What is the surface of a (dynamical) Black Hole?

José M M Senovilla

Department of Theoretical Physics and History of Science
University of the Basque Country, Bilbao, Spain

The Time Machine Factory, Torino, 23rd September 2019

Outline

Black Holes

- Black Holes (BH) are fundamental objects in Gravitation, and in Physics, predicted classically in General Relativity (GR) with intriguing consequences in quantum physics.

Black Holes

- Black Holes (BH) are fundamental objects in Gravitation, and in Physics, predicted classically in General Relativity (GR) with intriguing consequences in quantum physics.
- BHs turn out to have a finite, non-zero, temperature. Thus, they emit (in quantum theory) “Hawking” radiation with a perfect thermal spectrum.

Black Holes

- Black Holes (BH) are fundamental objects in Gravitation, and in Physics, predicted classically in General Relativity (GR) with intriguing consequences in quantum physics.
- BHs turn out to have a finite, non-zero, temperature. Thus, they emit (in quantum theory) "Hawking" radiation with a perfect thermal spectrum.
- One can actually attach an entropy to the BH:

$$S_{BH} = \frac{kc^3}{\hbar G} \frac{A}{4}$$

Black Holes

- Black Holes (BH) are fundamental objects in Gravitation, and in Physics, predicted classically in General Relativity (GR) with intriguing consequences in quantum physics.
- BHs turn out to have a finite, non-zero, temperature. Thus, they emit (in quantum theory) "Hawking" radiation with a perfect thermal spectrum.
- One can actually attach an entropy to the BH:

$$S_{BH} = \frac{kc^3}{\hbar G} \frac{A}{4}$$

- This shows a very deep, fundamental, relation between Gravitation, Thermodynamics, Geometry and Quantum Theory.

Black Holes

- Black Holes (BH) are fundamental objects in Gravitation, and in Physics, predicted classically in General Relativity (GR) with intriguing consequences in quantum physics.
- BHs turn out to have a finite, non-zero, temperature. Thus, they emit (in quantum theory) "Hawking" radiation with a perfect thermal spectrum.
- One can actually attach an entropy to the BH:

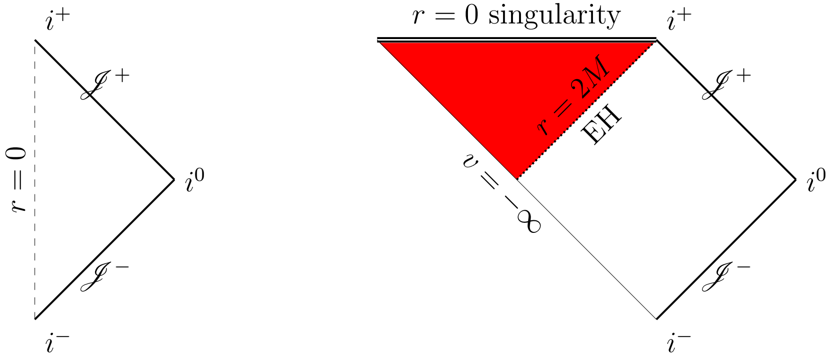
$$S_{BH} = \frac{kc^3}{\hbar G}\frac{A}{4}$$

- This shows a very deep, fundamental, relation between Gravitation, Thermodynamics, Geometry and Quantum Theory.
- But..., what do we mean by A?

Black Holes and the Event Horizon

- Classically, the characteristic feature of a BH is its event horizon EH: the boundary of the region from where one can send signals to infinity —one assumes infinity is well-defined.

The Event Horizon (EH)



Black Holes are teleological and too global

- The EH is commonly identified as the world-surface of stationary BHs, **and its area is** A (essentially entropy).

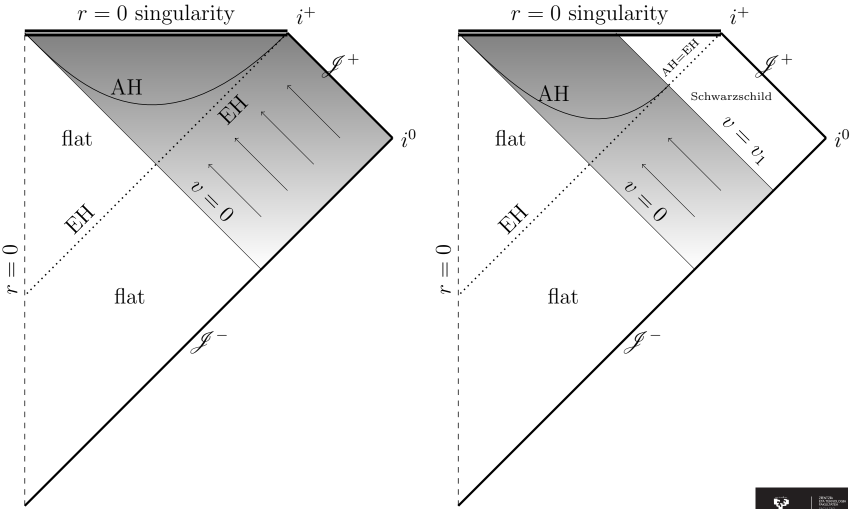
Black Holes are teleological and too global

- The EH is commonly identified as the world-surface of stationary BHs, **and its area is** A (essentially entropy).
- However, this leads to unsurmountable practical problems for dynamical BHs.

Black Holes are teleological and too global

- The EH is commonly identified as the world-surface of stationary BHs, **and its area is A** (essentially entropy).
- However, this leads to unsurmountable practical problems for dynamical BHs.
- EH depends on the *whole* future evolution of the spacetime. In fact, EHs can even start developing in flat regions of spacetime! EHs are *teleological*.

Teleology of the EH (observe central line!)



How to tell if something is a BH or not?

- How can one recognize, locally, the presence of a BH? For instance, in numerical GR. Or in Astrophysics: what do we mean by the sentence "there is a BH in the center of the Galaxy"?
- As a drastic example of the problems arising Hajicek (Phys. Rev. D 36 (1987) 1065) argued that the structure of the EH can be radically changed, and even fully destroyed, by changing the geometry of the spacetime in a Planck size neighbourhood.

