A time machine allowing travel to the past by free fall

Livio Pizzocchero

Dipartimento di Matematica, Università degli Studi di Milano
and Istituto Nazionale di Fisica Nucleare, Sezione di Milano

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(Classical and Quantum Gravity 35 (2018), 165003, 42 pp)

Background and motivations

- **Time machines** := **spacetimes** with **closed timelike curves (CTCs).** Literature on this topic, and on the (somehow related) subject of spacetimes with superluminal trajectories:

  Van Stockum (1938); Gödel (1949); Kerr (1963); Misner (1963); Tipler (1974); Morris, Thorne and Yurtsever (1988); Gott (1991); Alcubierre (1994); Ori (1993,1995,2007), Ori and Soen (1994); Li (1994); Low (1995); Everett, Everett and Roman (1996-97); Krasnikov (1998); Tippet and Tsang (2017); Mallary, Khanna and Price (2017); Fermi and P (2018); Fermi (2018); Krasnikov (2018); ...

  Analysis of problematic aspects:

  Friedman, Morris, Novikov, Echeverria, Klinkhammer, Thorne, Yurtsever (1990); Deutsch (1991); Deser, Jackiw and 't Hooft (1992); Hawking (1992); Ford and Roman (1995); Krasnikov (1998,2014,2018); Olum (1998); Visser (2003); Maeda, Ishibashi and Narita (1998), Lobo (2008); ...

- **Typical issues** affecting spacetimes with CTCs:
  - Infinitely extended structures (dust cylinders, cosmic strings, ...).
  - Naked curvature singularities, or CTCs beyond event horizons.
  - Huge tidal accelerations experienced along CTCs.
  - Violations of energy conditions (ECs); large, negative energy densities.
    - Still, some quantum systems do violate the ECs (e.g., Casimir effect).
    - Ori 2007 time machine: *ECs fulfilled*, perhaps singularities/horizons.
- **Chronology protection conjecture:** quantum backreactions forbid CTCs.

Our proposal for a time machine (Fermi and P, 2018):

- A deformation of Minkowski spacetime, flat outside a torus.
- *No singularities. No horizons. Time orientable. ECs are violated.*
- Symmetries ⇒ explicit computation of *geodesics* by quadratures.
- A *freely falling observer* (timelike geodesic) can start from the outer Minkowski region, move across the toric region and return to its *initial position* in the outer region *at an earlier time, arbitrarily far in the past*.
- Quantitative analysis of the above time travel including proper duration, tidal forces, energy densities (all of them can be made acceptable on a human scale).

The spacetime model (in units with $c = 1$)

- **Spacetime manifold** : it is $\mathbb{R}^4 = \mathbb{R} \times \mathbb{R}^3$.

$(t, \varphi, \rho, z) :=$ natural coordinate on $\mathbb{R}$ + cylindrical coordinates on $\mathbb{R}^3$.

- Main **building blocks** : two **concentric tori** $\mathbf{T}_\lambda$, $\mathbf{T}_\Lambda$ with minor radii λ, Λ and common major radius R ($0 < \lambda < \Lambda < R$; R = scale factor):

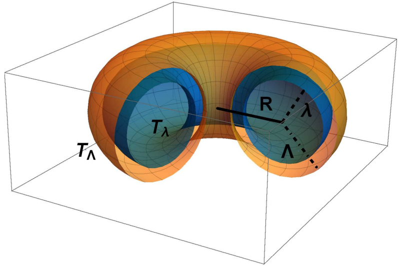
$$\mathbf{T}_\lambda := \left\{(\rho - R)^2 + z^2 = \lambda^2\right\} \quad . \quad \mathbf{T}_\Lambda := \left\{(\rho - R)^2 + z^2 = \Lambda^2\right\} ;$$

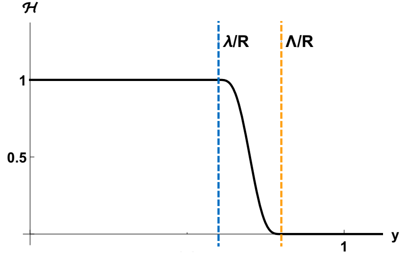
- We use the **shape function**

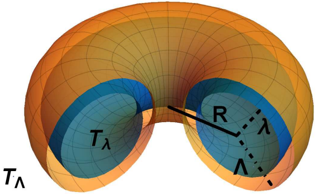
▸ DETAILS

$\mathcal{X}(\rho, z) := \mathcal{H}\left(\sqrt{(\rho/R - 1)^2 + (z/R)^2}\,\right)$, $\mathcal{H} \in C^k([0, +\infty))$ as below ($2 \leqslant k \leqslant \infty$).

