Causal set theory and quantum fields

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Outline

- Introduction to causal sets
- Causal sets and scalar fields
- Entanglement entropy
- Summary and comments

Causal Set Theory

S. Surya, Living Reviews in Relativity (2019)
L. Bombelli, J. Lee, D. Meyer, and R.D. Sorkin, **Phys. Rev. Lett. 59, 521**

A causal set (causet) is a locally finite ordered set.

It is a pair $(C, \preccurlyeq)$, given by a set C and a partial order relation $\preccurlyeq$ that is:

Reflexive	$\forall x \in C: x \preccurlyeq x$
Antisymmetric	$\forall x, y \in C: x \preccurlyeq y \preccurlyeq x \Rightarrow x = y$
Transitive	$\forall x, y, z \in C: x \preccurlyeq y \preccurlyeq z \Rightarrow x \preccurlyeq z$
Locally finite	$\forall x, y \in card\{z \in C: x \preccurlyeq z \preccurlyeq y\} < \infty$

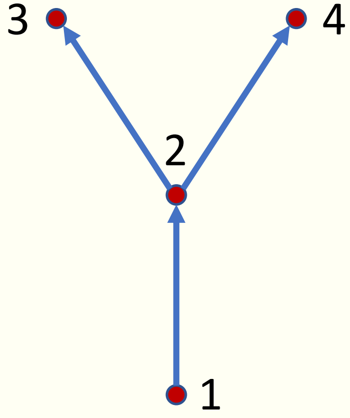
Causal structure gives 9/10 of the metric. Hawking, King, McCarthey, Malament, Levichev

The rest (a conformal factor) can be fixed by **volume** information.
In a causet the volume is naturally associated with the number of elements N

$$\text{VOLUME} + \text{ORDER} = \text{GEOMETRY} \rightarrow \text{NUMBER} + \text{ORDER} = \text{GEOMETRY}$$

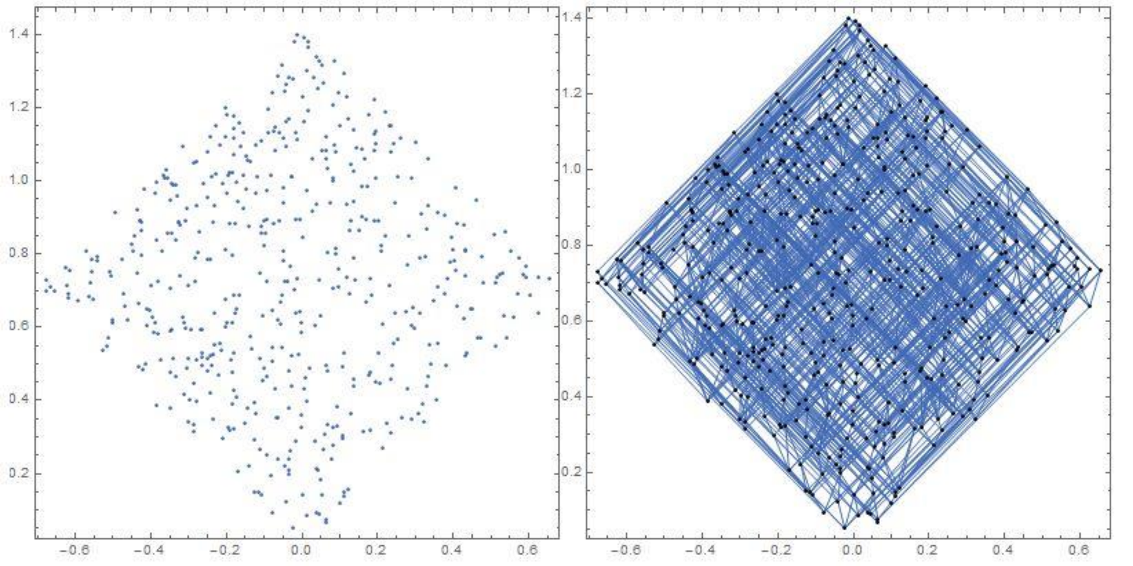
Causal Set Theory

S. Surya, Living Reviews in Relativity (2019)
L. Bombelli, J. Lee, D. Meyer, and R.D. Sorkin, **Phys. Rev. Lett. 59, 521**



$$L = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Sprinkling



- Density $\rho \propto \ell^{-1/d}$
- $< N >= \rho\, V$
- Spacetime volume $V_{st} \propto N$
- Nonlocality