Closed trapped surfaces

It is only natural to turn to closed trapped surfaces, the hallmark of gravitational collapse.

Trapped surfaces

The key idea in GR: trapped surfaces

- In 1965 Penrose introduced a new important concept: closed trapped surfaces.

The key idea in GR: trapped surfaces

- In 1965 Penrose introduced a new important concept: closed trapped surfaces.
- These are closed surfaces (usually topological spheres) such that their area tends to decrease locally along any possible *future* direction. (There is a dual definition to the past).

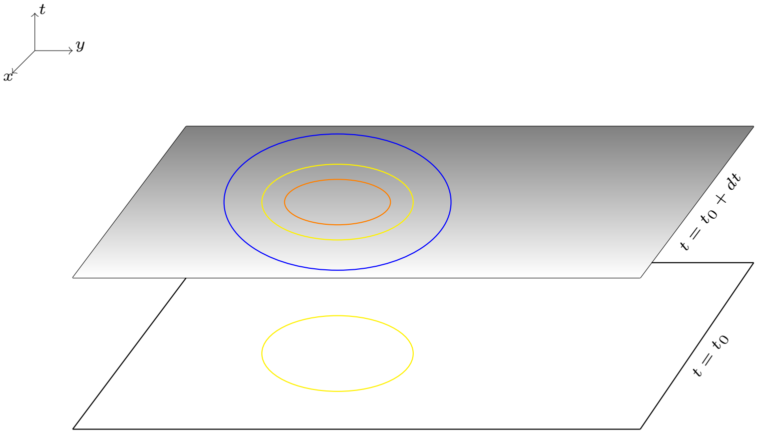
The key idea in GR: trapped surfaces

- In 1965 Penrose introduced a new important concept: closed trapped surfaces.
- These are closed surfaces (usually topological spheres) such that their area tends to decrease locally along any possible *future* direction. (There is a dual definition to the past).
- In the limit case where the area just does not increase (maybe staying the same along some future direction), the surface is called marginally trapped.

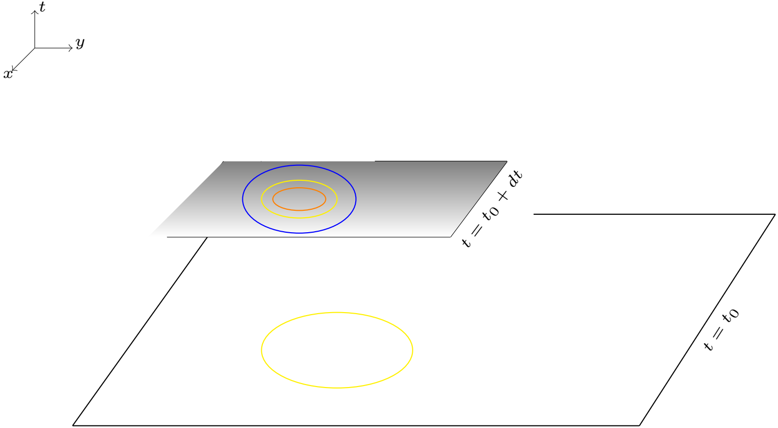
The key idea in GR: trapped surfaces

- In 1965 Penrose introduced a new important concept: closed trapped surfaces.
- These are closed surfaces (usually topological spheres) such that their area tends to decrease locally along any possible *future* direction. (There is a dual definition to the past).
- In the limit case where the area just does not increase (maybe staying the same along some future direction), the surface is called marginally trapped.
- The traditional stationary Black Hole solutions have closed trapped surfaces in the region inside the Event Horizon, and only there. And the EH is foliated by marginally trapped surfaces.

“Normal situation”



Possible trapping in contracting worlds



Marginally trapped tubes

An obvious alternative to EHs: Marginally trapped tubes

- Using closed trapped surfaces, the case has been made for quasi-local objects that might resolve the mentioned problems. These are the Marginally Trapped Tubes (MTT)

An obvious alternative to EHs: Marginally trapped tubes

- Using closed trapped surfaces, the case has been made for quasi-local objects that might resolve the mentioned problems. These are the Marginally Trapped Tubes (MTT)
- MTTs are hypersurfaces foliated by closed (compact without boundary) marginally trapped surfaces. If the MTT is spacelike, it is termed dynamical or trapping horizon.

An obvious alternative to EHs: Marginally trapped tubes

- Using closed trapped surfaces, the case has been made for quasi-local objects that might resolve the mentioned problems. These are the Marginally Trapped Tubes (MTT)
- MTTs are hypersurfaces foliated by closed (compact without boundary) marginally trapped surfaces. If the MTT is spacelike, it is termed dynamical or trapping horizon.
- It is widely believed that closed (marginally) trapped surfaces are the single most important ingredient in gravitational collapse, and in the formation of BHs. Thus the idea of using MTTs (or DHs) as a viable replacement for the EH looked very promising.

An obvious alternative to EHs: Marginally trapped tubes

- Using closed trapped surfaces, the case has been made for quasi-local objects that might resolve the mentioned problems. These are the Marginally Trapped Tubes (MTT)
- MTTs are hypersurfaces foliated by closed (compact without boundary) marginally trapped surfaces. If the MTT is spacelike, it is termed dynamical or trapping horizon.
- It is widely believed that closed (marginally) trapped surfaces are the single most important ingredient in gravitational collapse, and in the formation of BHs. Thus the idea of using MTTs (or DHs) as a viable replacement for the EH looked very promising.
- Actually, MTTs satisfy laws of thermodynamics similar to those of EH. In particular, their area ($\rightarrow$ entropy) grows during the collapse, and decreases with Hawking radiation.

The case of spherically symmetric spacetimes

- $ds^2 = g_{ab}(x^c)dx^a dx^b + r^2(x^c)d\Omega^2 \qquad (a, b, \cdots = 0, 1)$
  Here $d\Omega^2$ is the round metric on the 2-sphere and $\det g_{ab} < 0$.

