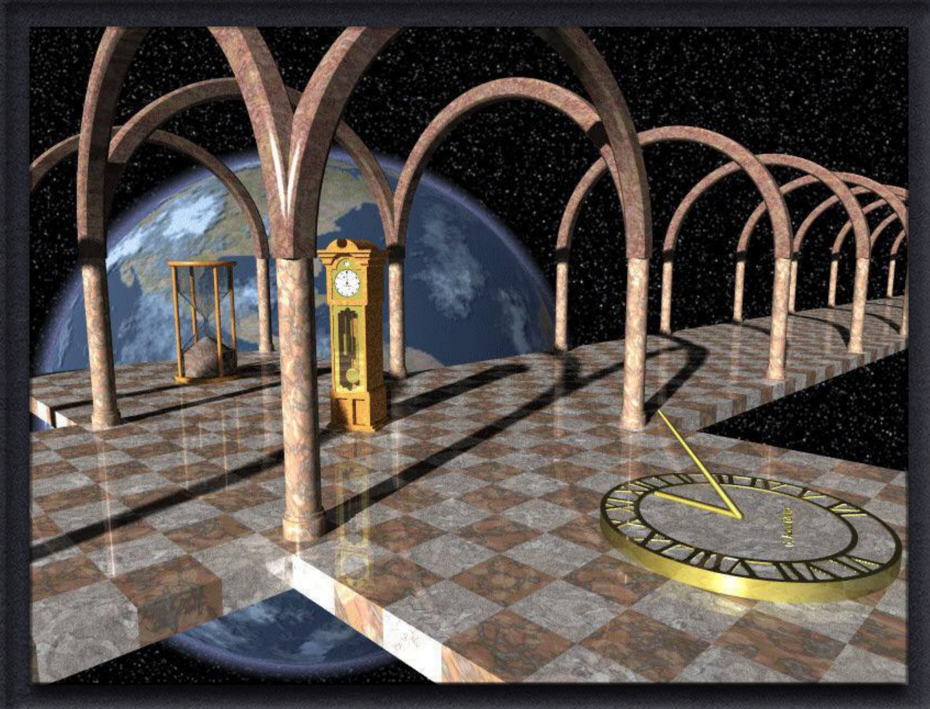


PROBING FASTER THAN LIGHT TRAVEL AND CHRONOLOGY PROTECTION WITH SUPERLUMINAL WARP DRIVES

Stefano Liberati

ifpU
INFN Istituto Nazionale di Fisica Nucleare
SISSA

S. Finazzi, SL, C. Barcelò,
Phys.Rev. D79 (2009) 124017

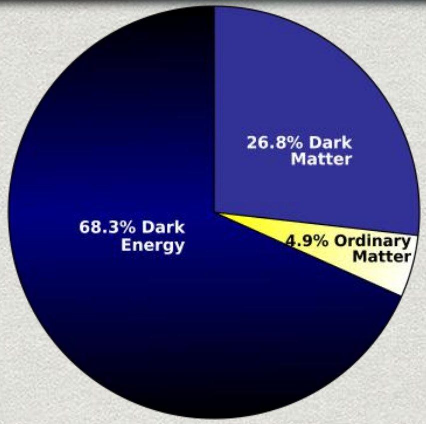
A. Coutant, S. Finazzi, S.L and R. Parentani,
Phys.Rev. D85 (2012) 064020)

It from the cracks that light gets in…

Leonard Cohen

Albeit we "use" GR everyday (e.g. GPS) still it has some tantalising features and it has resisted so far any attempt to be quantised…

- Singularities
- Critical phenomena in gravitational collapse
- Horizon thermodynamics
- Spacetime thermodynamics: Einstein equations as equations of state.
- The cosmological constant problem
- Faster than light and Time travel solutions
- AdS/CFT duality, holographic behaviour
- Gravity/fluid duality
- Information Problem in BH Physics
- The problem of Time



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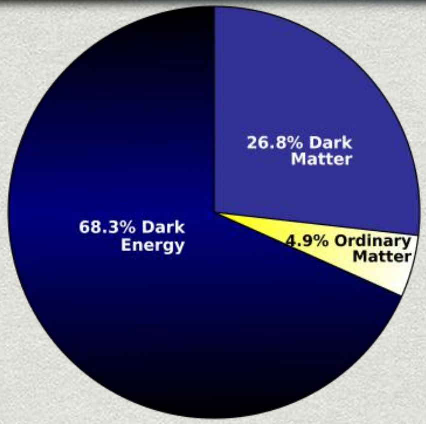


TIME MACHINES IN A NUT SHELL

As long as no one asks me [what time is] then I know it,
but if someone asks me to explain it, I don't know it.
Saint Augustine.

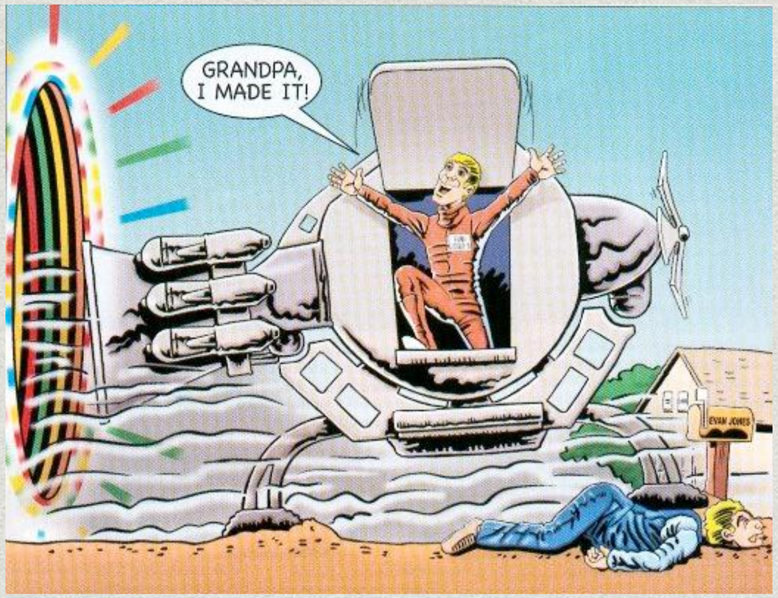
What is a Time Machine?

- Time is a local and observer dependent object in Einstein Relativity
- It is very easy to travel forward in time: e.g. go close to a black hole and then come back home…
- Much different is to go backward in time… we say that there is time machine if in some region of a spacetime it is possible to do so.

More rigorously (the boring professor version 🤓)

- One has a time machine when there is a chronology violating region in spacetime.
- A chronology violating region (CVR) is a region where closed timelike curves (CTC) are present.
- An event is in a chronology violating region if the intersection of his chronological past and future is not empty. $I^0(p)\equiv I^+(p)\cap I^-(p)\neq\varnothing$
- The total chronology violating region of a spacetime M is the union of all the chronology violating regions for all the events $I^0(M)\equiv\bigcup I^0(p)$ for all $p\in M$.
- The future chronological horizon is defined as the boundary of the chronological future of the chronology violating region $H^+(I)\equiv\partial[I^+(I^0(M))]$
  - Stationary chronological horizons are generated by null geodesics
  - They normally coincide also with causal horizons
  - They are a special case of Cauchy horizons, hence spacetime with a CVR is not globally hyperbolic

Why physicists do not like time travel…



Not so nice paradoxes

- Grandpa paradox
- Bootstrap paradox

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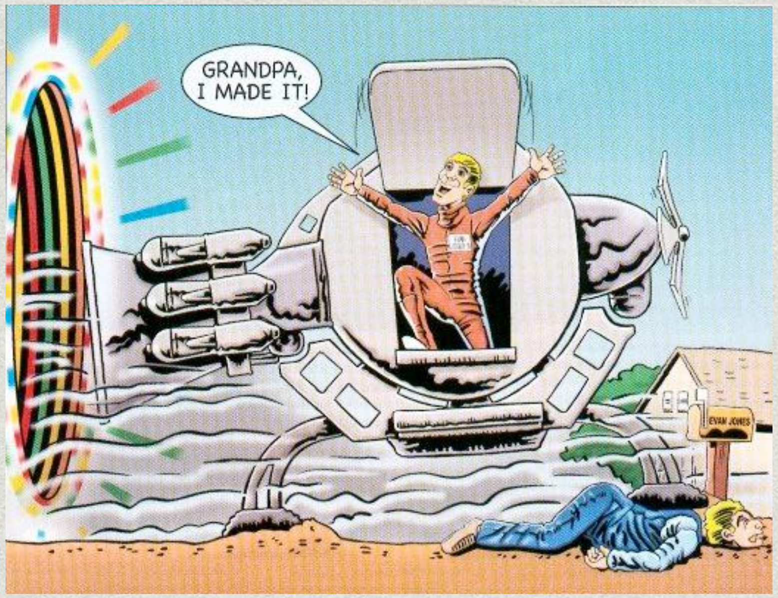

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Proposed solutions

Avoid a block universe

QG enforces classical spacetime to be always of the form $R \times \Sigma^3$

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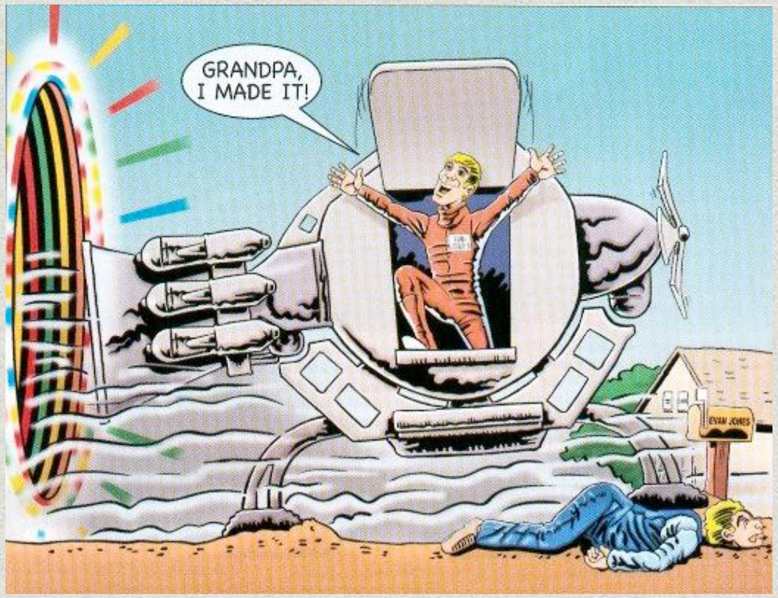

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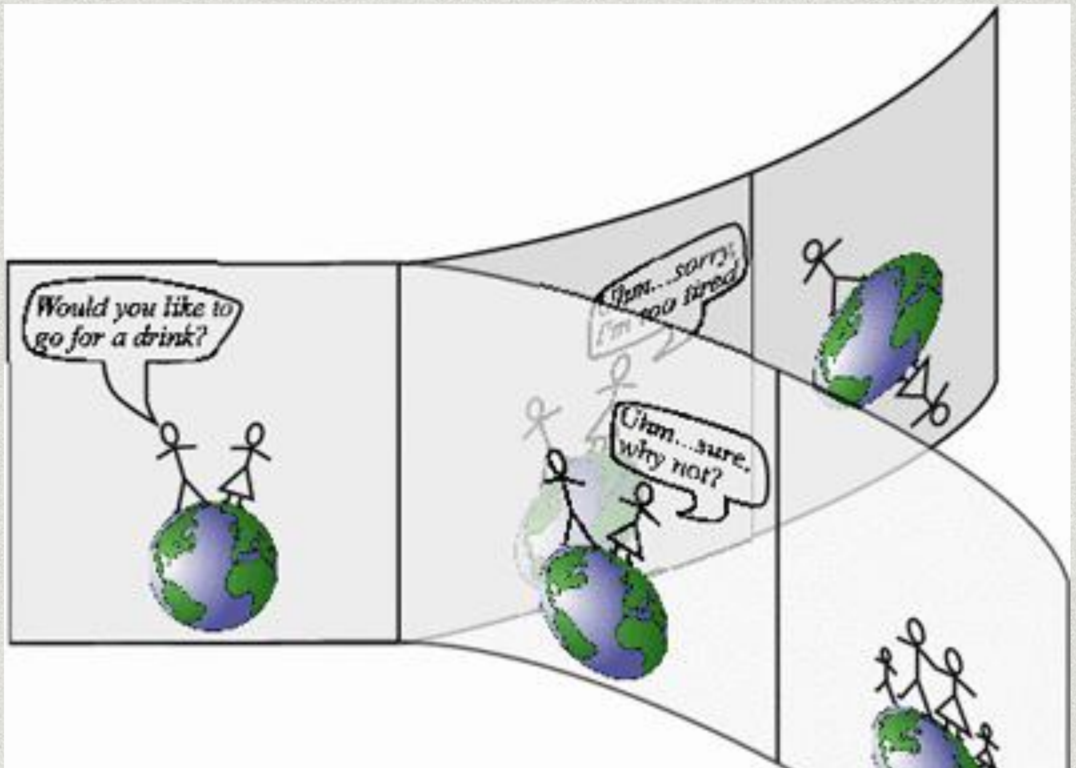

A representation of the split that occurs based on the possible outcomes for each action, according to Everett's Many-Worlds interpretation (courtesy of Max Tegmark).

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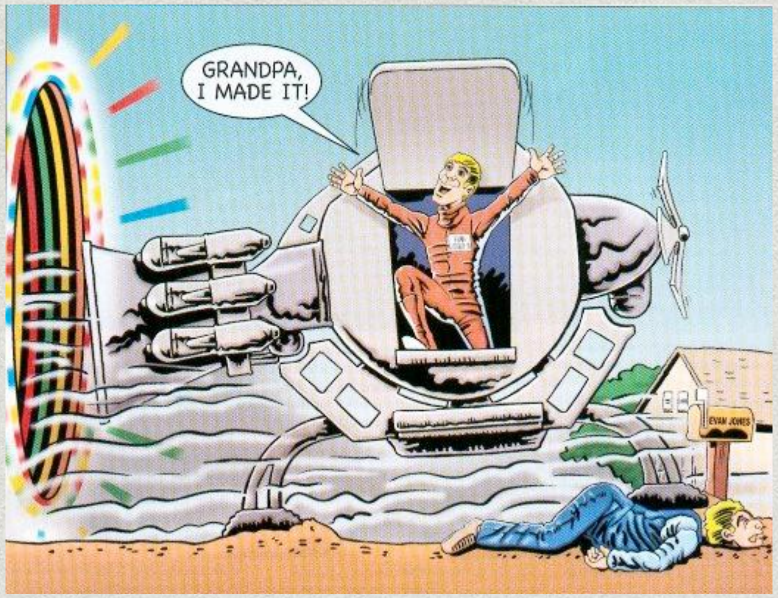

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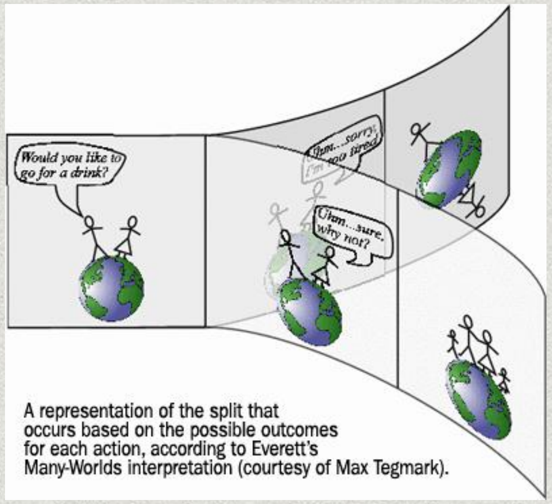

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Sliding doors

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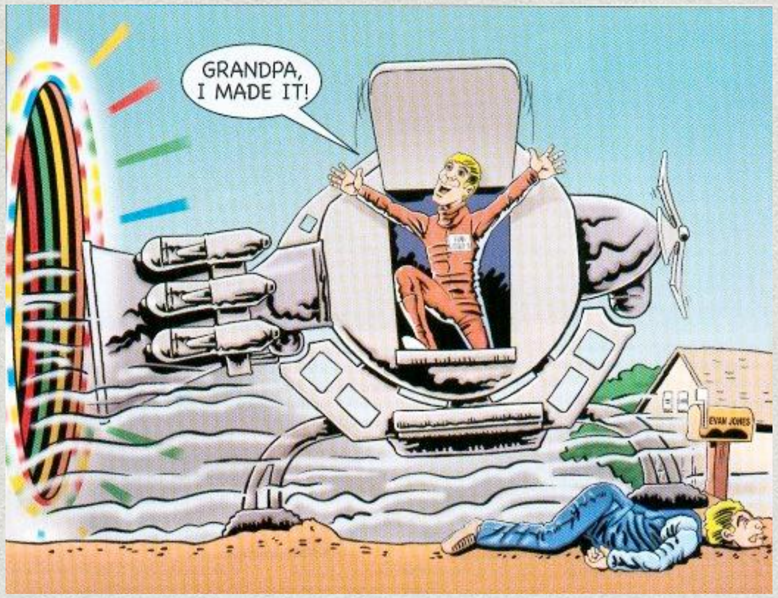