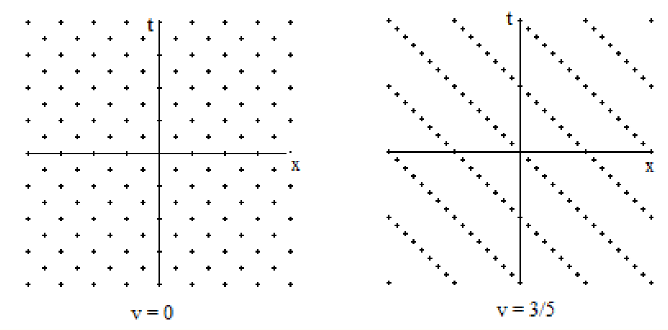
- Fixed background, no dynamics!

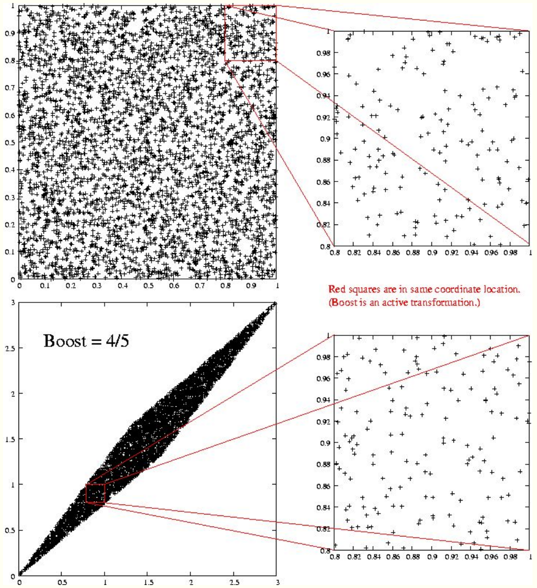
Causal Set Theory

S. Surya, Living Reviews in Relativity (2019)
L. Bombelli, J. Lee, D. Meyer, and R.D. Sorkin, **Phys. Rev. Lett. 59, 521**

A sprinkling of Minkowski st cannot determine a rest frame, an arrow of time, brake translation symmetry or Lorentz symmetry by endowing spacetime with a distinguished lattice.

F. Dowker and R.D. Sorkin arXiv:1909.06070 [gr-qc]; L. Bombelli, J. Henson and R. D. Sorkin, **Mod.Phys.Lett. A24 (2009) 2579-2587**.





Images from David P. Rideout's webpage.

Entanglement Entropy

- Black hole entropy
- Gravity and EE T. Jacobson (2012, 2016); N. Lashkari, M. B. McDermott and M. Van Raamsdonk (2014)

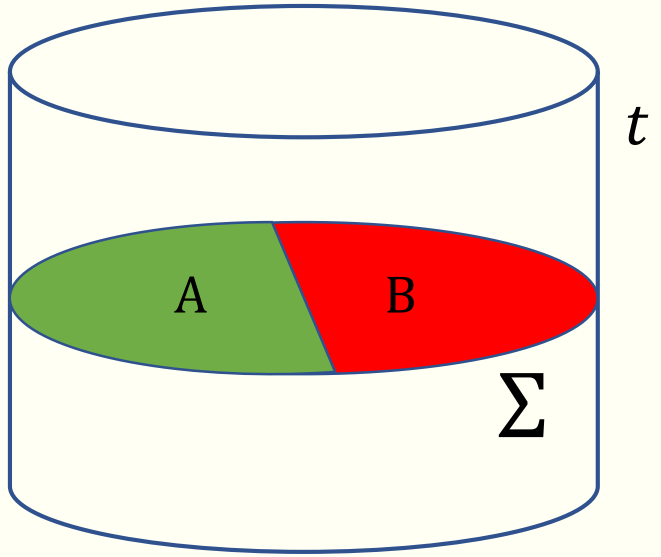
The study of entanglement entropy in QFT is the study of Planck scale physics