The case of spherically symmetric spacetimes

- $ds^2 = g_{ab}(x^c)dx^a dx^b + r^2(x^c)d\Omega^2 \qquad (a, b, \cdots = 0, 1)$
  Here $d\Omega^2$ is the round metric on the 2-sphere and $\det g_{ab} < 0$.
- Thus, the round spheres (orbits of the $SO(3)$ symmetry group, defined by x^c =consts.) have area $4\pi r^2(x^c)$

The case of spherically symmetric spacetimes

- $ds^2 = g_{ab}(x^c)dx^a dx^b + r^2(x^c)d\Omega^2 \qquad (a, b, \dots = 0, 1)$
  Here $d\Omega^2$ is the round metric on the 2-sphere and $\det g_{ab} < 0$.
- Thus, the round spheres (orbits of the $SO(3)$ symmetry group, defined by x^c =consts.) have area $4\pi r^2(x^c)$
- These round spheres are trapped iff $r < 2m$, and marginally trapped iff $r = 2m$, where m is the mass function:

$$2m \equiv r\left(1 - \nabla_\mu r \nabla^\mu r\right)$$

The case of spherically symmetric spacetimes

- $ds^2 = g_{ab}(x^c)dx^a dx^b + r^2(x^c)d\Omega^2 \qquad (a, b, \cdots = 0, 1)$
  Here $d\Omega^2$ is the round metric on the 2-sphere and $\det g_{ab} < 0$.
- Thus, the round spheres (orbits of the $SO(3)$ symmetry group, defined by $x^c =$consts.) have area $4\pi r^2(x^c)$
- These round spheres are trapped iff $r < 2m$, and marginally trapped iff $r = 2m$, where m is the mass function:

$$2m \equiv r\left(1 - \nabla_\mu r \nabla^\mu r\right)$$

- Thus, there is a MTT at $\{r = 2m\}$ (equivalently, where the gradient of r is null). We call it A3H.

The case of spherically symmetric spacetimes

- $ds^2 = g_{ab}(x^c)dx^a dx^b + r^2(x^c)d\Omega^2 \qquad (a, b, \cdots = 0, 1)$
  Here $d\Omega^2$ is the round metric on the 2-sphere and $\det g_{ab} < 0$.
- Thus, the round spheres (orbits of the $SO(3)$ symmetry group, defined by x^c =consts.) have area $4\pi r^2(x^c)$
- These round spheres are trapped iff $r < 2m$, and marginally trapped iff $r = 2m$, where m is the mass function:

$$2m \equiv r\left(1 - \nabla_\mu r \nabla^\mu r\right)$$

- Thus, there is a MTT at $\{r = 2m\}$ (equivalently, where the gradient of r is null). We call it A3H.
- A3H can be timelike, null or spacelike depending on the sign of

$$g^{ab} m_{,a} \left(r_{,b} - m_{,b}\right)\Big|_{A3H}$$

The case of spherically symmetric spacetimes

- $ds^2 = g_{ab}(x^c)dx^a dx^b + r^2(x^c)d\Omega^2 \qquad (a, b, \cdots = 0, 1)$
  Here $d\Omega^2$ is the round metric on the 2-sphere and $\det g_{ab} < 0$.
- Thus, the round spheres (orbits of the $SO(3)$ symmetry group, defined by x^c =consts.) have area $4\pi r^2(x^c)$
- These round spheres are trapped iff $r < 2m$, and marginally trapped iff $r = 2m$, where m is the mass function:

$$2m \equiv r\left(1 - \nabla_\mu r \nabla^\mu r\right)$$

- Thus, there is a MTT at $\{r = 2m\}$ (equivalently, where the gradient of r is null). We call it A3H.
- A3H can be timelike, null or spacelike depending on the sign of

$$g^{ab} m_{,a}\left(r_{,b} - m_{,b}\right)\Big|_{A3H}$$

- Wherever $dm \wedge dr = 0$, A3H is null and a portion of an isolated horizon: A3H^{iso}

The case of spherically symmetric spacetimes

- $ds^2 = g_{ab}(x^c)dx^a dx^b + r^2(x^c)d\Omega^2 \qquad (a, b, \dots = 0, 1)$
  Here $d\Omega^2$ is the round metric on the 2-sphere and $\det g_{ab} < 0$.
- Thus, the round spheres (orbits of the $SO(3)$ symmetry group, defined by x^c =consts.) have area $4\pi r^2(x^c)$
- These round spheres are trapped iff $r < 2m$, and marginally trapped iff $r = 2m$, where m is the mass function:

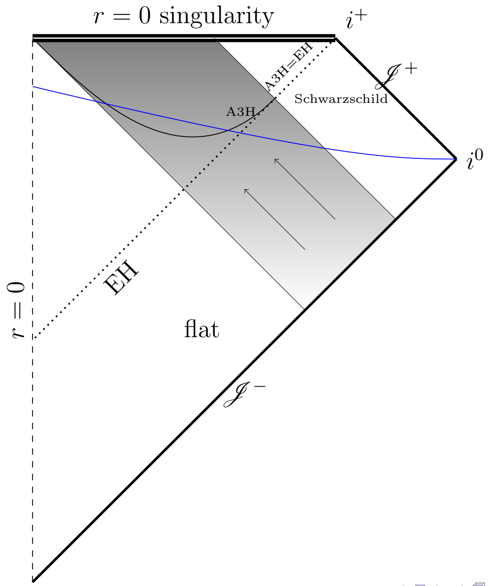
$$2m \equiv r\left(1 - \nabla_\mu r \nabla^\mu r\right)$$

- Thus, there is a MTT at $\{r = 2m\}$ (equivalently, where the gradient of r is null). We call it A3H.
- A3H can be timelike, null or spacelike depending on the sign of

