Quantum entanglement near open timelike curves: theory and experimental simulation

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Time travel

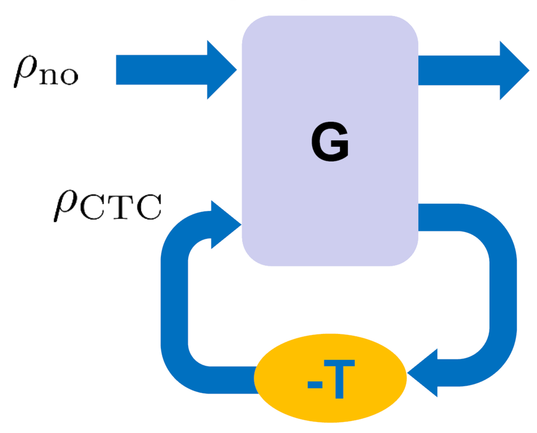
General relativity: time is a general spacetime coordinate. Time travel is (in principle) allowed.

Closed timelike curve (CTC): a particle travels back in time affecting its past self, giving rise to causality issues (e.g.: grandfather paradox)

Open timelike curve (OTC): special CTC case in which a particle travels back in time without causally affecting itself.

Time travel in quantum mechanics

CTC:



G: quantum gate causing the unitary evolution U on the joint subspace.
[Deutsch, Phys. Rev. D 44, 10 (1991)]

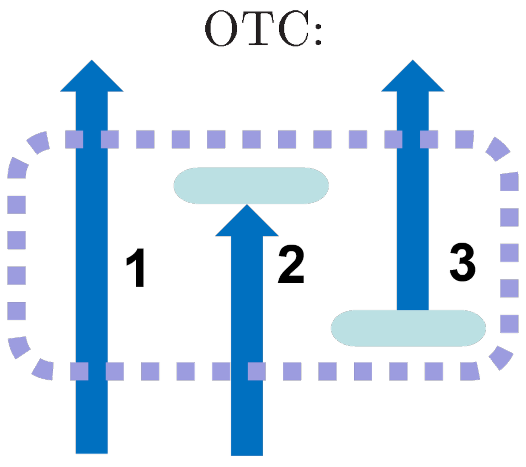
Consistency condition:

$$\mathrm{Tr}_{no}\left(U\left(\rho_{no}\otimes\rho_{CTC}\right)U^{\dagger}\right)=\rho_{CTC}$$

CTCs cause non- linearity!

Optical simulations: Lloyd et al., PRL 106 040403 (2011); Ringbauer et al., Nat. Comm. 5 4145 (2014)

OTC:



With the density matrix formalism, being qubits 1 and 2 initially maximally entangled, in the chronology violation region:

$$E_{12}+E_{13}>1$$

Entanglement monogamy is violated! Possible solutions:

- non-linear quantum evolution;
- pseudo-density operators.

