Sky localization with GW detector networks: Eliminating a hidden coordinate dependence

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Problem Overview

- Current data analysis tools process GW data in Earth-centered reference system (GCRS);
- LGWA has the capabilities of detecting the modulation in long-duration GW signal due to revolution around the Sun;
- Geo-centered coordinate system will destroy any information regarding the modulation;
- Additionally, GCRS is not inertial and solidal with the source.

→ New working coordinate reference system is needed

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Solar System Barycenter

OPTION 1: Solar System Barycenter (SSB)

Not really working because:

- Degeneracy between detection time and reference time, i.e time at which merger signal crosses center of reference frame;
- When converting to the SSB reference frame, the new reference time and the localization from detection time create a "divergence" in the phase information.

Basic Concept

Method to solve problem:

- Exploit the properties of the GW signal at the detector (in frequency domain) and of the Fisher matrix to prevent divergence

$$d_a(\nu) \approx A_a(\nu) \exp\left[2\pi i \nu\left(t_0 + \frac{\hat{n}\cdot\vec{r}_a(t_d - t_0 - \tau_a)}{c}\right)\right]$$

$$\Gamma^n_{ij} = \langle \partial_i d_a | \partial_j d_a \rangle$$

$$\frac{\partial d_a(\nu)}{\partial t_0} = d_a(\nu) 2\pi i \nu \left(1 - \frac{1}{c}\hat{n}\cdot\frac{\partial \vec{r}_a}{\partial t}\right)$$

GS
SI

Basic Concept

Method to solve problem:

$$d_a(\nu) \approx A_a(\nu) \exp\left[2\pi i \nu \left(t_0 + \frac{\hat{n}\cdot\vec{r}_a(t_d - t_0 - \tau_a)}{c}\right)\right]$$

Search for a change of reference frame, such that:

- the partial derivatives of the GW signal are small when $\nu \to \infty$ (i.e. t is approaching merger)

⇒ Idea for new coordinate reference system

Detector Trajectory Barycenter

OPTION 2: Wen&Chen Proposal

$$\Gamma_{jk} = \frac{1}{c^2}\left[\overline{r_j r_k} - \overline{r_j}\,\overline{r_k}\right]\sum_J \xi_J \quad \text{where} \quad \overline{r_k} = \frac{\sum_J \int_0^T dt\, r_J^k \dot{\xi}_J(t)}{\sum_J \xi_J(T)}$$

$$\overline{r_j r_k} = \frac{\sum_J \int_0^T dt\, r_J^j(t)\, r_J^k(t)\, \dot{\xi}_J(t)}{\sum_J \xi_J(T)}$$

$$\dot{\xi}_J(t) = 2\int d_j(t-\tau/2)\, w_J(\tau)\, d_J(t-\tau/2)\, d\tau$$

Properties:

- Rate of increase of SNR as function of time;
- Solidal with SSB system.

Ref: L. Wen, Y. Chen.
"Geometrical expression for the angular resolution of a network of gravitational-wave detectors."
Physical review D 81, 082001 (2010)

Detector Trajectory Barycenter

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$$\dot{\xi}_J(t)=2\int d_j(t-\tau/2)\, w_J(\tau)\, d_J(t-\tau/2)\, d\tau$$

- If $\dot{\xi}_J(t)=const$: centered on trajectory barycenter;
- If $\dot{\xi}_J(t)\neq const$: centered on weighted trajectory barycenter;

