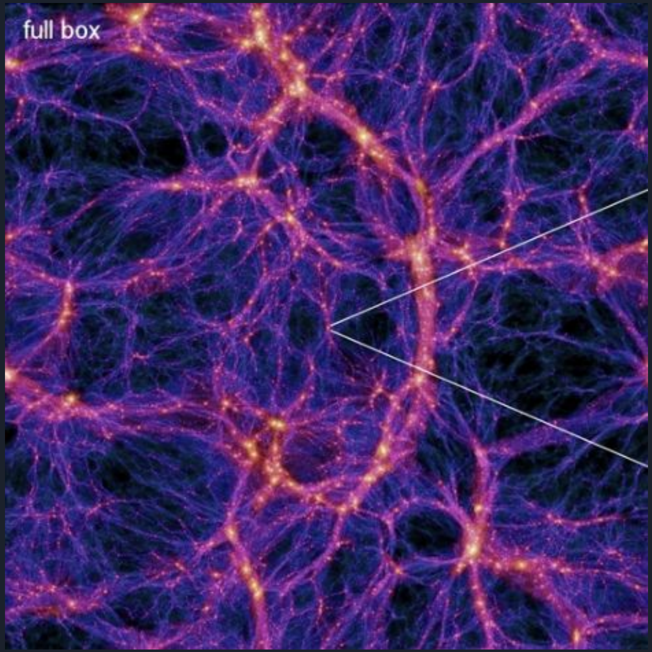
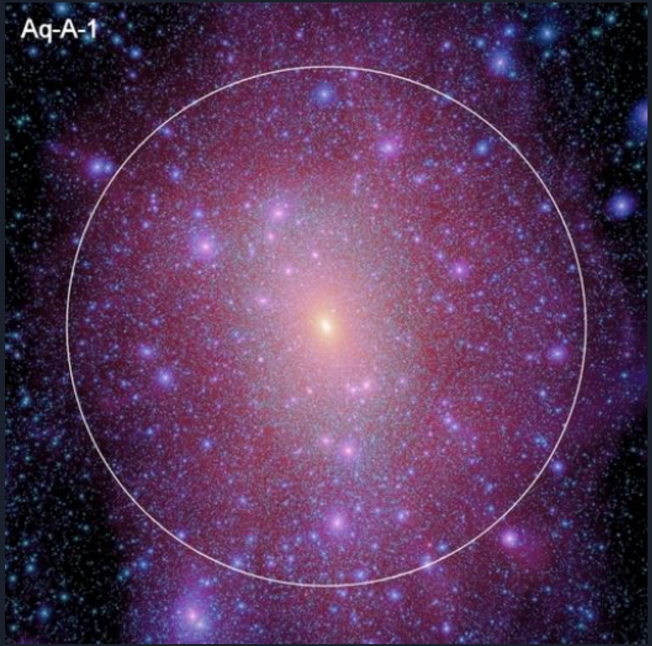
Tidal Stripping and the fate of dark substructures of the Milky Way

Jens Stücker
Feb. 19 2025
Dynamical DM tracers meeting

LCDM predicts structure on a vast range of scales

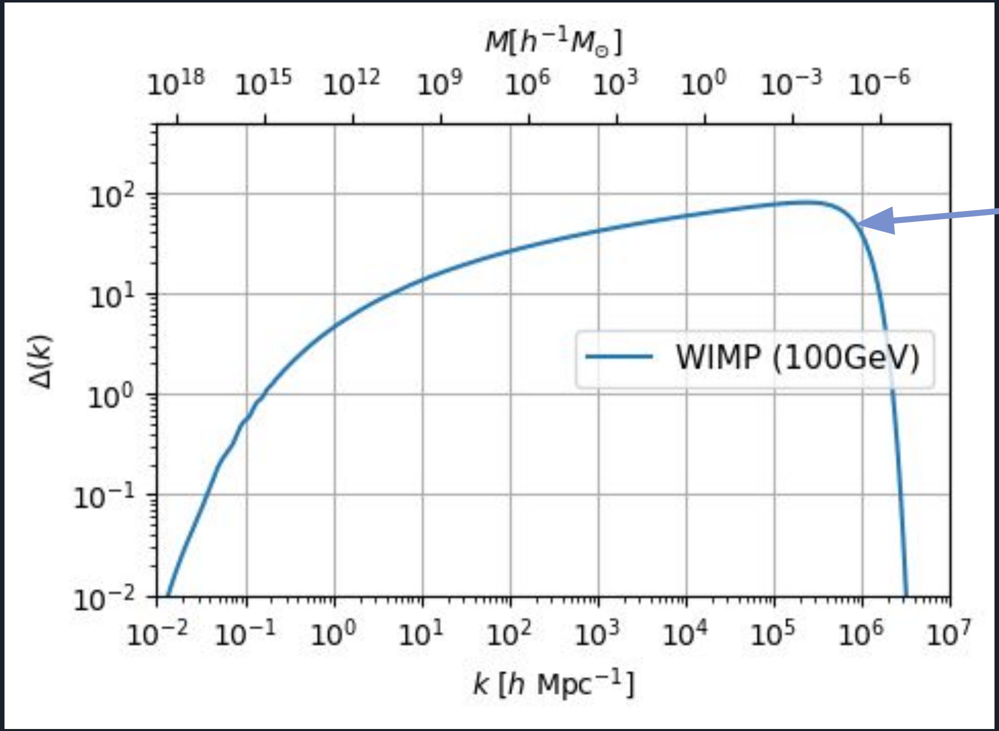




Aquarius Simulations (Springel et al 2008)

Haloes may exist as small as Earth masses

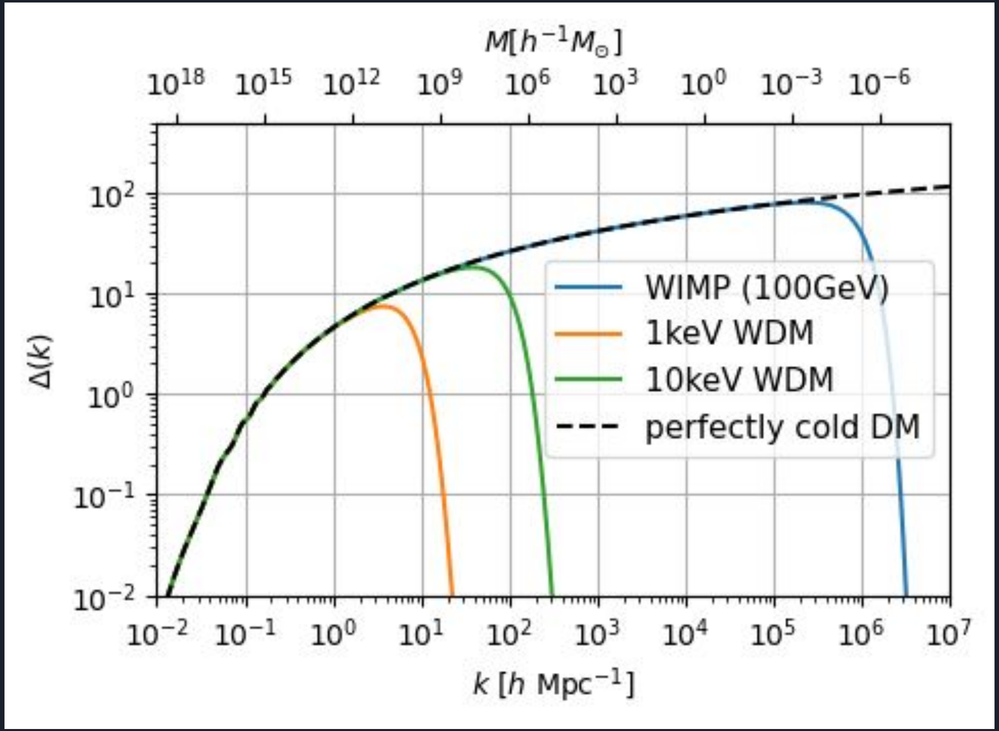
$\Delta(k) = d\sigma^2 / d\ln k$



~ parsec scale
Earth mass haloes
('prompt cusps')

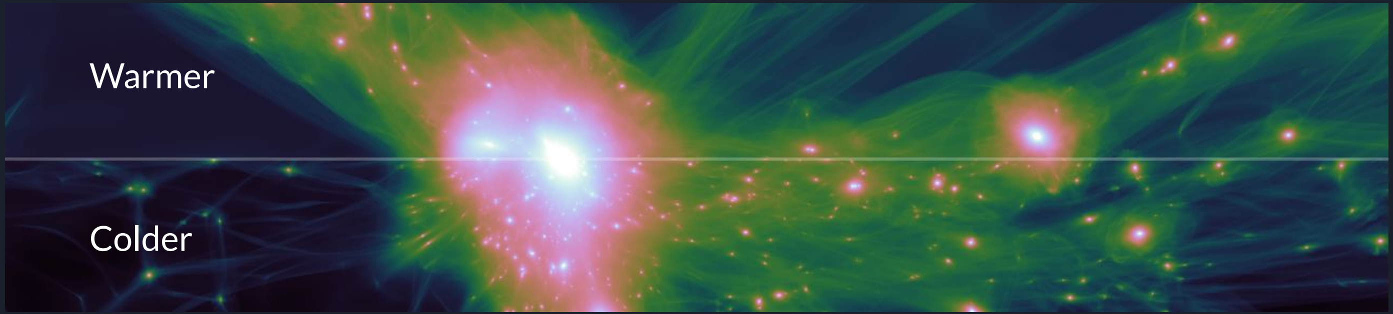
Haloes may exist as small as Earth masses

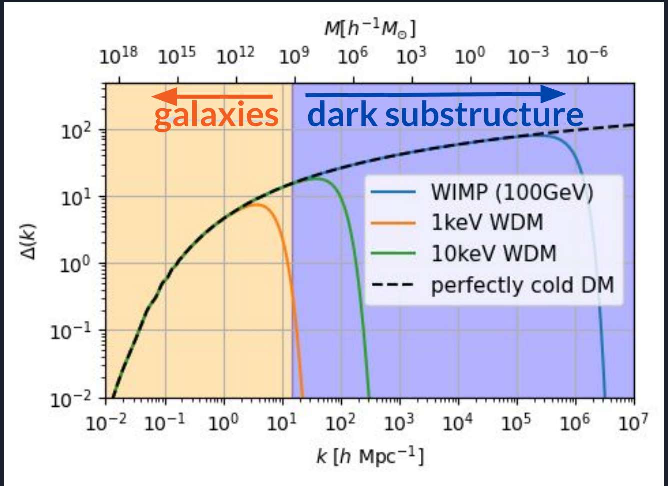
$\triangle(k) = d\sigma^2 / d\ln k$




a=0.02
Colder
Warmer

Constraining the Nature of DM

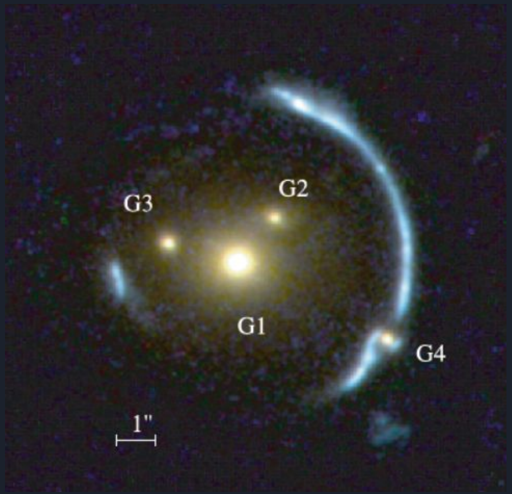




- Galaxies form only in DM haloes $\gtrsim 10^9 M_\odot$
- The presence of (lower mass) dark haloes is very sensitive to the nature of dark matter
- Finding or disproving their presence is a powerful probe of DM

How can we find dark substructures?

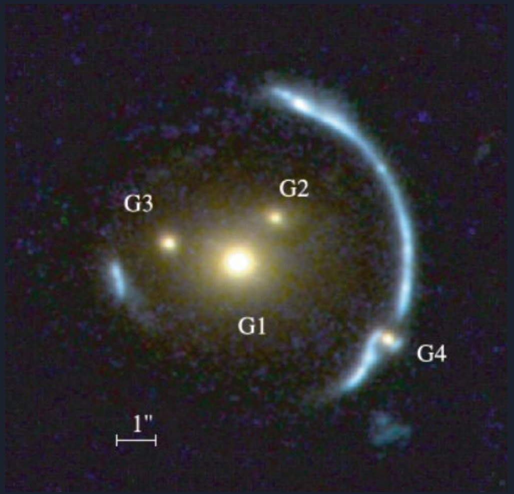
Example 1: Gravitational Lensing



Vegetti et al (2010) / Lin et al. (2009)

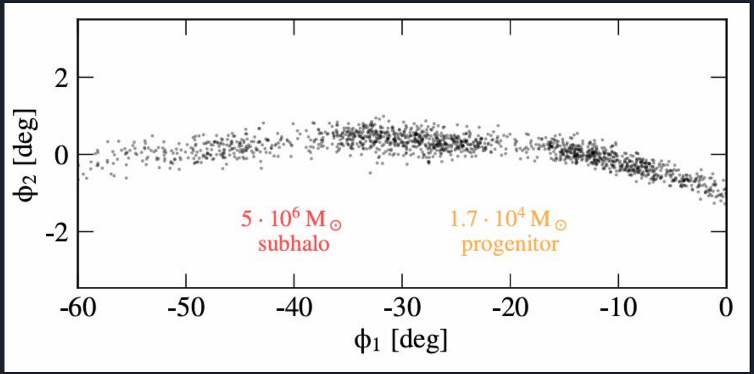
How can we find dark substructures?