$\chi = 0$ outside $\mathbf{T}_\Lambda$; $\chi = 1$ inside $\mathbf{T}_\lambda$; $0 < \chi < 1$ between $\mathbf{T}_\lambda$ and $\mathbf{T}_\Lambda$.







▶ We now *postulate* for $\mathbb{R}^4$ the **line element** ($a, b > 0$ dimensionless parameters)

$$ds^2 := -\left[(1-\mathcal{X})dt + a\,R\,\mathcal{X}\,d\varphi\right]^2 + \left[(1-\mathcal{X})\rho\,d\varphi - b\,\mathcal{X}\,dt\right]^2 + d\rho^2 + dz^2.$$

• Outside $\mathbf{T}_\Lambda$ $(\mathcal{X}=0)$: $ds^2 = -\,dt^2 + \rho^2 d\varphi^2 + d\rho^2 + dz^2$.
Usual, flat Minkowski metric in cylindrical coordinates; t a timelike coord.

• Inside $\mathbf{T}_\lambda$ $(\mathcal{X}=1)$: $ds^2 := -\,a^2R^2 d\varphi^2 + b^2 dt^2 + d\rho^2 + dz^2$.
A flat metric; φ a timelike coord.; CTCs with $t, \rho, z =$ const., $\varphi \in \mathbb{R}/(2\pi\mathbb{Z})$.

▶ ds^2 defines a *Lorentzian metric g of class C^k on $\mathbb{R}^4$* $(2 \leqslant k \leqslant \infty)$,
non flat between $\mathbf{T}_\Lambda$ and $\mathbf{T}_\lambda$. $\mathrm{Riem}_g \in C^{k-2}$; ***no** curvature singularity* .

▶ $\mathfrak{T} := (\mathbb{R}^4, g)$ **is our spacetime**. Spacetime region **outside**/**inside** $\mathbf{T}_\Lambda$ will be called the **outer Minkowski region**/**time machine**.

A tetrad. Time (and space) orientation

- The explicit expression of ds^2 suggests to consider the 1-forms

$$e^{(0)} := (1-\mathcal{X})\, dt + a\, R\, \mathcal{X}\, d\varphi\,, \quad e^{(1)} := (1-\mathcal{X})\, \rho\, d\varphi - b\, \mathcal{X}\, dt\,,$$
$$e^{(2)} := d\rho\,, \quad e^{(3)} := dz\,,$$

$$\text{s.t.} \quad ds^2 = -\left[e^{(0)}\right]^2 + \left[e^{(1)}\right]^2 + \left[e^{(2)}\right]^2 + \left[e^{(3)}\right]^2 .$$

- The dual vector fields $E_{(\nu)}$ s.t. $\langle e^{(\mu)}, E_{(\nu)} \rangle = \delta^{\mu}{}_{\nu}$ $(\mu, \nu = 0, 1, 2, 3)$ are C^k and form an **orthonormal tetrad**:

$$g(E_{(0)}, E_{(0)}) = -1\,, \quad g(E_{(i)}, E_{(j)}) = \delta_{ij}\,, \quad g(E_{(0)}, E_{(i)}) = 0 \quad (i, j = 1, 2, 3)\,.$$

  - $E_{(0)}$ is everywhere timelike, and $E_{(0)} = \partial_t$ in outer Minkowski region. We call $E_{(0)}$ the **fundamental timelike vector field**, and *define* **future** to be the **time orientation containing** E_0.

  - $E_{(i)}$ $(i = 1, 2, 3)$ are everywhere spacelike, spanning $E_0^{\perp}$, and $(E_{(1)}, E_{(2)}, E_{(3)}) = (\rho^{-1}\partial_{\varphi}, \partial_{\rho}, \partial_z)$ in outer Minkowski region. We can use $(E_{(1)}, E_{(2)}, E_{(3)})$ to *define* the **left-handed orientation** of $E_{(0)}^{\perp}$.

Remarks. (i) In principle each $E_{(\nu)}$ is defined on $\mathbb{R}^4 \setminus \{\rho = 0\}$, but $E_{(0)}$, $E_{(3)}$, $E_{(0)}^{\perp}$ have C^k extensions to the whole $\mathbb{R}^4$.
(ii) The distribution $E_{(0)}^{\perp}$ is *not involutive* $\Rightarrow$ $\nexists$ *spacelike hypersurfaces* $\perp E_{(0)}$.

