TMF 2019 Torino

Casimir Traversable Wormholes

Remo Garattini

I.N.F.N. - Sezione di Milano

UNIVERSITÀ DEGLI STUDI DI BERGAMO

Dipartimento di Ingegneria e Scienze Applicate

History of Wormhole Physics

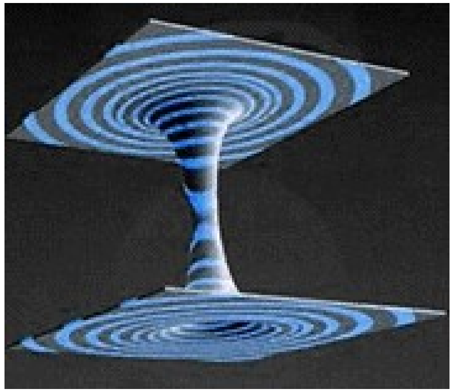
In 1916, L. Flamm recognized that the Schwarzschild solution of Einstein's field equations represents a wormhole.

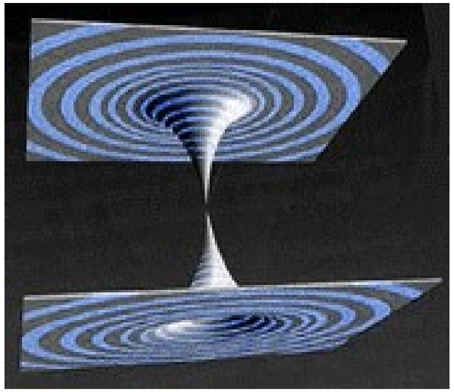
In 1935, A. Einstein and N. Rosen published a paper in Physical Review **48**, 73 (1935) showing that implicit in the general relativity formalism is a curved-space structure that can join two distant regions of space-time through a tunnel-like curved spatial shortcut. The purpose of the paper of was *not* to promote faster-than-light or inter- universe travel, but to attempt to explain fundamental particles like electrons as space-tunnels threaded by electric lines of force.
Their particle model was subsequently shown to be invalid when it was realized that the smallest possible mass-energy of such a curved-space topology is a Planck mass, far larger than the mass-energy of an electron. Their spatial short-cut subsequently became known as an ***Einstein-Rosen Bridge***, rechristened "***wormhole***" by John Wheeler.

History of Wormhole Physics

In 1962 John Wheeler and a collaborator discovered that the Einstein-Rosen bridge space-time structure, which Wheeler re-christened as a "wormhole," was dynamically unstable in field-free space.

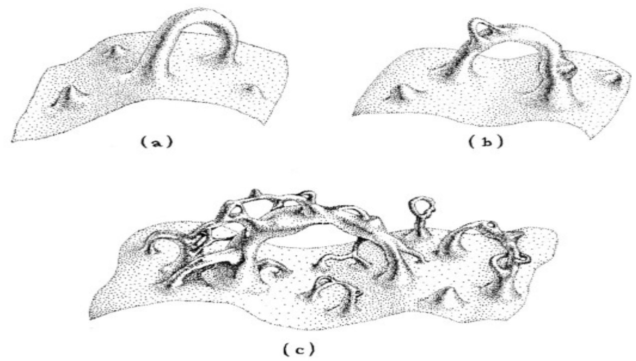
They showed that if such a wormhole somehow opened, it would close up again before even a single photon could be transmitted through it, thereby preserving Einsteinian causality.



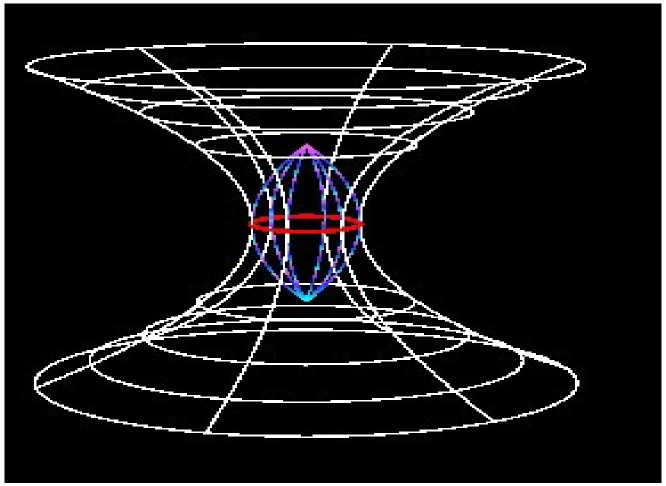


History of Wormhole Physics

In 1988 Kip Thorne and his graduate student Mike Morris showed that a wormhole might be snatched from the quantum foam and stabilized by a region of space containing negative mass-energy.



They suggested that an "advanced civilization" capable of manipulating planet-scale quantities of mass-energy might use the Casimir effect to produce such a region of negative mass energy and, starting with vacuum fluctuations, might create stable wormholes.



M. Visser, *Lorentzian Wormholes: From Einstein to Hawking (American Institute of* Physics, New York, 1995).

What is a Traversable Wormhole?!?

