Challenges of estimating sky-localization with multiband observations

LGWA meeting

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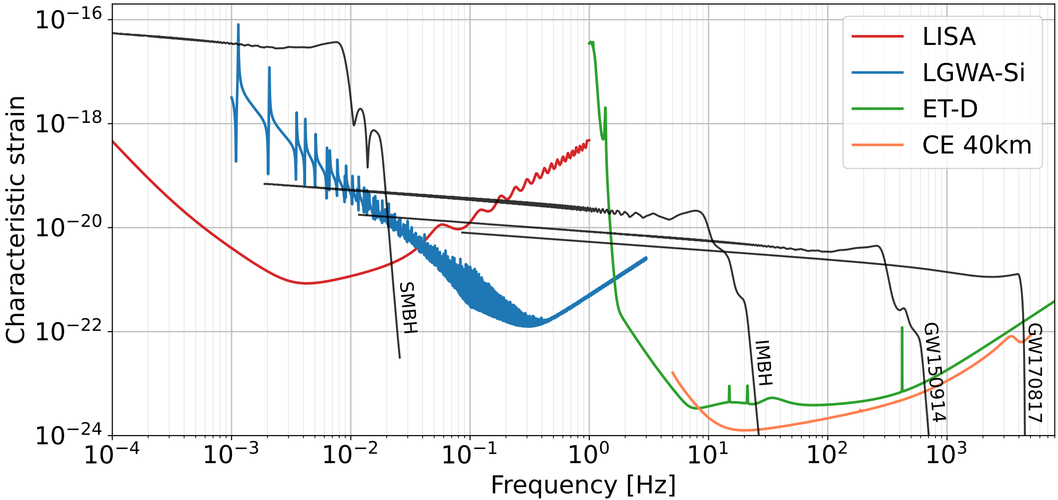
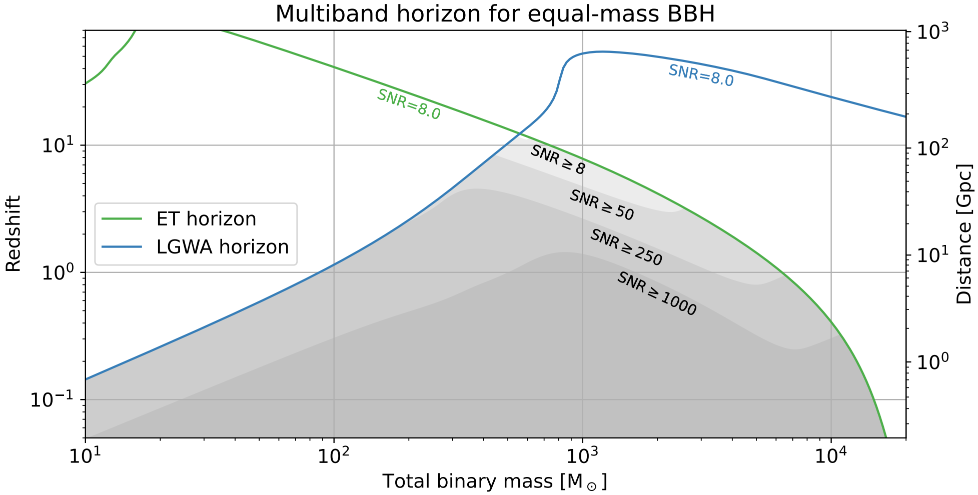
Multibanding with LISA + LGWA + ET



Horizon for ET + LGWA



Current assumptions: ground-based, Compact Binary Coalescences

- geocentric system of reference,
- reference (merger) time t_0,
- $f \sim 10 \div 1000$Hz
- chirping signal

Current assumptions: ground-based, Continuous Waves

- solar-centered system of reference,
- no reference time t_0,
- $f \sim 10 \div 1000$Hz
- quasi-monochromatic signal