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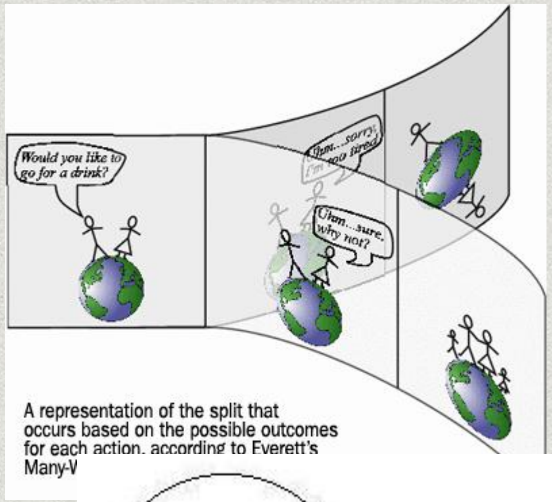
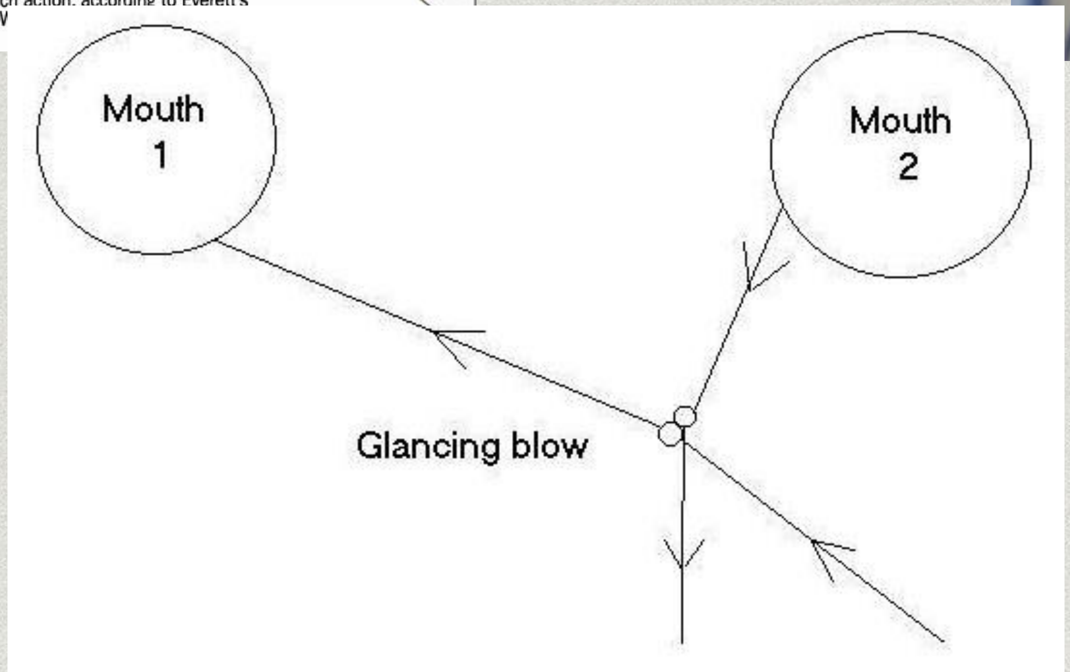


Sliding doors

Novikov Conjecture (consistent histories)



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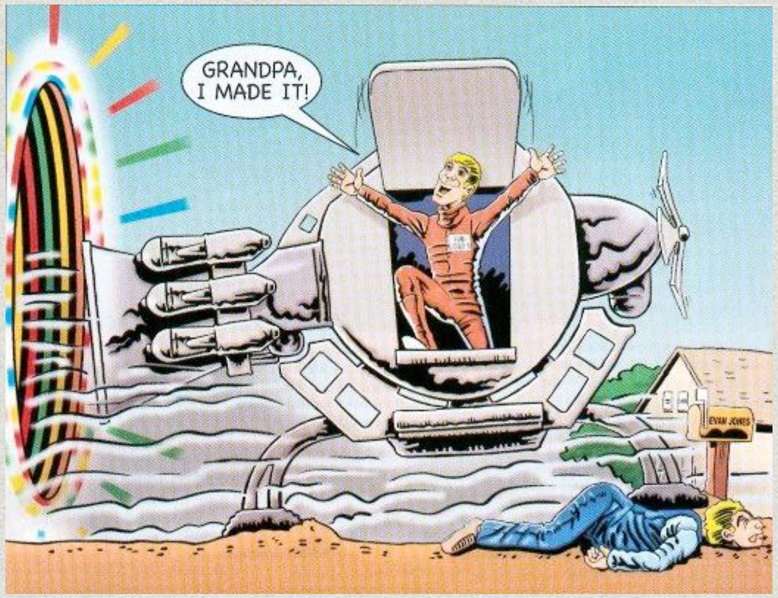

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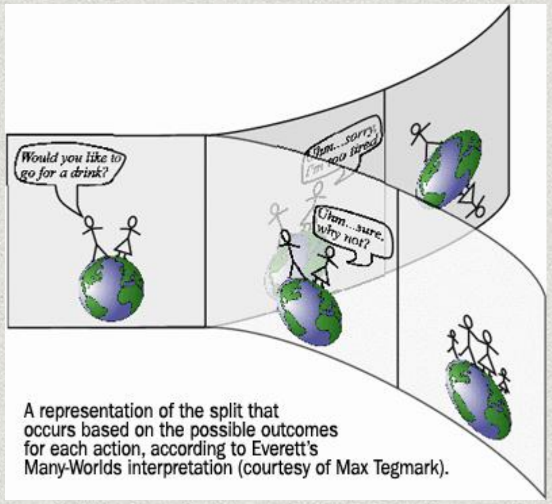

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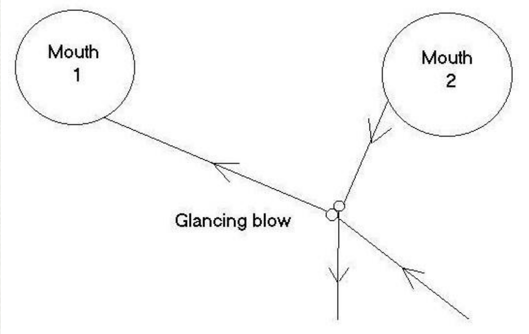

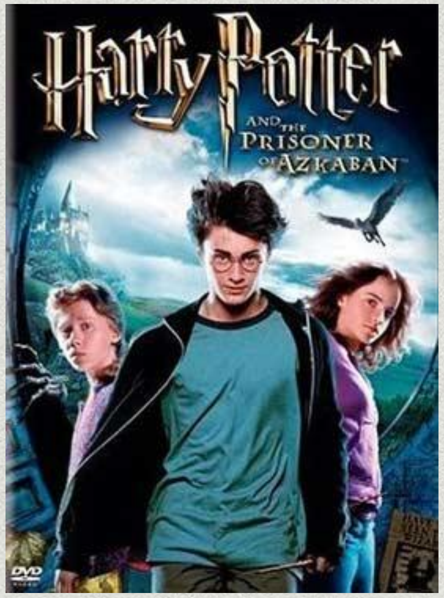

Harry Potter and the prisoner of Azkaban

Interstellar

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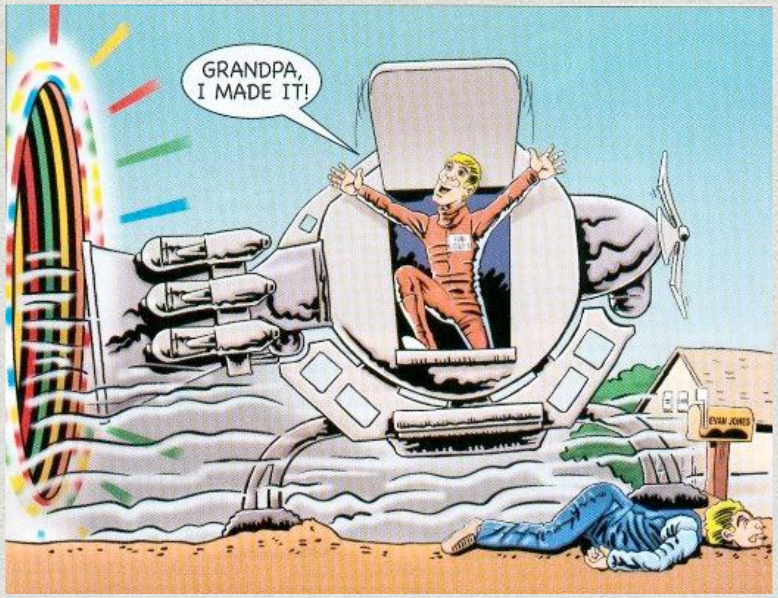

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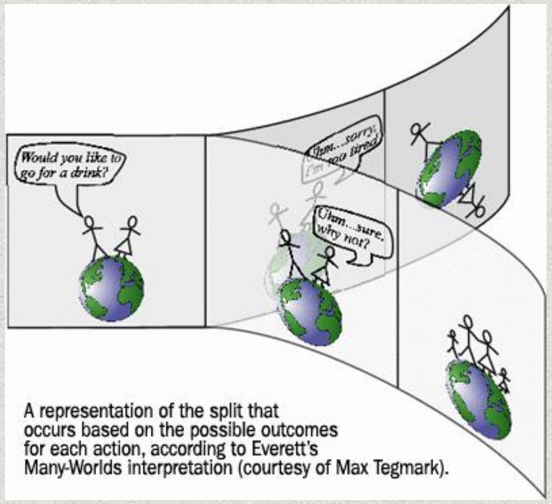

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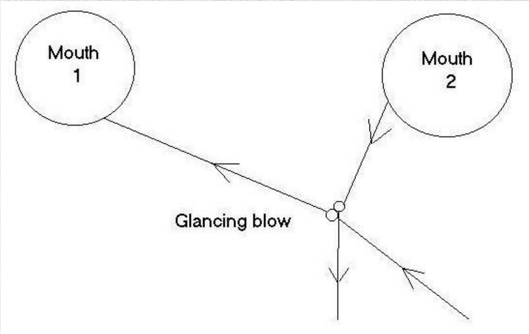

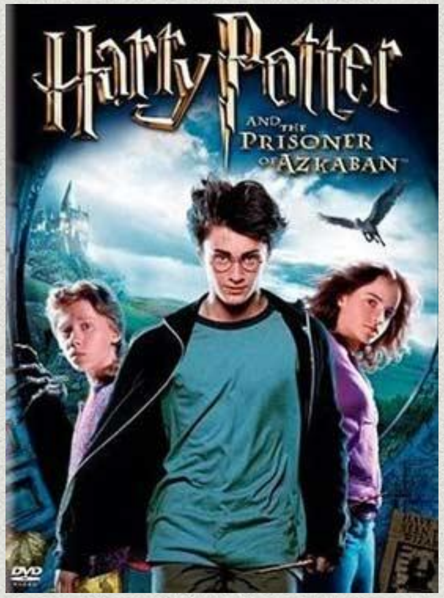

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Interstellar

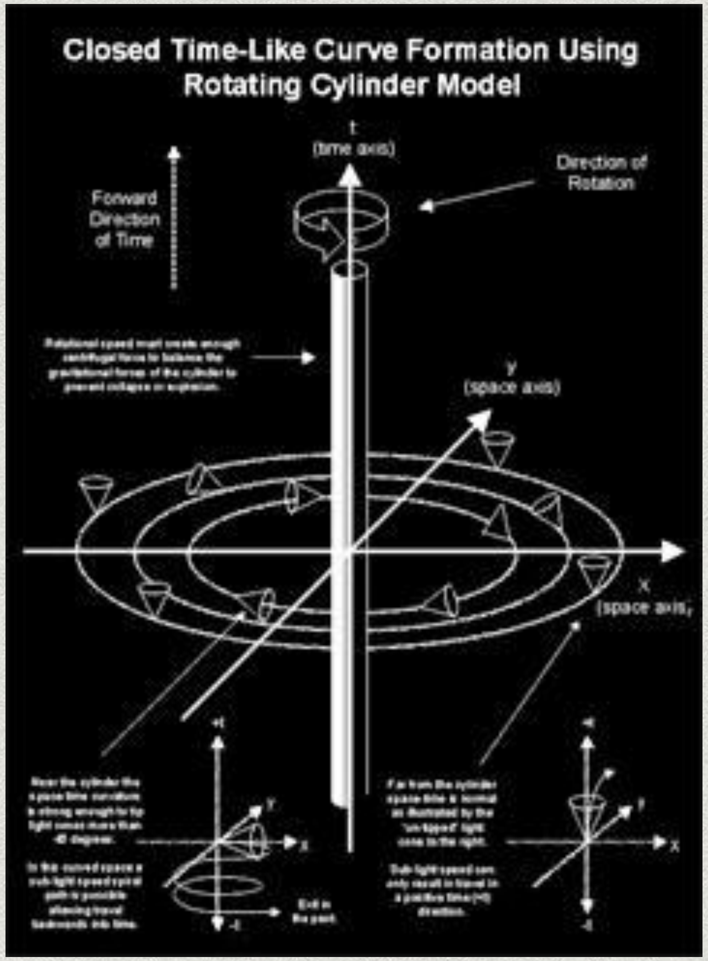
Chronology protection

Can physics save us from time machines?

Bad news: GR allows for "Causality challenged" Spacetimes

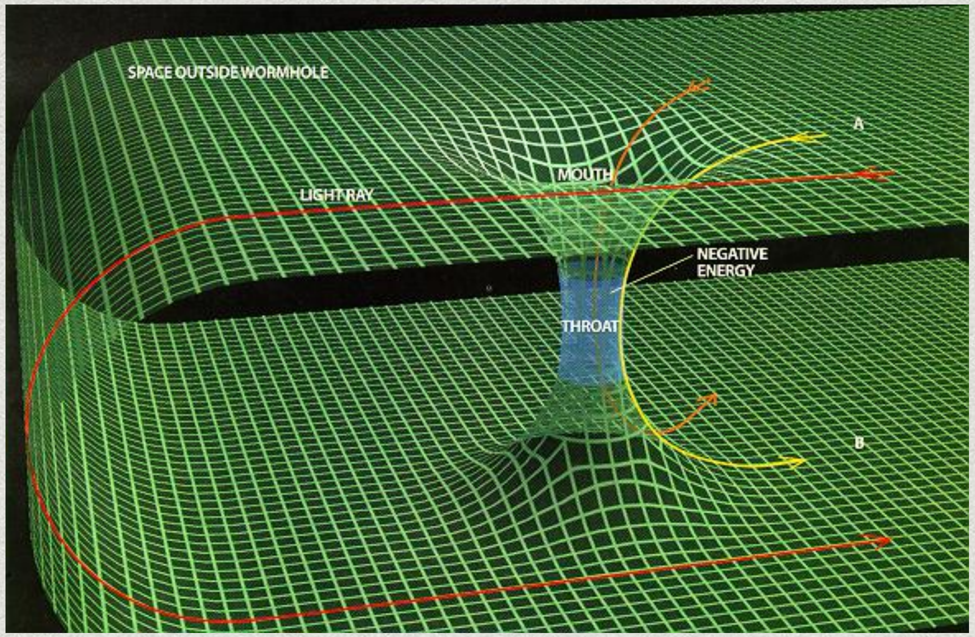
- Warped spacetimes by rotation, an incomplete list
  - Van Stockum/Gott
  - Goedel
  - Kerr interior (Inside Cauchy)