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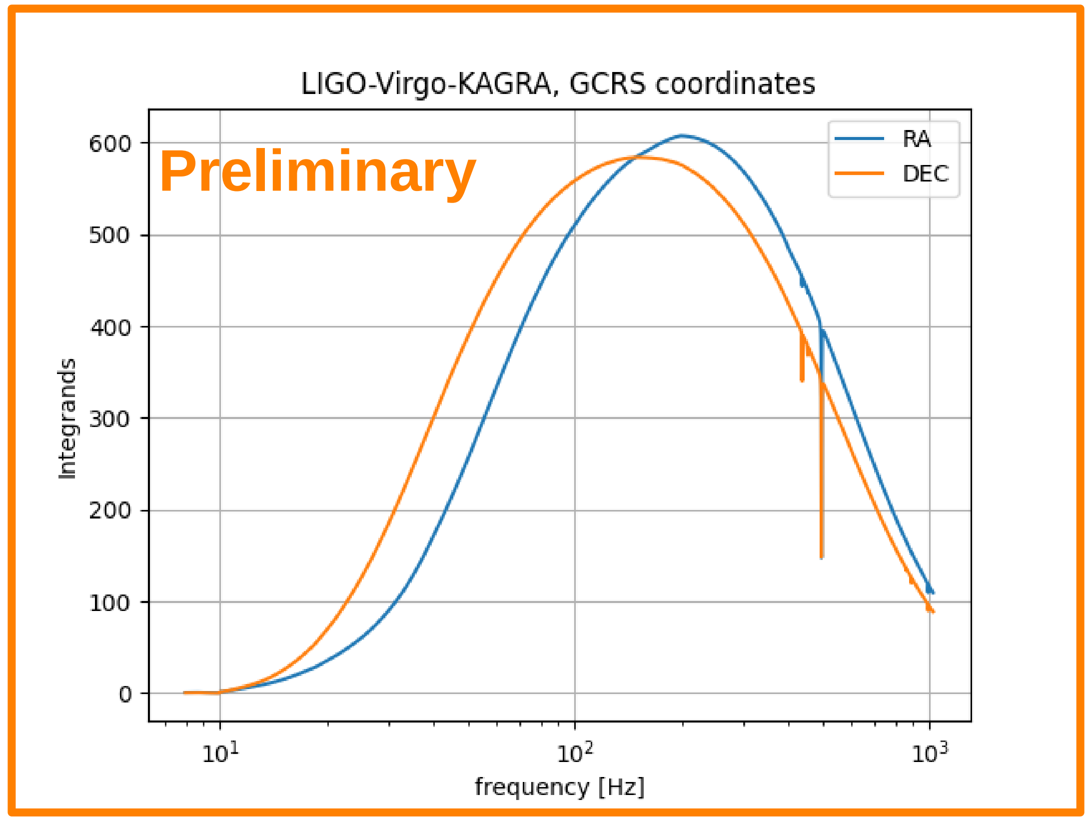
