Probing Regular Black Hole Spacetime with Scalar Field

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Motivation

- Having naked singularity solution or irregular horizon when scalar field is present was predicted already by J. E. Chase in 1970,

  "Chase Theorem":

  Any static spherically symmetric vacuum solution minimally coupled to scalar field can not have a regular horizon, if there exists any horizon it would also be the locus of a true singularity.
- Nonlinear Electrodynamics as a good candidate for non-vacuum situation, analysis of gravitating case rather than perturbative approach

J. E. Chase in 1970,Commun. math. Phys, 19, 276

Fields Equations

Lagrangian describing a scalar field minimally coupled to gravity and also Nonlinear Electrodynamics

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[\mathcal{R} + \nabla_\mu\varphi\nabla^\mu\varphi + \mathcal{L}(F)]$$

- φ Real Massless Scalar Field
- $F = F_{\mu\nu}F^{\mu\nu}$ Electromagnetic Field Invariant

$$\Downarrow$$

$$G^{\mu}{}_{\nu} = {}^{\mathrm{SF}}T^{\mu}{}_{\nu} + {}^{\mathrm{NE}}T^{\mu}{}_{\nu}$$

Static Scalar Field

Assuming the static spherically symmetric metric

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + R(r)^2\,(d\theta^2 + \sin^2\theta\,d\phi^2)$$

The energy momentum tensor

$${}^{\mathrm{SF}}T^{\mu}{}_{\nu} = \frac{f\,\varphi_{,r}^2}{2}\,diag\,\{-1, 1, -1, -1\}$$

The wave equations of Radial scalar field

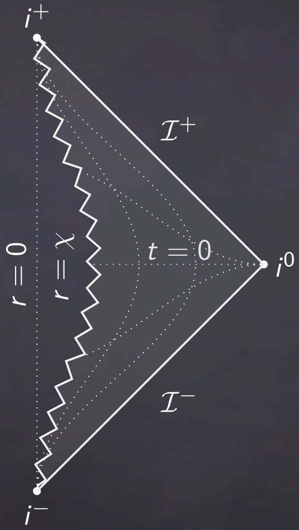
$$\Box\varphi = 0$$

where $\Box$ is a standard d'Alembert operator.

$$\begin{aligned} f(r) &= 1 \\ R(r) &= r^2 - \chi^2 \\ \varphi(r) &= \frac{1}{\sqrt{2}}\ln\left\{\frac{r-\chi}{r+\chi}\right\} \end{aligned}$$

Properties of Scalar Field Solution

- $r \to \infty$ the scalar field is vanishing
- Asymptotically flat
- The solution is representing a time–like naked singularity
- Quantization of the spacetime to remove the singularity
- In dynamic case for some parameters there exist event horizon!



i^+
$\mathcal{I}^+$
$r = 0$
$r = \chi$
$t = 0$
i^0
$\mathcal{I}^-$
i^-

Janis, Newman and Winicour Solution

$$\mathrm{d}s^2 = -f(\tilde{R})dt^2 + \frac{1}{f(\tilde{R})}\left\{d\tilde{R}^2 + (\tilde{R}^2 - M^2)d\Omega^2\right\},$$

in which

$$\begin{aligned} f(\tilde{R}) &= \left[\frac{\tilde{R}-M}{\tilde{R}+M}\right]^{\frac{1}{\mu}} \\ \phi &= \sqrt{(\mu^2-1)/2}\ \ln f(\tilde{R}) \end{aligned}$$

- When $\mu \to \infty$, it would be our solution
- The event horizon is a singular point

Phys. Rev. Lett. **20**, 878 (1968) ;
I.Z. Fisher, Zh. Eksp. Teor. Fiz. **18**, 636 (1948)

Introduction to Nonlinear Electrodynamics

The idea of Non-Linear Electrodynamics (NED) is about a century old but it was made popular in 1930s by Born and Infeld.
The main goal was solving the point charge singularity:

$$\frac{q}{r^2} \Rightarrow \frac{q}{r^2 + a^2}$$

- Resolve the spacetime singularity $\Rightarrow$ Regular Black Holes
- Wide application in different theories

Different forms of Nonlinear Electrodynamics

- Born-Infeld (BI) theory
- Hoffmann-Born-Infeld (HBI) theory
- Logarithmic Lagrangian
- Power Maxwell (PM)

and many many other models!