Example 1: Gravitational Lensing



Vegetti et al (2010) / Lin et al. (2009)

Example 2: Gaps in stellar streams



Bonaca & Price-Whelan (2024)

Both methods may probe dark matter substructures as small as $M \sim 10^6 M_\odot$

Constraining the Nature of DM through substructure

Observations

- The smallest galaxies
- Gaps in Stellar Streams
- Perturbations in gravitational lenses
- Dark Matter self-annihilation
- …

+

Predictions of subhalo abundance (and properties)

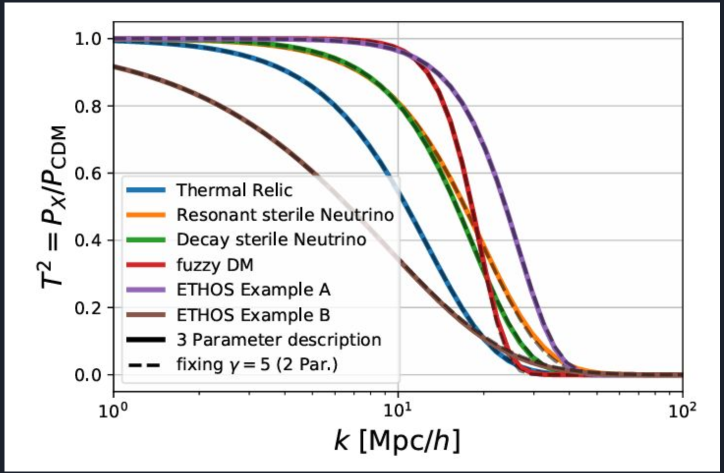
- Cosmological simulations
- Idealized simulations
- Analytical tools

Constraints on the Nature of DM

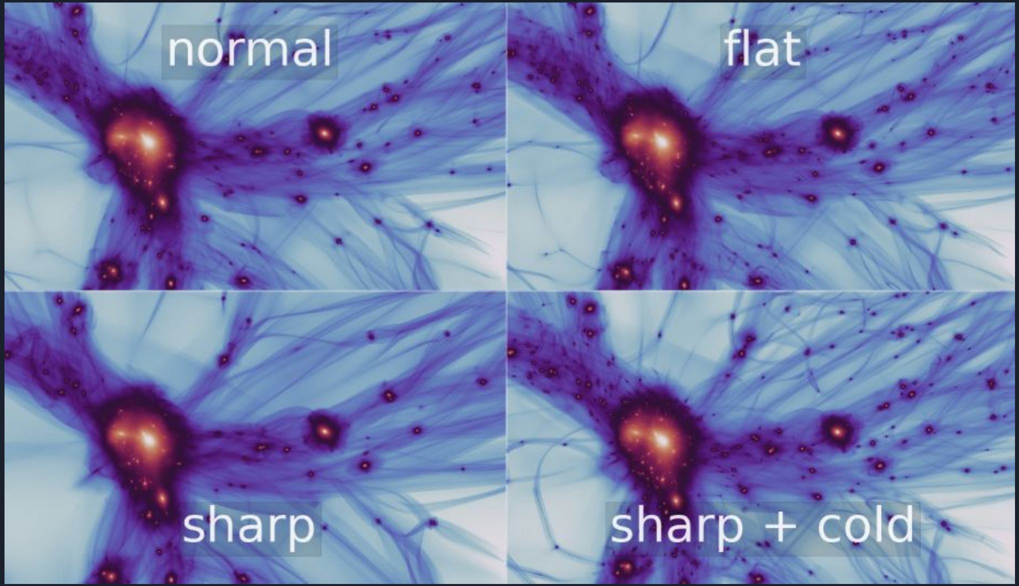
This Talk

Predictions of the abundance of haloes

Haloes and the Nature of Dark Matter



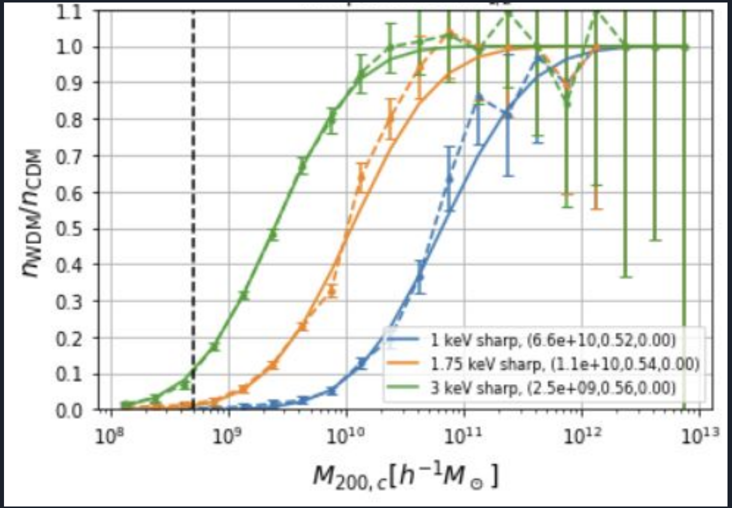
(Most DM models can be captured by a the scale and the sharpness of the cut-off)



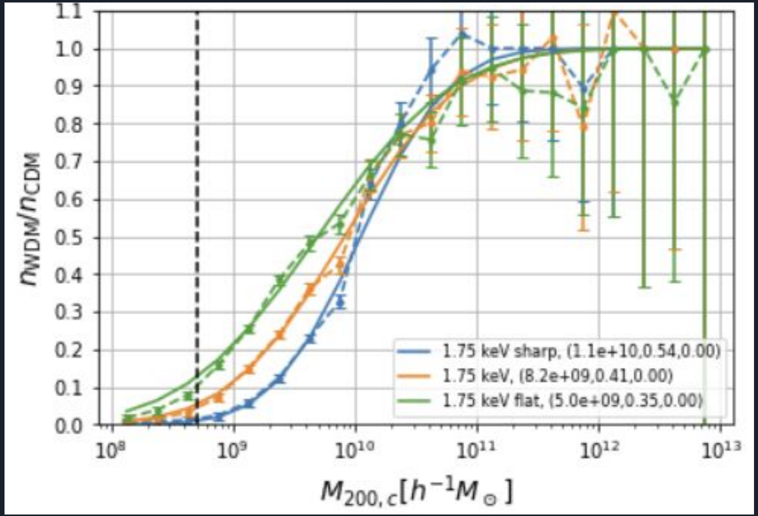
Stücker et al. (2021)

Haloes and the Nature of Dark Matter

Varying the **scale** of the cut-off

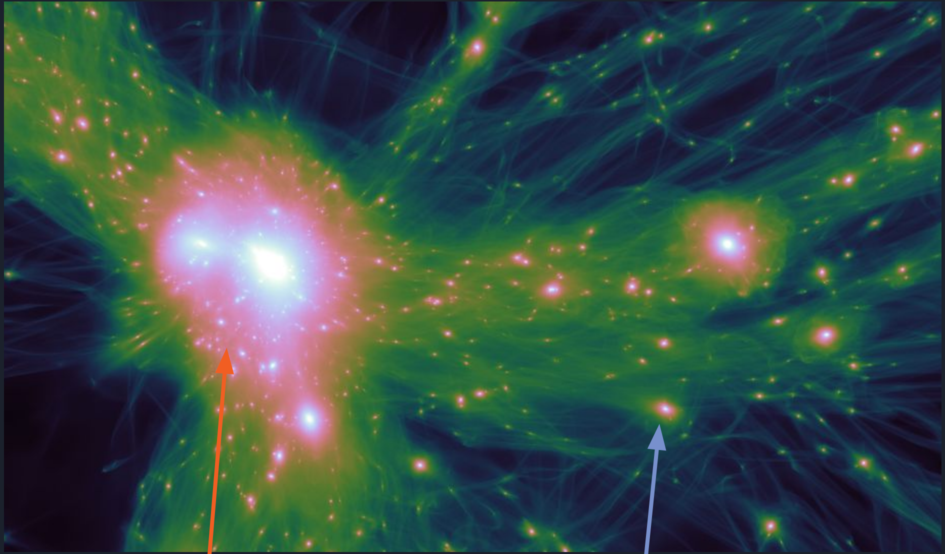


Varying the **sharpness** of the cut-off



Stücker et al. (2021)

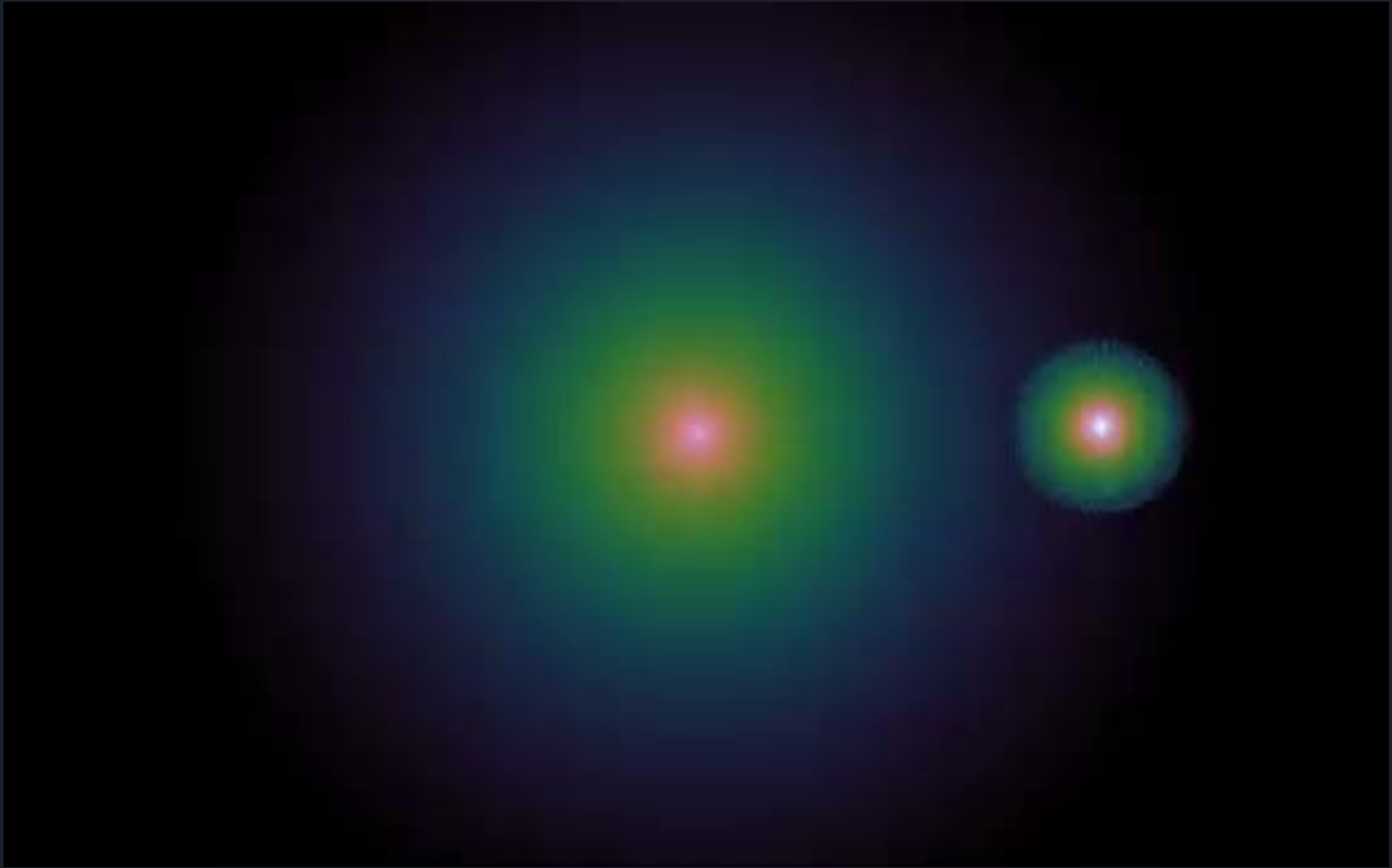
Haloes vs. Subhaloes



subhaloes

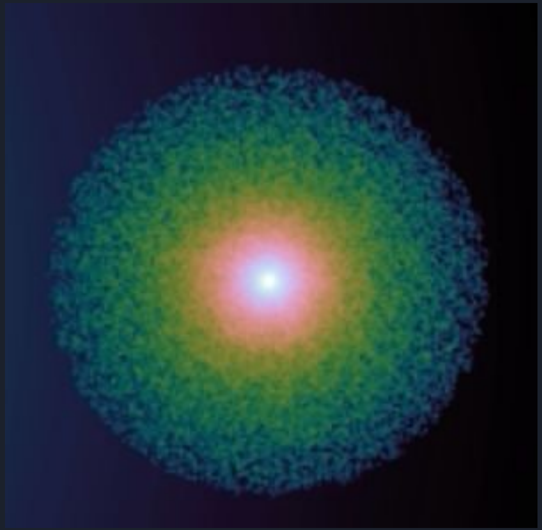
field haloes

Can we reliably predict the dark substructure with cosmological N-body simulations?