Results on free fall motions (timelike geodesics)

• **Geodesics** (with an affine parametrization) in our spacetime $\mathfrak{T}$ are the solutions of Euler-Lagrange eq.s associated to the **Lagrangian**

$$L(\xi,\dot{\xi}) := \frac{1}{2}\, g_\xi\big(\dot{\xi},\dot{\xi}\big) \qquad (\xi \in \mathfrak{T}, \dot{\xi} \in T_\xi \mathfrak{T})\ ;$$

$$L(t,\varphi,\rho,z,\dot{t},\dot{\varphi},\dot{\rho},\dot{z}) = \frac{1}{2}\left[-\Big[(1-\mathcal{X})\,\dot{t} + a\,R\,\mathcal{X}\,\dot{\varphi}\Big]^2 + \Big[(1-\mathcal{X})\,\rho\,\dot{\varphi} - b\,\mathcal{X}\,\dot{t}\Big]^2 + \dot{\rho}^2 + \dot{z}^2\right].$$

$(\mathcal{X} = \mathcal{X}(\rho,z))$. This has the following **constants of motion**:

- The **canonical momenta** associated to the coordinates t, φ, i.e.,

  $$p_t := \frac{\partial L}{\partial \dot{t}}\ , \qquad p_\varphi := \frac{\partial L}{\partial \dot{\varphi}}\ ,$$

  conserved since $\partial L/\partial t = 0, \partial L/\partial \varphi = 0$ ($\sim \partial_t, \partial_\varphi$ Killing vector fields).
- The **energy** $E := (\partial L/\partial \xi^\mu)\dot{\xi}^\mu - L$; due to the *purely kinetic* form of L,

  $$E(\xi,\dot{\xi}) = L(\xi,\dot{\xi}) = \frac{1}{2}\, g_\xi\big(\dot{\xi},\dot{\xi}\big)\ .$$

• **Free fall motions** are **future-directed, timelike geodesics** $\tau \mapsto \xi(\tau)$, that we parametrize with **proper time** τ so that

$$E(\xi(\tau),\dot{\xi}(\tau)) = \frac{1}{2}\, g_{\xi(\tau)}\big(\dot{\xi}(\tau),\dot{\xi}(\tau)\big) = -\frac{1}{2}\ , \qquad \dot{\xi}(\tau) \text{ future directed.}$$

• From now on **we only consider free fall motions in the plane $\{z=0\}$** (that do exist) parametrized by **proper time τ**, for which $\dot{} \equiv d/d\tau$.

• These have a **Lagrangian** $L(t,\varphi,\rho,\dot t,\dot\varphi,\dot\rho)$, as before with $(z,\dot z)=(0,0)$.

• The **canonical momenta** $p_t := \partial L/\partial \dot t$, $p_\varphi := \partial L/\partial\dot\varphi$ are **constants of motion**; we replace p_t, p_φ with the **dimensionless parameters**

$$\gamma := -p_t = \Big[(1-\mathcal{X})^2 - b^2\mathcal{X}^2\Big]\,\dot t + \Big[(a+b\,\rho/R)\mathcal{X}(1-\mathcal{X})\Big]R\,\dot\varphi\ ,$$

$$\omega := \frac{p_\varphi}{\gamma\,R} = \frac{1}{\gamma}\Big[(\rho/R)^2(1-\mathcal{X})^2 - a^2\mathcal{X}^2\Big]R\,\dot\varphi - \frac{1}{\gamma}\Big[(a+b\,\rho/R)\mathcal{X}(1-\mathcal{X})\Big]\,\dot t\,.$$

$(\mathcal{X}\equiv\mathcal{X}(\rho,0)=\mathcal{H}(|\rho/R-1|))$. In the **outer Minkowski region** $(\mathcal{X}=0)$:

- $\gamma = \dot t \equiv dt/d\tau \Rightarrow$ for future dir. motions γ is the *Lorentz factor, $\gamma\geqslant 1$*;
- $\omega = \frac{\rho^2}{\gamma\,R}\,\dot\varphi$, a sort of *rescaled angular momentum*.

