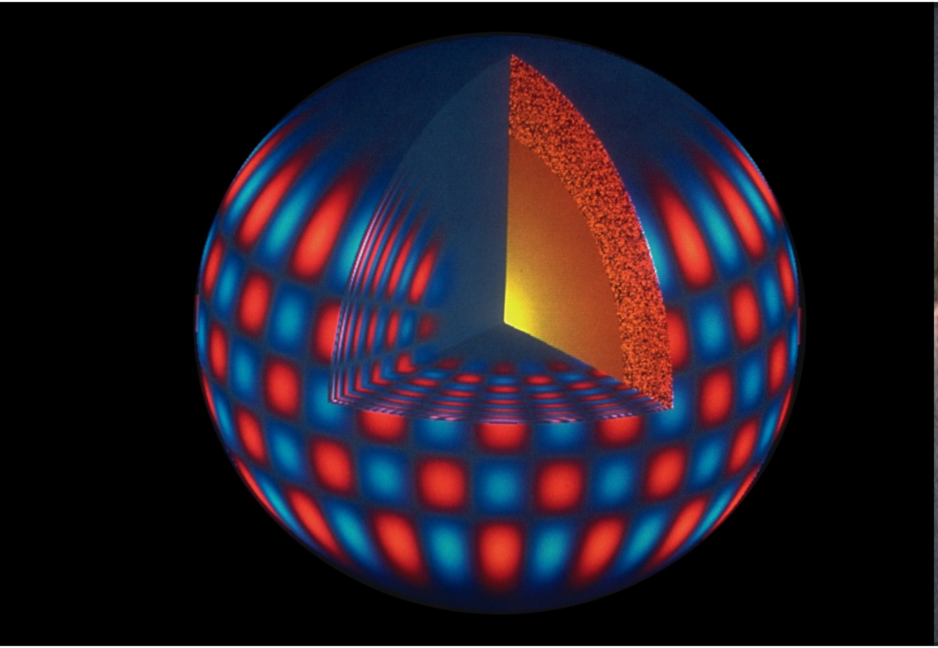
Principles of Asteroseismology



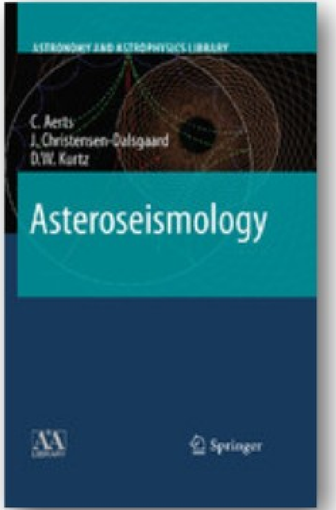
Diego Bossini

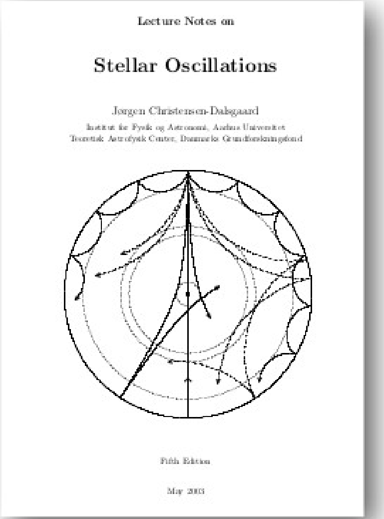
17th October 2024

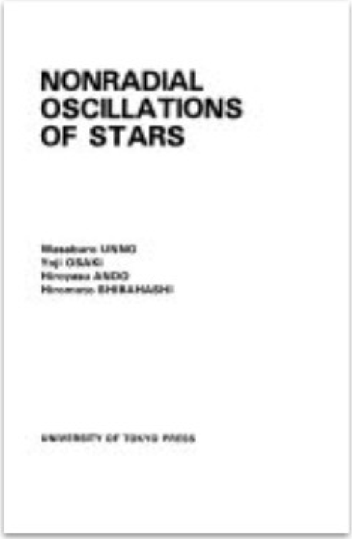
INAF
ISTITUTO NAZIONALE
DI ASTROFISICA

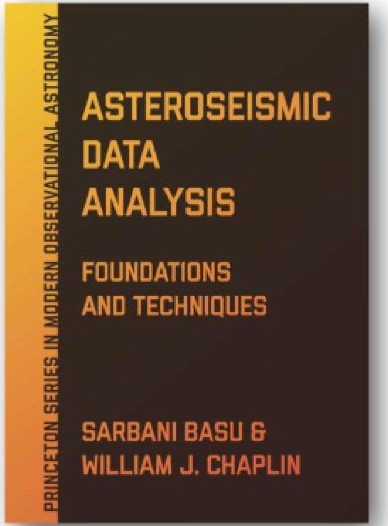
UNIVERSITÀ
DEGLI STUDI
DI PADOVA

Resources



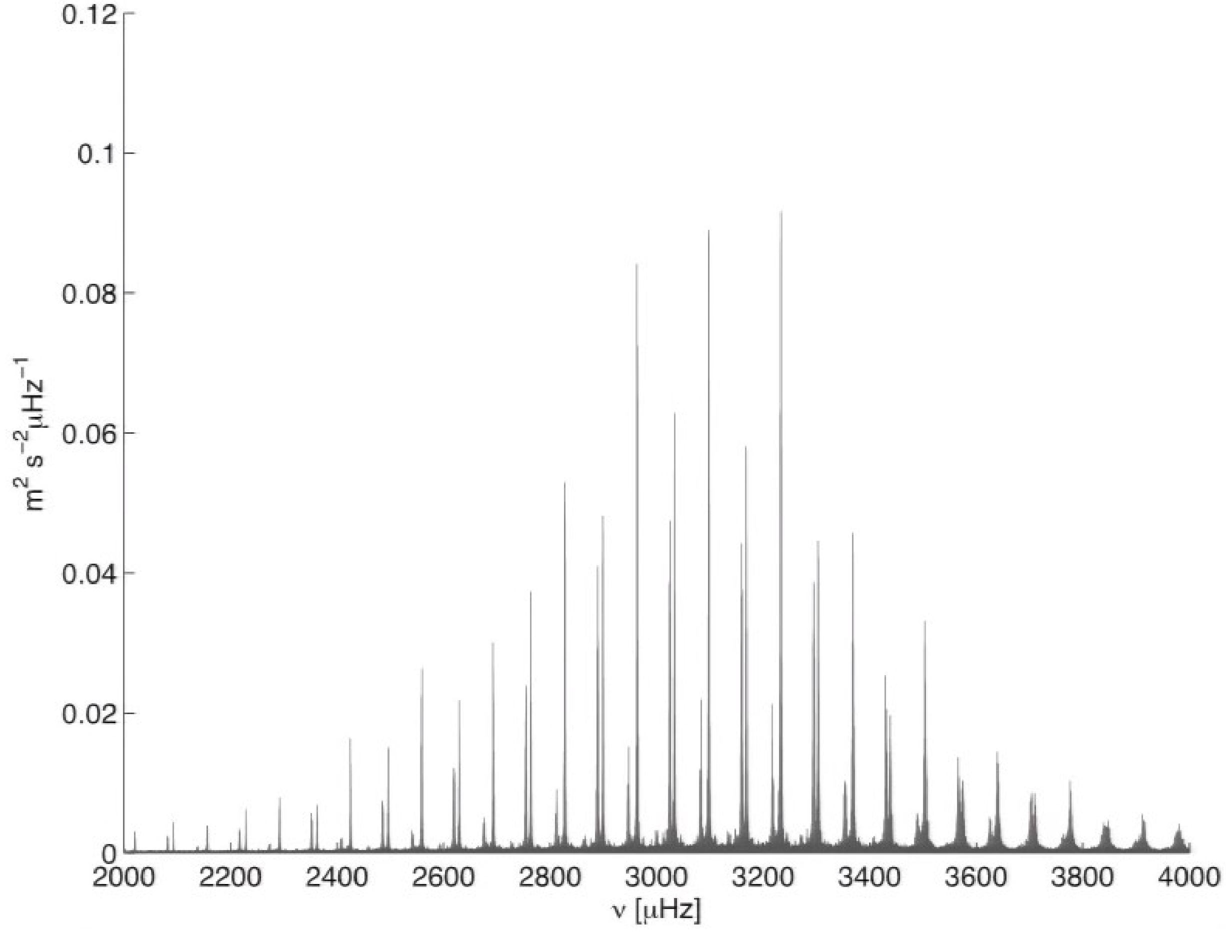






lecture notes by Christensen-Dalsgaard
http://users-phys.au.dk/jcd/oscilnotes/

What is this?



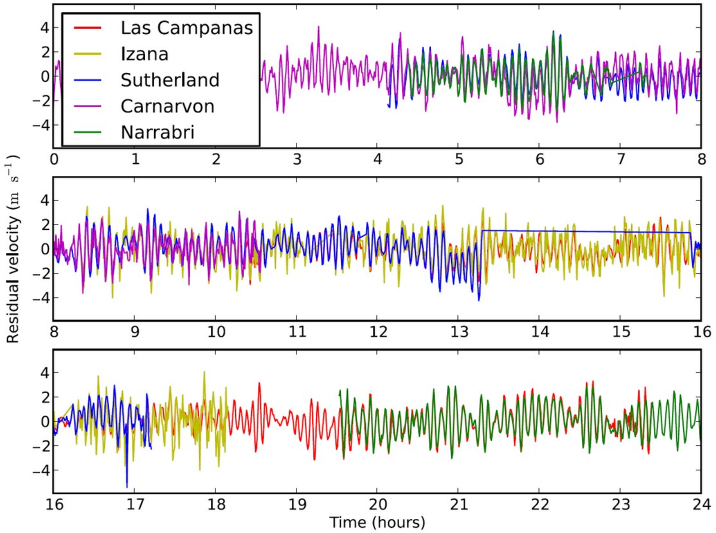
0.12
0.1
0.08
0.06
0.04
0.02
0
$m^2 s^{-2} \mu Hz^{-1}$
2000
2200
2400
2600
2800
3000
3200
3400
3600
3800
4000
ν [μHz]

Birmingham Solar Oscillation Network

A wide-world network of 6 telescopes for observe oscillation in the Sun.

Helioseismology

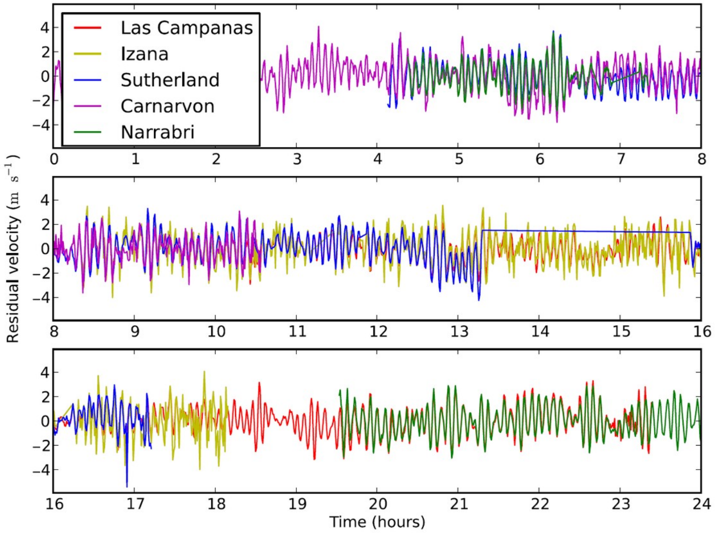
Time Series



Davies et al. 2014

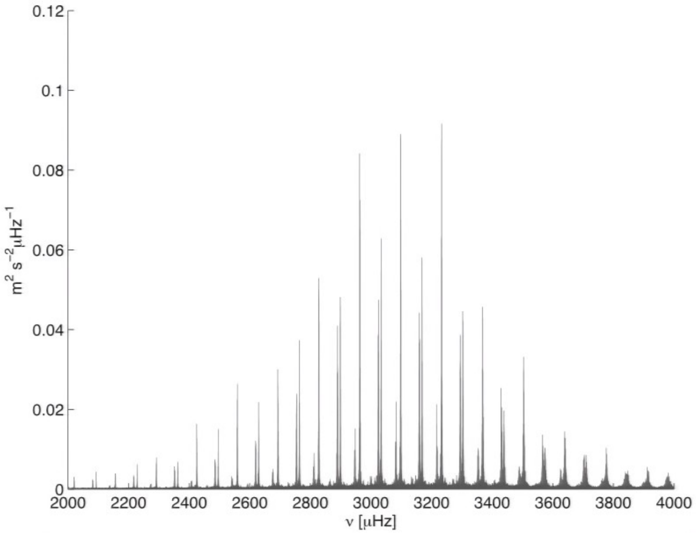
Helioseismology

Time Series



Fourier Transform

Power spectrum

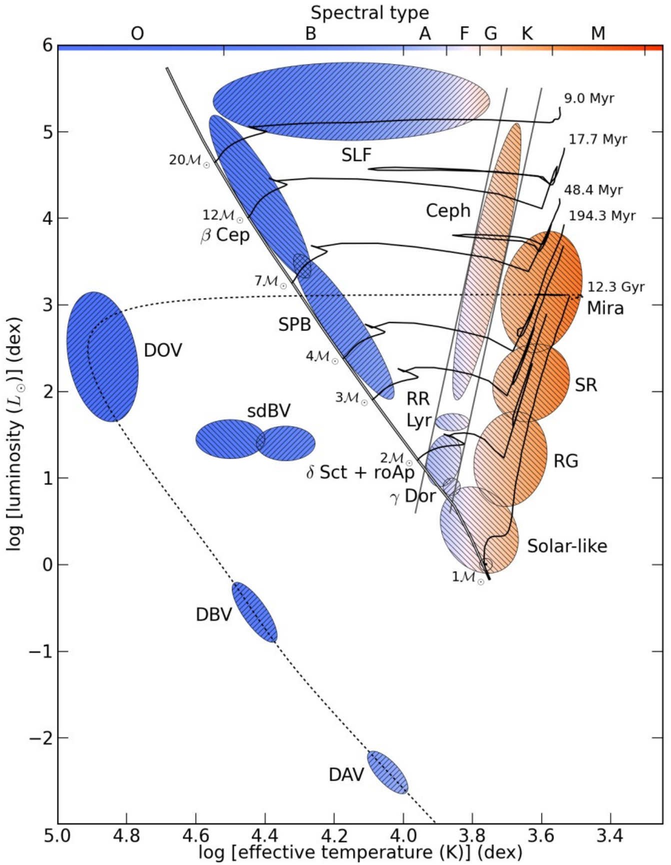


Davies et al. 2014

Why Do Stars Pulsate?

- **Self-exited oscillations**

unstable oscillations, e.g. κ-mechanism. They are typical in stars like Cepheids, RR Lyrae, Mira, δ Scuti, β Cephei, and other classical pulsators.

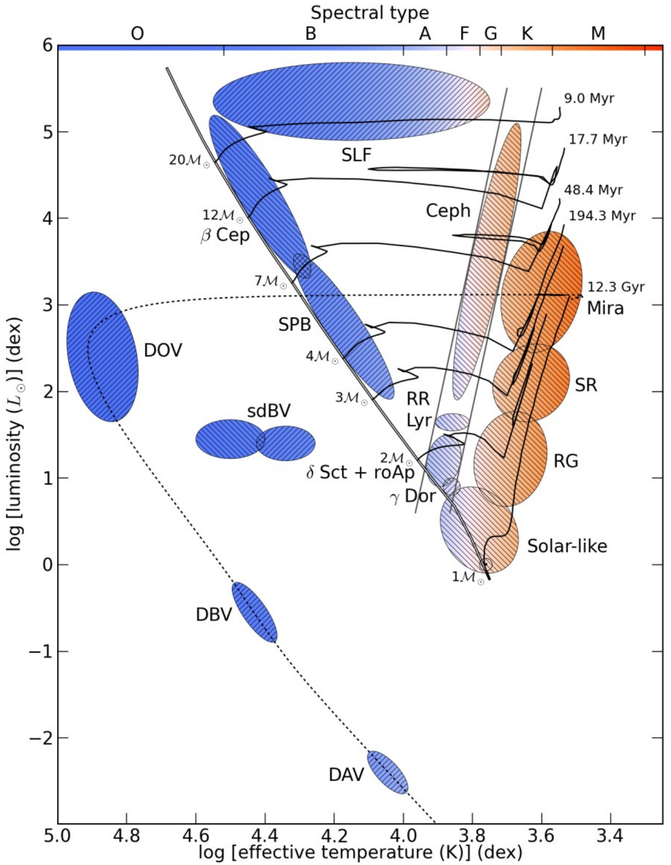


Aerts ,2021RvMP...93a5001A

Why Do Stars Pulsate?

• **Self-exited oscillations**
unstable oscillations, e.g. κ-mechanism. They are typical in stars like Cepheids, RR Lyrae, Mira, δ Scuti, β Cephei, and other classical pulsators.