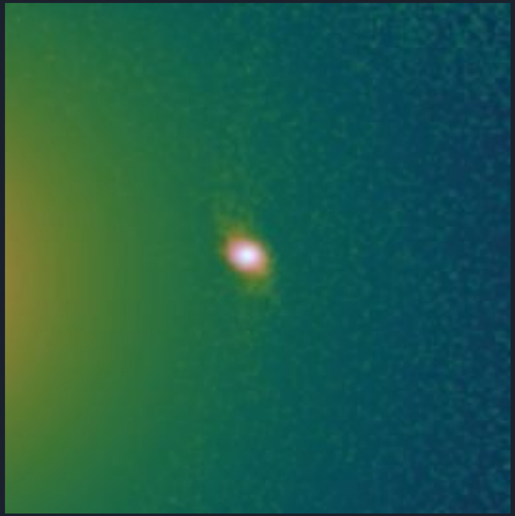
Tidal Stripping and mass loss

before $M \sim 10^9 M_\odot$



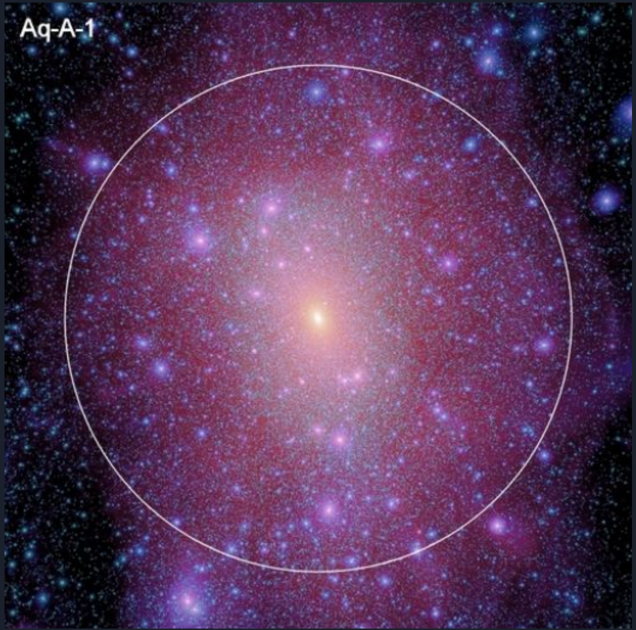
vs.

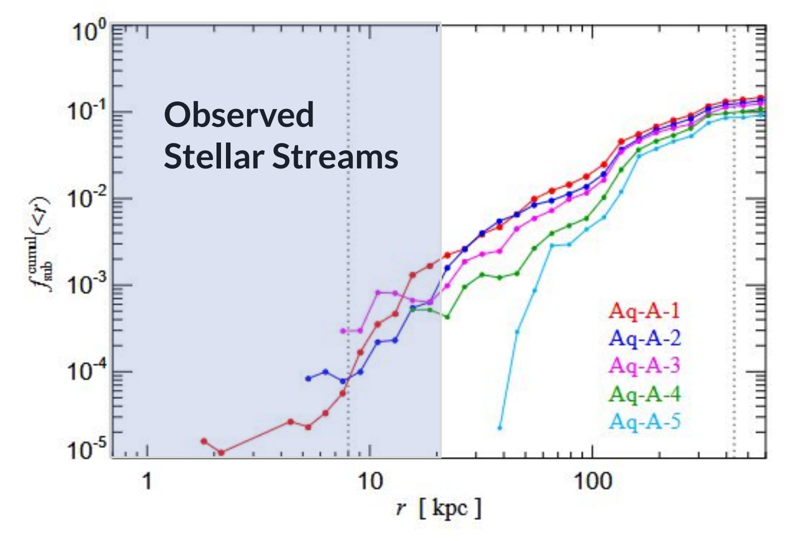
after $M \sim 10^8 M_\odot$



(This is a very moderate stripping scenario)

Is tidal stripping resolved in N-body simulations?

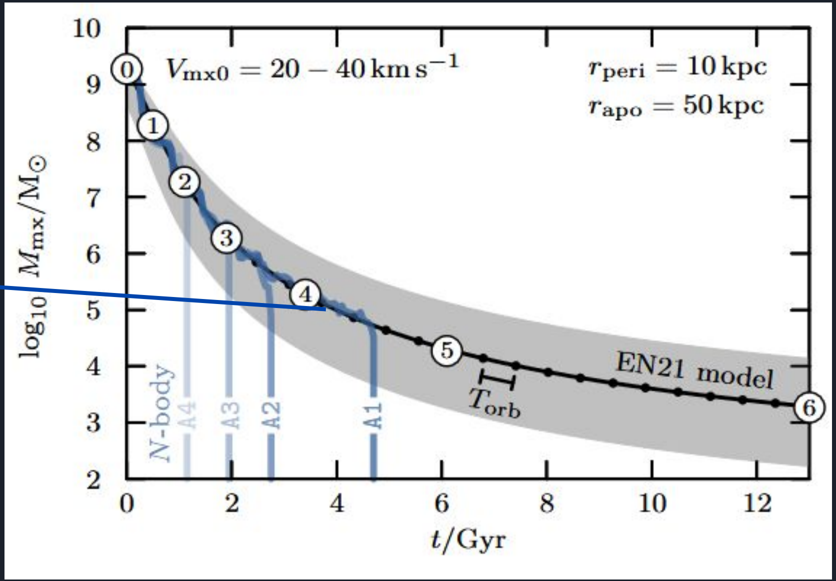




Aquarius Simulations (Springel et al 2008)

Is tidal stripping resolved in N-body simulations?

Factors of 8
in resolution!



Errani et al. (2024)

Disruption of dark matter substructure: fact or fiction? FREE

Frank C van den Bosch, Go Ogiya, Oliver Hahn, Andreas Burkert

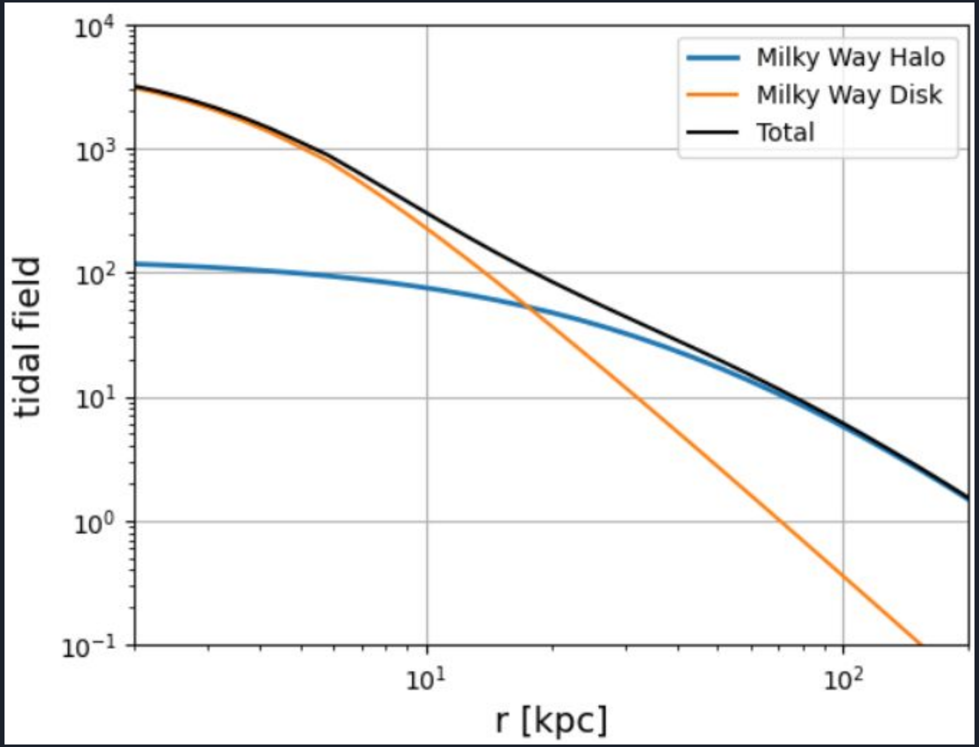
Monthly Notices of the Royal Astronomical Society, Volume 474, Issue 3, March 2018, Pages 3043–3066, https://doi.org/10.1093/mnras/stx2956

Published: 17 November 2017 **Article history** ▾

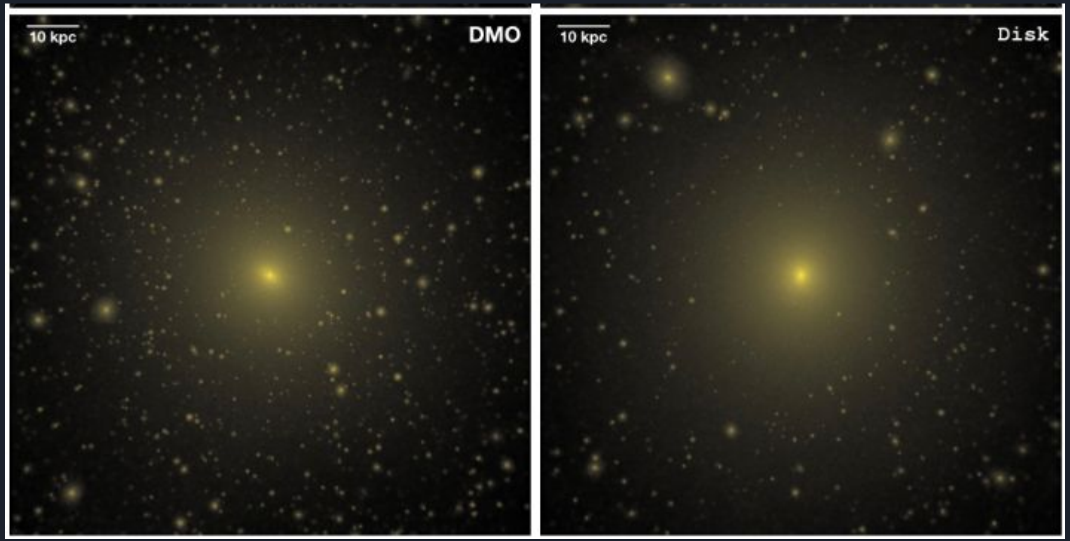
(See also)

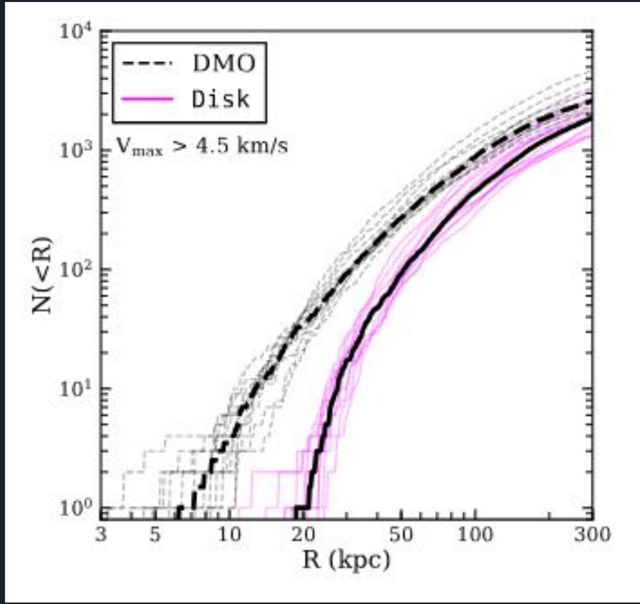
At small radii (e.g. <~ 50 kpc) convergence is very tricky!

Do N-body simulations have a realistic tidal field?



Baryons dominate the tides at r < 20kpc





Phat ELVIS simulation, Kelley et al. (2019)

Can we predict all of the dark substructure with cosmological N-body simulations?

