Angular Power Spectrum and Bispectrum for 21cm Intensity Mapping

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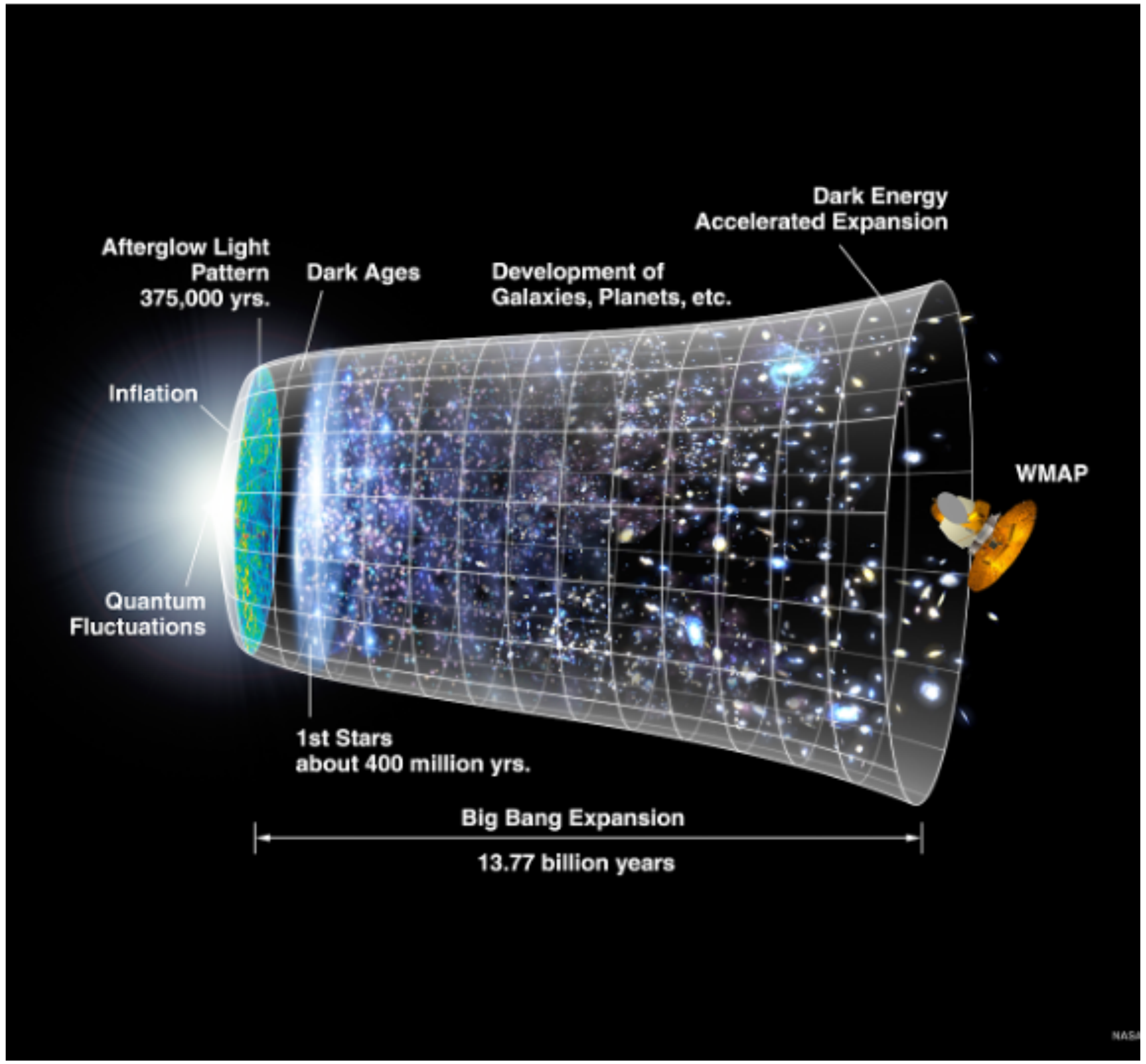
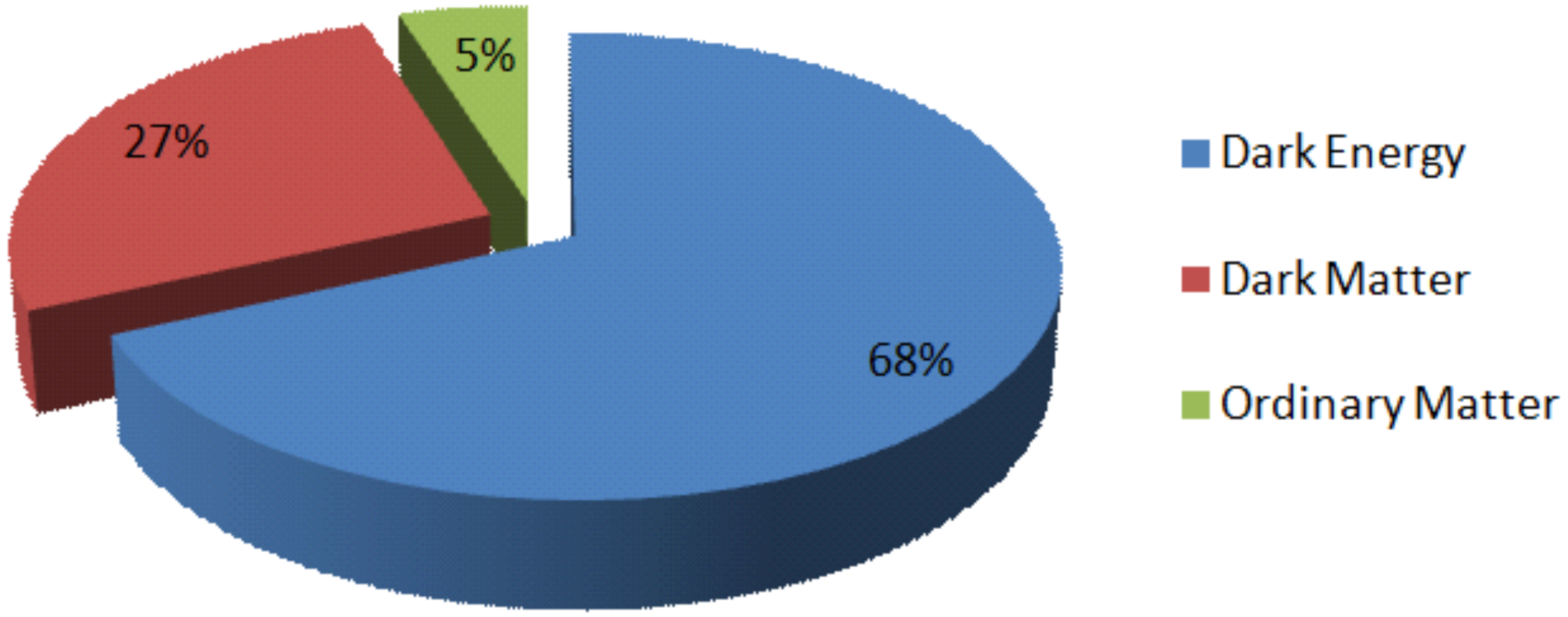
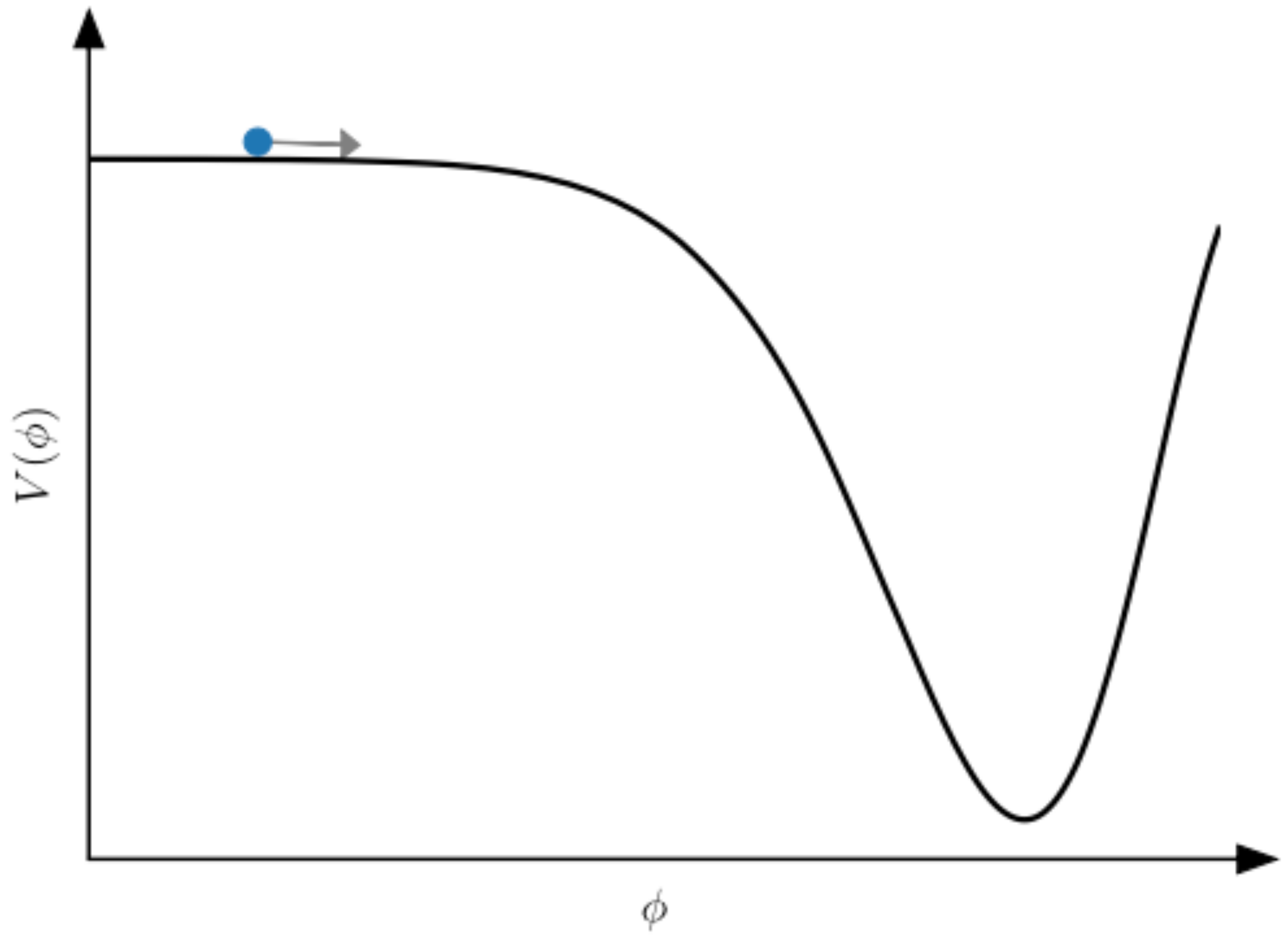
Introduction

- The standard cosmological model provides a beautiful description for the evolution of our universe.
- However several processes are still unknown.
- In particular, what is responsible for the two accelerating periods in the universe's history?



