The Contribution Path Picture of Flows for the s, i, and r Processes

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s, i & r Element Nucleosynthesis (sirEN)
CONFERENCE
Giulianova (Italy), 8-13 June 2025
Dedicated to the memory of Prof. Roberto Gallino
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INAF

Ulysses and the Sirens by John William Waterhouse (1891)


ODYSSEUS' TEN-YEAR JOURNEY HOME
(POSSIBLE ROUTE ACCORDING TO PETER STRUCK, UNIVERSITY OF PENNSYLVANIA)
1 TROY
ODYSSEUS LEAVES WITH 12 SHIPS
2 ISMARUS
THE CREW PILLAGES TOWN. KIDNAPS CICONES' WIVES. THE CICONES KILL 76
3 CAPE MALEA
ODYSSEUS IS BLOWN OFF COURSE
4 CYTHERA
STORMS FOR TEN DAYS
5 LOTOPHAGI
(LAND OF LOTUS-EATERS) THE CREW EATS ADDICTIVE FLOWERS AND WANTS TO STAY
6 CYCLOPES
ODYSSEUS BLINDS POLYPHEMUS, POSEIDON'S SON
7 AEOLIA
THE CREW OPENS BAG OF WINDS AND IS BLOWN OFF COURSE
8 LAMOS
LAESTRYGONIANS KILL AND EAT MOST OF THE CREW
9 AEAEA
GODDESS CIRCE TURNS THE CREW TO SWINE
10 CIMMERIANS
ODYSSEUS ENTERS THE UNDERWORLD
11 SIRENS
CREW BLOCKS EARS WITH WAX TO AVOID SONG MAGIC
12 SCYLLA
MONSTER EATS SIX OF THE CREW
13 CHARYBDIS
MONSTROUS WHIRLPOOL
14 THRINACIA
THE CREW EATS THE "OXEN OF THE SUN" AND IS PUNISHED BY ZEUS. ODYSSEUS IS THE LONE SURVIVOR
15 OGYGIA
ODYSSEUS HAS A 7-YEAR AFFAIR WITH THE NYMPH CALYPSO
16 SCHERIA
ODYSSEUS TELLS HIS STORY TO THE PHAEACIANS AND IS OFFERED A RIDE HOME
17 ITHACA
ODYSSEUS RETURNS HOME KILLS THE SUITORS WHO BESET PENELOPE, HIS WIFE.
MARE LIGUSTICUM
MARE ADRIATICUM
PONTUS EUXINUS
MARE TYRRHENUM
MARE AEGEUM
MARE IONIUM
MARE IBERICUM
MARIS MEDITERRENEI
WORLD HISTORY ENCYCLOPEDIA
CY ME ON

Roberto Gallino and Maurizio Busso (Pasadena 2004)

Don Clayton (1935-2024)

2008—Mount Washington

ANNALS OF PHYSICS: **12:** 331–408 (1961)

Neutron Capture Chains in Heavy Element Synthesis*

The Universe, too, loves to create
whatsoever is destined to be made.
—Marcus Aurelius, *Meditations IX*.

D. D. CLAYTON, W. A. FOWLER, T. E. HULL,† AND B. A. ZIMMERMAN

Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California

$$\frac{dN_{56}}{d\tau} = -\sigma_{56} N_{56} ,$$

$$\frac{dN_A}{d\tau} = -\sigma_A N_A + \sigma_{A-1} N_{A-1} , \qquad 57 \leqq A \leqq 209, \tag{6}$$

$$\frac{dN_{210}}{d\tau} = \sigma_{209} N_{209} .$$

The boundary conditions are

$$N_A(0) = \begin{cases} N_{56}(0) & A = 56 \\ 0 & A > 56 \end{cases} .$$

$$d\tau = n_n v_T \, dt, \qquad \tau = \int n_n(t) v_T \, dt.$$

$$N_k(\tau) = N_1(0) \sum_{i=1}^{k} C_{ki} e^{-\sigma_i \tau}. \tag{7}$$

By substituting back into the equations, it follows that

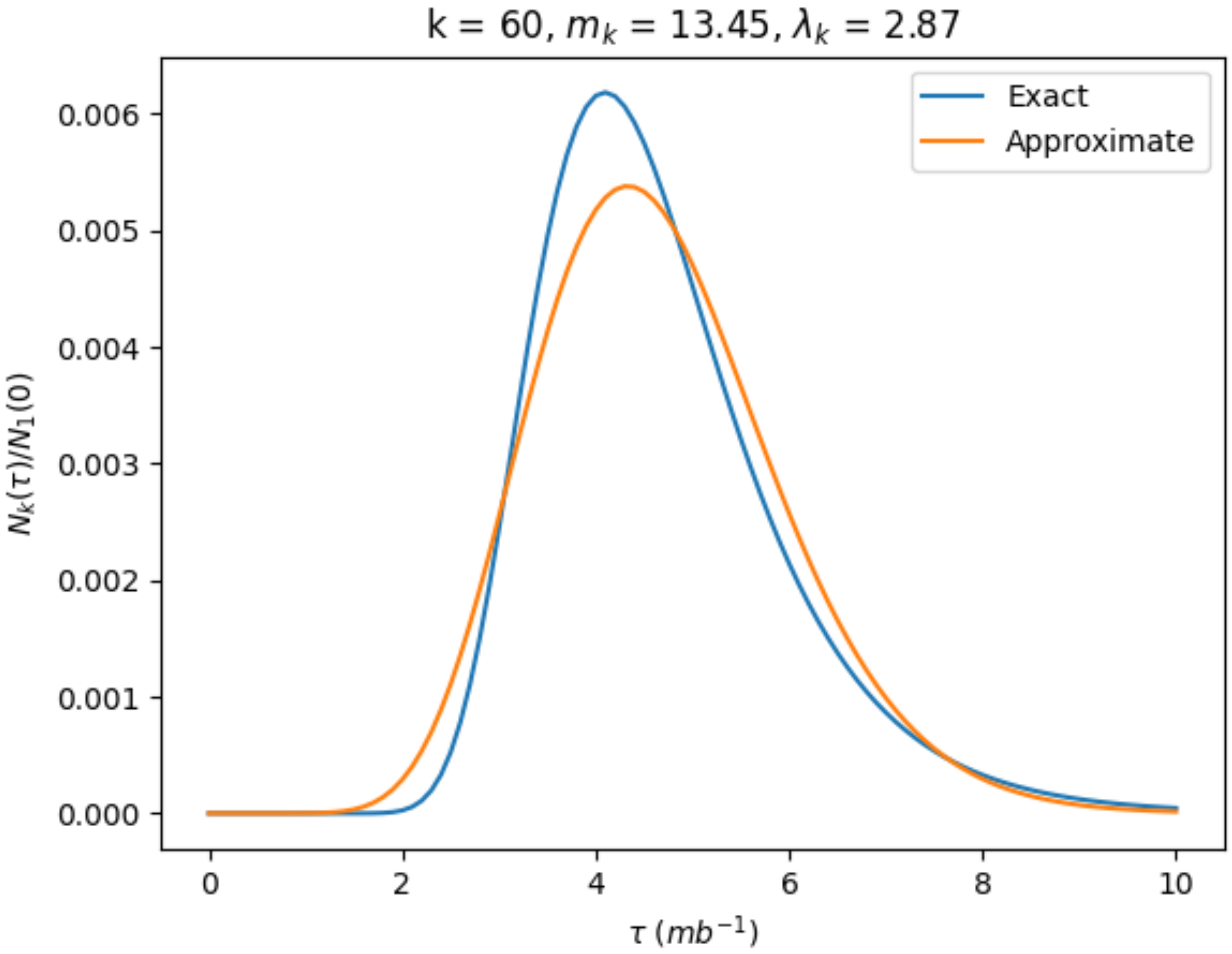
$$C_{ki} = \frac{\sigma_1 \sigma_2 \sigma_3 \cdots \sigma_{k-1}}{(\sigma_k - \sigma_i)(\sigma_{k-1} - \sigma_i) \cdots (\sigma_2 - \sigma_i)(\sigma_1 - \sigma_i)}, \quad \text{omitting} \quad \left(\frac{1}{\sigma_i - \sigma_i}\right). \tag{8}$$