Difficult, because of **tidal stripping**

- Requires extremely high **resolution** at small radii
- **Baryons** dominate the tidal field

➡ Can't trust N-body results at $r \lesssim 50$ kpc

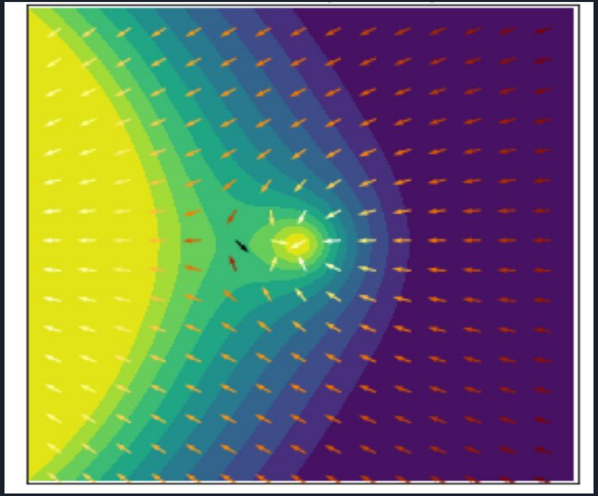
➡ **Theoretical understanding** of tidal stripping is important

➡ Analytical approaches desirable (for extrapolation and for corrections)

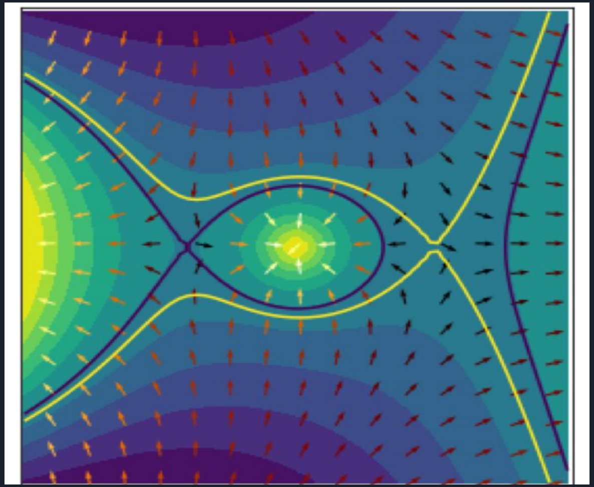
Why does tidal stripping happen?

The "boosted" potential

Full potential φ_{tot}



"Boosted" Potential φ_{boost}



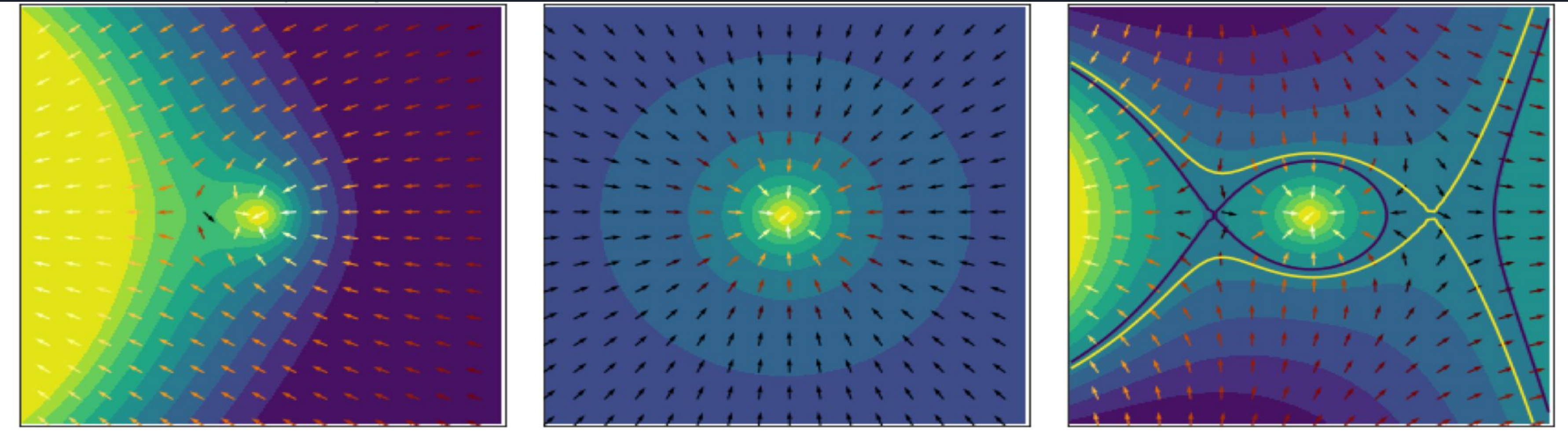
The potential as the subhalo "experiences" it

$$\varphi_{boost}(\mathbf{x}) = \varphi_{tot}(\mathbf{x}) + a_0\,\mathbf{x}$$

Stücker, Busch & Angulo (2022)

The “boosted” potential

Full potential φ_{tot} | Self-potential φ_{self} | “Boosted” Potential φ_{boost}



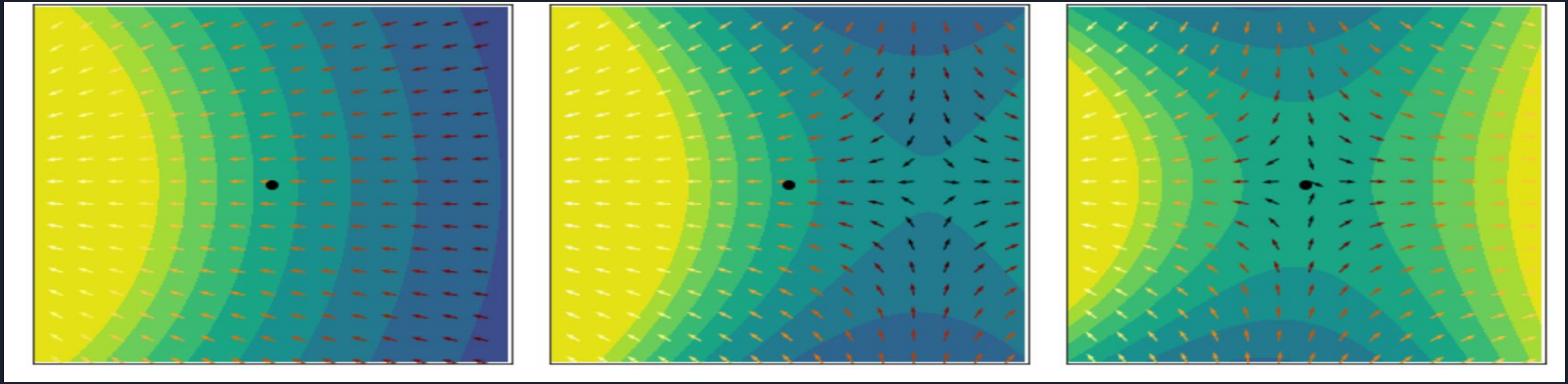
$$\varphi_{boost}(\mathbf{x}) = \varphi_{tot}(\mathbf{x}) + \mathbf{a}_0\, \mathbf{x}$$

The potential as the subhalo “experiences” it

Stücker, Busch & Angulo (2022)

The tidal tensor

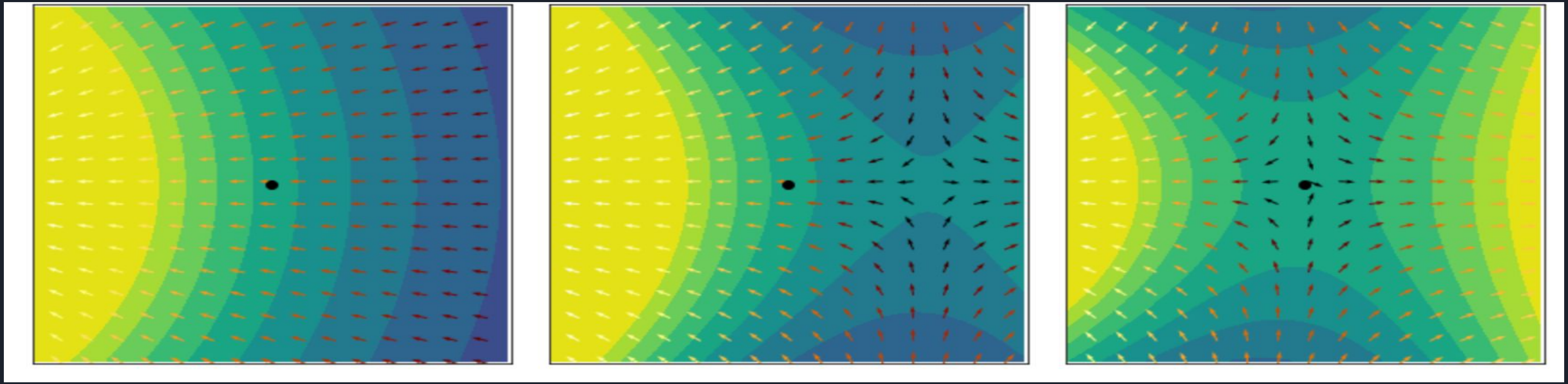
External Potential φ_{ext} | Expansion O(2) | Tidal Field φ_{tid}



$$\varphi_{ext,\,O(2)}(\mathbf{x}) = \varphi_0 - \mathbf{a}_0\,\mathbf{x} - \tfrac{1}{2}\,\mathbf{x}^T\,\mathbf{T}\,\mathbf{x}$$

The tidal tensor

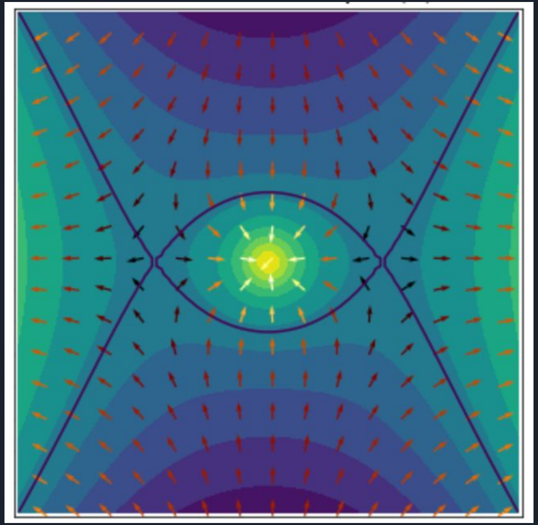
External Potential φ_{ext} | Expansion O(2) | Tidal Field φ_{tid}



$$\varphi_{tid}(x) = \varphi_0 - a_0\, x - \tfrac{1}{2}\, x^T T\, x$$

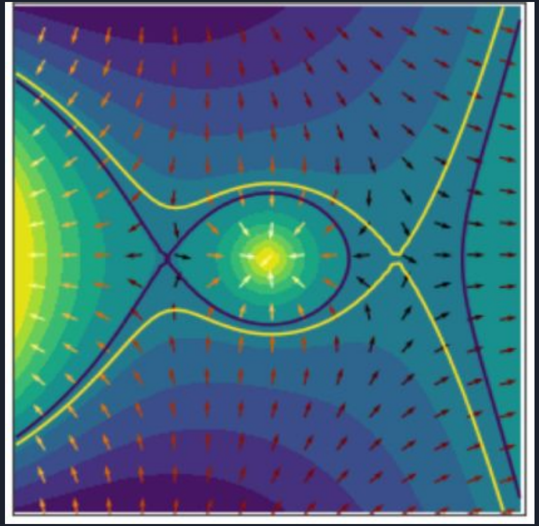
The "distant-tide" approximation

$\varphi_{self}(\mathbf{x}) + ½\ \mathbf{x}^T\mathbf{T}\,\mathbf{x}$



≈

"Boosted" Potential φ_{boost}



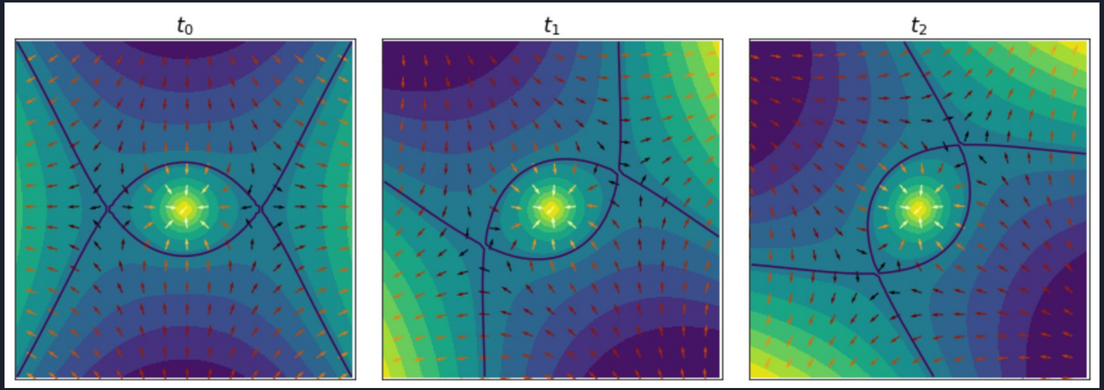
- In practice almost always accurate (roughly if $M_{sub} \lesssim 10^{-3} M_{host}$)
- Implies mass-invariance of tidal stripping

