Introduction to Stellar Evolution Models

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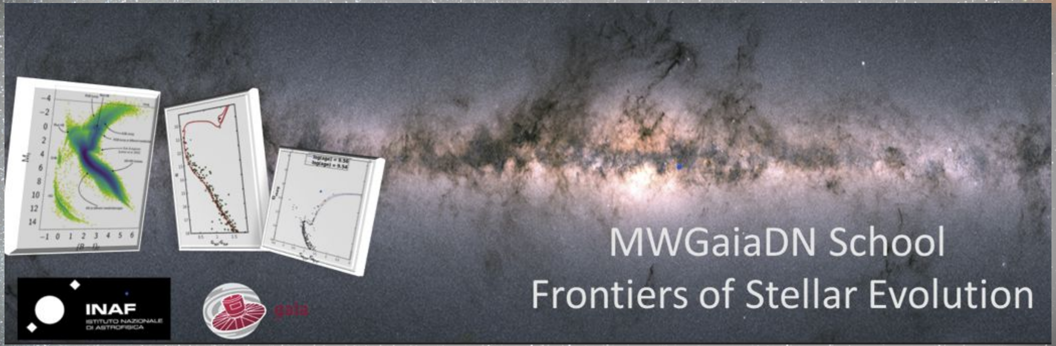
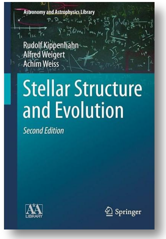
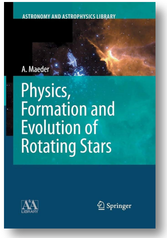
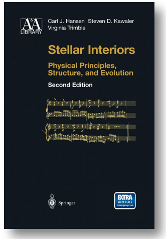


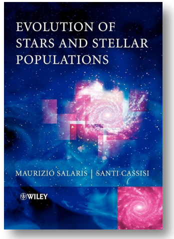
Suggested reading

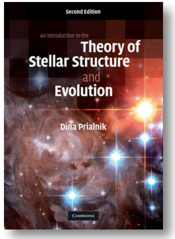
- [Onno Pols - Lecture Notes on Stellar Evolution]
- Kippenhahn, Weigert & Weiss
  Stellar Structure & Evolution
- Maeder
  Physics, Formation and Evolution of Rotating Stars
- Hansen, Kawaler & Trimble
  Stellar Interiors: Physical Principles, Structure, and Evolution
- Salaris & Cassisi
  Evolution of Stars and Stellar Populations
- Prialnik
  An Introduction to the Theory of Stellar Structure and Evolution
- Tassoul & Tassoul
  A Concise History of Solar and Stellar Physics

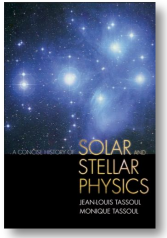












Outline

1. Basic Concepts → Stellar Models & Diagrams
2. Equations of Stellar Structure & Evolution → But With Little Math...
3. Stellar Evolution Codes → Solving the Equations: the Basics
4. Early Phases of Stellar Evolution → Interpreting the Output of Stellar Codes
5. The Role of Initial Mass → Stellar Classification & Advanced Evolution
6. Chemical Composition → Stellar Populations & Chemical Evolution
7. Advanced Topics → Star Formation, Pulsations, Mass-loss, Explosions, ...
8. Conclusions

1.
Basic Concepts

- The HR Diagram
- Stellar Evolutionary Tracks
- Stellar Isochrones
- Stellar Models
- The Kippenhahn Diagram

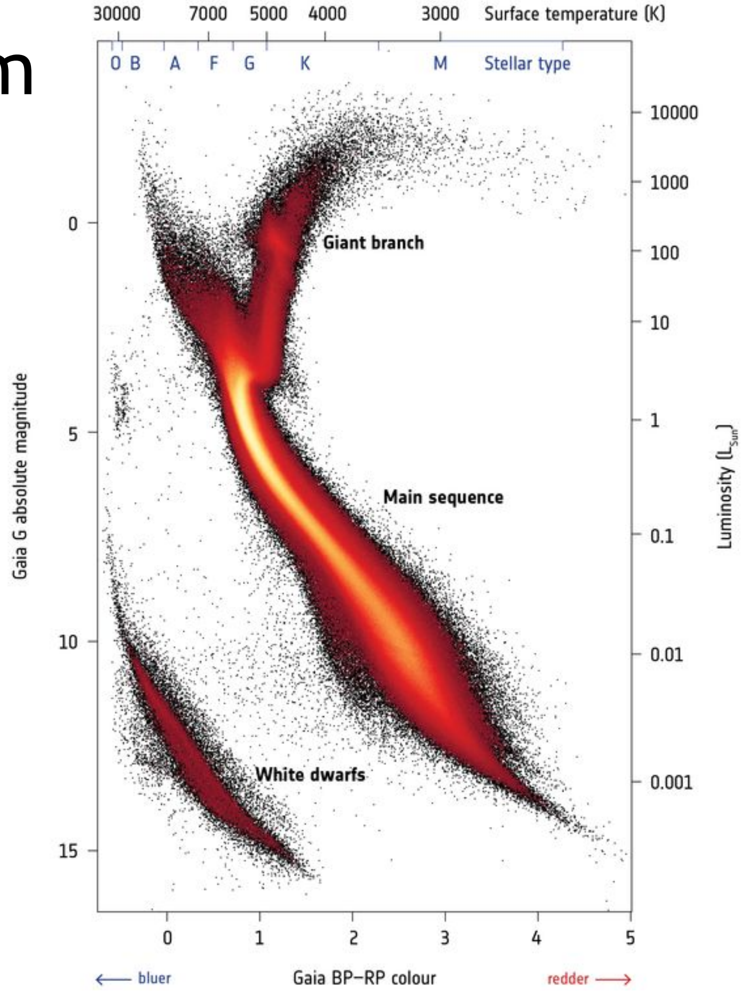
The Hertzsprung-Russell Diagram

For the first time Hertzsprung (1911) & Russell (1914) found out that stars do not randomly scatter in a Color-Magnitude diagram (CMD), but they lay in specific locations.
Important properties:

1. Magnitudes ⟶ Luminosity
   Colors ⟶ Surface temperature

2. The position on the HR depends on the stellar mass, chemical composition and evolutionary phase.

CMD example on the right:
A recent HR diagram, it contains more than 4 million stars within 5000 light-years from the Sun from GAIA satellite DR2.



Stellar Evolutionary Tracks

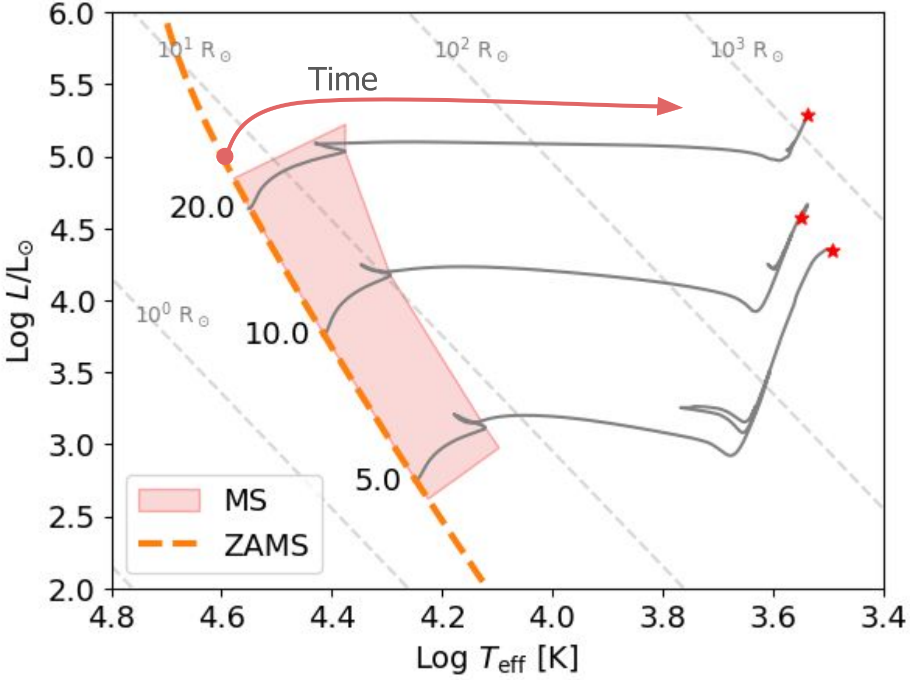
Why do we need stellar models?

Because they tell us about:
1. Inner stellar properties
2. Surface properties (comparable with data)
3. How stars evolve.

What are stellar tracks?

They are a time collection of stellar models of a single star with an initial mass and initial composition, which in mass fraction is
X = hydrogen
Y = helium
Z = metallicity (all the other elements).

Stellar tracks on the HRD



MS = Main Sequence
ZAMS = Zero-Age Main Sequence

Stellar Isochrones

Why stellar isochrones?

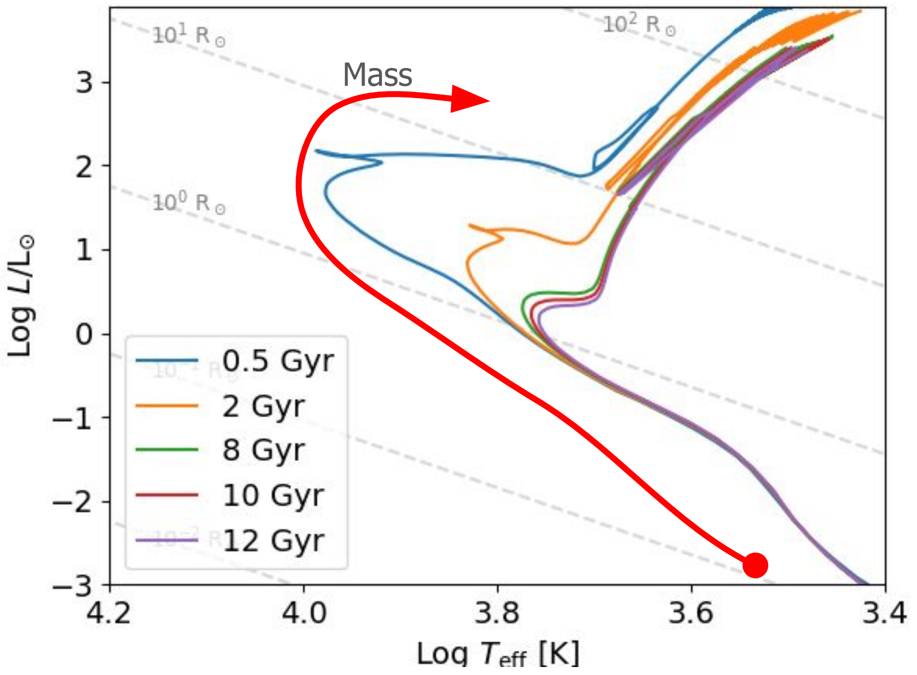
They help us to interpret observed stellar populations (ages, distances, etc.), assuming that:

- All stars are coeval (same age)
- They formed with the same initial composition.

What are isochrones?

They are a collection of stellar models representing different stars, with a different initial mass, at the same age. Isochrones are computed from grids of stellar tracks.