- A wormhole can be represented by two asymptotically flat regions joined by a bridge
- One very simple and at the same time fundamental example of wormhole is represented by the Schwarzschild solution of the Einstein's field equations.
- One of the prerogatives of a wormhole is its ability to connect two distant points in space-time. In this amazing perspective, it is immediate to recognize the possibility of traveling crossing wormholes as a short-cut in space and time.
- A Schwarzschild wormhole does not possess this property.

Traversable wormholes

The traversable wormhole metric

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$$ds^2 = -\exp\left(-2\phi(r)\right)dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2$$

b(r) is the shape function

ϕ***(r)*** is the redshift function

$$r \in \left[r_0, +\infty\right)$$

Condition

$$b_\pm(r_0) = r_0$$

$$b_\pm(r) < r$$

Proper radial distance

$$l(r) = \pm\int_{r_0}^{r} \frac{dr'}{\sqrt{1 - b_\pm(r')/r'}}$$

$$\lim_{r\to\infty} b_\pm(r) = b_\pm \qquad \text{Appropriate asymptotic}$$

$$\lim_{r\to\infty} \phi_\pm(r) = \phi_\pm \qquad \text{limits}$$

Einstein Field Equations

Orthonormal frame

$$b'(r) = 8\pi G \rho c^2 r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2\left(1 - b(r)/r\right)} \qquad \tau(r) = -p_r$$

$$p'_r(r) = \frac{2}{r}\left(p_t(r) - p_r(r)\right) - \left(\rho(r) + p_r(r)\right)\phi'(r)$$

Exotic Energy

$\rho(r) + p_r(r) < 0 \quad r \in [r_0, r_0 + \varepsilon]$ ⟺ $b'(r) < b(r)/r \quad r \in [r_0, r_0 + \varepsilon]$

Flare-Out Condition

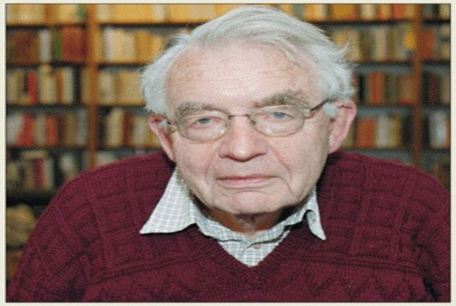
Candidate ⟹ Casimir Energy

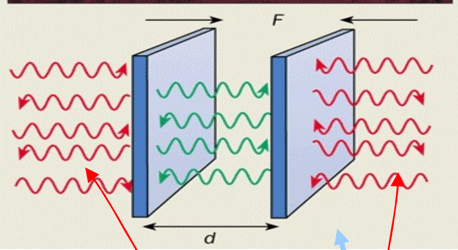
Casimir Effect

H.B.G.Casimir and D.Polder,

Phys. Rev., 73, 360, 1948

Hendrik Casimir 1909-2000





(ZPE) responsible for the Casimir effect. This was predicted by Casimir [1] and confirmed experimentally in the Philips laboratories†. This is induced when the presence of electrical conductors distorts the zero-point energy of the quantum electrodynamics vacuum. Two parallel conducting surfaces, in a vacuum environment, attract one another by a very weak force that varies inversely as the fourth power of the distance between them. This kind of energy is a purely quantum effect; no real particles are involved, only virtual ones. The difference between the stress–energy computed in the presence and in the absence of the plates with the same boundary conditions gives

$$\Delta\langle T^{\mu\nu}\rangle = \langle T^{\mu\nu}\rangle^{p}_{\rm vac} - \langle T^{\mu\nu}\rangle_{\rm vac} = \frac{\pi^2}{720a^4}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}. \qquad (1)$$

It is evident that separately, each contribution coming from the summation over all possible resonance frequencies of the cavities is divergent and devoid of physical meaning but the *difference* between them in the two situations (with and without the plates) is well defined. Note that the energy density

$$\rho = E/V = \Delta\langle T^{00}\rangle = -\frac{\pi^2}{720a^4} \qquad (2)$$

Only wavelength less than d

Any wavelength is possible

Parallel Conducting Planes

This example shows how the Q.E.D. vacuum reacts in presence of conducting surfaces. We consider two parallel perfectly conducting plates separated by a distance a at zero temperature.
To proceed we imagine the electromagnetic field as a infinite set of harmonic oscillators with frequencies

$$\begin{cases} \omega_J = c\sqrt{k^2} & J = (k_1, k_2, k_3) \quad \text{continuous} \\ \omega_J = \omega_{k_\perp, n} = c\sqrt{k_1^2 + k_2^2 + \left(\frac{\pi n}{a}\right)^2} & J = \left(k_1, k_2, \frac{\pi n}{a}\right) = \left(k_\perp^2, \frac{\pi n}{a}\right) \quad \text{boundaries} \end{cases}$$