Rotating spacetimes



- Dangerous Shortcuts
  - Wormholes
  - Warp drives
  - Krasnikov tubes

Faster than light travels

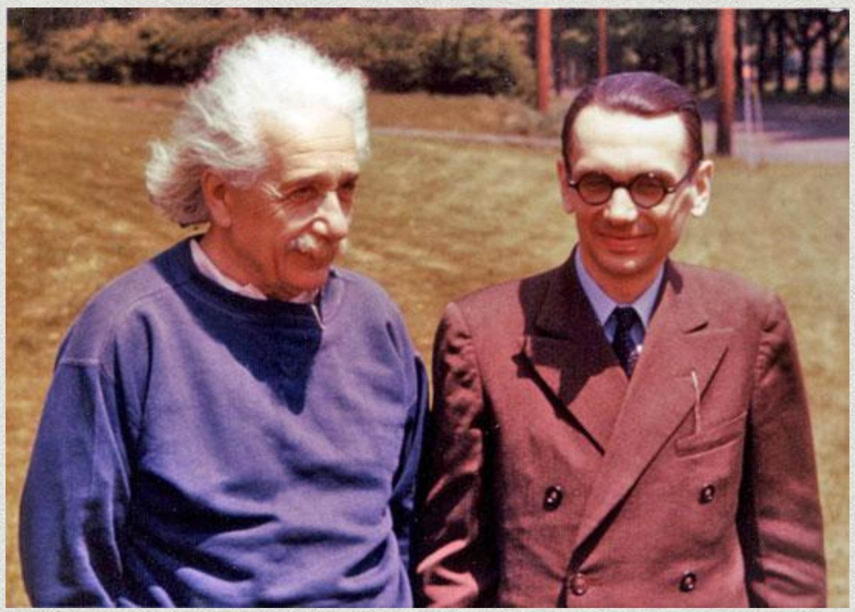


See M. Visser "Lorentzian wormholes" for an exhaustive review

Two rotating examples

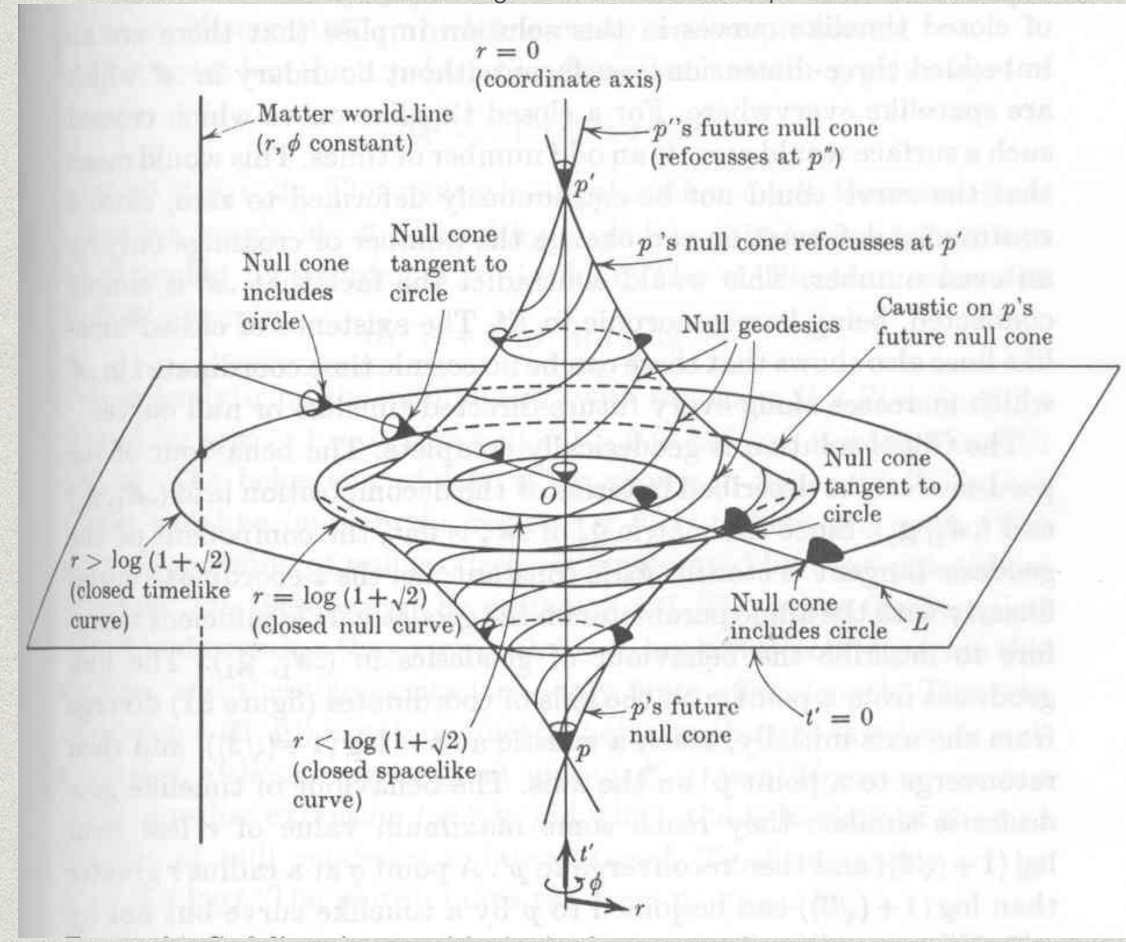
Gödel Universe

A homogeneous universe filled with dust and a cosmological constant



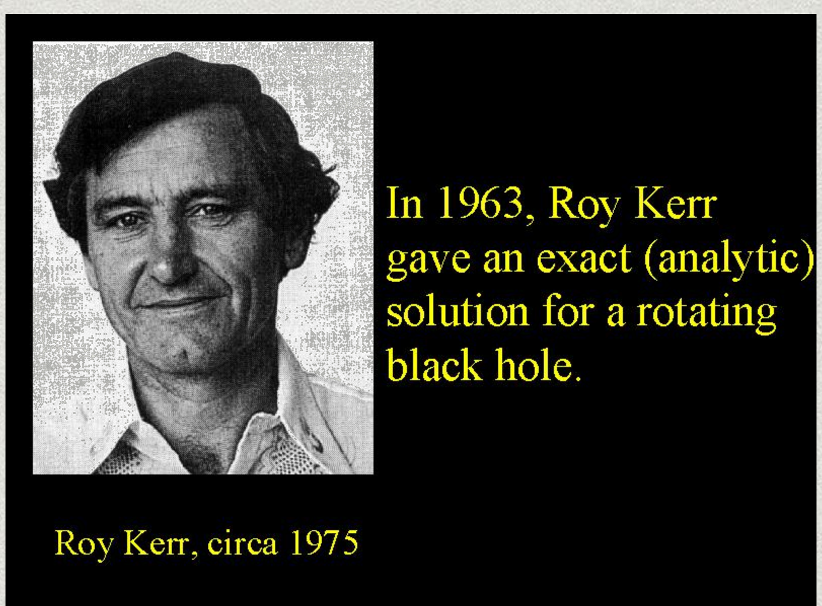
Einstein and Gödel. Princeton 1950

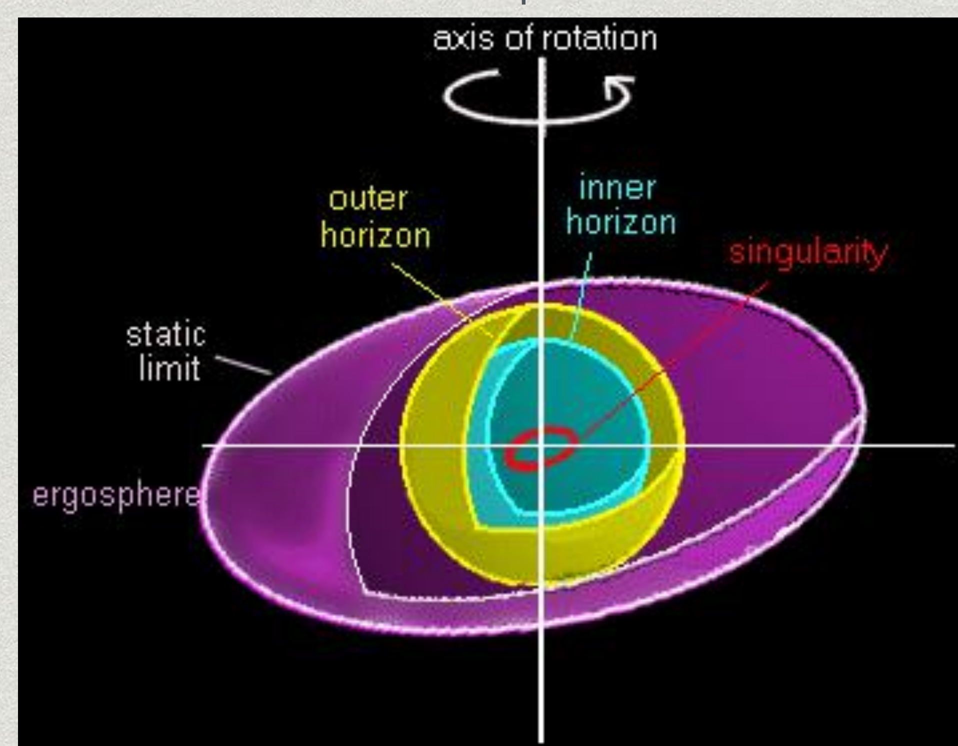
Hawking-Ellis.
The Large Scale Structure of Spacetime.



CTC inside Kerr Black Holes

The Killing vector associated to rotational invariance has closed orbits and inside the Inner horizon becomes spacelike.

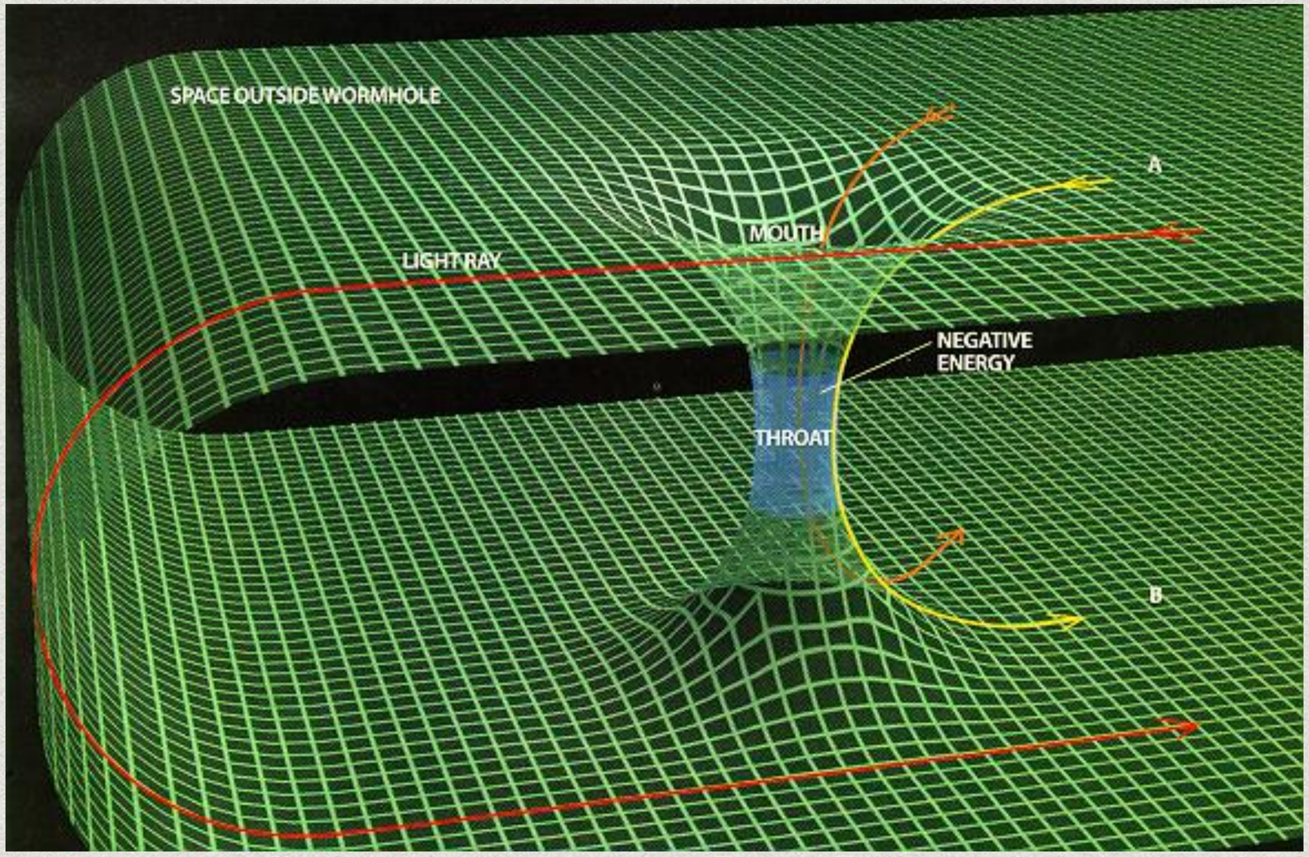


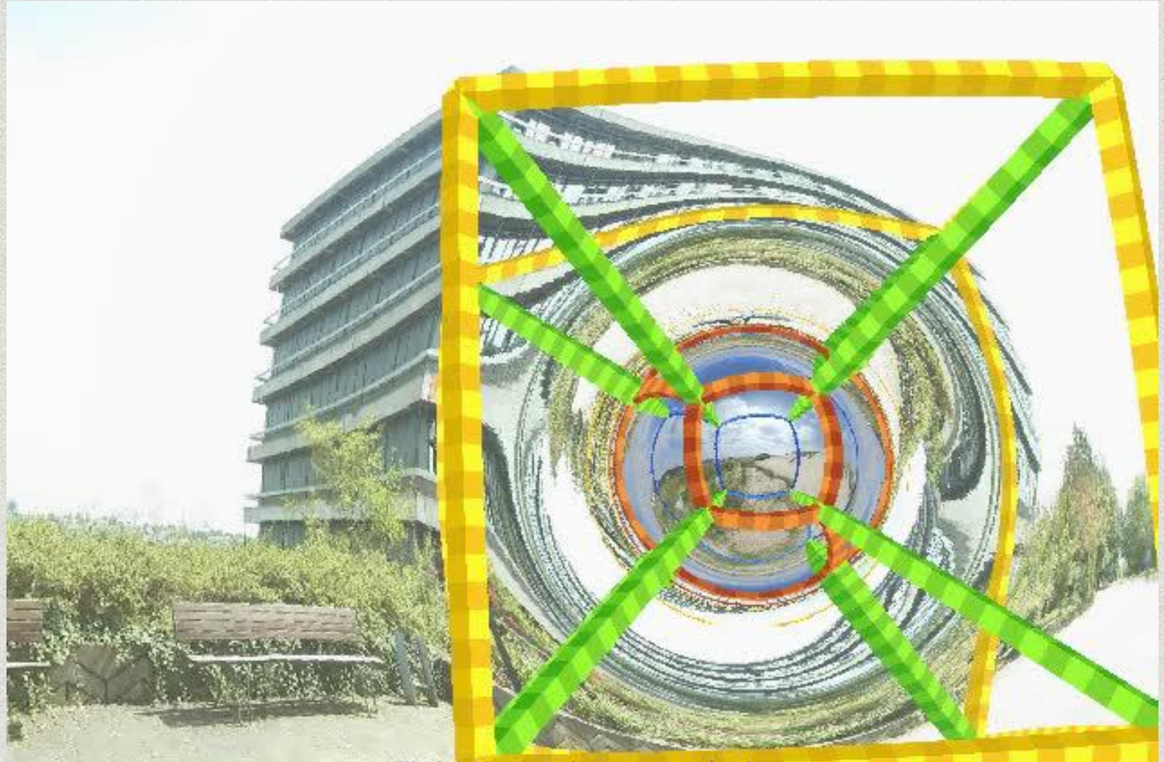


Shortcuts and faster than light propagation

Note: All of these solutions require at least violation of the Null Energy Condition
Actually FTL travel implies NEC violation: see Visser, Bassett, SL: Nucl.Phys.Proc.Suppl. 88 (2000) 267-270

Wormholes





flight through a wormhole
Corvin Zahn (2008), www.spacetimetravel.org

- Classically a topology change is incompatible with a global hyperbolic spacetime.
- Even worse Topology change is known to be unsustainable from QFT in Curved Spacetimes leading at a paroxysmal particle creation
- It seems that if we want to use wormholes for FTL travel we need to find them (early universe remnants) or grow them out of Wheeler's “spacetime foam” at the Planck scale…
- Still we cannot exclude there are wormholes out there as relic of the Planck era and then of inflation…