• Conversely:

$$\dot t = \gamma\,\frac{\Big[(\rho/R)^2(1-\mathcal{X})^2 - a^2\mathcal{X}^2\Big] - (a+b\,\rho/R)\,\mathcal{X}(1-\mathcal{X})\,\omega}{\Big[(\rho/R)(1-\mathcal{X})^2 + a\,b\,\mathcal{X}^2\Big]^2} \equiv \dot t(\rho,\gamma,\omega)\,,$$

$$\dot\varphi = \gamma\,\frac{(a+b\,\rho/R)\,\mathcal{X}(1-\mathcal{X}) + \Big[(1-\mathcal{X})^2 - b^2\,\mathcal{X}^2\Big]\,\omega}{R\,\Big[(\rho/R)(1-\mathcal{X})^2 + a\,b\,\mathcal{X}^2\Big]^2} \equiv \dot\varphi(\rho,\gamma,\omega)\ .$$

Inside $\mathbf{T}_\lambda$ $(\mathcal{X}=1)$: $\dot t = -\gamma/b^2$ (<0 for future dir. motions from Mink. region),
$\dot\varphi = -\gamma\,\omega/(a^2 R)\,.$

- Let's recall that another constant of motion is the **energy** $E = L$.
- Substituting $\dot{t}, \dot{\varphi}$ in the expression for E gives

$$E \;=\; \frac{1}{2}\,\dot{\rho}^2 + V_{\gamma,\omega}(\rho)\ ,$$

$$V_{\gamma,\omega}(\rho) \;:=\; \left(\frac{\gamma^2}{2}\right) \frac{\big[a\,\mathcal{X}+(1-\mathcal{X})\,\omega\big]^2 - \big[(\rho/R)\,(1-\mathcal{X})-b\,\mathcal{X}\,\omega\big]^2}{\big[(\rho/R)\,(1-\mathcal{X})^2 + a\,b\,\mathcal{X}^2\big]^2}\ .$$

This is the energy of an *effective* $1D$ *conservative system*: a particle of position $\rho \in (0,+\infty)$ with potential $V_{\gamma,\omega}(\rho)$.

In outer Minkowski region: $(\mathcal{X}=0)$: $V_{\gamma,\omega}(\rho) = \frac{\gamma^2}{2}\left(\frac{R^2\omega^2}{\rho^2} - 1\right)$.
Inside $\mathbf{T}_\lambda$ $(\mathcal{X}=1)$: $V_{\gamma,\omega}(\rho) = \text{const.} = \frac{\gamma^2}{2\,a^2}\left(\frac{a^2}{b^2} - \omega^2\right)$.
Between $\mathbf{T}_\lambda$ and $\mathbf{T}_\Lambda$ $(0<\mathcal{X}<1)$: $V_{\gamma,\omega}$ *depends sensibly on* $\operatorname{sign}\omega$.

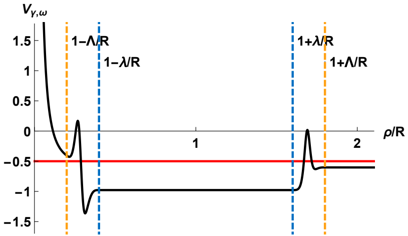
- Recall we are considering motions parametrized by proper time τ, so that

$$E = -1/2\ .$$

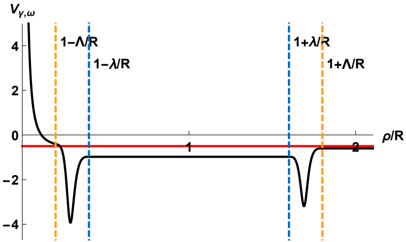
- **Qualitative analysis**: $\rho(\tau)$ is confined to a connected component of

$$\{\rho \in (0,+\infty)\ |\ V_{\gamma,\omega}(\rho) \leqslant -1/2\}.$$

Graphs of $V_{\gamma,\omega}$ for two choices of the parameters



$\omega = 0.08$



$\omega = -0.08$

$(\lambda = 0.6\,R,\ \Lambda = 0.8\,R,\ a = 0.09,\ b = 10,\ k = 3,\ \gamma = 1.1)$

- **Quantitative analysis:** the conservation laws for E and p_t, p_φ (or γ, ω) yield **quadrature formulas** for free fall motions $\tau \mapsto t(\tau), \varphi(\tau), \rho(\tau)$.
  If sign $\dot\rho(\tau) = \sigma \in \{\pm 1\}$ for $\tau_- < \tau < \tau_+$, $\rho_\pm := \rho(\tau_\pm)$ and so on, then

$$\tau_+ - \tau_- = \sigma \int_{\rho_-}^{\rho_+} \frac{d\rho}{\sqrt{2(-1/2 - V_{\gamma,\omega}(\rho))}} \ ,$$

$$t_+ - t_- = \sigma \int_{\rho_-}^{\rho_+} \frac{d\rho\, \dot t(\rho,\gamma,\omega)}{\sqrt{2(-1/2 - V_{\gamma,\omega}(\rho))}} \ , \ \ \varphi_+ - \varphi_- = \sigma \int_{\rho_-}^{\rho_+} \frac{d\rho\, \dot\varphi(\rho,\gamma,\omega)}{\sqrt{2(-1/2 - V_{\gamma,\omega}(\rho))}} .$$