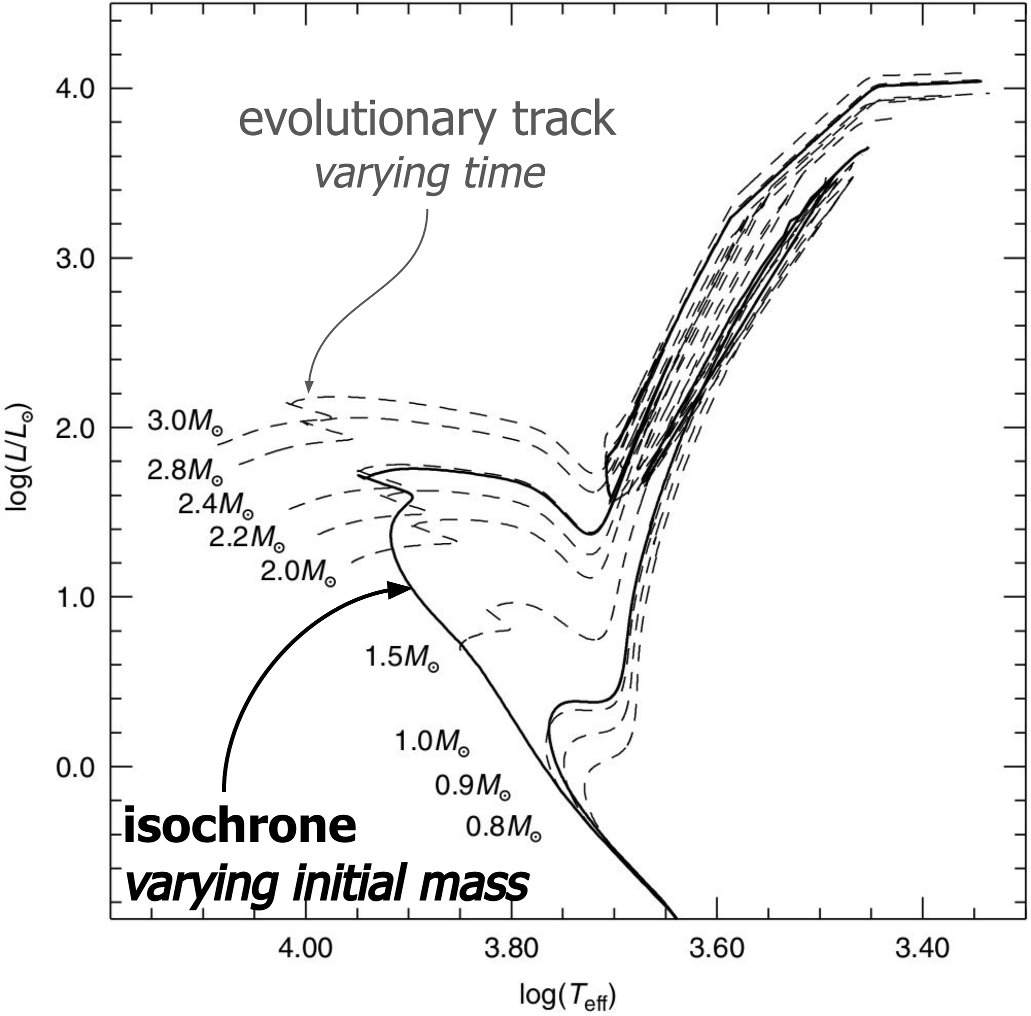
Isochrones on the HRD



Stellar Tracks & Stellar Isochrones

What are evolutionary stellar tracks?
They are a time collection of stellar models of a single star with an given initial mass and initial composition

What are stellar isochrones?
They are a collection of stellar models representing different stars, with a different initial mass, at the same age



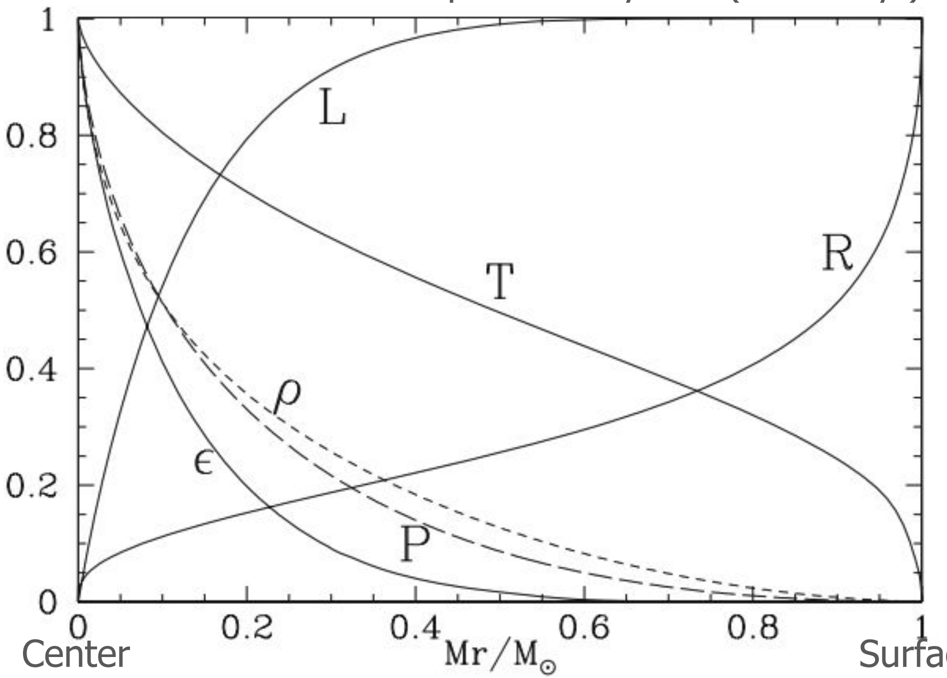
Stellar Models

What is a stellar model?

A stellar model give a complete description of the physical properties of a star at a certain age. Assuming certain (reasonable) approximations, we can model very accurate 1-dimensional models, and obtain the profile of:

Luminosity	→	$L = L(m)$
Temperature	→	$T = T(m)$
Density	→	$\varrho = \varrho(m)$
Pressure	→	$P = P(m)$
Energy generation	→	$\epsilon = \epsilon(m)$
Radius	→	$R = R(m)$
Chemical elements	→	$X_i = X_i(m)$ (not shown)

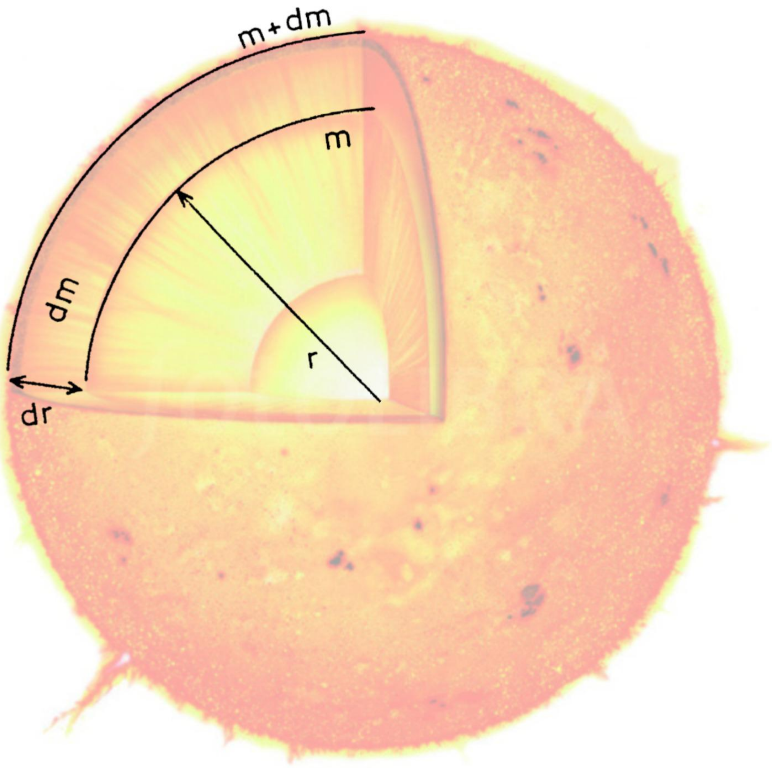
Inner structure of the present day Sun (~4.57 Gyr)



Each variable is normalized to its maximum value

$L = 3.85\times10^{33}$ erg g^{-1} s^{-1}, $T = 1.57\times10^{7}$ K, $\varrho = 1.54\times10^{2}$ g cm^{-3}, $P = 2.35\times10^{17}$ g cm^{-1} s^{-2}, $\epsilon = 1.7\times10$ erg g^{-1} s^{-1} and $R = 6.97\times10^{10}$ cm.

Stellar Models



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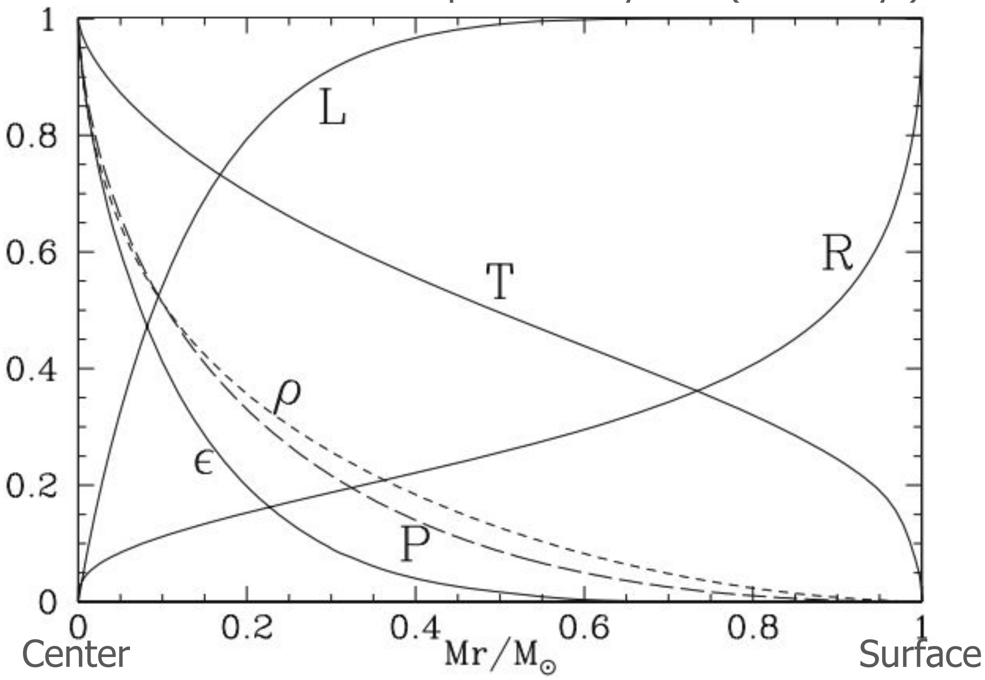

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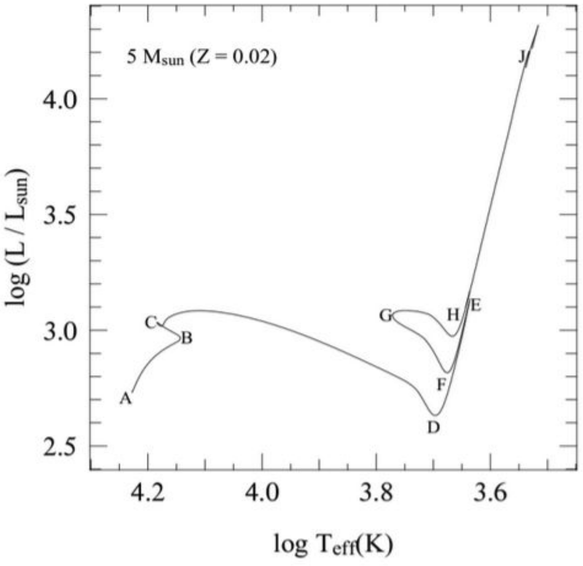
The Kippenhahn Diagram

What is a Kipp. Diagram?