Dark Energy & Inflation





$$\dot{\rho}_d + 3H(1+w)\rho_d = 0$$

$$w = -1$$

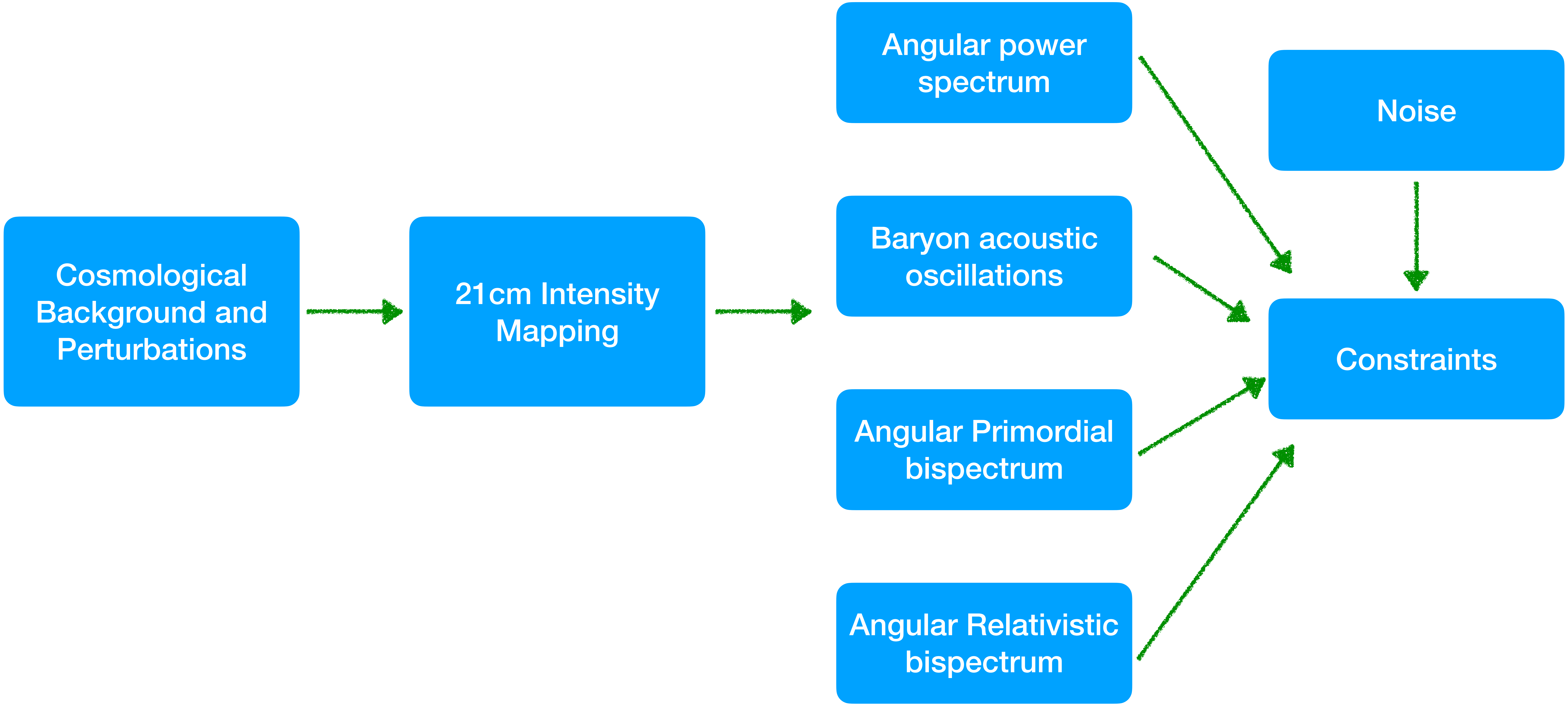
constant energy density

$$w = \frac{\mathcal{P}}{\rho} = \frac{\dot{\phi}^2 - V(\phi)}{\dot{\phi}^2 + V(\phi)}$$

gaussian primordial distribution


CAASTRO
ALL-SKY ASTROPHYSICS

Theoretical Pipeline



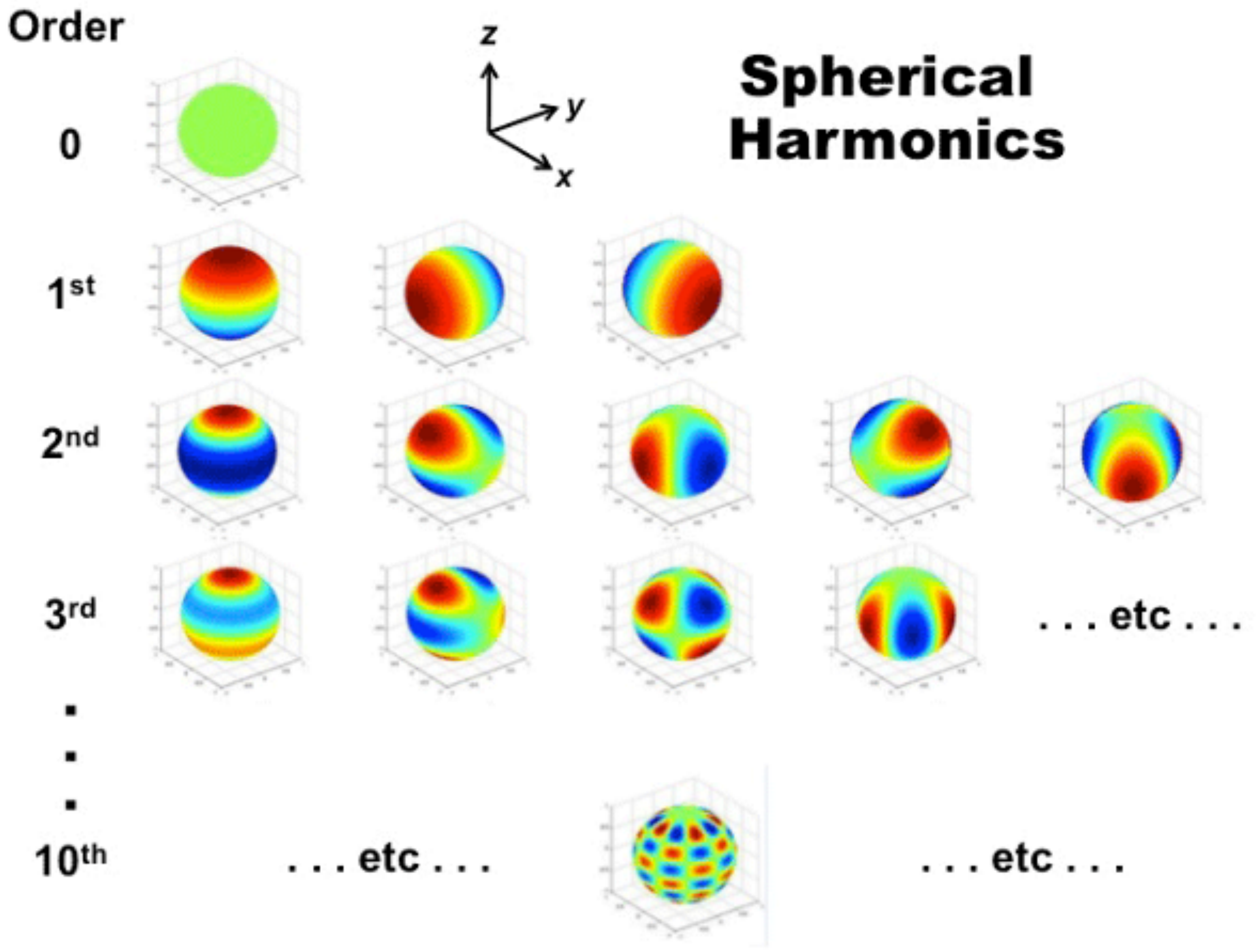
Angular Power Spectra

Following the same procedure used for CMB, we expand the brightness temperature in spherical harmonics

$$\Delta T_b(z,\hat{n}) = \sum_{lm} \Delta T_{b,lm}(z) Y_{lm}(\hat{n})$$

where

$$\Delta T_{b,lm}(z) = 4\pi i^l \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \Delta T_{b,l}(\mathbf{k},z) Y^*_{lm}(\hat{\mathbf{k}})$$



Including all relativistic effects in the post-reionization epoch

$$\Delta_{T_b,l}(\mathbf{k},z) = \delta_n j_l(k\chi) + \frac{kv}{\mathcal{H}} j_l''(k\chi) + \left(\frac{1}{\mathcal{H}}\dot{\Phi} + \Psi\right) j_l(k\chi) - \left[\frac{1}{\mathcal{H}}\frac{d\ln\left(a^3\bar{n}_{HI}\right)}{d\eta} - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - 2\right]\left[\Psi j_l(k\chi) + v j_l'(k\chi) + \int_0^\chi (\dot{\Psi} + \dot{\Psi}) j_l(k\chi') d\chi'\right]$$

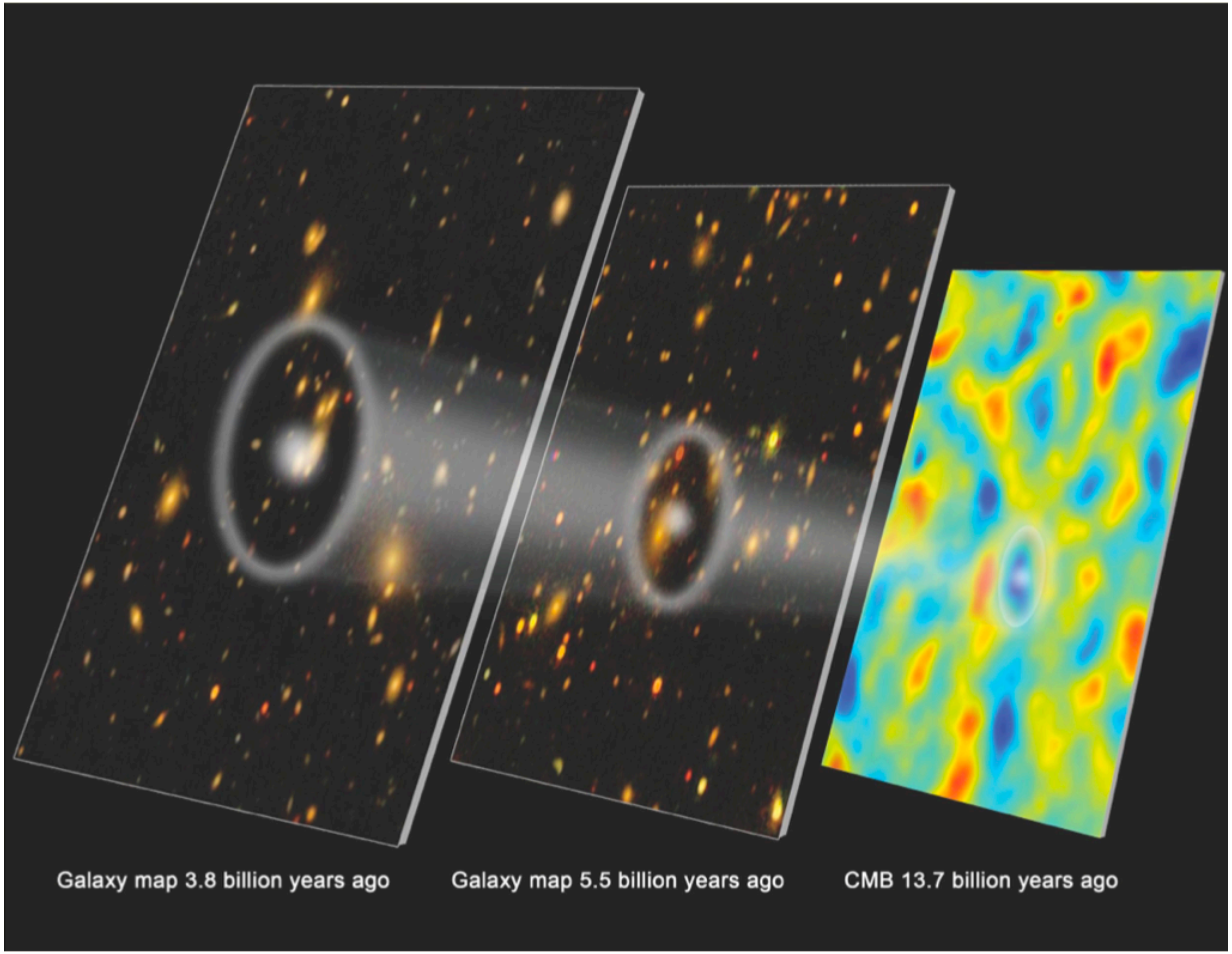
A. Hall et al. PRD (2013)