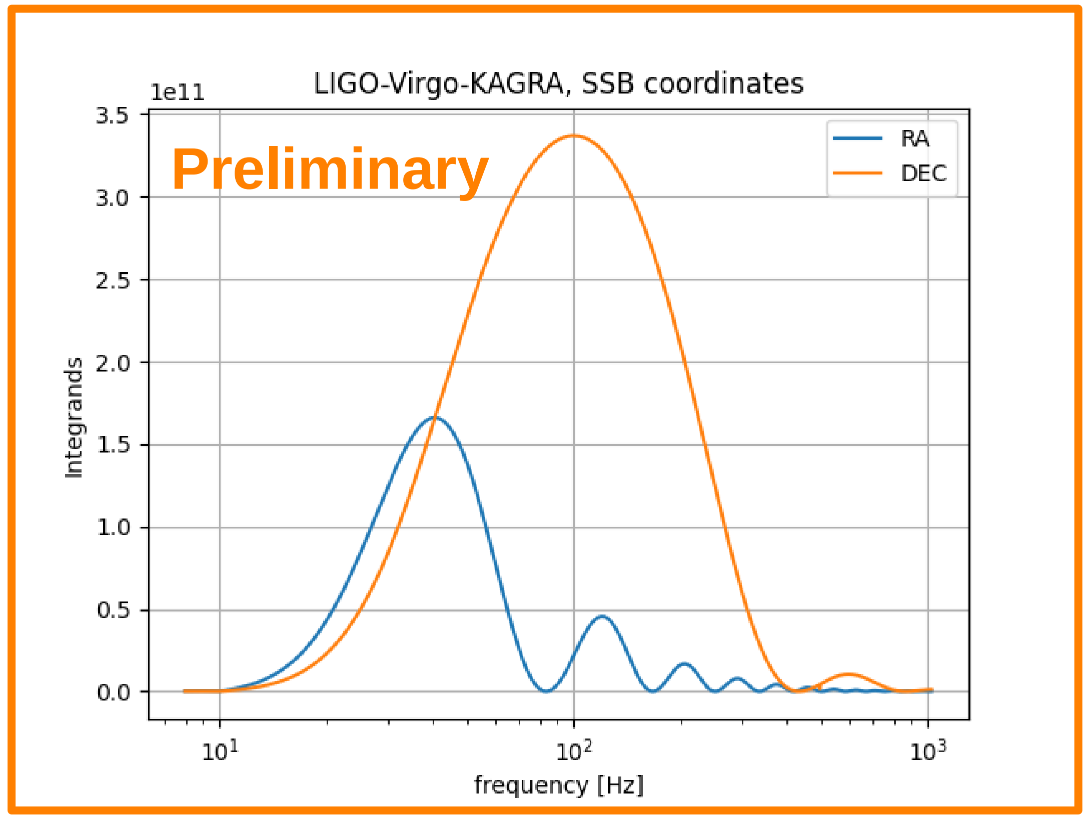
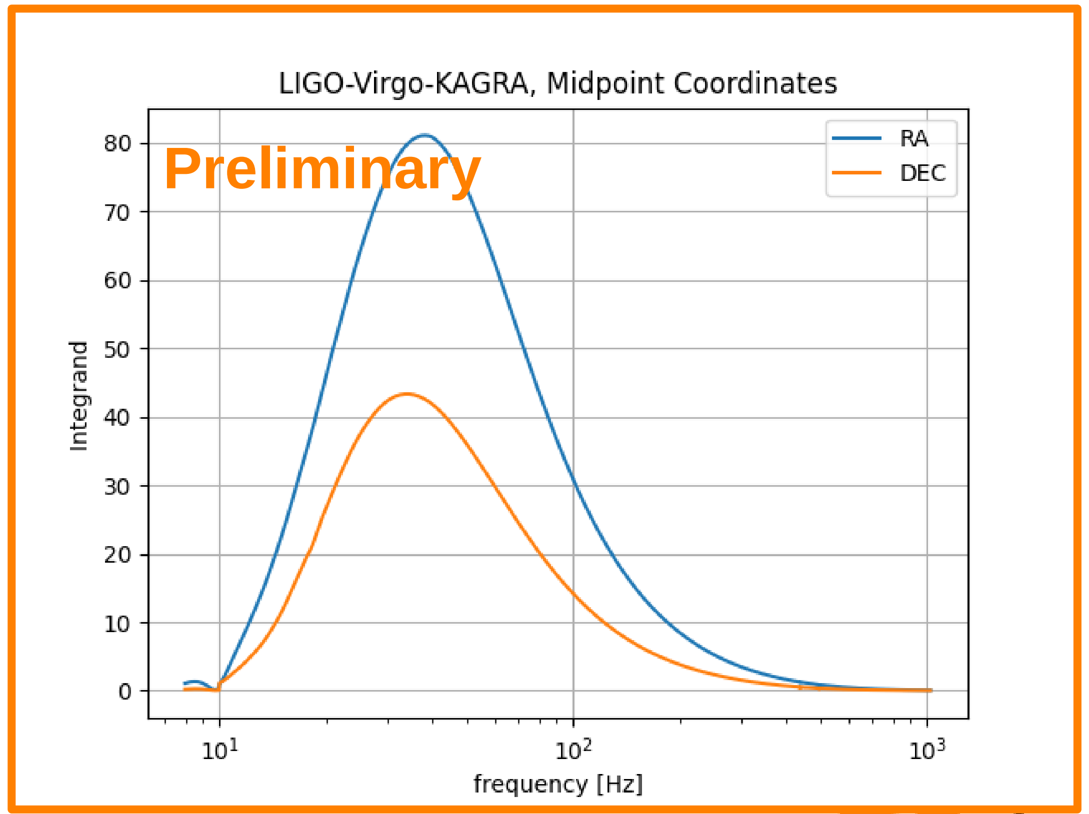
Detector Trajectory Barycenter