Parallel Conducting Planes

$$E_0^{\text{ren}}(a) = \lim_{\delta\to 0}\frac{\hbar}{2}\int\frac{\mathrm{d}k_1\,\mathrm{d}k_2}{(2\pi)^2}\left(\sum_{n=-\infty}^{\infty}\omega_{\mathbf{k}_\perp,n}\mathrm{e}^{-\delta\omega_{\mathbf{k}_\perp,n}} - 2a\int\frac{\mathrm{d}k_3}{2\pi}\omega_{\mathbf{k}}\mathrm{e}^{-\delta\omega_{\mathbf{k}}}\right)S$$

$$= \frac{c\hbar\pi}{a}\lim_{\delta\to 0}\int\frac{\mathrm{d}k_1\,\mathrm{d}k_2}{(2\pi)^2}\left(\sum_{n=0}^{\infty}\sqrt{\frac{k_\perp^2 a^2}{\pi^2}+n^2}\mathrm{e}^{\delta\omega_{\mathbf{k}_\perp,n}}\right.$$

$$\left.-\int_0^{\infty}\mathrm{d}t\sqrt{\frac{k_\perp^2 a^2}{\pi^2}+t^2}\,\mathrm{e}^{-\delta\omega_{\mathbf{k}}} - \frac{k_\perp a}{2\pi}\right)S\,,$$

where $k_\perp^2 \equiv k_1^2 + k_2^2$, $t \equiv ak_3/\pi$.

$$: -\frac{c\hbar\pi^2}{720a^3}S$$

- Casimir effect has a quantum origin but it predicts a force between macroscopic bodies
- $S = 1cm^2 \quad a = 1\mu m$
- $F(a) = -\frac{\pi^2 \hbar c S}{240 a^4} \cong 1.7 \times 10^{-7} N$
- Interesting for Nanomachines

The role of the Casimir effect in different fields of physics

☞In QFT, the Casimir effect gives contributions to the total nucleon energy in QCD bag model of hadrons.

Spontaneous compactification of extra spatial dimensions in Kaluza–Klein field theories.

Gauge theories, SUSY, SUGRA, and string theory.

☞In Condensed Matter Physics, the Casimir effect leads to attractive and repulsive forces between the closely spaced material boundaries which depend on the configuration geometry, on temperature, and on the electrical and mechanical properties of the boundary surface.

It is responsible for some properties of thin films and should be taken into account in investigations of surface tension and latent heat.

bulk and surface critical phenomena.

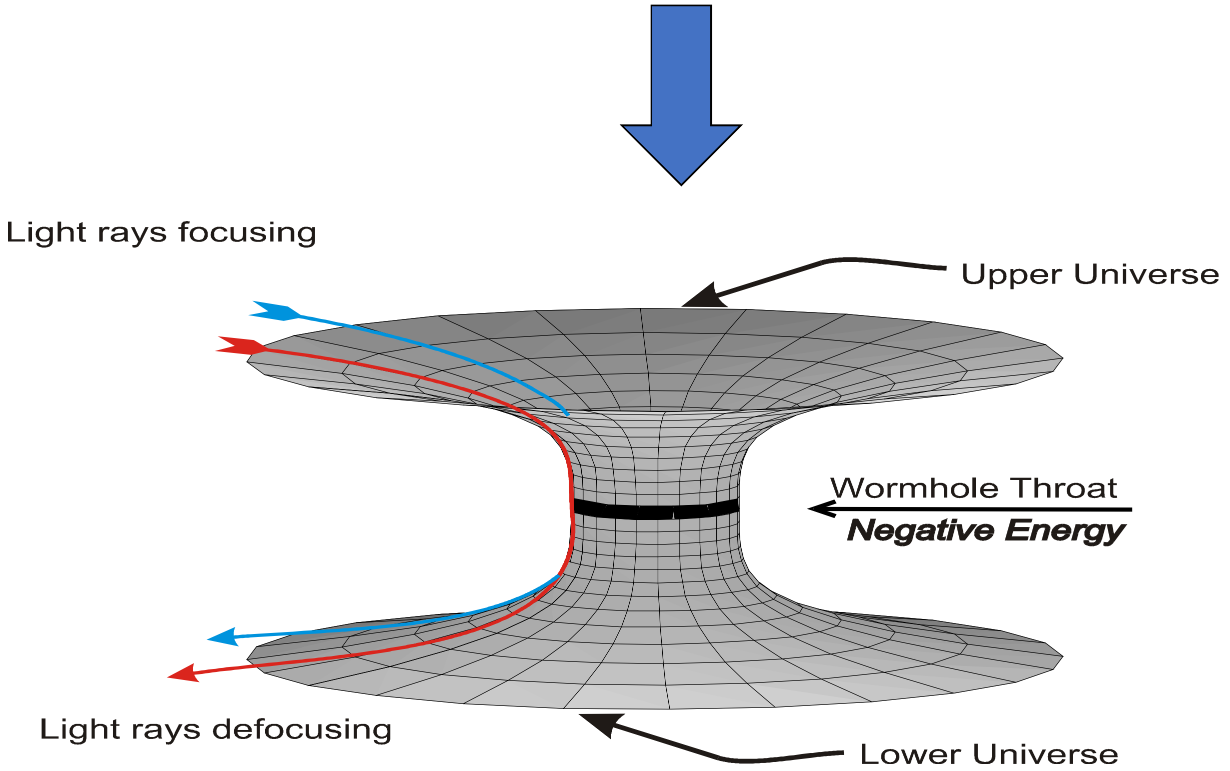
The role of the Casimir effect in different fields of physics

☞ In Gravitation, Astrophysics and Cosmology, the Casimir effect arises in space–times with nontrivial topology.The vacuum polarization resulting from the Casimir effect can drive the inflation process.In the theory of structure formation of the Universe due to topological defects, the Casimir vacuum polarization near the cosmic strings may play an important role.