$$\sigma_k = \gamma k$$

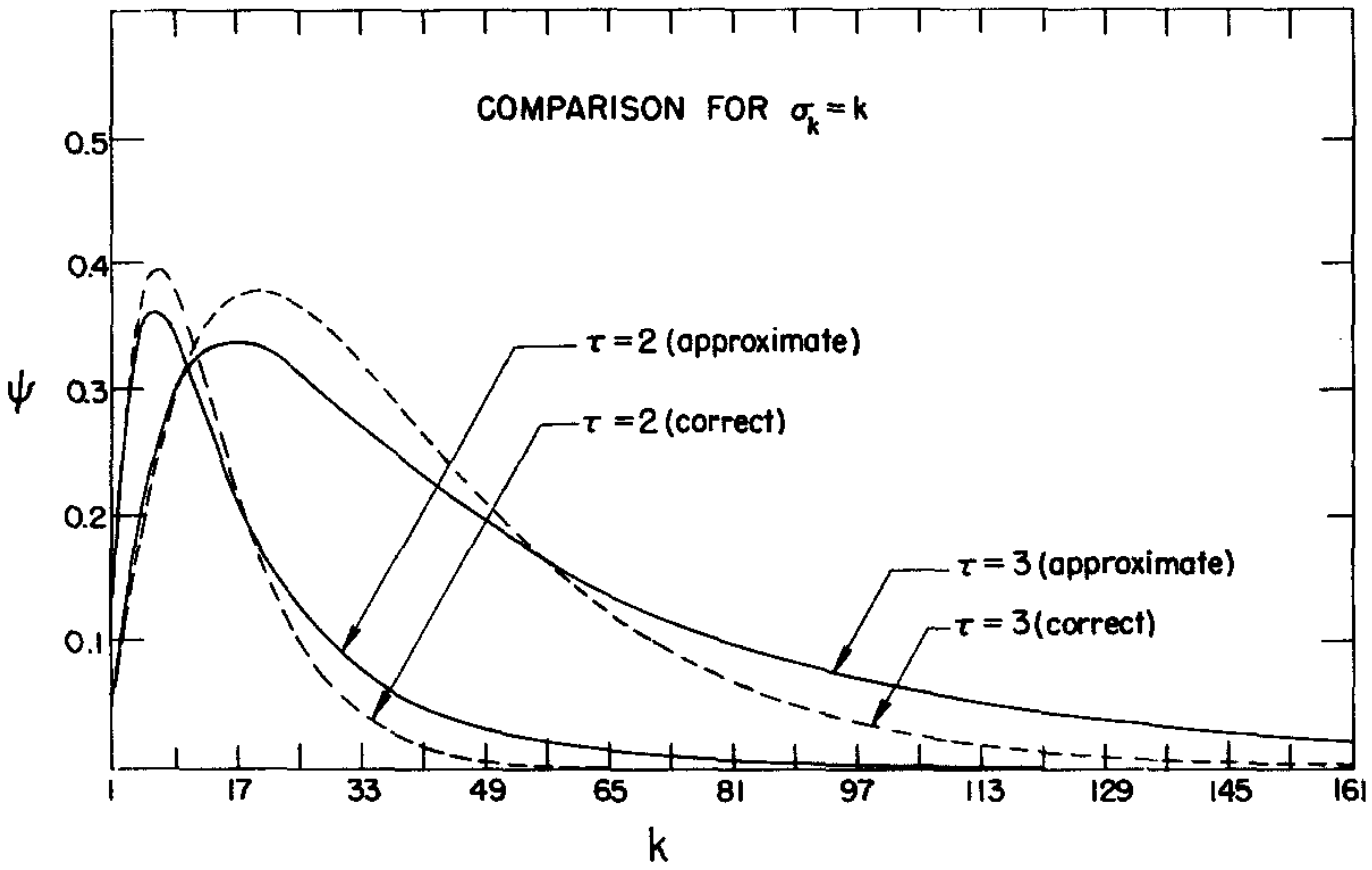
$$N_k(\tau) = N_1(0)(k-1)! \sum_{i=1}^{k} \frac{(-1)^{i-1} e^{-\gamma i \tau}}{(k-i)!(i-1)!} = N_1(0) e^{-\gamma \tau}(1 - e^{-\gamma \tau})^{k-1}.$$

$$\frac{\sigma_k N_k(\tau)}{N_1(0)} \simeq L^{-1} \frac{1}{\left(\dfrac{s}{\lambda_k} + 1\right)^{m_k}} = \lambda_k \frac{(\lambda_k \tau)^{m_k - 1}}{\Gamma(m_k)} e^{-\lambda_k \tau}$$

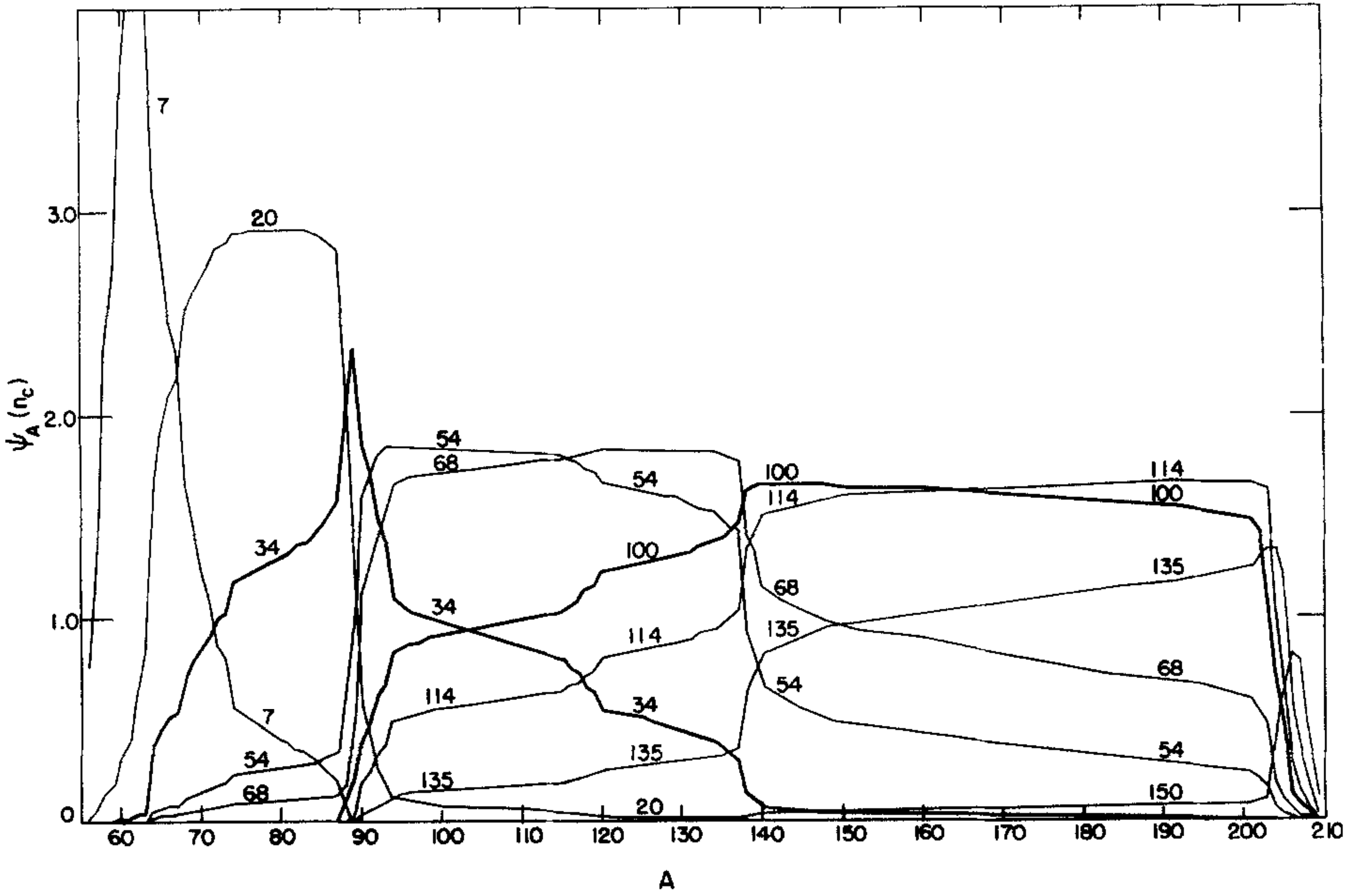
$$m_k = \frac{\left(\displaystyle\sum_{i=1}^{k} \frac{1}{\sigma_i}\right)^2}{\displaystyle\sum_{i=1}^{k} \frac{1}{\sigma_i{}^2}} = k \frac{\left\langle \dfrac{1}{\sigma} \right\rangle_k^2}{\left\langle \dfrac{1}{\sigma^2} \right\rangle_k}, \qquad \lambda_k = \frac{\displaystyle\sum_{i=1}^{k} \frac{1}{\sigma_i}}{\displaystyle\sum_{i=1}^{k} \frac{1}{\sigma_i{}^2}} = \frac{\left\langle \dfrac{1}{\sigma} \right\rangle_k}{\left\langle \dfrac{1}{\sigma^2} \right\rangle_k}.$$


k = 60, m_k = 13.45, λ_k = 2.87
Exact
Approximate
0.006
0.005
0.004
0.003
0.002
0.001
0.000
$N_k(\tau)/N_1(0)$
0
2
4
6
8
10
τ (mb^{-1})

