October 27, Tuesday

Session [III–A] The Mathematical Side of Causality

Chair G. Hörmann

09:00 Causality and solutions of Einstein's equations with closed timelike Curves

James Vickers [key-speaker]

Abstract

In this review I will start by looking at the difference between the notions of time and causality in Newtonian Theory and in General Relativity. I will then give some examples of solutions to Einstein's equations which contain closed timelike curves (CTCs) - so that by locally travelling within ones forward light-cone in spacetime, one can actually travel into ones own past. The possibility of travelling into ones own past leads to many potential causal paradoxes, however this does not necessarily mean that the existence of CTCs is incompatibile with the laws of physics. I will examine this issue by considering the Cauchy problem for a scalar field in such a spacetime. In particular I will consider the evolution of a scalar field in a spacetime which contains an initially globally hyperbolic region which evolves into one containing CTCs. I will end by discussing how these results relate to Hawking's Chronology Protection Conjecture.

9.45 Chronology protection *James Grant [invited]*

Abstract

There are exact solutions of the Einstein field equations of General Relativity that contain ``closed time-like curves'', i.e. paths that a physical observer could travel along and return to their own past. There is, however, some evidence that quantum mechanical effects will preclude the creation of such a scenario, and will, in effect, prevent the creation of any putative ``time machine''. The statement that quantum effects will forbid the creation of closed time-like curves is Hawking's ``Chronology protection conjecture''. I will discuss a simple example of such a space with closed time-like curves due to Gott, and show how, when quantum mechanical effects are taken account, the space-time will be distorted in such a way that the region with closed time-like curves will no longer form. I will also discuss some modern developments regarding quantum fields on space-times that admit closed time-like curves.

10.15 Singularity theorems in low regularity *Michael Kunzinger [invited]*

Abstract

The singularity theorems of General Relativity are important milestones in the understanding of solutions to the Einstein equations. Initiated by R. Penrose in [1] and continued by S. W. Hawking, R. Penrose, G.F.R. Ellis, R. Geroch and others (cf. [2]), the investigation of singularity theorems to this day constitutes a central research field in mathematical relativity. They show that under realistic assumptions on the spacetime (and independently of any symmetries) there necessarily exist incomplete geodesics, which may be interpreted as singularities.

One weakness of the classical singularity theorems is that they do not make any statement on the actual nature of the singularities themselves. In particular, they do not imply that the curvature blows up where a causal geodesic ceases to exist. Thus, in principle, they allow the possibility that the spacetime might be singular in the above sense merely due to the fact that the differentiability of the spacetime metric drops below C^2 (twice continuously

differentiable). Recent progress in low-regularity causality theory has allowed to show that, in fact, both the Penrose and the Hawking theorem remain valid for metrics of differentiability class $C^{1,1}$, the maximal class in which the geodesic equation still has unique solutions ([3, 4]). We will report on these developments and discuss open questions and further directions of research. This is joint work with Roland Steinbauer, Milena Stojkovíc, and James A. Vickers.

References
[1] R. Penrose. Gravitational collapse and space-time singularities. Phys. Rev. Lett., 14:57–59, 1965.
[2] S. W. Hawking and G. F. R. Ellis. The large scale structure of space-time. Cambridge University Press, 1973.
[3] M. Kunzinger, R. Steinbauer, M. Stojkoví c, and J. A. Vickers. Hawk- ing's singularity theorem for C^{1,1}-metrics. Classical and Quantum Gravity, 32(7):075012, 2015.
[4] M. Kunzinger, R. Steinbauer, and J. A. Vickers. The Penrose singularity theorem in regularity C^{1,1}. Classical and Quantum Gravity, to appear.

10:45 COFFE BREAK

Session [III–B] The Mathematical Side of Causality

Chair S. Coriasco

11:15 **Global hyperbolicity for spacetimes with continuous metrics** *Clemens Saemann*

Abstract
Global hyperbolicity is the strongest commonly used causality condition in general relativity.
It ensures well-posedness of the Cauchy problem for the wave equation, globally hyperbolic spacetimes are the class of spacetimes used in the initial value formulation of Einstein's equations and it plays an important role in the singularity theorems. These examples emphasize the importance of this notion in Lorentzian geometry and, in particular, in the theory of general relativity.
Classically (i.e., with smooth metric), there are four equivalent notions of global hyperbolicity. These are: compactness of the causal diamonds and (strong) causality, compactness of the space of causal curves connecting two points and causality, existence of a Cauchy hypersurface and the metric splitting of the spacetime.
We show that the definition of global hyperbolicity in terms of the compactness of the causal diamonds and non-total imprisonment can be extended to spacetimes with continuous metrics, while retaining the first three equivalences above. Furthermore, global hyperbolicity implies causal simplicity, stable causality and the existence of maximal curves connecting any two causally related points.

11:35 **Cauchy problem in General Relativity** *Simon Garruto*

Abstract
I'll review the initial value problem in (vacuum) GR discussing the role of coordinate choices. The work is part of a project to discuss physical structure of a theory in terms of algebraic properties of principal symbol of quasi-linear equations.

11:55 **A simple test about potential unitarity violation in the black hole firewall paradox**
Philippe Jacquet

Abstract

We propose a simple test designed to detect non unitary effects in the black hole evaporation. Since the effects are expected to be tiny, the test relies on information theory in order to discriminate between the non unitary hypothesis and the unitary hypothesis. We show that the modification of the (final) mass of an isolated black hole would imply a modification in the non-unitary effects within its evaporation. Thus the effect should affect any quantum measure standing on the causal past of any mass absorption event. However shooting a mass toward a black hole in our galaxy would not lead to a detectable non unitary effect since most of the matter in the galaxy would eventually end in the central black hole. Therefore the test is based on expelling flows of particles outward the galaxy. Before any mass expulsion, one creates a binary codeword from a sequence of spin measurements, and performs a protocol of mass expulsion based on the codeword. Given a fixed quantity of mass to be expelled, the protocol would maximize the likelihood of the discrimination between unitary and non unitary effect via the Kullback-Leibler divergence. In term of information theory, this is equivalent to consider the extraction of one bit of information from a received codeword, the bit corresponds to the a posteriori application of a protocol with mass expulsion versus the alternative protocol without mass expulsion. Assuming accumulation in non unitary effects, this channel has a larger capacity than classic symmetric channels.

12.15 **Wormholes and Time-Machines tuning** *Sergey Kozyrev*

Abstract

Composite static spherically symmetric wormholes are analyzed within the framework of general relativity. We construct the composite configuration by gluing the exterior vacuum metric on standard Hilbert gauge to the interior vacuum metric in different gauge at the junction hypersurface. It is shown that by tuning parameters of interior solution, this wormhole can be supported without exotic matter. Thus, we can conclude that well behaved time machine solutions are possible in general relativity models of composite wormhole. In this context, the idea of linkages for vacuum regions bounding other metrics may have broad applications.

12:35 LUNCH