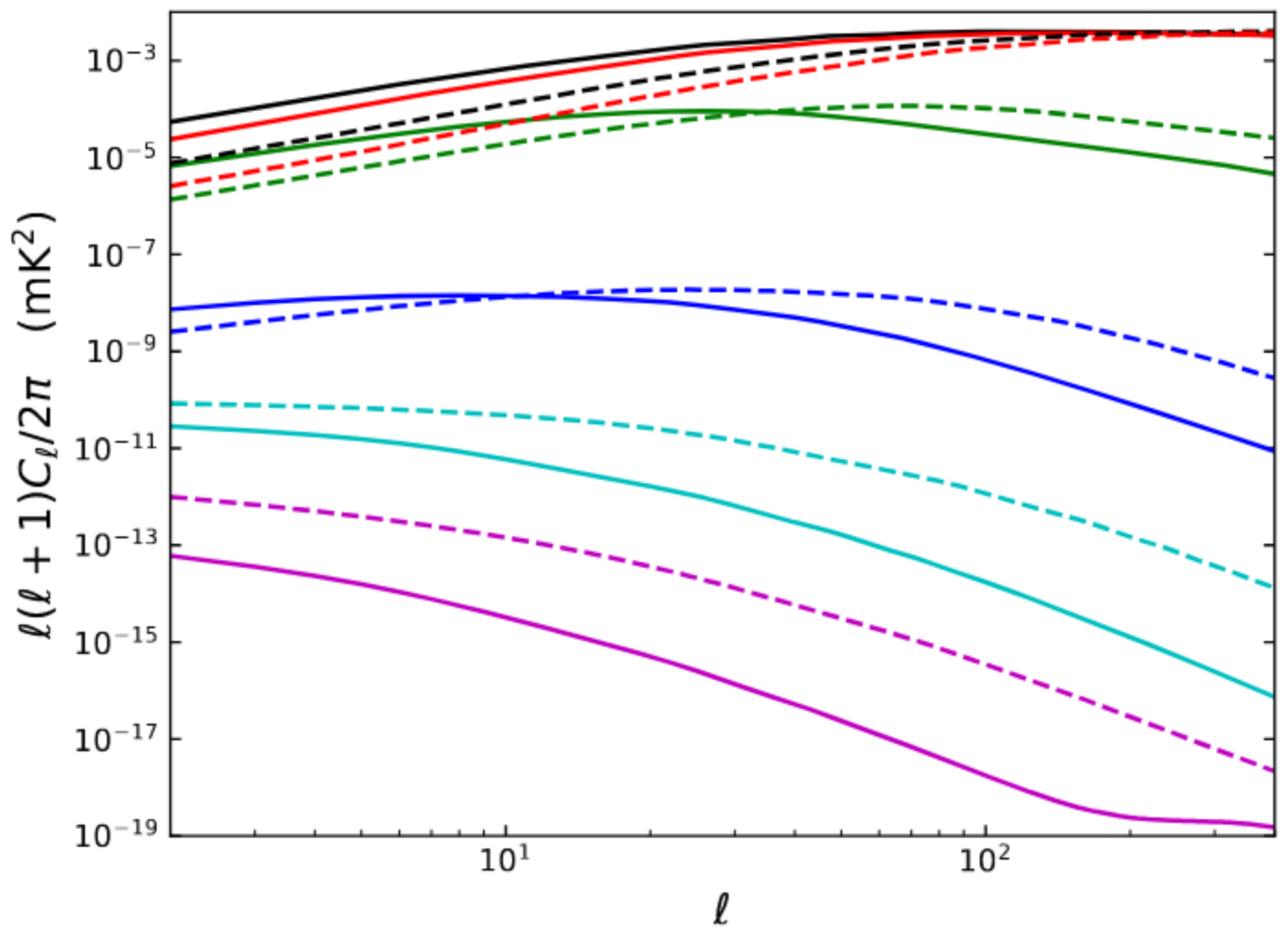
and integrating over a redshift normalized window function

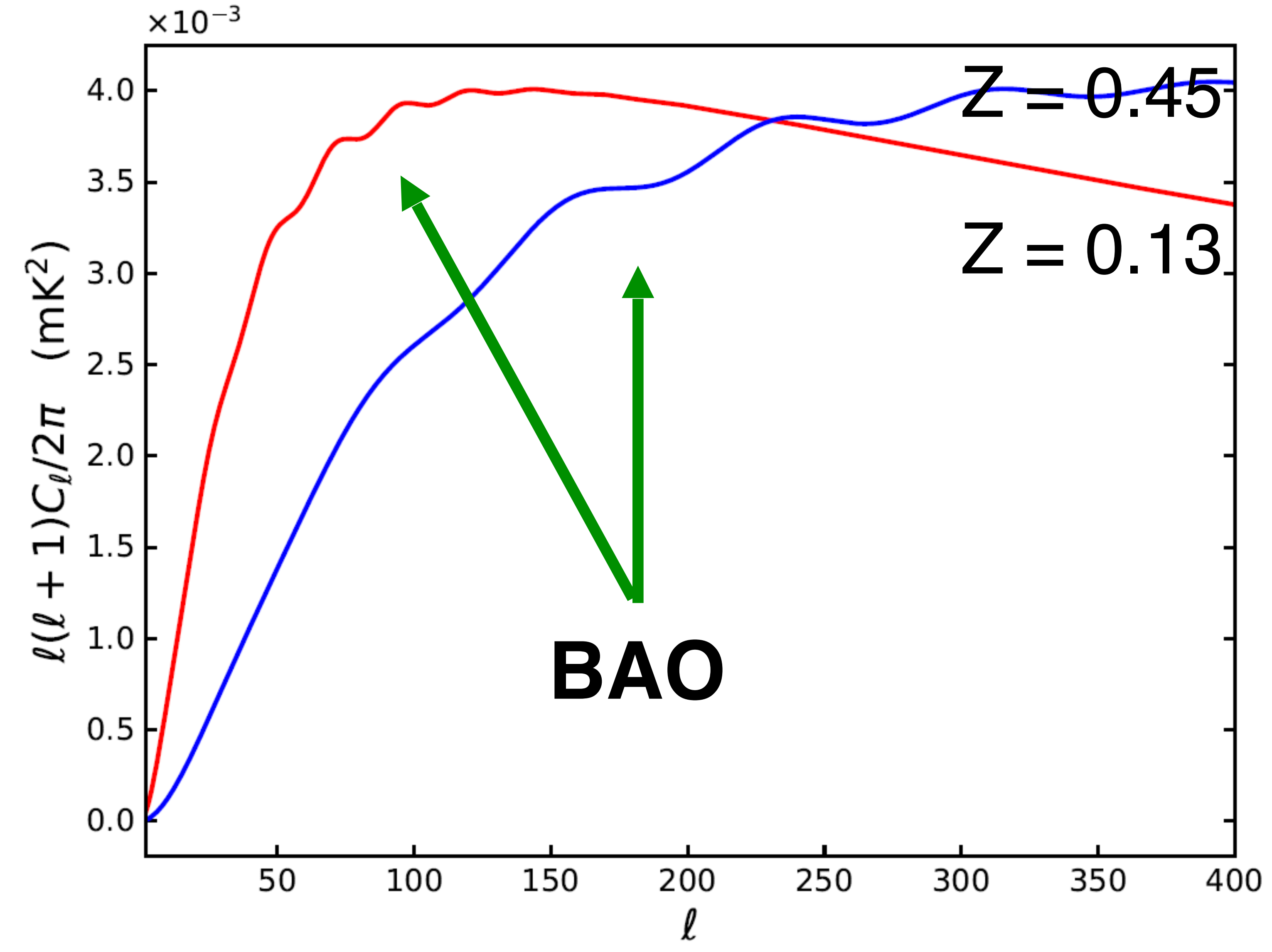
$$\Delta^W_{T_b,l}(\mathbf{k}) = \int_0^\infty dz W(z) \Delta_{T_b,l}(\mathbf{k},z)$$

The angular cross spectra between redshift windows is then

$$C_l^{WW'} = 4\pi \int d\ln k \mathcal{P}_{\mathcal{R}}(k) \Delta^W_{T_b,l}(\mathbf{k}) \Delta^{W'}_{T_b,l}(\mathbf{k}) \, .$$


Galaxy map 3.8 billion years ago
Galaxy map 5.5 billion years ago
CMB 13.7 billion years ago


$\ell(\ell+1)C_\ell/2\pi$ (mK^2)
ℓ


$\times10^{-3}$
Z = 0.45
Z = 0.13
BAO
$\ell(\ell+1)C_\ell/2\pi$ (mK^2)
ℓ

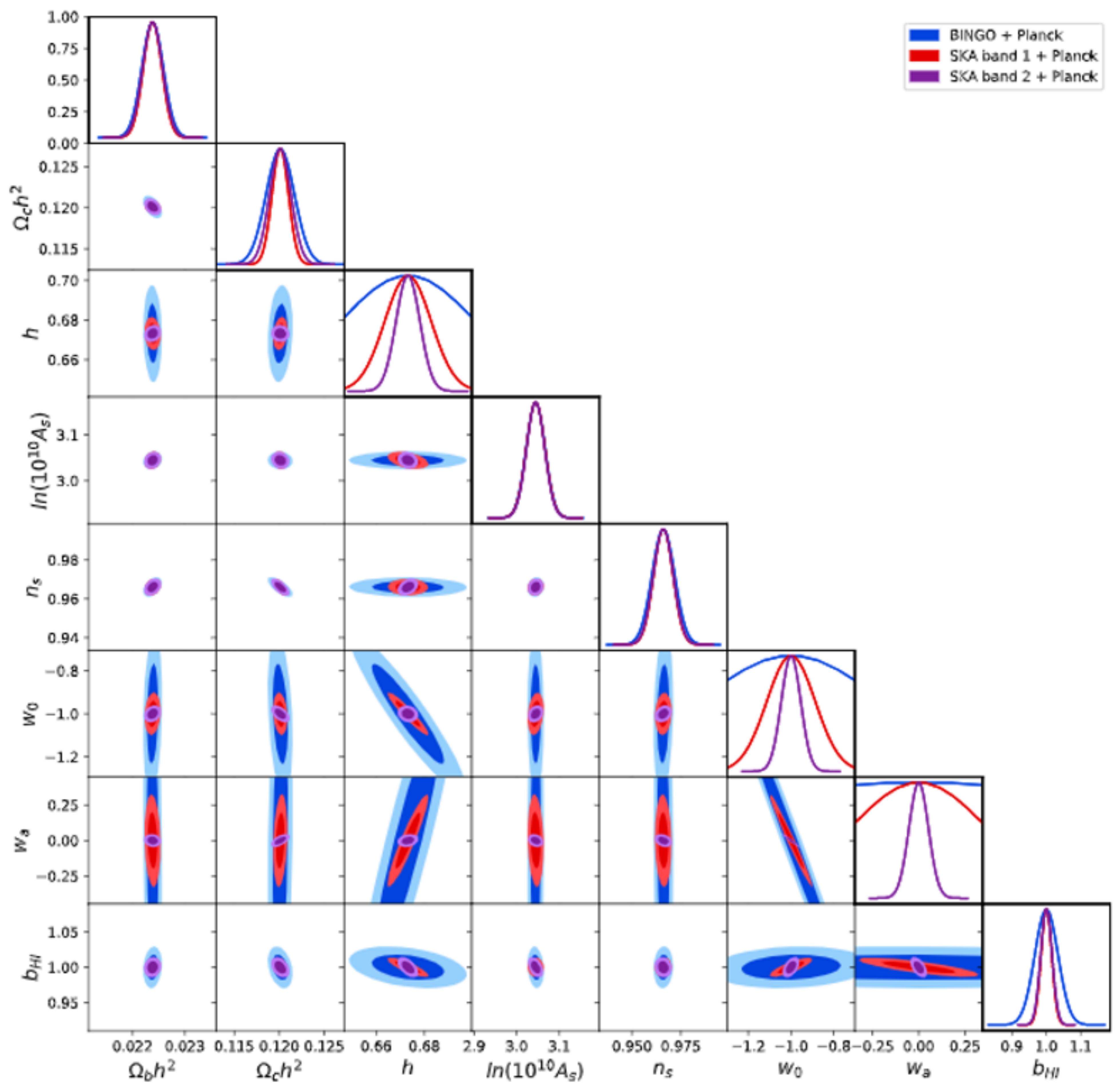
Forecasts

The CPL parameterization is a z-dependent Ansatz for the EoS of dark energy given by

$$w_{\rm CPL}(z) = w_0 + w_a \frac{z}{1+z} \quad \text{or} \quad w_{\rm CPL}(a) = w_0 + w_a(1-a)$$

