Dynamical wormholes in Robinson–Trautman class

O. Svítek[1] and T. Tahamtan[1,2]

[1]Institute of Theoretical Physics, Charles University, Czech Republic
[2]Silesian University, Opava, Czech Republic

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The thin-shell wormhole

- cutting and gluing spacetimes
- distributional source induced on the gluing hypersurface
- well-defined throat due to the process of creation

The bridge-type wormhole

- does not contain distributional source
- throat can be localized using techniques similar to quasilocal horizons

Vacuum Robinson–Trautman (RT) metric (beginning of ’60)

Twistfree, shearfree and diverging null geodesic congruence

$$\mathrm{d}s^2 = -2H\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + \frac{r^2}{P^2}(\mathrm{d}y^2 + \mathrm{d}x^2),$$

where $2H = \Delta(\ln P) - 2r(\ln P)_{,u} - 2m/r$
and $\Delta \equiv P^2(\partial_{xx} + \partial_{yy})$

- Gaussian curvature of x, y spaces $K = \Delta \ln P(u, x, y)$
- Minima of K constrain the position of horizon from above
- EFE $\Rightarrow$ Parabolic PDE

$$\Delta\Delta(\ln P) + 12m(\ln P)_{,u} = 0$$

constrains asymptotic behavior

Wormhole setup

- gluing together two identical copies of RT spacetime (its external part) along timelike hypersurface $r = f(u)$
- ensuring the wormhole throat is above the horizon
- computing extrinsic curvature on the throat to derive the induced energy momentum tensor
- interpreting the matter induced on the throat and investigating energy conditions

Induced metric on the throat

$$h_{ab} = g_{ab} - n_a n_b$$

n_a is normal to the hypersurface

$$n_a = (-af_{,u}, a, 0, 0)$$

and $a = (2H + 2f_{,u})^{-1/2}$

Extrinsic Curvature

$$\mathcal{K}_{ab} = h_a^c h_b^d \nabla_c n_d$$

and its nonzero components are

$$\{\mathcal{K}_{\tau\tau}, \mathcal{K}_{\tau x} = \mathcal{K}_{x\tau}, \mathcal{K}_{\tau y} = \mathcal{K}_{y\tau}, \mathcal{K}_{xx} = \mathcal{K}_{yy}\}$$

Surface EMT

According to Darmois–Israel formalism EMT $S_{\mu\nu}$ induced on the throat hypersurface is

$$8\pi S_{\mu\nu} = tr[\mathcal{K}]h_{\mu\nu} - [\mathcal{K}_{\mu\nu}]$$

Interpretation: two dust streams $T_{\mu\nu} = \sum_{i=1}^{2} \rho_i v_{i\mu} v_{i\nu}$

- normalization of velocities: $v_i^{\tau} v_{i\tau} + v_i^{x} v_{ix} + v_i^{y} v_{iy} = -1$
- matching $S_{\mu\nu}$ with $T_{\mu\nu}$

This leads to constraint on fluid EMT components

$$(T_{\tau x})^2 + (T_{\tau y})^2 = T_{\tau\tau} T_{xx}$$

Some Results

1. The constraint on EMT translates into the following condition for the function f

$$(\mathcal{K}_{\tau x})^2 + (\mathcal{K}_{\tau y})^2 = 2\mathcal{K}_{xx}(\mathcal{K}_{\tau\tau} - \frac{P^2}{f^2}\mathcal{K}_{xx})$$

Especially effective for asymptotic values of f

2. The energy densities for two streams with arbitrary directions

$$\begin{aligned} \rho_1 &= -\frac{4P^2\mathcal{K}_{xx}}{f^2}\left[\frac{(v_{2\tau}^2 - 1) - \beta^2(2\,v_{2\tau}^2 - 1)}{v_{2\tau}^2 - 1}\right] \\ \rho_2 &= -\frac{4P^2\,\beta^2}{f^2}\frac{\mathcal{K}_{xx}}{v_{2\tau}^2 - 1} \end{aligned}$$

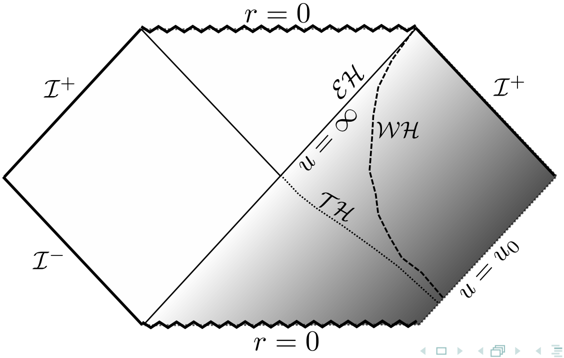
their sign is completely determined by the sign of $\mathcal{K}_{xx}$.
Bounds on horizon position $\Rightarrow$ both densities are negative

Asymptotic Analysis

Using

- Asymptotic behavior of vacuum RT spacetime (exponential decay of nonsphericity)
- Result 1 (constraint on the throat position)

wormhole asymptotically ($u \to \infty$) approaches the Schwarzschild one with the throat approaching $r = 2m$ from above



Higher dimensions

Generalization of RT (Podolský & Ortaggio 2006)

$$\mathrm{d}s^2 = -2H\,\mathrm{d}u^2 - 2\,\mathrm{d}u\mathrm{d}r + \frac{r^2}{P^2}\,\gamma_{ij}\,\mathrm{d}x^i\mathrm{d}x^j$$

with

$$2H = \frac{\mathcal{R}}{(D-2)(D-3)} - 2\,r(\ln P)_{,u} - \frac{2\Lambda}{(D-2)(D-1)}\,r^2 - \frac{\mu(u)}{r^{D-3}}$$

Transversal spaces are Einstein and $\mathcal{R}$ their curvature scalar.