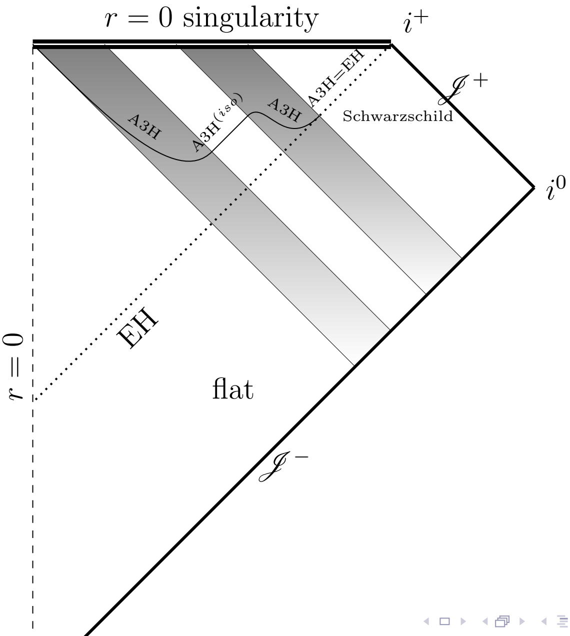
$$g^{ab} m_{,a} \left(r_{,b} - m_{,b}\right)\Big|_{A3H}$$

- Wherever $dm \wedge dr = 0$, A3H is null and a portion of an isolated horizon: A3H^{iso}
- In Schwarzschild, EH $\equiv$ A3H $\equiv$ A3H^{iso}

Example: The spherically symmetric MTT (A3H)



Case with a portion of Isolated Horizon A3H$^{(iso)}$



The spherically symmetric MTT: A3H

- One can prove (Bengtsson & JMMS, PRD **83** (2011) 044012) that

The spherically symmetric MTT: A3H

- One can prove (Bengtsson & JMMS, PRD **83** (2011) 044012) that
  1. A3H is actually the only spherically symmetric MTT : the only spherically symmetric hypersurface foliated by MTSs —be they round spheres or not. (This is with respect to the given $SO(3)$ group).

The spherically symmetric MTT: A3H

- One can prove (Bengtsson & JMMS, PRD **83** (2011) 044012) that
  1. A3H is actually the only spherically symmetric MTT : the only spherically symmetric hypersurface foliated by MTSs —be they round spheres or not. (This is with respect to the given $SO(3)$ group).
  2. Any closed trapped surface cannot be fully contained in a region with $r > 2m$.

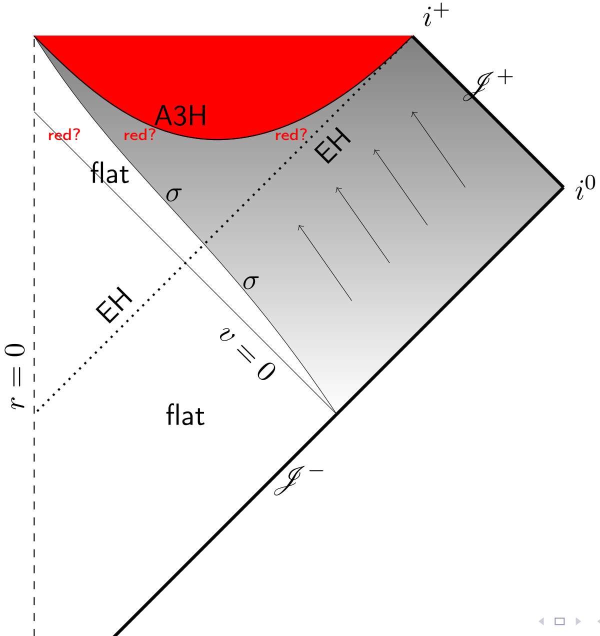
The spherically symmetric MTT: A3H

- One can prove (Bengtsson & JMMS, PRD 83 (2011) 044012) that
  1. A3H is actually the only spherically symmetric MTT : the only spherically symmetric hypersurface foliated by MTSs —be they round spheres or not. (This is with respect to the given $SO(3)$ group).
  2. Any closed trapped surface cannot be fully contained in a region with $r > 2m$.
  3. Thus, all possible closed trapped surfaces must intersect the region with $r < 2m$.

The spherically symmetric MTT: A3H

- One can prove (Bengtsson & JMMS, PRD 83 (2011) 044012) that
  1. A3H is actually the only spherically symmetric MTT : the only spherically symmetric hypersurface foliated by MTSs —be they round spheres or not. (This is with respect to the given $SO(3)$ group).
  2. Any closed trapped surface cannot be fully contained in a region with $r > 2m$.
  3. Thus, all possible closed trapped surfaces must intersect the region with $r < 2m$.
- But, can closed trapped surfaces —other than round spheres— penetrate outside A3H? How far can they go?

A dynamical situation



A3H does not bound general trapped surfaces

- By doing perturbations on the A3H, one can also prove (Bengtsson & JMMS, PRD **83** (2011) 044012) that

A3H does not bound general trapped surfaces

- By doing perturbations on the A3H, one can also prove (Bengtsson & JMMS, PRD 83 (2011) 044012) that
  1. There are trapped surfaces (topological spheres) penetrating both sides of A3H$\setminus$A3H^{iso}

A3H does not bound general trapped surfaces

- By doing perturbations on the A3H, one can also prove (Bengtsson & JMMS, PRD **83** (2011) 044012) that
  1. There are trapped surfaces (topological spheres) penetrating both sides of A3H$\setminus$A3H^{iso}
  2. There exist MTTs penetrating both sides of A3H$\setminus$A3H^{iso}

A3H does not bound general trapped surfaces

- By doing perturbations on the A3H, one can also prove (Bengtsson & JMMS, PRD 83 (2011) 044012) that
  1. There are trapped surfaces (topological spheres) penetrating both sides of A3H$\setminus$A3H^{iso}
  2. There exist MTTs penetrating both sides of A3H$\setminus$A3H^{iso}
- How far can these closed trapped surfaces go away from A3H?

MTTs have problems of their own!