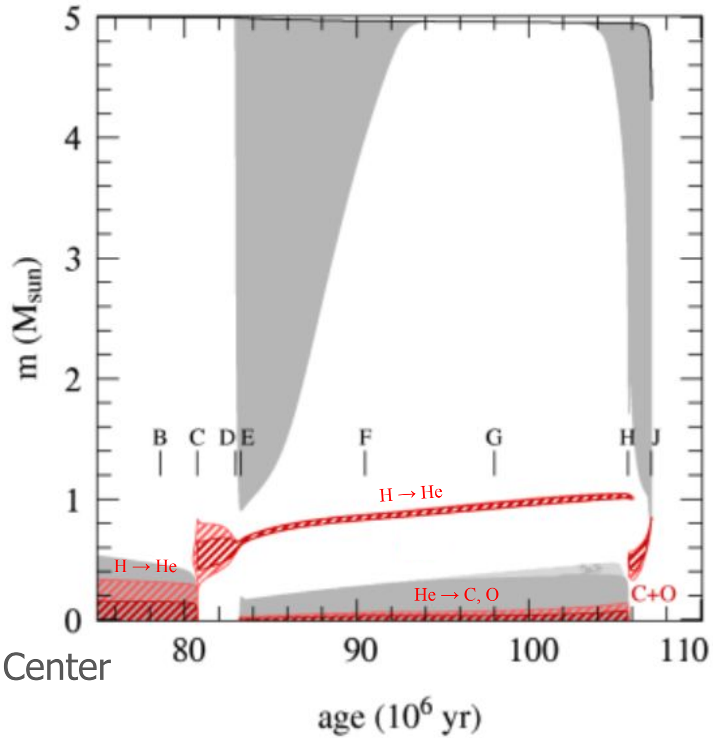
It is a useful plot that shows some inner properties of the stellar models along the evolution. Usually, it shows the convective regions and the active burning layers, in gray and red, respectively, in the plot on the right.

Each evolutionary point in the HRD correspond to a column in the Kippenhahn diagram!

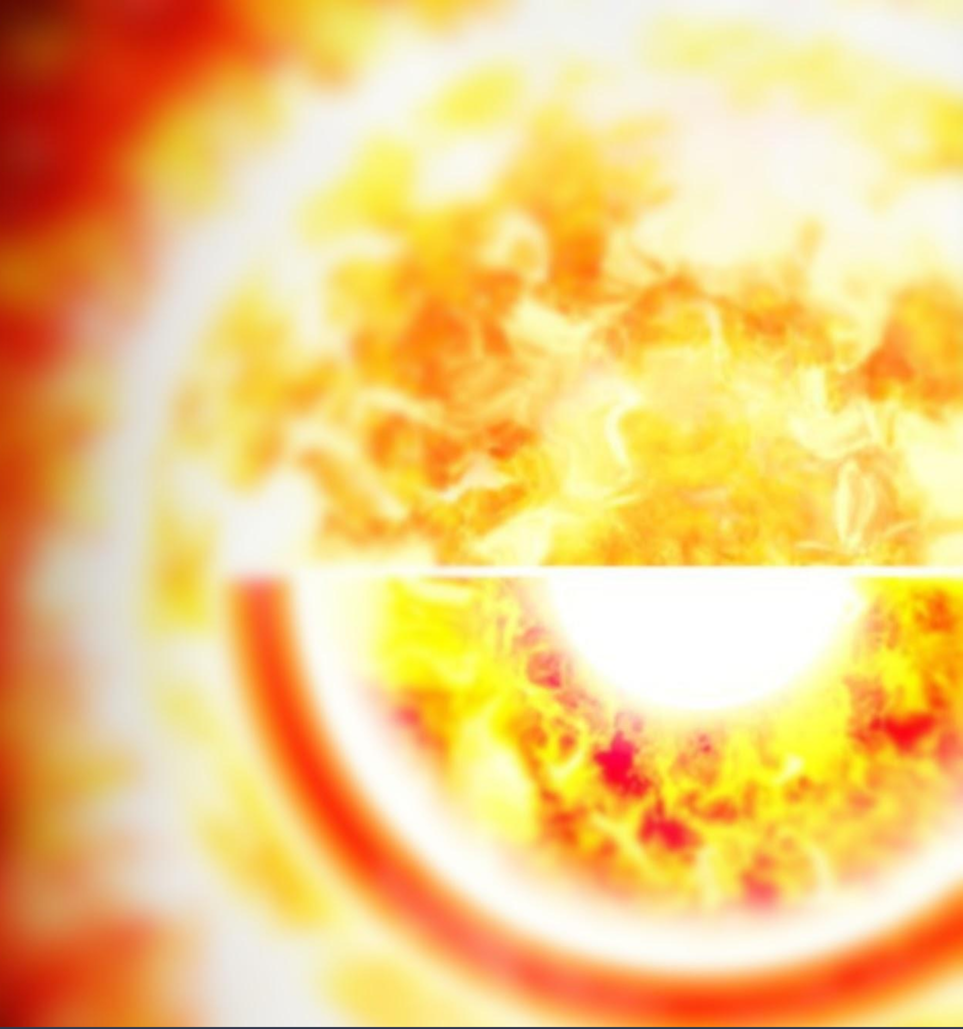
5 $M_\odot$ track in the HRD



Surface Kippenhahn diagram



2.
The Equations of Stellar Structure & Evolution



- Conservation Laws
- Chemical Evolution
- Timescales
- Constitutive Equations

The Equations of **Stellar Structure** & Evolution

Lagrangian Coordinates

$r = r(m)$	Radius
$\rho = \rho(m)$	Density
$T = T(m)$	Temperature
$P = P(m)$	Pressure
$l = l(m)$	Luminosity

Mass Conservation
$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

Momentum Conservation
$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

Energy Conservation
$$\frac{\partial l}{\partial m} = \varepsilon_{\rm nuc} - \varepsilon_\nu - T\frac{\partial s}{\partial t}$$

Energy Transport
$$\frac{\partial T}{\partial m} = -\frac{Gm}{4\pi r^4}\nabla \quad \text{with} \quad \nabla = \begin{cases} \nabla_{\rm rad} = \dfrac{3}{16\pi acG}\dfrac{\kappa l}{m}\dfrac{P}{T^4} & \text{if} \quad \nabla_{\rm rad} \le \nabla_{\rm ad} \\ \nabla_{\rm ad} + \Delta\nabla & \text{if} \quad \nabla_{\rm rad} > \nabla_{\rm ad} \end{cases}$$

The Equations of Stellar Structure & **Evolution**

Lagrangian Coordinates

$r = r(m)$	Radius
$\rho = \rho(m)$	Density
$T = T(m)$	Temperature
$P = P(m)$	Pressure
$l = l(m)$	Luminosity

Mass Conservation
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$$\frac{\partial T}{\partial m} = -\frac{Gm}{4\pi r^4}\nabla \quad \text{with} \quad \nabla = \begin{cases} \nabla_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{\kappa l}{m}\frac{P}{T^4} & \text{if} \quad \nabla_{\mathrm{rad}} \le \nabla_{\mathrm{ad}} \\ \nabla_{\mathrm{ad}} + \Delta\nabla & \text{if} \quad \nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}} \end{cases}$$

Chemical Evolution
$$\frac{\partial X_i}{\partial t} = \frac{A_i m_{\mathrm{u}}}{\rho}\left[-\sum_j \left(1 + \delta_{ij}\right) r_{ij} + \sum_{k,l} r_{kl,i}\right] (+\,\mathrm{mix}) \qquad i = 1, \ldots, N$$

Extra Ingredients: the Constitutive Equations

- The Equation of State (EoS)
- Stellar Opacity
- Nuclear Processes

Linking dependent variables with extra quantities, to close the system of equations.

Although, in principle, some analytical formulae can be retrieved, they never hold for the whole star. Therefore, when solving for stellar structure equations, one uses EOS and opacity numerical tables.

To follow the nucleosynthesis of the element and the energy generated, a nuclear reaction network is coupled with the structure equations.

Extra Ingredients: the Constitutive Equations

- The Equation of State (EoS)
- Stellar Opacity
- Nuclear Processes

The equation of state (EoS) describes the stellar matter's microscopic properties, at a certain density ρ, temperature T, and chemical composition X_i.

Classical Ideal Gas

$$P = nk_{\mathrm{B}}T = \frac{k_{\mathrm{B}}}{\mu m_{\mathrm{u}}}\rho T$$

Radiation Pressure

$$P_{\mathrm{rad}} = aT^4/3$$

Quantum Effects: Electron Degeneracy

Non-Rel.: $P_e \propto (\varrho/\mu_e)^{5/3}$

Extreme Rel.: $P_e \propto (\varrho/\mu_e)^{4/3}$

Independent of temperature!

Extra Ingredients: the Constitutive Equations

- The Equation of State (EoS)
- Stellar Opacity – $\kappa(\varrho, T)$
- Nuclear Processes

The opacity of stellar matter determines the speed at which a star spends its energy, i.e., at which rate the light goes out of a star (Maeder, 2009).

Matter-Radiation Interaction

Stellar opacities are mainly due to
– line absorption or bound–bound transitions
– electron scattering
– photoionization or bound–free transitions
– free–free transitions.

Diffusive Energy Transport

It defines the maximum luminosity that radiation can transport within a hydro-static region of the star.

$$l < l_{\mathrm{Edd}} = \frac{4\pi c G m}{\kappa}$$ Eddington luminosity

Convective Instability

$$\nabla_{\mathrm{rad}} = \frac{3}{4\pi a c G}\frac{P}{T^4}\frac{\kappa l}{m} > \nabla_{\mathrm{ad}}$$ Schwarzschild criterion

Extra Ingredients: the Constitutive Equations

- The Equation of State (EoS)
- Stellar Opacity
- **Nuclear Processes**

Nuclear reactions determine how long a star is able to live radiating at the luminosity set by opacity. They transform the original elements in new heavier ones.

Nuclear energy generation rate

Of a single reaction

$$\varepsilon_{ij} = \varepsilon_{0,ij} X_i X_j \rho T^{\nu}$$

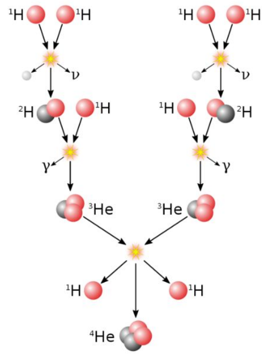
Total rate per unit mass

$$\varepsilon_{\rm nuc} = \sum_{i,j} \varepsilon_{ij}$$

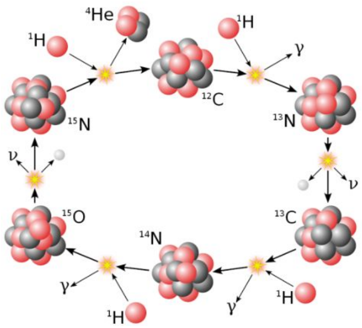
Nuclear networks

Collect all the important reactions that act within stars at different ages, defining the evolutionary stage. For instance:

Proton - proton chain



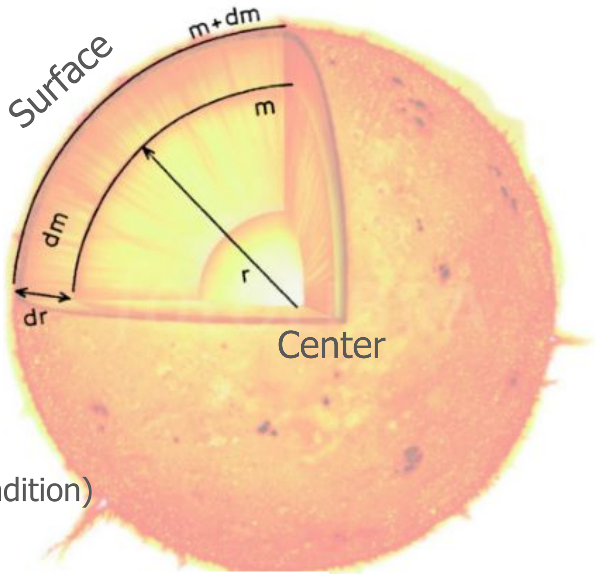
CNO - cycle



3.
Stellar Evolution Codes

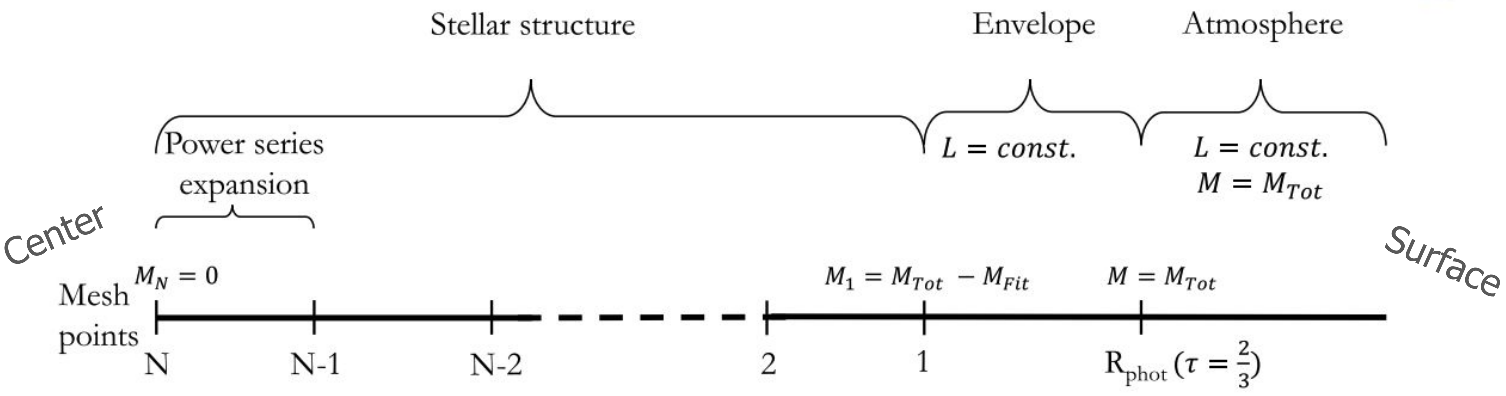
- Setting the Stage
- The Solving Method
- Results

Setting the Stage



Standard approach:

- Assumption of spherical symmetry (so we can treat just 1-D profiles)
- Splitting the star in N shells
- Three main regions:
  - Stellar interior (complete solution of the stellar structure equations)
  - Envelope (no nuclear reactions, thus the luminosity is assumed constant)
  - Atmosphere, mass is equal to the stellar total mass (external boundary condition)
- Boundary condition at the center



The Solving Method - Basic concepts

The equations of stellar structure are a system of ordinary differential equations, and to solve them we use the **Henyey (1964) method**.

1- We can write the equations as follows $\longrightarrow$ $$\frac{dy_i}{dm} = f_i(y_1, \ldots, y_4), \quad i = 1, \ldots, 4\,,$$

2- The differential equations are replaced by four finite differences, that can be written as $\longrightarrow$ $$A_i^j := \frac{y_i^j - y_i^{j+1}}{m^j - m^{j+1}} - f_i\left(y_1^{j+1/2}, \ldots, y_4^{j+1/2}\right), \quad i = 1, \ldots, 4\,,$$

3- If we have the hydrostatic solution, we have that $\longrightarrow$ $$A_i^j = 0, \quad i = 1, \ldots, 4, \quad j = 1, \ldots,$$

4- But, we start with an approximate solution $\longrightarrow$ $$A_i^j(1) = \delta A_i^j \neq 0$$

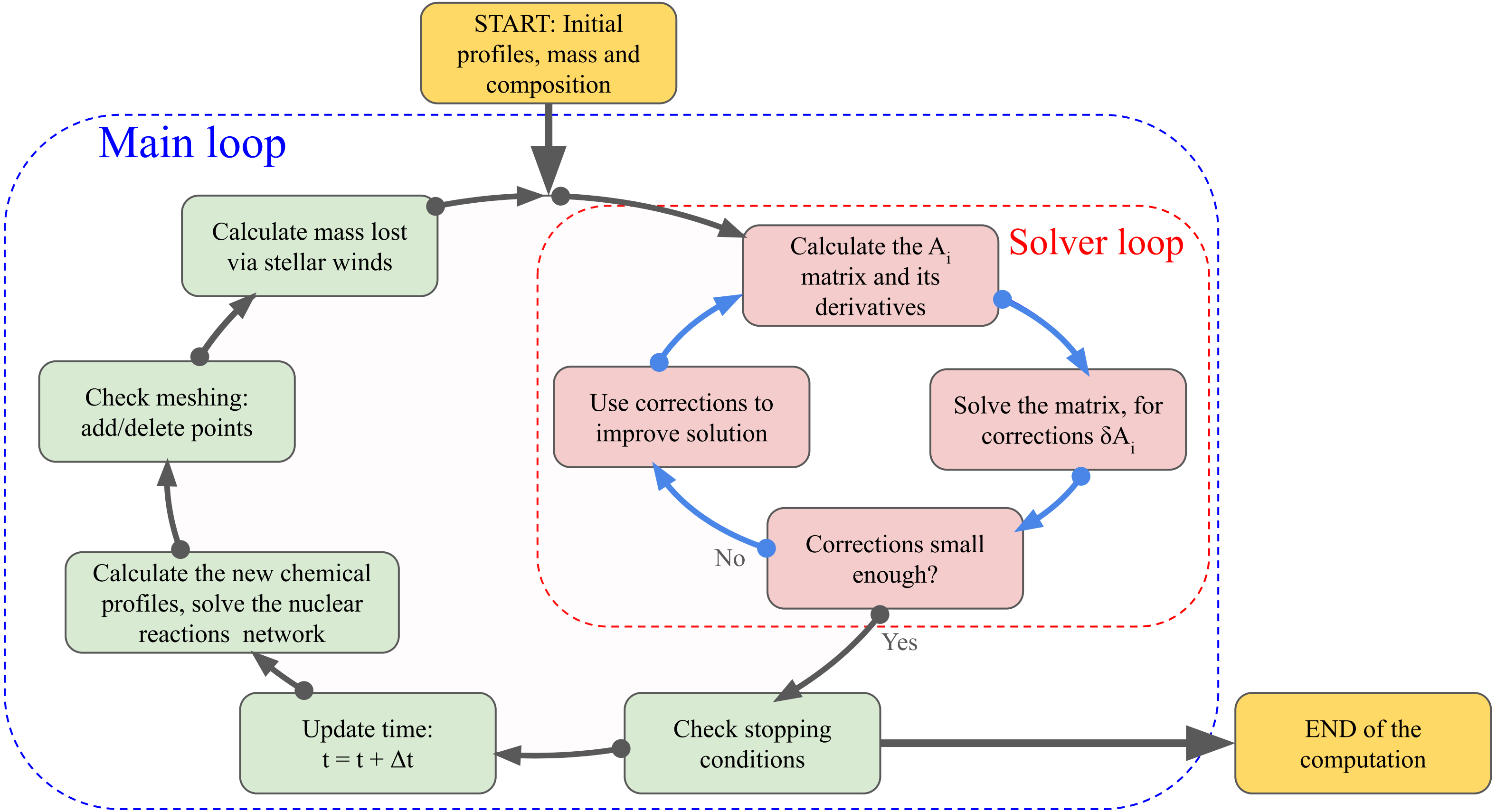
5- We use the first approximation to correct the solution $\longrightarrow$ $$A_i^j(1) + \delta A_i^j = 0$$

Iterations

The (1) is the number of the iterations.

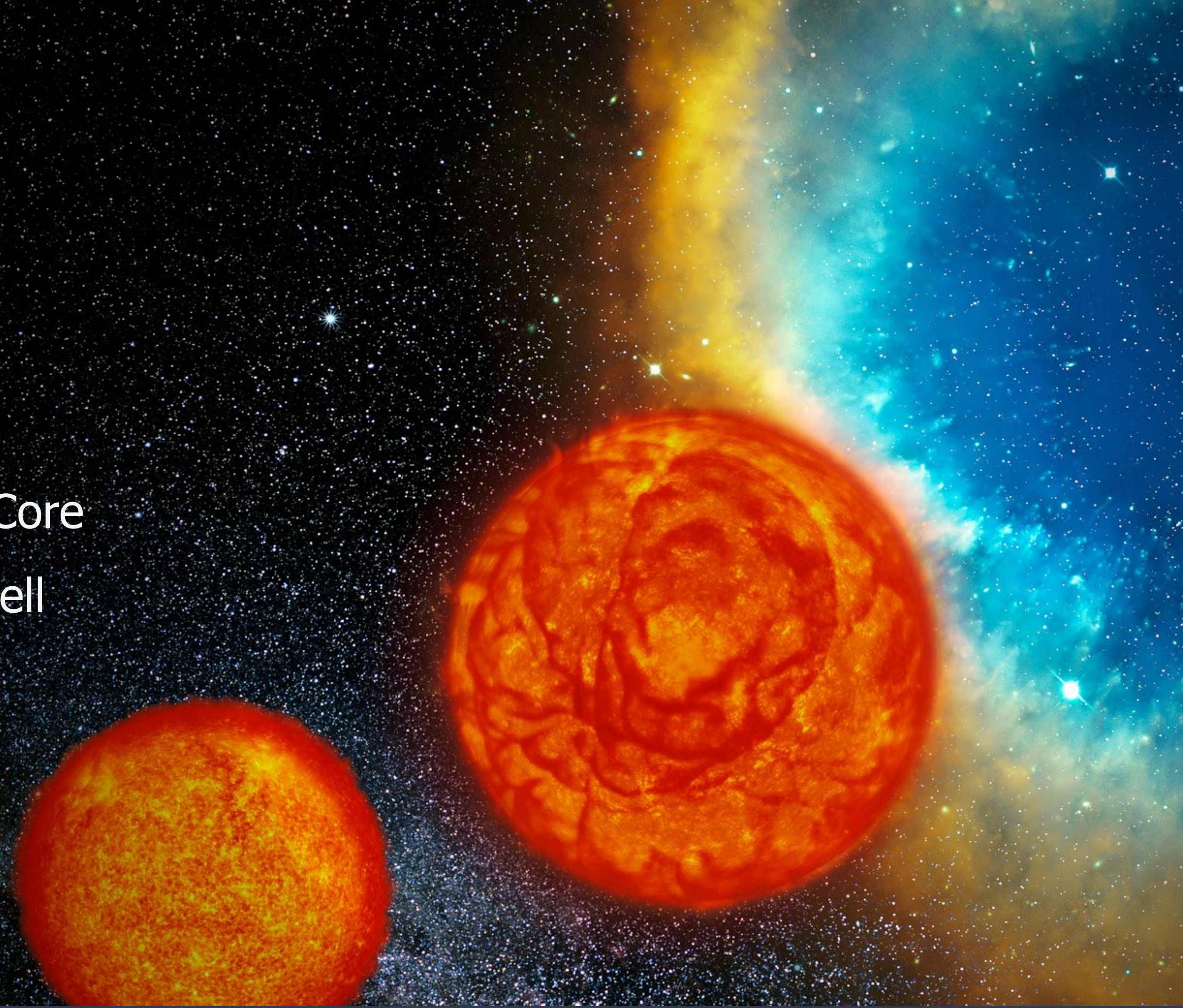
We continue to iterate this way until the solution reaches the required precision!