In higher dimensions compact Einstein spaces with $\mathcal{R} \leq 0$ exist

$$\Downarrow$$

Wormholes with positive energy density can be constructed

Partial Conclusion

- the wormhole is dynamical and has no symmetries
- good candidate for stability analysis:
  1. asymptotically goes to Schwarzschild-type wormhole
  2. transition to spherical symmetry by GW emission indistinguishable from BH version of RT spacetime
- it is possible to generalize the solutions to higher dimensions

RT solution with minimally coupled imaginary scalar field

RT metric sourced by a purely imaginary scalar field becomes

$$\mathrm{d}s^2 = -\left[(\ln U)_{,u}\, r + \frac{K(x,y)}{U}\right] \mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + \left(U\, r^2 + \frac{C_0^2}{U}\right) \frac{(\mathrm{d}x^2 + \mathrm{d}y^2)}{P(x,y)^2}$$

with $C_0 = const.$, $K = \Delta \ln P$

$$R(u,r) = \sqrt{\frac{U(u)^2 r^2 + C_0^2}{U(u)}} \quad , \quad U(u) = \gamma e^{-\frac{\alpha^2}{4\,C_0^2} u^2 + \eta u}$$

Scalar field

$$\varphi(u,r) = \frac{1}{\sqrt{2}} \ln\left\{\frac{U(u)r - i\,C_0}{U(u)r + i\,C_0}\right\} = \frac{i}{\sqrt{2}} \arg\left(\frac{U\,r - i\,C_0}{U\,r + i\,C_0}\right)$$

The stress energy tensor $\sim (\nabla\varphi)^2 \Rightarrow$ violates all energy conditions.

Curvature scalars

$$RicciSc = \frac{2C_0^2\,U\,(U_{,u}r - k)}{(U^2r^2 + C_0^2)^2}$$

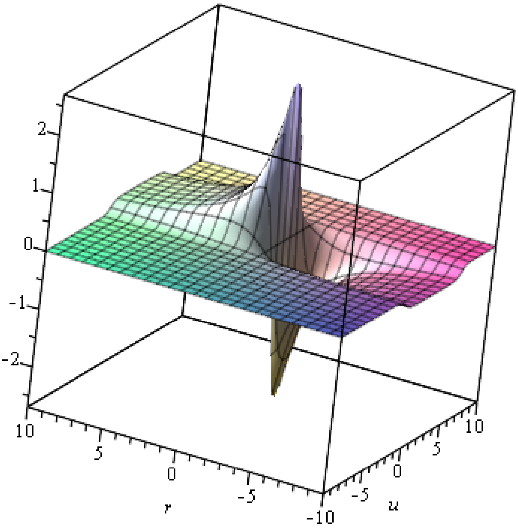
$$Kretschmann = 3(RicciSc)^2$$

- no divergence ⇒ curvature singularities are absent
- $r = 0$ is neither a curvature singularity nor a coordinate one
- we can continue to negative r

Expansion

$$\Theta_{\partial_r} = \frac{2U^2r}{U^2r^2 + C_0^2}$$

- the expansion of congruence ∂_r changes sign at $r = 0$
- on the surface $r = 0, u = const$ (which has a nonzero area) we have both $\Theta_{\partial_r} = 0$ and $\partial_r\Theta_{\partial_r} > 0 \Rightarrow$ genuine wormhole throat satisfying the flare-out condition (Visser & Hochberg)


2
1
0
-1
-2
10
5
0
-5
-10
r
10
5
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u

Asymptotics

The asymptotic form of the metric as $u \to \pm\infty$

$$\mathrm{d}s^2 \sim -\left[\frac{K(x,y)}{U}\right] \mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + \left(\frac{C_0^2}{U\,P^2}\right)(\mathrm{d}x^2 + \mathrm{d}y^2)$$

The two-dimensional metric (for x, y) independent of $r \Rightarrow \Theta_{\partial_r} = 0$

$$\Downarrow$$

The wormhole has genuine Robinson-Trautman behavior for finite times but asymptotically transforms into a Kundt geometry

The asymptotic geometry is characterized by nonzero Ψ_0 and Φ_{00}

$$\Downarrow$$

Kundt-type gravitational and scalar waves near $u = \pm\infty$.

THANK YOU

References

- Nonsymmetric dynamical thin shell wormhole, O. Svitek and T. Tahamtan, Eur. Phys. J. C 78 (2018) 167
- Properties of Robinson–Trautman solution with scalar hair, T. Tahamtan and O. Svitek, Phys. Rev. D 94, 064031 (2016)