• **Stochastic oscillations**
Stable dumped oscillations, excited by external convective layers. Present in stars with T_{eff} low enough to have a superficial convective region, typical, e.g. low main sequence, sub-giant and red-giant.



Aerts ,2021RvMP...93a5001A

Why Do Stars Pulsate?

• **Self-exited oscillations**

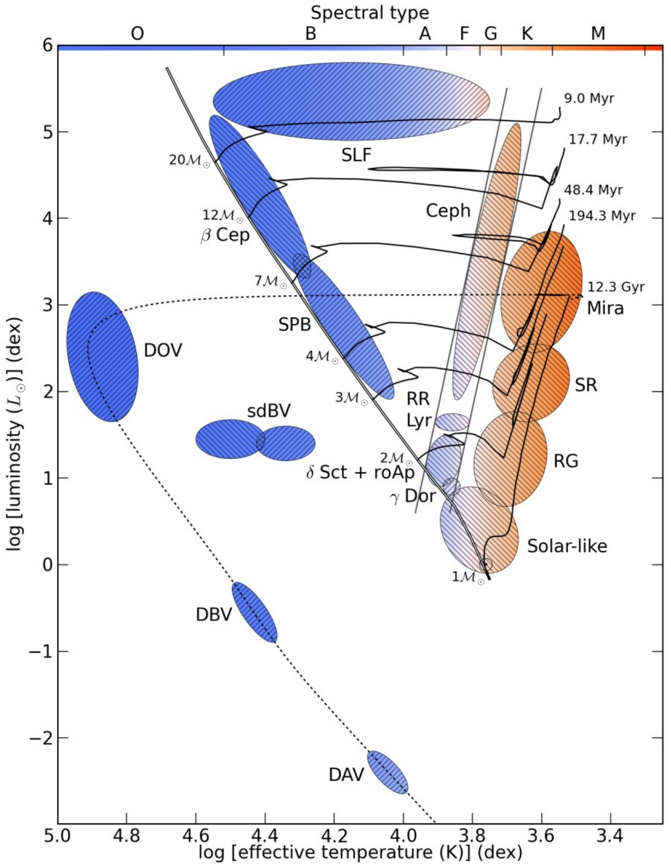
unstable oscillations, e.g. κ-mechanism. They are typical in stars like Cepheids, RR Lyrae, Mira, δ Scuti, β Cephei, and other classical pulsators.

• **Stochastic oscillations**

Stable dumped oscillations, excited by external convective layers. Present in stars with T_{eff} low enough to have a superficial convective region, typical, e.g. low main sequence, sub-giant and red-giant.

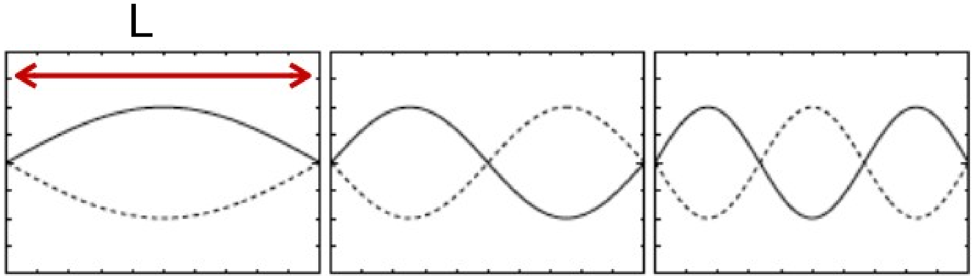
First observed in the Sun

Solar-like pulsations



Aerts ,2021RvMP...93a5001A

Little bit of theory…
Standing Waves in a String



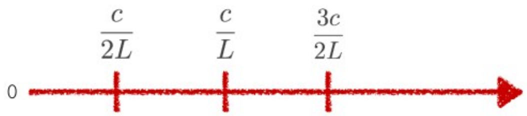
Wave equation:

$$\frac{\partial^2 \Psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \Psi}{\partial t^2} = 0$$

Solution in the form of monocromatic wave: $\Psi(x,t) = e^{i2\pi\nu t}\psi(x)$

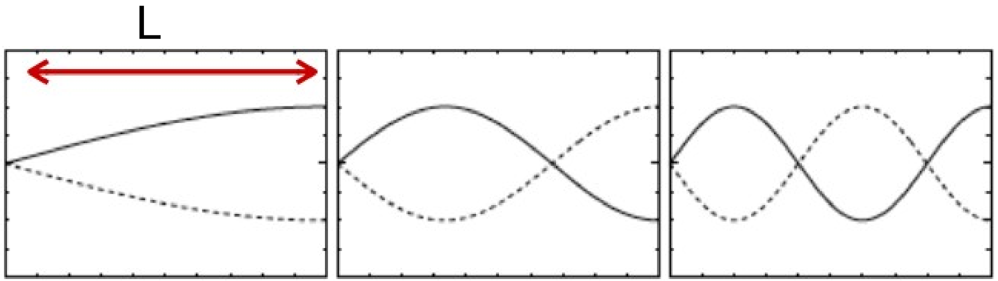
$\frac{\partial^2 \psi}{\partial x^2} + \frac{2\pi\nu^2}{c^2}\psi = 0$ with boundaries: $\psi(0) = \phi(L) = 0$

$\psi_n = A\sin(k_n x)$ Where: $\nu_n = k_n c/2\pi, k_n = \frac{n\pi}{L}, n = 1, 2, ...$



$$\Delta\nu = \nu_n - \nu_{n-1} = \frac{c}{2L}$$

Little bit of theory…
Acoustic Standing Waves in an Organ Pipe



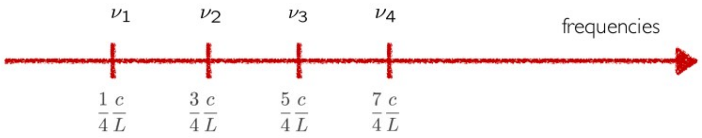
Acoustic wave equation:

$$\frac{\partial^2 P}{\partial x^2} + \frac{2\pi\nu^2}{c^2} P = 0$$

Solution in the form of monocromatic wave: $P(x,t) = A\cos(kx/c)e^{i2\pi\nu t}$

with boundaries: $P(0) = \frac{dP}{dx}(L) = 0$ and: $c^2 = \gamma_1 \frac{P}{\rho}$

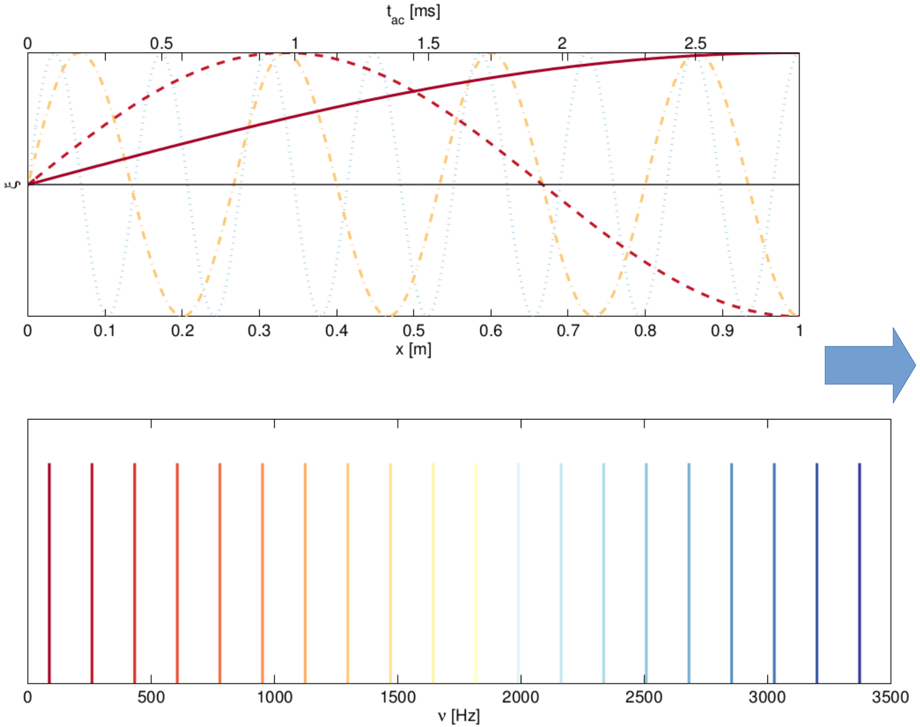
$$k_n = \frac{n\pi}{L}, \nu_n = (n - 1/2)\frac{c}{2L}, n = 1, 2, ...$$

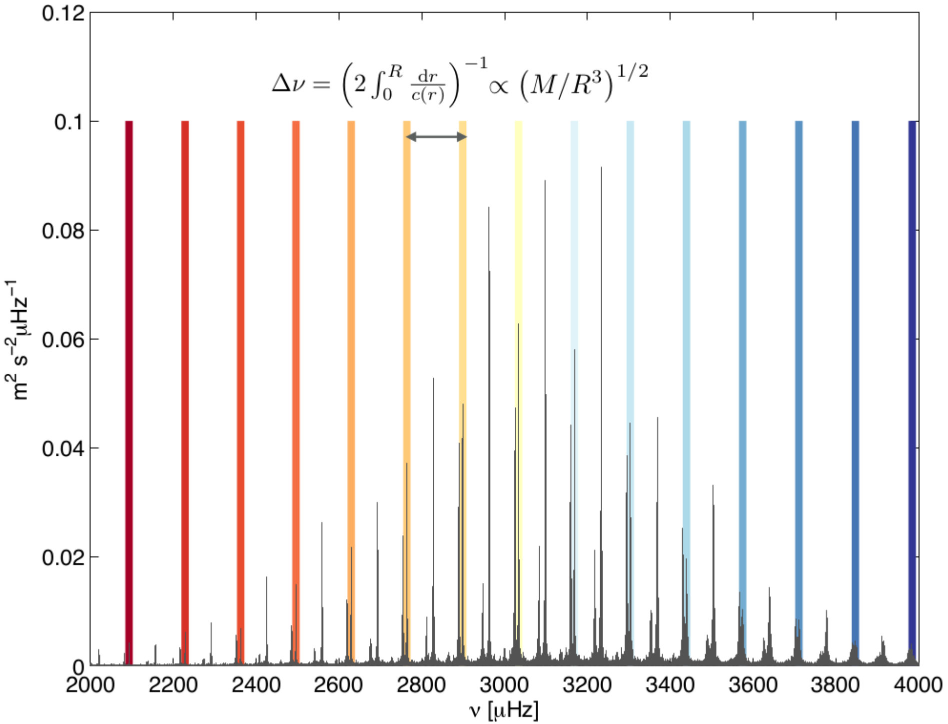


$$\Delta\nu = \nu_n - \nu_{n-1} = 1/2\frac{c}{L}$$

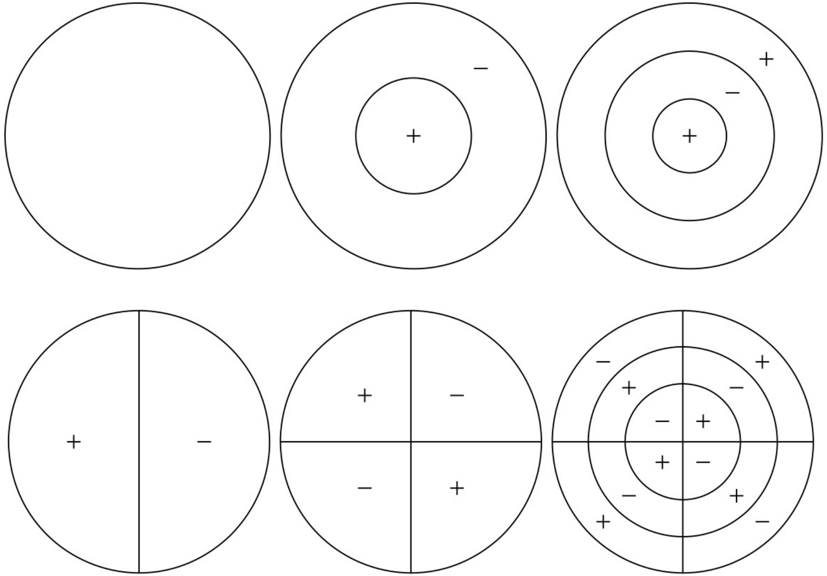
Applying the theory…

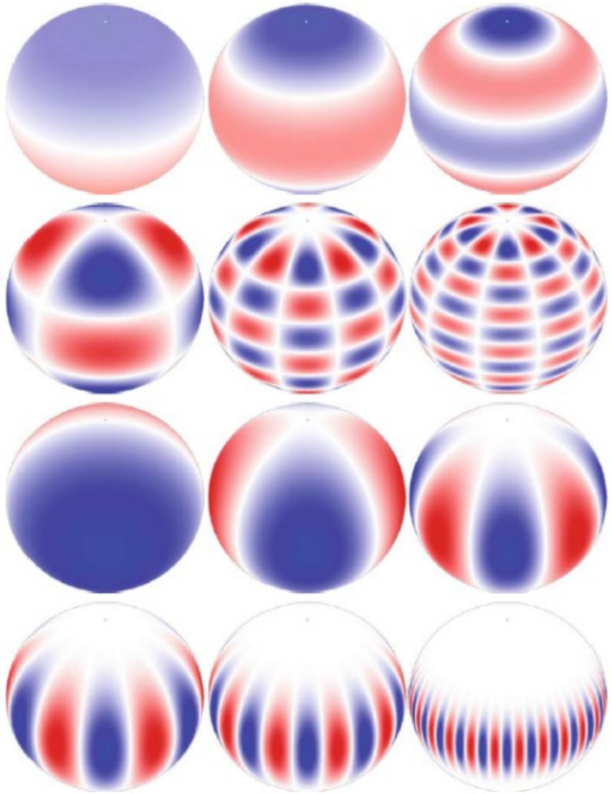
Ulrich 1986



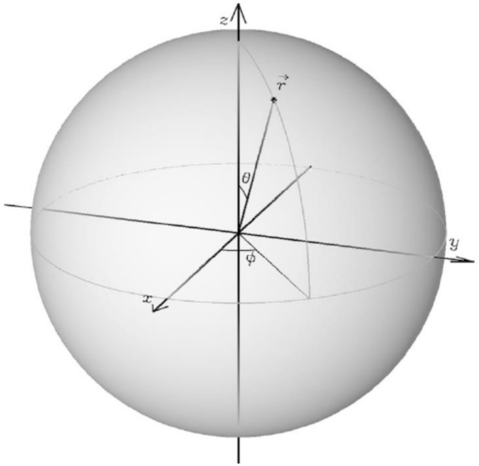


Again a little bit of theory…
2D and 3D





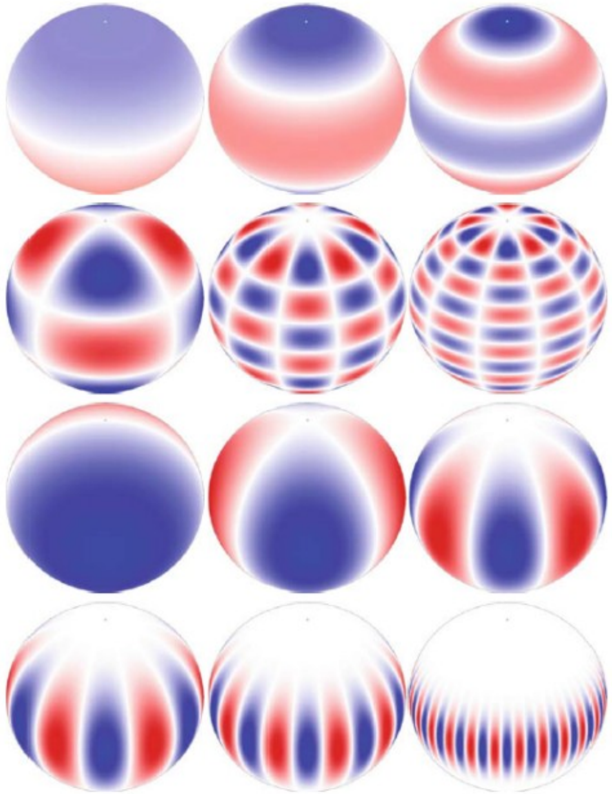
Again a little bit of theory…
Spherical harmonics



$$\xi_r(r,\theta,\phi,t) = a(r)\, Y_l^m(\theta,\phi)\exp(-\mathrm{i}\,2\pi\nu t)\,,$$

$$\xi_\theta(r,\theta,\phi,t) = b(r)\,\frac{\partial Y_l^m(\theta,\phi)}{\partial\theta}\exp(-\mathrm{i}\,2\pi\nu t)\,,$$

$$\xi_\phi(r,\theta,\phi,t) = \frac{b(r)}{\sin\theta}\frac{\partial Y_l^m(\theta,\phi)}{\partial\phi}\exp(-\mathrm{i}\,2\pi\nu t)\,,$$