☞ In Atomic Physics, the long-range Casimir interaction leads to corrections to the energy levels of Rydberg states.A number of the Casimir-type effects arise in cavity Q.E.D.when the radiative processes and associated energy shifts are modified by the presence of the cavity walls.

☞ In Mathematical Physics, the Casimir effect has contributed to develop zeta functions and heat kernel expansion techniques for the regularization and renormalization of infinities.

Traversable Wormhole



Traversable Wormholes
Simple Example

$$ds^2 = -dt^2 + \frac{dr^2}{1-\frac{r_0}{r}} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$b(r) = r_0 \quad \Phi(r) = 0 \quad \Longrightarrow \quad \rho(r) = 0 \quad p_r(r) = -\frac{r_0}{\kappa r^3} \quad p_t(r) = \frac{r_0}{2\kappa r^3}$

SET is traceless

Very interesting but the Volume Integral Quantifier measuring the ANEC

$$I_V = \int [\rho(r) + p_r(r)]dV \longrightarrow \infty$$

Traversable Wormholes
Example

$$ds^2 = -dt^2 + dl^2 + \left(r_0^2 + l^2\right)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).
H. G. Ellis, J. Math. Phys. 14, 104 (1973). K.A. Bronnikov, Acta Phys. Pol. B 4, 251 (1973).

The new coordinate *l* covers the range $-\infty < l < +\infty$. The constant time hypersurface Σ is an Einstein-Rosen bridge with wormhole topology $S^2 \times R^1$. The Einstein-Rosen bridge defines a bifurcation surface dividing Σ in two parts denoted by Σ_+ and Σ_-.

In Schwarzschild coordinates becomes

$$ds^2 = -dt^2 + \frac{dr^2}{1 - \frac{r_0^2}{r^2}} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$b(r_0) = r_0$ Throat Condition

$$\text{Minimum at the throat} \Rightarrow \frac{d^2 r}{dl^2} > 0 \Leftrightarrow \frac{b(r)}{r} > b'(r)$$

Features of the EB Wormhole

$\mathrm{M}(r) = \frac{r_0^2}{2Gr} \to 0 \;\; when\, r \to \infty$ Zero Mass Wormhole

But $M^P(r) = \pm \int_{r_0}^{r} \frac{4\pi\rho(r\prime)r\prime^2}{\sqrt{1 - b(r\prime)/_{r\prime}}} dr' \to \mp \frac{\pi r_0}{4G} \;\; when\; r \to \infty$

$$E_G(r) = M(r) - M^P(r) = \pm \frac{\pi r_0}{4G}$$ Total Energy

$$\mathrm{l}(r) = \pm\sqrt{r^2 - r_0^2}$$ Proper length

$$p_r(r) = -\frac{b(r)}{8\pi G r^3} = -\frac{r_0^2}{8\pi G r^4} \Longrightarrow -\frac{1}{8\pi G r_0^2}$$ On the throat

$$p_t(r) = -\frac{b'(r)r - b(r)}{16\pi G r^3} = \frac{r_0^2}{8\pi G r^4} \Longrightarrow \frac{1}{8\pi G r_0^2}$$ On the throat

Traversable Wormholes

Einstein Field Equations

Eq. of State

$p_r = \omega\rho$

$$b'(r) = 8\pi G\rho(r) r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2\left(1 - b(r)/r\right)} = \frac{b + \omega b' r}{2r^2\left(1 - b(r)/r\right)}$$

$N.E.C.$ Violation $\rho + p < 0$

$EoS \ \ \rho(1+\omega) < 0$

Asymptotic flatness

$$\lim_{r\to\infty}\frac{b(r)}{r} = 0 \Rightarrow \frac{\omega+1}{\omega} > 0 \begin{cases}\omega > 0 \\ \omega < -1\end{cases}$$

$$b(r) = r_0\left(\frac{r_0}{r}\right)^{\frac{1}{\omega}}$$

$$\phi'(r) = 0$$

$$ds^2 = -Adt^2 + \frac{dr^2}{1-\left(\frac{r_0}{r}\right)^{1+\frac{1}{\omega}}} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$b'(r)_{|r=r_0} = -\frac{1}{\omega} < 1$$

Take seriously the Casimir Energy

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \Longrightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^4} \langle T_{\mu\nu} \rangle^{Ren}$$

See also

M.S. Morris, K.S. Thorne, U. Yurtsever (Caltech). 1988. 4 pp.
Published in Phys.Rev.Lett. 61 (1988) 1446-1449

AND

M. Visser, Lorentzian Wormholes: From Einstein to Hawking
(American Institute of Physics, New York), 1995.

Take seriously the Casimir Energy

$$\rho(a) = -\frac{\hbar c \pi^2}{720 a^4} \quad p_r(a) = -3\frac{\hbar c \pi^2}{720 a^4} \quad p_t(a) = \frac{\hbar c \pi^2}{720 a^4}$$