Current assumptions: space-based, Compact Binary Coalescences

- solar-centered system of reference,
- no reference time t_0,
- $f \sim 0.1 \div 100$mHz
- chirping signal

What to do? LGWA, Compact Binary Coalescences

- solar-centered system of reference,
- reference time t_0,
- $f \sim 0.01 \div$ few Hz
- chirping signal

Multibanding: adding Fisher matrices

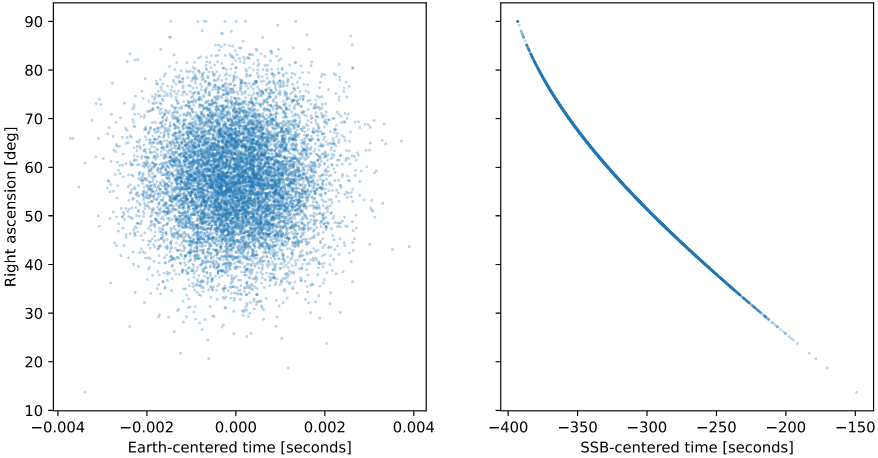
Typical approach:

$$\mathcal{F}_{ij} = \sum_{k \in \text{detectors}} \mathcal{F}_{ij}^{(k)}$$

But the analyisis parameters must be the same!

The reference time is always the one at which the merger wavefront reaches the **center of the coordinate frame**.

Changing variable in the posterior



Time definitions

$$t_{\mathsf{SSB}} = t_{\mathsf{det}} + \Delta_{R\odot}(t_{\mathsf{det}}) + \text{relativistic}$$

where $\Delta_{R\odot}(t_{\mathsf{det}}) = \vec{r} \cdot \hat{n}/c$ is the Roemer time delay.

Time-varying detector response

We define a $t(f)$ mapping using some PN expression, and then compute the FD response using the Stationary Phase Approximation:

$$
\begin{aligned}
h(f) =& (F_+(t(f))A_+(f) + F_\times(t(f))A_\times(f))\times \\
&\times \exp\left(2\pi i f t_0 + 2\pi i f \Delta(t) + i\phi(f) + i\phi_0 - i\pi/4\right)
\end{aligned}
$$

where $\Delta(t)$ is the time delay from the center of the reference frame to the detector position:

- GCRS (earth center): $\Delta(t)$ is $\sim$ tens of milliseconds
- ICRS (SSB): $\Delta(t)$ is $\sim$ minutes

The "diverging" phase term

For **ground-based** detectors and **LGWA**, the phase contribution $2\pi f \Delta$ becomes huge: taking $\Delta \sim \text{AU}/c$,

$$2\pi f \Delta \sim 3000\text{rad} \left(\frac{f}{\text{Hz}} \right)$$

Which means that signals from slightly different sky locations are extremely different (since we are fixing t_{SSB}).

LGWA sky localization

From Wen and Chen (1992):

$$\Delta\Omega = \frac{c^2}{(f\rho_T)^2 A(T) |\sin \iota|}$$

where f is the frequency of a monochromatic signal, and $A(T)$ is the area drawn by its motion.

So, we know that the localization capabilities will be good!