Solutions we search are $\varphi(\boldsymbol{r},t) = R(r)a(\theta,\phi)\tau(t).$

Radial functions (n)

+

Spherical harmonics

$$Y_l^m(\theta,\phi) \equiv\ (-1)^m c_{lm} P_l^m(\cos\theta)\exp(\mathrm{i}m\phi)$$

Again a little bit of theory…
System of Equations for Adiabatic Oscillations

In most of the star, oscillations are adiabatic

Equation of continuity

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,(\rho \boldsymbol{v}) = 0$$

Poisson's equation

$$\nabla^2 \Phi = 4\pi G \rho$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p + \rho \boldsymbol{f}$$

Equations of motion

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\Gamma_1 p}{\rho} \frac{\mathrm{d}\rho}{\mathrm{d}t}$$

Energy equation

Oscillations typically have small amplitudes compared with the characteristic scales of the star, and so they can be treated as small perturbations around a static equilibrium state

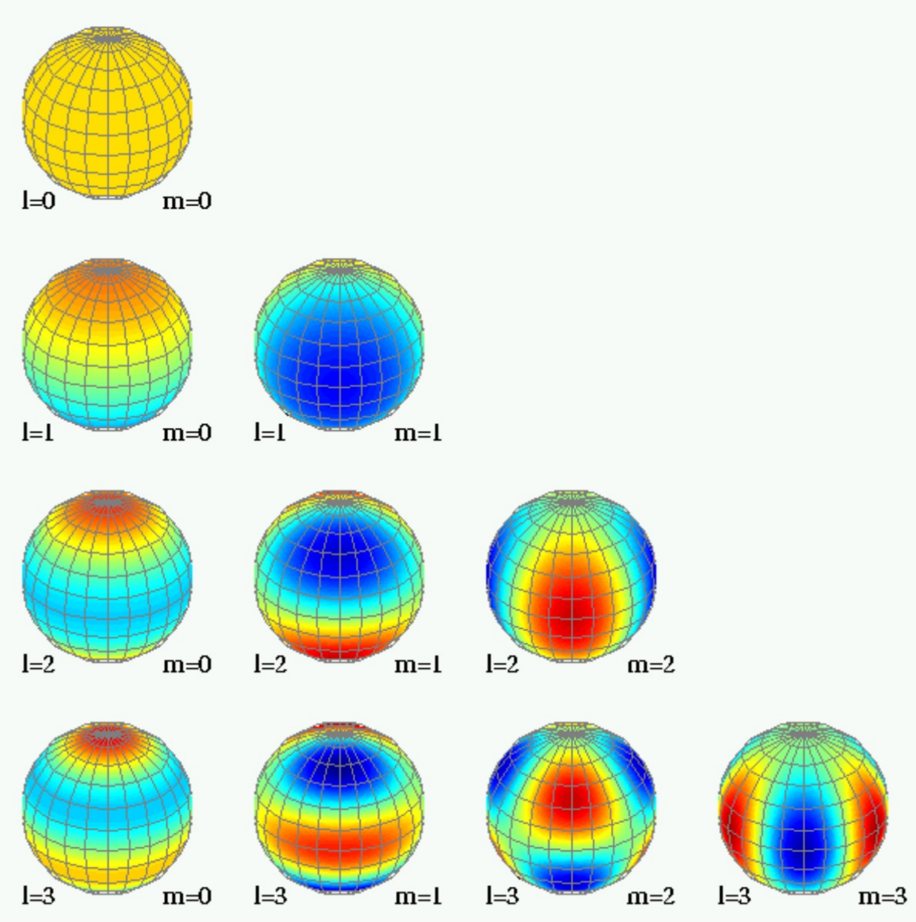
Again a little bit of theory…
A lot of maths later….

$$\begin{cases} \dfrac{\mathrm{d}\xi_r}{\mathrm{d}r} = -\left(\dfrac{2}{r} + \dfrac{1}{\Gamma_1 H_p}\right)\xi_r + \dfrac{1}{\rho c^2}\left(\dfrac{S_l^2}{\omega^2} - 1\right)p' + \dfrac{l(l+1)}{\omega^2 r^2}\Phi', \\ \dfrac{\mathrm{d}p'}{\mathrm{d}r} = \rho\omega^2\left((1 - \dfrac{N^2}{\omega^2}\right)\xi_r + \dfrac{1}{\Gamma_1 H_p}p' + \rho\dfrac{\mathrm{d}\Phi'}{\mathrm{d}r}, \\ \dfrac{1}{r^2}\dfrac{\mathrm{d}}{\mathrm{d}r}\left(r^2\dfrac{\mathrm{d}\Phi^2}{\mathrm{d}r}\right) = 4\pi G\left(\dfrac{p'}{c^2} + \dfrac{\rho\xi_r}{g}N^2\right) + \dfrac{l(l+1)}{r^2}\Phi', \end{cases}$$

Solving the system of equations is the primary aim of oscillation codes (like e.g. GYRE, Townsend & Teitler, 2013 or LOSC, Scuflaire et al., 2008).

The solutions consist in a set of discrete eigenfunctions which describe the properties of oscillation modes. Each solution/mode can be identified by three integers numbers defining the spherical harmonics (n,l,m)

Modes Properties



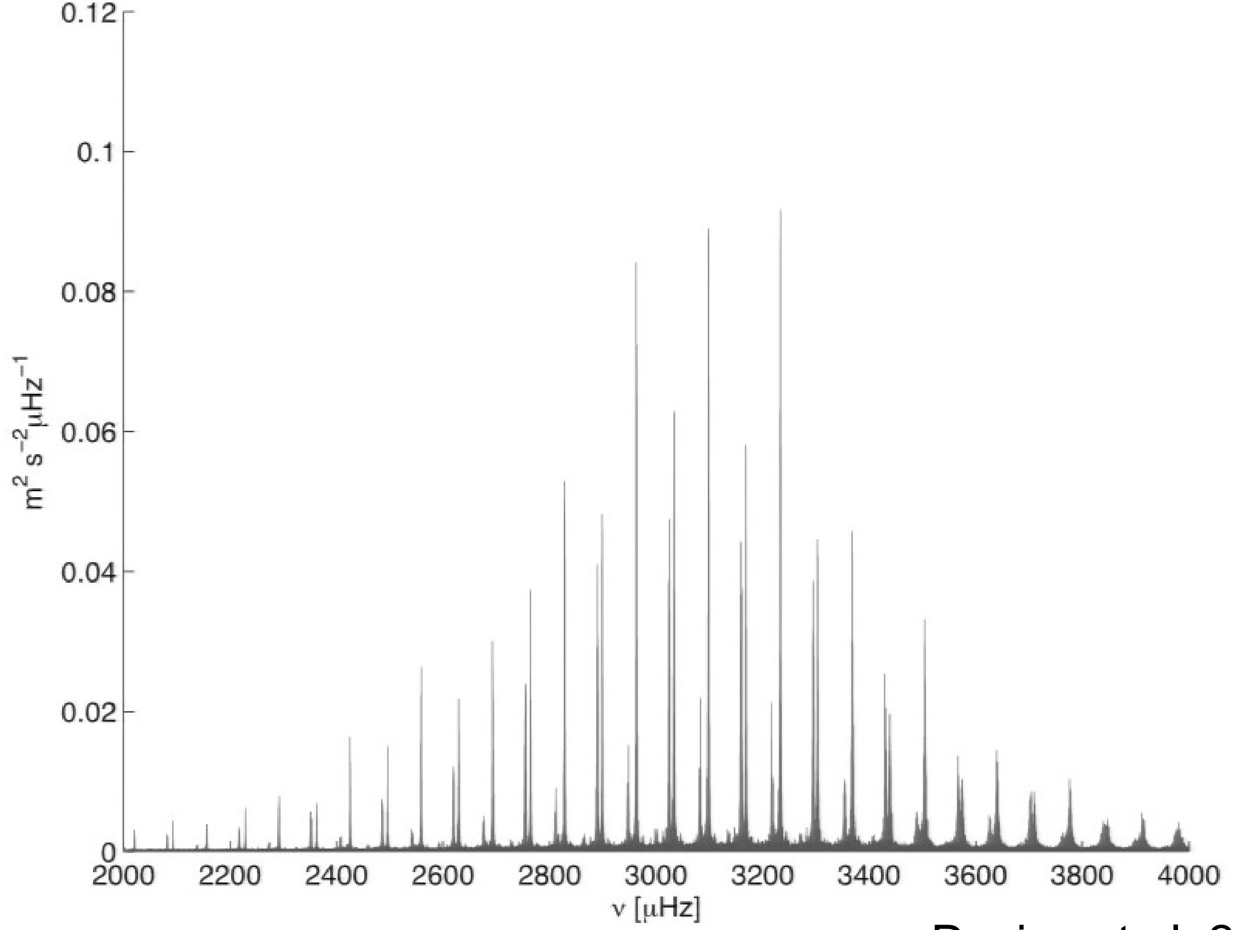
The radial order n. The radial order corresponds to the numbers of nodes of the mode between the centre and the surface.

The angular order l (or degree). The angular order represents the total number of nodal lines on the stellar surface

The azimuthal order m. The azimuthal order indicates how many of these surface nodal lines cross the equator.

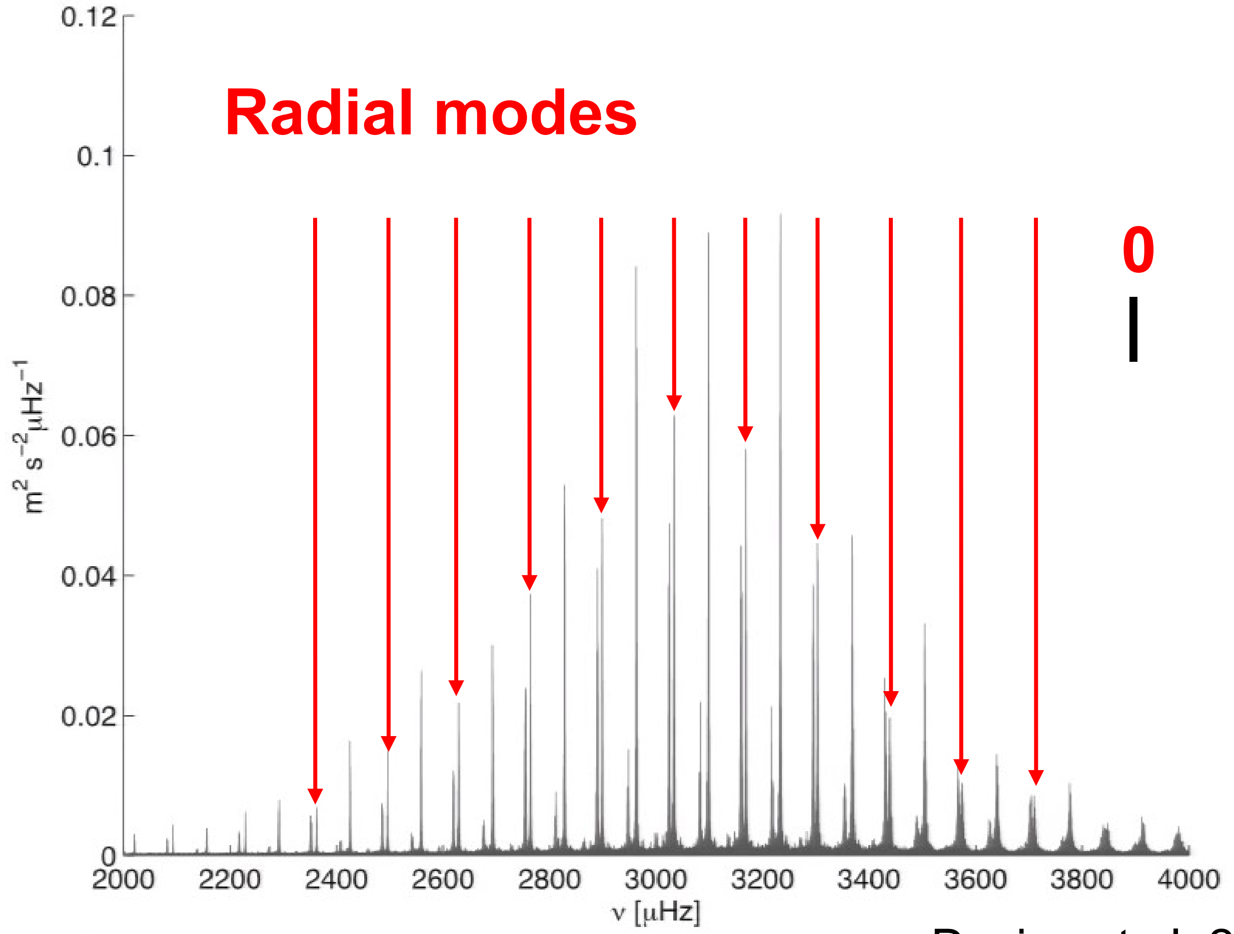
In absence of rotation or other features that break the spherical symmetry of the star, m does not affect the frequencies.