The Solving Method - Code flow chart



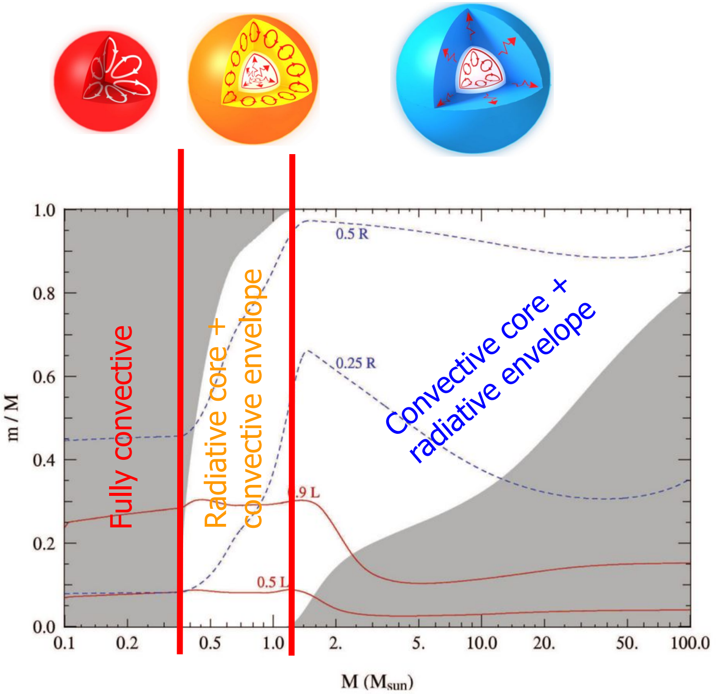
4.
The Phases of Stellar Evolution

- The Main Sequence
- Hydrogen Burning in the Core
- Hydrogen Burning in a Shell



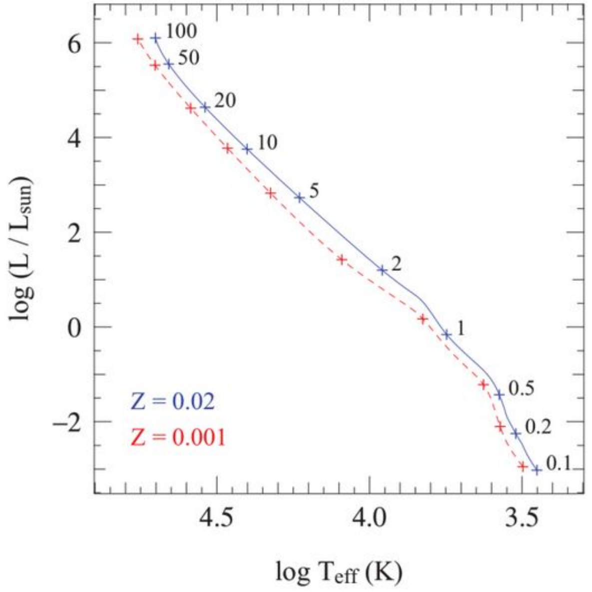
The Main Sequence

This is the first (and longest) phase of stars. They are in hydrostatic and thermodynamic equilibrium, and are sustained by the hydrogen burning in the core. At the beginning (Zero-Age Main-Sequence, ZAMS) stars are homogeneous and can be in three different configurations depending on mass.



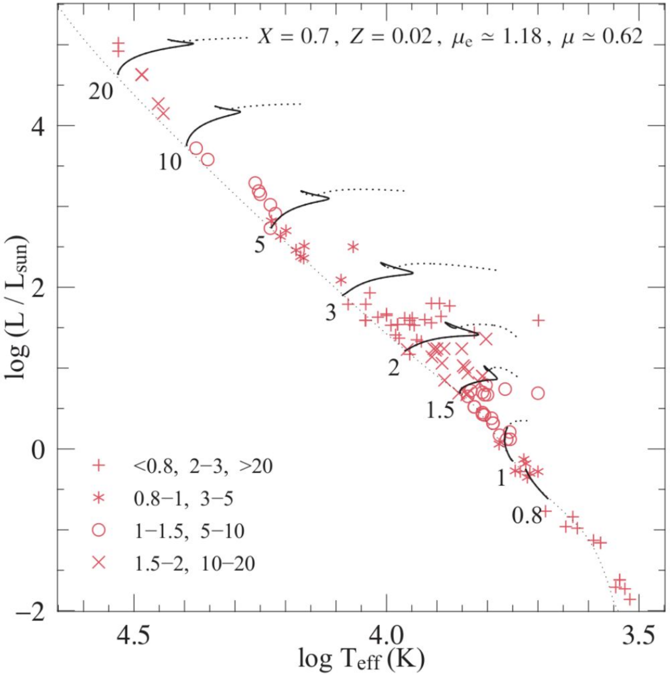
The location of the ZAMS on the HRD depends mainly on:

1. Initial Mass
2. Composition (Z)



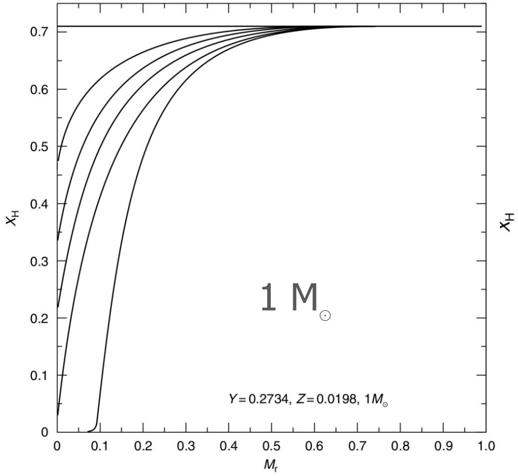
Hydrogen Burning in the Core

During the core H-burning phase stars move from the ZAMS toward higher luminosities and bigger radii.

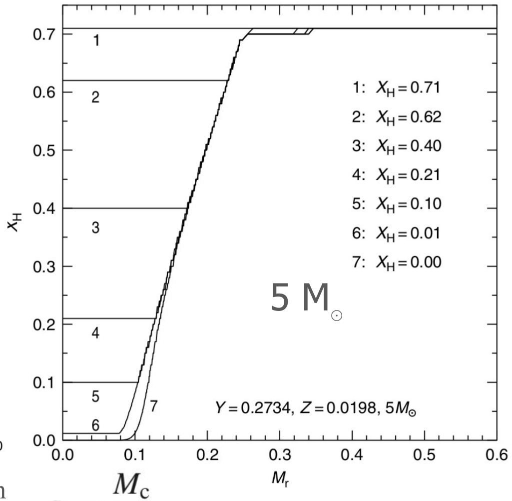


Depending on the core configuration, hydrogen profiles are different as the star evolves.

Radiative core



Convective core



relative mass defect per nucleon

$$q_c = \frac{M_c}{M}$$

$$t_H \approx \frac{\text{fuel available}}{\text{power}} \approx \frac{\Delta M c^2}{\bar{L}} \approx \frac{0.007\, q_c M c^2}{\bar{L}}$$

For the Sun

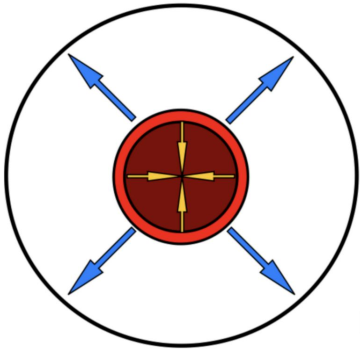
$$t_H \approx 3 \times 10^{17}\,\mathrm{s} \approx 10^{10}\,\mathrm{yr}$$

Hydrogen Burning in a Shell

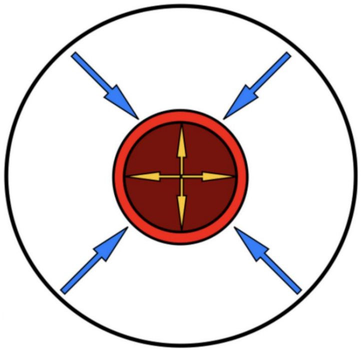
When hydrogen is exhausted in the core, the core contracts and hydrogen ignites in a shell above the core, leading the envelope to expand.

This is the so-called **mirror effect**!

$r_{\mathrm{core}} \downarrow \Rightarrow \; r_{\mathrm{env}} \uparrow$



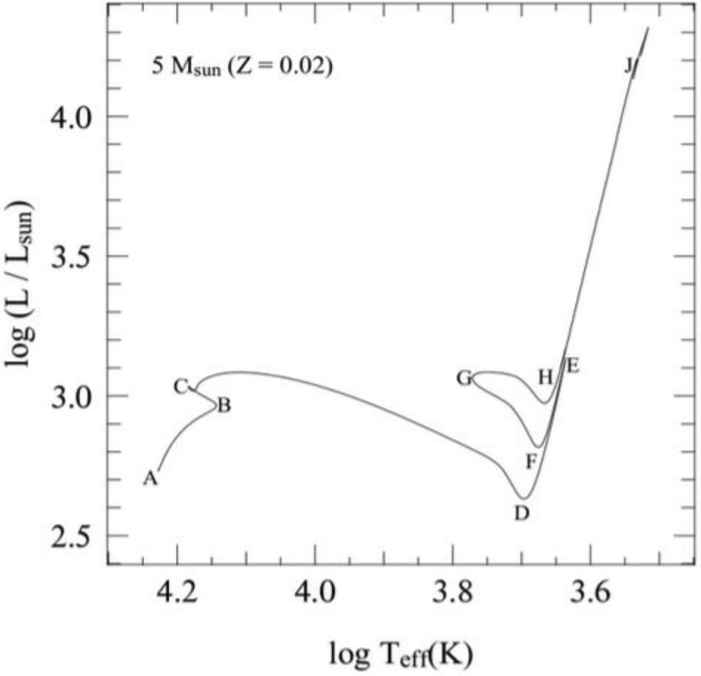
$r_{\mathrm{core}} \uparrow \Rightarrow \; r_{\mathrm{env}} \downarrow$



(this happens later!)

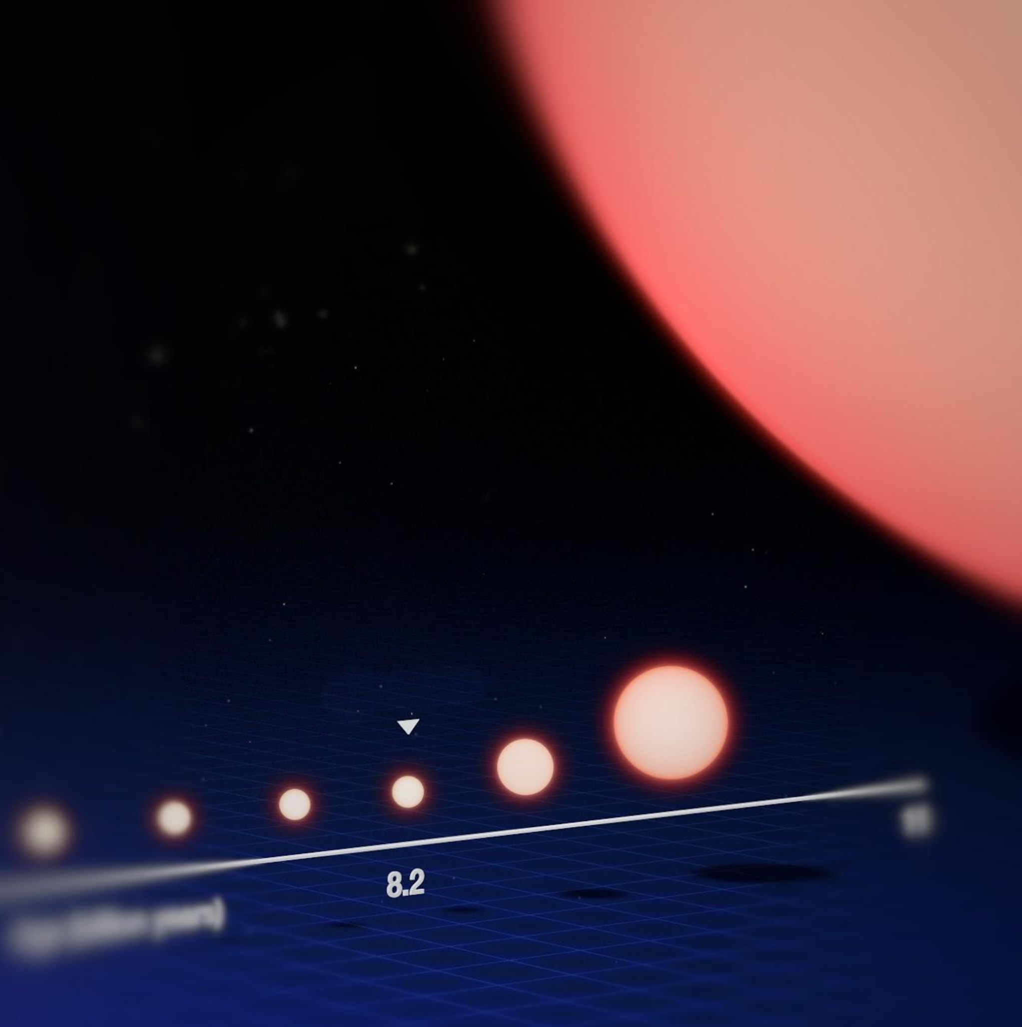
While the star expands, it moves toward colder temperatures and becomes a Red Giant (RG).