Parameter	Fiducial value
$\Omega_b h^2$	0.022383
$\Omega_c h^2$	0.12011
h	0.6732
n_s	0.96605
A_s	2.1×10^{-9}
w_0	-1
w_a	0
$b_{\rm HI}$	1
$\Omega_{\rm HI}$	6.2×10^{-4}

	BINGO + *Planck*	SKA Band 1 + *Planck*	SKA Band 2 + *Planck*
Parameter	$\pm 1\sigma$ $(100\% \times \sigma/\theta_i^{\rm fid})$	$\pm 1\sigma$ $(100\% \times \sigma/\theta_i^{\rm fid})$	$\pm 1\sigma$ $(100\% \times \sigma/\theta_i^{\rm fid})$
$\Omega_b h^2$	0.000 15 (0.7%)	0.000 13 (0.6%)	0.000 13 (0.6%)
$\Omega_c h^2$	0.001 12 (0.9%)	0.000 65 (0.5%)	0.000 84 (0.7%)
h	0.019 70 (2.9%)	0.006 79 (1%)	0.003 50 (0.5%)
$\ln(10^{10}A_s)$	0.015 69 (0.5%)	0.015 41 (0.5%)	0.015 07 (0.5%)
n_s	0.004 10 (0.4%)	0.003 50 (0.4%)	0.003 61 (0.4%)
w_0	0.305 38 (30.5%)	0.080 20 (8%)	0.032 30 (3.2%)
w_a	1.223 89	0.261 91	0.037 29
$b_{\rm HI}$	0.023 84 (2.4%)	0.010 99 (1.1%)	0.011 72 (1.2%)



A. A. Costa et al. A&A (2022)

We also considered constraints in interacting dark energy models

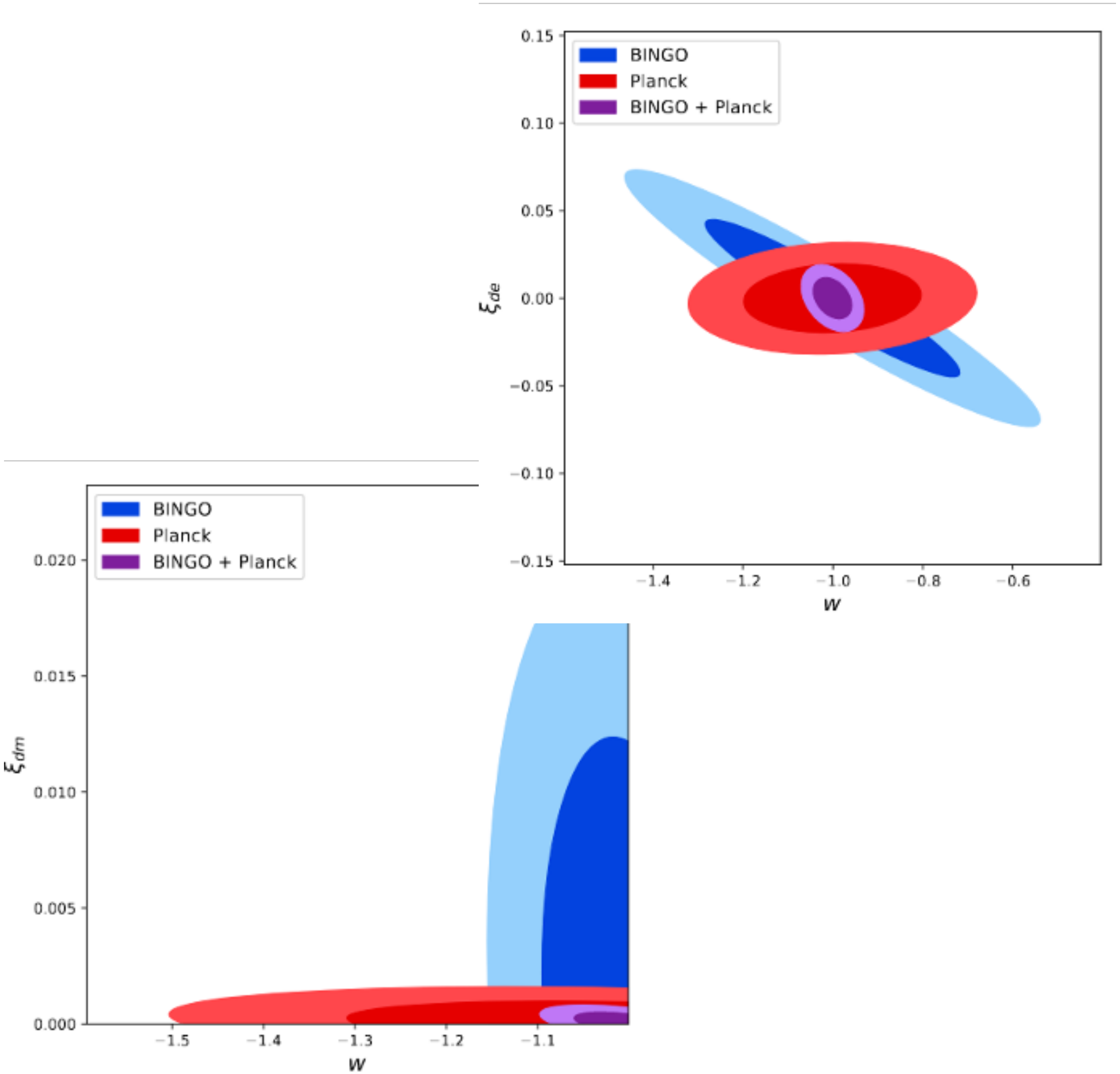
$$\nabla_\mu T^{\mu\nu}_{(i)} = Q^{\nu}_{(i)}$$

which in the background yields

$$\dot{\rho}_{dm} + 3H\rho_{dm} = +\,Q$$
$$\dot{\rho}_{de} + 3H\,(1 + w_{de})\,\rho_{de} = -\,Q$$

and we parameterize the interaction as

$$Q = 3H(\xi_{dm}\rho_{dm} + \xi_{de}\rho_{de})$$



A. A. Costa et al. A&A (2022)

$$\sigma_{B_0} = 3.07 \times 10^{-5} \quad \text{(BINGO)}$$
$$\sigma_{B_0} = 5.31 \times 10^{-2} \quad (Planck)$$
$$\sigma_{B_0} = 1.09 \times 10^{-5} \quad (\text{BINGO} + Planck)\,.$$

We also considered constraints on modified theories of gravity given by

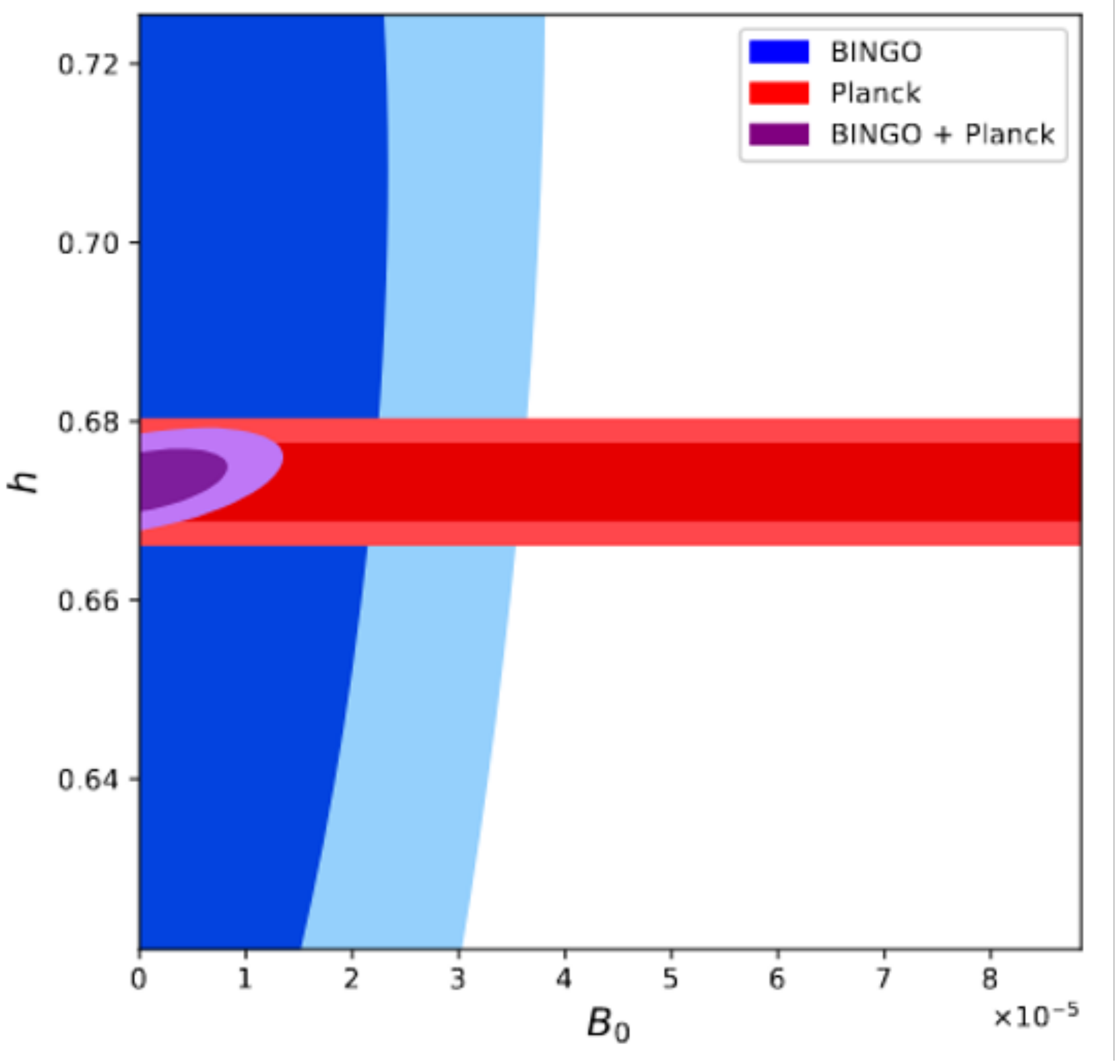
$$-k^2\Psi = 4\pi G a^2 \mu(a,k)\bar{\rho}\Delta$$

$$\Phi = \gamma(a,k)\Psi$$

In particular, the B_0-parameterization of $f(R)$ gravity

$$\mu(a,k) = \frac{1}{1 - 1.4\times 10^{-8}|\lambda/\mathrm{Mpc}|^2 a^3}\,\frac{1 + 4\lambda^2k^2a^4/3}{1+\lambda^2k^2a^4}$$
$$\gamma(a,k) = \frac{1+2\lambda^2k^2a^4/3}{1+4\lambda^2k^2a^4/3}\,,$$

where $B_0 = 2(H_0\lambda)^2$.