Solar Oscillations



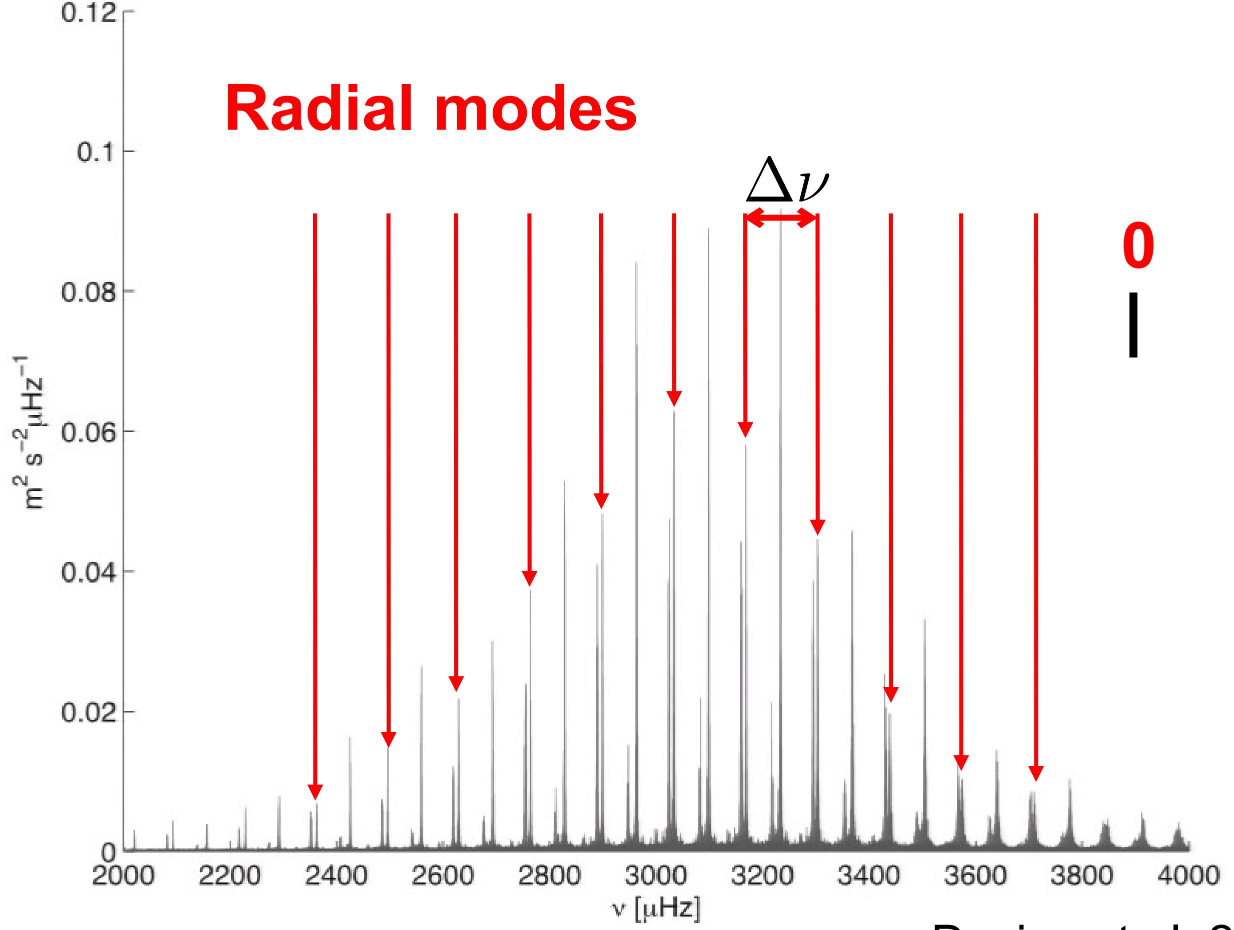
Davies et al. 2014

Solar Oscillations



Davies et al. 2014

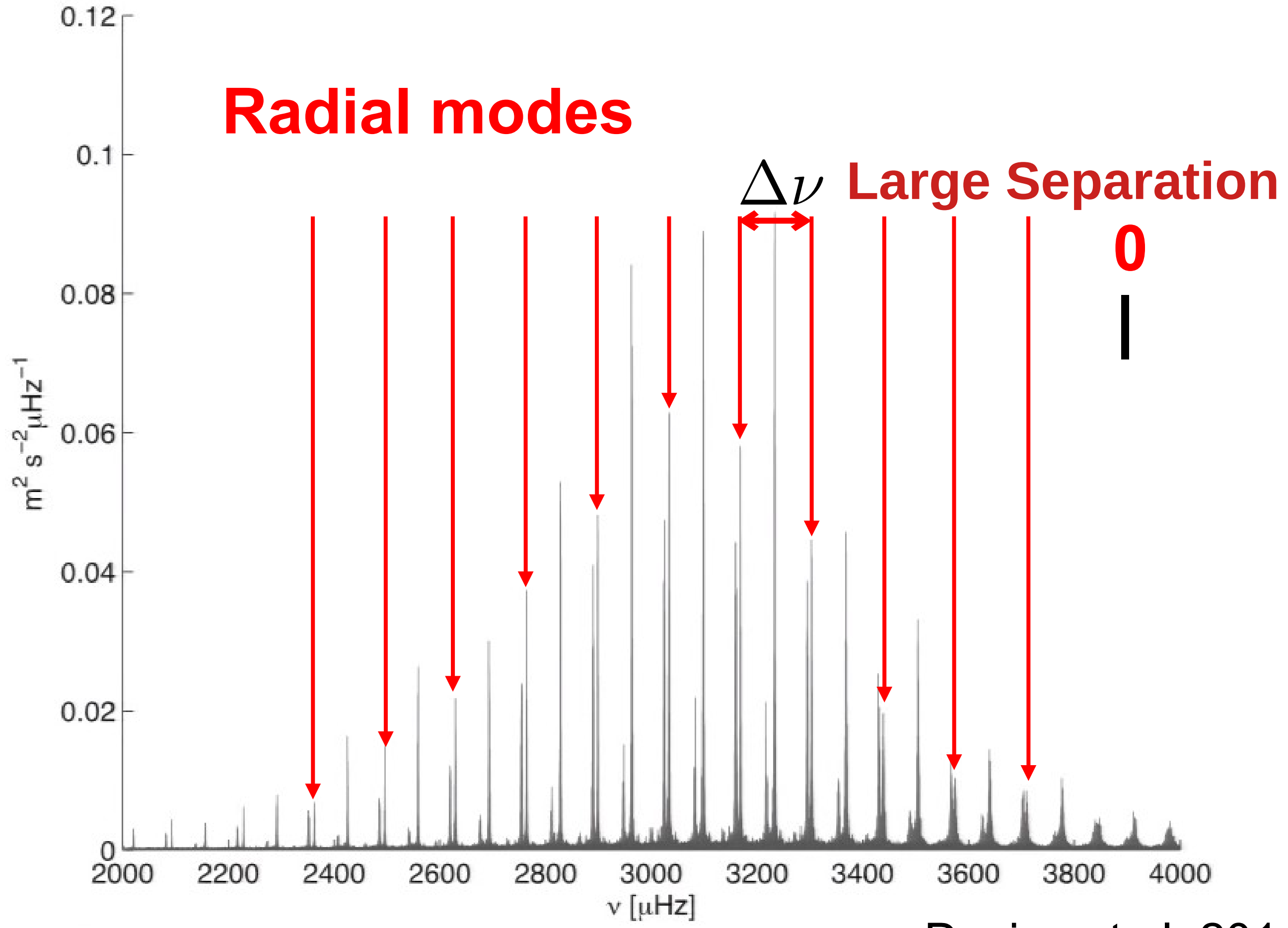
Solar Oscillations



Davies et al. 2014

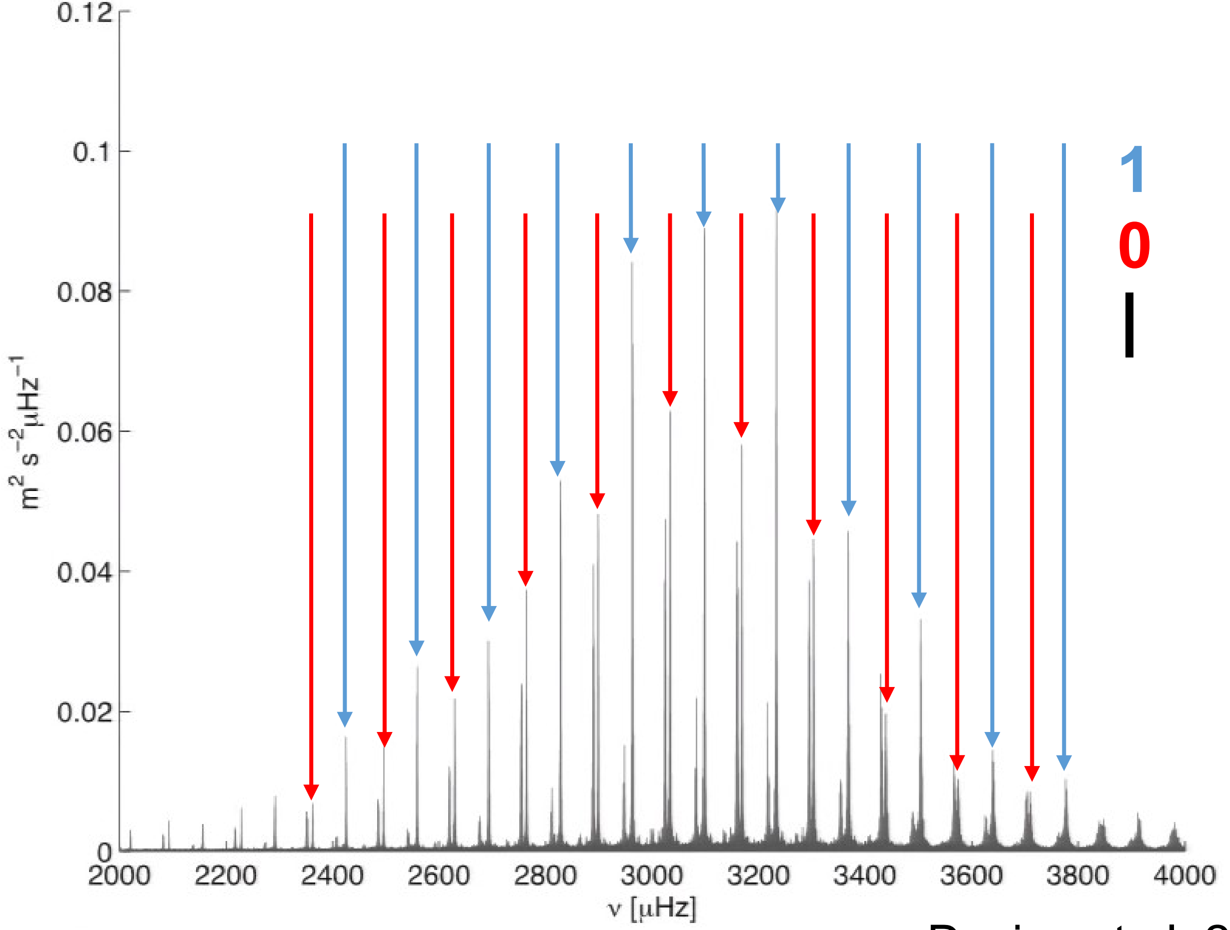
Solar Oscillations

$$\Delta\nu \propto \sqrt{\frac{M}{R^3}}$$



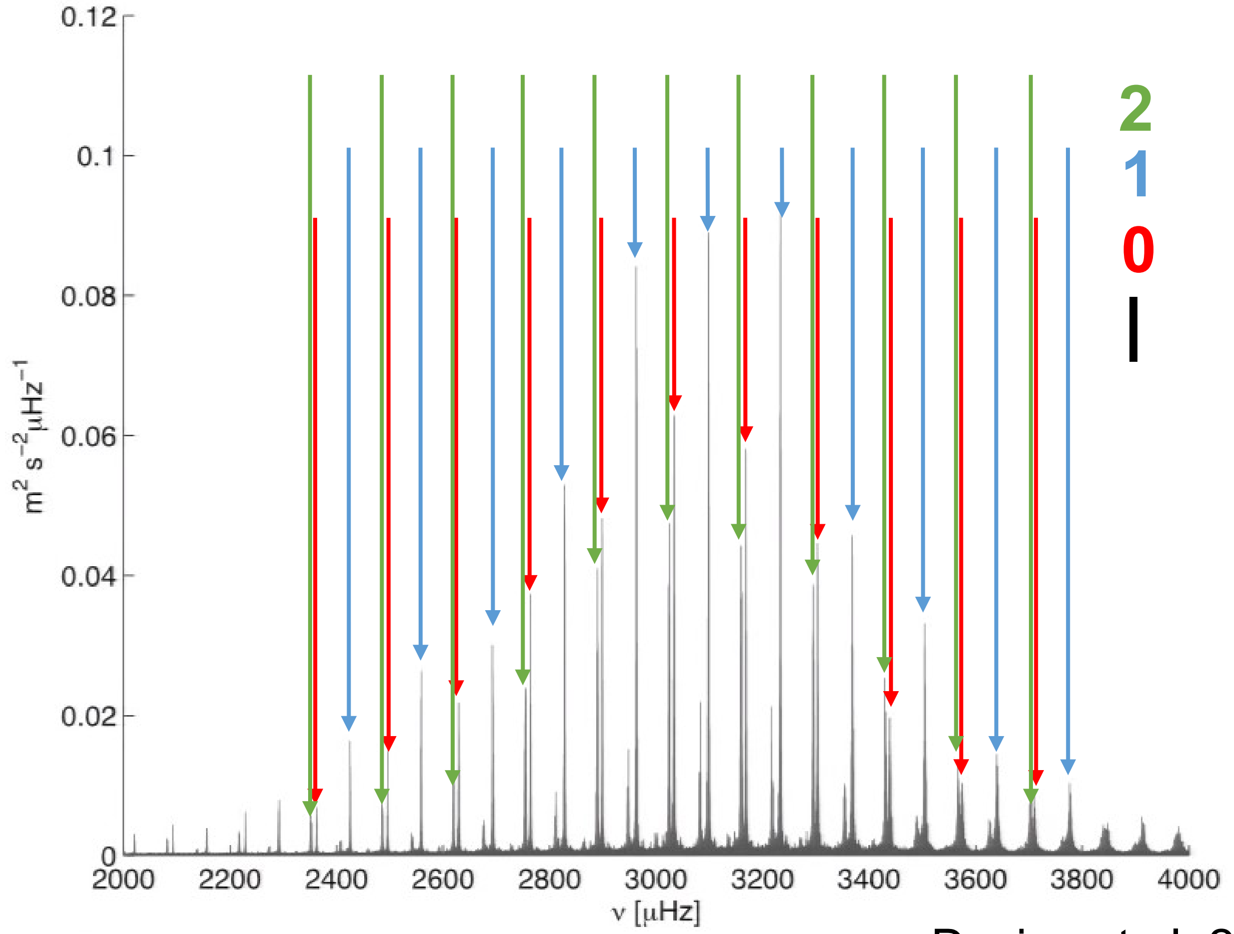
Davies et al. 2014

Solar Oscillations



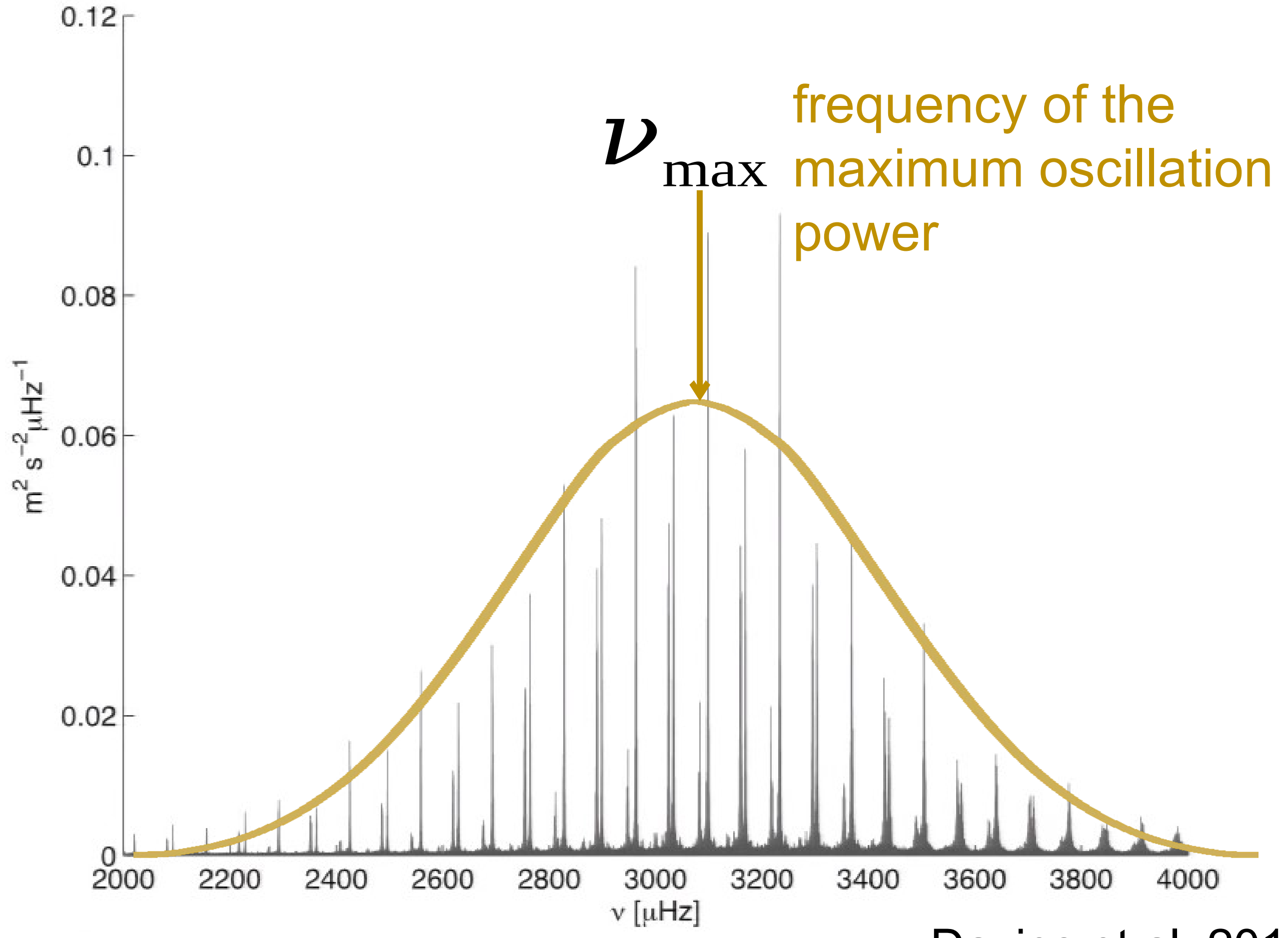
Davies et al. 2014

Solar Oscillations



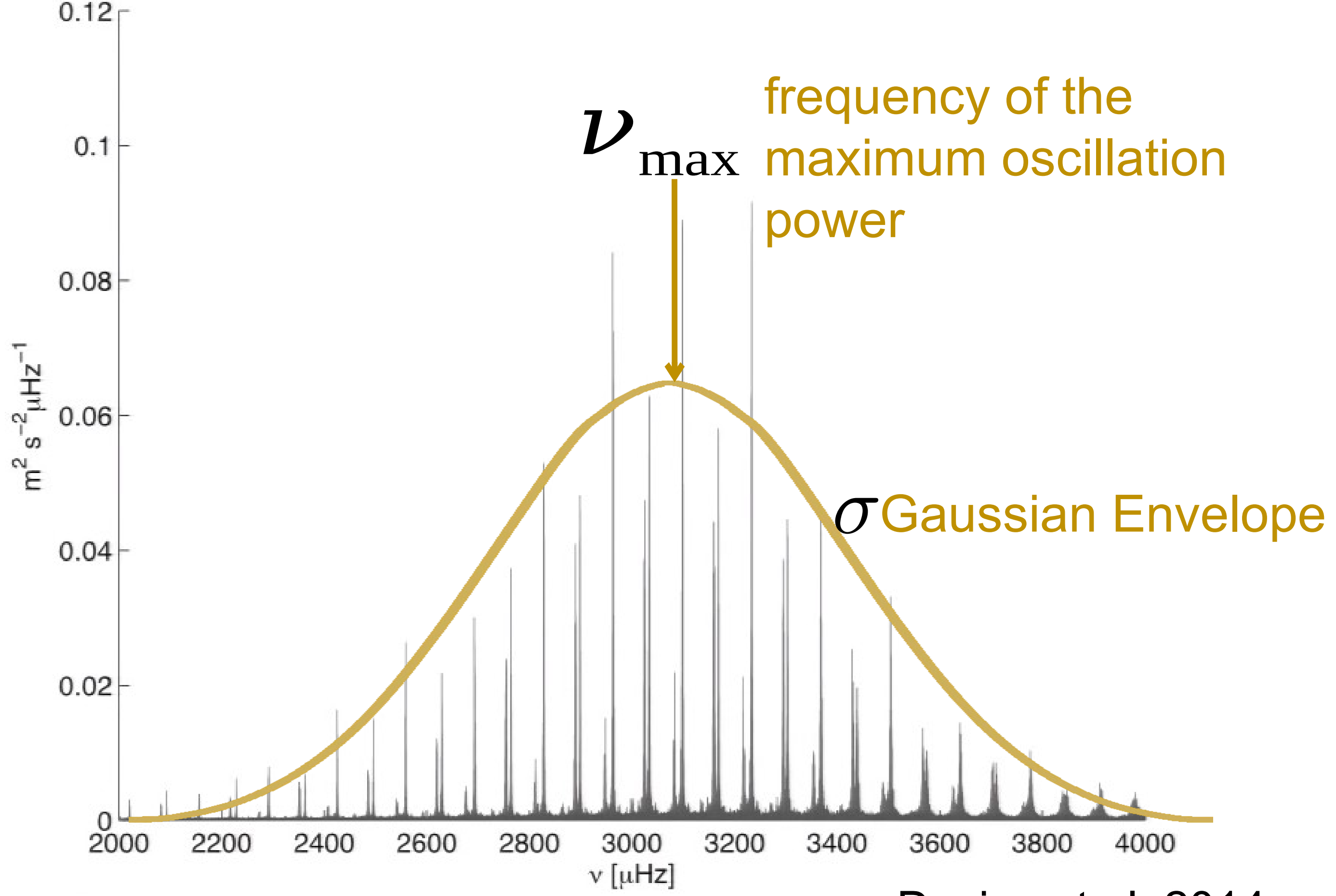
Davies et al. 2014

Solar Oscillations



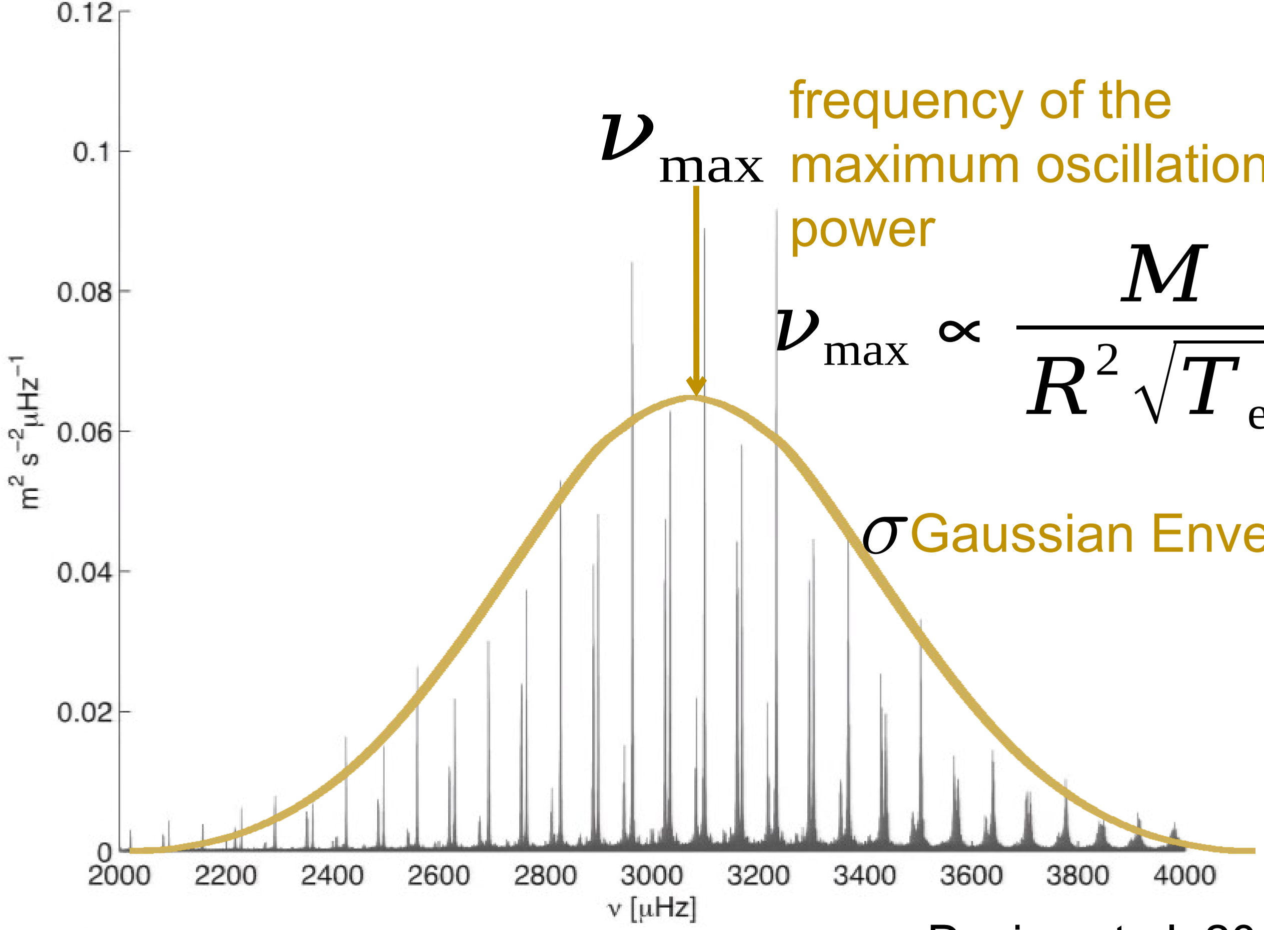
Davies et al. 2014

Solar Oscillations



Davies et al. 2014

Solar Oscillations



$\nu_{\max}$ frequency of the maximum oscillation power

$$\nu_{\max} \propto \frac{M}{R^2 \sqrt{T_{\text{eff}}}}$$

σ Gaussian Envelope

$$\sigma = 0.66 \cdot \nu_{\max}^{0.88}.$$

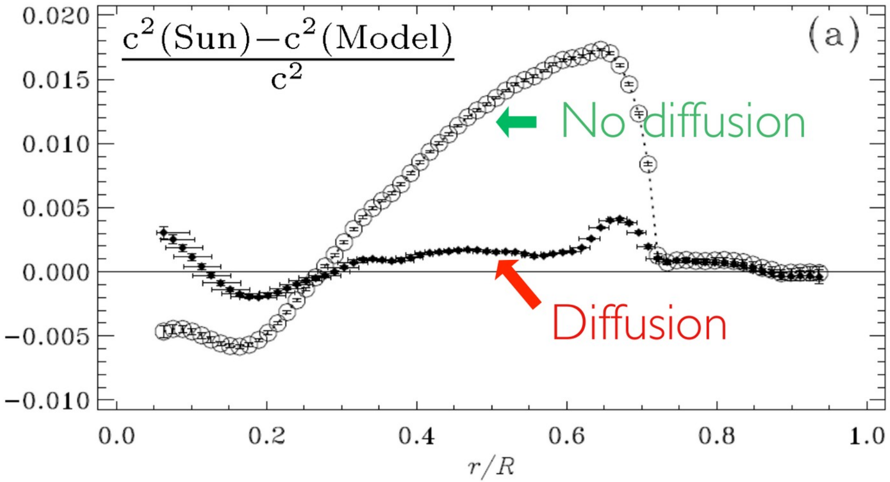
Davies et al. 2014

Helioseismology

Constraints on the solar structure by model – data comparison

e.g. depth of convective envelope: 0.713 ± 0.003 R