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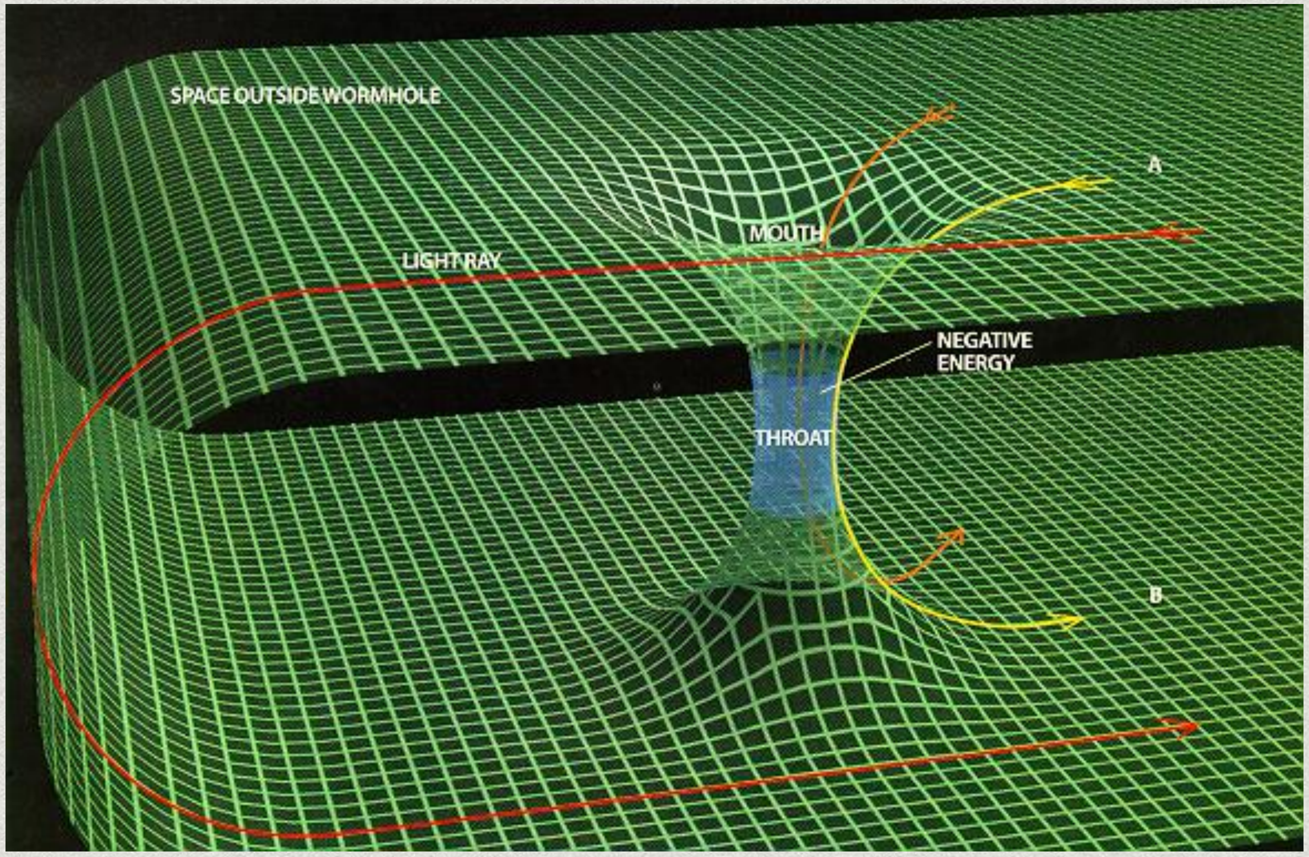

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Note: All of these solutions require at large violations of the Null/Weak/Dominant Energy Condition
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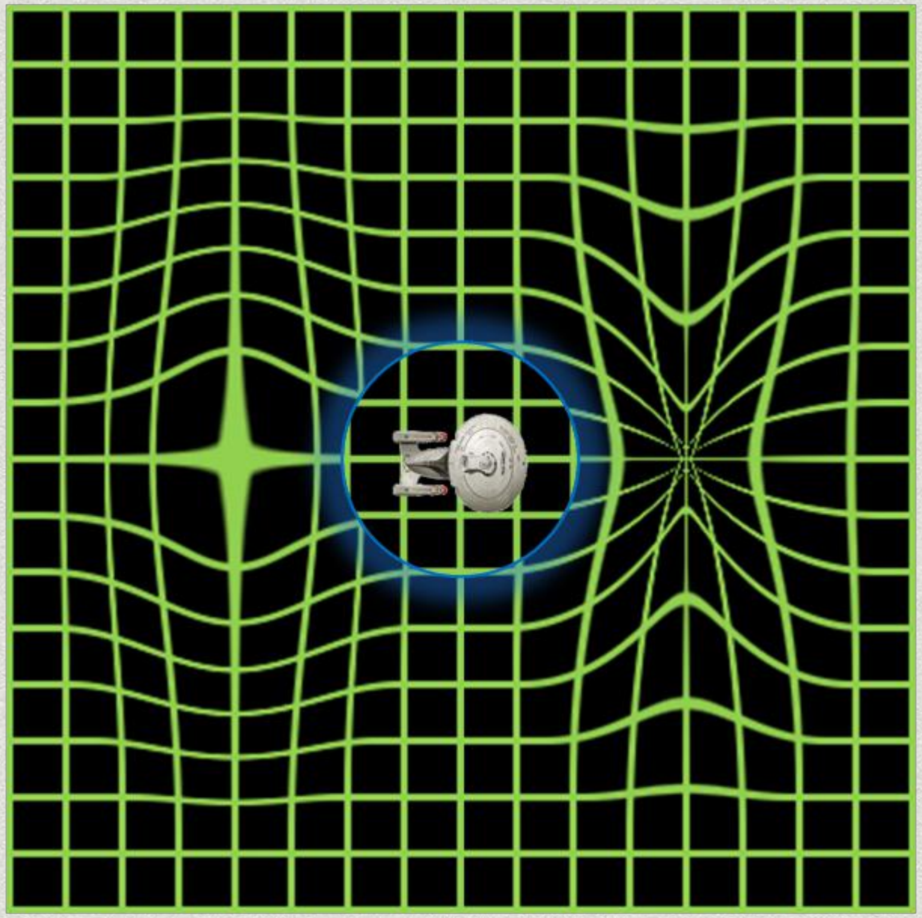
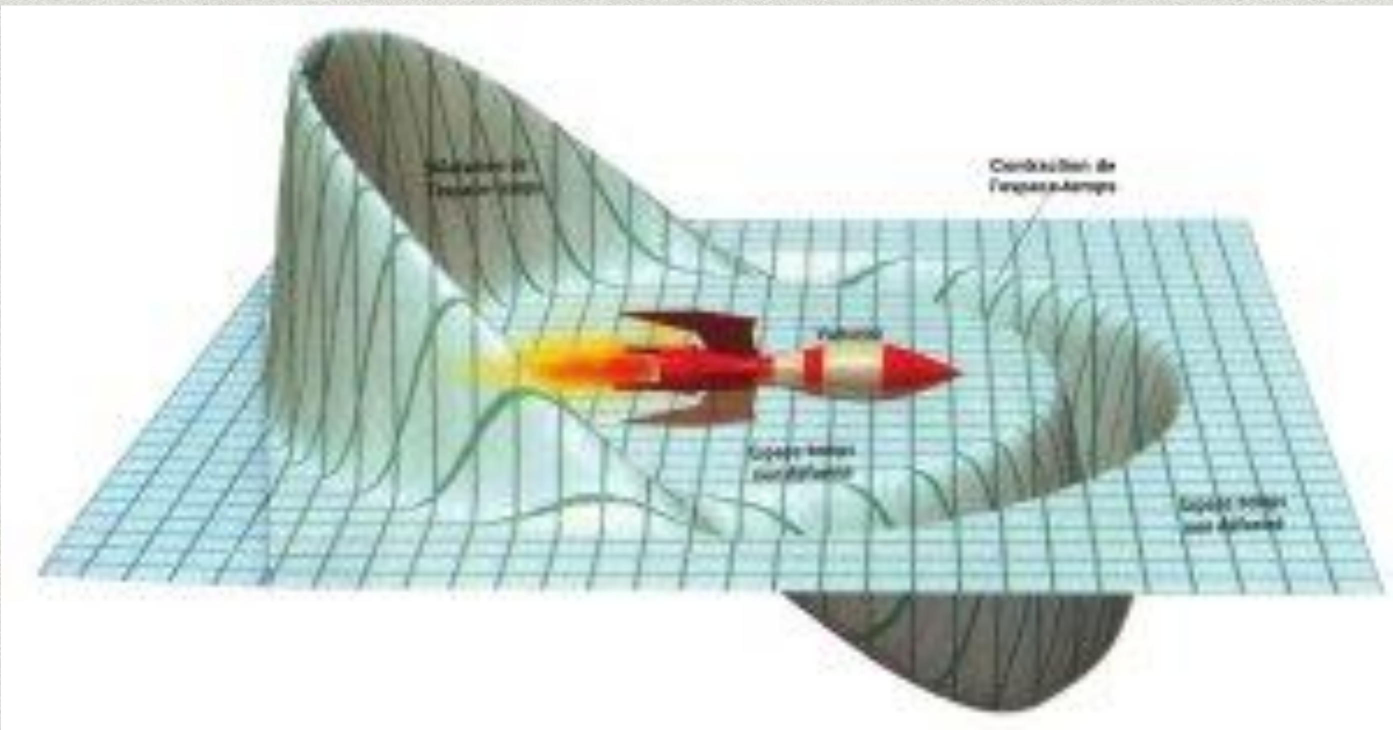
Alcubierre Warp Drive





- The spaceship is causally discontented by the exterior of the bubble
- So one must construct a sort of “railway” generating the right amount of NEC violating matter so to produce then the synchronised expansion of the rear wall and contraction at the front one.
- Quantum Inequalities imply that the walls are of Planck thickness so to allow the existence of negative energy densities.
- No time delay inside/outside bulle as the center follows a geodesic in flat space.

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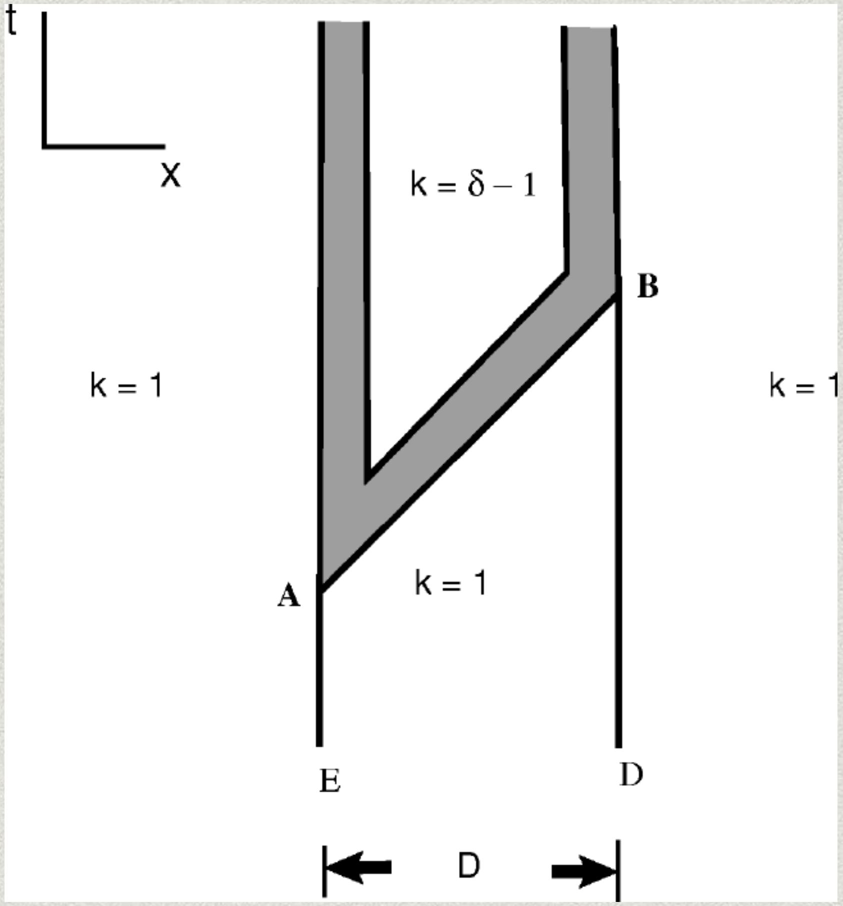
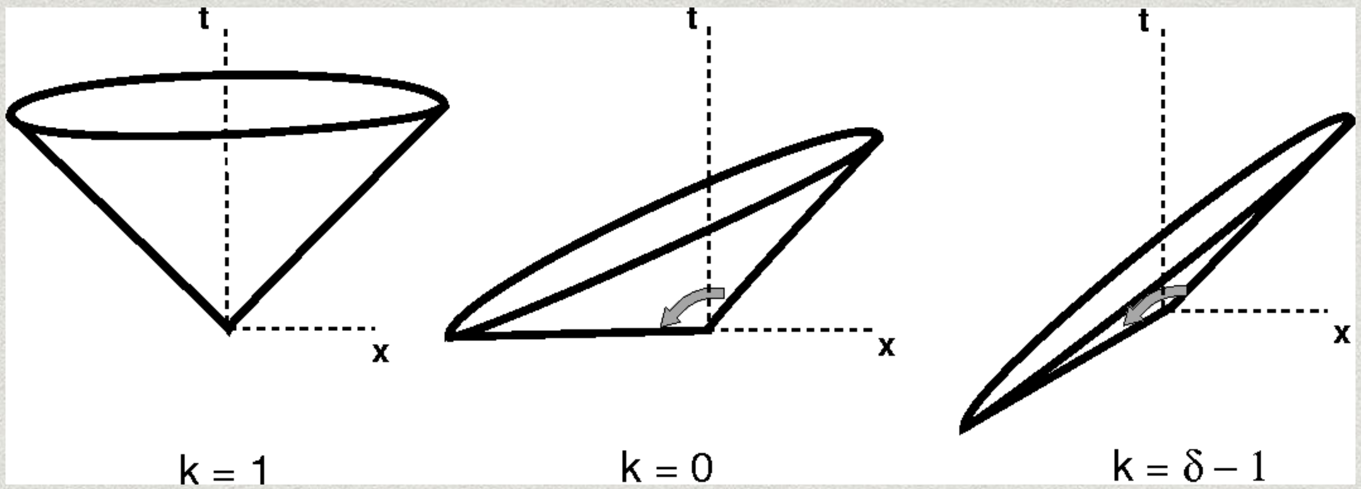
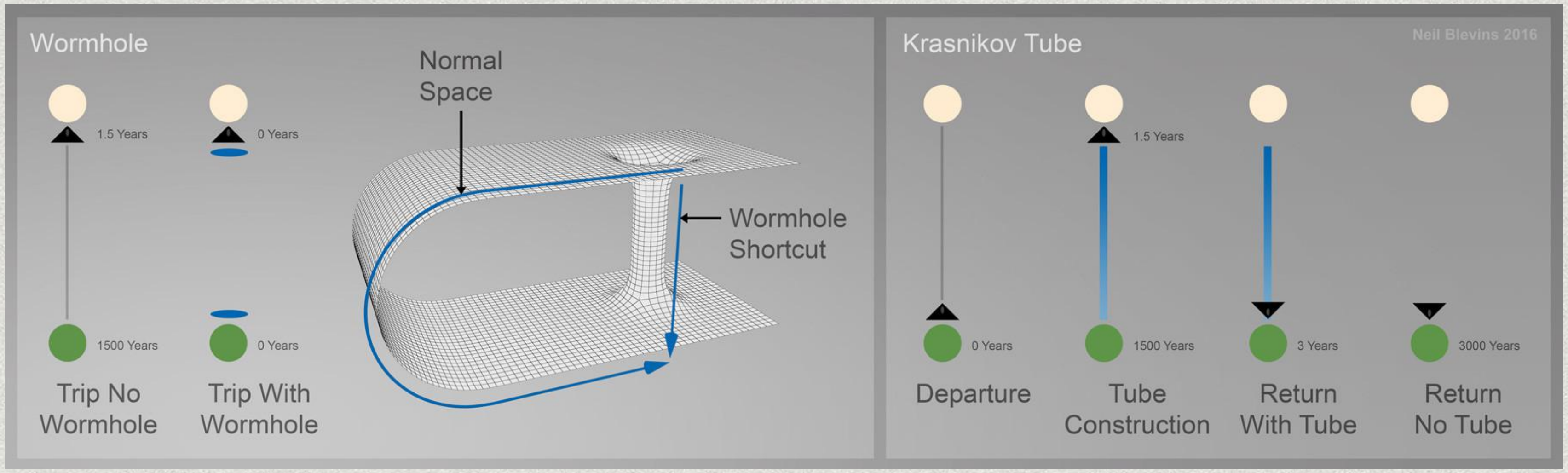
Krasnikov tube



From Allen E. Everett, Thomas A. Roman 1997 DOI:10.1103/physrevd.56.2100





- This is tube where the light cones are opened and tilted so to allow FTL in one direction.
- Differently from Warp drive it can be generated by spaceship itself. Still one needs a first slow trip.
- Even worse the amount of required NEC/DEC violation is huge as it increases with the length of the tube.