Vacuum state ρ_Σ

$$\rho_B = \mathrm{Tr}_A \rho_\Sigma$$

$$S_B = -\mathrm{Tr}\rho_B \log \rho_B$$

$$S_A = S_B = \infty$$



Correlations at arbitrarily small scales.

UV cutoff ℓ $\quad S_{A,B} \propto \frac{\mathrm{Area}(\partial A)}{\ell^{d-2}}$

Entanglement Entropy

- Area law from QG-inspired models:
  - Hard spatial cutoff;
  - Bandlimited QFT (also volume-law) J. Pye, W. Donnelly, A. Kempf, Phys.Rev. D92 (2015) no.10, 105022;
  - Lorentz violating theories $(\partial_t + f(\partial_x))\phi = 0$, D. Nesterov, S.N. Solodukhin, Nucl.Phys. B842 (2011) 141-171;
  - Nonlocal scalar fields $f(\Box)\,\phi = 0$ (continuum limit of CS d'Alembertian, string-field theory)

    D. Nesterov, S.N. Solodukhin, Nucl.Phys. B842 (2011) 141-171

    D.M.T. Belenchia, A. Benincasa, ML, S. Liberati, Class.Quant.Grav. 35 (2018) no.7, 074002
  - New examples? ML, J. Pye (work in progress)

Free Scalar Fields and Entanglement Entropy

A causal set is intrinsically spacetime discrete without the analogue of a Cauchy hypersurface where to define states and initial data.

- New algorithm to compute EE
- New method to quantize (free) fields

Free Scalar Fields and Entanglement Entropy

A causal set is intrinsically spacetime discrete without the analogue of a Cauchy hypersurface where to define states and initial data.

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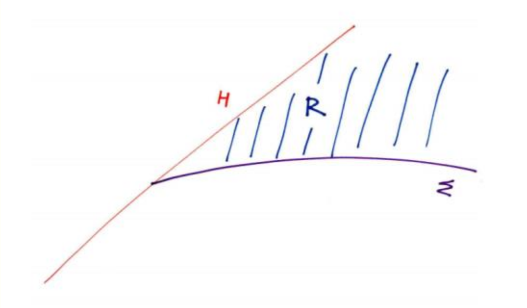
Spacetime entropy R.D. Sorkin, J.Phys.Conf.Ser. 484 (2014) 012004

- $i\Delta = \mathrm{G^R} - (\mathrm{G^R})^T$ Pauli-Jordan, $\Delta = [\phi, \phi]$
- $W = \langle\phi\phi\rangle$ Wightman,
  $\Delta = 2\,\Im(W)$; $\Re(W)$ specifies the vacuum.

$W \cdot v = i\lambda\, \Delta \cdot v$ $\quad\quad$ $\Delta\, v \neq 0 \;\rightarrow S = \sum \lambda \log|\lambda|$



SJ quantization R.D. Sorkin, Int.J.Geom.Meth.Mod.Phys. 14 (2017) no.08, 1740007

Cannot split positive/negative frequencies.

$$\Box \rightarrow G^R \rightarrow \Delta \rightarrow [\phi, \phi] \xrightarrow{freq} a \rightarrow |0\rangle \rightarrow W$$

- $G^R \rightarrow \Delta \rightarrow W_{SJ}$

$$W_{SJ} = Pos(i\Delta)$$

- $W - W^* = i\Delta$
- $W \geq 0$
- $W\, W^* = 0$

W specifies completely the free theory.