Christensen-Dalsgaard, et al 1991 ApJ 378

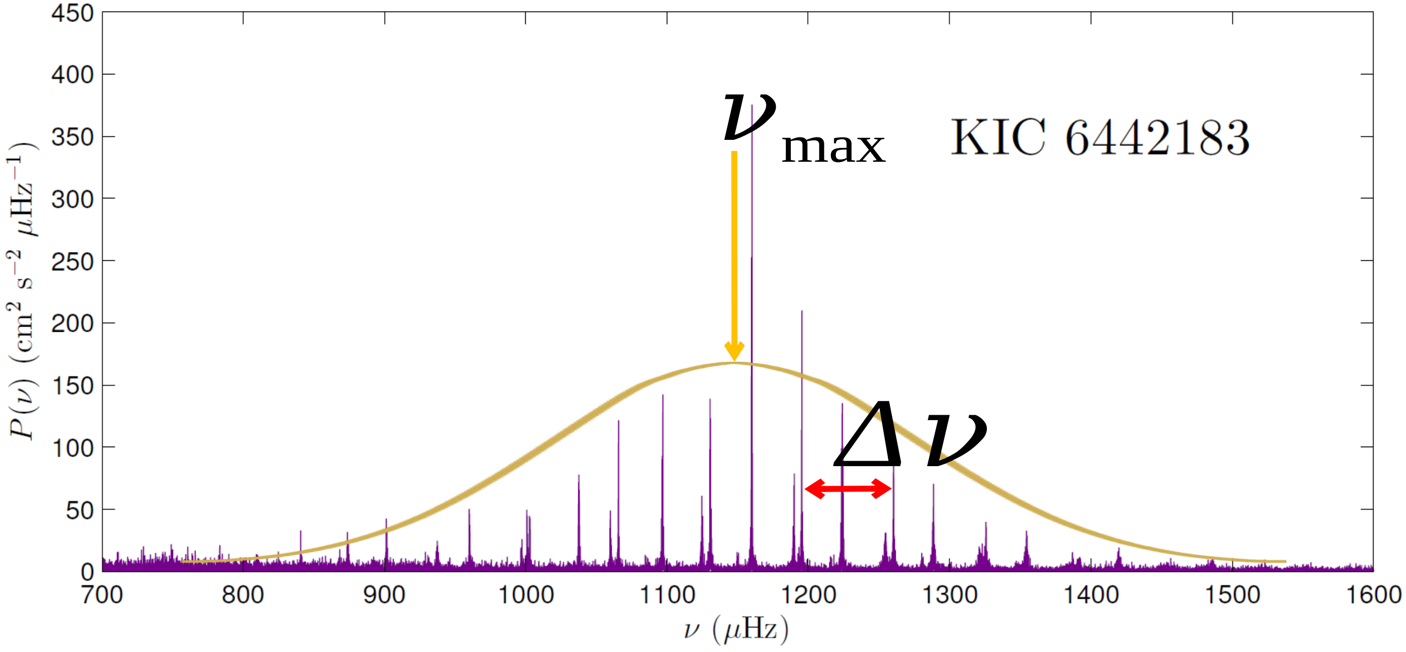


Christensen-Dalsgaard, 2002 Rev. Mod. Phy. 74

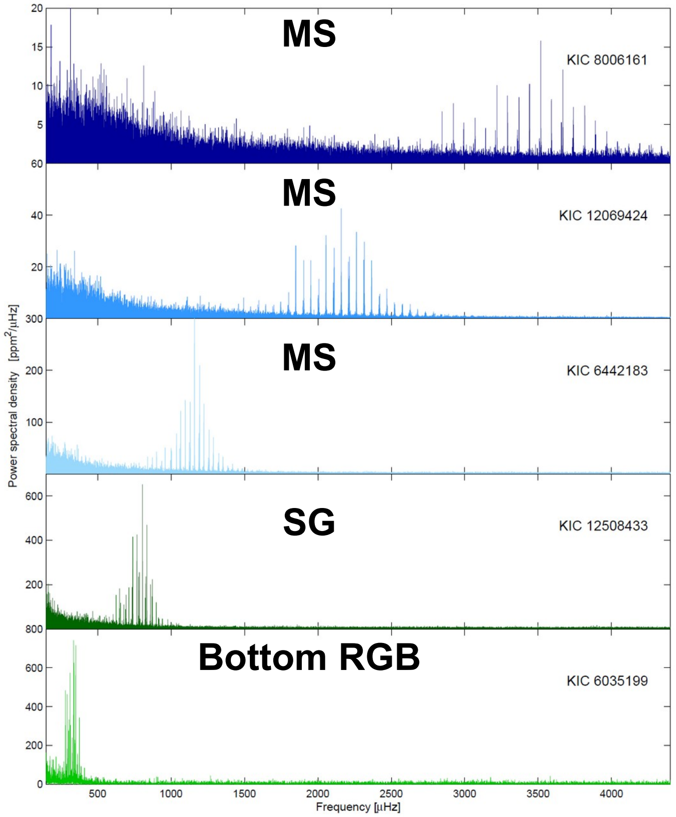
From Solar to Stellar Oscillations

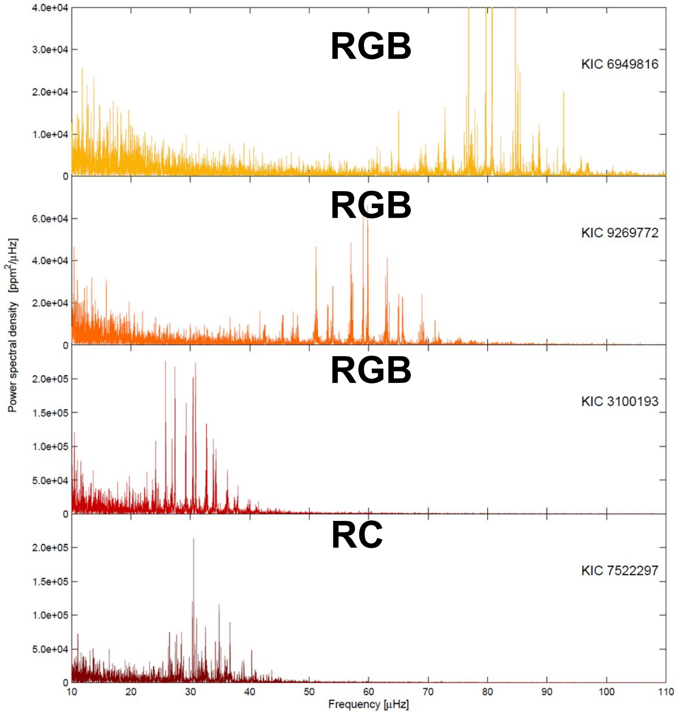
From Solar to Stellar Oscillations

Using the light curve Fourier transformation



Asteroseismology in *Kepler*





Chaplin & Miglio 2013

Scaling Relations

$$\Delta\nu = \Delta\nu_{\odot}\sqrt{\frac{M}{R^3}}$$

$$\nu_{\max} = \nu_{\max,\odot}\frac{M}{R^2\sqrt{T_{\rm eff}}}$$

DIRECT METHOD

Inverting the scaling relations

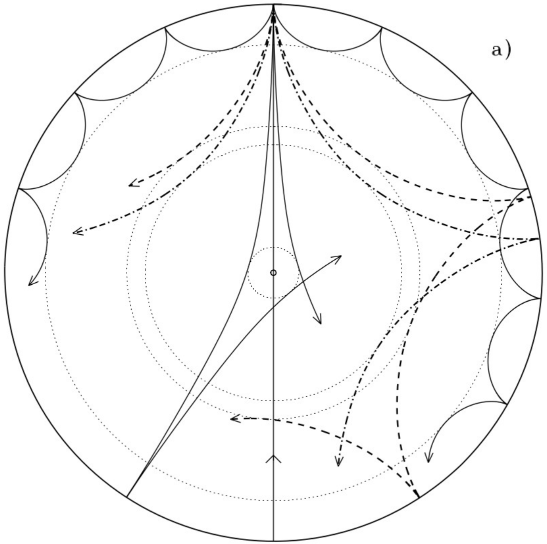
$$\frac{M}{\mathrm{M}_{\odot}} = \left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right)^{3}\left(\frac{\Delta\nu}{\Delta\nu_{\odot}}\right)^{-4}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{3/2}$$

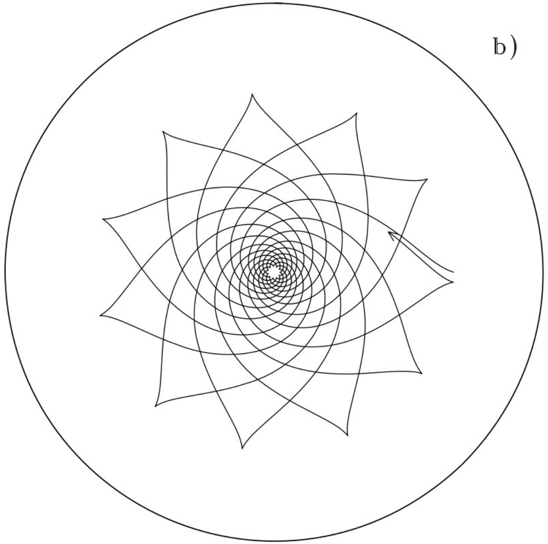
$$\frac{R}{\mathrm{R}_{\odot}} = \left(\frac{\nu_{\max}}{\nu_{\max,\odot}}\right)\left(\frac{\Delta\nu}{\Delta\nu_{\odot}}\right)^{-2}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{1/2}$$