FTL travel spells troubles…

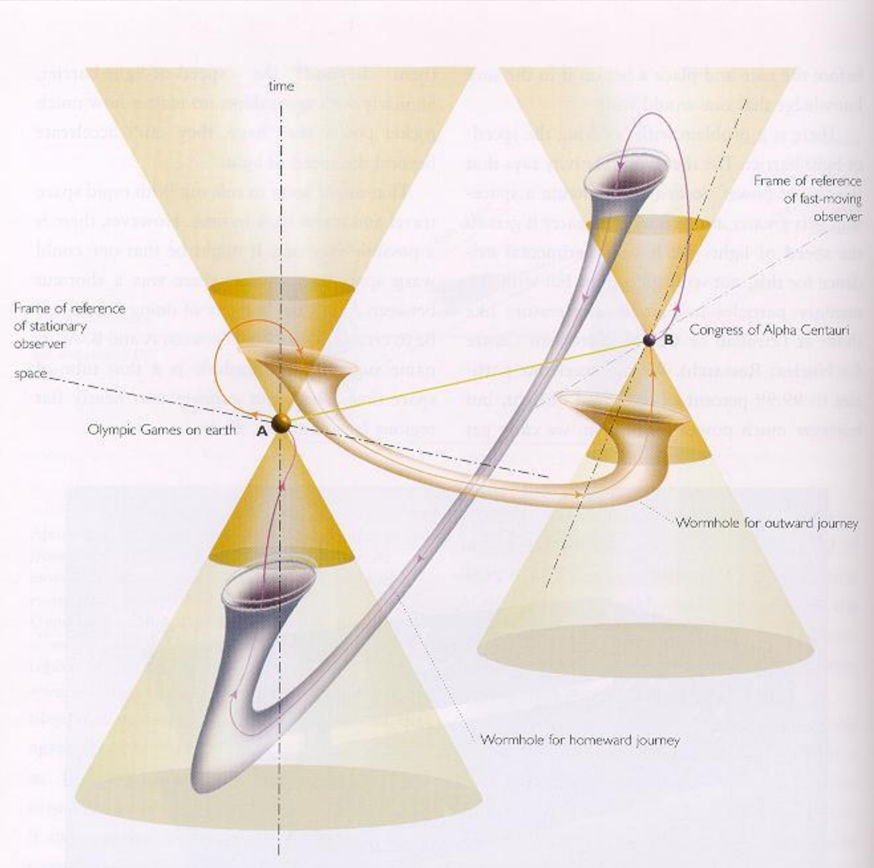
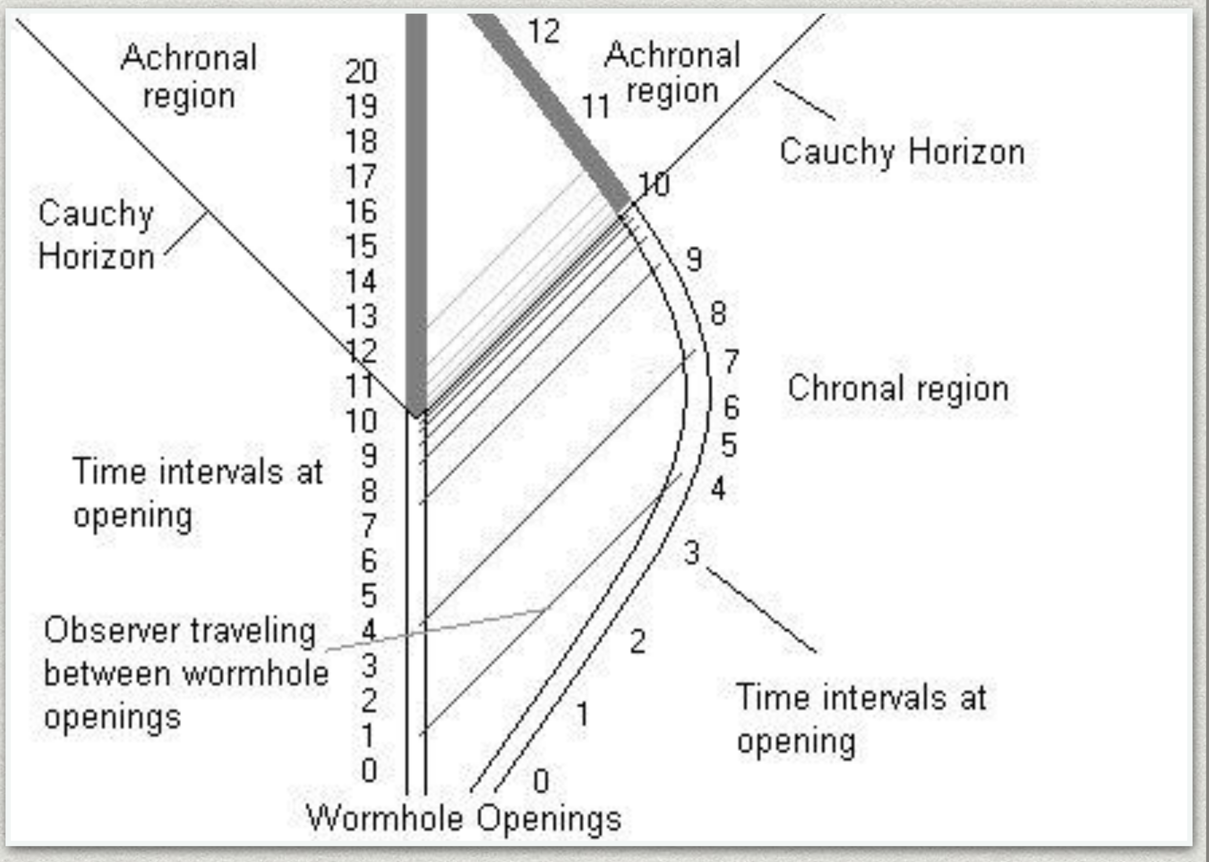


Fig. 10.5 *A space traveler could use a wormhole, which is stationary with respect to earth, as a shortcut to get from event A to B and then come back through a moving wormhole and return to earth before he set out.*

If you can have dragon egg then you can cook a dragon omelette!

Spacelike connection+Lorentz invariance=Time Travel



I.e. FTL travel allows for the dynamical generation of a time machine.

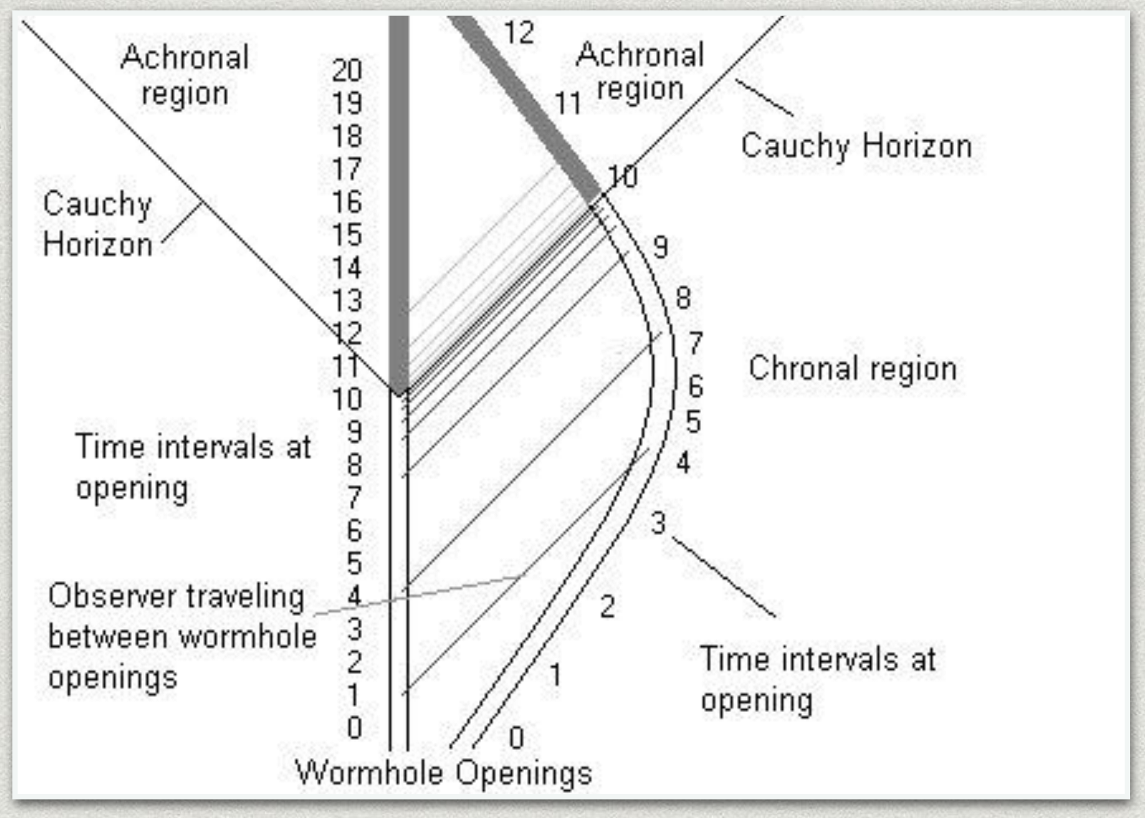
Key point: Dynamical TM generation

- **Spacetime must be time orientable**
- **Must have a definitive time orientation**
- **Must have a "causally innocuous past" (no CTC in the past)**

Then if in the future you form CTC then you have produced a time-machine

Almost yes…

QFT in Curved Spacetime calculations shows clearly a paroxysmal growth of the renormalised stress energy tensor close to the Chronology horizon (which is also a Cauchy horizon)



But the real obstruction is the Kay, Radzikowski, and Wald (1997) theorem which states that the quantum state fails to be Hadamard (UV structure like in flat spacetime) somewhere on the Chronological horizon, hence we do not know how to renormalise the SET!

It seems that only a full fledged QG theory will be able to rule out Time Machines…

For the moment let's assume GR+SR: it seems we cannot exclude TM
But we saw that Time Machines lead to paradoxes
Can perhaps QFT on CS save us from time machines?
I.e. is there a Chronology Protection?

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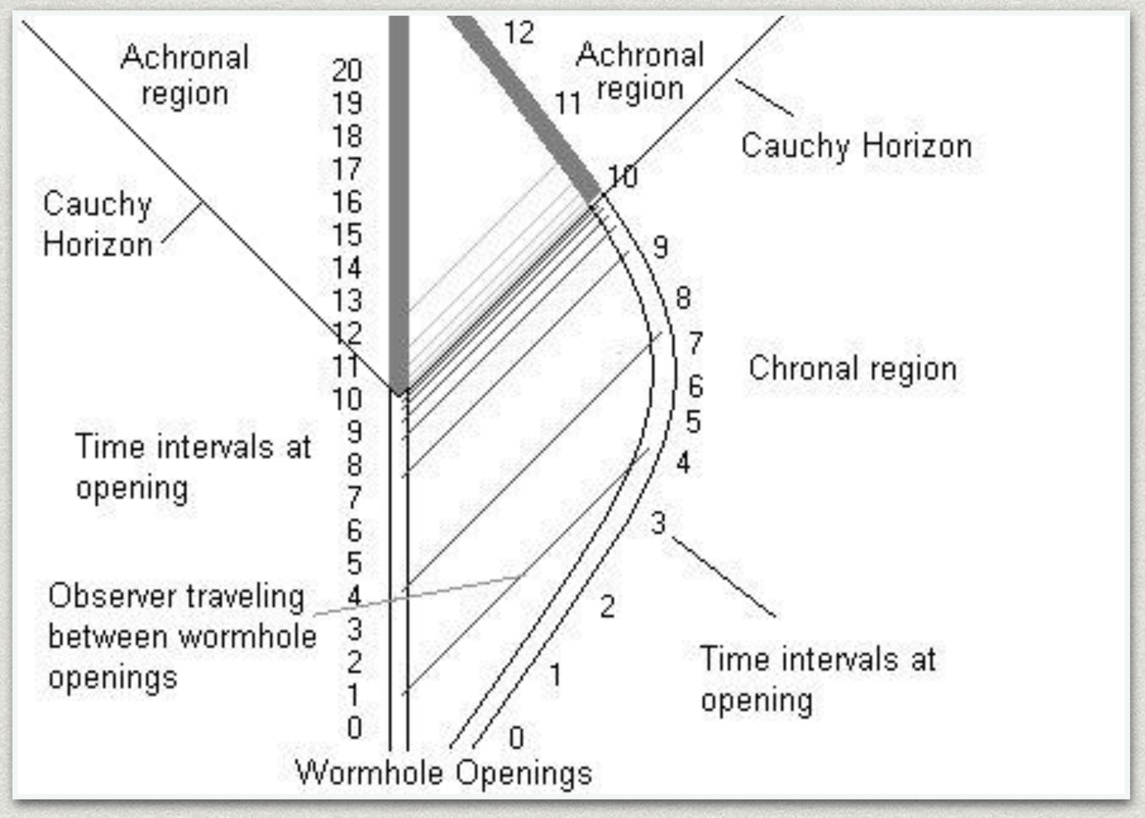

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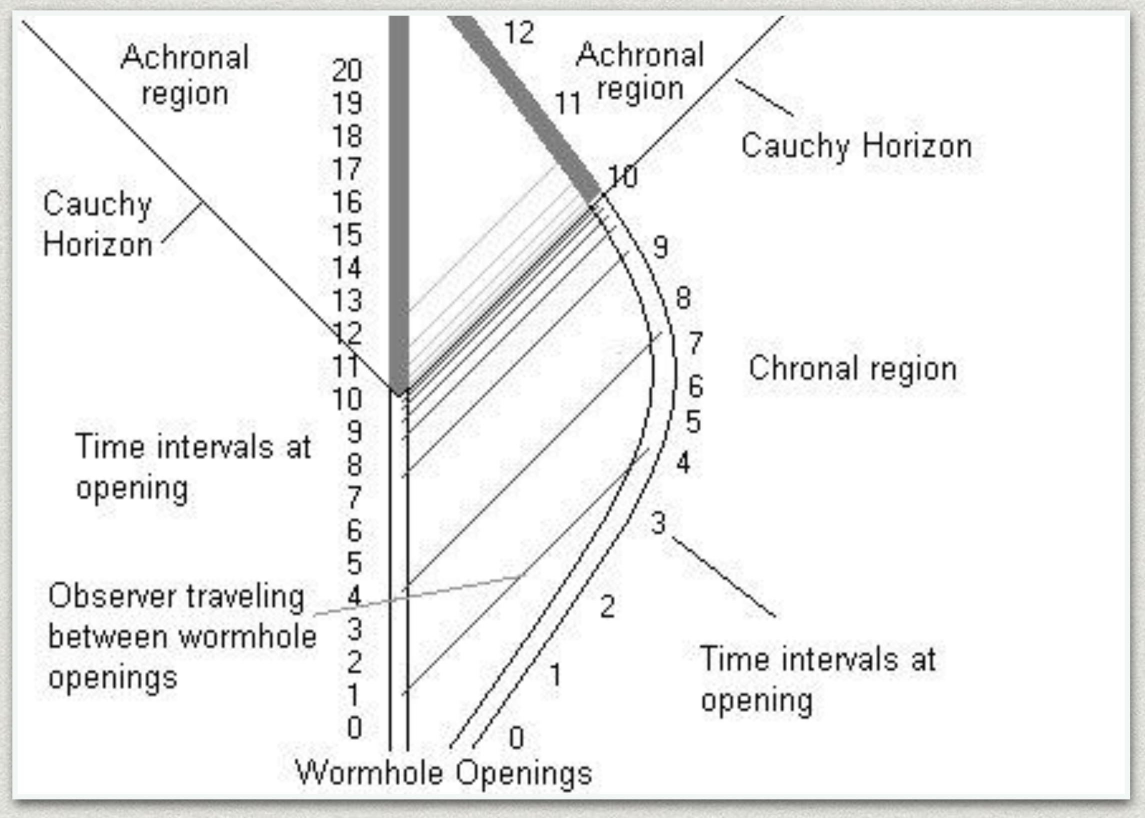

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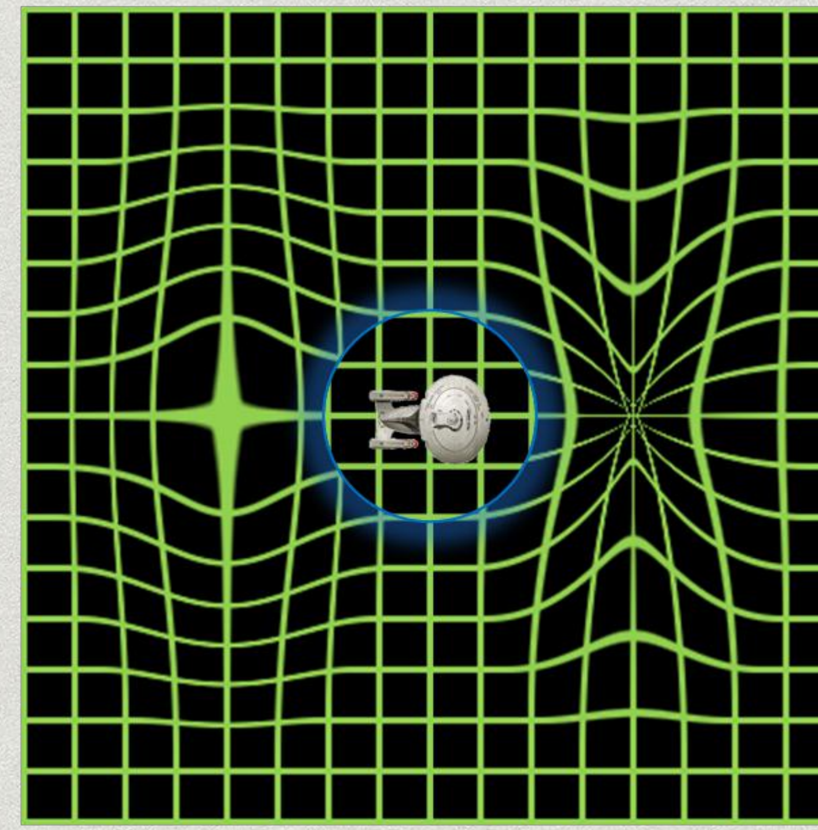
Really?