Points C → D



5.
The Role of Initial Mass

- Stellar Classification
- Helium Burning in the Core
- Double-Shell Burning and the AGB
- Advanced Burning Stages
- Remnants of Stellar Evolution



Stellar Classification by Initial Mass

The most important initial property that determines the stars' evolution and fate is the mass. Therefore, it is interesting to classify them by their mass.

Low-mass (LM) stars : $0.8\ M_\odot \lesssim M \lesssim 2\ M_\odot$

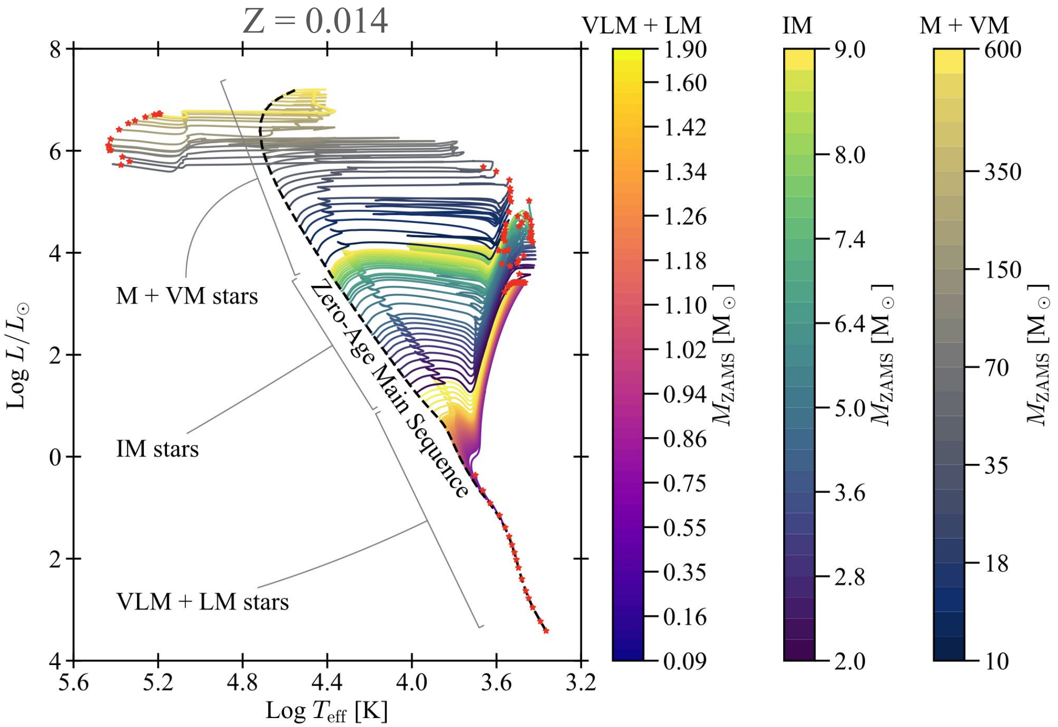
- They form a degenerate He-core after MS
- Ignite explosively He → He flash!
- They will form a degenerate CO core → WD

Intermediate-mass (IM) stars: $2\ M_\odot \lesssim M \lesssim 9\ M_\odot$

- Form a non-degenerate He core after MS
- They ignite He 'smoothly'
- End their evolution with a degenerate CO → WD

Massive (M) stars: $M \gtrsim 10\ M_\odot$

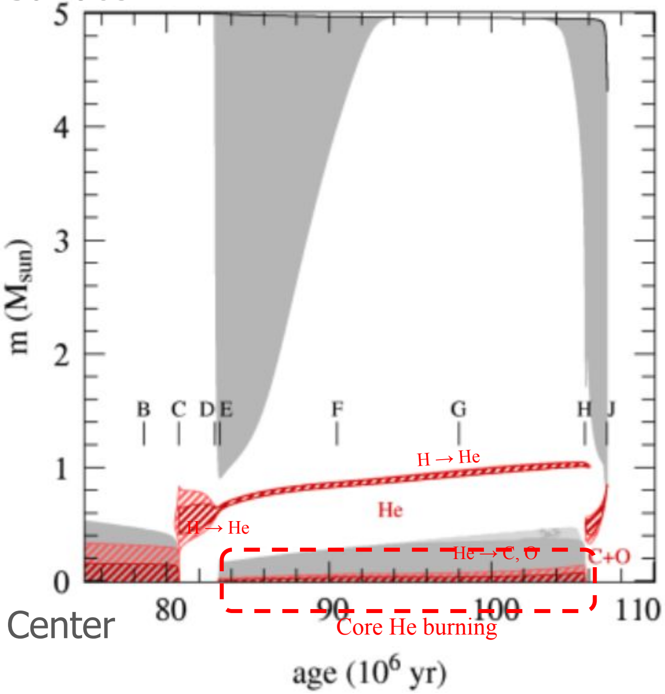
- They are able to ignite C in their core after CHeB
- Nuclear cycles up to Si burning
- They form a Fe core and explode as supernova



Helium Burning in the Core

After reaching the Red Giant phase, the core continues to contract, reaching the temperature to ignite helium (points D → E).

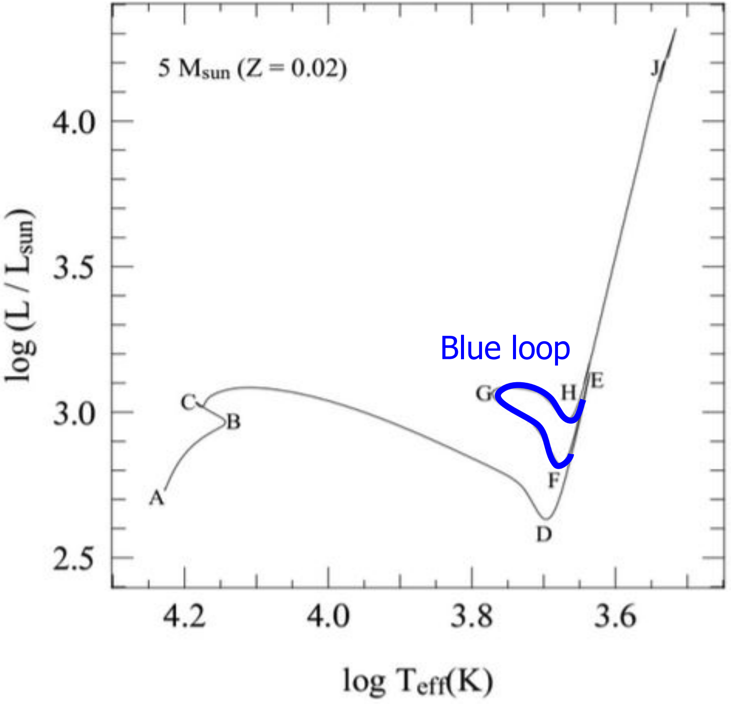
At this point the core expands and the envelope contracts (the mirror effect again!), and happens the contrary when He is exhausted. In this phase the star run a loop in the HRD called blue loop (Points E → H).



Main reactions during CHeB

1. $3\,{}^4He \rightarrow {}^{12}C + \gamma$
2. ${}^{12}C + {}^4He \rightarrow {}^{16}O + \gamma$
3. ${}^{16}O + {}^4He \rightarrow {}^{20}Ne + \gamma$

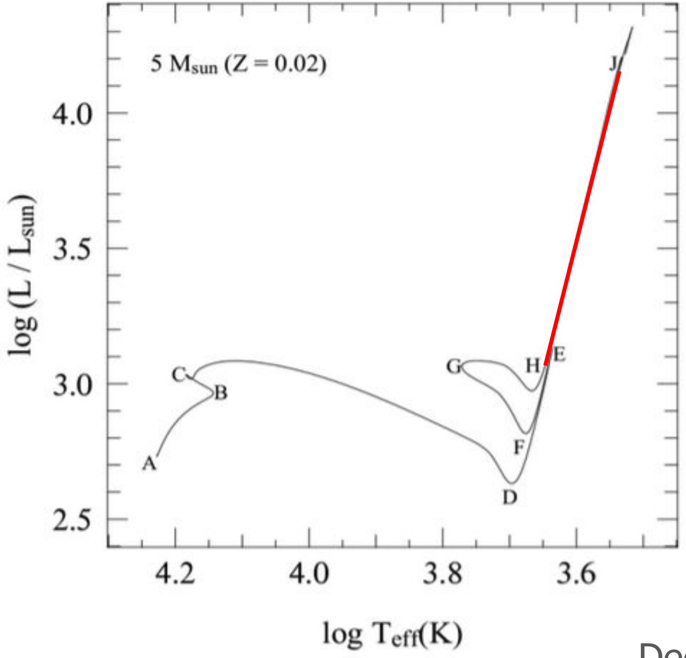
At the end of this phase there is a carbon-oxygen (CO) core at the center of the star.

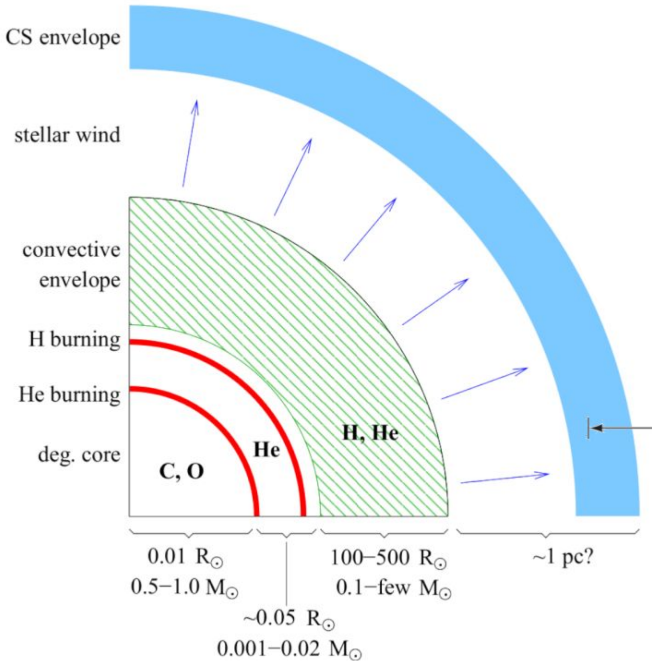


Double-Shell Burning - The Asymptotic Giant Branch

After CHeB, low- and intermediate mass stars undergo a contraction of the core and move toward the early-AGB (E-AGB). Points H → J

In this phase the star has two burning shells acting at the same time.