The energy momentum tensor

$$T^{\mu}_{\nu} = \frac{1}{2}\{\delta^{\mu}_{\nu}\mathcal{L} - (F_{\nu\lambda}F^{\mu\lambda})\mathcal{L}_F\}$$

$$\mathcal{L}_F = \frac{d\mathcal{L}(F)}{dF}$$

The modified Maxwell field equations

$$\partial_{\mu}(\sqrt{-g}\mathcal{L}_F F^{\mu\nu}) = 0$$

Electromagnetic Field Invariant

$$F = F_{\mu\nu}F^{\mu\nu}$$

Born-Infeld

Born and Infeld Lagrangian:

$$\mathcal{L}_{BI} = 4\beta^2 \left(1 - \sqrt{1 + \frac{F}{2\beta^2}}\right)$$

- β is BI parameter
- $\lim_{\beta\to\infty} \mathcal{L}_{BI} = -F$ (Maxwell limit)
- strong field limit $\to \mathcal{L}_{BI} \sim \sqrt{F}$
- for Electric Charge q_e, $F = -\frac{2{q_e}^4}{r^4 + {q_e}^4/\beta^2} \Rightarrow$ "*Regular*"

Regular Black Holes

Generic solution for having RBH

$$f(r) = 1 - \frac{2C_1\, r^{\sigma-1}}{(r^{\beta} + q)^{\frac{\sigma}{\beta}}}$$

where $\sigma > 1, \beta > 0$.

- If $q = 0$ the solution is Schwarzschild
- If $\sigma = 3$ and $\beta = 2$ the solution is Bardeen
- If $\sigma = 3$ and $\beta = 3$ the solution is Hayward

- $F \Rightarrow$ "Singular"
- Spacetime and the EMT $\Rightarrow$ "Regular"

Square Root Model $\mathcal{L} = -\sqrt{F}$

Maxwell 2-form

$$\mathbf{F} = F_{\theta\phi}\,\mathrm{d}\theta \wedge \mathrm{d}\phi,$$

where $F_{\theta\phi} = q_m \sin\theta$ and $F = \frac{2\,{q_m}^2}{R^4}$

The energy momentum tensor

$${}^{\mathrm{NE}}T^{\mu}{}_{\nu} = diag\left\{-\frac{\sqrt{F}}{2}, -\frac{\sqrt{F}}{2}, 0, 0\right\}$$

The corresponding solution is

$$\begin{aligned} f(r) &= \alpha - \frac{2m}{r} \\ R(r) &= r \end{aligned}$$

where $\alpha = 1 - \sqrt{2}\,q_m$

Properties of NE Solution

- $F = \frac{2\, {q_m}^2}{r^4} \Rightarrow$ "Singular"
- The corresponding metric is not asymptotically flat
- It is a Black Hole solution
- Similar to the solution of geometry outside the core of so-called global monopole

Scalar Field and NE

Metric functions

$$\begin{aligned} f(r) &= \frac{C_0}{\sqrt{2}\,\chi} \\ R(r) &= r^2 - \chi^2 \end{aligned}$$

Constraint Eq: $G^t{}_t - ({}^{\mathrm{NE}}T^t{}_t + {}^{\mathrm{SF}}T^t{}_t) = 0,$

$$f\left(\frac{R_{,r}}{R}\right)^2 + \frac{R_{,r}}{R}\,f_{,r} - \frac{1}{R^2} - f\,\frac{R_{,rr}}{R} + \frac{q_m}{\sqrt{2}}\frac{1}{R^2} = 0$$

which leads to

$$q_m = \frac{C_0}{\chi} - \sqrt{2}$$

Properties: Scalar Field and NE

- If $f = 1$ then $q_m = 0$
- No Horizon unless scalar field vanishes identically
- Kretschmann scalar is diverging at $r = \chi$

Different Model of NE

Scalar Field + Regular Models (magnetic charge)

{Bardeen, Hayward, Generic Model}

Long Eq.s

Not Explicit Solu.