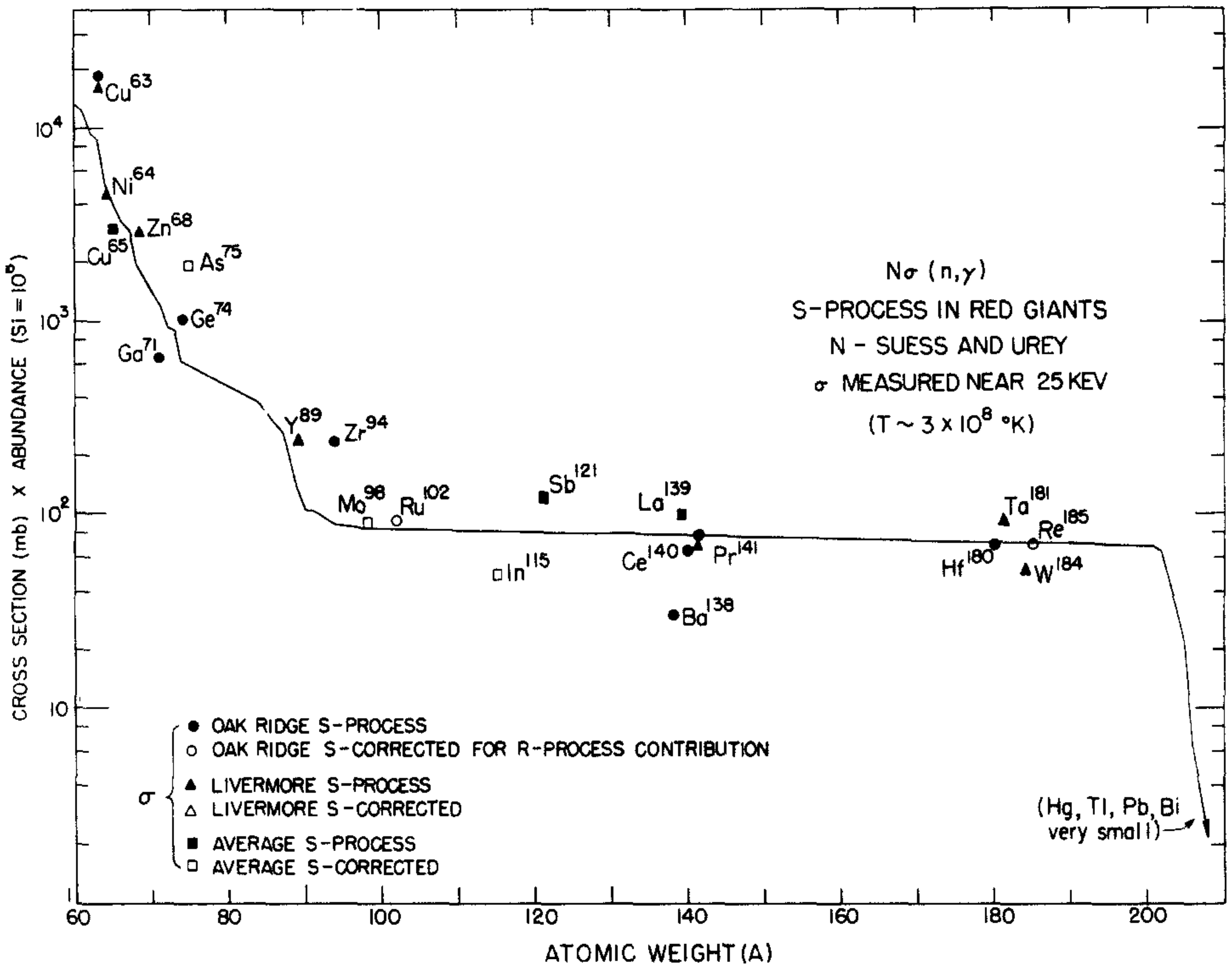
$\sigma_k = k$



COMPARISON FOR $\sigma_k = k$
ψ
0.5
0.4
0.3
0.2
0.1
$\tau = 2$ (approximate)
$\tau = 2$ (correct)
$\tau = 3$ (approximate)
$\tau = 3$ (correct)
1
17
33
49
65
81
97
113
129
145
161
k



$\psi_A(n_c)$
A
3.0
2.0
1.0
0
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
7
20
34
54
68
100
114
135
150



$$\sigma N = 2160\psi(\tau = 0.1) + 990\psi(\tau = 0.2) + 45\psi(\tau = 0.6) + 45\psi(\tau = 1.1)$$

Time evolution of a linear, directed network

$$\frac{dY}{dt} = AY$$

$$A_{ij} = \begin{cases} \lambda_{ji}, & i \neq j, 1 \leq i, j \leq n \\ -\Lambda_i = -\sum_{k=1, k\neq i}^{n} \lambda_{ik}, & i = j, 1 \leq i \leq n \end{cases}$$

$$Y(t) = MY(0)$$

$$M = \mathscr{L}^{-1}\left((s - A)^{-1}\right)$$

$$D = det\left(\begin{bmatrix} v_{11} + v_{21} & -v_{12} \\ -v_{21} & v_{22} + v_{12} \end{bmatrix}\right)$$

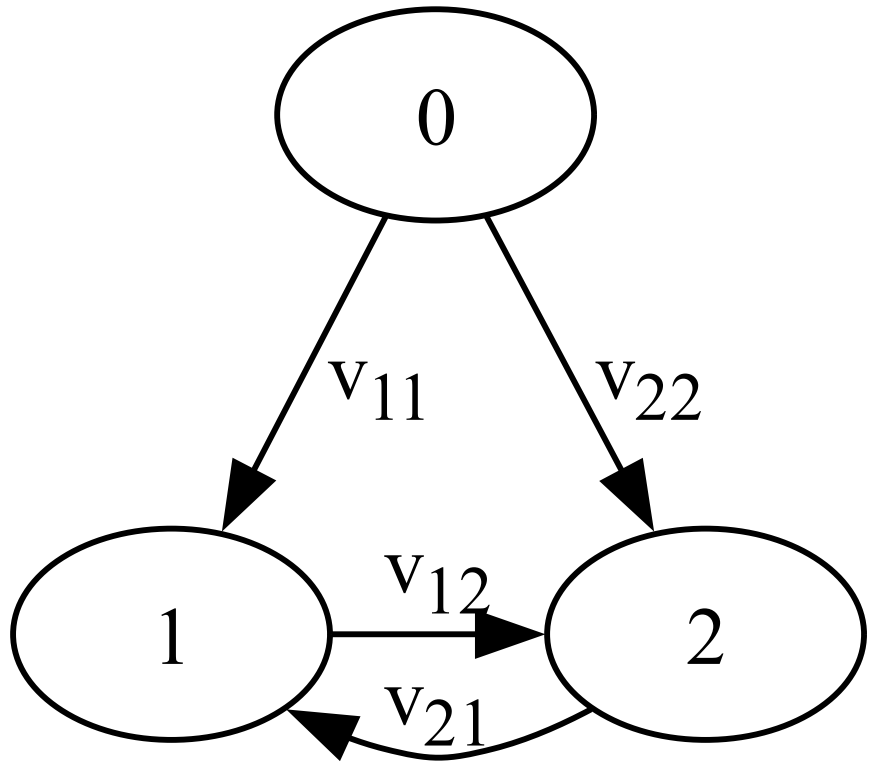
$$D = (v_{11} + v_{21})(v_{22} + v_{12}) - v_{21}v_{12} = v_{11}v_{22} + v_{11}v_{21} + v_{21}v_{22} + \cancel{v_{21}v_{12}} - \cancel{v_{21}v_{12}}$$

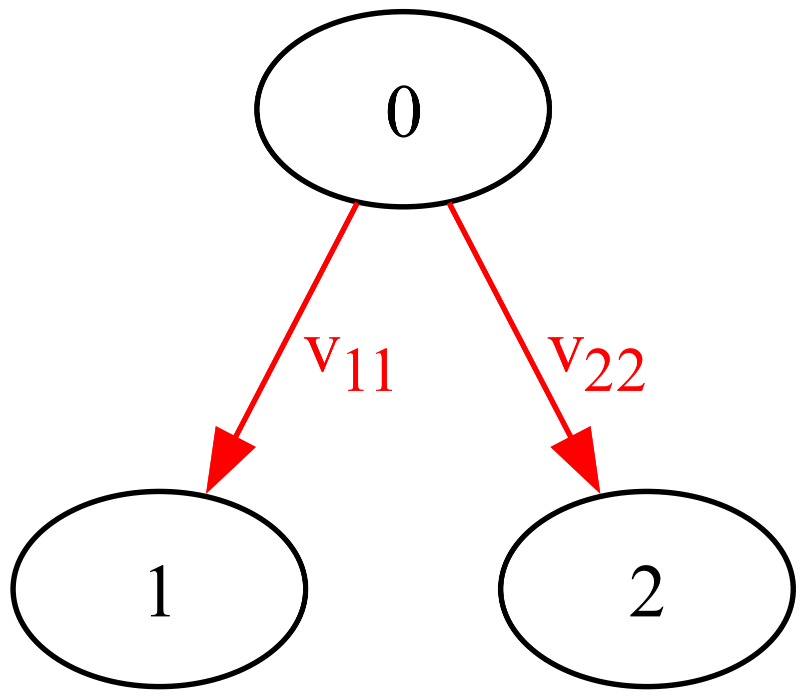
$$D = v_{11}v_{22} + v_{11}v_{12} + v_{22}v_{11}$$

Matrix-Tree Theorem (Tutte 1948)