SUPERLUMINAL WARP DRIVES: PRE-EMPTIVE CHRONOLOGY PROTECTION?

"L'éternité, c'est long … surtout vers la fin."
"Eternity is long… especially towards the end"
Franz Kafka

Let's focus on Warp-drives

Finazzi, SL, Barcelò,
Phys.Rev. D79 (2009) 124017



$$ds^2 = -c^2 dt^2 + [dx - v(r)dt]^2 + dy^2 + dz^2$$

$r \equiv \sqrt{(x - v_0 t)^2 + y^2 + z^2}$ is the distance from the center of the bubble
v_0 the warp-drive velocity $v = v_0 f(r)$
f is a suitable smooth function satisfying $f(0) = 1$ and $f(r) \to 0$ for $r \to \infty$.

Independently for being sub or super-luminal, any macroscopic warp-drive geometry require large violations of the energy conditions, i.e. exotic matter.
If this matter is provided by a quantum field then the so called quantum inequalities imply very sharp warp drive walls of the order of the Planck scale. Still these are problem for engineers…

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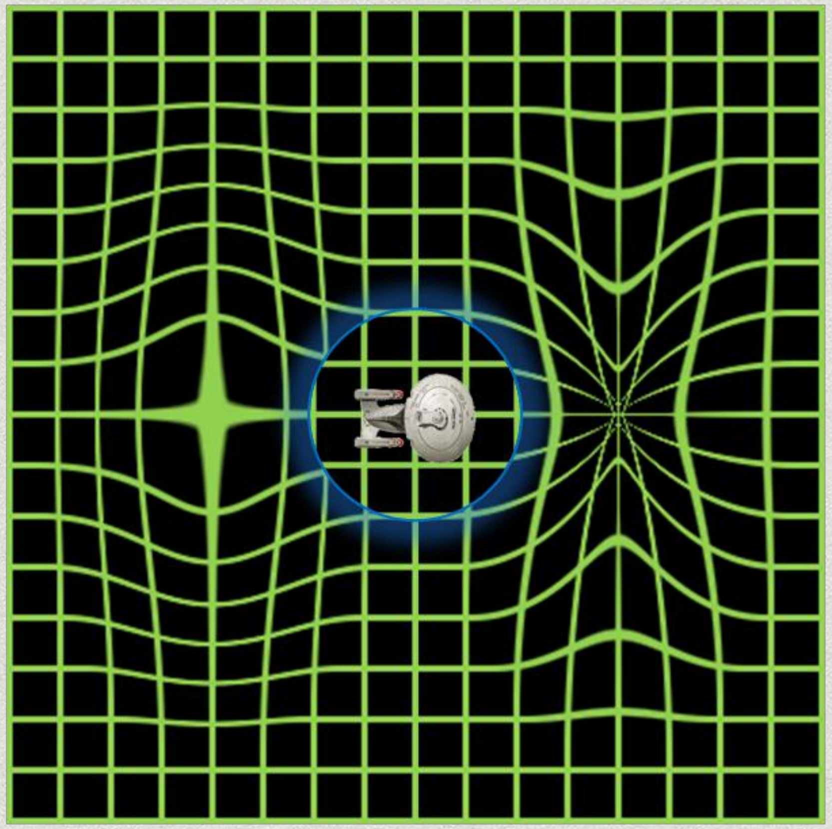

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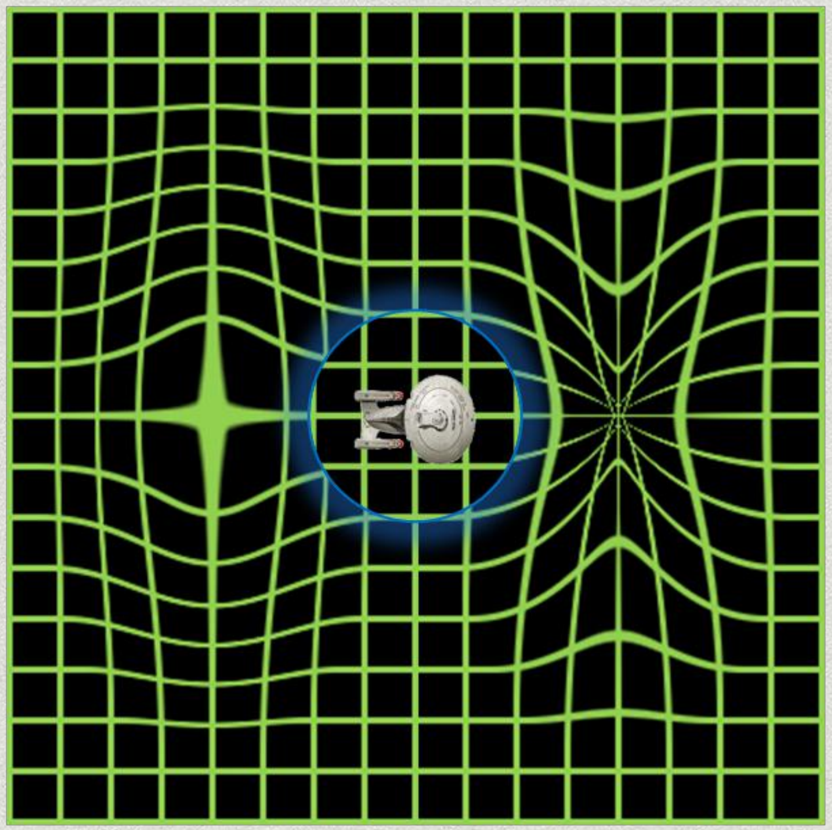

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If this matter is provided by a quantum field then the so called quantum inequalities imply very sharp warp drive walls of the order of the Planck scale. Still these are problem for engineers…

Superluminal Warp drive generation

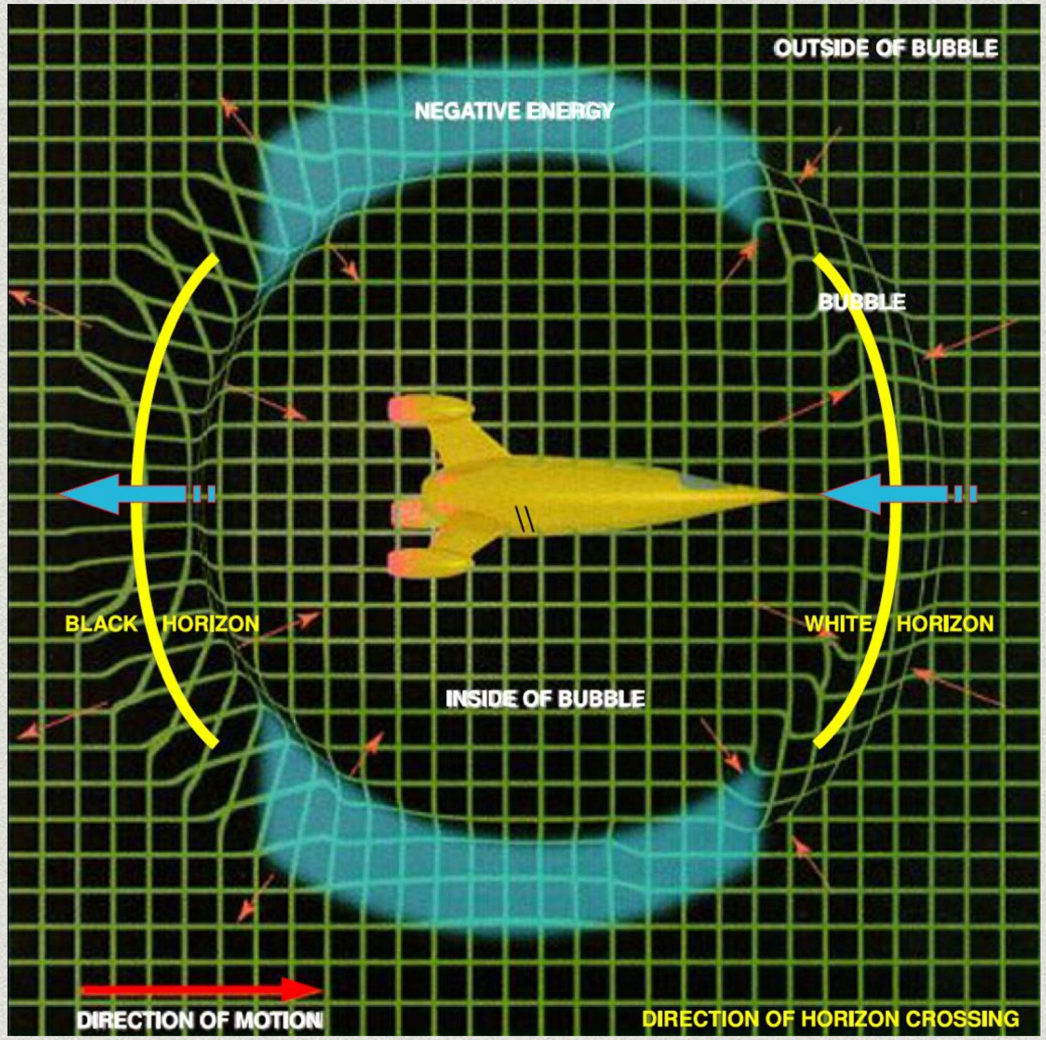
We want to describe a superluminal Alcubierre warp drive dynamically created at some time t=0

$$ds^2 = -c^2 dt^2 + [dr - \hat{v}(r,t)dt]^2$$
$$\hat{v}(r,t) = v_0 \delta(t) [f(r) - 1]$$

with

$$\delta(t) \equiv \begin{cases} e^{t/\tau} & \text{if} \quad t < 0 \\ 1 & \text{if} \quad t \geq 0 \; . \end{cases}$$

Remarkably the causal structure of a superluminal warp drive shows the presence of black hole and white hole like horizons...



Let's compute see what QFT on this spacetimes predicts in this case

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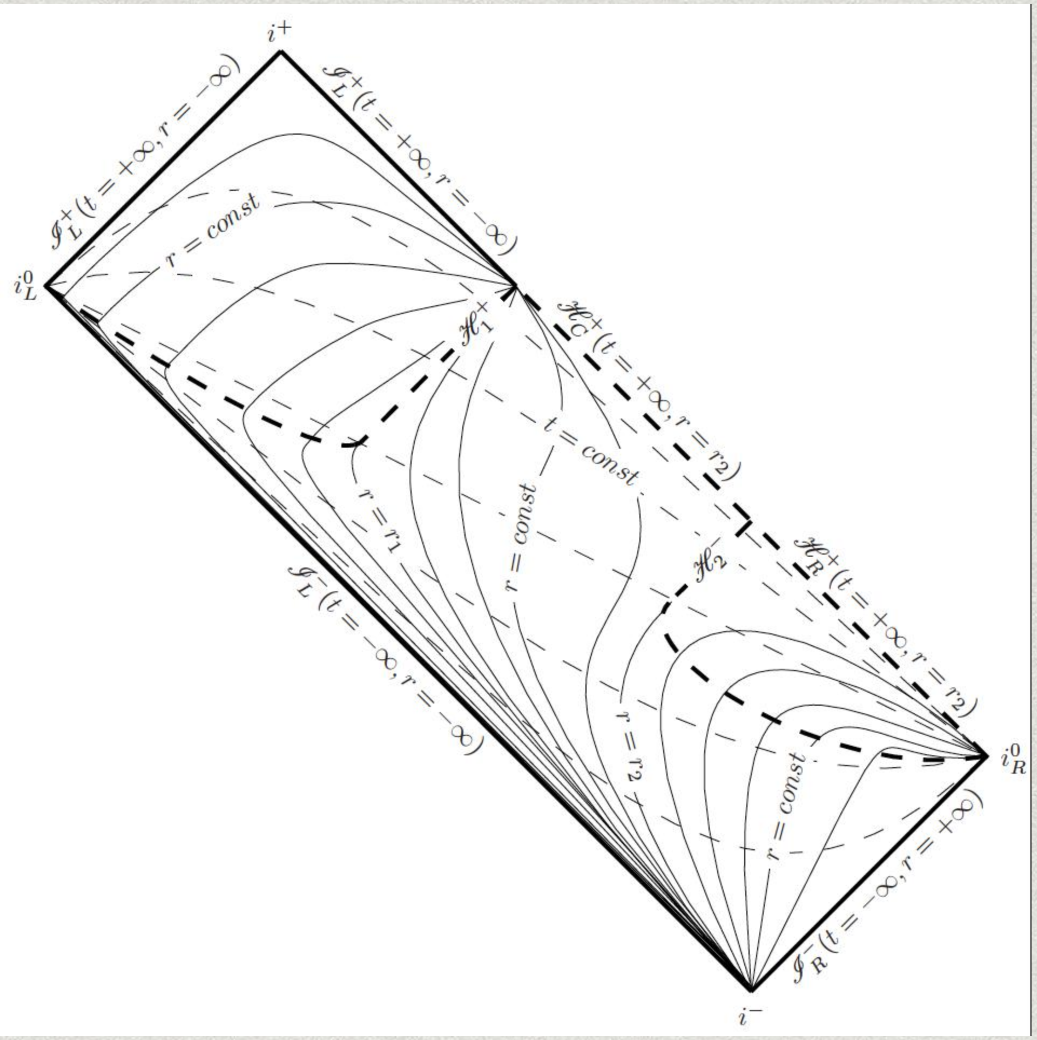