Solar-like oscillations

Pressure Modes
- acoustic waves
- high frequencies
- Equally spaced in frequency ($\Delta\nu$)
- Fundamental mode with the lowest frequency
- Can be radial

Gravity modes
- restoring force: buoyancy
- low frequencies
- Equally spaced in period ($\Delta\Pi$)
- Fundamental mode with the highest frequency
- Cannot be radial





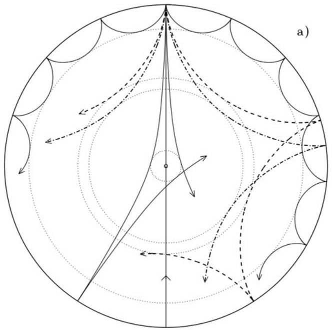
Solar-like oscillations

Pressure Modes

- acoustic waves
- high frequencies
- Equally spaced in frequency ($\Delta\nu$)
- Fundamental mode with the lowest frequency
- Can be radial

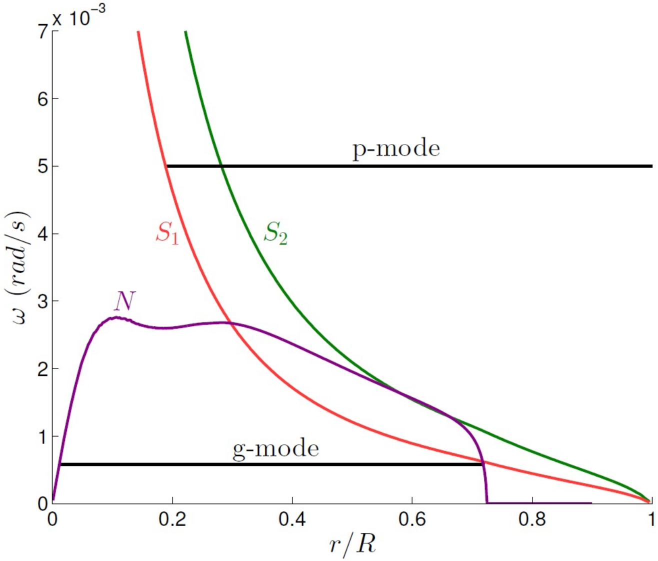
Gravity modes

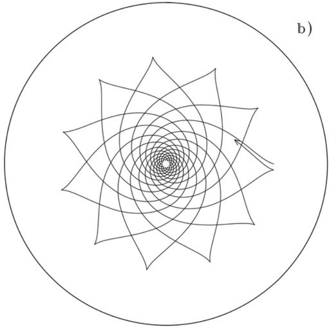
- restoring force: buoyancy
- low frequencies
- Equally spaced in period ($\Delta\Pi$)
- Fundamental mode with the highest frequency
- Cannot be radial



$$S_l = l(l+1)\frac{c^2}{r^2}.$$

Lamb Frequency





$$N^2 = g_0\left(\frac{1}{\Gamma_{1_0}}\frac{\mathrm{d}\ln p_0}{\mathrm{d}r} - \frac{\mathrm{d}\ln \rho_0}{\mathrm{d}r}\right)$$

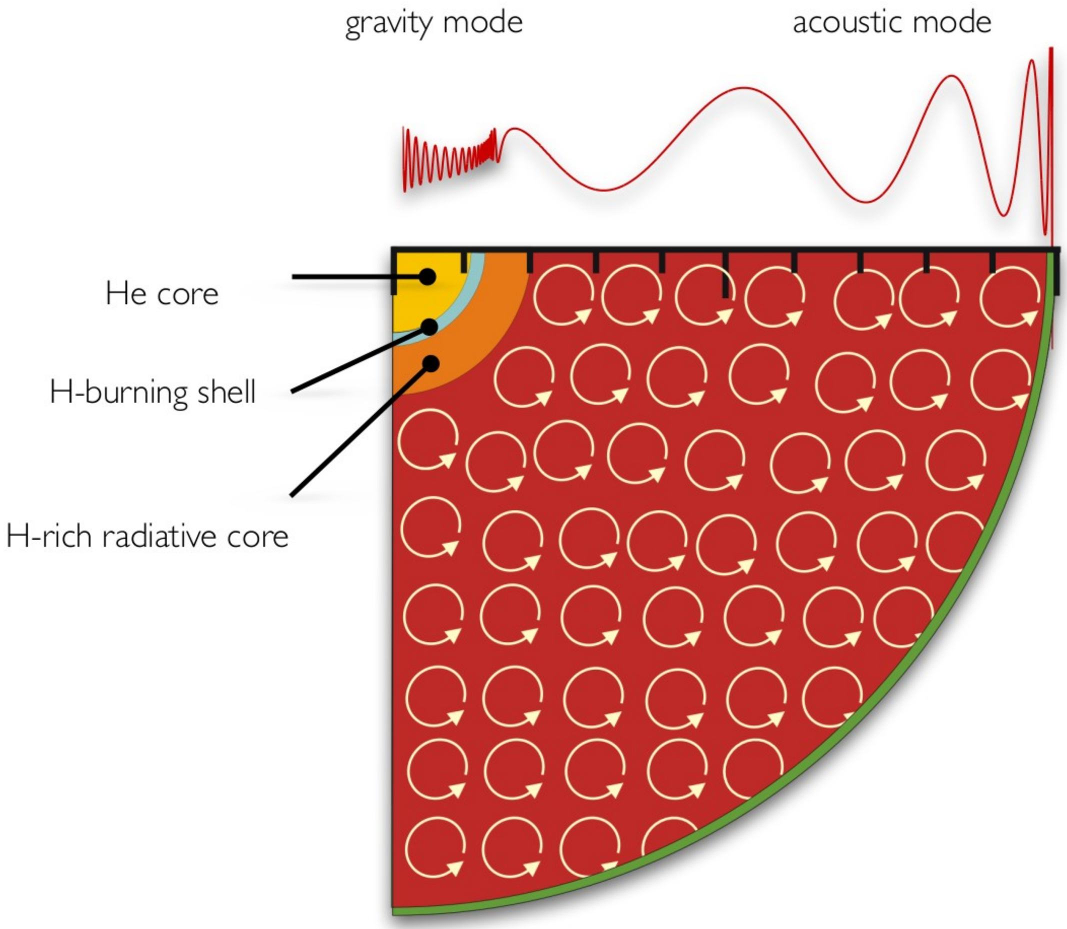
Brunt-Väisäla Frequency

Solar-like oscillations in evolved stars

Solar-like oscillations in red giants

Mixed modes:

- Acoustic waves coupled with gravity waves
- Cannot be radial

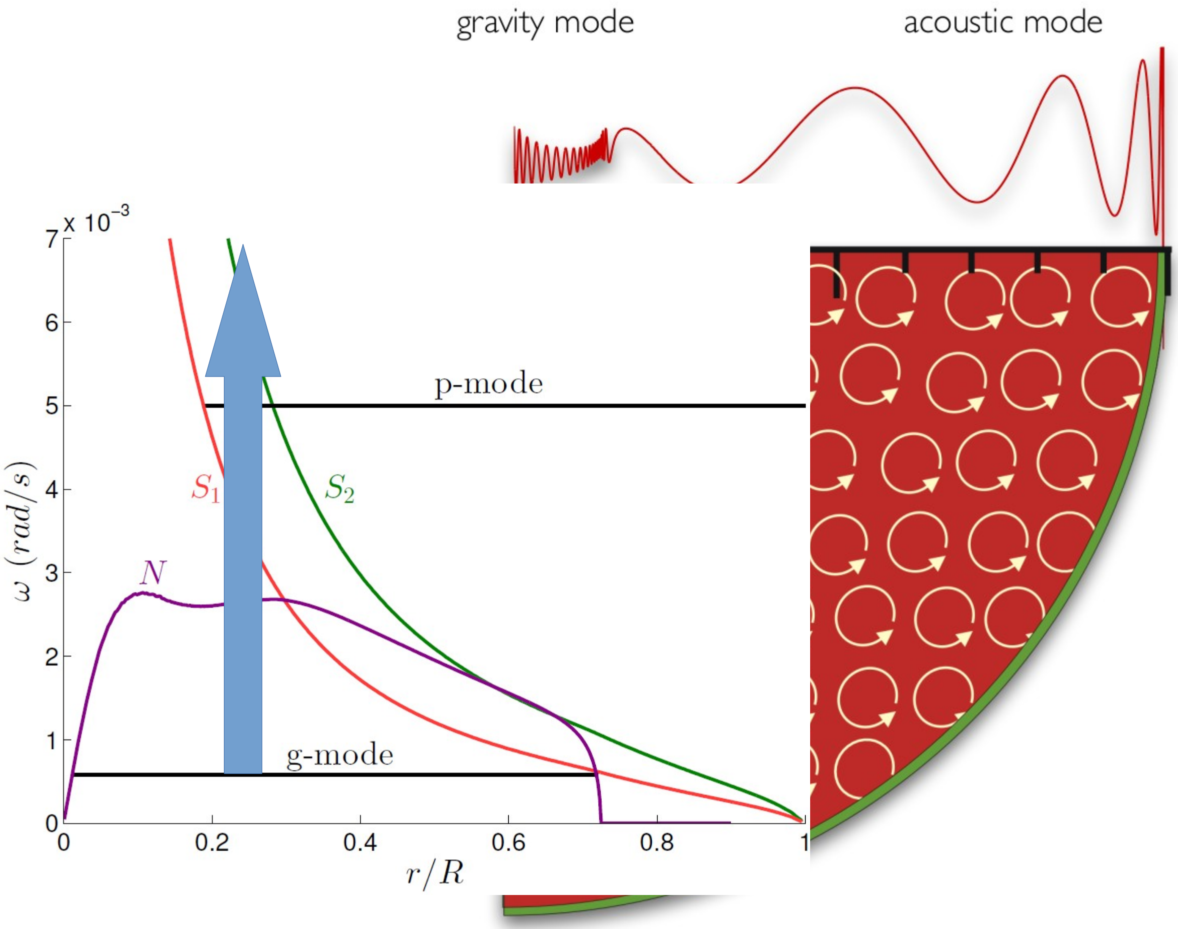


Solar-like oscillations in evolved stars

Solar-like oscillations in red giants

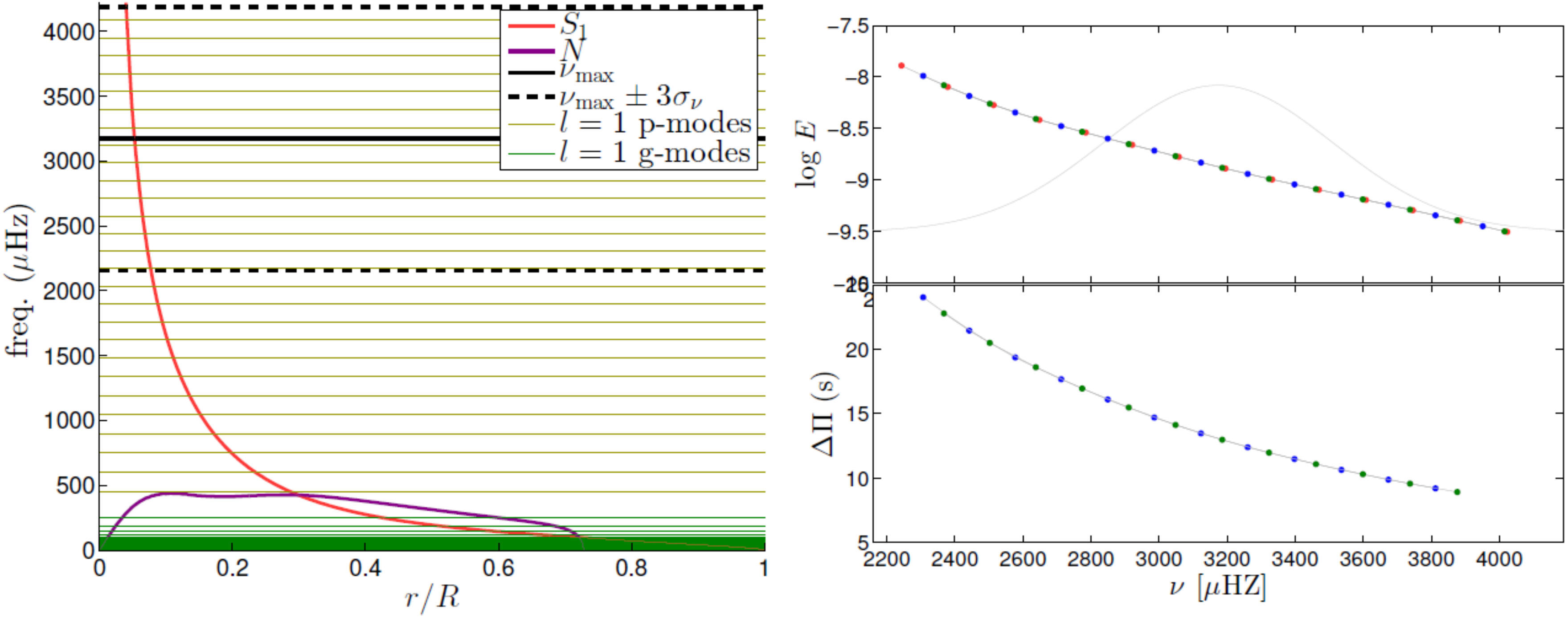
Mixed modes:

- Acoustic waves coupled with gravity waves
- Cannot be radial



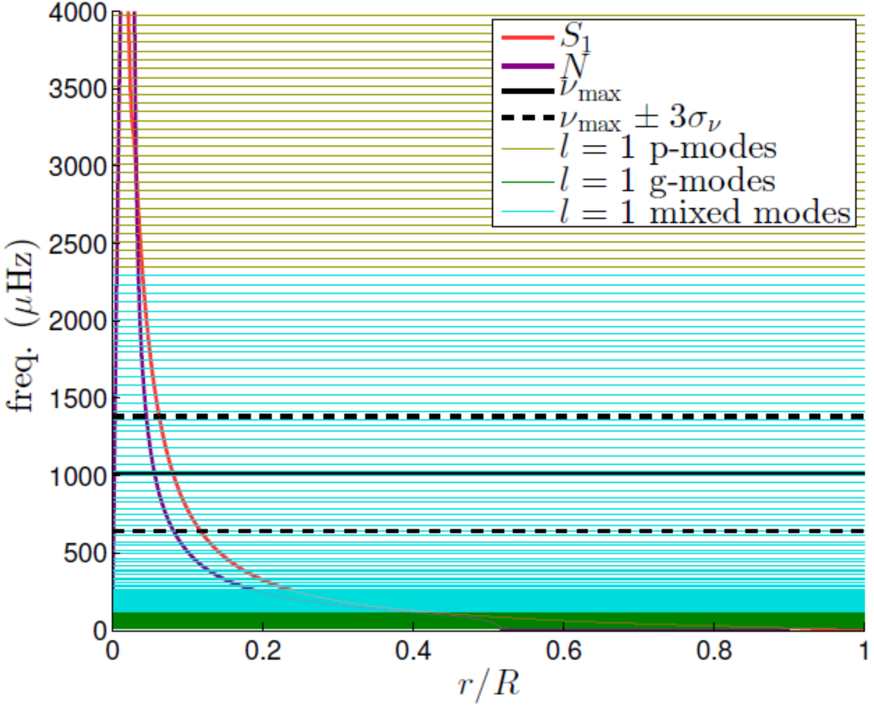
Solar-like oscillations Across the HR

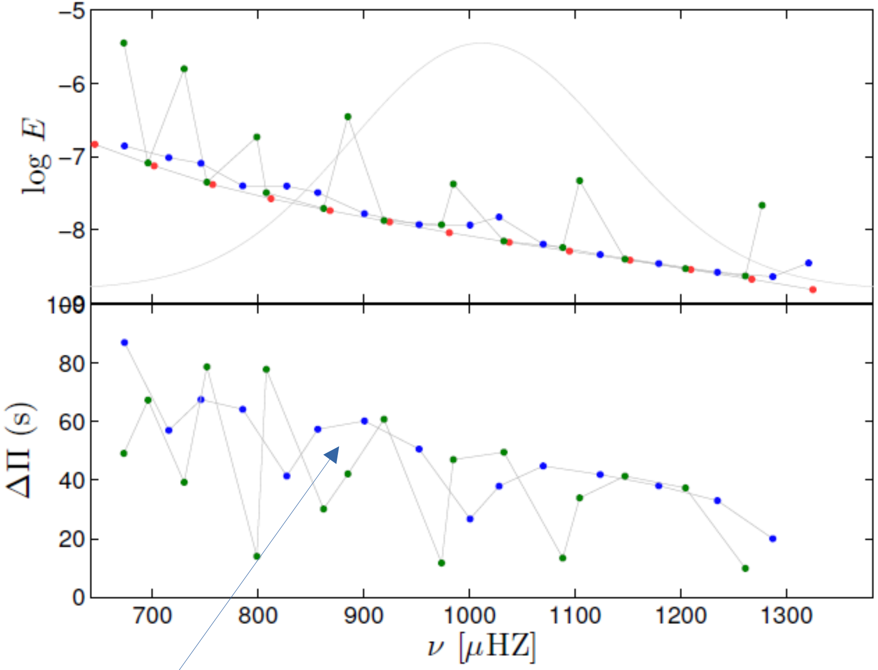
MAIN SEQUENCE



Solar-like oscillations Across the HR

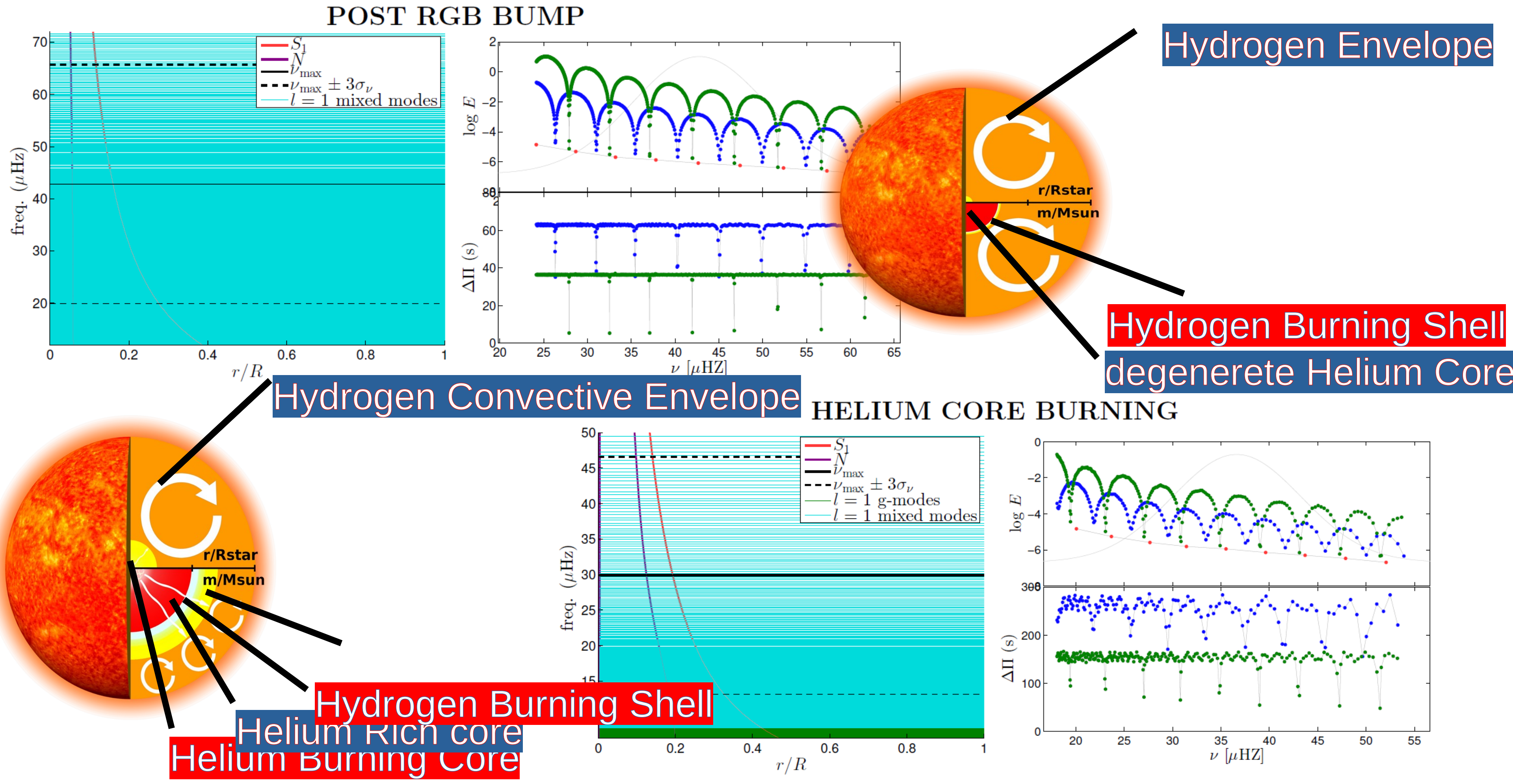
SUB GIANT BRANCH



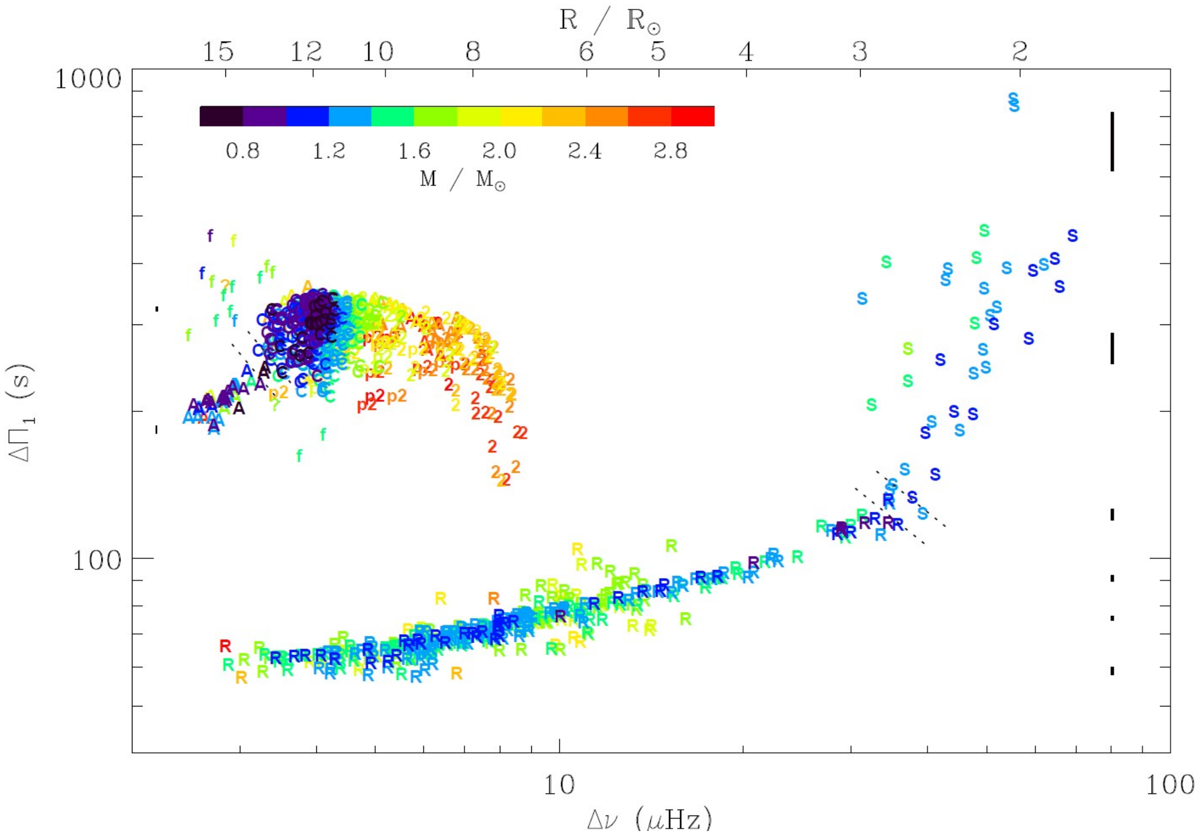


First mixed mode appears

Solar-like oscillations Across the HR

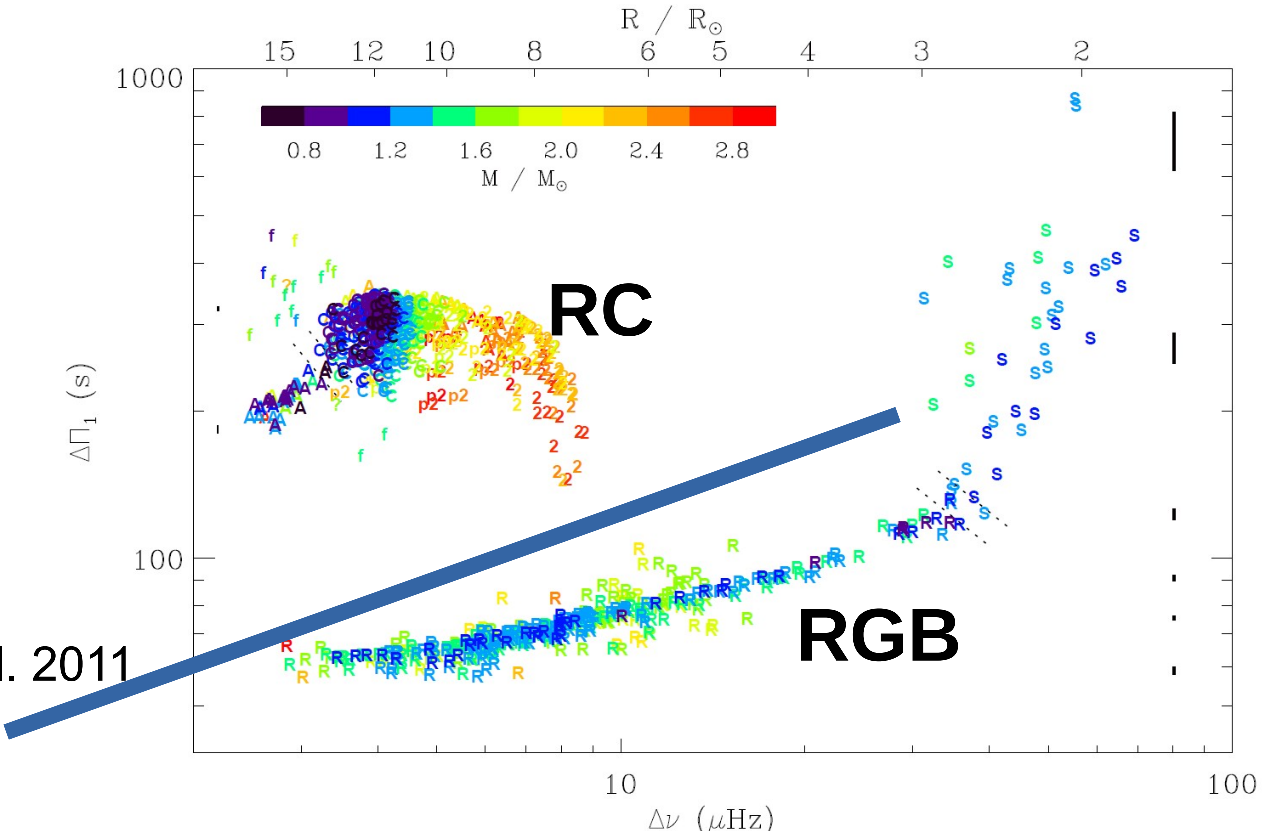


Period Spacing in Kepler



Mosser et al. 2014

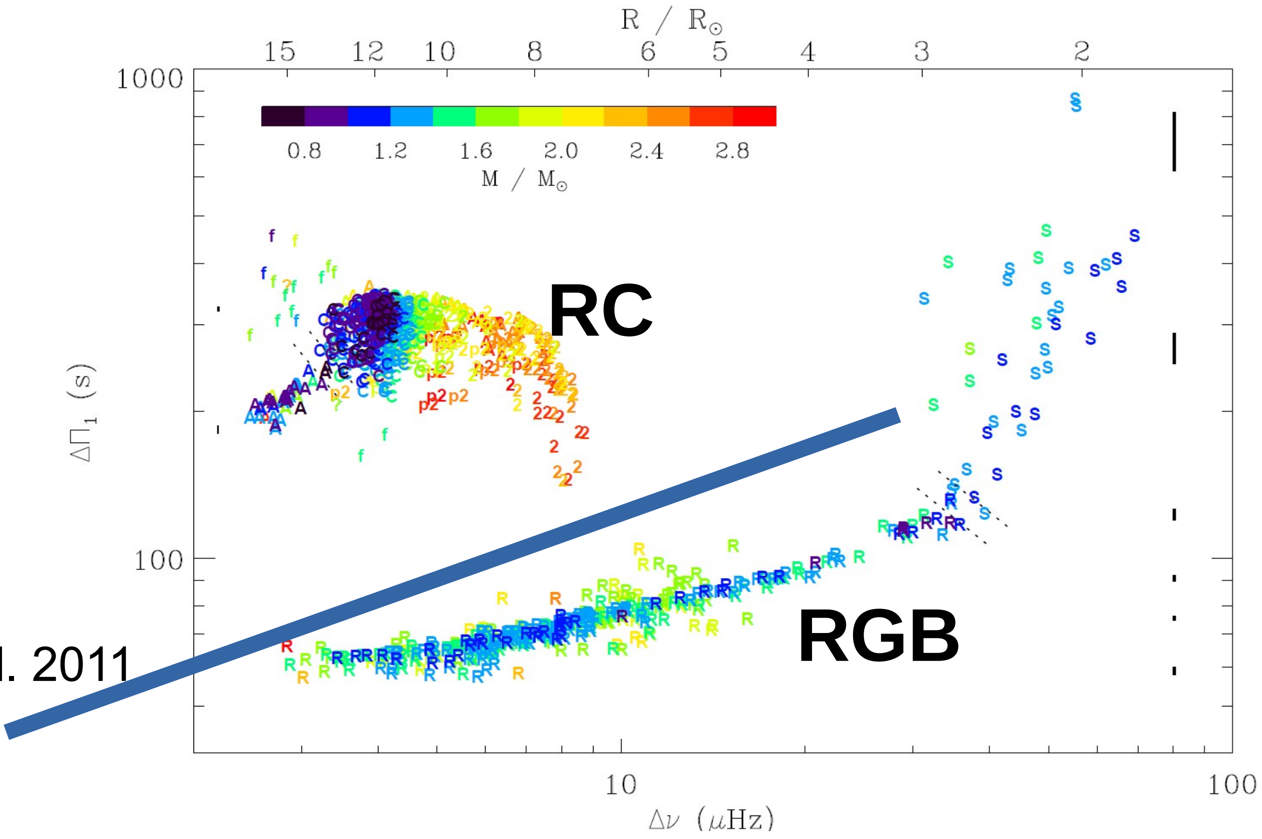
Period Spacing in Kepler



Bedding et al. 2011

Mosser et al. 2014

Period Spacing in Kepler



Bedding et al. 2011

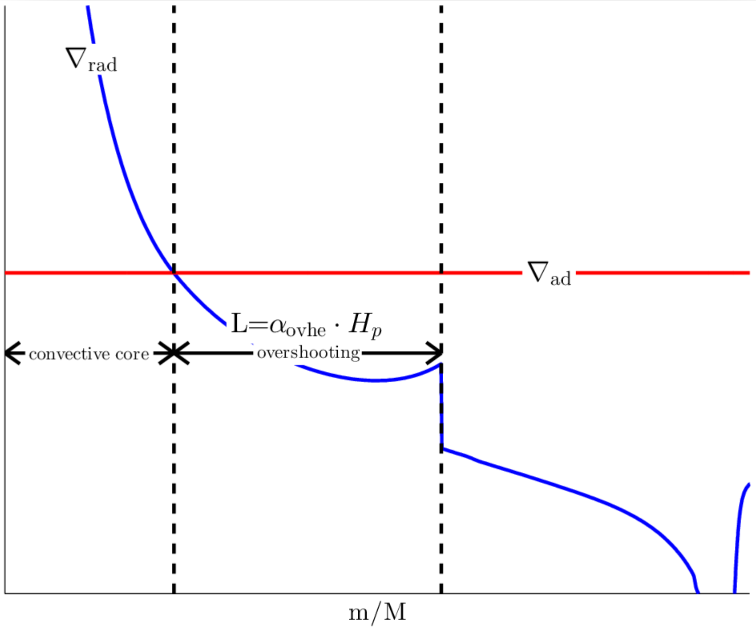
Mosser et al. 2014

Period Spacing in Red Clump

Period Spacing is sensible to the inner layers of the stars. It can be used to test e.g. Core-convection.