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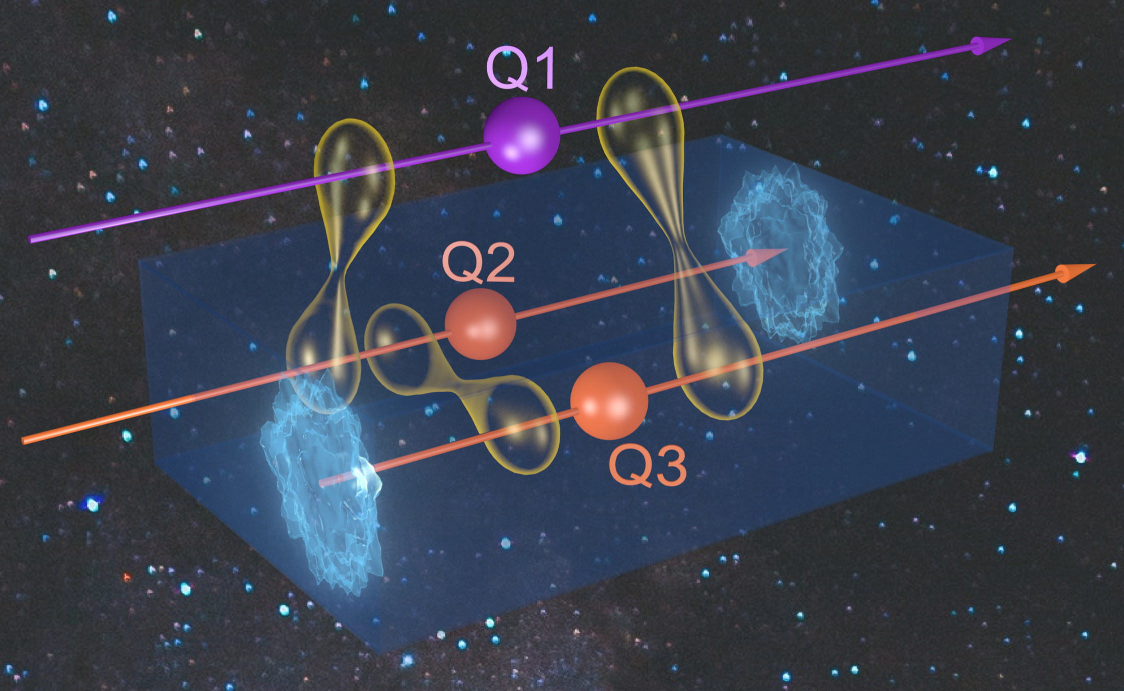
Pseudo-density operator

The Pseudo-Density Operator (PDO) [Fitzsimmons et al., Sci. Rep. 5, 18281 (2015)]is a generalization of the density operator, it is defined for n qubits as:

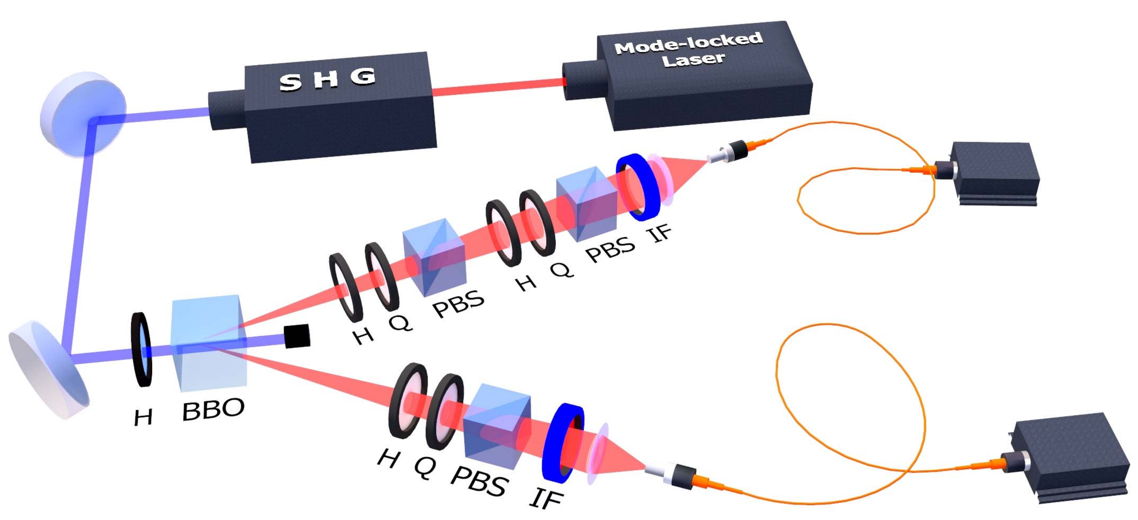
$$R=\frac{1}{2^n}\sum_{i_1=0}^{3}\cdots\sum_{i_n=0}^{3}\left\langle\{\sigma_{i_j}\}_{j=1}^{n}\right\rangle\otimes_{j=1}^{n}\sigma_{i_j}\qquad\begin{bmatrix}\sigma_0=\mathbb{1} & \sigma_1=X\\ \sigma_2=Y & \sigma_3=Z\end{bmatrix}$$