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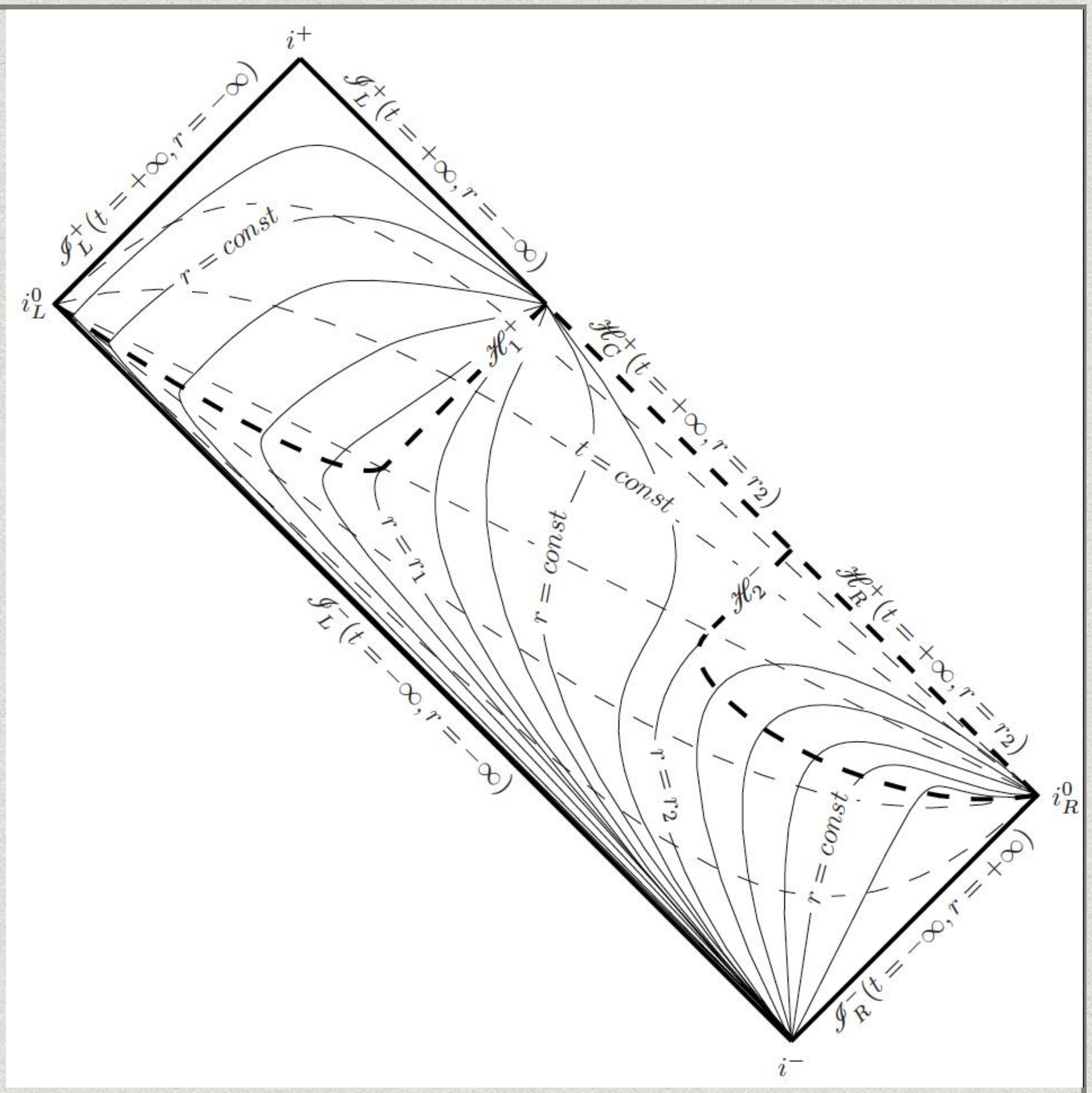
Let's compute see what QFT on this spacetimes predicts in this case

RSET on a dynamical Warp drive

- One can restrict the computation to the 1+1 case.
- Let us consider light ray propagation in this geometry.
- Like in the BH collapse case the relation between affine coordinates on $\mathfrak{I}^-_L$ (U), $\mathfrak{I}^-_R$ (W) and H^+_C (u), and $\mathfrak{I}^+_L$ ($\tilde{w}$) will determine the universal features of the RSET.



then using U=p(u) and W=q($\tilde{w}$) we get for the energy density ρ as measured by a set of FF observers with four velocity (1,v):

$\rho_{total}=\rho_{st}+\rho_{dyn-u}+\rho_{dyn-\tilde{w}}$

$$\rho_{\rm st} \equiv -\frac{1}{24\pi}\left[\frac{(\bar{v}^4-\bar{v}^2+2)}{(1-\bar{v}^2)^2}\bar{v}'^{\,2}+\frac{2\bar{v}}{1-\bar{v}^2}\bar{v}''\right]$$

$$\rho_{{\rm dyn}-u} \equiv \frac{1}{48\pi}\frac{f(u)}{(1+\bar{v})^2}$$

$$\rho_{{\rm dyn}-\tilde{w}} \equiv \frac{1}{48\pi}\frac{g(\tilde{w})}{(1-\bar{v})^2}$$

This term is a static contribution which vanishes at the very center of the bubble where v=v'=0

This term is right-going radiation

This term is transient left going radiation which, for sufficiently smooth transitions Minkowski->Warp Drive, goes to zero at late times.

Local RSETs

$$\bar{v}_{\pm}(r) = -1 \pm \kappa\,(r - r_{1,2}) + \frac{1}{2}\sigma\,(r - r_{1,2})^2 + \mathcal{O}\left((r - r_{1,2})^3\right)$$

- **RSET @ center of the bubble**

ρ_{st} vanishes identically here because v=v'=0.

$\rho_{dyn\text{-}u}$ --> $\kappa^2/48\pi$ at late times --> thermal bath with $T=\kappa/2\pi$

- **RSET @ bubble horizons**

The leading contributions are

$$\rho_{\rm st}\,(r \simeq r_{1,2}) = -\frac{1}{48\pi}\left[\frac{1}{(r - r_{1,2})^2} \mp \frac{\sigma}{\kappa\,(r - r_{1,2})}\right] + \mathcal{O}(1)$$

$$\rho_{{\rm dyn}-u}\,(r \simeq r_{1,2}) = \frac{1}{48\pi}\left[\frac{1}{(r - r_{1,2})^2} \mp \frac{\sigma}{\kappa\,(r - r_{1,2})}\right] + \mathcal{O}(1)$$

they both diverge at the horizons but also cancels identically

the sub-leading contributions are

$$\rho\,(r \simeq r_{1,2}) = \frac{1}{48\pi}\left(3A_{\pm}^2 - 2B_{\pm}\right)e^{\mp 2\kappa t} + C_{\pm} + \mathcal{O}\,(r - r_{1,2}) \qquad A_{\pm}, B_{\pm}, C_{\pm} = \text{constants}$$

at the black-horizon the energy density as seen by a free-falling observer is damped exponentially with time

at the white-horizon however the energy density exponentially blows up in time. In about a time $1/\kappa$ from the formation of the WH the backreaction will not be negligible and will destabilize the WD

Superluminal Warp drive instability

Hence one finds that

Finazzi, SL, Barcelò,
Phys.Rev. D79 (2009) 124017

- The observer in the bubble center will detect a thermal flux at the “Hawking temperature” of the black-horizon.
  - if quantum inequalities apply the surface gravity, being related to the thickness of the bubble walls, will be Planck scale. Hence the observer will be suddenly “boiled”!
- At the white horizon there is an exponential accumulation of energy density that will rapidly destabilise the WD
- At the Cauchy horizon (not discussed) there is also a divergence of the energy density if the WD last forever (but still a very rapid growing energy density even in a short time).

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So it seems that GR+QFT “know” that FTL is dangerous…

Can we escape this?

FTL propagation and Lorentz invariance

Faster than c signals, special relativity, and causality
Stefano Liberati, Sebastiano Sonego, Matt Visser
Annals Phys. 298 (2002) 167-185
DOI: 10.1006/aphy.2002.6233

Faster than light propagation does not necessarily implies problems with Causality. It is its union with Lorentz invariance which is the problem.

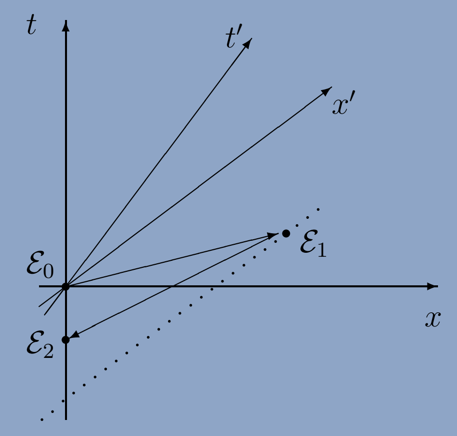

Figure 2: A causal paradox using tachyons. The dotted line represents the set of events which are simultaneous with $\mathcal{E}_1$ according to the reference frame $\mathcal{K}'$. The tachyonic signal from $\mathcal{E}_1$ to $\mathcal{E}_2$ travels to the future with respect to $\mathcal{K}'$, and to the past with respect to $\mathcal{K}$.

Paradoxes of this type require not only that tachyons exist, but also that, given an arbitrary reference frame, it is always possible to send a tachyon backward in time in that frame.

Obviously, there can be no paradox if, *in one particular reference frame*, tachyons can only propagate forward in time.

I.e. if the Relativity Principle is not always valid.

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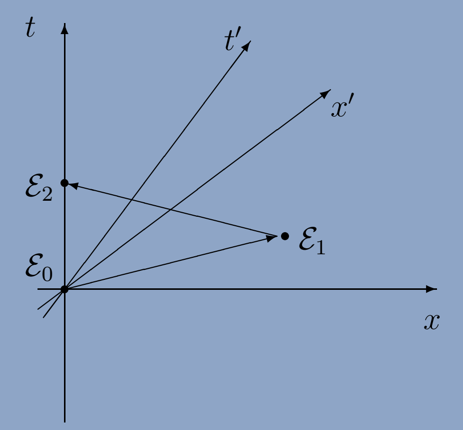

Figure 3: Tachyon propagation without causal paradoxes. Both signals travel to the future in at least one reference frame.

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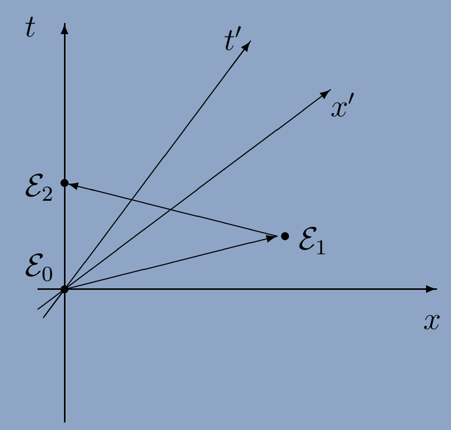

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I.e. if the Relativity Principle is not always valid.

Can this be the case in nature?

And in this case can we travel faster than light?

INTERLUDIUM

(DIGGING IN THE DIRT, AKA CONDENSED MATTER PHYSICS)

“Here we are, trapped in the amber of the moment. There is no why.”
Kurt Vonnegut

Analogue models of gravity

And I cherish more than anything else the Analogies, my most trustworthy masters.
They know all the secrets of Nature, and they ought least to be neglected in Geometry.
Johannes Kepler

An analogue system of gravity is a generic dynamical system where the propagation of linearised perturbations can be described via hyperbolic equations of motion on some curved spacetime possibly characterized be one single metric element for all the perturbations.

analogue models

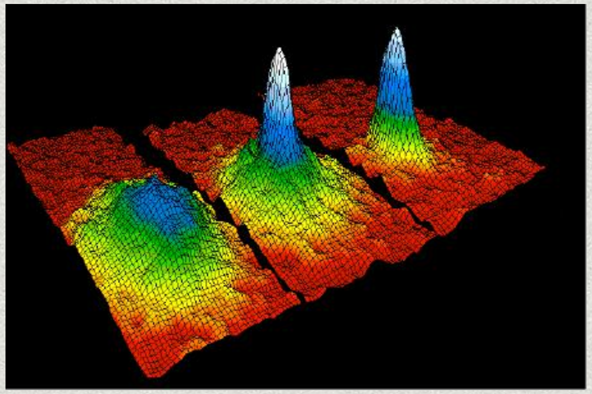
- Dielectric media
- Acoustic in moving fluids
- Gravity waves
- High-refractive index dielectric fluids: “slow light”
- Optic Fibers analogues
- Quasi-particle excitations: fermionic or bosonic quasi-particles in He3
- Non-linear electrodynamics
- “Solid states black holes”
- Perturbation in Bose-Einstein condensates
- Graphene

Hydrodynamical models
Theorem: linearised perturbations on a inviscid, irrotational fluid with barotropic EOS move like fields on a curved spacetime

C.Barcelo, S.L and M.Visser,
“Analogue gravity”
Living Rev.Rel.8,12 (2005-2011).

Bose Einstein Condensates as an Analogue Gravity example

A BEC is quantum system of N interacting bosons in which most of them lie in the same single-particle quantum state ($T<T_c \sim 100$ nK, $N_{atoms} \sim 10^5 \div 10^6$)



It is described by a many-body Hamiltonian which in the limit of dilute condensates gives a non-linear Schrödinger equation

(a=s-wave scattering length)

$$i\hbar \frac{\partial}{\partial t}\hat{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\hat{\Psi} - \mu\hat{\Psi} + \kappa|\hat{\Psi}|^2\hat{\Psi}.$$

$$\kappa(a) = \frac{4\pi a\hbar^2}{m}.$$

This is still a very complicate system, so let's adopt a mean field approximation

$$\text{Mean field approximation: } \hat{\Psi}(t,\mathbf{x}) = \psi(t,\mathbf{x}) + \hat{\chi}(t,\mathbf{x}) \quad \text{where} \quad |\psi(t,\mathbf{x})|^2 = n_c(t,\mathbf{x}) = N/V$$

$$\psi(t,\mathbf{x}) = \langle \hat{\Psi}(t,\mathbf{x}) \rangle = \text{classical wave function of the BEC}, \qquad \hat{\chi}(t,\mathbf{x}) = \text{excited atoms}$$

Note that: $\hat{\Psi}|0\rangle = 0$ (atomic Fock vacuum) $\qquad \hat{\Psi}|\Omega\rangle \neq 0$ (ground state)

The ground state is the vacuum for the collective excitations of the condensate (quasi-particles) but this an inequivalent state w.r.t. the atomic vacuum. They are linked by Bogoliubov transformations.

BEC PERTURBATION THEORY

By direct substitution of the mean field ansatz in the non-linear Schrödinger equation gives

$$i\hbar\,\frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + \kappa|\psi|^2\right)\psi + 2\kappa\left(\tilde{n}\psi + \tilde{m}\psi^*\right)$$

Background dynamics

$$i\hbar\,\frac{\partial}{\partial t}\widehat{\chi} = \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + 2\kappa n_T\right)\widehat{\chi} + \kappa m_T\,\widehat{\chi}^\dagger$$

Excitations dynamics

$$n_c \equiv |\psi(t,\mathbf{x})|^2\,; \qquad m_c \equiv \psi^2(t,\mathbf{x});$$
$$\tilde{n} \equiv \langle\widehat{\chi}^\dagger\widehat{\chi}\rangle; \qquad \tilde{m} \equiv \langle\widehat{\chi}\widehat{\chi}\rangle;$$
$$n_T = n_c + \tilde{n}; \qquad m_T = m_c + \tilde{m}.$$

These are the so called Bogoliubov-de Gennes equations
The first one encodes the BEC background dynamics
The second one encodes the dynamics for the quantum excitations

The equations are coupled via the so called anomalous mass m and density ñ.
Which we shall neglect for the moment…

LET'S CONSIDER QUANTUM PERTURBATIONS OVER THE BEC BACKGROUND AND ADOPT THE "QUANTUM ACOUSTIC REPRESENTATION" (BOGOLIUBOV TRANSFORMATION)

$$\widehat{\chi}(t,\mathbf{x}) = e^{-i\theta/\hbar}\left(\frac{1}{2\sqrt{n_c}}\,\widehat{n}_1 - i\frac{\sqrt{n_c}}{\hbar}\,\widehat{\theta}_1\right)$$

FOR THE PERTURBATIONS ONE GETS THE SYSTEM OF EQUATIONS

$$\partial_t\widehat{n}_1 + \frac{1}{m}\boldsymbol{\nabla}\cdot\left(\widehat{n}_1\,\boldsymbol{\nabla}\theta + n_c\,\boldsymbol{\nabla}\widehat{\theta}_1\right) = 0,$$
$$\partial_t\widehat{\theta}_1 + \frac{1}{m}\boldsymbol{\nabla}\theta\cdot\boldsymbol{\nabla}\widehat{\theta}_1 + \kappa(a)\,n_1 - \frac{\hbar^2}{2m}\,D_2\widehat{n}_1 = 0.$$

WHERE D_2 IS A REPRESENTS A SECOND-ORDER DIFFERENTIAL OPERATOR: THE LINEARIZED QUANTUM POTENTIAL

$$D_2\,\widehat{n}_1 \equiv -\frac{1}{2}n_c^{-3/2}\left[\nabla^2(n_c^{+1/2})\right]\widehat{n}_1 + \frac{1}{2}n_c^{-1/2}\,\nabla^2(n_c^{-1/2}\,\widehat{n}_1).$$

Acoustic geometries and the fate of Lorentz invariance

For very long wavelengths the terms coming from the linearized quantum potential D_2 can be neglected.

$$\Delta\theta_1 \equiv \frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\,\partial_\nu\right)\widehat{\theta}_1 = 0,$$

So the metric is

$$c_s = \frac{\hbar}{m}\sqrt{4\pi\rho a}$$

$$g_{\mu\nu}(t,\mathbf{x}) \equiv \frac{c_s}{\bar{\bar{\lambda}}}\begin{bmatrix} -\left(c_s^{\ 2} - \mathrm{v}_0^2\right) & \vdots & -\left(\mathrm{v}_0\right)_j \\ \cdots & \cdot & \cdots \\ -\left(\mathrm{v}_0\right)_i & \vdots & \delta_{ij} \end{bmatrix} = \frac{n_0}{c_s m}\begin{bmatrix} -\left(c_s^{\ 2} - \mathrm{v}_0^2\right) & \vdots & -\left(\mathrm{v}_0\right)_j \\ \cdots & \cdot & \cdots \\ -\left(\mathrm{v}_0\right)_i & \vdots & \delta_{ij} \end{bmatrix}$$

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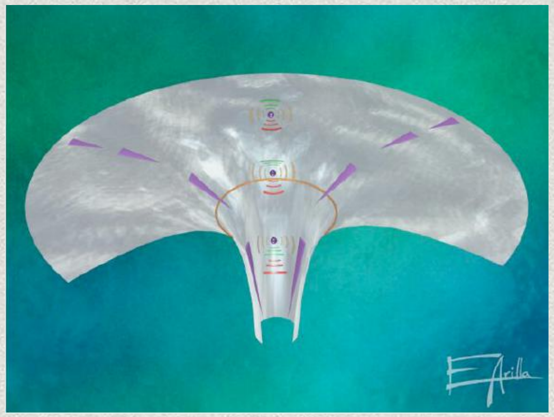

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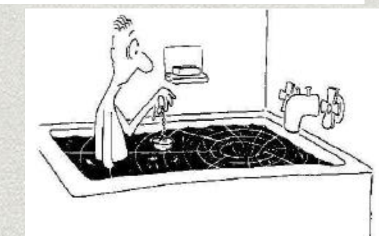
This opened up the possibility for lab analogue of Black holes!