To extend the fully mixed region overshoot is usually used. Let consider a **step function overshooting:** the overall radius of the mixed core is given by

$$r_{\text{mixed core}} = r_{\text{classical border}} + \alpha_{\text{ovhe}} H_P$$



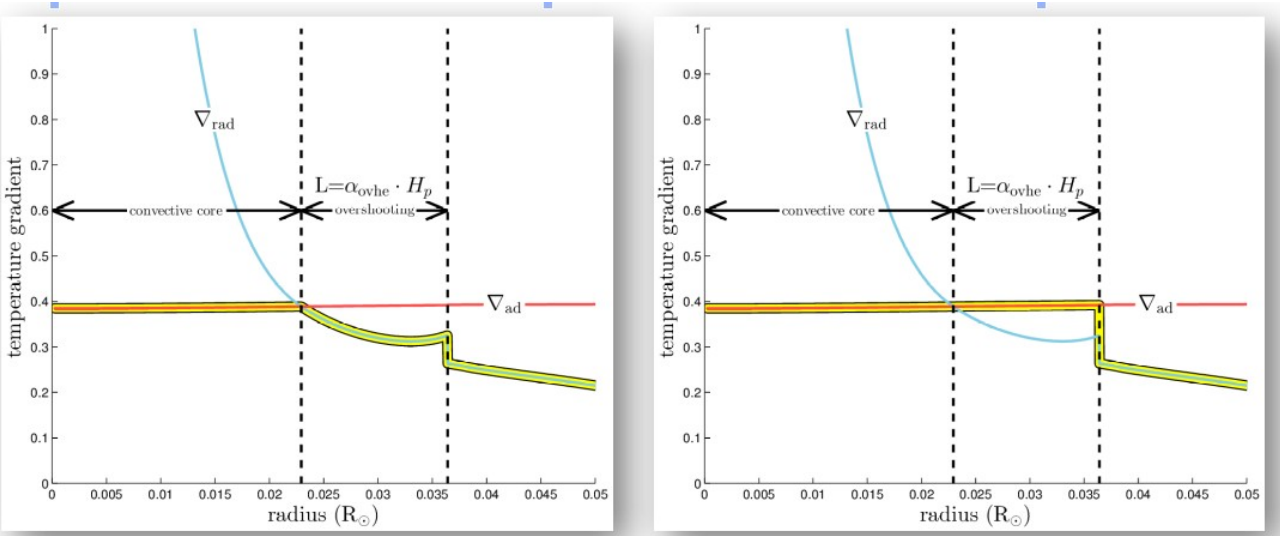
Period Spacing in Red Clump

Period Spacing is sensible to the inner layers of the stars. It can be used to test e.g. Core-convection.

To extend the fully mixed region overshoot is usually used. Let consider a **step function overshooting:** the overall radius of the mixed core is given by

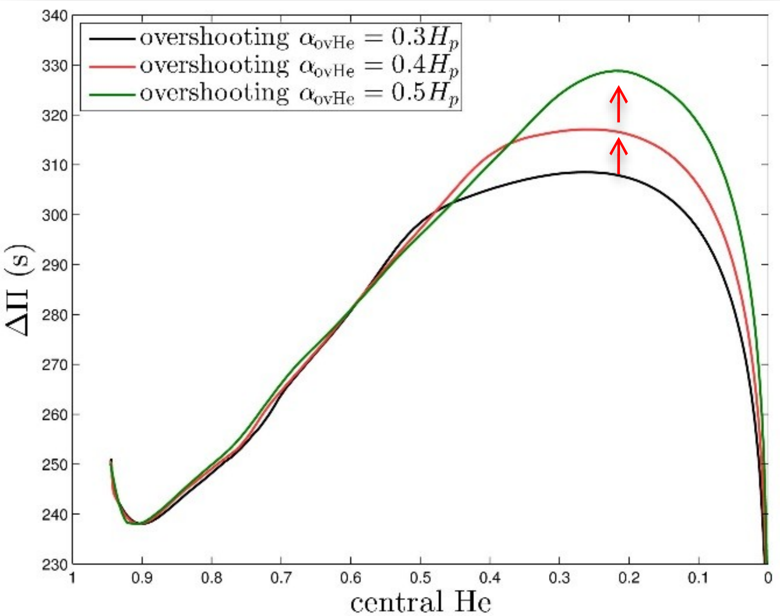
$$r_{\text{mixed core}} = r_{\text{classical border}} + \alpha_{\text{ovhe}} H_P$$

classic ***OV**ershooting*　　　　***P**enetrative **C**onvection*

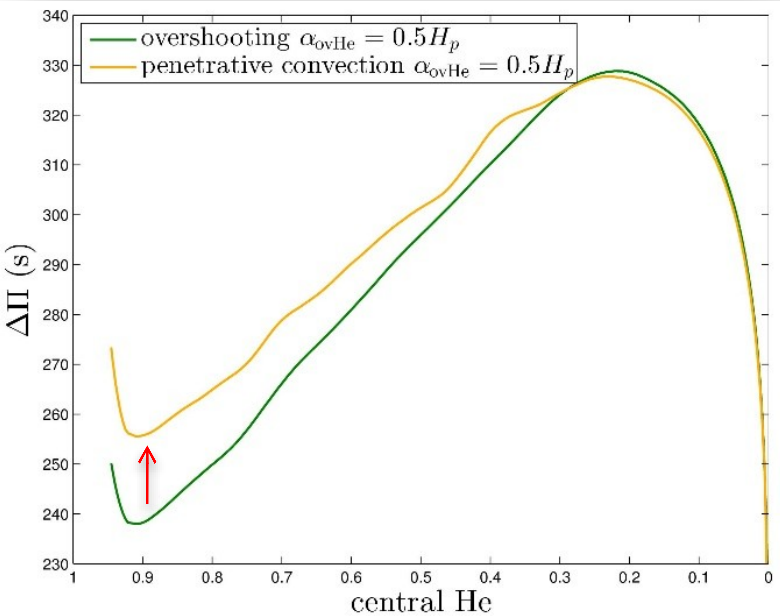


Period Spacing in Red Clump

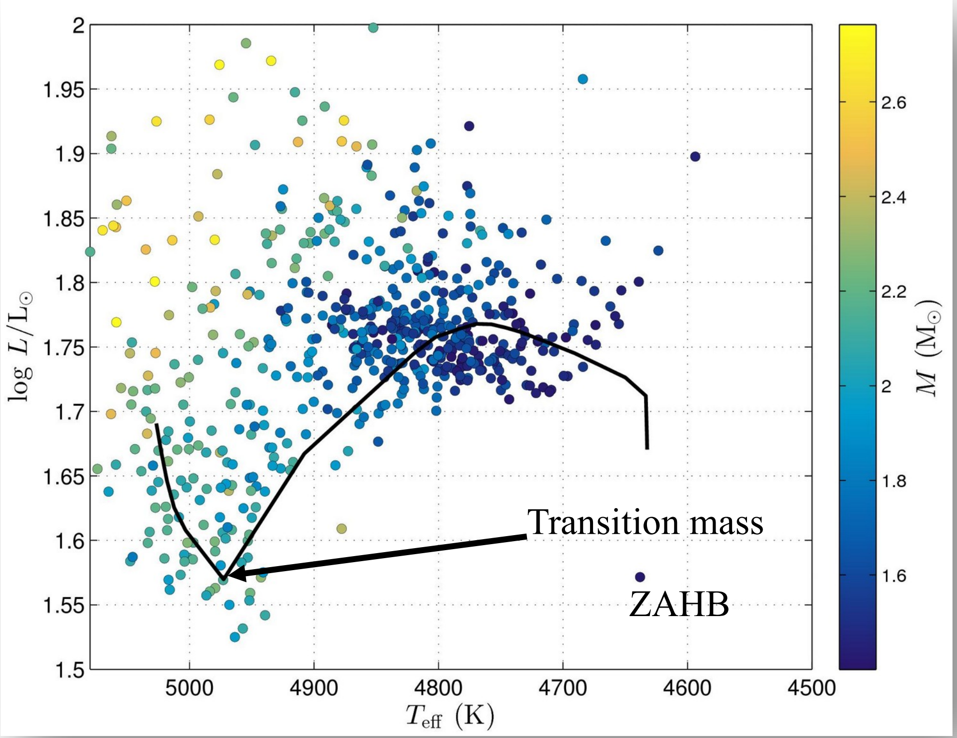
Overshooting changes the maximum period spacing



Thermal stratification changes the minimum period spacing

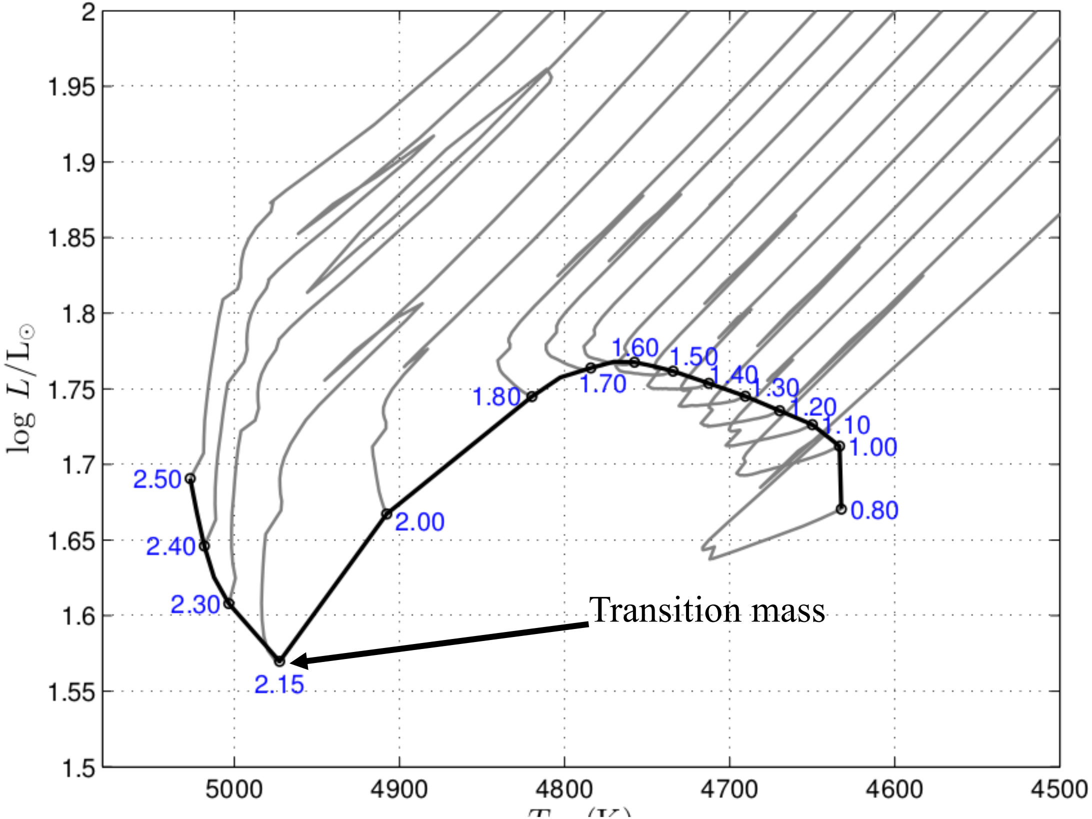


Period Spacing in Secondary Clump

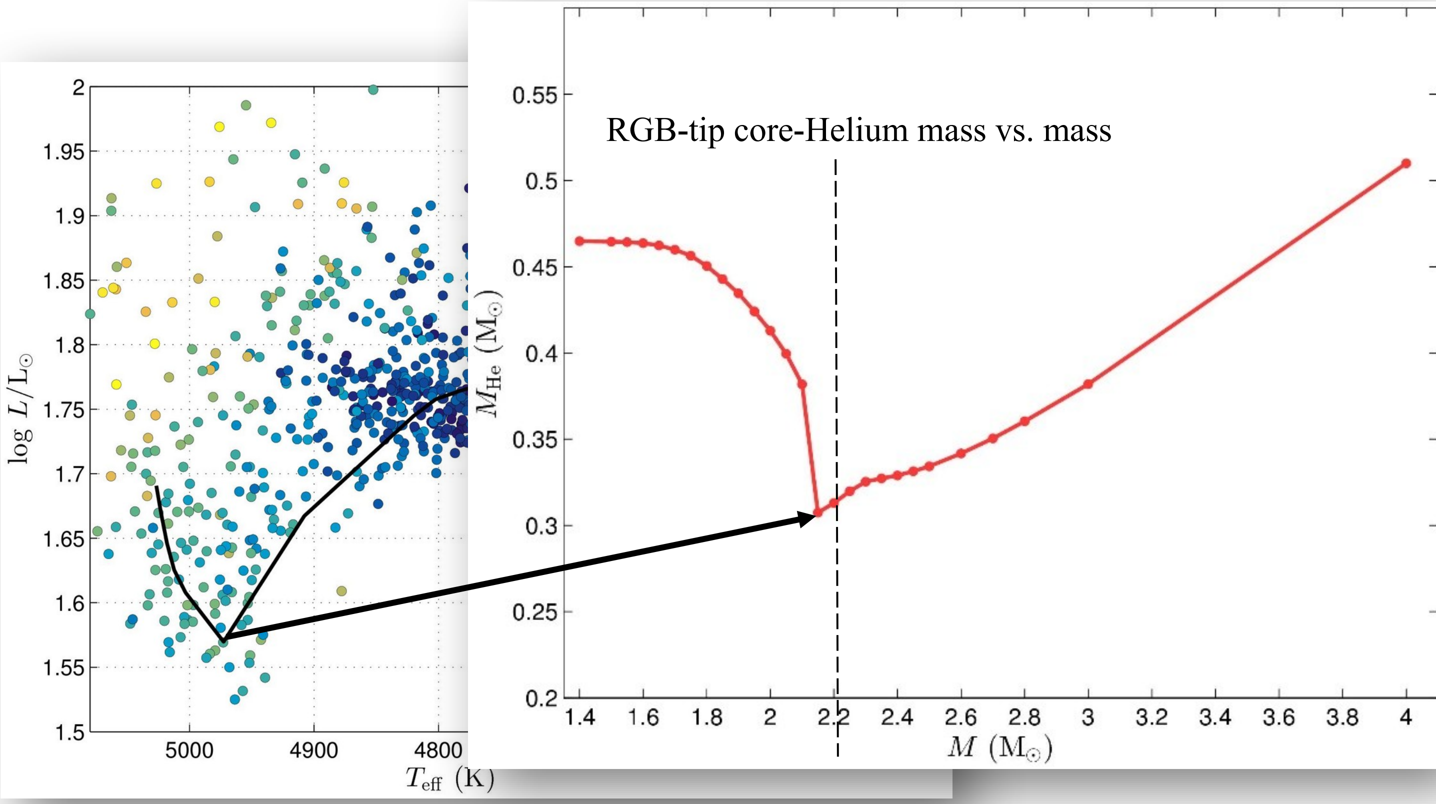


Field red giant stars Vrard et al. 2016 catalogue + APOGEE metallicity

Period Spacing in Secondary Clump

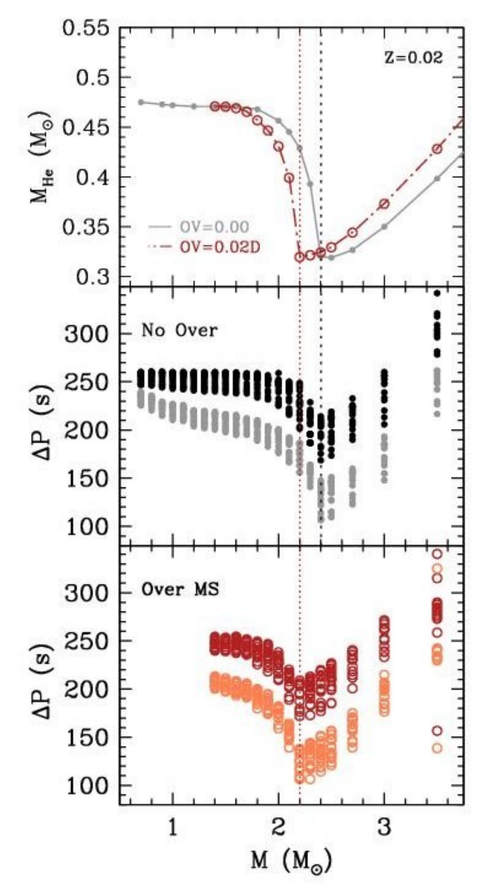


Period Spacing in Secondary Clump



Field red giant stars Vrard et al. 2016 catalogue + APOGEE metallicity

Period Spacing in Secondary Clump



Montalbán et al. (2013) used the observed ΔΠg provided by Mosser et al. (2012) as diagnostic for studying the central properties of secondary clump stars.

They showed that at the same mass M_{tr} , the predicted average RC period spacing presents a minimum as well.

They pointing out that in stellar models increasing MS-overshooting modifies M_{tr} , shifting the expected minimum to lower mass values.