The “distant-tide” view of tidal stripping

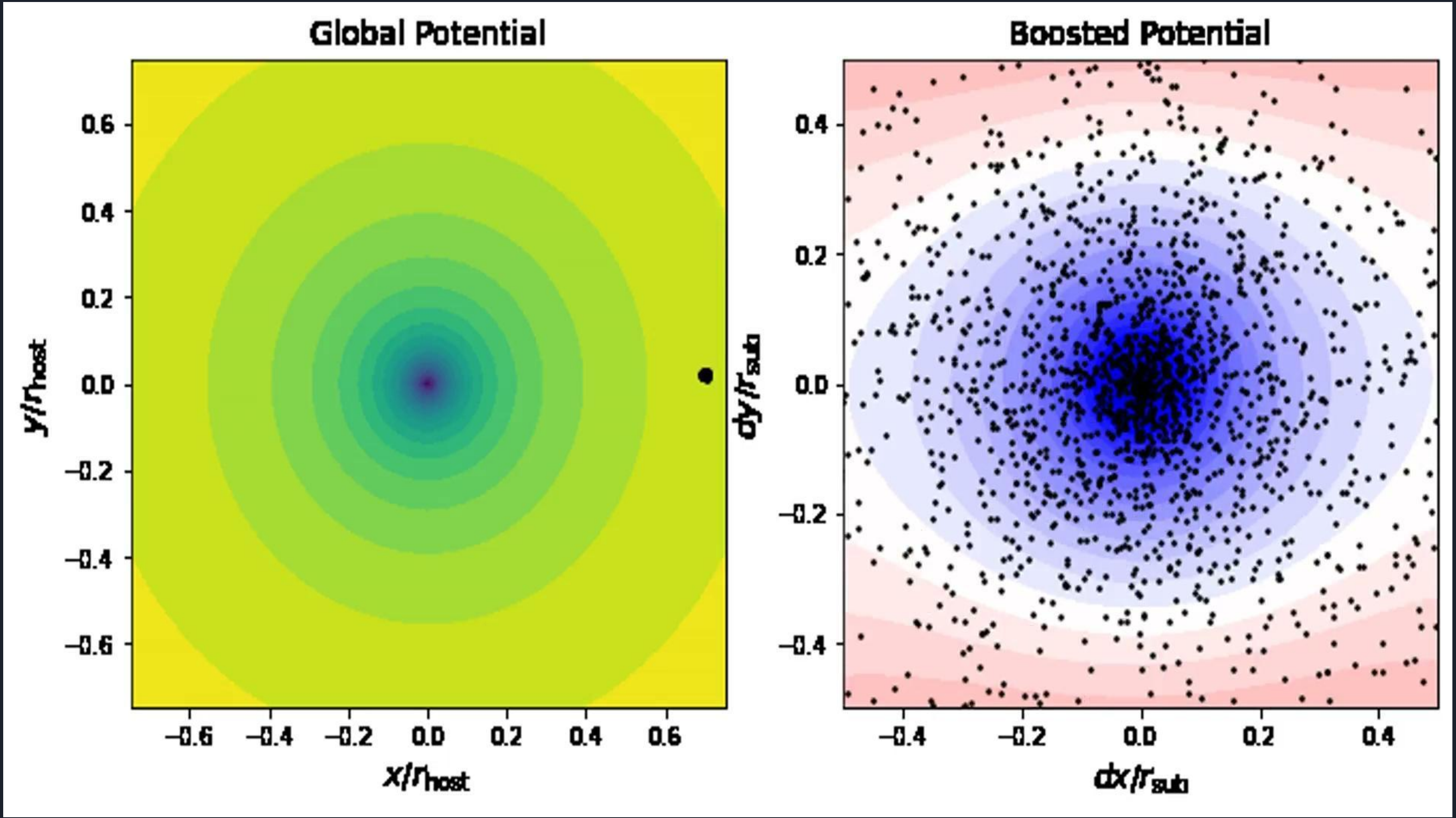
Particles move in the **time-dependent potential** landscape

$$\varphi(\mathbf{x}, t) = \varphi_{self}(\mathbf{x}, t) + \tfrac{1}{2}\, \mathbf{x}^T\, \mathbf{T}(t)\, \mathbf{x}$$

The subhalo’s orbit and the host potential matter only as they determine the “**tidal history**” **T**(t)

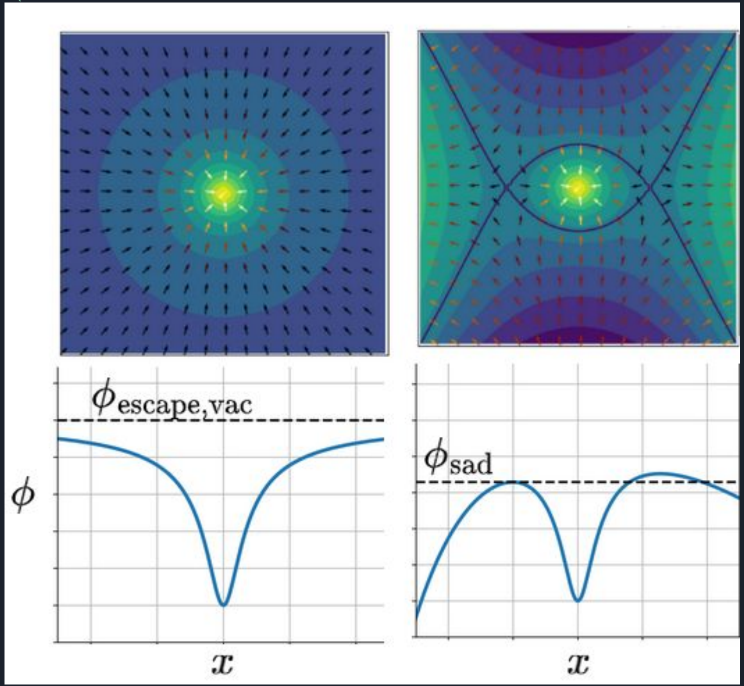


$$\mathbf{T}(t) = -\nabla \otimes \nabla\, \varphi_{ext}(\mathbf{x}_{sub}(t))$$



Global Potential
Boosted Potential
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
y/r_host
x/r_host
dy/r_sub
dx/r_sub

Why does tidal stripping happen?
1) Tides create a saddle-point in the potential



- Often referred to as the “tidal radius” or “Jacobi radius”
- The saddle-point corresponds to a reduced escape energy level

$$\varphi_{\rm sad} \ll \varphi_{\rm escape,\,vac}$$

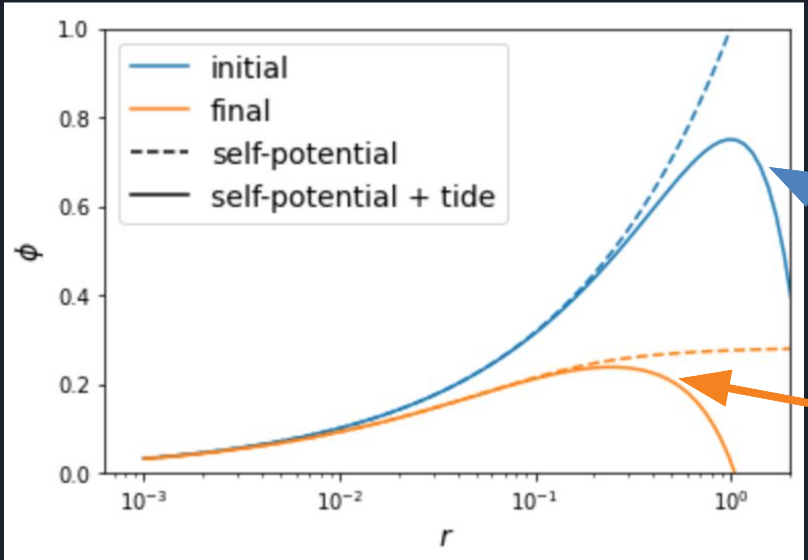
Why does tidal stripping happen?
2) The time-dependent tidal field injects energy

in the impulsive limit: $\Delta \mathbf{v} = \int T(t)\, \mathbf{x}\, dt$

- Particles that are raised beyond the escape energy level will escape
- Side-note: This is only relevant when the tidal field changes quicker than the orbital time-scale of particles, otherwise the system is adiabatically-shielded

Why does tidal stripping happen?
3) Mass-loss facilitates further mass-loss

Example: $\rho \propto r^{-1.5}$ powerlaw profile



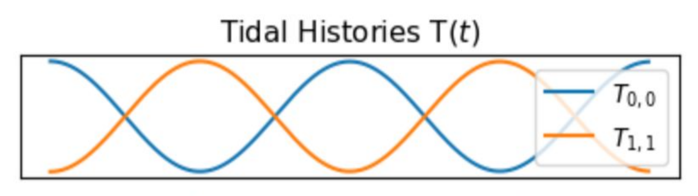
$\downarrow M$ → $\downarrow \varphi_{self}$ → $\downarrow \varphi_{escape}$ → $\downarrow M$