- Hence (and unfortunately) MTTs have important problems: first of all, they are highly non-unique, and they interweave each other in complicated ways (A Ashtekar and G Galloway, Adv. Theor. Math. Phys. **9** (2005) 1-30)

MTTs have problems of their own!

- Hence (and unfortunately) MTTs have important problems: first of all, they are highly non-unique, and they interweave each other in complicated ways (A Ashtekar and G Galloway, Adv. Theor. Math. Phys. **9** (2005) 1-30)
- I am going to show that they also suffer from problems similar to that of EHs: they are too much global.

MTTs have problems of their own!

- Hence (and unfortunately) MTTs have important problems: first of all, they are highly non-unique, and they interweave each other in complicated ways (A Ashtekar and G Galloway, Adv. Theor. Math. Phys. **9** (2005) 1-30)
- I am going to show that they also suffer from problems similar to that of EHs: they are too much global.
- This is linked to a fundamental property of closed trapped surfaces: they are clairvoyant.

MTTs have problems of their own!

- Hence (and unfortunately) MTTs have important problems: first of all, they are highly non-unique, and they interweave each other in complicated ways (A Ashtekar and G Galloway, Adv. Theor. Math. Phys. **9** (2005) 1-30)
- I am going to show that they also suffer from problems similar to that of EHs: they are too much global.
- This is linked to a fundamental property of closed trapped surfaces: they are clairvoyant.
- Can closed trapped surfaces penetrate outside *all* MTTs? Or are they confined by (at least one of) them?

MTTs have problems of their own!

- Hence (and unfortunately) MTTs have important problems: first of all, they are highly non-unique, and they interweave each other in complicated ways (A Ashtekar and G Galloway, Adv. Theor. Math. Phys. **9** (2005) 1-30)
- I am going to show that they also suffer from problems similar to that of EHs: they are too much global.
- This is linked to a fundamental property of closed trapped surfaces: they are clairvoyant.
- Can closed trapped surfaces penetrate outside *all* MTTs? Or are they confined by (at least one of) them?
- Can closed trapped surfaces actually penetrate into flat regions?

MTTs have problems of their own!

- Hence (and unfortunately) MTTs have important problems: first of all, they are highly non-unique, and they interweave each other in complicated ways (A Ashtekar and G Galloway, Adv. Theor. Math. Phys. **9** (2005) 1-30)
- I am going to show that they also suffer from problems similar to that of EHs: they are too much global.
- This is linked to a fundamental property of closed trapped surfaces: they are clairvoyant.
- Can closed trapped surfaces penetrate outside *all* MTTs? Or are they confined by (at least one of) them?
- Can closed trapped surfaces actually penetrate into flat regions?
- Where can there be closed trapped surfaces?

Trapped surfaces behave wildly

Trapped surfaces even penetrate flat portions!

Example: imploding Vaidya spacetime

Vaidya

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

Trapped surfaces even penetrate flat portions!

Example: imploding Vaidya spacetime

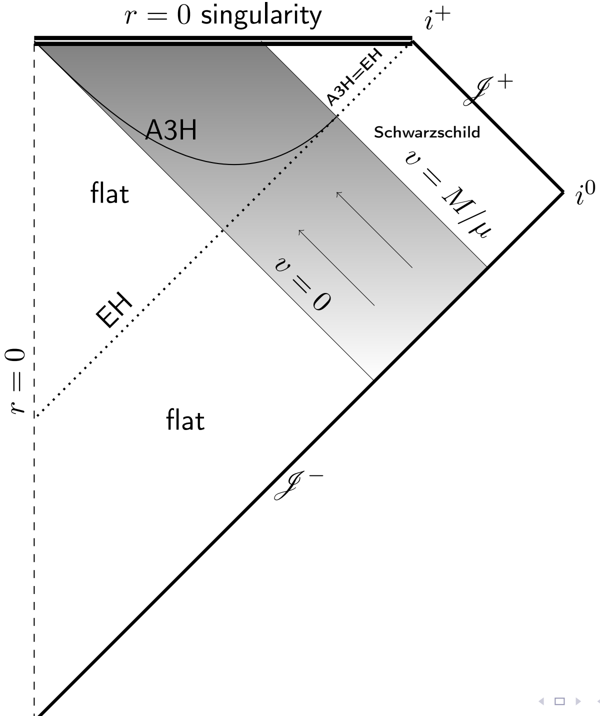
Vaidya

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

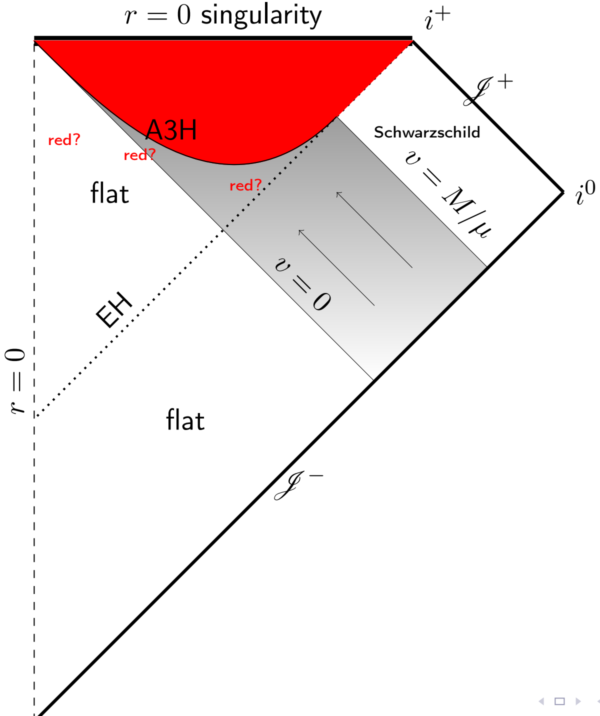
Consider the following simple mass function

$$m(v) = \begin{cases} 0 & v < 0 \\ \mu v & 0 \leq v \leq M/\mu \\ M & v > \mu \end{cases}$$

Thus, this is flat for $v < 0$, it ends in a Schwarzschild region with mass M ($v > M/\mu$), and it is self-similar in the intermediate Vaidya region for $0 < v < M/\mu$.