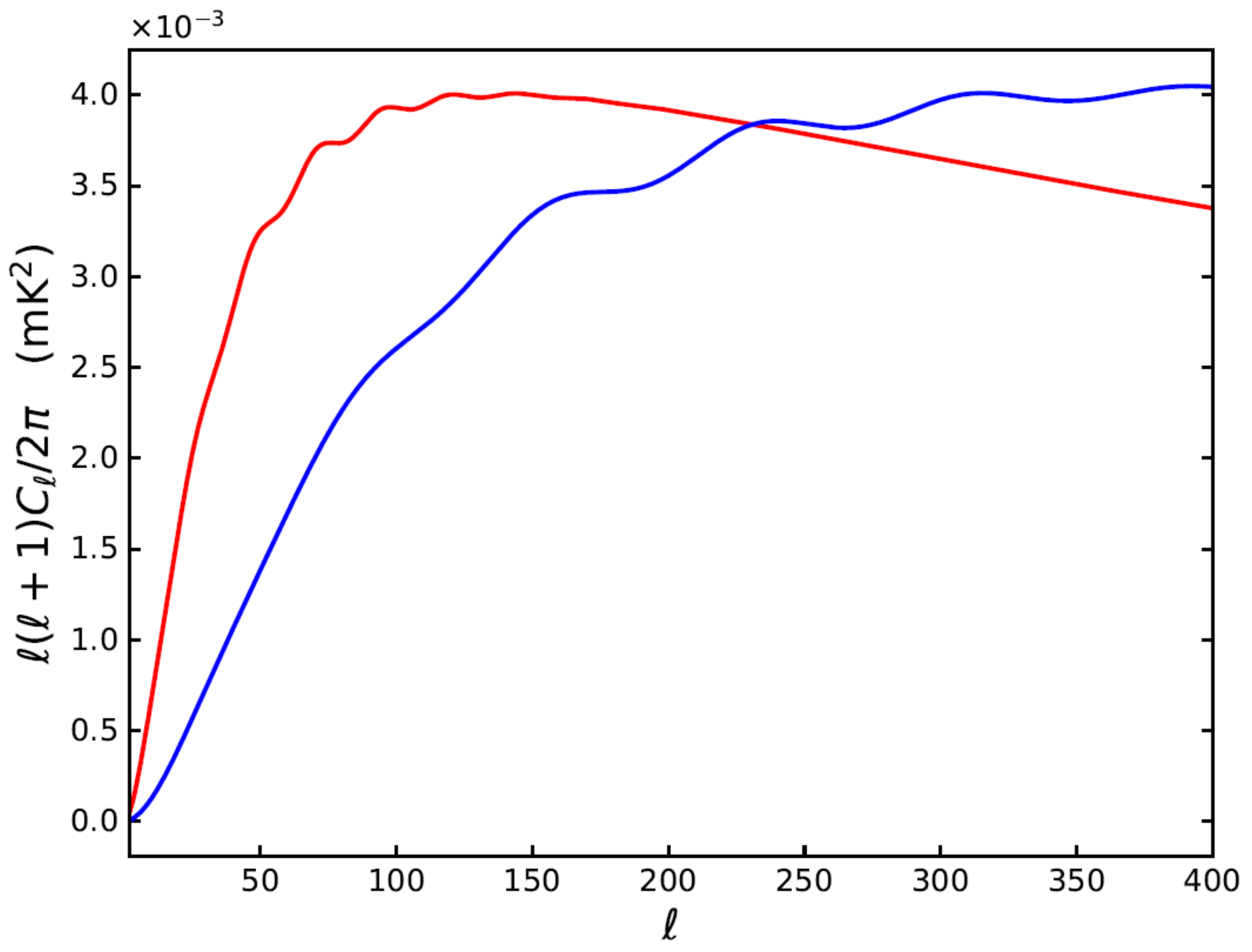
A. A. Costa et al. A&A (2022)

Baryon Acoustic Oscillations (BAO)

$$P^{\rm temp}(k) = [P(k)^{\rm lin} - P^{\rm nw}(k)]e^{-k^2\Sigma_{nl}^2} + P^{\rm nw}(k)$$

$$C(\ell) \quad = \quad B_0\, C^{\rm temp}(\ell/\alpha) + A_0 + A_1\ell + A_2/\ell^2$$

$$\alpha = \frac{(D_A(z)/r_d)}{(D_A(z)/r_d)_{\rm fid}}$$



H. Camacho et al. MNRAS (2019)

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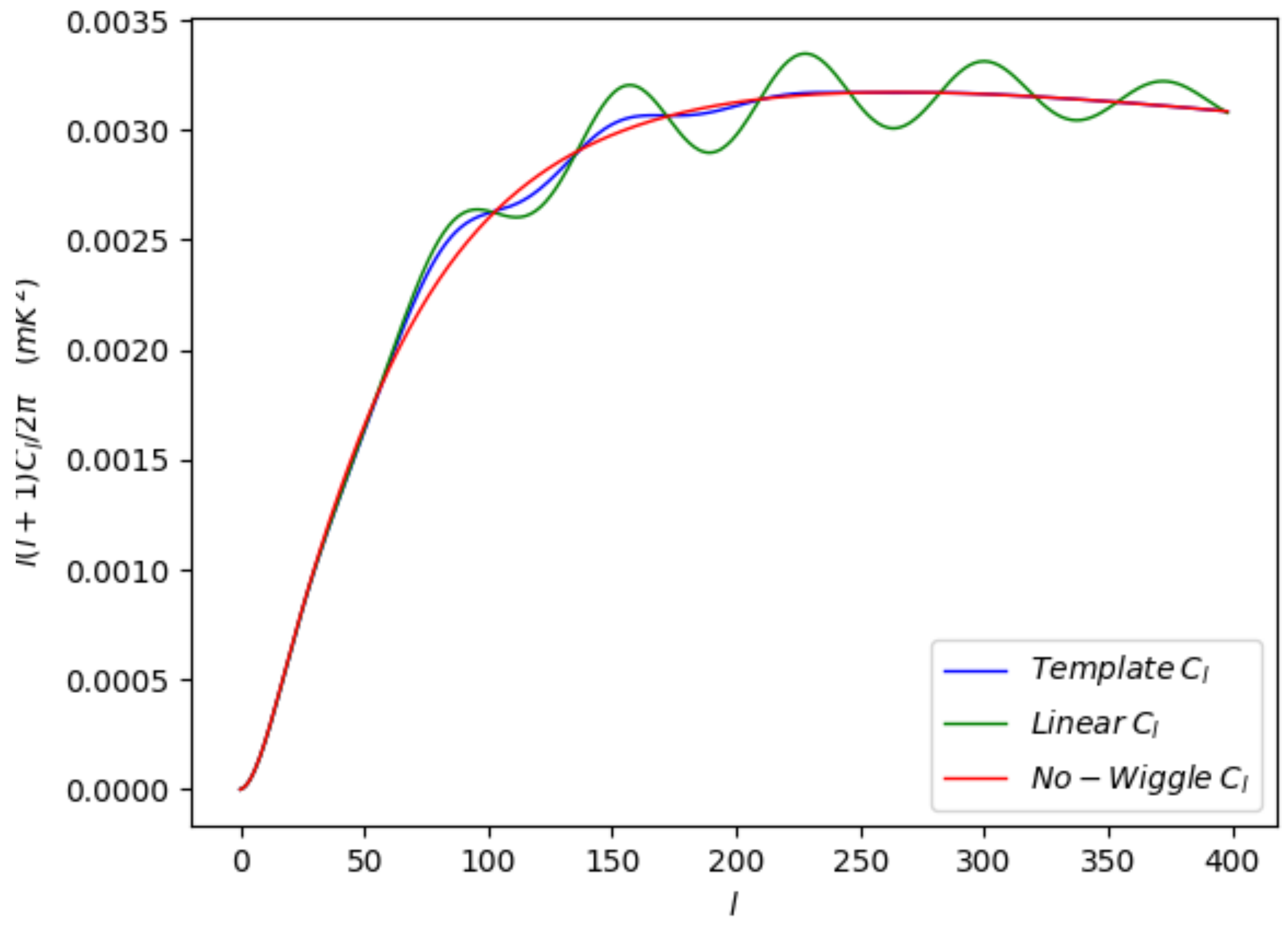

B. Ostergaard, in preparation.

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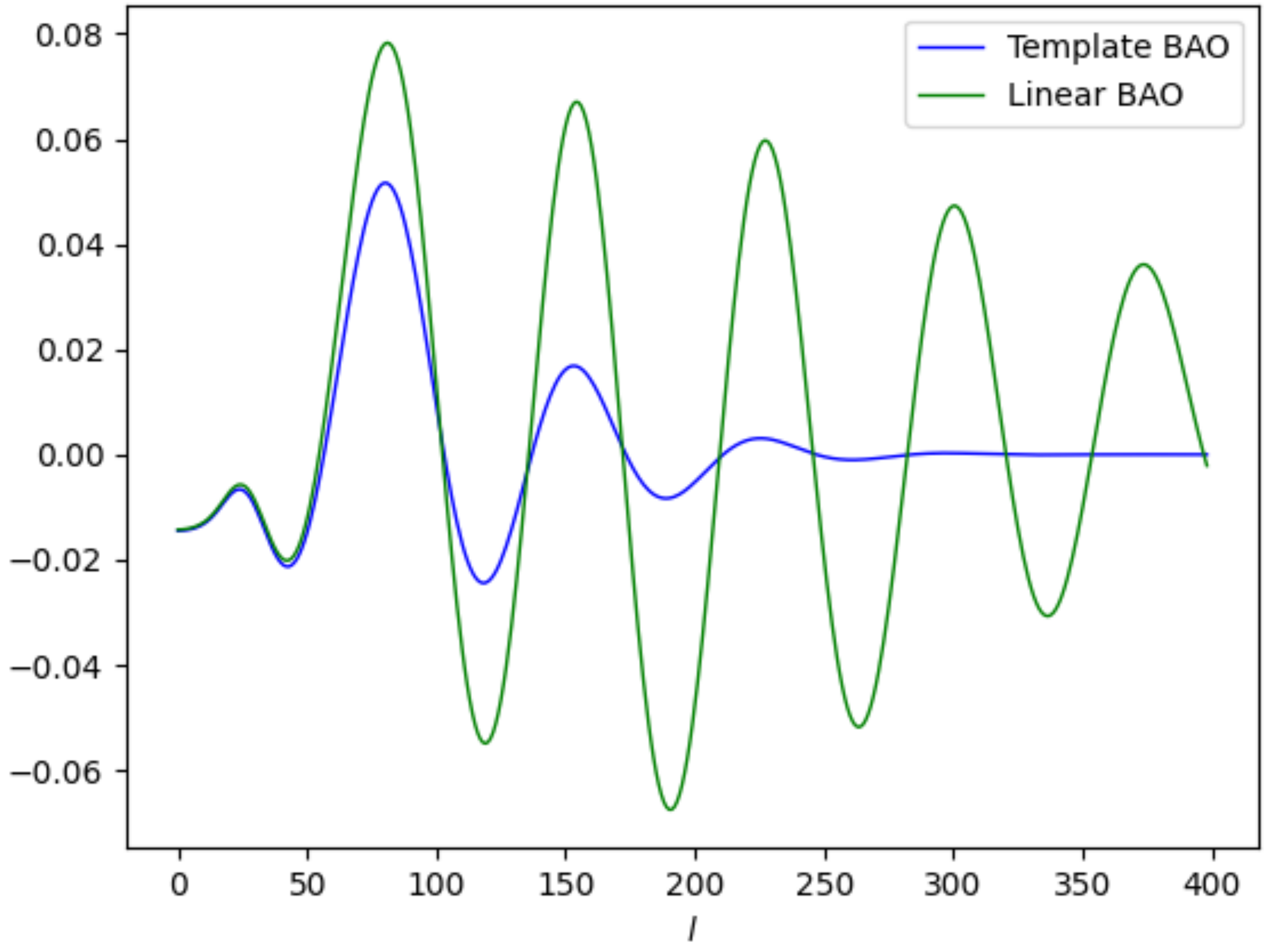