Free fall into the past

We now search for a future directed, **free fall "travel"** $\tau \in [0, \tau_2] \mapsto \xi(\tau)$ with $z = 0$, constants of motion γ, ω and with the following features, where $\tau_1 := \frac{\tau_2}{2}$ and $_{0,1,2}$ indicate evaluation at $\tau = 0, \tau_1, \tau_2$:

1. $\rho_0 > R + \Lambda$ (**departure in the outer Minkowski region**); $\varphi_0 = 0,\ t_0 = 0$ (conventional choices).
2. $\rho(\tau) \searrow$ for $0 \leqslant \tau \leqslant \tau_1$; $\rho_1 < R - \Lambda$ (**motion across both tori**). $\rho(\tau) \nearrow$ for $\tau_1 \leqslant \tau \leqslant \tau_2$; $\rho_2 = \rho_0$ (return to initial radius, after crossing again both tori).
3. $\varphi_2 = 0$ (i.e., $\varphi_2 = \varphi_0$; with $\rho_2 = \rho_0$, this indicates the **return to the initial position, in the outer Minkowski region**).
4. Consider $t_2 =$ **final value of Minkowski time coordinate $=$ endtime of the travel, according to a clock fixed at $\rho = \rho_0, \varphi = 0, z = 0$ and indicating time 0 at the departure**. **It is required that $t_2 < 0$ (travel to the past!) and $|t_2|$ be arbitrarily large**.
5. Consider $\tau_2 =$ **duration of the travel according to the traveller's clock**. $\tau_2 / |t_2|$ **is required to be small**.

In [Fermi and P, 2018] it is shown that **all the previous conditions 1-5 can be fulfilled choosing suitably the quantities** $a, b, \lambda/R, \Lambda/R$ **associated to the spacetime metric** g **and the constants** γ, ω **of the free fall motion**. Indeed, using some qualititative analysis and the previous quadrature formulas (with some asymptotic expansions derived from then), one infers the following: ▸ DETAILS

- **Conditions 1-4 are fulfilled if**

$$\frac{a}{b} < 1 - \frac{\Lambda}{R}\ ;\ \gamma > \max\left(1, \sqrt{\frac{(1-\frac{\Lambda}{R})^2 + a^2}{(1-\frac{\Lambda}{R})^2 - \frac{a^2}{b^2}}}\right)\ ;\ \omega = -\sqrt{\frac{a^2}{b^2}\left(1+\frac{b^2}{\gamma^2}\right) + \varpi^2}\,,\ 0 < \varpi \ll 1$$

and the small value of ϖ is *fine tuned* to grant that $\varphi_2 = 0$ (return to the initial angle). Moreover, for $\varpi \to 0^+$:

$$t_2 \sim -\left(\frac{4a}{b^2}\lambda\right)\frac{1}{\varpi}\,, \quad \tau_2 \sim \left(\frac{4a}{\gamma}\lambda\right)\frac{1}{\varpi} \quad \Longrightarrow \quad \frac{\tau_2}{|t_2|} \sim \frac{b^2}{\gamma}\ ;$$

so, $\tau_2/|t_2|$ **is small (condition 5) if** γ **is large** (this essentially reflects the standard phenomenon of time dilatation in special relativity).

Tidal accelerations along free fall to the past

- For any geodesic ξ of the previous type and for any fixed $\tau \in \mathbb{R}$, set

$$\dot\xi(\tau)^\perp := \left\{ X \in T_{\xi(\tau)}\mathfrak{T} \,\middle|\, g_{\xi(\tau)}(X, \dot\xi(\tau)) = 0 \right\}, \quad \mathcal{A}_\tau X := -\,\mathrm{Riem}_{\xi(\tau)}(X, \dot\xi(\tau))\,\dot\xi(\tau)\,.$$