Estimate for LGWA

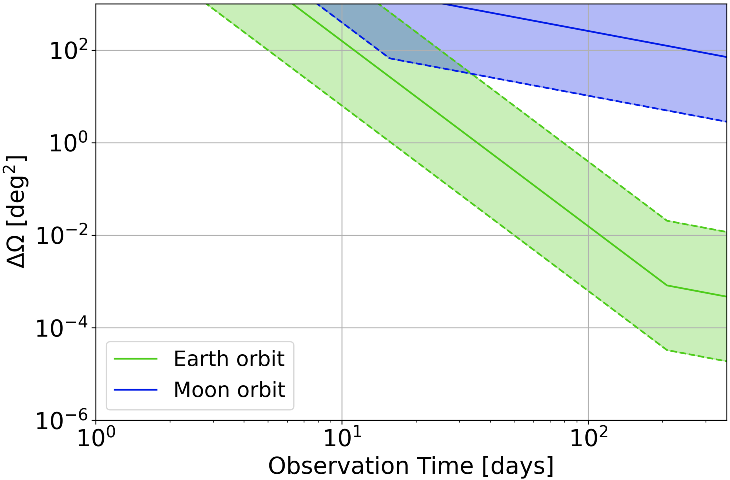
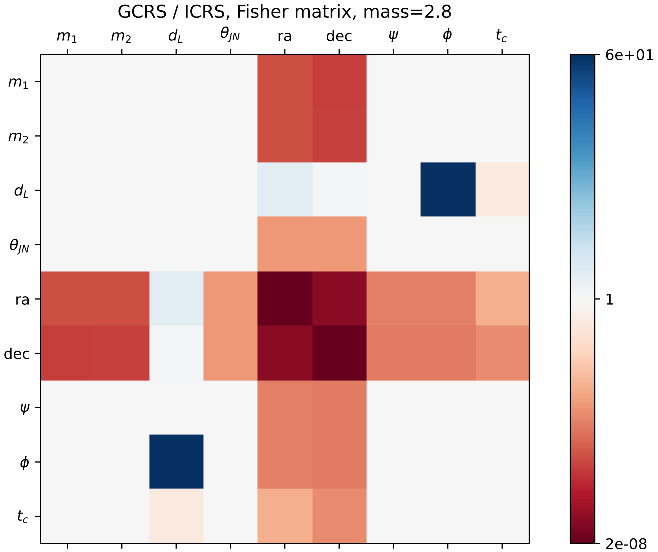
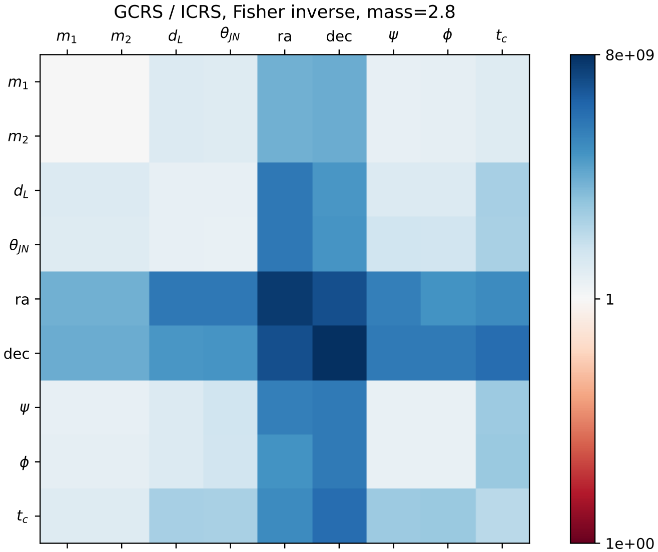


Figure from the LGWA whitepaper.

Fisher matrix comparison (Einstein Telescope)



Fisher inverse comparison (Einstein Telescope)



Summary

- We want to capture the motion of the Moon around the Sun, it is the basis for LGWA's localization capabilities;
- we want to model a multi-band detection with LGWA plus other detectors, but the references must be self-consistent;
- the obvious reference frame choices have issues: how should we handle this?