B. Ostergaard, in preparation.

Angular Bispectra

The bispectrum is the Fourrier transform of the three point correlation function

$$
\begin{aligned}
B^{\ell_1\ell_2\ell_3}_{m_1m_2m_3}(z_1, z_2, z_3) &= \langle \Delta T_{b,\ell_1m_1}(z_1)\Delta T_{b,\ell_2m_2}(z_2)\Delta T_{b,\ell_3m_3}(z_3)\rangle \\
&= (4\pi)^3(-i)^{\ell_1+\ell_2+\ell_3}\int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\Delta_{\ell_1}(k_1, z_1)\Delta_{\ell_2}(k_2, z_2)\Delta_{\ell_3}(k_3, z_3)\langle\Phi(\mathbf{k_1})\Phi(\mathbf{k_2})\Phi(\mathbf{k_3})\rangle Y_{\ell_1m_1}(\hat{\mathbf{k}}_1)Y_{\ell_2m_2}(\hat{\mathbf{k}}_2)Y_{\ell_3m_3}(\hat{\mathbf{k}}_3) \\
&= \left(\frac{2}{\pi}\right)^3\int x^2dx\int dk_1dk_2dk_3(k_1k_2k_3)^2B_\Phi(k_1, k_2, k_3)\Delta_{\ell_1}(k_1, z_1)\Delta_{\ell_2}(k_2, z_2)\Delta_{\ell_3}(k_3, z_3)j_{\ell_1}(k_1x)j_{\ell_2}(k_2x)j_{\ell_3}(k_3x)\int d\Omega_{\hat{\mathbf{x}}}Y_{\ell_1m_1}(\hat{\mathbf{x}})Y_{\ell_2m_2}(\hat{\mathbf{x}})Y_{\ell_3m_3}(\hat{\mathbf{x}}) \\
&= b_{\ell_1\ell_2\ell_3}(z_1, z_2, z_3)\mathcal{G}^{\ell_1\ell_2\ell_3}_{m_1m_2m_3}
\end{aligned}
$$

where

$$
\mathcal{G}^{\ell_1\ell_2\ell_3}_{m_1m_2m_3} = \int d\Omega_{\hat{\mathbf{x}}}Y_{\ell_1m_1}(\hat{\mathbf{x}})Y_{\ell_2m_2}(\hat{\mathbf{x}})Y_{\ell_3m_3}(\hat{\mathbf{x}}) = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}}\begin{pmatrix}\ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3\end{pmatrix}
$$

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$$
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&= (4\pi)^3(-i)^{\ell_1+\ell_2+\ell_3}\int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\Delta_{\ell_1}(k_1,z_1)\Delta_{\ell_2}(k_2,z_2)\Delta_{\ell_3}(k_3,z_3)\langle\Phi(\mathbf{k_1})\Phi(\mathbf{k_2})\Phi(\mathbf{k_3})\rangle Y_{\ell_1m_1}(\hat{\mathbf{k}}_1)Y_{\ell_2m_2}(\hat{\mathbf{k}}_2)Y_{\ell_3m_3}(\hat{\mathbf{k}}_3) \\
&= \left(\frac{2}{\pi}\right)^3\int x^2dx\int dk_1dk_2dk_3(k_1k_2k_3)^2B_\Phi(k_1,k_2,k_3)\Delta_{\ell_1}(k_1,z_1)\Delta_{\ell_2}(k_2,z_2)\Delta_{\ell_3}(k_3,z_3)j_{\ell_1}(k_1x)j_{\ell_2}(k_2x)j_{\ell_3}(k_3x)\int d\Omega_{\hat{\mathbf{x}}}Y_{\ell_1m_1}(\hat{\mathbf{x}})Y_{\ell_2m_2}(\hat{\mathbf{x}})Y_{\ell_3m_3}(\hat{\mathbf{x}}) \\
&= b_{\ell_1\ell_2\ell_3}(z_1,z_2,z_3)\mathcal{G}^{\ell_1\ell_2\ell_3}_{m_1m_2m_3}
\end{aligned}
$$

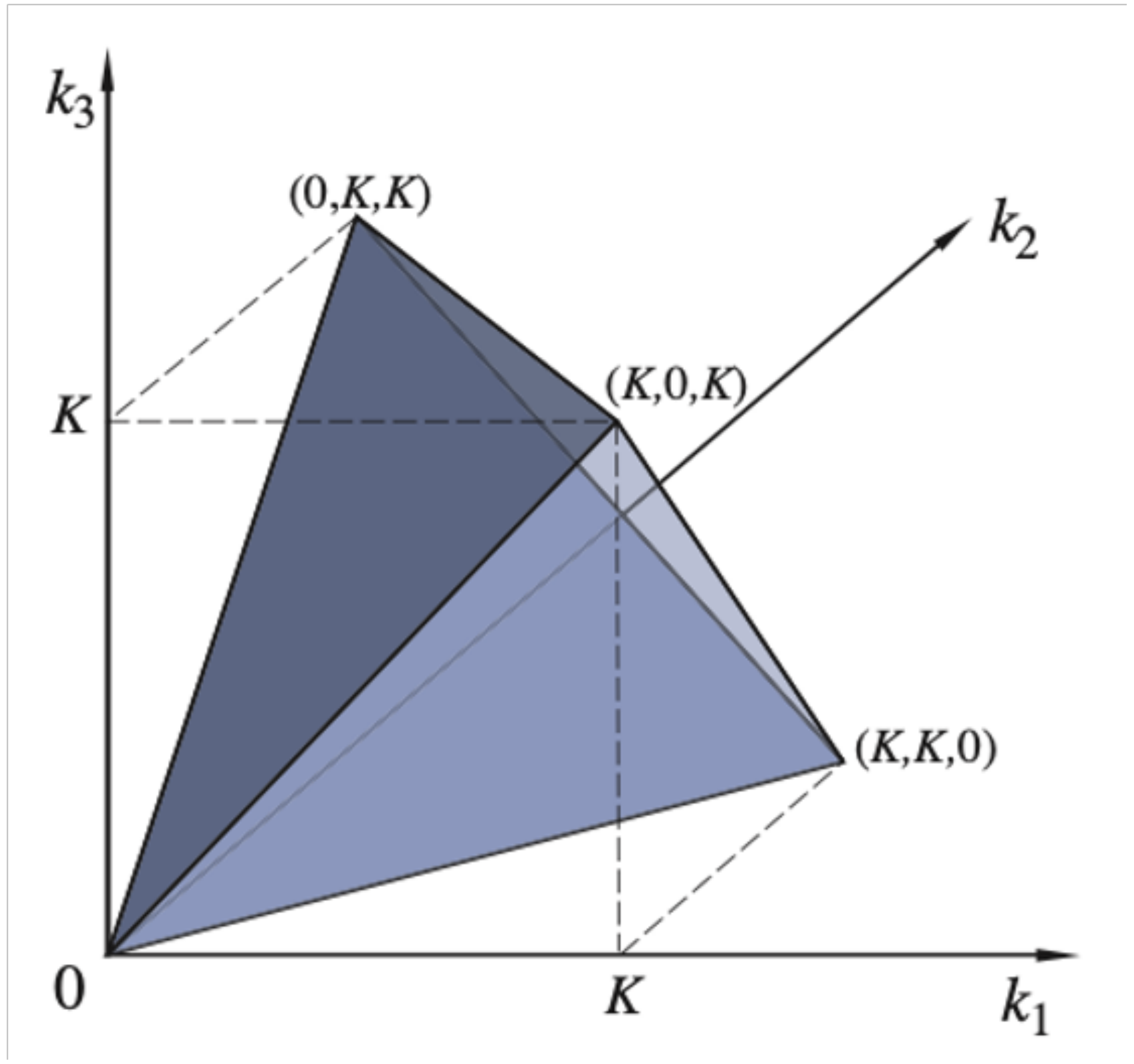
Three different contributions could produce a bispectral signal

- Primordial non-gaussianities
- Non-linear structure formation
- Subtracting the foregrounds with a component separation technique

Primordial

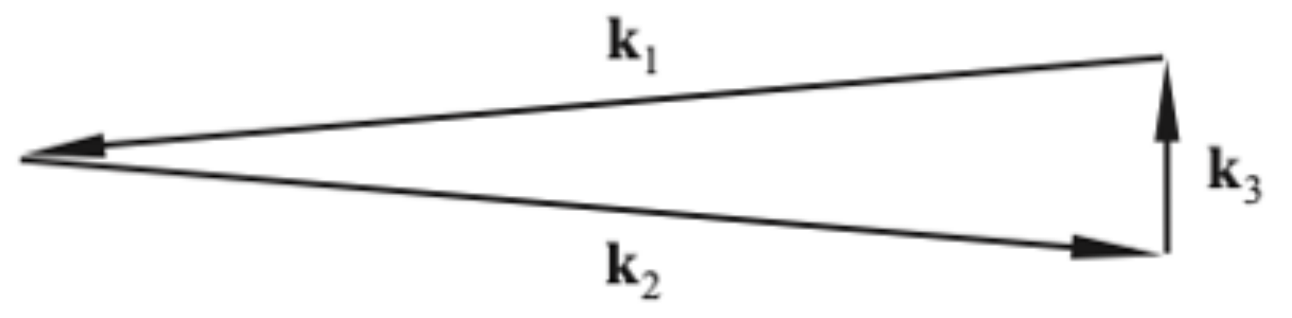
Bispectrum of primordial perturbations

$$\langle\Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\Phi(k_1,k_2,k_3)$$



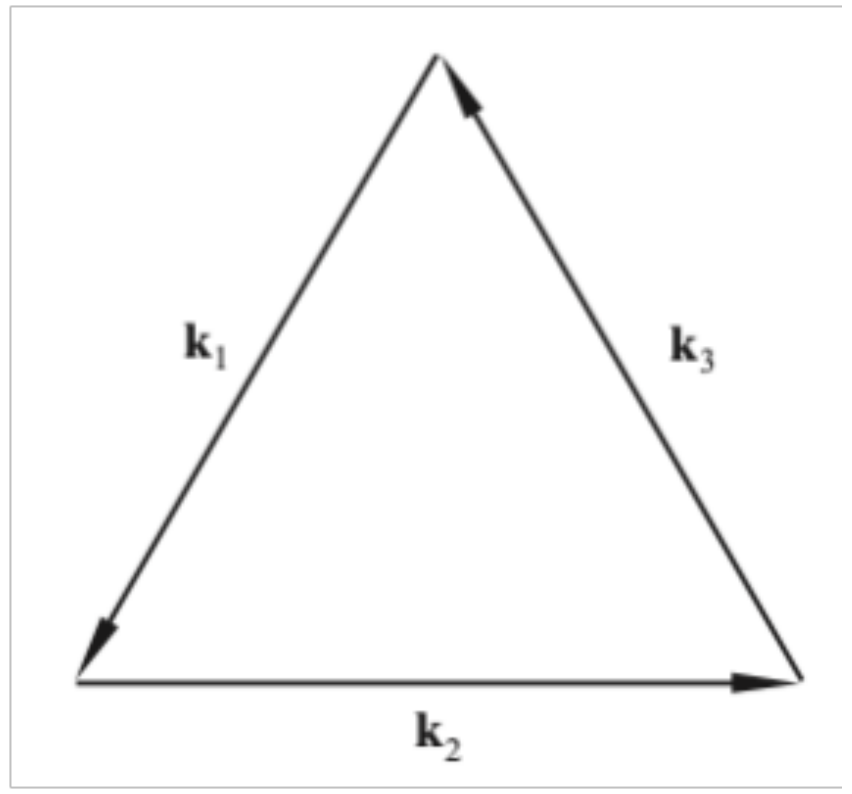
local or squeezed

$k_3 \ll k_1 \sim k_2$



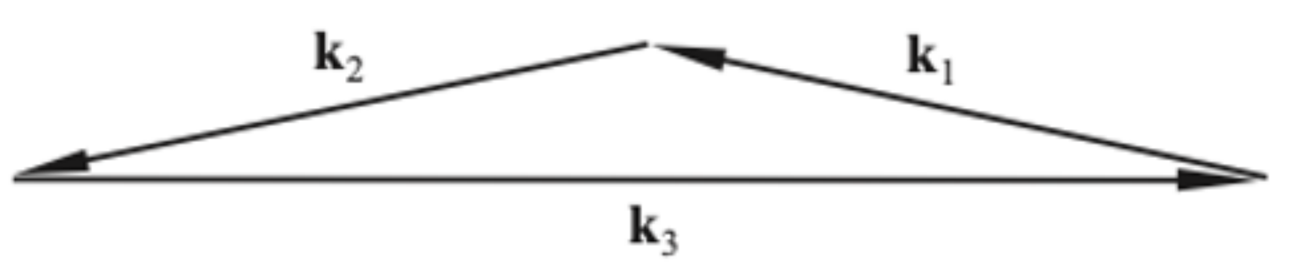
equilateral

$k_1 = k_2 = k_3$



flattened or folded

$k_1 = k_2 + k_3$



The local shape primordial bispectrum

$$\begin{aligned}\Phi(\mathbf{x}) &= \Phi_L(\mathbf{x}) + \Phi_{NL}(\mathbf{x}) \\ &= \Phi_L(\mathbf{x}) + f_{NL}[\Phi_L^2(\mathbf{x}) - \langle\Phi_L^2(\mathbf{x})\rangle]\end{aligned}$$

$$B_\Phi(k_1, k_2, k_3) = 2 f_{NL}\,[P_\Phi(k_1)P_\Phi(k_2) + P_\Phi(k_2)P_\Phi(k_3) + P_\Phi(k_3)P_\Phi(k_1)]$$