$\varphi_{sad} \sim 0.75$, $f_{bound} \sim 24\%$

$\varphi_{sad} \sim 0.24$, $f_{bound} \sim 1.4\%$

Examples of Tidal Histories

Circular Orbit



ϕ_{sad}	ΔE
✔	✖

Examples of Tidal Histories

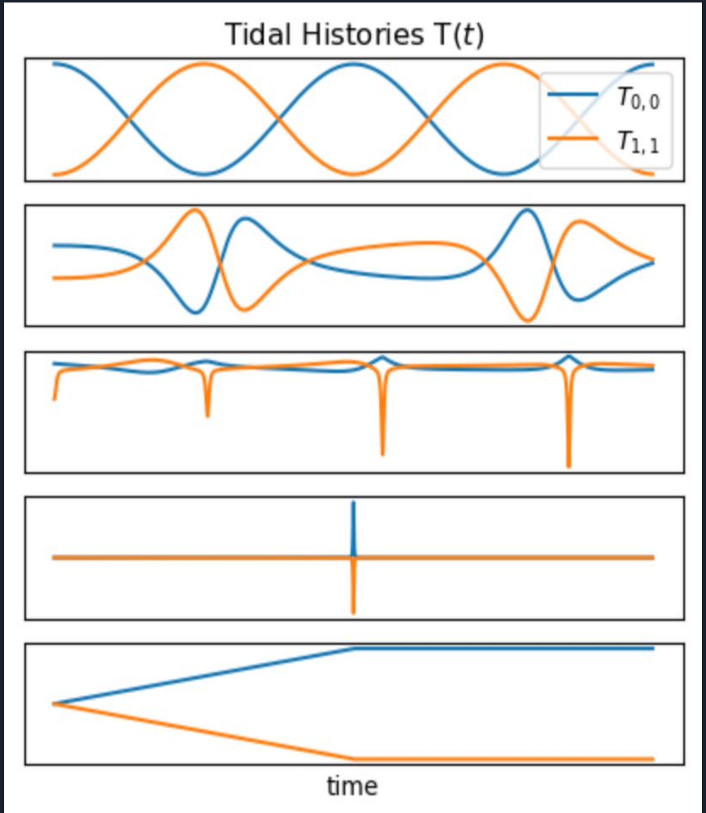
Circular Orbit

Non-Circular Orbit

Galactic Disk

Stellar Encounter

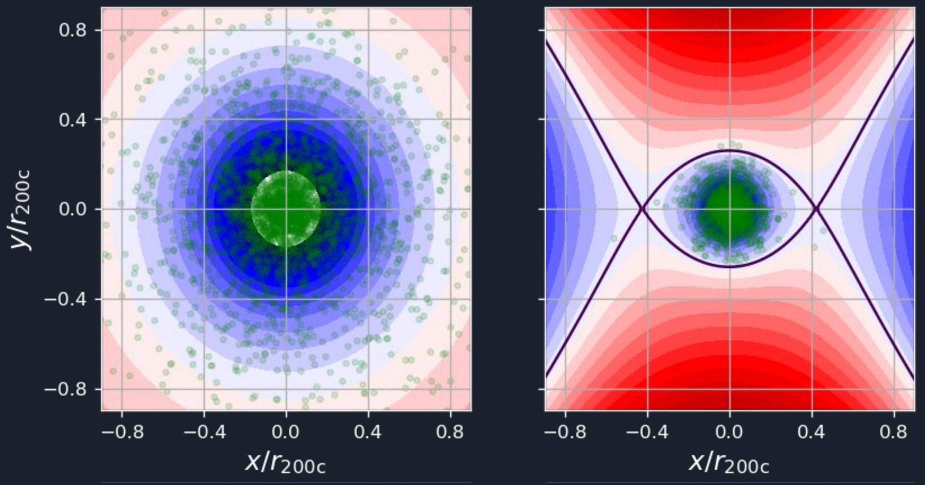
Adiabatic Limit



$\phi_{\rm sad}$	ΔE	$M\downarrow M\downarrow$
✔	✖	✔
✔	~	✔
~	✔	✔
✖	✔	~
✔	✖	✔

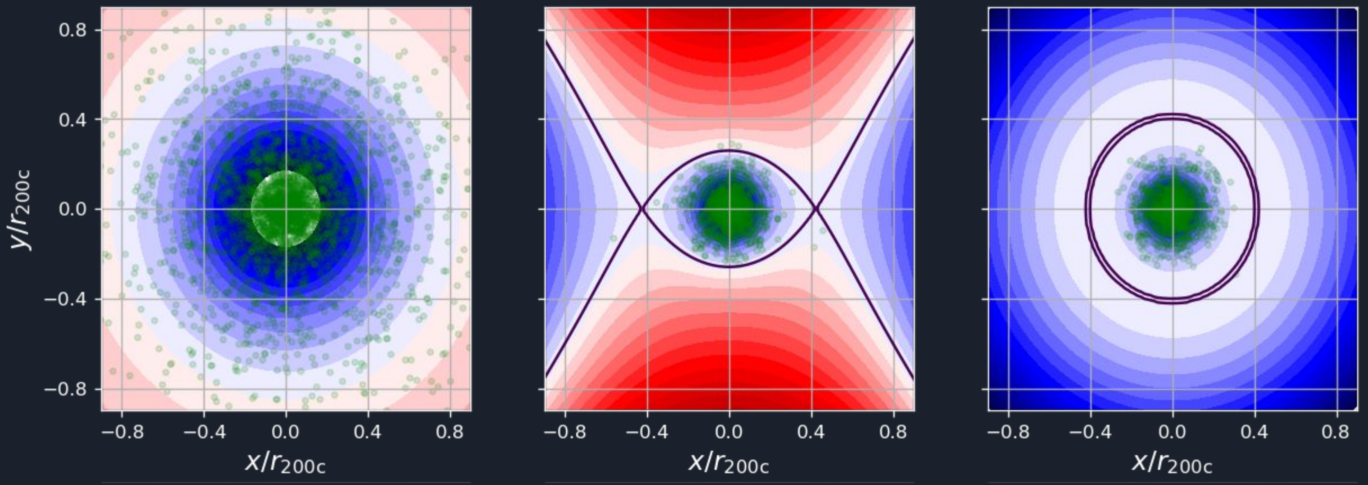
Tidal Stripping in the Adiabatic Limit

Tidal Stripping in the Adiabatic Limit

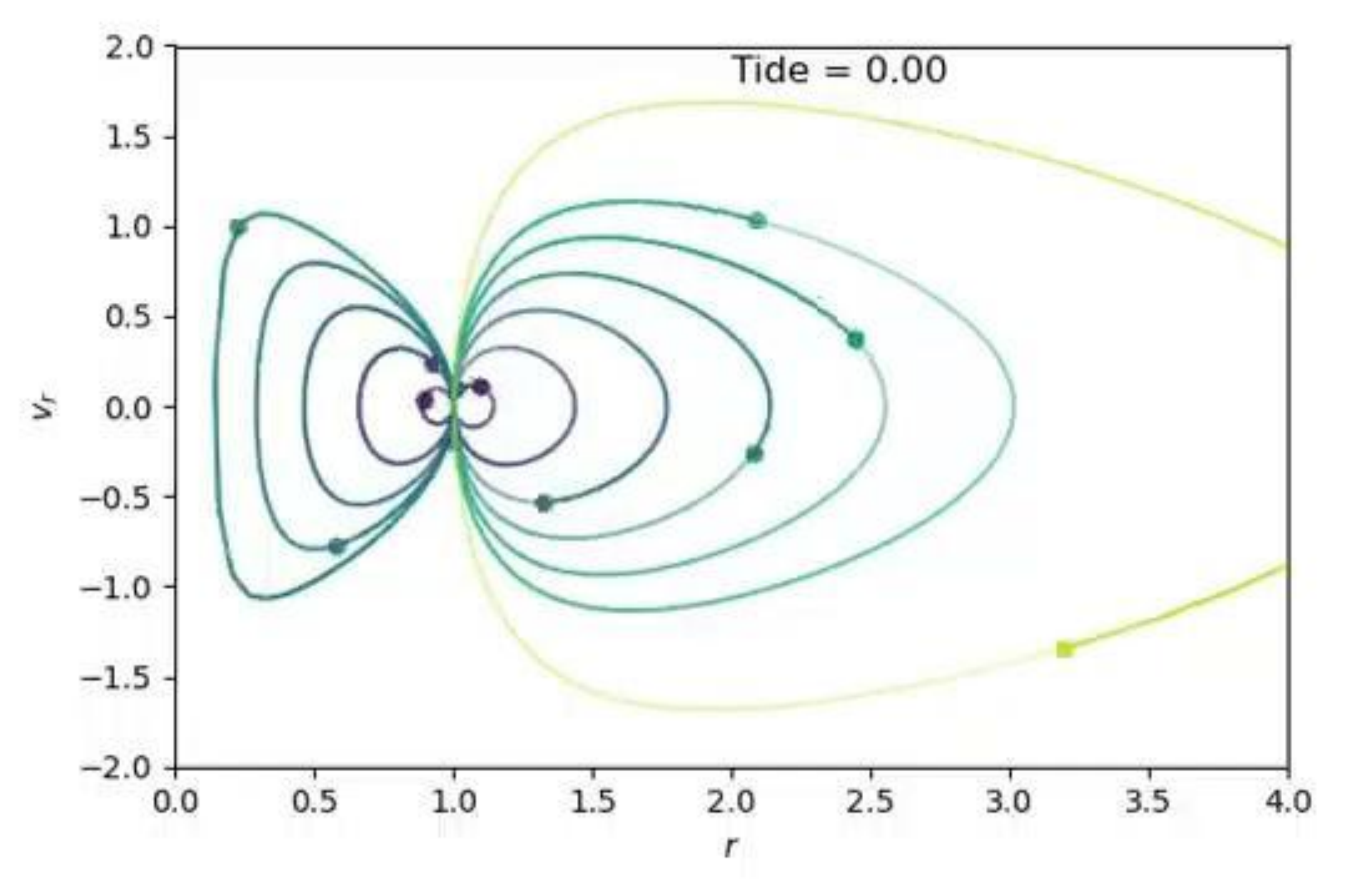


- Start with system in equilibrium
- Increase a **tidal field** extremely **slowly**
- The system will react **adiabatically**

Tidal Stripping in the Adiabatic Limit

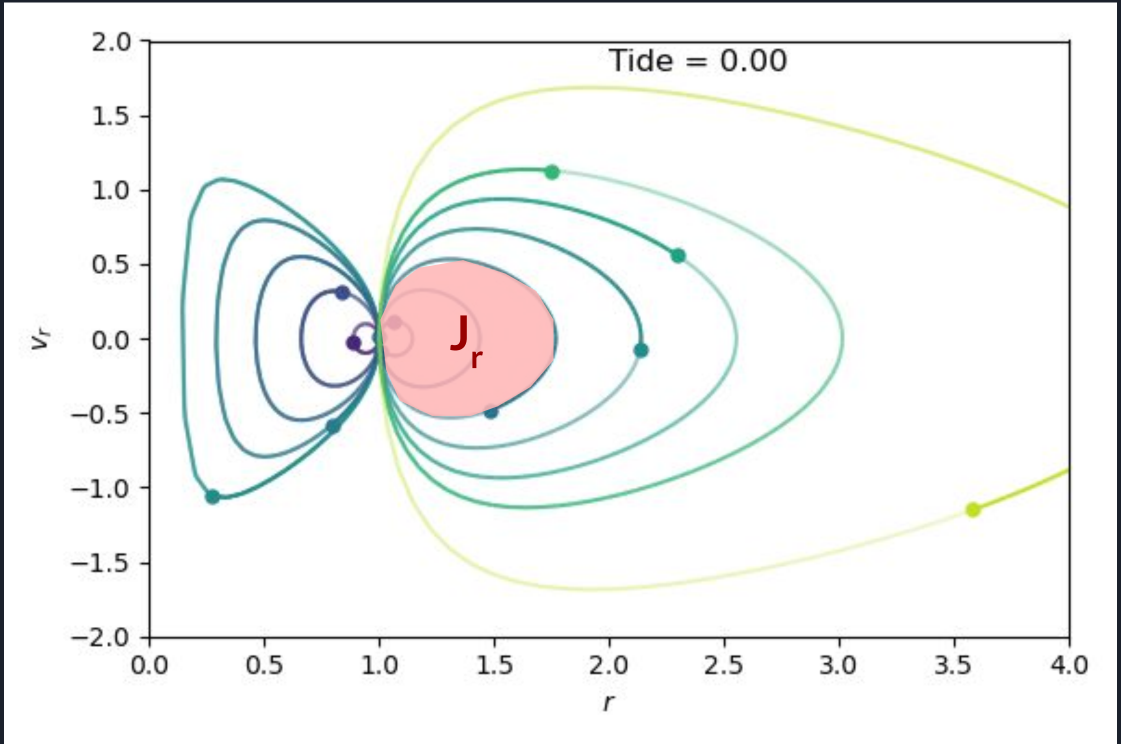


- Start with system in equilibrium
- Increase a **tidal field** extremely **slowly**
- The system will react **adiabatically**
- Further simplification: spherical tide T = diag(λ_r, λ_r, λ_r)



Tide = 0.00
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
v_r
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
r

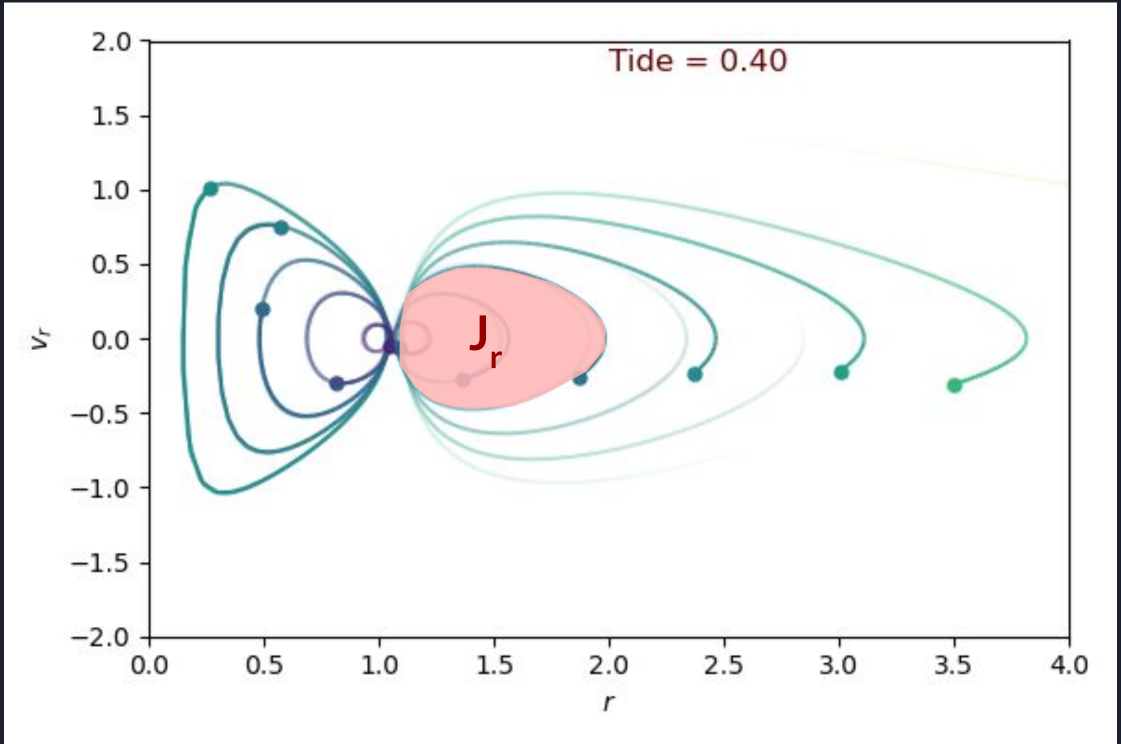
The Conservation of Actions



$J_r = \int v_r dr$

The **radial Action** is the enclosed area and it is conserved for adiabatic transitions