The Casimir Tensor is traceless and divergenceless

$$b'(r) = \frac{8\pi G}{c^4}\rho(a) r^2 \quad \Rightarrow \quad b(r) - b(r_0) = \frac{8\pi G}{c^4}\int_{r_0}^{r}\left(-\frac{c\hbar\pi^2}{720 a^4}\right) r'^2 dr'$$

$$\Rightarrow \quad b(r) = r_0 - \frac{\pi^3 G\hbar}{270 c^3 a^4}\left(r^3 - r_0^3\right)$$

$\rho(a)$ is a tiny cosmological constant

This is not a TW because there is no A. Flatness. It is Asymptotically de Sitter

It can be transformed into a TW with the junction condition method matching the solution with the Schwarzschild metric at some point r=c

Take seriously the Casimir Energy

$$\rho(a) = -\frac{\hbar c \pi^2}{720 a^4} = -\rho_0 \quad p_r(a) = -3\frac{\hbar c \pi^2}{720 a^4} \quad p_t(a) = \frac{\hbar c \pi^2}{720 a^4}$$

The Casimir Tensor is traceless and divergenceless

$$b(r) = r_0 - \frac{\kappa \rho_0}{3}\left(r^3 - r_0^3\right) \qquad \kappa = \frac{8\pi G}{c^4}$$

Possible approaches:

- Connection with other profiles
- Modify the energy density
- Promote the plate separation *a* to be a radial variable

Close to the throat

$$b(r) = r_0 - \frac{\kappa \rho_0}{3}(r - r_0)\left(r^2 + r r_0 + r_0^2\right)$$
$$\cong r_0\left(1 - r_0 \kappa \rho_0 (r - r_0)\right)$$

Other Profile → Generalized Absurdly Benign TW

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$b(r) = r_0\left(1-\mu(r-r_0)\right)^{\alpha}$ $\quad \Phi(r) = 0 \quad r \in \left[r_0, r_0 + {}^{1}/_{\mu}\right]$ For $\alpha = 2$ we have the absurdly benign TW

$b(r) = 0 \quad \Phi(r) = 0 \quad r \in \left[r_0 + {}^{1}/_{\mu}, \infty\right]$ → Minkowski

$$\rho(r) = \frac{b'(r)}{\kappa r^2} = -\frac{r_0\alpha\mu}{\kappa r^2}\left(1-\mu(r-r_0)\right)^{\alpha-1} \qquad p_r(r) = -\frac{b(r)}{\kappa r^3} = -\frac{r_0\left(1-\mu(r-r_0)\right)^{\alpha}}{\kappa r^3}$$

$$p_t(r) = \frac{b(r)-b'(r)r}{2\kappa r^3} = \frac{r_0\left(1-\mu(r-r_0)\right)^{\alpha-1}(1+\alpha\mu r)}{2\kappa r^3}$$

$$\kappa = \frac{8\pi G}{c^4}$$

Close to the throat $\quad \rho(r_0) = -\frac{\alpha\mu}{\kappa r_0} \quad p_r(r_0) = -\frac{1}{\kappa r_0^2} \quad p_t(r_0) = \frac{1+\alpha\mu r_0}{2\kappa\ {}_0^2}$

Choose $\mu = \frac{1}{3\alpha r_0}$ → $\rho(r_0) = -\frac{1}{3\kappa r_0^2} \quad p_r(r_0) = -\frac{1}{\kappa r_0^2} \quad p_t(r_0) = \frac{2}{3\kappa r_0^2}$

Other Profile → *Generalized Absurdly Benign TW*

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

Identify the Casimir Energy Density

$$\rho_0 = -\frac{1}{3\kappa r_0^2} \qquad p_r(r_0) = -3\rho_0 \qquad p_t(r_0) = 2\rho_0$$

⇒ $$SET \quad T_{\mu\nu} = \left(\frac{1}{3\kappa r_0^2}\right)[diag(-1,-3,1,1) + diag(0,0,1,1)]$$

$$\frac{\hbar G}{c^3}\frac{\pi^3}{30a^4} = \frac{1}{r_0^2}$$ ⇒ $$r_0 = \frac{a^2}{l_P}\sqrt{\frac{30}{\pi^3}}$$

Plate separation $a{\sim}\mu m$ ⇒ $r_0{\sim}10^{22}m$ Plate separation $a{\sim}fm$ ⇒ $r_0{\sim}10^5 m$

Take seriously the Casimir Energy

R.G. ArXiv:1907:03623 [gr-qc] Submitted to EPJC

Strategy $\Longrightarrow$ *Impose an EoS* *Zero Tidal Forces* $\phi(r) = 0$ *Promote* $a \longrightarrow r$

$$\rho(r) = -\frac{\hbar c \pi^2}{720 r^4} \quad p_r(r) = -3\frac{\hbar c \pi^2}{720 r^4} \quad p_t(r) = \frac{\hbar c \pi^2}{720 r^4}$$

$$p_r(r) = \omega \rho(r) \qquad \omega = 3$$

$$\Longrightarrow ds^2 = -dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{\frac{4}{3}}} + r^2 d\Omega^2$$