$r = 0$ singularity
i^+
A3H=EH
$\mathscr{I}^+$
A3H
Schwarzschild
$v = M/\mu$
i^0
flat
$v = 0$
EH
$r = 0$
flat
$\mathscr{I}^-$


$r = 0$ singularity
i^+
$\mathscr{I}^+$
Schwarzschild
A3H
red?
red?
red?
flat
i^0
$v = M/\mu$
$v = 0$
EH
$r = 0$
flat
$\mathscr{I}^-$

A closed trapped surface penetrating outside A3H and into the flat region!

We constructed, analytically and explicitly, examples of closed future-trapped surfaces going far away from $r \leq 2m$ and entering well inside the flat region in the (self-similar) Vaidya spacetime.

The closed future-trapped surface is composed of:

The closed future-trapped surface is composed of:

- Flat region: a topological disk on an equatorial plane

The closed future-trapped surface is composed of:

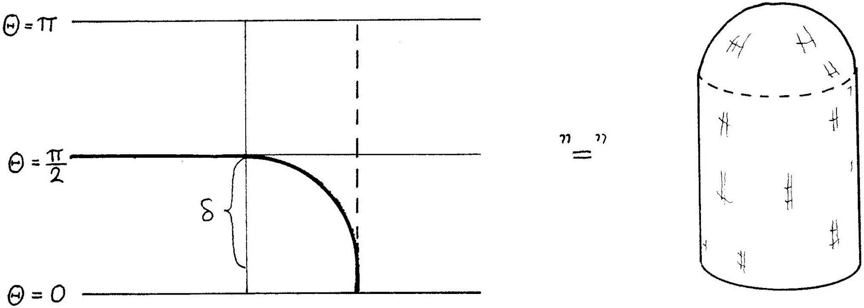
- Flat region: a topological disk on an equatorial plane
- Vaidya region: a topological cylinder on that equatorial plane

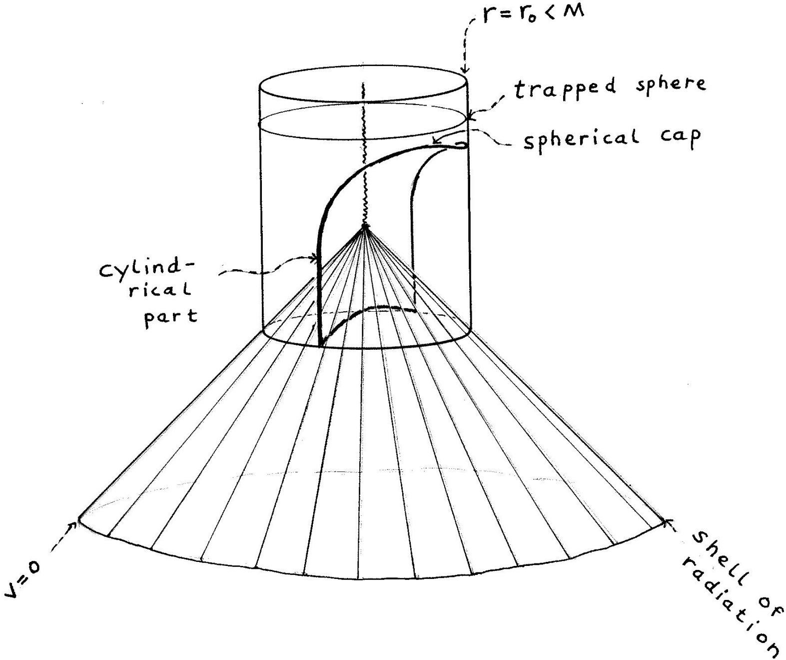
The closed future-trapped surface is composed of:

- Flat region: a topological disk on an equatorial plane
- Vaidya region: a topological cylinder on that equatorial plane
- Schwarzschild region: another disk composed of two parts
  - a cylinder on that equatorial plane with $r = \gamma M$

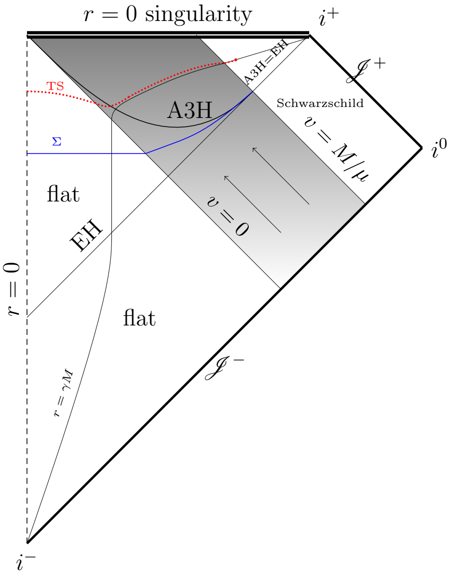
The closed future-trapped surface is composed of:

- Flat region: a topological disk on an equatorial plane
- Vaidya region: a topological cylinder on that equatorial plane
- Schwarzschild region: another disk composed of two parts
  - a cylinder on that equatorial plane with $r = \gamma M$
  - a final "capping" disk defined on $r = \gamma M$
- Here $\gamma < 0.68514$ is a constant, and $\mu > \frac{1}{4\gamma}$ is required.


Θ = π
Θ = π/2
Θ = 0
δ
"="


$r = r_0 < M$
trapped sphere
spherical cap
cylind-
rical
part
v=0
shell of
radiation

Bengtsson & Senovilla , Phys. Rev. D 79 (2009) 024027



Closed trapped surfaces are clairvoyant

Closed trapped surfaces may intersect the flat region

They do enter into the flat region (if the mass function rises fast enough).

Closed trapped surfaces are clairvoyant

Closed trapped surfaces may intersect the flat region

They do enter into the flat region (if the mass function rises fast enough).