Rewriting the reduced bispectrum

$$b_{\ell_1\ell_2\ell_3}(z_1, z_2, z_3) = \left(\frac{2}{\pi}\right)^3 \int x^2 dx \int dk_1 dk_2 dk_3\ (k_1k_2k_3)^2\ B_\Phi(k_1, k_2, k_3) \times \Delta_{\ell_1}(k_1, z_1)\,\Delta_{\ell_2}(k_2, z_2)\,\Delta_{\ell_3}(k_3, z_3)\ j_{\ell_1}(k_1 x)\ j_{\ell_2}(k_2 x)\ j_{\ell_3}(k_3 x)$$

$$\Downarrow$$

$$b^{\text{local}}_{\ell_1\ell_2\ell_3}(z_1, z_2, z_3) = \int x^2 dx \left[\boldsymbol{\alpha}_{\ell_1}(x, z_1)\boldsymbol{\beta}_{\ell_2}(x, z_2)\boldsymbol{\beta}_{\ell_3}(x, z_3) + 2 \text{ perm.}\right]$$

$$\boldsymbol{\alpha}_\ell(x, z) = \frac{2}{\pi}\int dk\, k^2\, \Delta_\ell(k, z)\, j_\ell(kx)$$

$$\boldsymbol{\beta}_\ell(x, z) = \frac{2}{\pi}\int dk\, k^2 P_\Phi(k)\, \Delta_\ell(k, z)\, j_\ell(kx)$$

For the equilateral shape

$$B_\Phi^{\rm eq}(k_1,k_2,k_3) = 6 f_{\rm NL}^{\rm equil}\Big\{ -\Big[P_\Phi(k_1)P_\Phi(k_2) + 2\ \text{perms}\Big] - 2\Big[P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3)\Big]^{2/3} + \Big[P_\Phi^{1/3}(k_1)P_\Phi^{2/3}(k_2)P_\Phi(k_3) + 5\ \text{perms}\Big]\Big\},$$

and the reduced bispectrum is

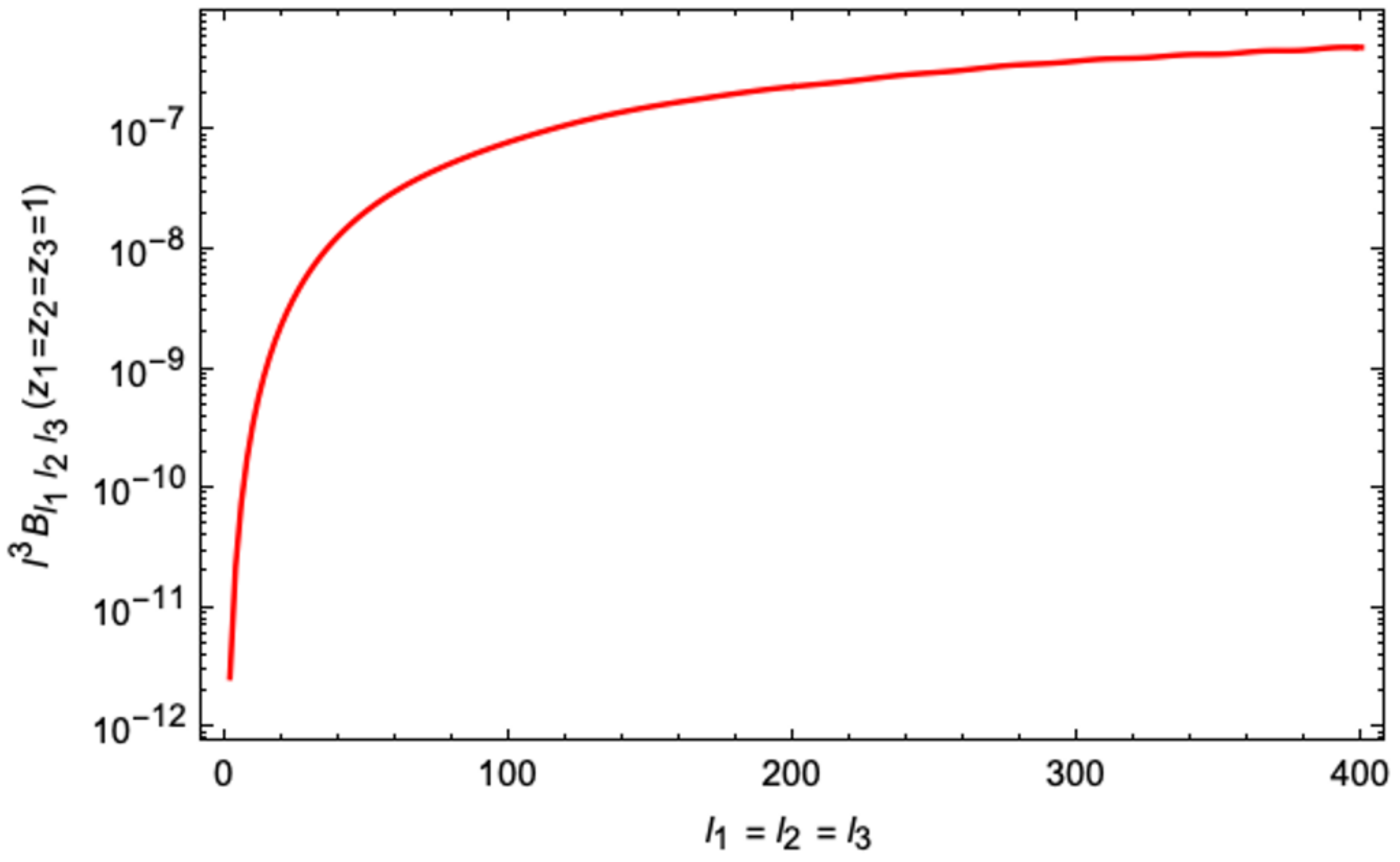
$$b^{\rm equil}_{\ell_1\ell_2\ell_3}(z_1,z_2,z_3) = 6 f_{\rm NL}^{\rm equil}\int x^2 dx\Big[-\big(\alpha_{\ell_1}(x,z_1)\beta_{\ell_2}(x,z_2)\beta_{\ell_3}(x,z_3) + 2\ \text{perms.}\big) - 2\delta_{\ell_1}(x,z_1)\delta_{\ell_2}(x,z_2)\delta_{\ell_3}(x,z_3) + \big(\beta_{\ell_1}(x,z_1)\gamma_{\ell_2}(x,z_2)\delta_{\ell_3}(x,z_3) + 5\ \text{perms.}\big)\Big]$$

with

$$\gamma_\ell(x,z) = \frac{2}{\pi}\int dk k^2 P_\Phi(k)^{1/3}\Delta_\ell(k,z)\, j_\ell(kx) \qquad \delta_\ell(x,z) = \frac{2}{\pi}\int dk k^2 P_\Phi(k)^{2/3}\Delta_\ell(k,z)\, j_\ell(kx)$$

$$B_{\ell_1\ell_2\ell_3}(z_1,z_2,z_3) = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} b_{\ell_1\ell_2\ell_3}(z_1,z_2,z_3)$$

Survey	SKA-MID Band 1	SKA-MID Band 2	BINGO	PUMA
Frequency (Mhz)	350 − 1050	950 − 1410	980 − 1260	200 − 1100
$\Delta\nu$ (Mhz)	10	10	10	10
$N_{\rm bins}$	70	46	28	90
$f_{\rm sky}$	0.48	0.12	0.07	0.5
$t_{\rm obs}$ (yr)	1	1	1	1
$\sigma_{f_{NL}^{\rm loc}}$	41.1	54.0	161.0	6.0
$\sigma_{f_{NL}^{\rm eq}}$	109.6	180.9	518.7	17.0
$\sigma_{f_{NL}^{\rm orth}}$	129.2	134.5	406.8	16.4



Y. Sang, in preparation.

Relativistic

$$
\begin{aligned}
B^{\ell_1\ell_2\ell_3}_{m_1m_2m_3}(z_1,z_2,z_3) &= \langle \Delta T_{b,\ell_1m_1}(z_1)\Delta T_{b,\ell_2m_2}(z_2)\Delta T_{b,\ell_3m_3}(z_3)\rangle \\
&= (4\pi)^3(-i)^{\ell_1+\ell_2+\ell_3}\int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\Delta_{\ell_1}(k_1,z_1)\Delta_{\ell_2}(k_2,z_2)\Delta_{\ell_3}(k_3,z_3)\langle\Phi(\mathbf{k_1})\Phi(\mathbf{k_2})\Phi(\mathbf{k_3})\rangle Y_{\ell_1m_1}(\hat{\mathbf{k}}_1)Y_{\ell_2m_2}(\hat{\mathbf{k}}_2)Y_{\ell_3m_3}(\hat{\mathbf{k}}_3) \\
&= \left(\frac{2}{\pi}\right)^3\int x^2dx\int dk_1dk_2dk_3(k_1k_2k_3)^2B_\Phi(k_1,k_2,k_3)\Delta_{\ell_1}(k_1,z_1)\Delta_{\ell_2}(k_2,z_2)\Delta_{\ell_3}(k_3,z_3)j_{\ell_1}(k_1x)j_{\ell_2}(k_2x)j_{\ell_3}(k_3x)\int d\Omega_{\hat{\mathbf{x}}}Y_{\ell_1m_1}(\hat{\mathbf{x}})Y_{\ell_2m_2}(\hat{\mathbf{x}})Y_{\ell_3m_3}(\hat{\mathbf{x}}) \\
&= b_{\ell_1\ell_2\ell_3}(z_1,z_2,z_3)\mathcal{G}^{\ell_1\ell_2\ell_3}_{m_1m_2m_3}
\end{aligned}
$$

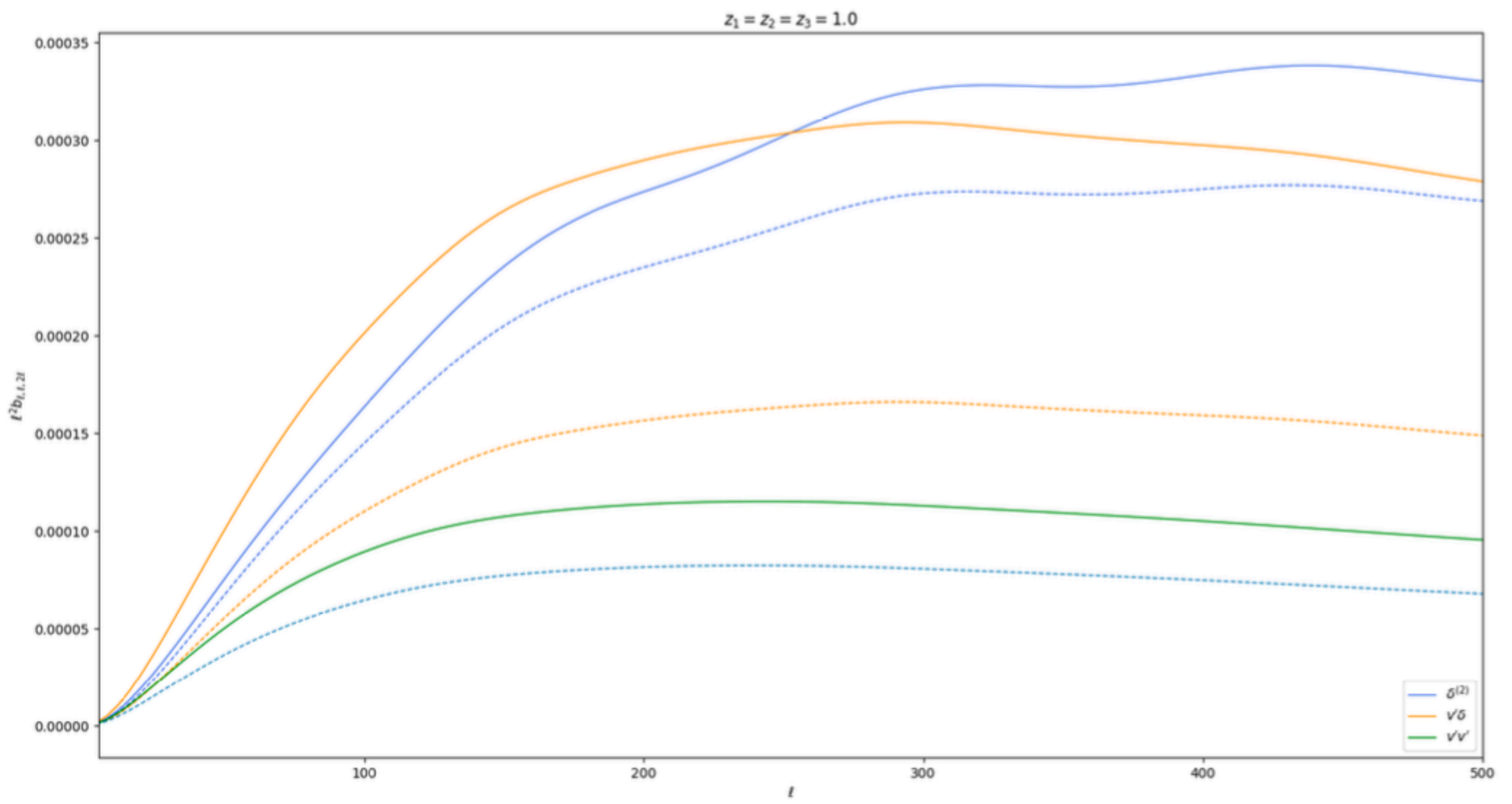
$$\Delta T_b(z,\hat{n}) = \sum_{lm}\Delta T_{b,lm}(z)Y_{lm}(\hat{n})$$