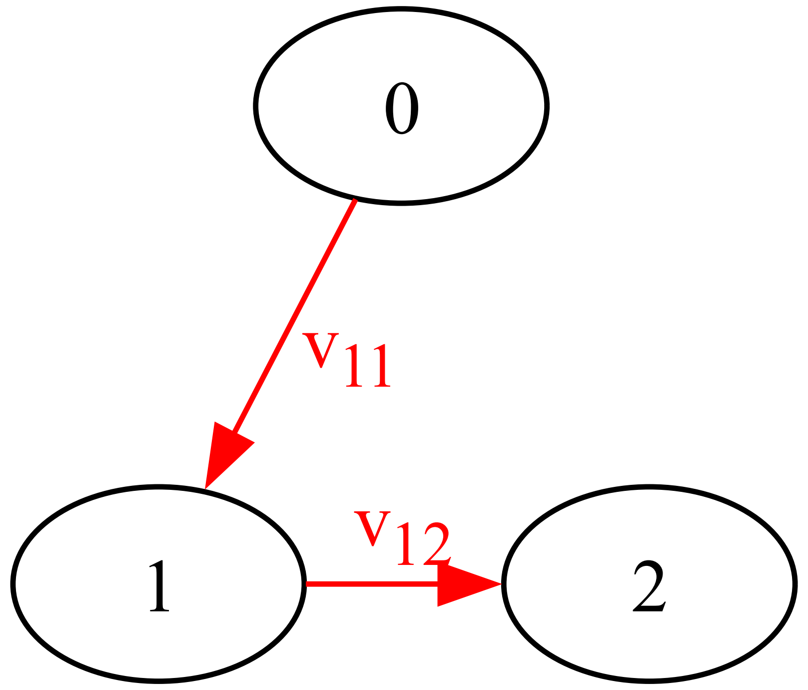
$$D = det\left(\begin{bmatrix} v_{11} + v_{21} & -v_{12} \\ -v_{21} & v_{22} + v_{12} \end{bmatrix}\right)$$

$$D = v_{11}v_{22} + v_{11}v_{12} + v_{22}v_{11}$$

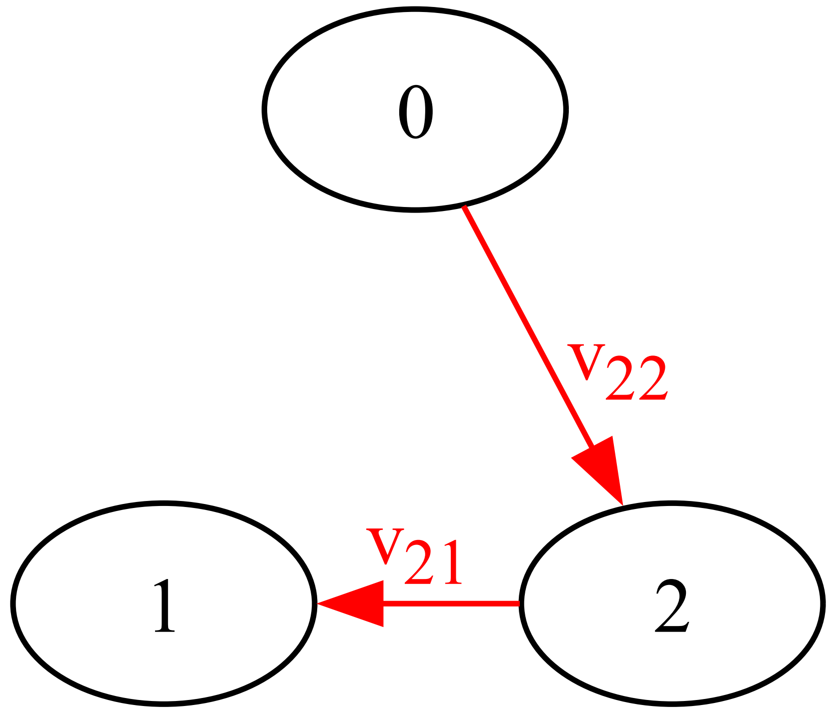




$v_{11}v_{22}$



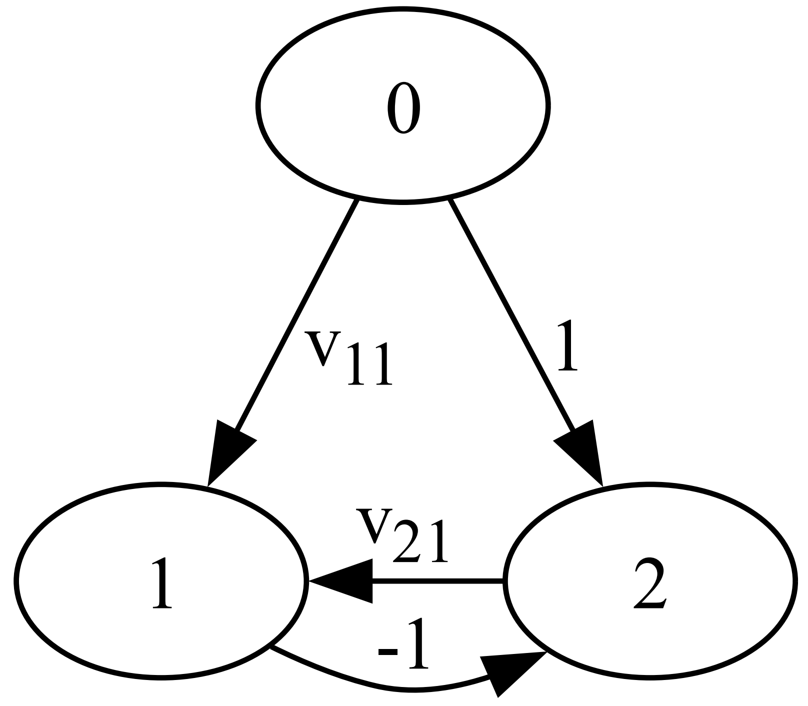
$v_{11}v_{12}$



$v_{22}v_{21}$

Matrix-Forest Theorem

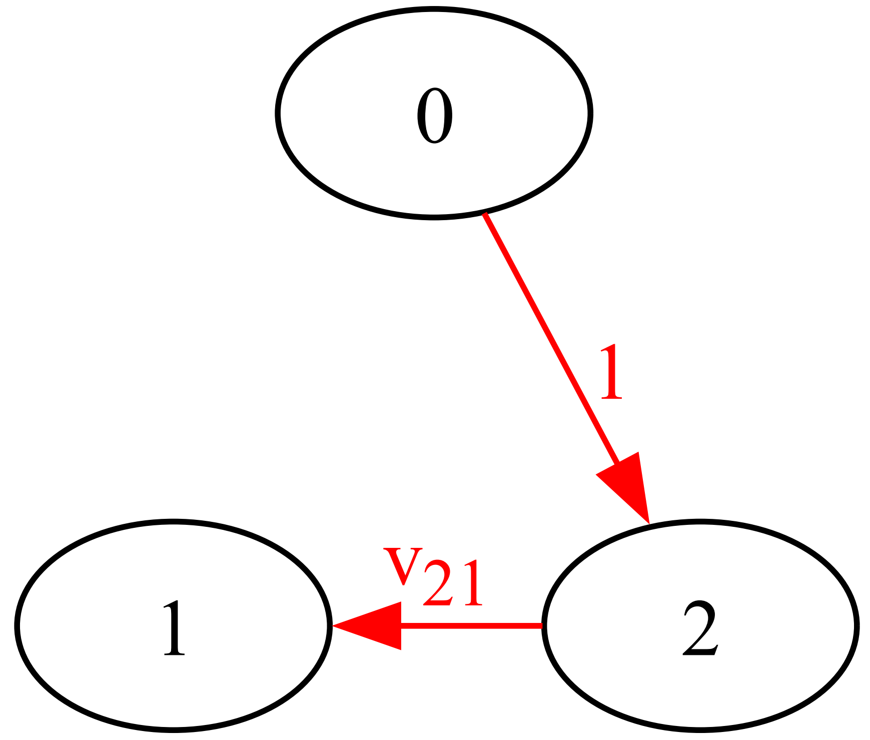
$$D = det\left(\begin{bmatrix} v_{11} + v_{21} & 1 \\ -v_{21} & 0 \end{bmatrix}\right)$$



Chaiken (1982)

Ghosh and Meyer (2025)

$$D = v_{21}$$



v_{21}

Analytic solution for contribution of species i to species j over time t in a directed, linear network (Ghosh and Meyer 2025)

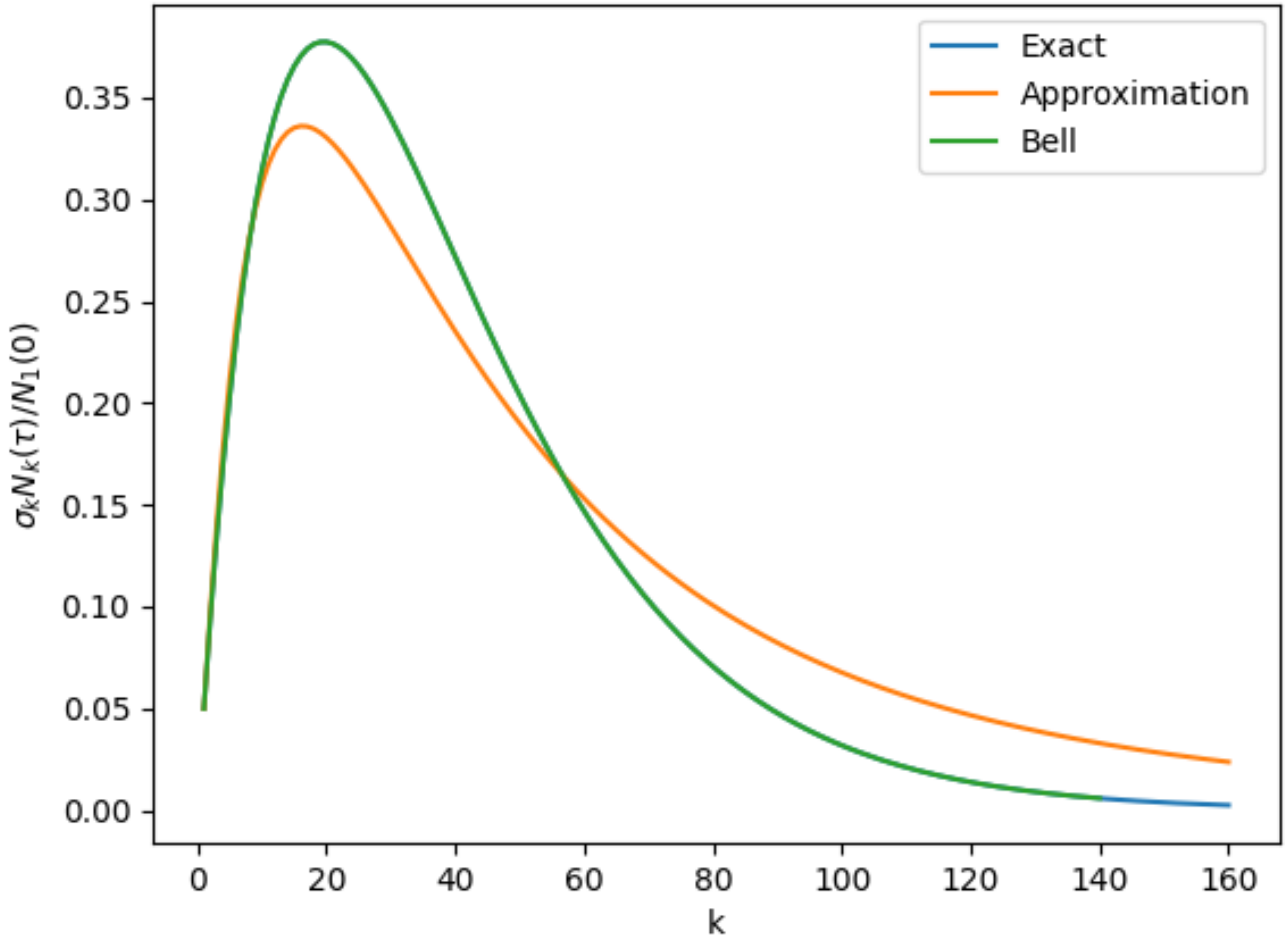
$$Y(t) = MY(0) \Rightarrow Y_j(t) = \sum_i M_{ji} Y_i(0) \qquad M_{ji} = \sum_{\mathscr{P}} G^{(\mathscr{P})}(j, t; i, 0)$$