Closed trapped surfaces are highly non-local

They can have portions in a flat region of spacetime whose *whole* past is also flat in clairvoyance of energy that crosses them elsewhere to make their compactness feasible.

The boundary $\mathscr{B}$ of the region containing closed trapped surfaces?

$\mathscr{B}$ is not an MTT!

- One could have hoped that the boundary of the region containing closed future-trapped surfaces has something to do with the surface of the black hole.

$\mathscr{B}$ is not an MTT!

- One could have hoped that the boundary of the region containing closed future-trapped surfaces has something to do with the surface of the black hole.
- However, the previous results imply that

$\mathscr{B}$ is not an MTT!

- One could have hoped that the boundary of the region containing closed future-trapped surfaces has something to do with the surface of the black hole.
- However, the previous results imply that
  1. Such a boundary $\mathscr{B}$ can penetrate flat portions of the spacetime

$\mathscr{B}$ is not an MTT!

- One could have hoped that the boundary of the region containing closed future-trapped surfaces has something to do with the surface of the black hole.
- However, the previous results imply that
    1. Such a boundary $\mathscr{B}$ can penetrate flat portions of the spacetime
    2. In spherical symmetry, $\mathscr{B}$ is contained in the region $r \geq 2m$, and actually in the region $r > 2m$ if it is not $A3H^{iso}$.

$\mathscr{B}$ is not an MTT!

- One could have hoped that the boundary of the region containing closed future-trapped surfaces has something to do with the surface of the black hole.
- However, the previous results imply that
  1. Such a boundary $\mathscr{B}$ can penetrate flat portions of the spacetime
  2. In spherical symmetry, $\mathscr{B}$ is contained in the region $r \geq 2m$, and actually in the region $r > 2m$ if it is not $A3H^{iso}$.
  3. Therefore, $\mathscr{B}$ cannot be an MTT in general (because there are no marginal trapped surfaces fully contained in the region $r > 2m$ where $\mathscr{B}$ is confined!).

$\mathscr{B}$ is not an MTT!

- One could have hoped that the boundary of the region containing closed future-trapped surfaces has something to do with the surface of the black hole.
- However, the previous results imply that
  1. Such a boundary $\mathscr{B}$ can penetrate flat portions of the spacetime
  2. In spherical symmetry, $\mathscr{B}$ is contained in the region $r \geq 2m$, and actually in the region $r > 2m$ if it is not $A3H^{iso}$.
  3. Therefore, $\mathscr{B}$ cannot be an MTT in general (because there are no marginal trapped surfaces fully contained in the region $r > 2m$ where $\mathscr{B}$ is confined!).
- This invalidates a claimed "theorem" by Hayward (PRD 49 (1994) 6467)

$\mathscr{B}$ is not an MTT!

- One could have hoped that the boundary of the region containing closed future-trapped surfaces has something to do with the surface of the black hole.
- However, the previous results imply that
  1. Such a boundary $\mathscr{B}$ can penetrate flat portions of the spacetime
  2. In spherical symmetry, $\mathscr{B}$ is contained in the region $r \geq 2m$, and actually in the region $r > 2m$ if it is not $A3H^{iso}$.
  3. Therefore, $\mathscr{B}$ cannot be an MTT in general (because there are no marginal trapped surfaces fully contained in the region $r > 2m$ where $\mathscr{B}$ is confined!).
- This invalidates a claimed "theorem" by Hayward (PRD 49 (1994) 6467)
- In general, one does not know where is $\mathscr{B}$, not even for spherical symmetry !!

Black Hole Cores

The core of the trapped region

- Going back to MTTs, we have put forward a novel strategy in order to try and find a possible unique MTT.

The core of the trapped region

- Going back to MTTs, we have put forward a novel strategy in order to try and find a possible unique MTT.
- The idea is based on the simple question:
  what part of the spacetime is absolutely indispensable for the existence of the black hole?

The core of the trapped region

- Going back to MTTs, we have put forward a novel strategy in order to try and find a possible unique MTT.
- The idea is based on the simple question:
  what part of the spacetime is absolutely indispensable for the existence of the black hole?
- Surely enough, any flat region is certainly not essential for the existence of the black hole.

The core of the trapped region

- Going back to MTTs, we have put forward a novel strategy in order to try and find a possible unique MTT.
- The idea is based on the simple question:
  what part of the spacetime is absolutely indispensable for the existence of the black hole?
- Surely enough, any flat region is certainly not essential for the existence of the black hole.
- What is?

Definition of Core

Definition

A region $\mathscr{Z}$ is called the core of the region with closed future-trapped surfaces if it is a minimal closed connected set that needs to be removed from the spacetime in order to get rid of all such closed future-trapped surfaces, [and such that any point on the boundary $\partial\mathscr{Z}$ is connected to $\mathscr{B}$ in the closure of the remainder].

Definition of Core

Definition

A region $\mathscr{Z}$ is called the core of the region with closed future-trapped surfaces if it is a minimal closed connected set that needs to be removed from the spacetime in order to get rid of all such closed future-trapped surfaces, [and such that any point on the boundary $\partial\mathscr{Z}$ is connected to $\mathscr{B}$ in the closure of the remainder].

- Here, "minimal" means that there is no other set $\mathscr{Z}'$ with the same properties and properly contained in $\mathscr{Z}$.

Definition of Core

Definition

A region $\mathscr{Z}$ is called the core of the region with closed future-trapped surfaces if it is a minimal closed connected set that needs to be removed from the spacetime in order to get rid of all such closed future-trapped surfaces, [and such that any point on the boundary $\partial\mathscr{Z}$ is connected to $\mathscr{B}$ in the closure of the remainder].

- Here, "minimal" means that there is no other set $\mathscr{Z}'$ with the same properties and properly contained in $\mathscr{Z}$.
- The final technical condition is needed because one could identify a particular removable region to eliminate the future-trapped surfaces, excise it, but then put back a tiny but central isolated portion to make it smaller. However, this is not what one wants to cover with the definition.

A surprising theorem!!

- We already know that closed future TSs must intersect the region $\{r < 2m\}$.