Black holes can be made in a lab!!!

(Steinhauer, first test of Hawking radiation in BEC black hole. Nat. Phys. 2014)



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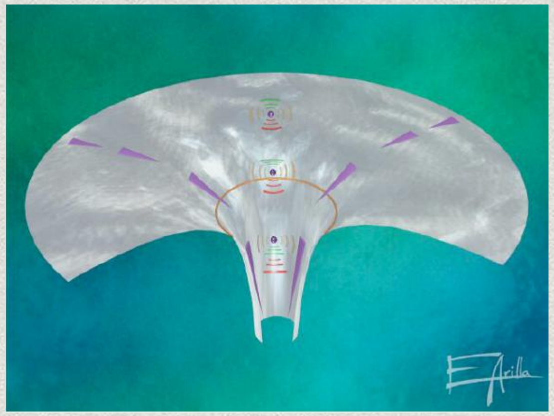

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$$g_{\mu\nu}(t,\mathbf{x}) \equiv \frac{c_s}{\lambda}\begin{bmatrix} -\left(c_s{}^2 - v_0^2\right) & \vdots & -(v_0)_j \\ \cdots & \cdot & \cdots \\ -(v_0)_i & \vdots & \delta_{ij}\end{bmatrix} = \frac{n_0}{c_s m}\begin{bmatrix} -\left(c_s{}^2 - v_0^2\right) & \vdots & -(v_0)_j \\ \cdots & \cdot & \cdots \\ -(v_0)_i & \vdots & \delta_{ij}\end{bmatrix}$$

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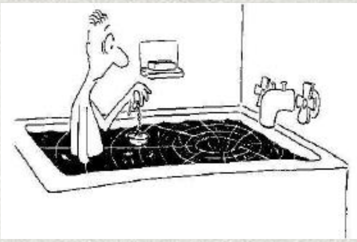


If instead of neglecting the quantum potential we adopt the eikonal approximation (high-momentum approximation) we find, as expected, deviations from the Lorentz invariant physics of the low energy phonons.

E.g. the dispersion relation for the BEC quasi-particles is

$$\omega^2 = c_s^2 k^2 + \left(\frac{\hbar}{2m}\right)^2 k^4$$

This (Bogoliubov) dispersion relation (experimentally observed) actually interpolates between two different regimes depending on the value of the fluctuations wavelength

$\lambda = 2\pi/|k|$ with respect to

the "acoustic Planck wavelength"

$\lambda_C = h/(2m_{cs}) = \pi\xi$ with ξ=healing length of BEC$=1/(8\pi\varrho a)1/2$

For $\lambda \gg \lambda_C$ one gets the standard phonon dispersion relation $\omega \approx c|k|$

For $\lambda \ll \lambda_C$ one gets instead the dispersion relation for an individual gas particle $\omega \approx (\hbar^2 k^2)/(2m)$ (breakdown of the continuous medium approximation)

SUPERLUMINAL WARP DRIVES IN A LORENTZ BREAKING UNIVERSE.

“As if you could kill time without injuring eternity.”
Henry David Thoreau, Walden

WD STABILITY IN A UV LORENTZ BREAKING WORLD

A. Coutant, S. Finazzi, S. Liberati and R. Parentani,
``On the impossibility of superluminal travel in Lorentz violating theories,''
arXiv:1111.4356 [gr-qc].
Phys.Rev. D85 (2012) 064020)

The calculation we performed for the warp drive was based on Relativistic QFT

What if we are indeed living in an emergent spacetime and Local Lorentz Invariance is broken in the UV? Or if we simulate a supersonic warp drive in a lab?

It is by know understood that modified LIV dispersion relations

- Remove Cauchy horizons
- Regulate fluxes emitted by white holes

What we can say about superluminal travel and chronology protection in a LIV world?

A SUPERLUMINAL WARP DRIVE IN A LIV QFT

$$ds^2 = -c^2dt^2 + [dx - v(r)dt]^2 + dy^2 + dz^2$$

$r \equiv \sqrt{(x - v_0 t)^2 + y^2 + z^2}$ is the distance from the center of the bubble

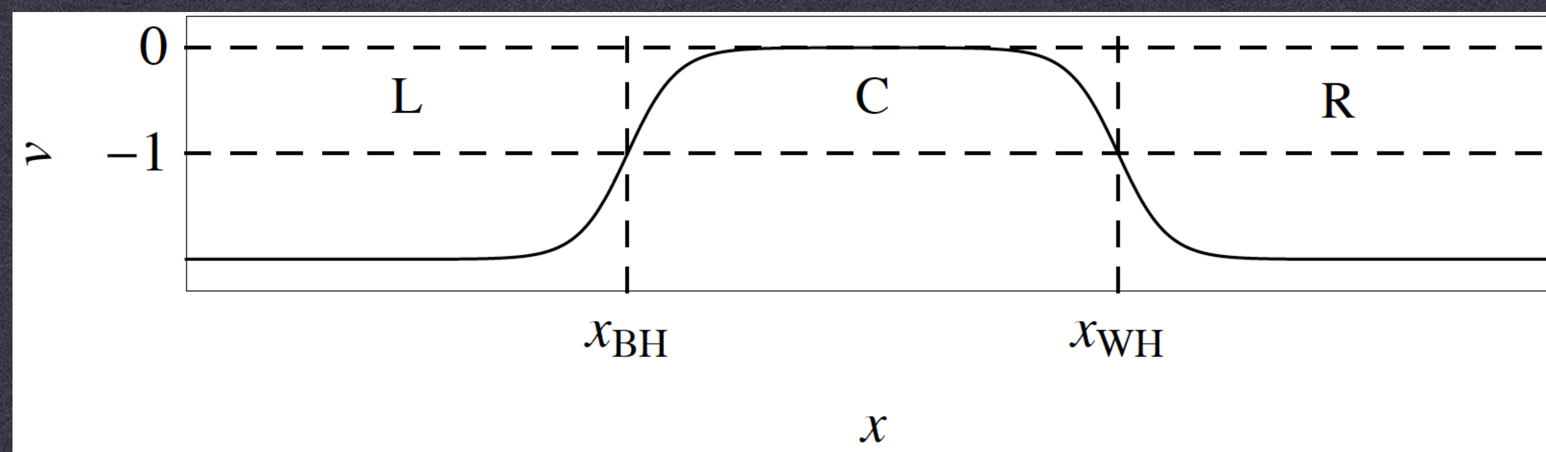
v_0 the warp-drive velocity $v = v_0 f(r)$

f is a suitable smooth function satisfying $f(0) = 1$ and $f(r) \to 0$ for $r \to \infty$.

1+1 →

$$ds^2 = -c^2dt^2 + [dX - V(X)dt]^2$$

$$X = x - v_0\, t,\ V(X) = v_0(f(X) - 1) < 0.$$



$$S_\pm = \frac{1}{2}\int d^2x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \pm \frac{(h^{\mu\nu}\partial_\mu\partial_\nu\phi)^2}{\Lambda^2}\right]$$

$$\text{where } h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu.$$

- (NOTE Λ here is not the CC is just UV Lorentz breaking scale!)

EQUATION OF MOTION AND MODE ANALYSIS

$$\left[(\partial_t + \partial_X V)(\partial_t + V\partial_X) - \partial_X^2 \pm \frac{1}{\Lambda^2}\partial_X^4\right]\phi = 0$$

- and the dispersion relation

$$(\omega - V k_\omega)^2 = k_\omega^2 \pm \frac{k_\omega^4}{\Lambda^2} \equiv \Omega_\pm^2$$

$$\omega = \text{Killing frequency}$$

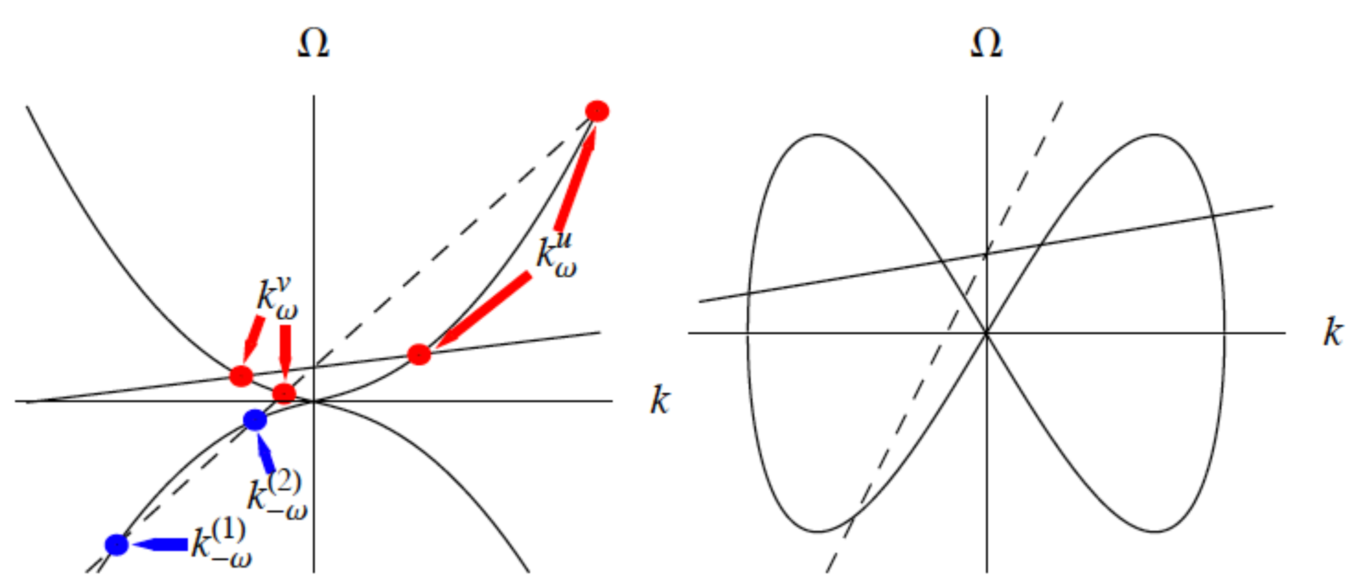


FIG. 2. Graphical solution of Eq. (5) for super (left panel), and subluminal dispersion (right panel). In both panels, the straight lines represent $\omega - Vk$ for $|V| < 1$ (solid) and $|V| > 1$ (dashed). The curved lines represents $\pm\Omega_\pm(k)$. On the left, red (blue) dots refer to roots with positive (negative) Ω_+ which correspond to positive (negative) norm modes.

Superluminal case
κ<<Λ

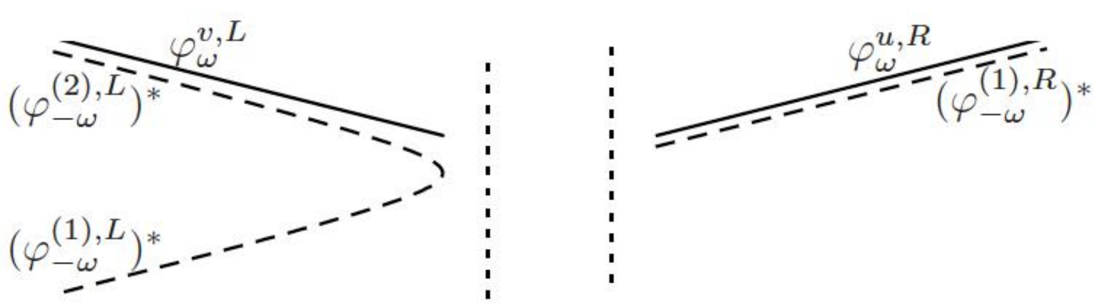


FIG. 3. Asymptotic decomposition in plane waves $\varphi^{(i),L/R}_\omega$ of the *in* mode $(\phi^{(1),\text{in}}_{-\omega})^*$. Note that only $\varphi^{(1),L}_{-\omega}$ has group velocity directed toward the horizons, with wavevector $k^{(1)}_\omega$.

A NEW DIVERGENCE IN THE IR!

$$\begin{pmatrix} \phi_\omega^{u,\mathrm{in}} \\ \left(\phi_{-\omega}^{(1),\mathrm{in}}\right)^* \\ \left(\phi_{-\omega}^{(2),\mathrm{in}}\right)^* \end{pmatrix} = \begin{pmatrix} \alpha_\omega^u & \beta_{-\omega}^{(1)} & \beta_{-\omega}^{(2)} \\ \beta_\omega^{(1)} & \alpha_{-\omega}^{(1)} & A_{-\omega} \\ \beta_\omega^{(2)} & \tilde{A}_{-\omega} & \alpha_{-\omega}^{(2)} \end{pmatrix} \begin{pmatrix} \phi_\omega^{u,\mathrm{out}} \\ \left(\phi_{-\omega}^{(1),\mathrm{out}}\right)^* \\ \left(\phi_{-\omega}^{(2),\mathrm{out}}\right)^* \end{pmatrix}$$

- **the above Bogoliubov coefficients can be computed via standard techniques. At sufficiently high frequencies the beta coefficients go to zero (no mode mixing) because $k^{(1)}$ and $k^{(2)}$ become complex. No UV divergencies in the SET.**

- **One can compute the Stress Energy Tensor which will be the standard relativistic one plus a Lorentz breaking term**

$$T_{\mu\nu}^{(\Lambda)} = \frac{1}{\Lambda^2}\left[h^{\alpha\beta}\left(\phi_{,\alpha\beta}\phi_{,\mu\nu} + \phi_{,\mu\nu}\phi_{,\alpha\beta}\right) - \frac{1}{2}\left(h^{\alpha\beta}\phi_{,\alpha\beta}\right)^2 g_{\mu\nu}\right]$$

- **For computing the Renormalized Stress Energy Tensor one can look at the asymptotic region on the right of the WH horizon where the geometry is stationary and homogeneous. Hence the RSET can be computed by simple normal ordering of the “out” creation and destruction operators.**

- **The final outcome is that the energy density in the right asymptotic region grows as**

$$\mathcal{E} \propto \Lambda \int_{1/T} d\omega \left[\bar{n}_\omega^{(u)} + \bar{n}_{-\omega}^{(1)}\right] \propto \Lambda\kappa^2 T.$$

Where T is the lapse of time since the WD creation. Now, by quantum inequalities $\kappa \gtrsim 10^{-2}\, T_{Pl}$ so, unless Λ is very far away from the Planck scale, the WD is again unstable.

Summary for LIV Warp-drives

- The result for the superluminal case can be understood as a sort of emission from the WH stimulated by the Hawking flux emitted by the BH. Similar to a Cherenkov instability.
- The case of subluminal dispersion relation (not explicitly shown here) is even more unstable, exponentially unstable, due to the so called BH laser instability (the WD acts as a resonant cavity for positive norm modes)
- Hence it seems that even in LIV world superluminal travel is semiclassically unstable and WD cannot be used to built time machines.
- Even as a low energy limit the "speed of light postulate" seems to strongly constrain the stability of FTL devices…

CONCLUSIONS

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Is this a "preemptive" chronology protection at work?

- This seem to suggest that QG will not only enforce a trivial structure $R \times \Sigma^3$ (no time machines) but also forbids the stability of superluminal warp drives dynamically generated from flat spacetime. Why?

  And what about wormholes and Krasnikov tubes?
- Maybe once we shall understand QG we shall see this has to be the case…

“Only time (whatever that may be) will tell…”

Stephen Hawking, A Brief History of Time