The Conservation of Actions



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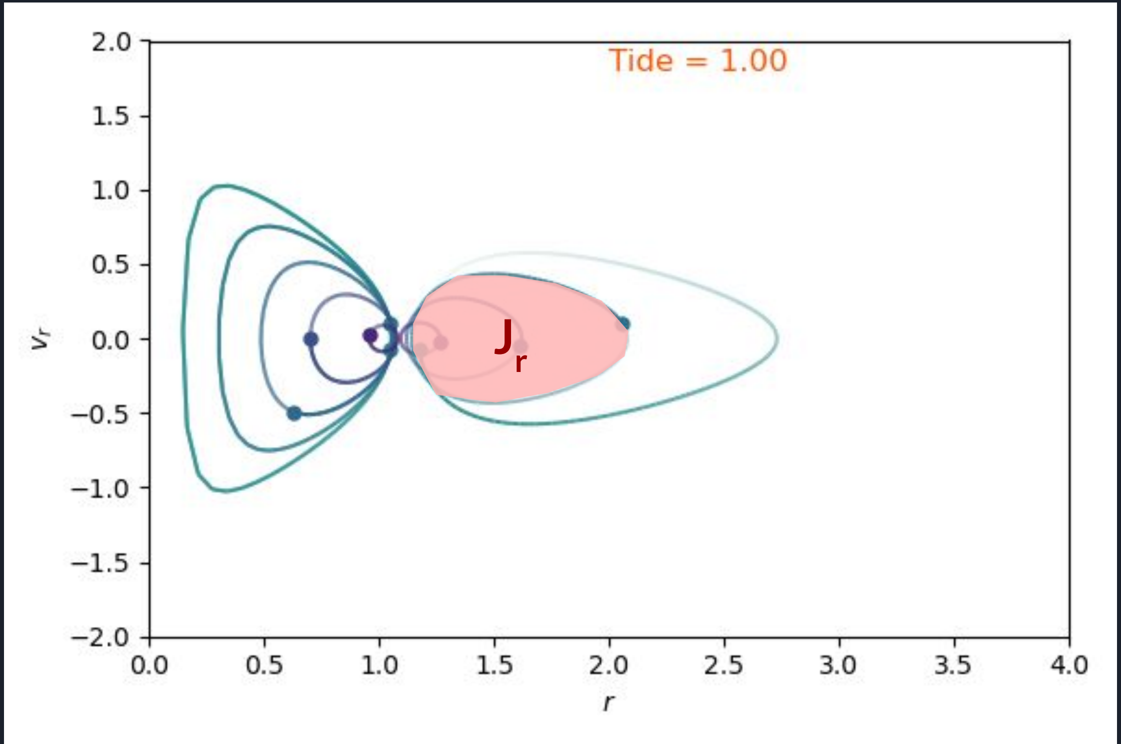
The Conservation of Actions



$J_r = \int v_r dr$

The **radial Action** is the enclosed area and it is conserved for adiabatic transitions

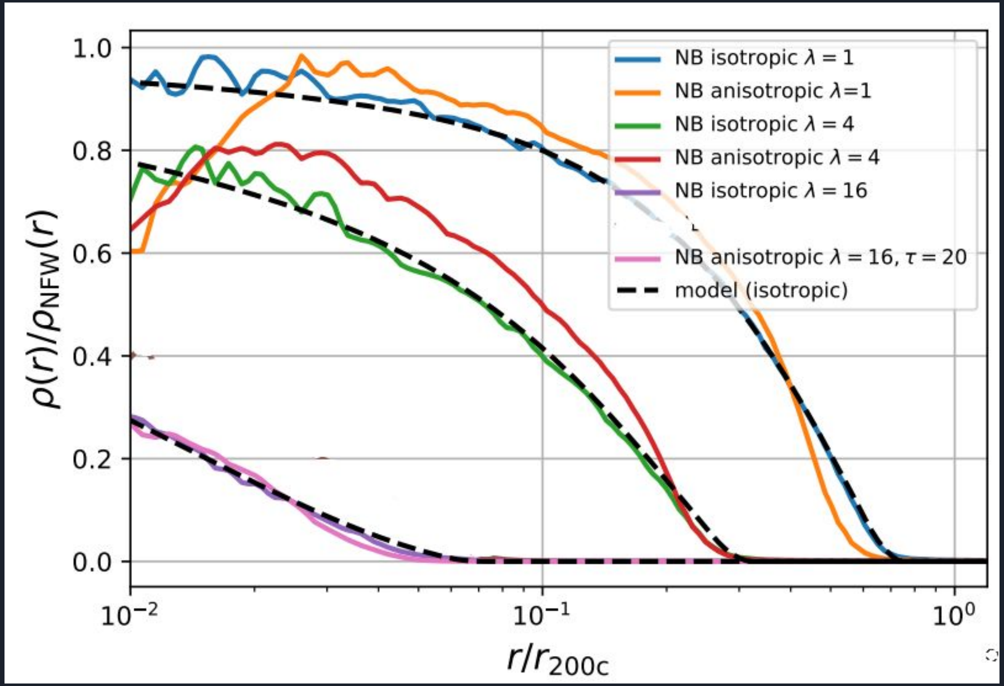
The Conservation of Actions

Actions are conserved for adiabatic transitions

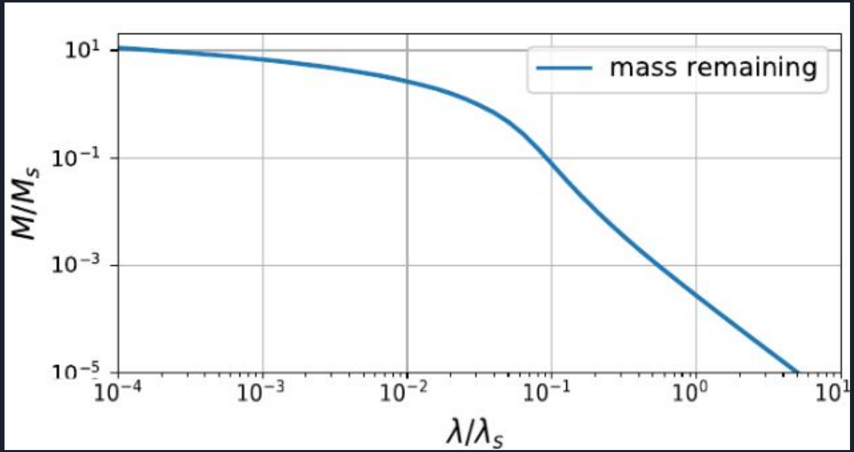
$$f(J,L) = \begin{cases} f_0(J,L) & \text{for bound orbits} \\ 0 & \text{for unbound orbits} \end{cases}$$

-> This allows to calculate the remnant analytically!

Predicted density profiles



Predicted Mass-loss of NFW Haloes

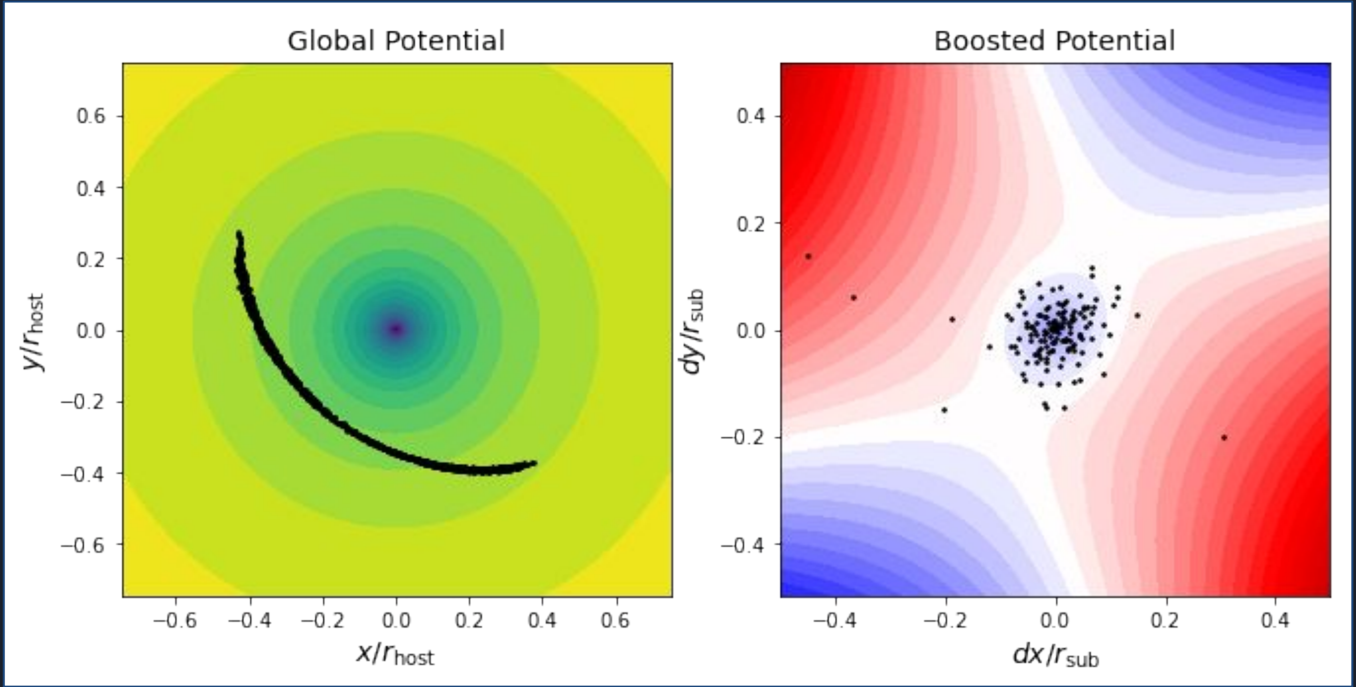


$\lambda_s = \partial_r \varphi_{NFW}(r_s) / r_s$

higher tidal field

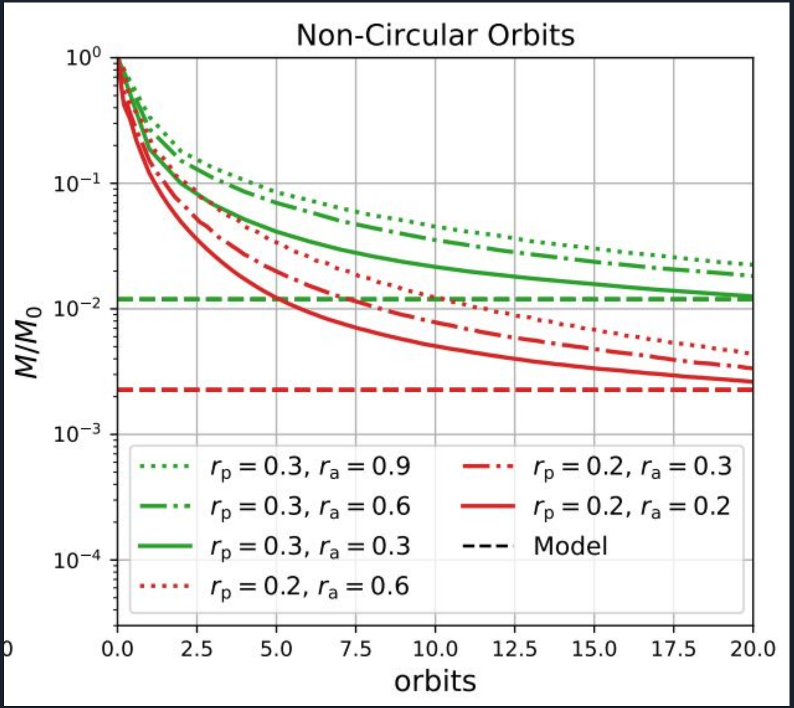
higher concentration

Adiabatic Tides & Non-Circular Orbits



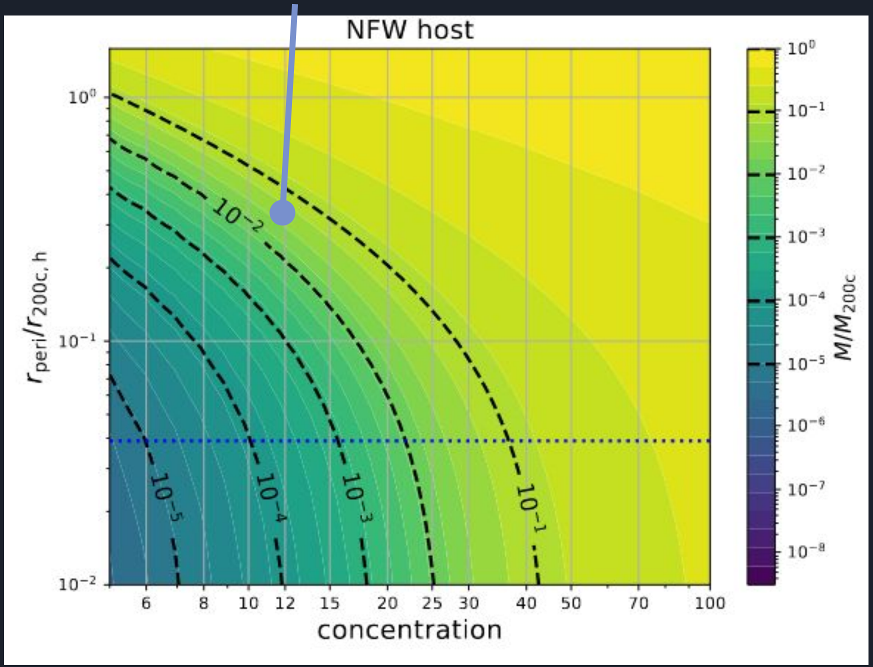
The tidal field at peri-center determines the asymptotic structure

Adiabatic Tides & Asymptotic Remnants



The effect of baryons

The halo I showed in the beginning



Outlook:
A general analytical model of tidal stripping

Initial Structure ρ(r)
e.g. NFW Halo, Prompt Cusp, Dwarf Galaxy, Globular Cluster …

Tidal History T(t), e.g.

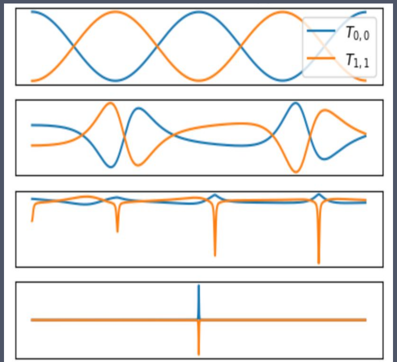
Circular orb.