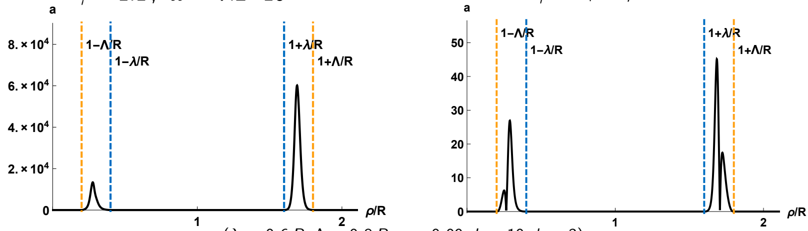
  - $\dot\xi(\tau)^\perp$ is a Euclidean space with the inner product $g_{\xi(\tau)}(\cdot,\cdot)$.
  - $\mathcal{A}_\tau : \dot\xi(\tau)^\perp \to \dot\xi(\tau)^\perp$ is well defined, linear and selfadjoint.
  - The *Jacobi equation for geodesic deviation* reads $\dfrac{\nabla^2 \delta\xi}{d\tau^2}(\tau) = \mathcal{A}_\tau \delta\xi(\tau)\,.$

- The **maximal tidal acceleration per unit length** along ξ at τ is

$$\alpha(\tau) := \sup \left\{ \frac{\|\mathcal{A}_\tau X\|}{\|X\|} \,\middle|\, 0 \neq X \in \dot\xi(\tau)^\perp \right\}, \quad \|\cdot\| := \sqrt{g_{\xi(\tau)}(\cdot,\cdot)}\,.$$

$\alpha(\tau) \neq 0$ **between $\mathbf{T}_\Lambda$ and $\mathbf{T}_\lambda$**. $\exists\, \mathrm{a}(r)$ dimensionless, s.t. $\alpha(\tau) = \dfrac{\gamma^2}{R^2}\, \mathrm{a}\big(\rho(\tau)/R\big)$

. $\gamma = 1.1\,,\ \varpi = 7.2 \cdot 10^{-10}$ $\qquad$ $\gamma \to +\infty\,,\ \varpi \to 0^+$



$(\lambda = 0.6\,R,\ \Lambda = 0.8\,R,\ a = 0.09,\ b = 10,\ k = 3)$

Energy density and ECs violations

- Einstein's eq.s can be used to *define* the stress-energy tensor as

$$\mathrm{T}_g := \frac{1}{8\pi G}\left(\mathrm{Ric}_g - \frac{1}{2}\,\mathrm{R}_g\, g\right) \qquad (\mathrm{T}_g \neq 0 \textbf{ bewteen the tori!}).$$

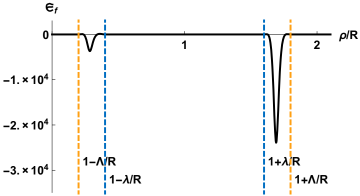
- The **energy density** measured at a spacetime point p by a **fundamental observer** (:= one with 4-velocity $E_{(0)}$), or measured at τ by a **freely falling traveller** with worldline ξ (and $z = 0$) are:

$$\mathcal{E}_f(p) := \mathrm{T}_g\left(E_{(0)}\big|_p, E_{(0)}\big|_p\right)\,, \qquad \mathcal{E}(\tau) := \mathrm{T}_g\left(\dot{\xi}(\tau), \dot{\xi}(\tau)\right).$$

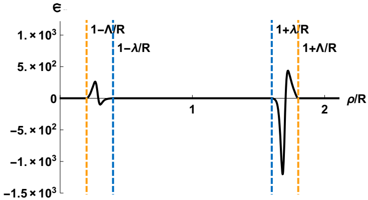
It is $\mathcal{E}_f(p) = \frac{1}{8\pi G\, R^2}\, \mathfrak{E}_f(\rho(p)/R, z(p)/R)$, $\mathcal{E}(\tau) = \frac{\gamma^2}{8\pi G\, R^2}\, \mathfrak{E}\left(\rho(\tau)/R\right)$,

($\mathfrak{E}_f, \mathfrak{E}$ dimensionless). Hereafter $\lambda = 0.6\, R$, $\Lambda = 0.8\, R$, $a = 0.09$, $b = 10$, $k = 3$:

$z = 0$



$\gamma \to +\infty\,,\ \varpi \to 0^+$



▶ $\exists\, p,\, \tau$ s.t. $\mathcal{E}_f(p) < 0$, $\mathcal{E}(\tau) < 0$ $\Rightarrow$ **weak energy cond. is violated**.