Dedicated lecture by G. Pastorelli

Advanced Nuclear Burning Stages

After CHeB, massive stars proceed toward advanced burning stages, i.e.:

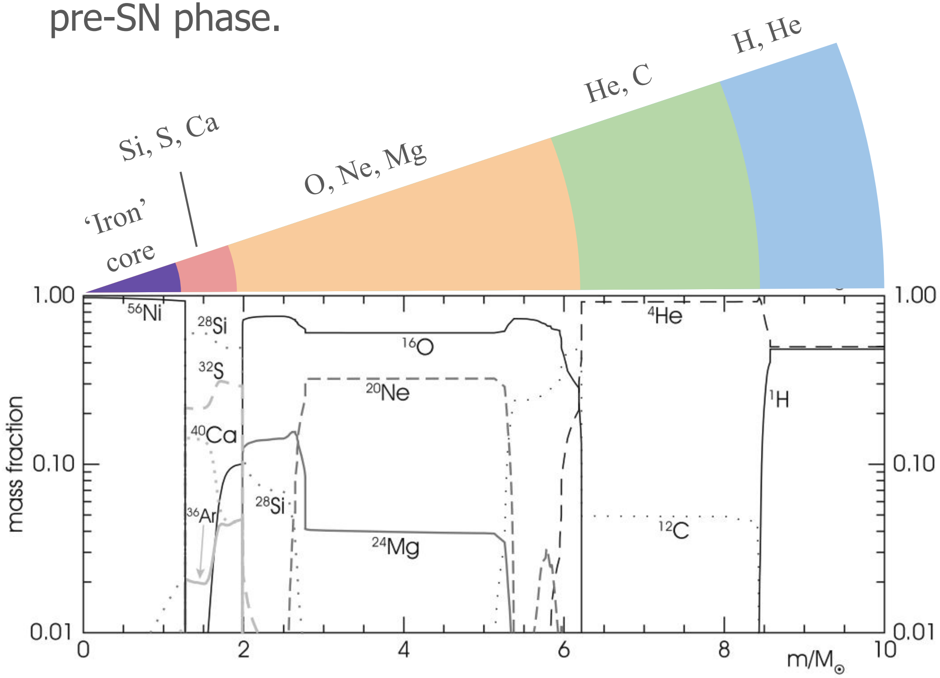
1. Carbon burning ($T_c \sim 5 - 8 \times 10^8$ K)
   $^{12}C + ^{12}C \rightarrow ^{24}Mg, ^{20}Ne, ^{23}Na$

2. Neon photodisintegration ($T_c \sim 1.2 - 1.5 \times 10^9$ K)
   $^{20}Ne \rightarrow ^{16}O, ^{24}Mg$

3. Oxygen burning ($T_c \sim 1.9 \times 10^9$ K)
   $^{16}O + ^{16}O \rightarrow ^{32}S, ^{28}Si, ^{31}P$

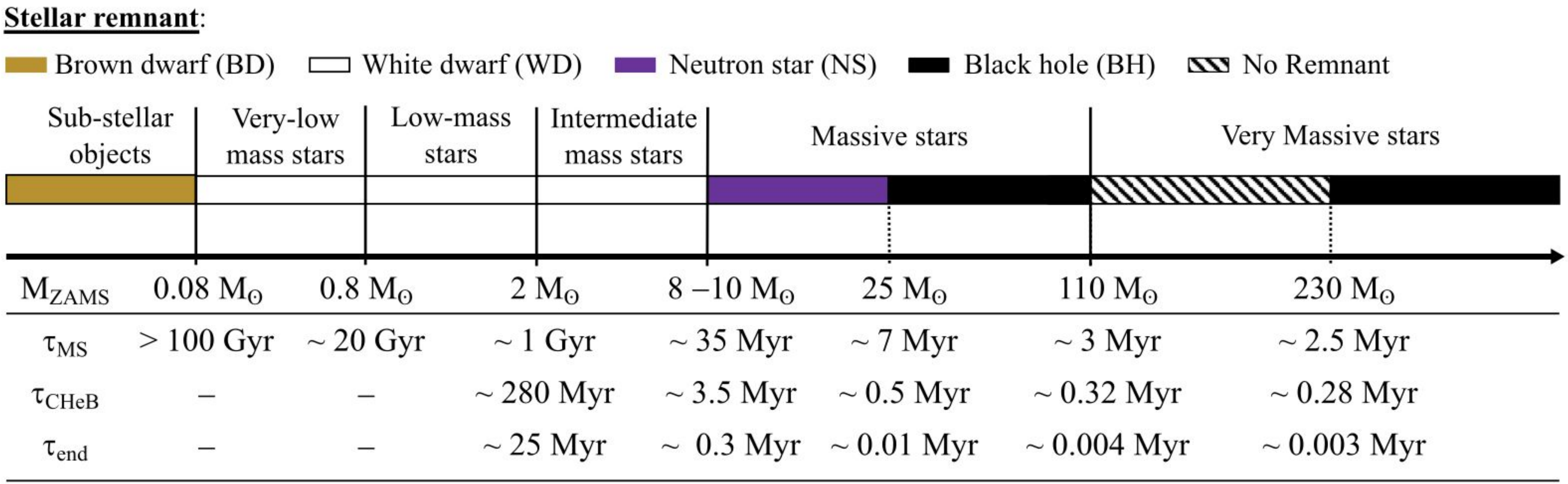
4. Silicon burning ($T_c \sim 3 \times 10^9$ K)
   $^{28}Si \rightarrow ^{56}Fe, ^{52}Cr$

These burning stages take massive stars to reach the so-called **onion-skin** configuration in the pre-SN phase.



Evolutionary Endpoints: Stellar Remnants

Each star have a final fate, which is mainly determined by its initial mass.



6.
Chemical Composition

- The Lifecycle of Stars
- Stellar Populations

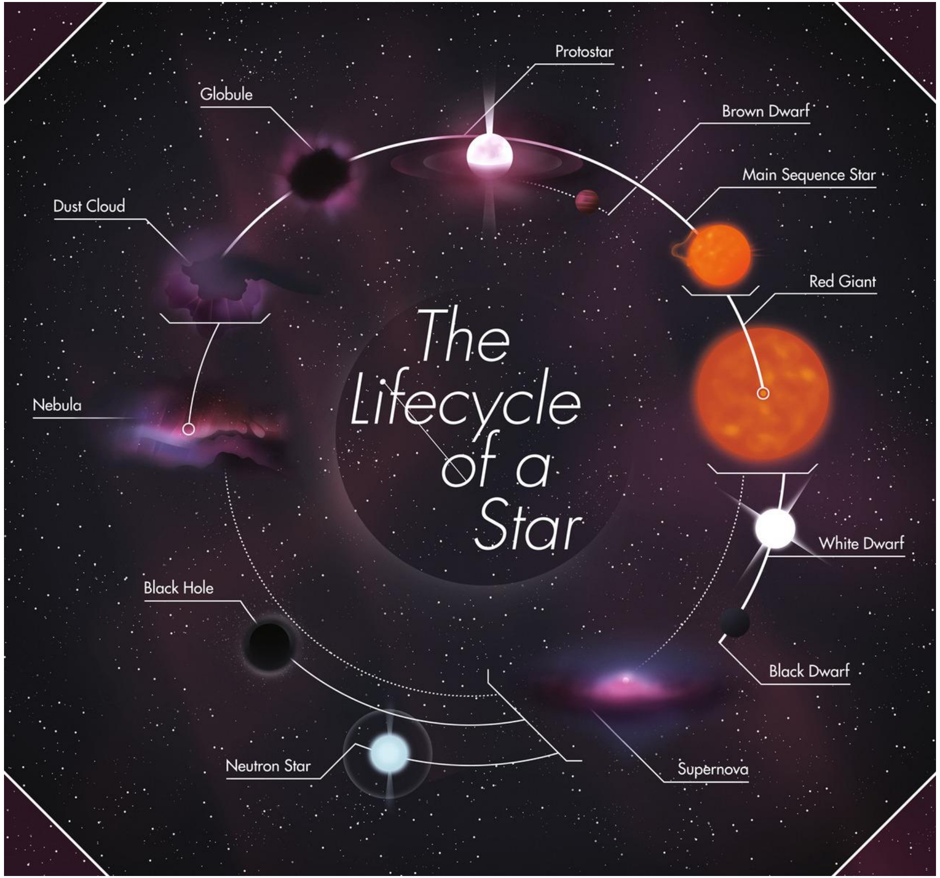
The Lifecycle of Stars

Stars are formed from matter of the interstellar medium, and only a part of it is returned to the medium via stellar winds and by Supernova explosions.

This given back matter is however enriched by heavier elements produced in stars during their evolution. As a result, each new generation of stars will contain more of these heavy elements.

This lifecycle generates different populations of stars which are generally labelled as follows:

- Population III – First stars of the Universe ($Z = 0$)
- Population II – Metal poor stars ($Z > 0$)
- Population I – Metal rich stars, solar-like ($Z \sim Z_\odot$)



Stellar Populations

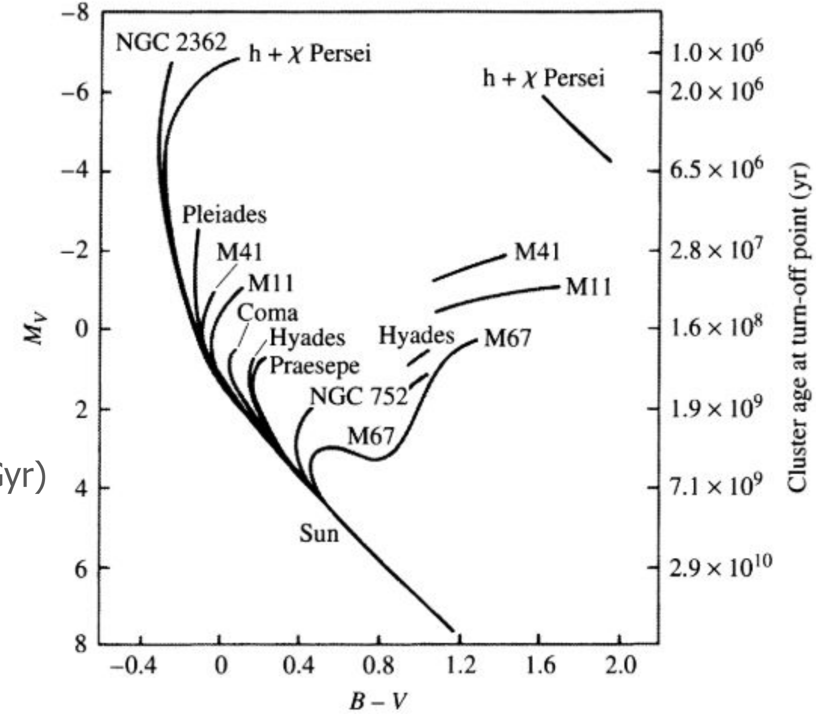
Clusters **turn-off** and **age** determination.

Stars do not form alone, but in groups. From the collapse of the progenitor cloud many stars could form, ranging from tens to hundreds of thousands.

Such groups of stars are called stellar clusters. Each population of stars share the same initial composition and formation age (coeval). → Isochrones!!!

Open clusters
- Few stars ($<10^5$)
- Small
- Generally young (< 1 Gyr)

Globular clusters
- Many stars ($>10^5$)
- Big
- Can be very old (10 Gyr)



7. Advanced Topics

- Star Formation
- Stellar Pulsations
- Stellar Winds & Mass Loss
- Stellar Explosions

Star Formation

Stars form in giant molecular clouds, with:

- Mass ~ 10^5 $M_\odot$
- Diameter ~ 10 parsec
- Temperature ~ 10 - 100 K
- Number density ~ 10 - 300 molecules/cm^3
- Formed by dust ~ 1 %

The cloud's hydrostatic equilibrium can be locally perturbed by various processes. The local equilibrium holds if the **Jeans mass** stability criterion is satisfied.