OPTION 3: End-Point Barycenter

- Easier and straight-forward implementation;
- Largest the SNR at the end of the signal, thus weighted trajectory position approximately end-point position.







Network of Detectors

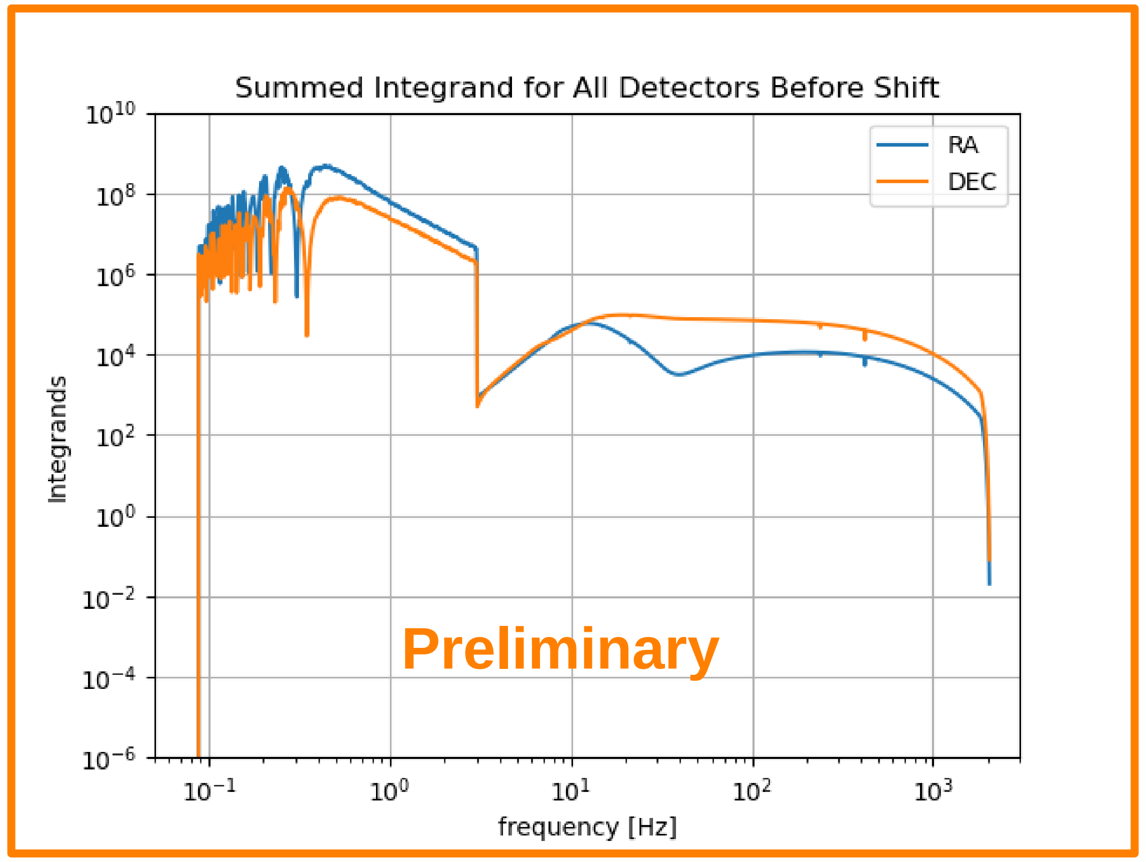
- Explore synergy of network of detectors;
- Each detector suffers of same reference frame shift problem;
- Combining the information of arrival times of different detectors at very large distances provides additional challenge in shifting the reference frame.