Playing with numbers in our model

TABLE 1 :	**R = 10^2 m**			min $\mathcal{E}_f = -1.28\ldots \cdot 10^{23}$ gr/cm^3	
γ	ϖ	t_2	τ_2	max α ($g_{\oplus}$/m)	min $\mathcal{E}$ (gr/cm^3)
1.1	$7.20\ldots \cdot 10^{-10}$	$-1\ s$	$90.97\ s$	$6.679 \cdot 10^{16}$	$-1.347 \cdot 10^{23}$
10^2	$7.20\ldots \cdot 10^{-10}$	$-1\ s$	$1\ s$	$4.144 \cdot 10^{17}$	$-6.779 \cdot 10^{25}$
10^2	$2.28\ldots \cdot 10^{-17}$	$-1\ y$	$1\ y$	$4.144 \cdot 10^{17}$	$-6.779 \cdot 10^{25}$
10^5	$2.28\ldots \cdot 10^{-20}$	$-10^3\ y$	$1\ y$	$4.143 \cdot 10^{23}$	$-6.768 \cdot 10^{31}$
10^8	$2.28\ldots \cdot 10^{-23}$	$-10^6\ y$	$1\ y$	$4.143 \cdot 10^{29}$	$-6.768 \cdot 10^{37}$

TABLE 2 :	**R = 10^{11} m** $\sim$ Earth-Sun dist.			min $\mathcal{E}_f = -1.28\ldots \cdot 10^{5}$ gr/cm^3	
γ	ϖ	t_2	τ_2	max α ($g_{\oplus}$/m)	min $\mathcal{E}$ (gr/cm^3)
10^2	$2.28\ldots \cdot 10^{-8}$	$-1\ y$	$1\ y$	0.4144	$-6.779 \cdot 10^{7}$
10^5	$2.28\ldots \cdot 10^{-11}$	$-10^3\ y$	$1\ y$	$4.143 \cdot 10^{5}$	$-6.768 \cdot 10^{13}$
10^8	$2.28\ldots \cdot 10^{-14}$	$-10^6\ y$	$1\ y$	$4.143 \cdot 10^{11}$	$-6.768 \cdot 10^{19}$

TABLE 3 :	**R = 10^{18} m** $\sim$ 100 light years			min $\mathcal{E}_f = -1.28\ldots \cdot 10^{-9}$ gr/cm^3	
γ	ϖ	t_2	τ_2	max α ($g_{\oplus}$/m)	min $\mathcal{E}$ (gr/cm^3)
10^5	$2.28\ldots \cdot 10^{-4}$	$-925\ y$	$1.02\ y$	$4.143 \cdot 10^{-9}$	-0.6778
10^8	$2.28\ldots \cdot 10^{-7}$	$-10^6\ y$	$1\ y$	$4.143 \cdot 10^{-3}$	$-6.768 \cdot 10^{5}$

$$\left(\begin{array}{c} \lambda = 0.6\,R,\ \Lambda = 0.8\,R,\ a = 0.09,\ b = 10,\ k = 3\,;\ \rho_0 = (1+10^{-3})(R+\Lambda)\,; \\ \min \mathcal{E}_f := \min\{\mathcal{E}_f(p) \mid p \in \mathfrak{T}\}\ ,\quad \min \mathcal{E} := \min\{\mathcal{E}(\tau) \mid 0 \leqslant \tau \leqslant \tau_2\}; \\ g_{\oplus} := \text{Earth's grav. accel.} = 9.8\,\text{m/s}^2 \end{array}\right)$$

For comparison : $\gamma_{LHC} \sim 10^4$, $\gamma_{LEP} \sim 10^5$, $d_{H_2O} \sim 1\text{gr/cm}^3$, $d_{\text{Planck}} \sim 10^{93}\text{gr/cm}^3$.

Developments

• Light signals emitted by freely falling time travellers and their frequency shifts were studied in [Fermi, 2018].

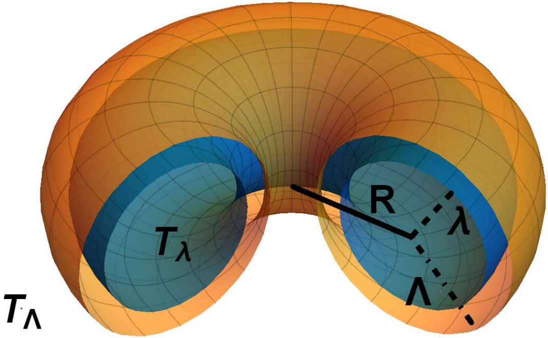
Open problems:

• Studying the *propagation of classical and quantum fields* on the background of the present spacetime $\mathfrak{T}$.

• Investigating the presence of *singularities* in the renormalized stress-energy tensor of such quantum fields (say, on vacuum states); the occurring of such singularities is suggested by the *chronology protection conjecture*.