Free Scalar Fields and Entanglement Entropy

- Discretized retarded Green functions. S. Johnston (2008)

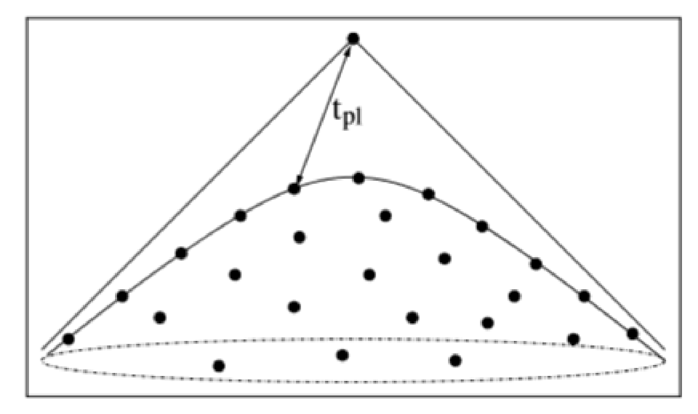
$$G^{(2)}_{xy} = \frac{1}{2}\, C_{xy}$$
$$G^{(3)}_{xy} = \frac{1}{2\pi}\left(\frac{\pi\rho}{12}\right)^{1/3}\left((C+\mathbb{I})^2\right)^{-1/3}_{xy}$$
$$G^{(4)}_{xy} = \frac{\sqrt{\rho}}{2\pi\sqrt{6}} L_{xy}$$

- Causal set d'Alembertian operators.

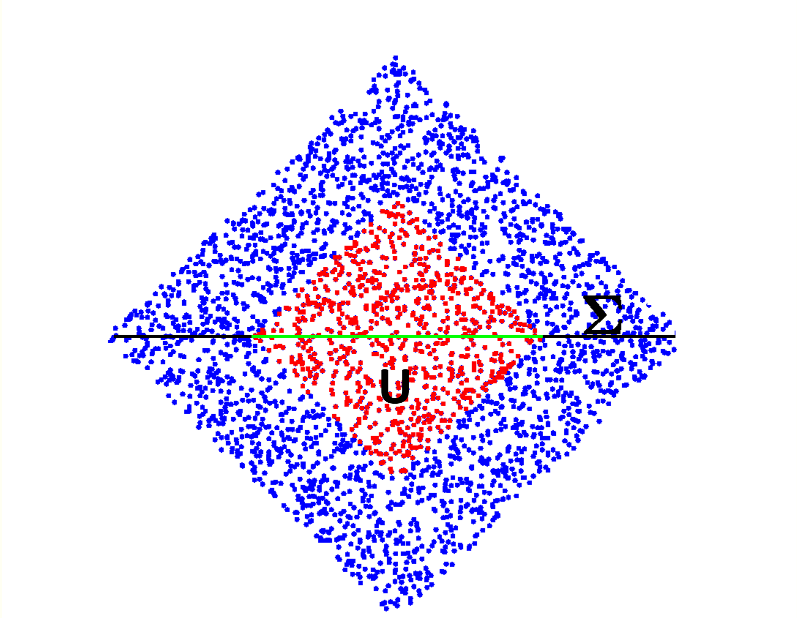
R.D. Sorkin, In *Oriti, D. (ed.): Approaches to quantum gravity* 26-43
D.M.T Benincasa, F. Dowker, Phys.Rev.Lett. 104 (2010) 181301
L. Glaser, Class.Quant.Grav. 31 (2014) 095007

