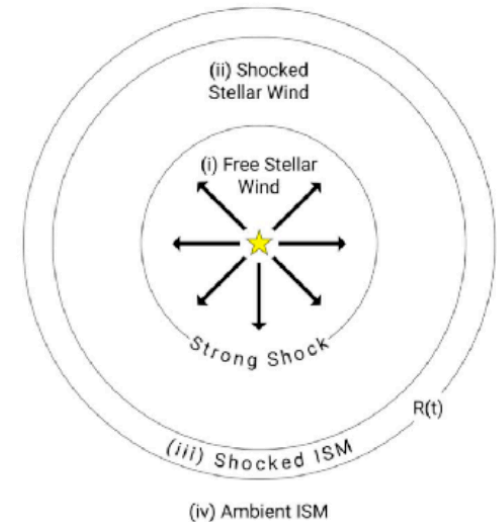
$\dot{M}$  scaling predictions available for line-driven winds. Also for RSGs, though not fundamental (turbulent pressure free parameter).

## BE AWARE:

Stars Close to Eddington Limit might be very chaotic, making predictions of global wind properties very challenging (no good 'recipes' to date). When evolving toward Cooler HRD, metallicity scaling might change drastically (He / H recombination).

If you're interested in e.g. feedback, be mindful of scales. (No wind acceleration zone needed?)

Wind Bubble Structure for Isotropic Wind



From yesterday's talk by Rosen