$$G^{(\mathscr{P})}(j, t; i, 0) = F^{(\mathscr{P})} \frac{(\Lambda t)^{N(\mathscr{P})-1}}{\big(N(\mathscr{P}) - 1\big)!} \exp\left[\sum_{j=1}^{\infty} x_j \frac{(-t)^j}{j!}\right]$$

$$F^{(\mathscr{P})} = \prod_{i=1}^{N(\mathscr{P})-1} \frac{\Lambda_i^{(\mathscr{P})}}{\Lambda_i} \qquad \Lambda = \left(\prod_{i=1}^{N(\mathscr{P})-1} \Lambda_i\right)^{1/(N(\mathscr{P})-1)}$$

$$B_n(x_1, \ldots, x_n) = \bar{h}_n(\Lambda_1, \ldots, \Lambda_{N(\mathscr{P})})$$

$$\tau = 3\ mb^{-1}$$



$$\sigma_k = k$$

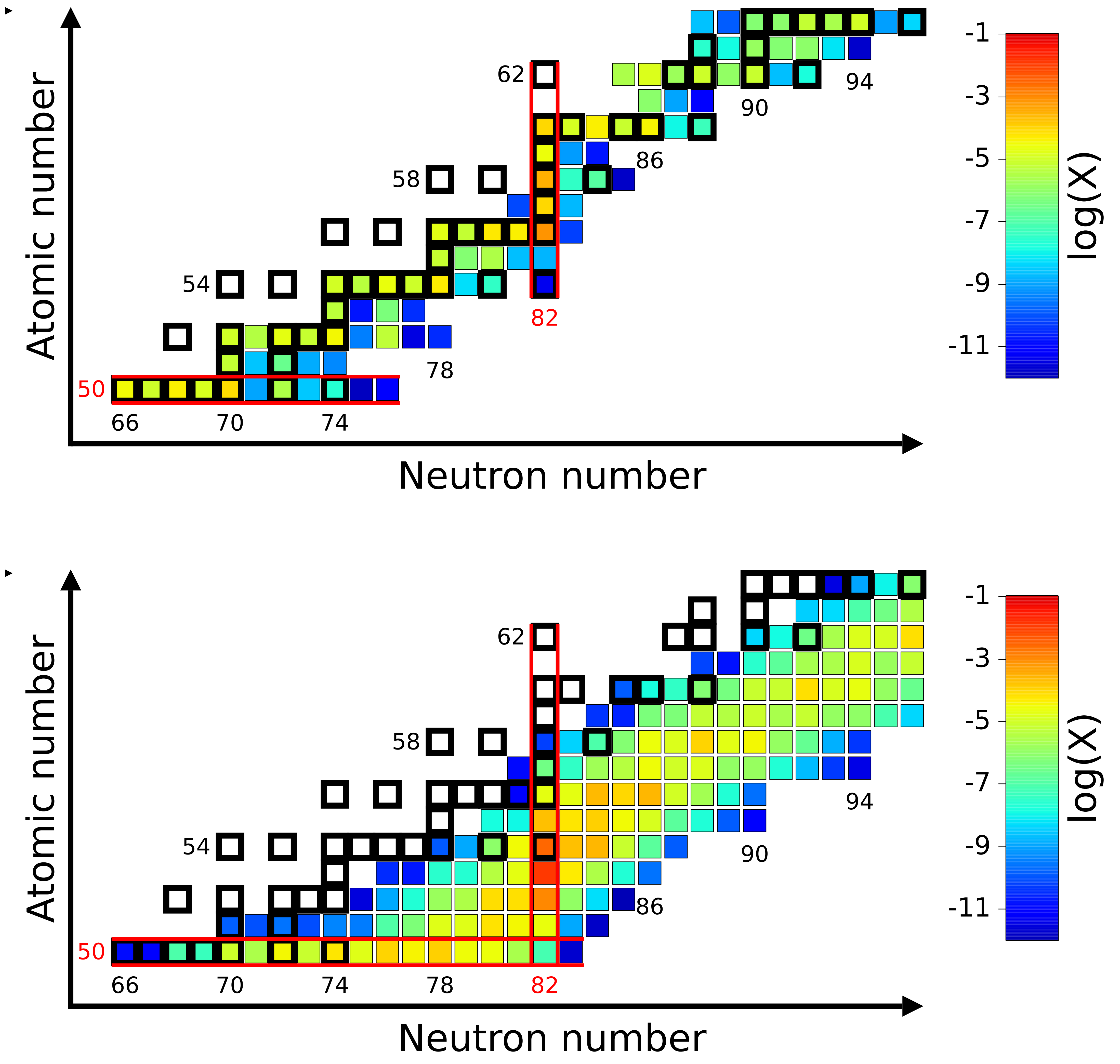
SIREN

sIREN

siREN

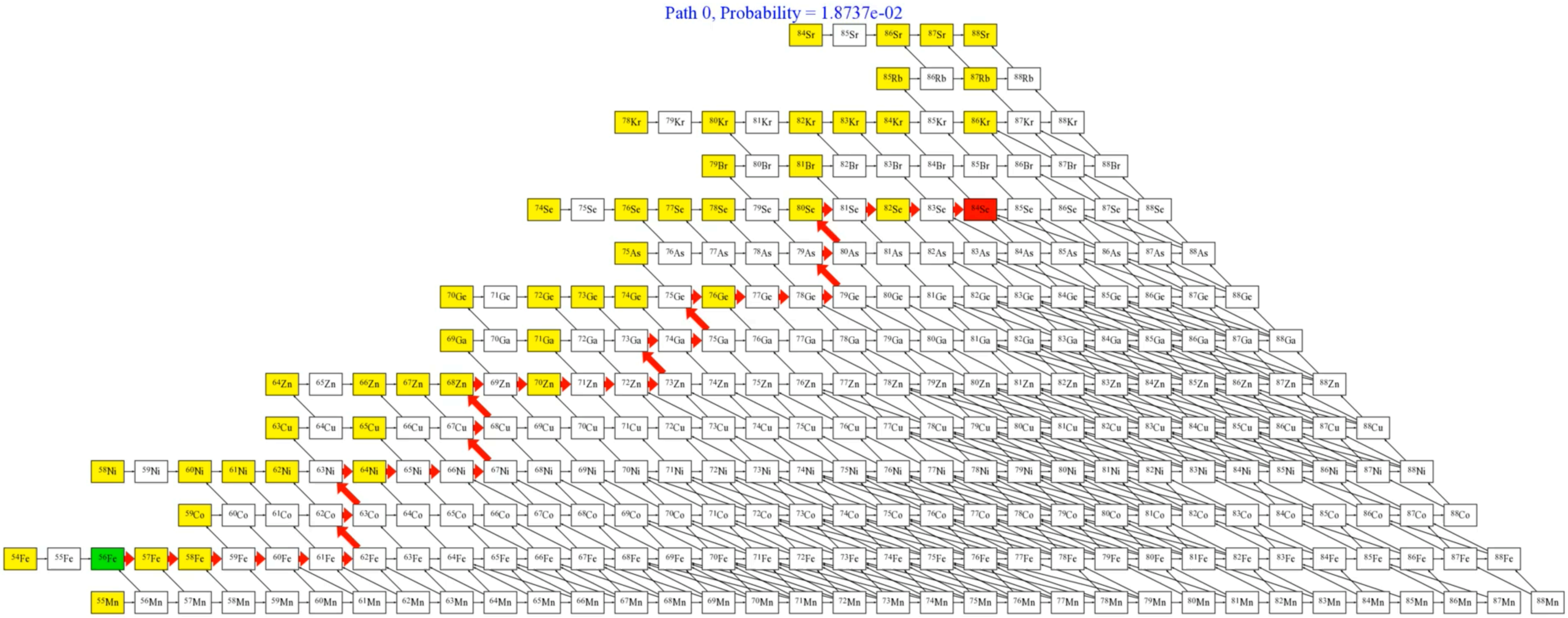
$n_n = 10^7\ cm^{-3}$

$n_n = 10^{15}\ cm^{-3}$

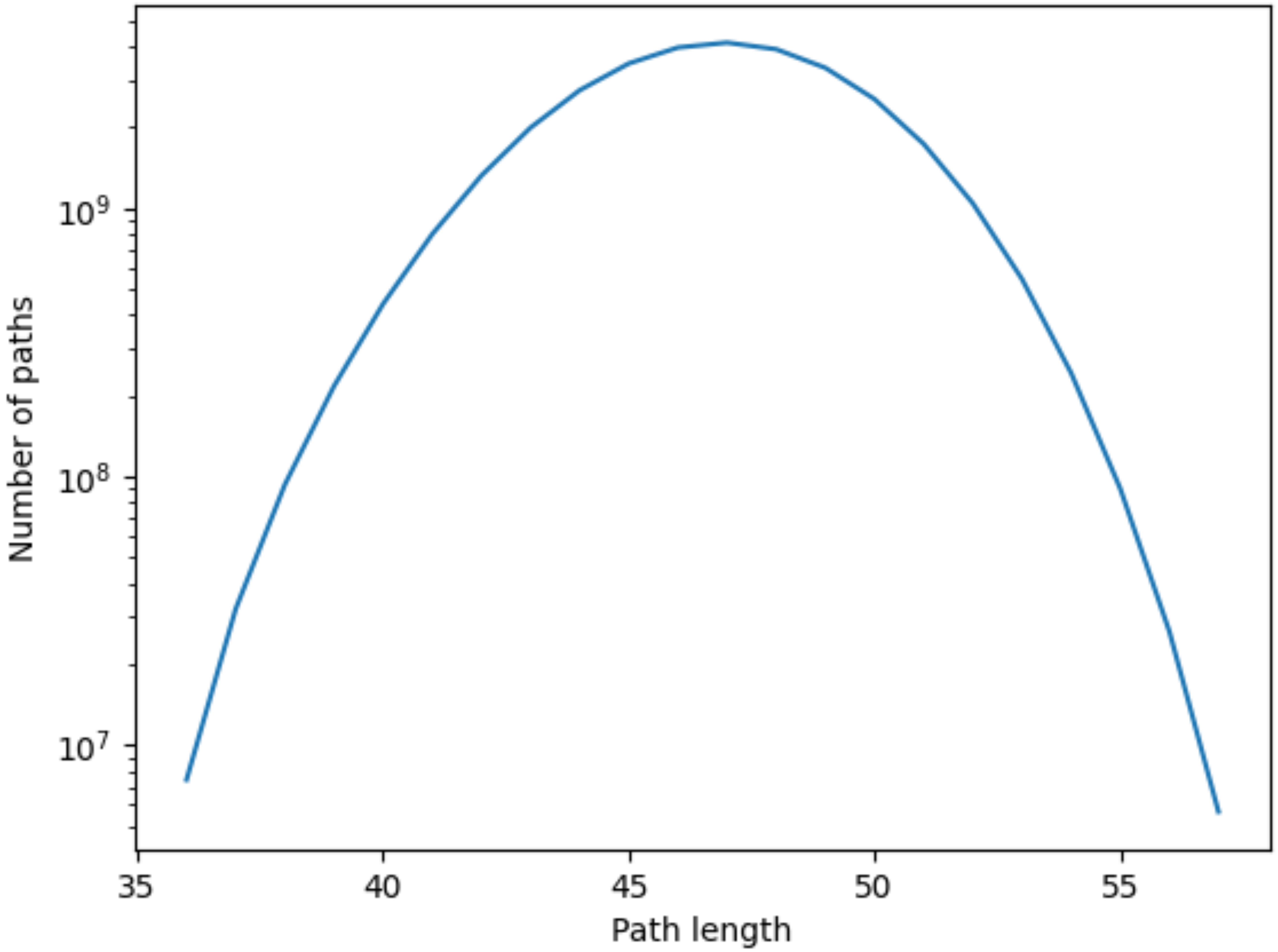


Hampel et al. (2016)

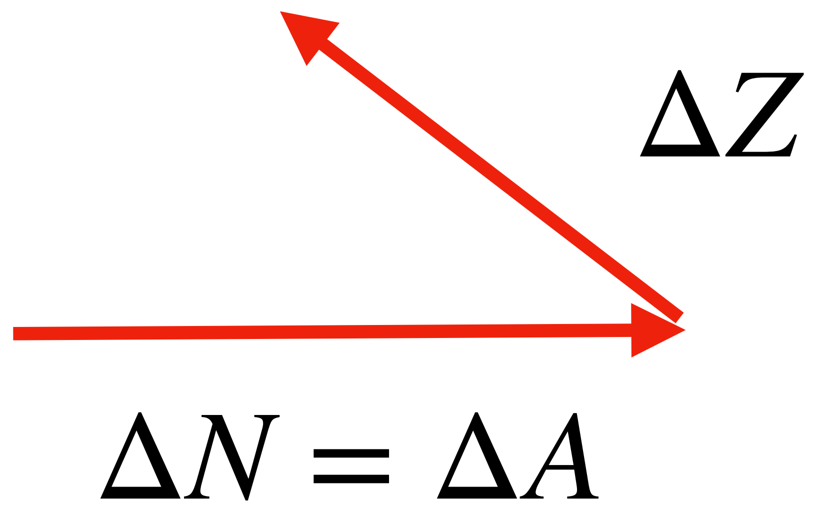
Figure 1. Neutron capture paths of the models with a constant neutron density of $n = 10^7$ cm^{-3} (upper panel) and $n = 10^{15}$ cm^{-3} (lower panel) shown in the