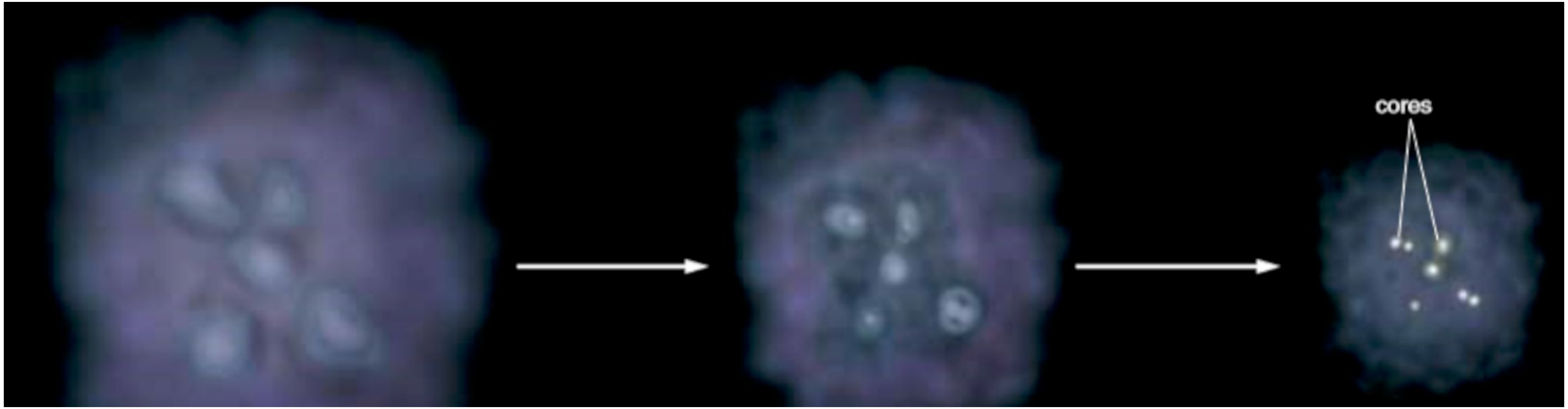
$$M < M_\mathrm{J} \simeq 4 \cdot 10^4\,\mathrm{M}_\odot \left(\frac{T}{100\,\mathrm{K}}\right)^{3/2} \left(\frac{n}{\mathrm{cm}^{-3}}\right)^{-1/2}$$

$M_\mathrm{J} \sim 10^3 - 10^4\ \mathrm{M}_\odot$, for typical T and n.

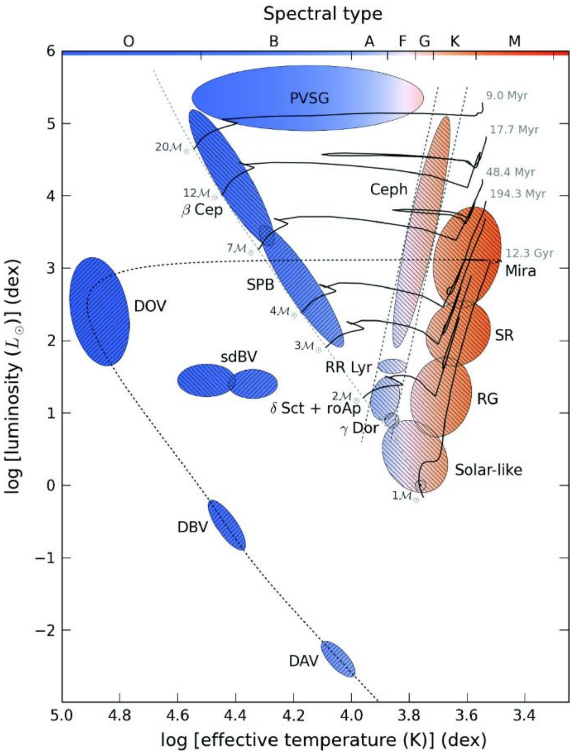
1- Portions of the cloud are perturbed

2- The collapse begins

3- Formation of stellar cores



Stellar Pulsations



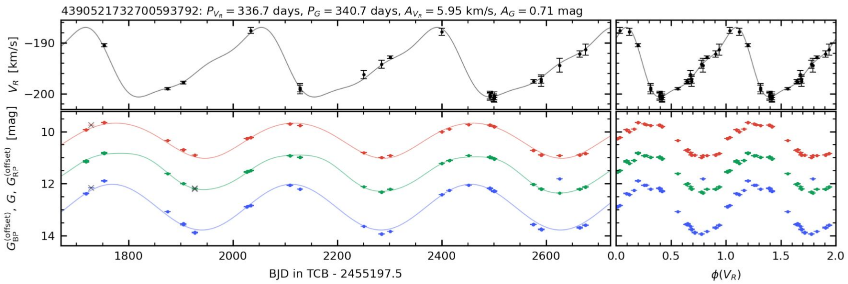
Ubiquitous across evolutionary phases

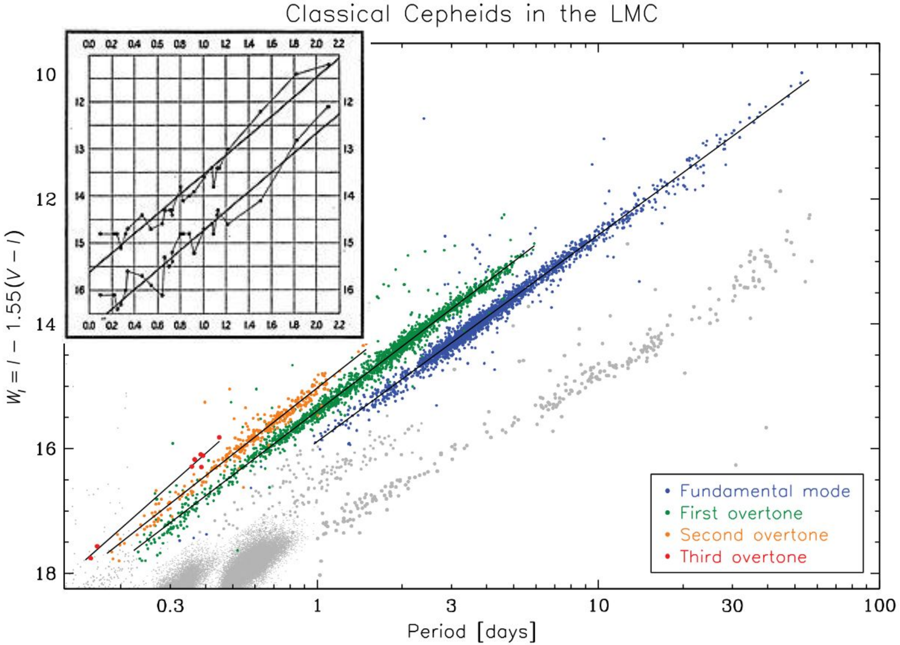
Observed changes: photometry, rad. vel.

Different mechanisms: self-excited / stochastic

Period determined by stellar structure:

- absolute luminosity, distance measures
- probe stellar interiors





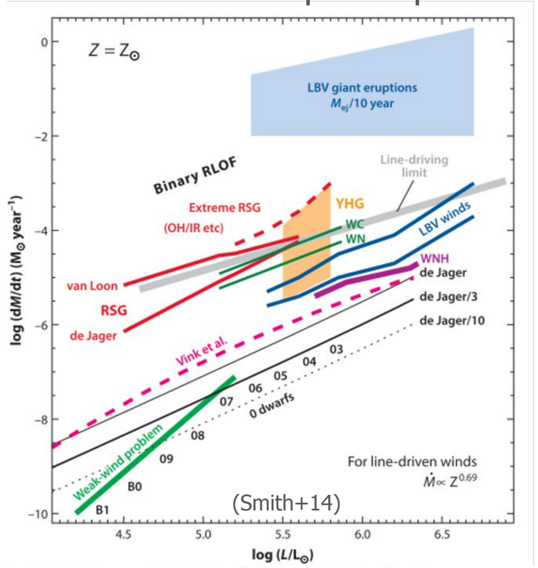
Stellar Winds & Mass Loss

Massive stars winds prescriptions zoo

Momentum transfer: radiation → matter

Matter outflow = mass loss: shapes stellar evolution

Mechanisms: dust-driven, line-driven, coronal winds, LBVs



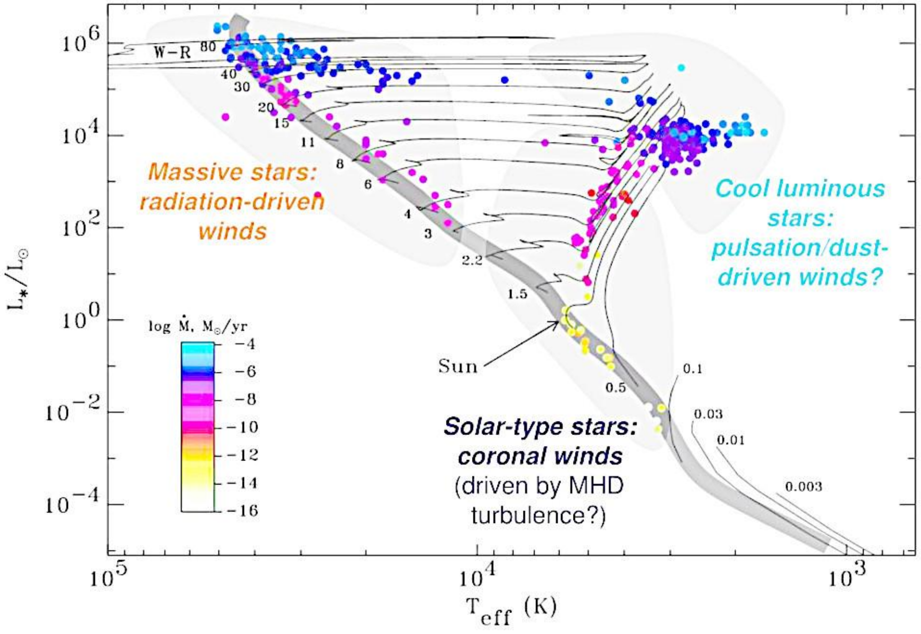


Table 1 Typical wind parameters for different stellar Types (Vink 2022)

Type	$T_{\rm eff}$ (kK)	M ($M_\odot$)	v_∞ (km/s)	$\dot{M}$ ($M_\odot {\rm yr}^{-1}$)	SN Type (speculative)
O	30-45	20-60	2000-3500	$10^{-7} - 10^{-5}$	-
WNh	35-50	80-300	1500-3000	10^{-4}	-
BSG	15-25	15-30	500-1500	$10^{-7} - 10^{-5}$	IIb/IIP-pec
YSG	5-10	10-25	50-200	$10^{-6} - 10^{-4}$	IIb
RSG	3-5	10-25	10-30	$10^{-7} - 10^{-4}$	IIP/IIL
LBV low-L	10-15	15-25	100-200	10^{-5}	IIb
LBV high-L	10-30	40-	200-500	$10^{-4} - 10^{-3}$	IIn
cWR	90-200	10-30	1500-6000	$10^{-5} - 10^{-4}$	Ic
Stripped He	50-80	1-5	1000	10^{-8}	Ib

The numbers in this Table should only be taken as typical, with the terms referring to broad evolutionary groupings. They are not used in a strict spectroscopic sense. E.g. very late WN-type stars such as WN10 here could fall in the "BSG" category despite having a WR emission-line spectrum.

Stellar Explosions

Massive stars end their evolution in different ways, depending on M_{ZAMS} :

$10 \lesssim M_{ZAMS}/M_\odot \lesssim 70$
core collapse (CC)

$70 \lesssim M_{ZAMS}/M_\odot \lesssim 230$
enter the **pair-instability** regime.

$M_{ZAMS} \gtrsim 230\ M_\odot$
direct core collapse.

Final phases

The iron core becomes unstable due to electron capture and later photodisintegration processes. A **proto-neutron star** forms and the infalling envelope bounce on its surface, creating a shock wave that propagate outwards.

After the C/Ne-burning, the core becomes unstable due to pair creation process. The collapse triggers the explosive ignition of the Oxygen.

The core becomes unstable due to pair creation process, but the explosive ignition of the Oxygen and Silicon is not enough to stop the collapse.

Final outcome

If the shock wave ejects the envelope → **Core Collapse Supernova (CCSN)**.
Remnant: **Neutron Star**

If the shock wave stops → **failed supernova**.
Remnant: **Black hole**

O-burning causes pulsations that eject matter from the envelope. At the end of the cycle the star is stable again. → **Pulsational pair instability supernova (PPISN)**.
Remnant: **Black hole**

If the explosion is powerful enough the whole star is destroyed → **Pair instability supernova (PISN)**.
Remnant: **Nothing**

Remnant: **Black hole**

8.
Conclusions

- Theory $\leftrightarrow$ Observations
- Conservation laws + auxiliary relations $\rightarrow$ stellar models

- Nuclear chemical changes $\rightarrow$ stellar evolution
- Numerical solution $\rightarrow$ interpretation
- Complications, advanced effects $\rightarrow$ frontiers of stellar evolution