• Model the *creation of* $\mathfrak{T}$ starting from Minkowski spacetime, in terms of a time-dependent shape function $\mathcal{X} = \mathcal{X}(t, \rho, z)$.

Thanks a lot for your attention!



Basic references for this talk

- D. Fermi, L. Pizzocchero, *A time machine for free fall into the past*, Classical and Quantum Gravity **35** (2018), 165003 (42pp).
  Click here for a link to the paper

- D. Fermi, *Some remarks on a new exotic spacetime for time travel by free fall*, arXiv:1812.09021v1 [gr-qc] (2018).
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  Click here for a link to the arXiv version

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Appendix. On the shape function

• Let's recall that $\mathcal{X}(\rho,z) := \mathcal{H}\big(\sqrt{(\rho/R-1)^2+(z/R)^2}\,\big)$, where $\mathcal{H}\equiv\mathcal{H}_k\in C^k([0,+\infty)$ has a graph as on page 4, and $k\in\{2,3,\dots,\infty\}$.

• Throughout [Fermi and P, 2018] and in the present talk, the following choice is considered for $\mathcal{H}_k$:

$$\mathcal{H}_k(y) := \mathfrak{H}_{(k)}\left(\frac{\Lambda/R-y}{\Lambda/R-\lambda/R}\right) \qquad \text{for } y\in[0,+\infty)\ ,$$

$$\mathfrak{H}_{(k)}(w) := \begin{cases} 0 & \text{for}\quad w\in(-\infty,0]\ , \\ \displaystyle\sum_{j=0}^{k}\binom{2k+1}{j+k+1}\,w^{j+k+1}\,(1-w)^{k-j} & \text{for}\quad k<\infty,\, w\in(0,1)\ , \\ \left(\displaystyle\int_0^w dv\, e^{-\frac{1}{v(1-v)}}\right)\Big/\left(\displaystyle\int_0^1 dv\, e^{-\frac{1}{v(1-v)}}\right) & \text{for}\quad k=\infty,\, w\in(0,1)\ , \\ 1 & \text{for}\quad w\in[1,+\infty)\ . \end{cases}$$

◦ In particular, for $k=3$:

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$$\mathfrak{H}_{(3)}(w) = \begin{cases} 0 & \text{for } w\in(-\infty,0]\,, \\ 35\,w^4(1-w)^3+21\,w^5(1-w)^2+7\,w^6(1-w)+w^7 & \text{for } w\in(0,1)\,, \\ 1 & \text{for } w\in[1,+\infty)\,. \end{cases}$$

Appendix. On the quantities $\rho_1, \varphi_2, t_2, \tau_2$

On pages 11 and 12 we describe a **time travel** in terms of the quantities ρ_0, ρ_1 (max and min radius during the travel, φ_2 (final angle), t_2 (final value of the Minkowski time coordinate), τ_2 (proper duration of the travel).

- ρ_0 must fulfill $\rho_0 > R + \Lambda$, and for the rest is arbitrary. GO BACK
- $\rho_1 \in (0, R - \Lambda)$ is found solving the equation $V_{\gamma,\omega}(\rho_1) = -1/2$, where $V_{\gamma,\omega}$ is the effective potential of page 9. One finds $\rho_1 = -R\omega/\sqrt{1 - 1/\gamma^2}$, provided that the r.h.s be in $(0, R - \Lambda)$.
- φ_2, t_2, τ_2 can be expressed via the following quadrature formulas (following from the results on page 10):

$$\varphi_2 = 2\int_{\rho_1}^{\rho_0} \frac{d\rho\, \dot{\varphi}(\rho, \gamma, \omega)}{\sqrt{2(-1/2 - V_{\gamma,\omega}(\rho))}} \;, \quad t_2 = 2\int_{\rho_1}^{\rho_0} \frac{d\rho\, \dot{t}(\rho, \gamma, \omega)}{\sqrt{2(-1/2 - V_{\gamma,\omega}(\rho))}} \;,$$

$$\tau_2 = 2\int_{\rho_1}^{\rho_0} \frac{d\rho}{\sqrt{2(-1/2 - V_{\gamma,\omega}(\rho))}} \;, \quad \dot{\varphi}(\rho, \gamma, \omega), \dot{t}(\rho, \gamma, \omega) \text{ as on page } 8\,.$$

- Let's recall that we want $\varphi_2 = 0 \pmod{2\pi}$, this can be seen as a fine tuning condition on the parameter ϖ defined on page 12.
- The $\varpi \to 0^+$ asymptotics of t_2, τ_2 reported on page 12 are derived in [Fermi and P, 2018] starting from the above quadrature formulas.