$$n_n = 10^{15}\ cm^{-3} \quad T_9 = 0.15$$



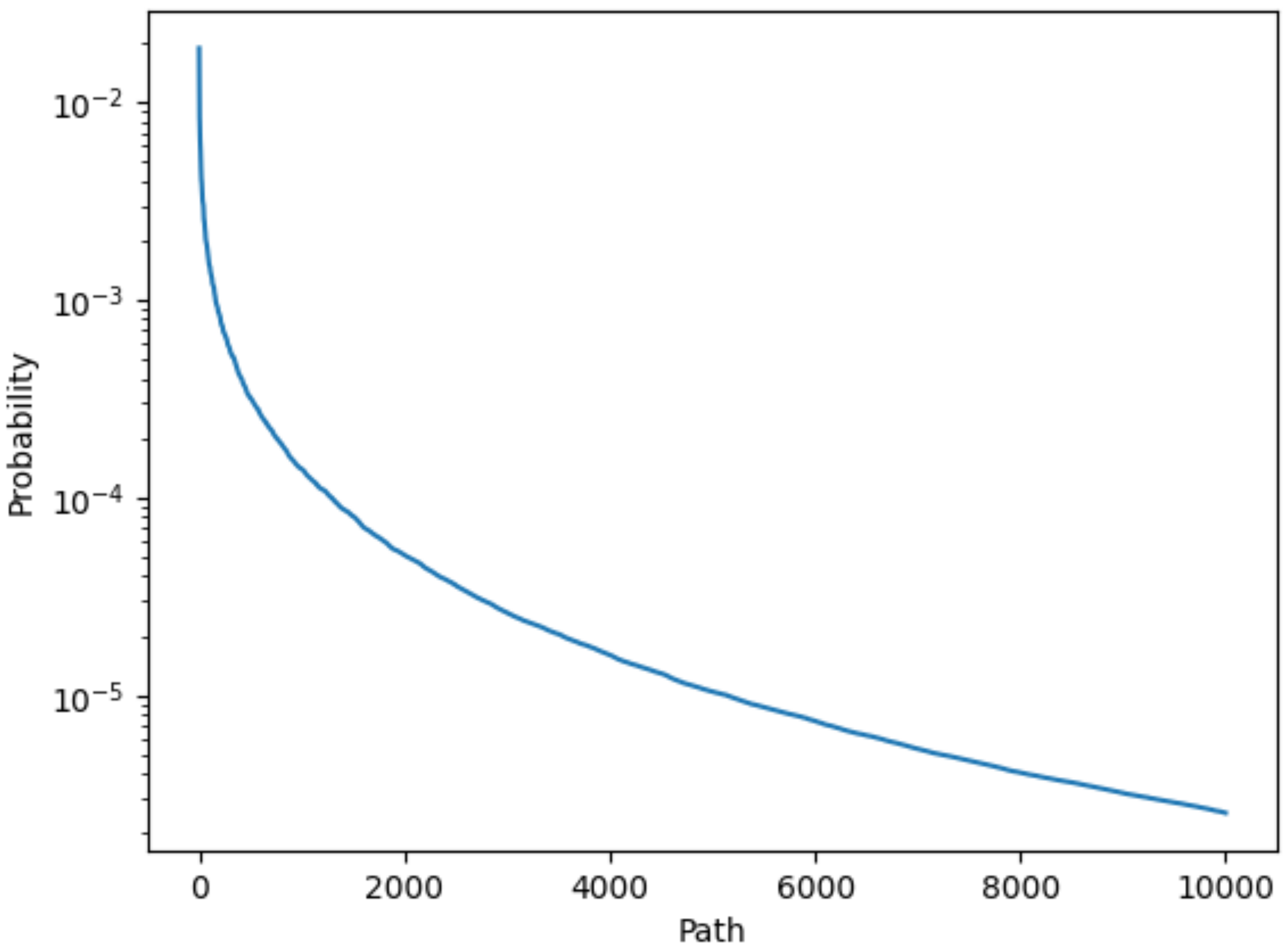
Total number of paths = 33 billion



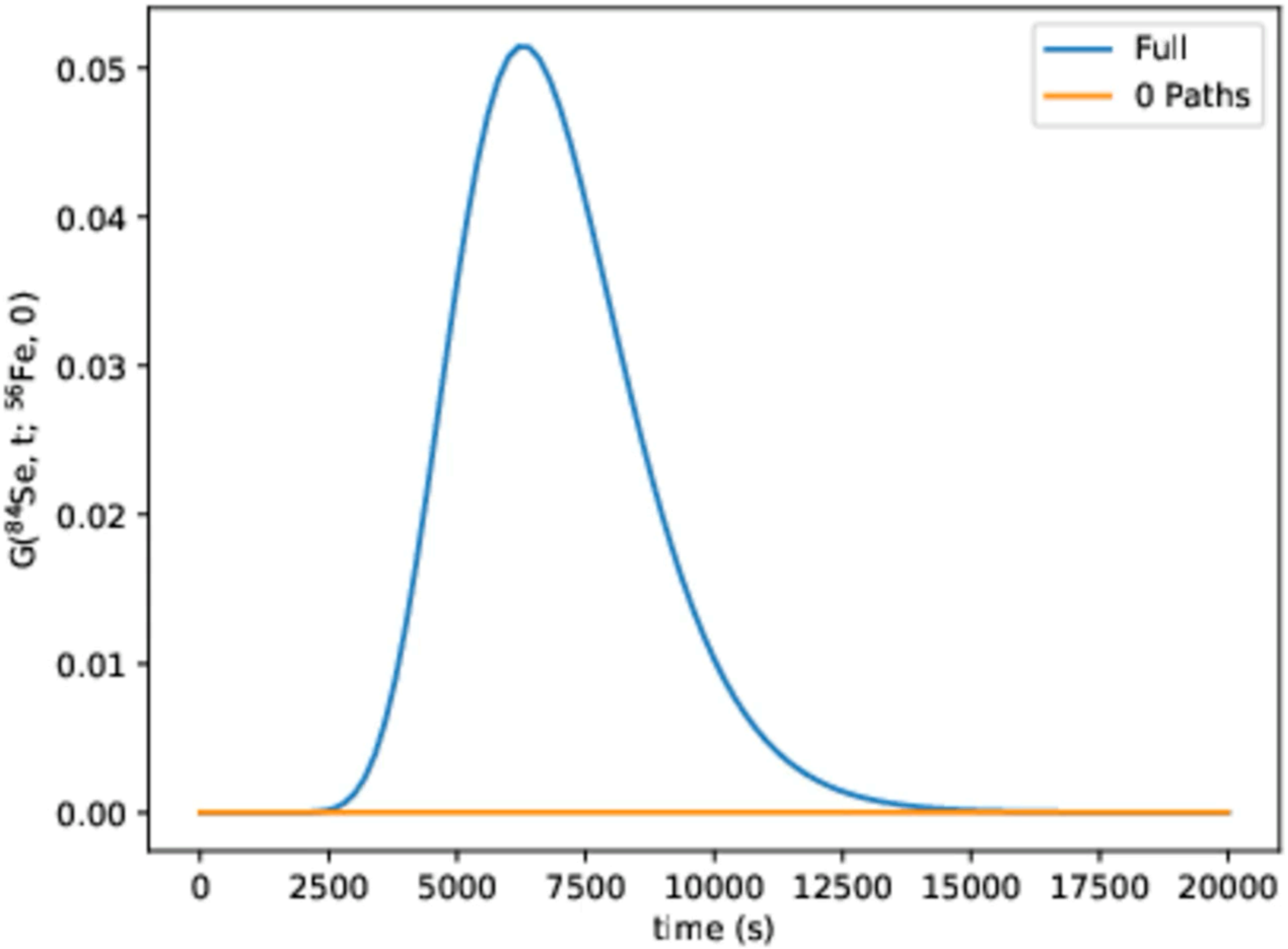
$$L \geq \Delta A + \Delta Z$$

$$^{56}Fe \rightarrow ^{84}Sr$$

$$\Rightarrow L \geq 28 + 8 = 36$$

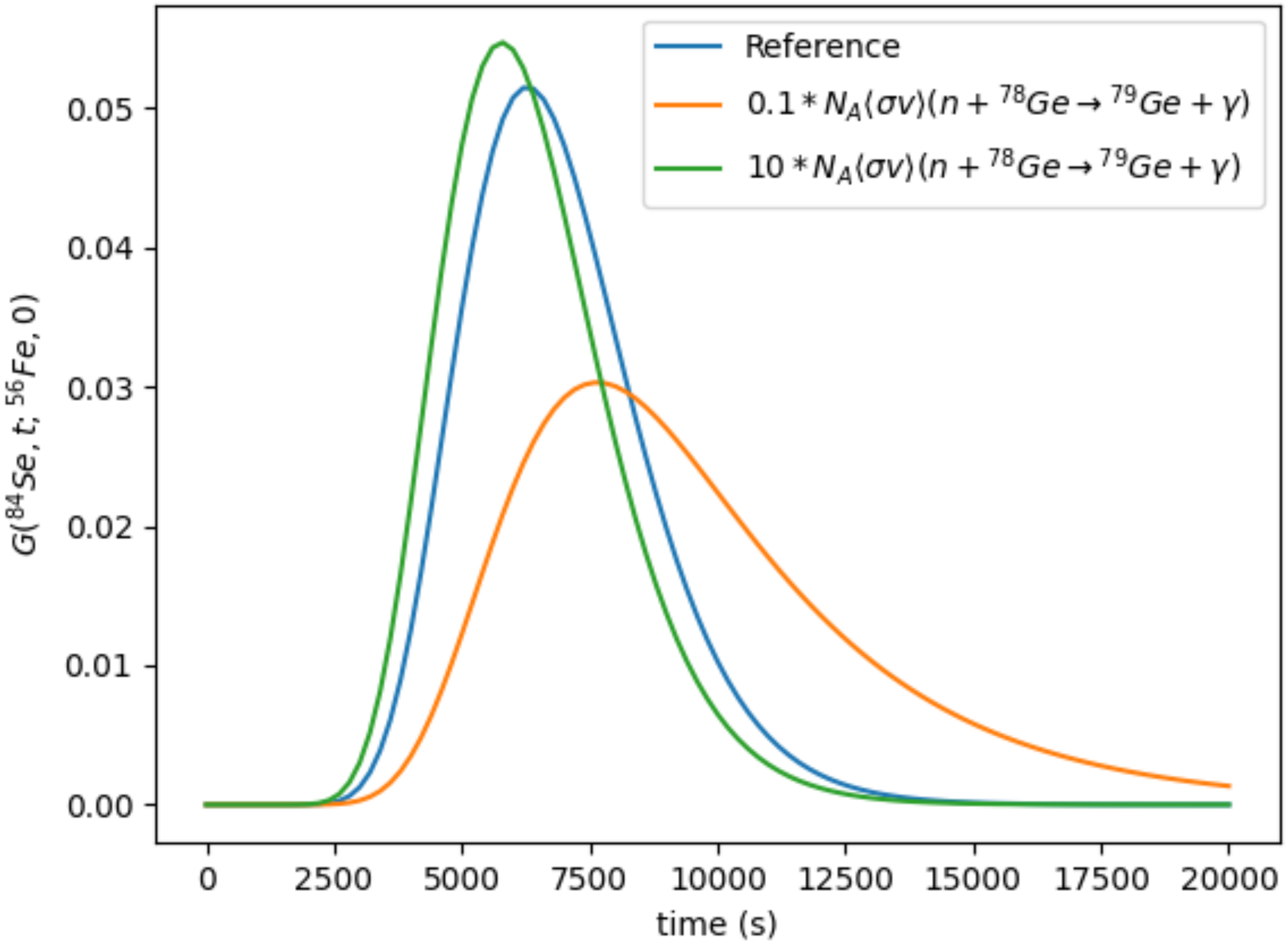


Probability
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
2000
4000
6000
8000
10000
Path



Full
0 Paths
G(84Se, t; 56Fe, 0)
0.00
0.01
0.02
0.03
0.04
0.05
0
2500
5000
7500
10000
12500
15000
17500
20000
time (s)

Reaction	Path Occurrences	Path Percentage
n + fe56 -> fe57 + gamma	10000	100.00%
n + fe57 -> fe58 + gamma	10000	100.00%
n + fe58 -> fe59 + gamma	10000	100.00%
n + fe59 -> fe60 + gamma	10000	100.00%
n + fe60 -> fe61 + gamma	10000	100.00%
n + ni64 -> ni65 + gamma	9974	99.74%
n + se83 -> se84 + gamma	9973	99.73%
n + ni65 -> ni66 + gamma	9253	92.53%
n + ge77 -> ge78 + gamma	9187	91.87%
n + ni66 -> ni67 + gamma	9012	90.12%
n + se82 -> se83 + gamma	9002	90.02%
n + ga74 -> ga75 + gamma	8415	84.15%
n + ge78 -> ge79 + gamma	7883	78.83%