$\Longrightarrow$ No Connection with the original Casimir Stress-Energy tensor

$$T_{\mu\nu} = \frac{1}{\kappa r^2}\left(\frac{r_0}{r}\right)^{\frac{\omega+1}{\omega}} \left(-\frac{1}{\omega}, -1, \frac{\omega+1}{\omega}, \frac{\omega+1}{\omega}, \right)$$

Take seriously the Casimir Energy

R.G. ArXiv:1907:03623 [gr-qc]   Submitted to EPJC

$$b'(r) = \frac{8\pi G}{c^4}\rho(r)r^2 \qquad \rho(r) = -\frac{c\hbar\pi^2}{720r^4}$$ Energy Density

$$\Rightarrow \quad b(r) = b(r_0) + \frac{8\pi G}{c^4}\int_{r_0}^{r}\left(-\frac{c\hbar\pi^2}{720r'^4}\right)r'^2 dr' = r_0 - \frac{\pi^3 G\hbar}{90c^3}\left(\frac{1}{r_0} - \frac{1}{r}\right)$$

$$\Rightarrow \quad r_1^2 = \frac{\pi^3}{90}\frac{G\hbar}{c^3} = \frac{\pi^3}{90}l_p^2 \quad \Rightarrow \quad b(r) = r_0 - r_1^2\left(\frac{1}{r_0} - \frac{1}{r}\right)$$

$$\Rightarrow \quad p_r(r) = \omega\rho(r) \qquad EoS$$

With the second EFE, we can compute $\phi(r)$ $\Rightarrow$ Depending on the value of ω we can have BH, TW or a singularity

Take seriously the Casimir Energy

R.G. ArXiv:1907:03623 [gr-qc] Submitted to EPJC

When $\omega = \frac{r_0^2}{r_1^2}$ → Traversable Wormhole

$$\phi(r) = \frac{1}{2}(\omega - 1) ln\left(\frac{r(\omega + 1)}{(\omega r + r_0)}\right) \xrightarrow{\omega = 3} \phi(r) = ln\left(\frac{4r}{3r + r_0}\right)$$ Planckian

$$\rightarrow \quad b(r) = \left(1 - \frac{1}{\omega}\right) r_0 + \frac{r_0^2}{\omega r} \xrightarrow{\omega = 3} b(r) = \frac{2}{3} r_0 + \frac{r_0^2}{3r}$$

$$SET \quad T_{\mu\nu} = \left(\frac{r_0^2}{3\kappa r^4}\right)\left[diag(-1, -3,1,1) + \left(\frac{6r}{3r + r_0}\right) diag(0,0,1,1)\right]$$

Take seriously the Casimir Energy

R.G. ArXiv:1907:03623 [gr-qc] Submitted to EPJC

When $\omega = \frac{r_0^2}{r_1^2}$ ⇒ Traversable Wormhole

$$\phi(r) = \frac{1}{2}(\omega - 1)ln\left(\frac{r(\omega + 1)}{(\omega r + r_0)}\right) \xrightarrow{\omega = 1} \phi(r) = 0$$

Sub-Planckian

$$\Rightarrow \quad b(r) = \left(1 - \frac{1}{\omega}\right) r_0 + \frac{r_0^2}{\omega r} \xrightarrow{\omega = 1} b(r) = \frac{r_0^2}{r}$$

EB Wormhole

$$SET \quad T_{\mu\nu} = \left(\frac{r_0^2}{\kappa r^4}\right)[diag(-1,-3,1,1) + 2\, diag(0,1,0,0)] = \left(\frac{r_0^2}{\kappa r^4}\right)[diag(-1,-1,1,1)]$$

Take seriously the Casimir Energy

R.G. ArXiv:1907:03623 [gr-qc]   Submitted to EPJC

When $\omega = \frac{r_0^2}{r_1^2}$ ⇒ Traversable Wormhole

$$\phi(r) = \frac{1}{2}(\omega - 1)ln\left(\frac{r(\omega+1)}{(\omega r + r_0)}\right) \xrightarrow{\omega = 1} \phi(r) = 0$$

Sub-Planckian

$$\Rightarrow \quad b(r) = \left(1 - \frac{1}{\omega}\right)r_0 + \frac{r_0^2}{\omega r} \xrightarrow{\omega = 3} b(r) = \frac{r_0^2}{r}$$

EB Wormhole

$$SET \quad T_{\mu\nu} = \left(\frac{r_0^2}{2\kappa r^4}\right)[diag(-1,-3,1,1) + diag(-1,1,1,1)] = \left(\frac{r_0^2}{\kappa r^4}\right)[diag(-1,-1,1,1)]$$

Canonical Decomposition

QWEC Equation of State

P. Martin-Moruno and M. Visser, JHEP 1309 (2013) 050; arXiv:1306.2076 [gr-qc].
M. Bouhmadi-Lopez, F. S. N. Lobo and P. Martin-Moruno, JCAP 1411 (2014) 007 [arXiv:1407.7758 [gr-qc]]

$$p_r(r)+\rho(r)=-f(r) \qquad f(r)\ \text{is an energy density}$$

$$\frac{b'(r)}{r}+\left[2\left(1-\frac{b(r)}{r}\right)\phi'(r)-\frac{b(r)}{r^2}\right]=-(8\pi G)rf(r)$$

$$\text{Introduce } u(r)=1-\frac{b(r)}{r}\Longrightarrow b(r)=r\left[1-(8\pi G)\exp\big(2\phi(r)\big)\int_{r_0}^{r}\exp\big(-2\phi(r')\big)f(r')r'dr'\right]$$