$$\Delta T_b = \Delta T_b^{(1)} + \Delta T_b^{(2)}$$

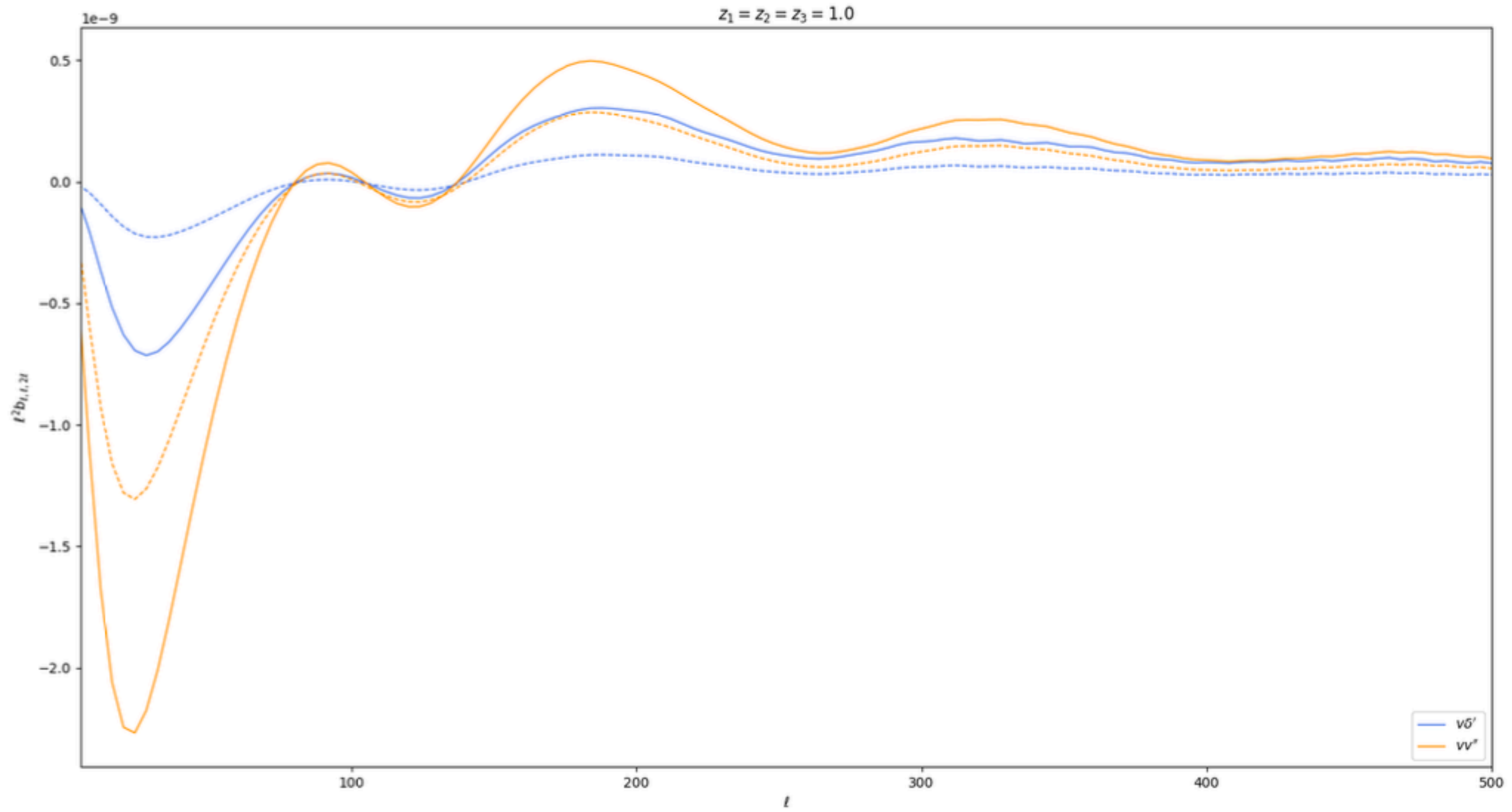
$$\Delta T_b^{(1)} = b_1\delta^{(1)} + \mathcal{H}^{-1}\partial_r^2 v^{(1)}$$

$$
\begin{aligned}
\Delta T_b^{(2)} = b_1\delta^{(2)} + \frac{1}{2}b_2(\delta^{(1)})^2 + b_s s^2 + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \\
+\mathcal{H}^{-2}\left[(\partial_r^2 v^{(1)})^2 + \partial_r v^{(1)}\partial_r^3 v^{(1)}\right] + \mathcal{H}^{-1}\left[\partial_r v^{(1)}\partial_r\delta^{(1)} + \partial_r^2 v^{(1)}\delta^{(1)}\right]
\end{aligned}
$$

$$B_{tree}(\mathbf{n_1}, \mathbf{n_2}, \mathbf{n_3}, z_1, z_2, z_3) \equiv \langle \Delta^{(2)}(\mathbf{n_1}, z_1)\Delta^{(1)}(\mathbf{n_2}, z_2)\Delta^{(1)}(\mathbf{n_3}, z_3)\rangle + \circlearrowleft$$

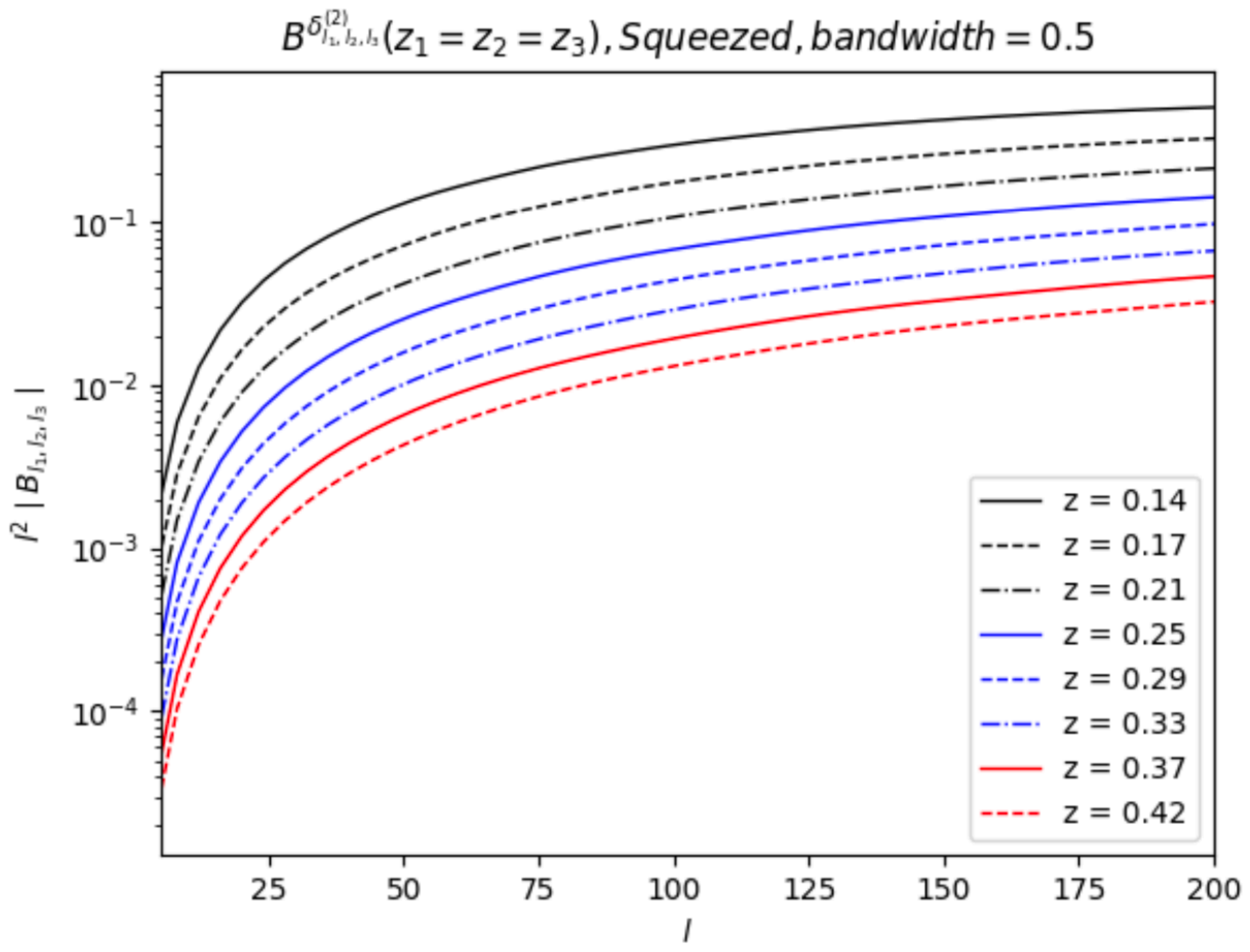


$$b_{\ell_1\ell_2\ell_3}(z_1, z_2, z_3) = b^{\delta^{(2)}}_{\ell_1\ell_2\ell_3}(z_1, z_2, z_3) + b^{v^{(2)'}}_{\ell_1\ell_2\ell_3}(z_1, z_2, z_3) + b^{\delta v'}_{\ell_1\ell_2\ell_3}(z_1, z_2, z_3) + b^{v'^2}_{\ell_1\ell_2\ell_3}(z_1, z_2, z_3) + b^{\delta' v}_{\ell_1\ell_2\ell_3}(z_1, z_2, z_3) + b^{v'' v}_{\ell_1\ell_2\ell_3}(z_1, z_2, z_3).$$



R. Pinheiro, in preparation.

$$B_{\ell_1,\ell_2,\ell_3}(z_i, z_j, z_k) = \int dz_1 W_i(z_1) \int dz_2 W_j(z_2) \int dz_3 W_k(z_3) B_{\ell_1,\ell_2,\ell_3}(z_1, z_2, z_3)$$



R. Pinheiro, in preparation.

Next Steps

- Finish forecasts for BAO only and Bispectrum;
- Improve code's speed;
- Combine the primordial angular bispectrum and the relativistic one;
- Include the effect of foreground residuals;
- Use MCMC for cosmological constraints.

Obrigado!