$\left\langle\{\sigma_{i_j}\}_{j=1}^{n}\right\rangle$ being the expectation value of the product of the results of an ensemble of separate von Neumann measurement events $(E_1,\dots,E_n)$ on the $(\sigma_1,\dots,\sigma_n)$ qubits forming the state. PDO properties are:

- Hermitian, trace-one operator, with $R_B=\mathrm{Tr}_A\left(R_{AB}\right)$;
- Non-positive: negative eigenvalues are allowed, both spatial and temporal correlations can be described;



Quantum optical simulation



SHG: Second-harmonic generation; BBO: β-barium borate crystal; H: half-wave plate; Q: quarter-wave plate; PBS: polarising beam splitter; IF: interference filter.

- entangled photons in the state $|\psi_-\rangle=\frac{1}{\sqrt{2}}\left(|HV\rangle-|VH\rangle\right)$ are produced via type-II parametric down-conversion.
- two polarization measurements ($Q2$ and $Q3$) can be performed in sequence on branch A; one ($Q1$) can be performed on branch B.
- photons are detected by two single photons avalanche diodes.

PDO reconstruction rules

Traditional quantum tomography fails to reconstruct R_{123}: new rules are needed.

I. Measure the whole set $\{X,Y,Z\}$ on Q_2 and Q_3, including all cross-correlations between different observables): $R_{23}=\frac{1}{4}\left(I_{23}+\Sigma_{23}\right)$ (Unphysical, negative eigenvalues)
II. Measure the whole set $\{X,Y,Z\}$ on Q_1 and Q_2: $R_{12}=\frac{1}{4}\left(I_{12}-\Sigma_{12}\right)$
III. Measure the whole set $\{X,Y,Z\}$ on Q_1 and Q_3: $R_{13}=\frac{1}{4}\left(I_{13}-\Sigma_{13}\right)$
IV. Measure the whole set $\{X,Y,Z\}$ on Q_1 and Q_2), followed by a measurement on Q_3 identical to the one occurred on Q_2.

$R_{13}=\mathrm{Tr}_2\left[R_{123}\right]$ cannot be reconstructed with the canonical method of acquiring measurements for the three-point correlations and averaging over $Q2$. This because:

$$\Pi_\psi=|\psi\rangle\langle\psi|:\mathrm{Tr}\left[\Pi_\psi R_{123}\right]<0$$

Not a real probability!