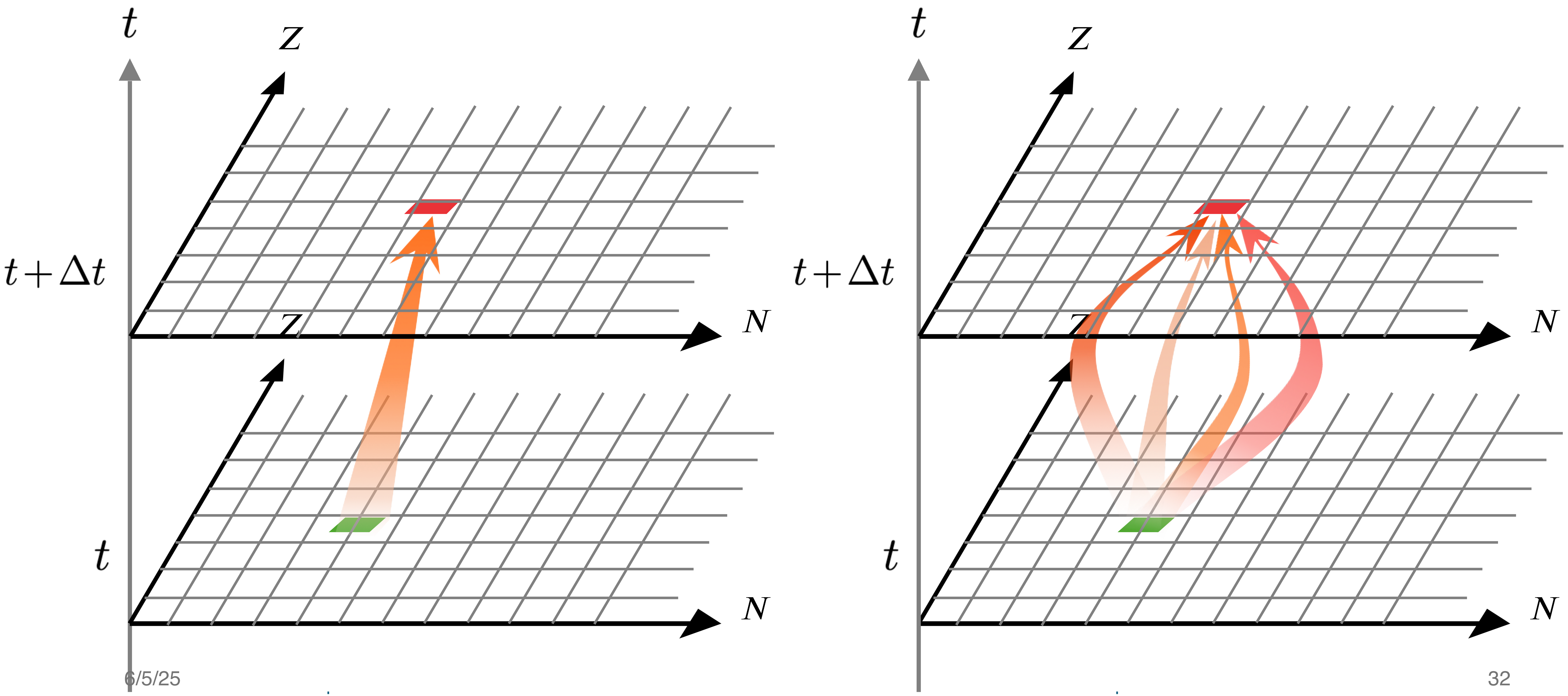


Reference
$0.1 * N_A\langle\sigma v\rangle(n + {}^{78}Ge \to {}^{79}Ge + \gamma)$
$10 * N_A\langle\sigma v\rangle(n + {}^{78}Ge \to {}^{79}Ge + \gamma)$
$G({}^{84}Se, t; {}^{56}Fe, 0)$
0.00
0.01
0.02
0.03
0.04
0.05
0
2500
5000
7500
10000
12500
15000
17500
20000
time (s)

sirEN

Contribution pathway

Li and Meyer (2025)



$$
\begin{aligned}
F_\ell(Z,N) &= \frac{\tilde{D}_\ell(Z,N)}{D_\ell(Z,N)} \\
&= \frac{1+\lambda_n(Z,N-1)\Delta t F_\ell(Z,N-1)}{\lambda_\gamma(Z,N)\Delta t + w(Z,N)[1+\lambda_n(Z,N-1)\Delta t F_\ell(Z,N-1)]}.
\end{aligned}
$$

$$
\begin{aligned}
F_u(Z,N) &= \frac{\tilde{D}_u(Z,N)}{D_u(Z,N)} \\
&= \frac{1+\lambda_\gamma(Z,N+1)\Delta t F_u(Z,N+1)}{\lambda_n(Z,N)\Delta t + w(Z,N)[1+\lambda_\gamma(Z,N+1)\Delta t F_u(Z,N+1)]}.
\end{aligned}
$$

$$
\begin{aligned}
\lambda_n^{'}(Z,N-1) &= \lambda_n(Z,N-1)F_\ell(Z,N-1) \\
\lambda_\gamma^{'}(Z,N+1) &= \lambda_\gamma(Z,N+1)F_u(Z,N+1),
\end{aligned}
$$

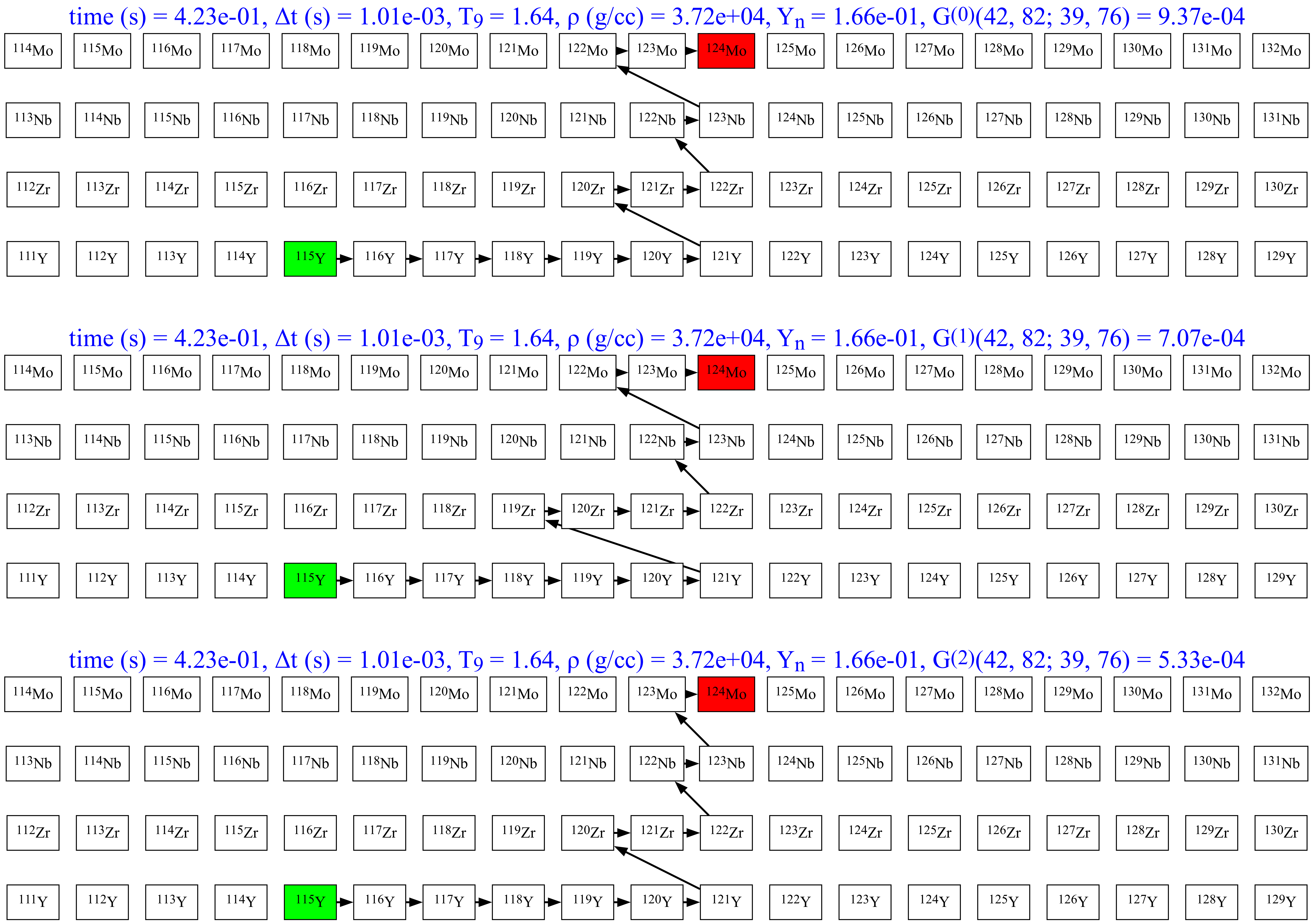


Figure 23. The contribution path from ^{115}Y to ^{124}Mo in equilibrium phase.

Conclusion

$$\frac{dY}{dt} = AY$$

$$\Rightarrow Y_j(t) = \sum_i \left(\sum_{\mathscr{P}} G^{(\mathscr{P})}(j, t; i, 0)] \right) Y_i(0)$$