Impose the ZTF $\quad \text{Example: Assume the Casimir Profile } f(r)=-\dfrac{4c\hbar\pi^2}{720r^4}$

$$\Longrightarrow b(r)=r\left[1+\frac{G\pi^3}{45}\left(\frac{1}{r^2}-\frac{r^2}{r_0^2}\right)\right] \qquad \text{When } r\to\infty \quad b(r)\backsimeq r\left[1-\frac{G\pi^3}{45r_0^2}\right]$$

$$\text{Global monopole} \Longrightarrow \text{Excess of the Solid Angle} \qquad \text{Rescale } \bar{r}^2=\frac{45r^2}{G\pi^3}$$

$$\text{Asymptotic limit} \Longrightarrow ds^2=-dt^2+d\bar{r}^2+\frac{G\pi^3}{45r_0^2}\bar{r}^2d\Omega^2$$

QWEC Equation of State

TW returns if $\frac{G\pi^3}{45 r_0^2} = 1 \quad \Rightarrow ds^2 = -dt^2 + \frac{dr^2}{\left[1 - \frac{r_0^2}{r^2}\right]} + r^2 d\Omega^2$

Abandon the ZTF ⇒ $\phi(r) = ln\left(\frac{4r}{3r + r_0}\right)$

⇒ $b(r) = r\left(1 - \frac{\hbar G\pi^3}{30 r_0^2 c^3}\right) + \frac{\hbar G\pi^3}{45 r_0 c^3} + \frac{\hbar G\pi^3}{90 r c^3}$

$When\ r \to \infty \quad b(r) \simeq r\left[1 - \frac{\hbar G\pi^3}{30 r_0^2 c^3}\right]$ *Global monopole* ⇒ *Excess of the Solid Angle*

TW returns if $\frac{\hbar G\pi^3}{30 r_0^2 c^3} = 1$ $\Rightarrow ds^2 = -\left(\frac{4r}{3r + r_0}\right)^2 dt^2 + \frac{dr^2}{\left[1 - \frac{2r_0}{3r} - \frac{r_0^2}{3r^2}\right]} + r^2 d\Omega^2$

Different Point of View

$b(r) = \frac{r_0^2}{r} \; EB\ Wormhole$ ⟺ $b(r) = \lim_{\mu - 0} \frac{r_0^2}{r} e^{-\mu(r-r_0)}$ Yukawa Wormhole

Motivations

- Nuclear Physics
- Yukawa constraints on the recent measurement of the Casimir force

(Bordag,Gillies and Mostepanenko Phys.Rev.D56:6-10,1997 arXiv:hep-th/9705101)

- Van der Waals forces described in a Yukawa form

(*Milonni The Quantum Vacuum: An Introduction to Quantum Electrodynamics*)

- *Black Holes in Modified Gravity (MOG)*

(J.W. Moffat, arXiv:1412.5424 [gr-qc])

- *Modified Theory of Gravity*
- *Many other contexts!!!*

Different Point of View

$b(r) = \frac{r_0^2}{r} e^{-\mu(r-r_0)}$ Yukawa Wormhole

$\mathrm{M}(r) = \frac{r_0^2 e^{-\mu(r-r_0)}}{2Gr} \to 0 \;\; when\; r \to \infty$

Zero Mass Wormhole

$$\rho(r) = -\frac{r_0^2}{8\pi G r^4} e^{-\mu(r-r_0)}(1+\mu r) \longrightarrow -\frac{1+\mu r_0}{8\pi \quad {}_0^2}$$

$$p_r(r) = -\frac{r_0^2}{8\pi G r^4} e^{-\mu(r-r_0)} \longrightarrow -\frac{1}{8\pi G r_0^2}$$

$$p_t(r) = \frac{r_0^2}{16\pi G r^4} e^{-\mu(r-r_0)}(2+\mu r) \longrightarrow \frac{2+\mu r_0}{16\pi G \quad {}_0^2}$$

On the throat

But $\mp\frac{\pi r_0}{4G} \lesseqgtr M^P(r) \lesseqgtr 0 \;\; when\; r \to \infty$

$$\pm\frac{\pi r_0}{4G} \gtreqless E_G(r) \gtreqless 0$$

Total Energy

Out of the throat $b(r)$ and $b'(r) \to 0$ for $\mu \to \infty$

Effective Einstein Equations

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

- $G_{\mu\nu}$ is the Einstein tensor,
- $\kappa = 8\pi G$,
- $T_{\mu\nu}$ is the stress-energy tensor.