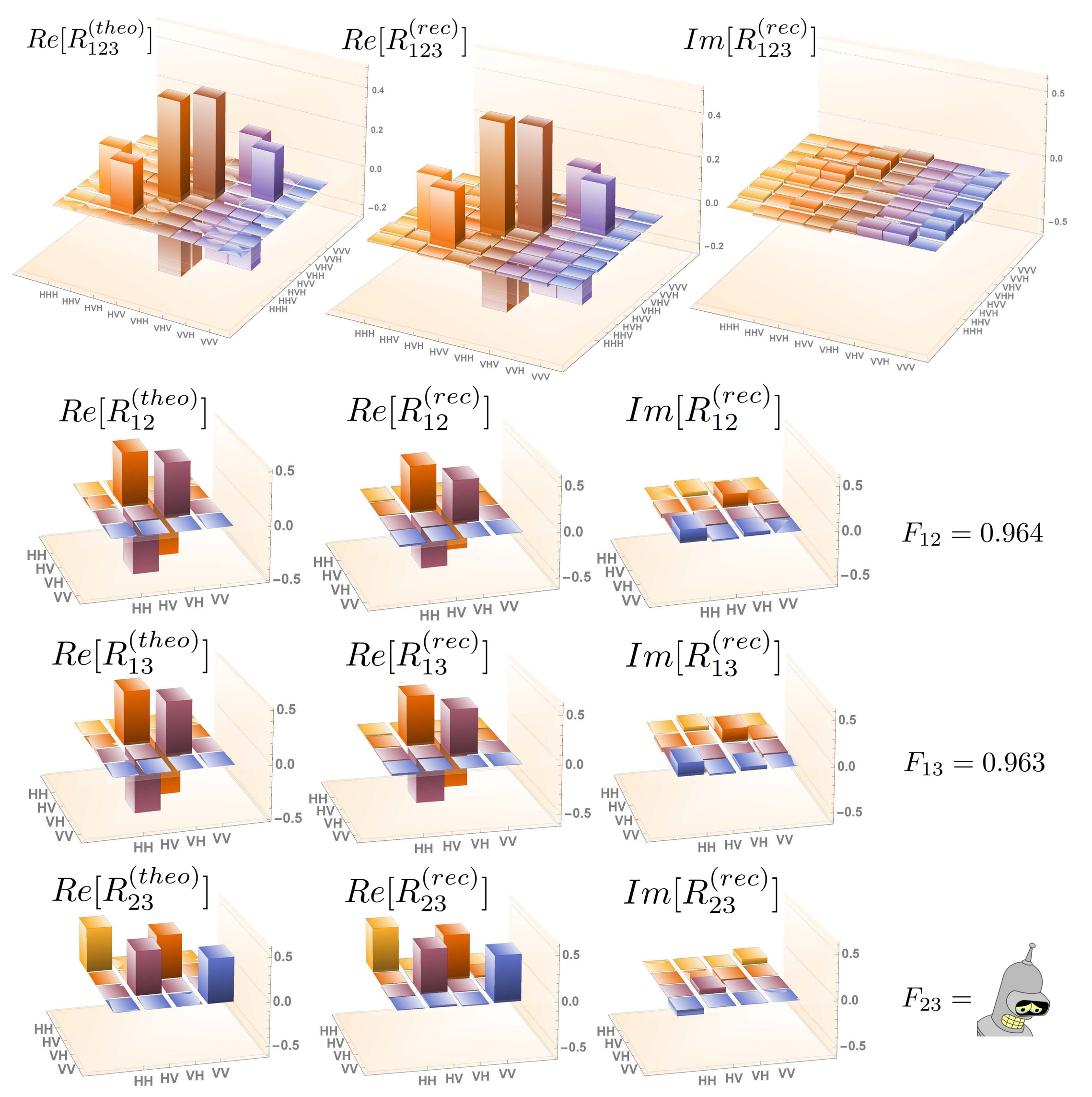
Model

For this OTC, the pseudo-density matrix is:

$$R_{123}=\frac{1}{8}\left(\mathbb{1}_{123}-\Sigma_{12}+\Sigma_{23}-\Sigma_{13}\right)$$

with $\Sigma_{ij}=X_iX_j\mathbb{1}_k+Y_iY_j\mathbb{1}_k+Z_iZ_j\mathbb{1}_k$

Reconstructed PDO



Monogamy violation

Entanglement monogamy:

$$C_{nm}+C_{mk}\leq 4$$

with C_{nm} being the CHSH inequality value for the qubits n and m. Our results:

$$C_{12}^{(exp)}+C_{23}^{(exp)}=5.52\pm0.03$$
$$C_{12}^{(exp)}+C_{13}^{(exp)}=5.42\pm0.07$$
$$C_{23}^{(exp)}+C_{13}^{(exp)}=5.55\pm0.07$$

More than 20 standard deviations violation of the classical bound!

Conclusions

- We implemented the first quantum optical simulation of an OTC, recostructing the pseudo-density operator R_{123};
- The reconstructed R_{123} and its reduced matrices are all in good agreement with the theoretical expectations;
- We measured a strong violation of the monogamy of entanglement within the OTC, something forbidden with the traditional density matrix description, but predicted by the PDO.
- Extension to other frameworks, e.g.: entangled particle falling into an evaporating black hole.

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