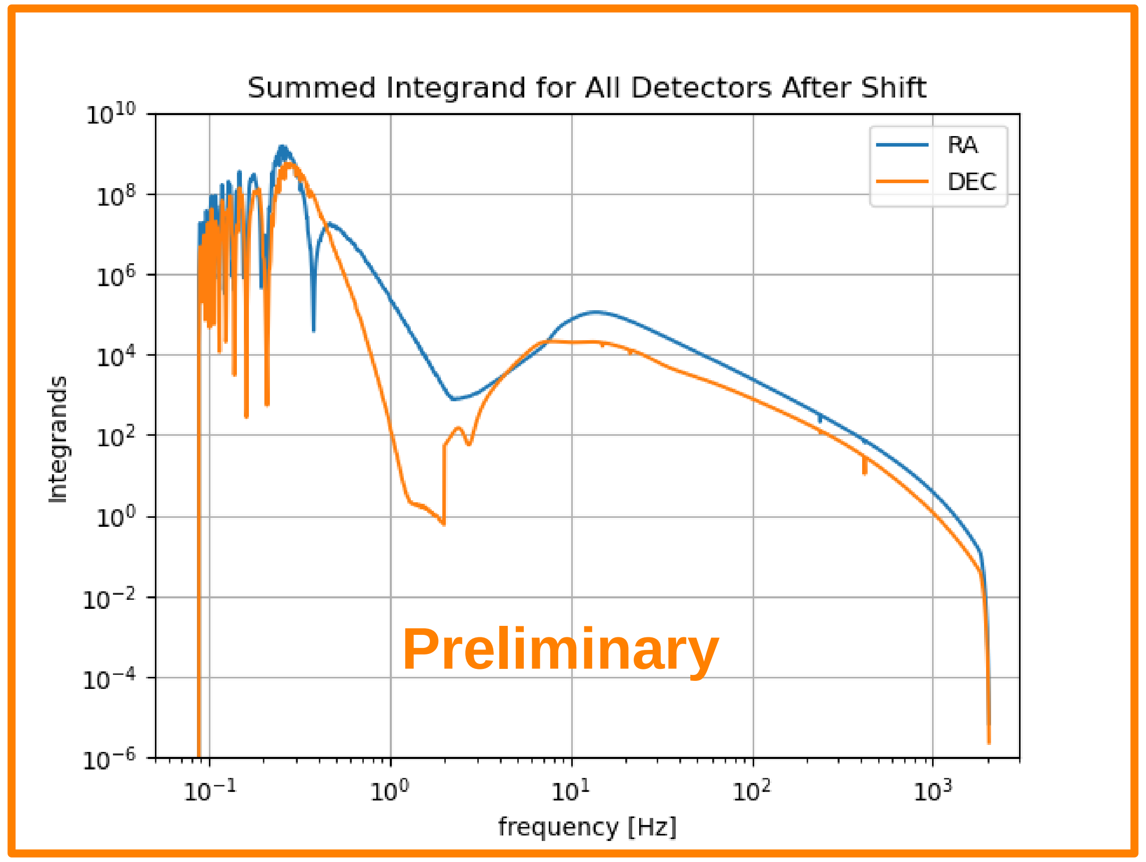
Detector Trajectory Barycenter

OPTION 3: End-Point Barycenter

- Easier and straight-forward implementation;
- Largest the SNR at the end of the signal, thus weighted trajectory position approximately end-point position.

All detectors:
- LGWA;
- LISA;
- KAGRA;
- LIGO;
- Virgo;
- CE;
- ET.





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Thanks for the attention

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Independent Detector

OPTION 4: Independent Detector Evaluation

- Problem arises from combination of information from different reference systems into the same Fisher matrix;
- Evaluate the Fisher matrix for each detector singularly;
- Combine the obtained constraints as final step.

Still to be implemented and explored