Several constraints

$\Longrightarrow$ NOT EVEN BLACK HOLE SOLUTION

No Black hole solutions

Different coordinates:

$$ds^2 = -f(r)\,dt^2 + \frac{h(r)}{f(r)}\,dr^2 + r^2 d\Omega^2$$

Solution:

$$f = \frac{f_0\,h}{\sqrt{r^3\,h_{,r}}}$$

$$\sqrt{-g} = \sqrt{h}\,r^2 \sin\theta$$

Ricci Scalar:

$$Ricci = \frac{2}{r^2} + \frac{f_0}{4\sqrt{r^5\,h_{,r}}}\left\{2\,r\left(\frac{h_{,r}}{h}\right)^2 + \frac{h_{,r} - r\,h_{,rr}}{h}\right.$$

$$\left. + \frac{1}{r3\,(h_{,r})^2}\left[r^2\left(2(h_{,rrr})^3\,h_{,r} - 3(h_{,rr})^2\right) + (r\,h_{,r}{}^2)_{,r}\right]\right\}$$

Assumptions:

- having horizon at $r = r_0 \to h(r_0) = 0$
- $h_{,r}(r_0)$ is finite and nonzero

Results:

- Vanishing h means diverging curvature scalars
- Highlighted terms would be zero:

$$h = -\frac{h_0}{r^2 + h_1} \Rightarrow \begin{cases} \text{Imaginary metric} \\ f \sim 1/r^2 \end{cases}$$

Wave equation for Black hole solutions close to event horizon

Scalar perturbation obeying the Klein-Gordon equation,

$$\Box \Psi(t, r, \theta, \phi) = 0$$

for the Static Spherical Symmetric (with $R = r$)

$$f\,\Psi_{,rr} + \left(f_{,r} + \frac{2f}{r}\right)\Psi_{,r} - \frac{\Psi_{,tt}}{f} + \frac{1}{r^2}\left\{\Psi_{,\theta\theta} + \cot\theta\,\Psi_{,\theta} + \frac{\Psi_{,\phi\phi}}{\sin^2\theta}\right\} = 0$$

By applying separation variables

$$\Psi(t, r, \theta, \phi) = e^{-i\,\omega t}\, \frac{\psi(r)}{r}\, Y_l^m(\theta, \phi)$$

Radial equation is

$$f\,\psi_{,rr} + f_{,r}\,\psi_{,r} - \left(\frac{l(l+1)}{r^2} - \frac{\omega^2}{f} + \frac{f_{,r}}{r}\right)\psi = 0$$

Since we are interested in perturbation around horizon, we assume

$$f = A\,(r - r_0) + O((r - r_0)^2)$$

In the end

$$\psi = (r - r_0)^{\frac{-I\omega}{A}} \left[\psi_0 r^{n_1} {}_2F_1\left(a_1, b_1; n_1; \frac{r}{r_0}\right) + \psi_1 r^{n_2} {}_2F_1\left(a_2, b_2; n_2; \frac{r}{r_0}\right)\right]$$

- $(r - r_0)^{\frac{-I\omega}{A}}$ is the dominant term, so $\psi_{,r} \sim (r - r_0)^{-1}$
- diverging stress energy momentum tensor of the scalar field
- generic test scalar field energy momentum tensor blows up on the horizon

Conclusions

- Chase theorem applies for NE sources
- test scalar field EMT blows up generally on event horizon
- gravitating scalar fields with NE produce singular horizons
- explicit solution for square root Lagrangian mimicks global monopole with modified singularity

THANK YOU