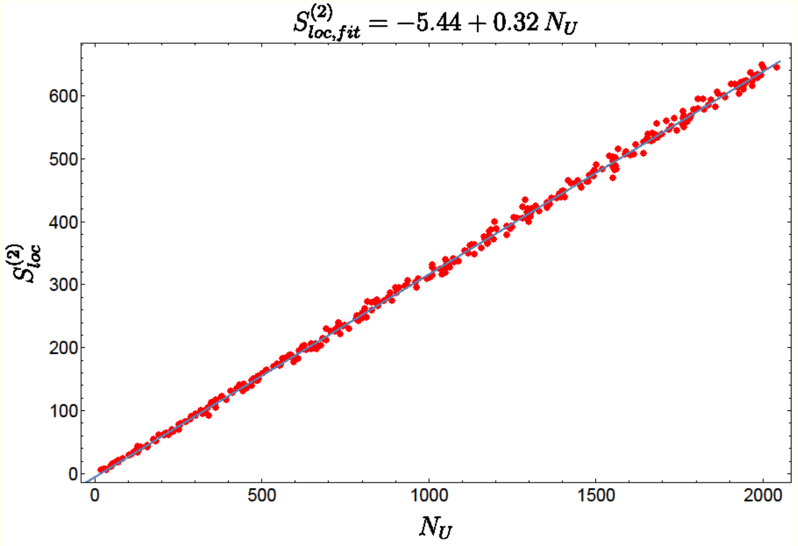
$$B^{(d)}_{\rho}\,\phi(x) = (\epsilon\rho)^{2/d}\left(a\phi(x) + \sum_{n=0}^{\infty} b_n \sum_{y\in I_n(x)} \phi(y)\right)$$

$$\epsilon = \left(\frac{\ell}{l_k}\right)^{d}, l_k \geq \ell$$



Free Scalar Fields and Entanglement Entropy





$$\Delta,\ W_{SJ} = Pos(i\Delta),$$

$$W_{SJ}\Big|_U v = i\lambda\, \Delta\Big|_U v, \qquad S = \sum \lambda \log|\lambda|, \qquad \ell \propto \rho^{-1/d} \nearrow$$

$$N \propto V_{st} \Rightarrow \boxed{S(U) \propto V_{st}}$$

R.D. Sorkin, Y. Yazdi, Class.Quant.Grav. 35 (2018) no.7, 074004
D.M.T. Belenchia, A. Benincasa, ML, S. Liberati, Class.Quant.Grav. 35 (2018) no.7, 074002

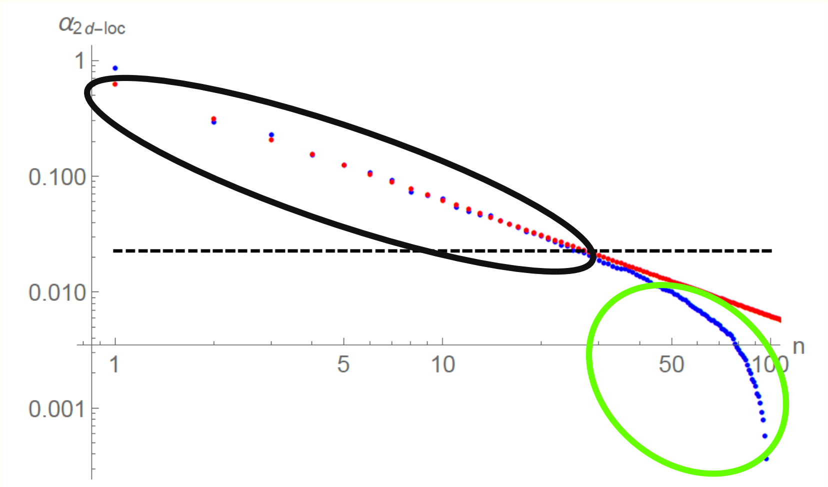
Free Scalar Fields and Entanglement Entropy