Non-circ. orb.

Galactic Disk

Stellar Encounter

…



predict

Remnant Structure
(including phase space structure, mass-loss history, density profile, J-factors, …)

Helps with:

- Understand tidal stripping
- Alleviate confusions about “disruption”
- Extrapolate simulations to **unresolved regime**
- Correct for **baryonic effects**

Will use for:

- Comprehensive **predictions of substructure** (all the way to Earth mass haloes)

Important for:

- DM. annihilation, Subhalo lensing, Stellar Streams…

Take-Away Points

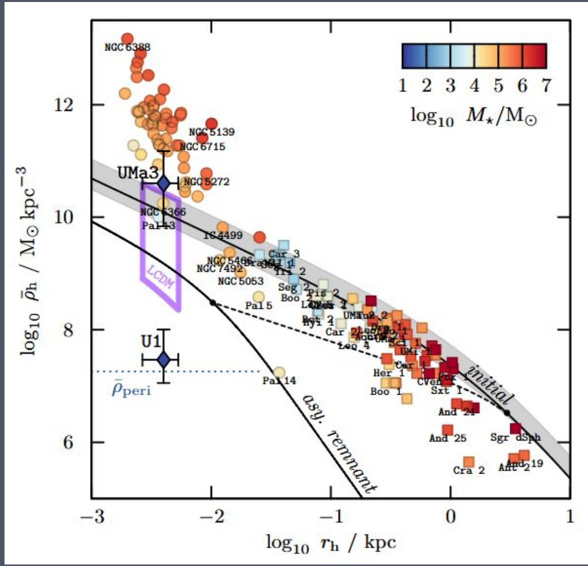
- Detecting the presence or absence of **dark substructure** is a powerful probe of the **nature of DM**
- Most substructures are affected by **tidal stripping**
- Don’t trust the substructure of your N-body simulation (at r < 50kpc for a Milky Way host)
  - Resolving tidal stripping requires large **resolution**
  - **Baryons** have a large impact on substructure
- New **analytical approach** for tidal stripping through **conservation of actions**
  - Allows to predict **asymptotic remnants**
  - NFW haloes don’t ‘disrupt’
  - Will be generalized to other scenarios

Appendix

Interesting Developments

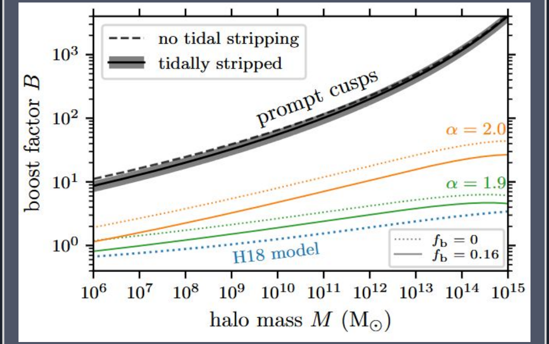
The faintest galaxy ever discovered?

$M_{*} \sim 20\, M_{\odot}$



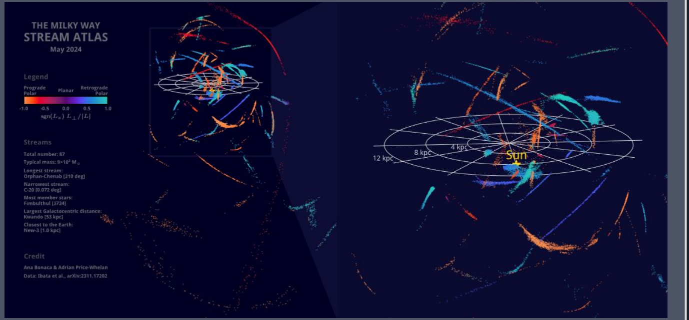
Errani et al. (2024)

Prompt Cusps & DM annihilation



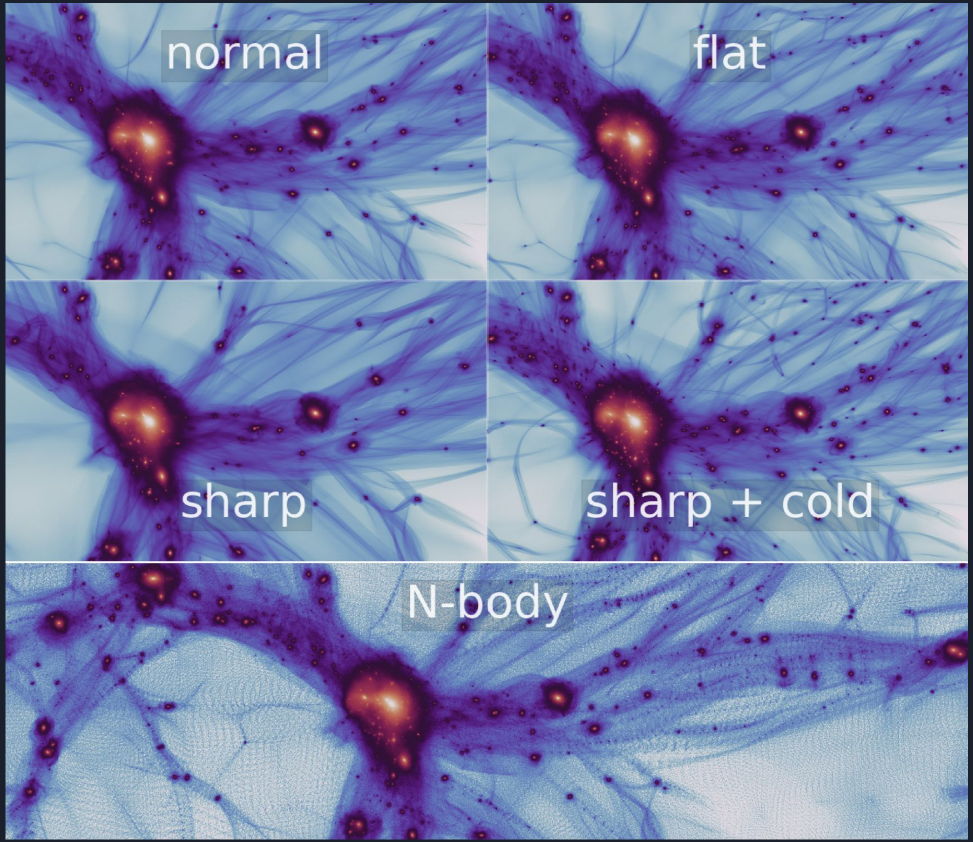
Delos & White (2022)

Stellar Streams & DM substructure in the Gaia Era

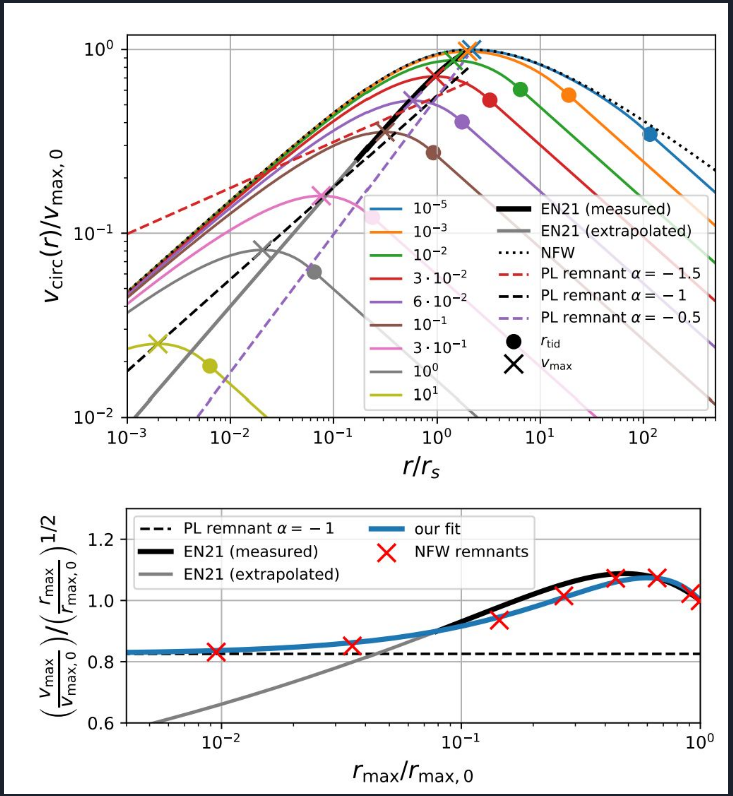


Bonaca & Price-Whelan (2024)

Artificial Fragmentation

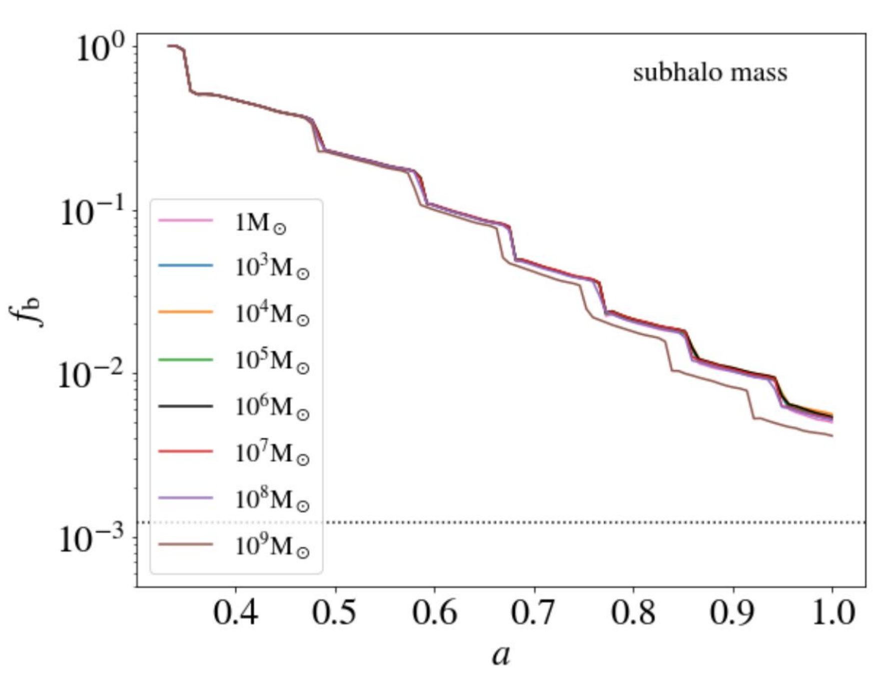


Tidal Track



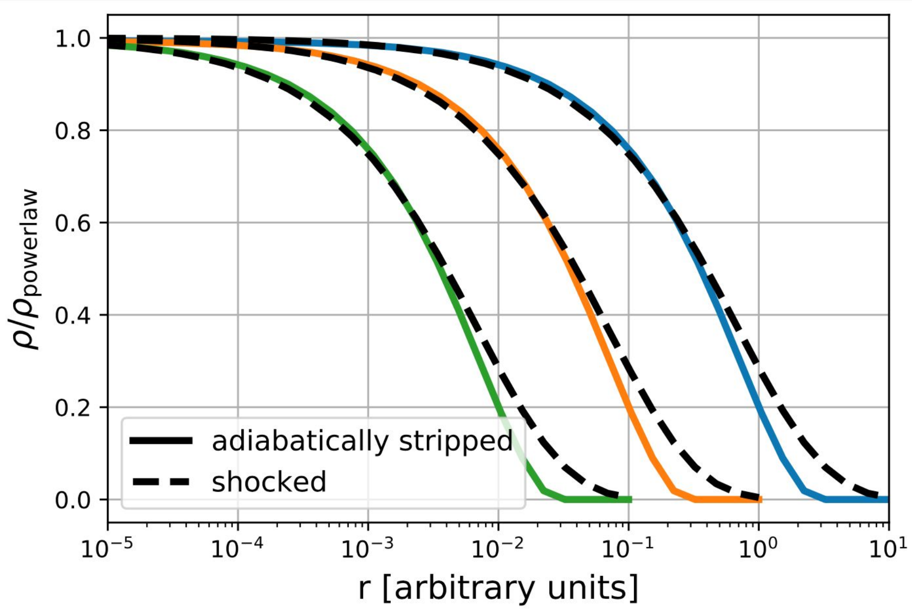
Stücker et al. (2023)

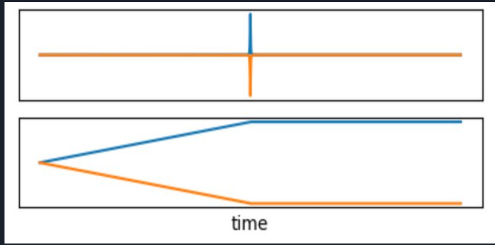
Mass independence of tidal stripping



Aguirre-Santaella et al. (2022)

The simplicity of Tidal Stripping





imp. shock

adiabatic limit

Polar opposite scenarios lead to similar remnants!!

Hypothesis: Tidal remnants **relax adiabatically**