A surprising theorem!!

- We already know that closed future TSs must intersect the region $\{r < 2m\}$.
- However, closed future-trapped surfaces may lie on $\{r \geq 2m\}$ almost completely.

A surprising theorem!!

- We already know that closed future TSs must intersect the region $\{r < 2m\}$.
- However, closed future-trapped surfaces may lie on $\{r \geq 2m\}$ almost completely.
- More precisely:

A surprising theorem!!

- We already know that closed future TSs must intersect the region $\{r < 2m\}$.
- However, closed future-trapped surfaces may lie on $\{r \geq 2m\}$ almost completely.
- More precisely:

Theorem (Bengtsson and JMMS, PRD 83 (2011) 044012)

In spherically symmetric spacetimes, there are closed future-trapped surfaces (topological spheres) penetrating both sides of the apparent 3-horizon $A3H \setminus A3H^{iso}$ with arbitrarily small portions inside the region $\{r < 2m\}$.

Cores in spherical symmetry

From this surprising theorem one derives:

Result

The region $\mathscr{Z} \equiv \{r \leq 2m(v,r)\}$ *is a core.*

Cores in spherical symmetry

From this surprising theorem one derives:

Result

The region $\mathscr{Z} \equiv \{r \leq 2m(v,r)\}$ *is a core.*

Result

In spherically symmetric spacetimes, $\mathscr{Z} = \{r \leq 2m\}$ *are the only spherically symmetric cores of* $\mathscr{T}$. *Therefore,* $\partial\mathscr{Z} =$ *A3H are the only spherically symmetric boundaries of a core.*

An important remark

It should be observed that the concept of core is global, and requires full knowledge of the future.

An important remark

It should be observed that the concept of core is global, and requires full knowledge of the future.

However, A3H is quasilocal and can be defined and identified by observing just around it.

An important remark

It should be observed that the concept of core is global, and requires full knowledge of the future.

However, A3H is quasilocal and can be defined and identified by observing just around it.

It is thus surprising, and perhaps with a deep meaning, that A3H$= \partial \mathscr{Z}$ can happen

Non-spherically symmetric cores

Proposition

There exist non-spherically symmetric cores of the future-trapped region in spherically symmetric spacetimes.

Non-spherically symmetric cores

Proposition

There exist non-spherically symmetric cores of the future-trapped region in spherically symmetric spacetimes.

- Still, the identified core $\mathscr{Z} = \{r \leq 2m\}$ might be unique in the sense that its boundary $\partial\mathscr{Z} =$ A3H is a marginally trapped tube.

Non-spherically symmetric cores

Proposition

There exist non-spherically symmetric cores of the future-trapped region in spherically symmetric spacetimes.

- Still, the identified core $\mathscr{Z} = \{r \leq 2m\}$ might be unique in the sense that its boundary $\partial\mathscr{Z} =$ A3H is a marginally trapped tube.
- This would happen if any MTT H other than A3H is such that its causal future $J^+(H)$ is *not* a core —the core being a proper subset of $J^+(H)$.

Non-spherically symmetric cores

Proposition

There exist non-spherically symmetric cores of the future-trapped region in spherically symmetric spacetimes.

- Still, the identified core $\mathscr{Z} = \{r \leq 2m\}$ might be unique in the sense that its boundary $\partial\mathscr{Z} =$ A3H is a marginally trapped tube.
- This would happen if any MTT H other than A3H is such that its causal future $J^+(H)$ is *not* a core —the core being a proper subset of $J^+(H)$.
- Then A3H would be selected as the unique MTT which is the boundary of a core of the future-trapped region $\mathscr{T}$.

Non-spherically symmetric cores

Proposition

There exist non-spherically symmetric cores of the future-trapped region in spherically symmetric spacetimes.

- Still, the identified core $\mathscr{Z} = \{r \leq 2m\}$ might be unique in the sense that its boundary $\partial\mathscr{Z} =$ A3H is a marginally trapped tube.
- This would happen if any MTT H other than A3H is such that its causal future $J^+(H)$ is *not* a core —the core being a proper subset of $J^+(H)$.
- Then A3H would be selected as the unique MTT which is the boundary of a core of the future-trapped region $\mathscr{T}$.
- And this could serve as a general definition, for non-spherically symmetric situations.

Non-spherically symmetric cores

Proposition

There exist non-spherically symmetric cores of the future-trapped region in spherically symmetric spacetimes.

- Still, the identified core $\mathscr{Z} = \{r \leq 2m\}$ might be unique in the sense that its boundary $\partial\mathscr{Z} =$ A3H is a marginally trapped tube.
- This would happen if any MTT H other than A3H is such that its causal future $J^+(H)$ is *not* a core —the core being a proper subset of $J^+(H)$.
- Then A3H would be selected as the unique MTT which is the boundary of a core of the future-trapped region $\mathscr{T}$.
- And this could serve as a general definition, for non-spherically symmetric situations.
- However, it may also happen that all boundaries of cores are MTTs...

Thanks!

Grazie!

Thank you very much!

References.

- I Bengtsson and JMMS, *Note on trapped surfaces in the Vaidya solution*, Phys. Rev. D **79** 024027 (2009) [arXiv:0809.2213]
- JMMS, *On the boundary of the region containing trapped surfaces* AIP Conf. Proc. **1122** 72 (2009) [arXiv:0812.2767]
- J Åman, I Bengtsson and JMMS *Where are the trapped surfaces?* J. Phys.: Conf. Ser. **229** (2010) 012021 [arXiv:0912.3691]
- I Bengtsson and JMMS, *The region with trapped surfaces in spherical symmetry, its core, and their boundaries* Phys. Rev. D **83** 044012 (2011) [arXiv:1009.0225]
- *Black Holes. New Horizons* edited by S.A. Hayward, World Scientific (2013)
- I Bengtsson, E Jakobsson and JMMS *Trapped surfaces in Oppenheimer-Snyder black holes* Phys. Rev. D **88** (2013) 064012 [arXiv:1306.6486]