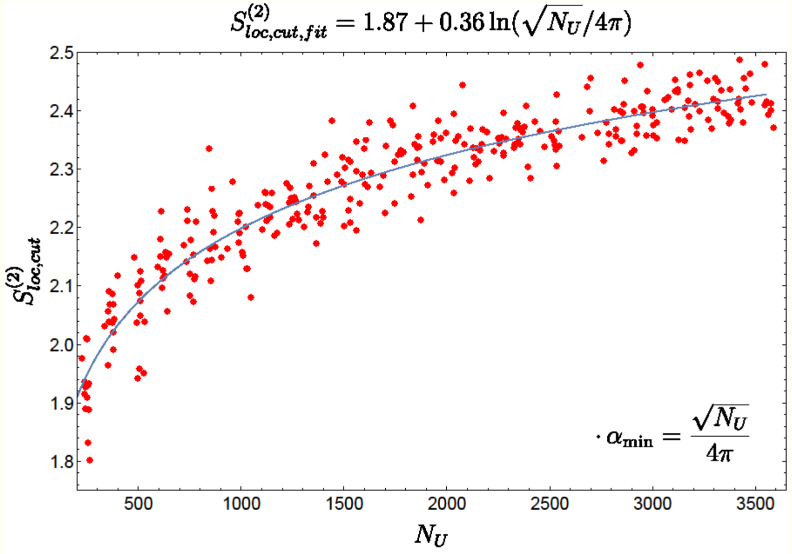
$$W\Big|_U v = i\lambda\,\Delta\Big|_U v, \qquad \Delta\Big|_U v \neq 0$$

Enlarge $\ker\Delta$ by imposing a cutoff $\to k_{max} \propto \ell^{-1} = \rho^{1/d}$

$$\alpha^{\min} = c_d\, V^{2d-1}\, N^{1-1/d}$$

$$S \propto \frac{A}{\ell^{d-2}}$$





R.D. Sorkin, Y. Yazdi, Class.Quant.Grav. 35 (2018) no.7, 074004
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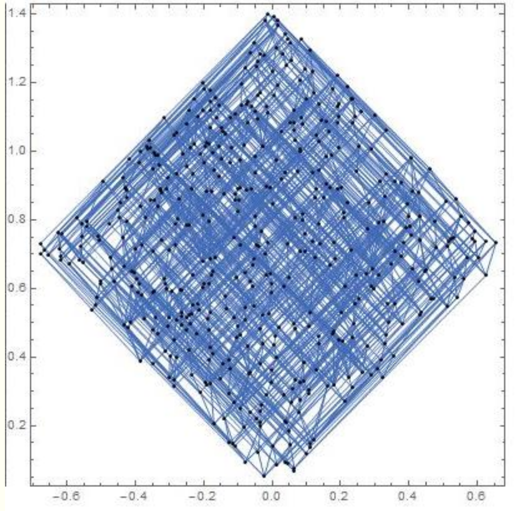
What is causing the spacetime volume law?



- Nonlocality

Nearest neighbours can be arbitrarily far in any given reference frame.

→ not localized close to the boundary.

- Equations of motion

In the continuum, it can be proven that $\mathrm{Im}(\Delta) = \ker(\Box)$.

Take a vector v_j in the kernel $\sum \Delta^{ij} v_j = 0$, then $v_j \phi^j = 0$ → eom!

In a causet, the kernel is very small → $\dim \ker \Delta = k \sim O(\log N) \ll N$ in 2d → highly underdetermined system.

The cutoff gives better equations of motion by enlarging the kernel.

Remember: $\Delta = [\phi, \phi]$ becomes a fully populated matrix! → causality violations!

R.D. Sorkin, Y. Yazdi, Class.Quant.Grav. 35 (2018) no.7, 074004
D.M.T. Belenchia, A. Benincasa, ML, S. Liberati, Class.Quant.Grav. 35 (2018) no.7, 074002

Summary and Comments

- Entanglement Entropy:
  - Black hole entropy
  - Gravity from EE
  - UV structure of spacetime

- Causal sets and scalar fields:
  - Spacetime volume law!
  - What is its origin?
  - Should we and can we recover the area law?
  - What does it tell us about constructing a QFT in a causal set?

- What else can we do with scalar fields on causal sets?
  - Phenomenology in the continuum approximation, A Belenchia, DMT Benincasa, S Liberati, JHEP 2015 (3), 36
  - Spectral geometry, Y. Yazdi, A. Kempf, Class.Quant.Grav. 34 (2017) no.9, 094001; A. Kempf, ML, Y. Yazdi (in preparation)