Hochberg, Popov and Sushkov considered a self-consistent solution of the semiclassical Einstein equations corresponding to a Lorentzian wormhole coupled with a quantum scalar field
[Hochberg D, Popov A and Sushkov S V 1997 *Phys. Rev. Lett.* 78 2050 (*Preprint* gr-qc/9701064)]

Khusnutdinov and Sushkov fixed their attention to the computation of the ground state of a massive scalar field in a wormhole background. They tried to see if a self-consistent solution restricted to the energy component appears in this configuration
[Khusnutdinov N R and Sushkov S V 2002 *Phys. Rev.* D 65 084028 (*Preprint* hep-th/0202068)]

General setting for self sustained traversable wormholes

R.Garattini, C.Q.Grav. 22 (2005) 1105 ArXiV:gr-qc/0501105

Instead of $G_{\mu\nu} = \kappa T_{\mu\nu}$ consider $G_{\mu\nu} = \kappa \left\langle T_{\mu\nu} \right\rangle^{ren}$

S.S.T.W.

$$E^{Classical} = -E^{TT}$$

where $\left\langle T_{\mu\nu} \right\rangle^{ren}$ renormalized expectation value of the stress-energy tensor operator of the quantized field

If the matter field source is absent $\left\langle T_{\mu\nu} \right\rangle^{ren} = -\frac{1}{\kappa}\left\langle \Delta G_{\mu\nu}\left(\tilde{g}_{\alpha\beta}, h_{\alpha\beta}\right)\right\rangle^{ren}$

$$G_{\mu\nu}\left(g_{\alpha\beta}\right) = G_{\mu\nu}\left(\tilde{g}_{\alpha\beta}\right) + \Delta G_{\mu\nu}\left(\tilde{g}_{\alpha\beta}, h_{\alpha\beta}\right) \qquad g_{\alpha\beta} = \tilde{g}_{\alpha\beta} + h_{\alpha\beta}$$

$\Delta G_{\mu\nu}\left(\tilde{g}_{\alpha\beta}, h_{\alpha\beta}\right)$ is a perturbation series in terms of $g_{\alpha\beta}$

The Einstein tensor $\boldsymbol{G}_{\mu\nu}$ can be divided into a part which is unperturbed related to the background geometry and a part related to quantum fluctuations like the metric $g_{\alpha\beta}$

Finding the wormhole radius with phantom energy

[R.G. Class.Quant.Grav.24:1189-1210,2007 gr-qc/0701019]

Eq. of State

$p_r = \omega\rho \quad \omega < 0$

$$b(r) = r_0\left(\frac{r_0}{r}\right)^{\frac{1}{\omega}}$$

Solution

$$\phi'(r) = \frac{b(r) + \omega b'(r) r}{2r^2\left(1 - b(r)/r\right)}$$

Asymptotic flatness

$$\lim_{r\to\infty}\frac{b(r)}{r} = 0 \quad \Rightarrow \quad \frac{\omega+1}{\omega} > 0 \quad \begin{cases}\omega > 0\\ \omega < -1\end{cases}$$

For $\omega = 1$ one simply gets

$$ds^2 = -dt^2 + dl^2 + \left(r_0^2 + l^2\right)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

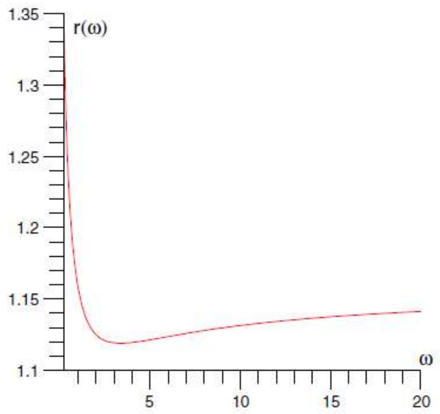


Figure 1. Plot of the wormhole throat $\bar{r}_t$ as a function of ω in the positive range.

Conclusions and Perspectives

- Casimir energy is the only source of exotic matter that can be generated in laboratory.
- Traversable wormholes can be sustained by Casimir Energy.
- The Wormhole is traversable in principle but not in practice.
- The QWEC condition supports the Casimir wormhole.
- For appropriate choices of the parameters we have global monopoles carried by TW. For other choices of the same parameters we describe black holes, traversable wormholes or singularities.
- Yukawa Wormholes generalize the Casimir wormhole.
- At this stage the TW is completely useless, we need an amplification mechanism.
- Self Sustained Casimir TW?!?
- TW relevant for GW as BH mimickers.

Thank You for Your Attention

Other Traversable Wormholes Papers

- R. G. and F. S. N. Lobo *Self sustained phantom wormholes in semi-classical gravity* CQG 24 (2007) 2401 gr-qc/0701020
- R. G. and F. S. N. Lobo *Self-sustained traversable wormholes in noncommutative geometry* PLB 671 (2009) 146 arXiv:0811.0919 [gr-qc]
- R. G. and F. S. N. Lobo *Self-sustained wormholes in modified dispersion relations* PRD 85 (2012) 024043 arXiv:1111.5729 [gr-qc]
- R.G. and F.S.N. Lobo, *Gravity`s Rainbow induces topology change* Eur.Phys.J. C74 (2014) 2884 arXiv:1303.5566 [gr-qc]
- R. G. and F. S. N. Lobo *Self-Sustained Traversable Wormholes* Fundam.Theor.Phys. 189 (2017) 111-135

Outlook