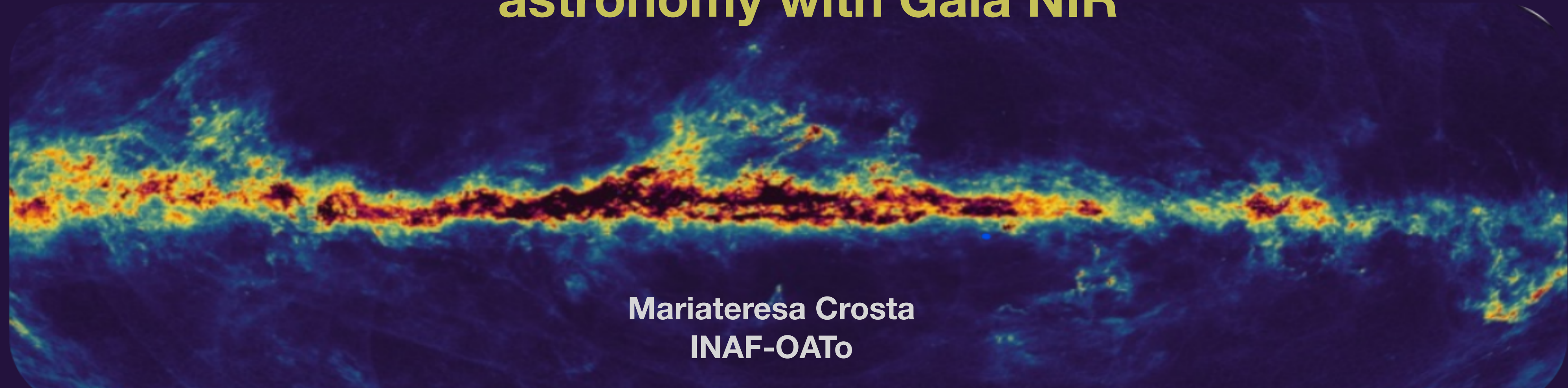


The Gaia legacy: Gravitation and multi-messenger astronomy with Gaia NIR



Mariateresa Crosta
INAF-OATo

Collaborators: U. Abbas, V. Akhmetov, W. Beordo, S. Bertone, B. Bucciarelli, D. Busonero,
M. Gai, M.G. Lattanzi, R. Morbidelli, P. Re Fiorentin, A. Spagna, F. Santucci, A. Vecchiato
INAF-OATo

17-18 January 2024
INAF, Bologna OAS

Gaia&GaiaNIR : The Era of Relativistic Astrometry

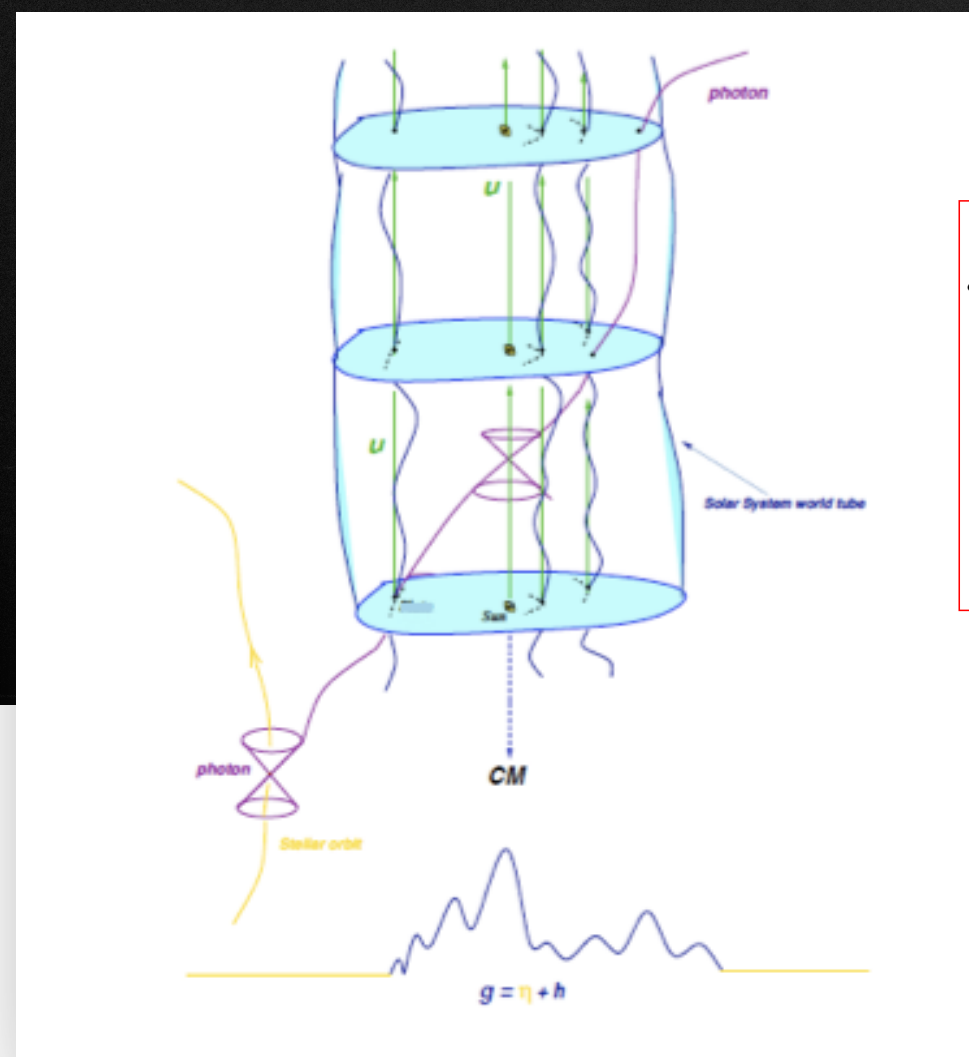
theoretical, analytical and/or numerical models, completely based on General Relativity (GR), and relativistic attitude (satellite or ground based observers) for increasingly accurate astronomical data

Source count maps based on the Gaia DR3 data.
Image credit: ESA/Gaia/DPAC
Image license: CC BY-SA 3.0 IGO

Acknowledgement: Images were created by André Moitinho and Márcia Barros, University of Lisbon, Portugal

micro-arcsecond accuracy + dynamic gravitational fields = relativistic models to reconstruct the propagation of light, from the observer to the star

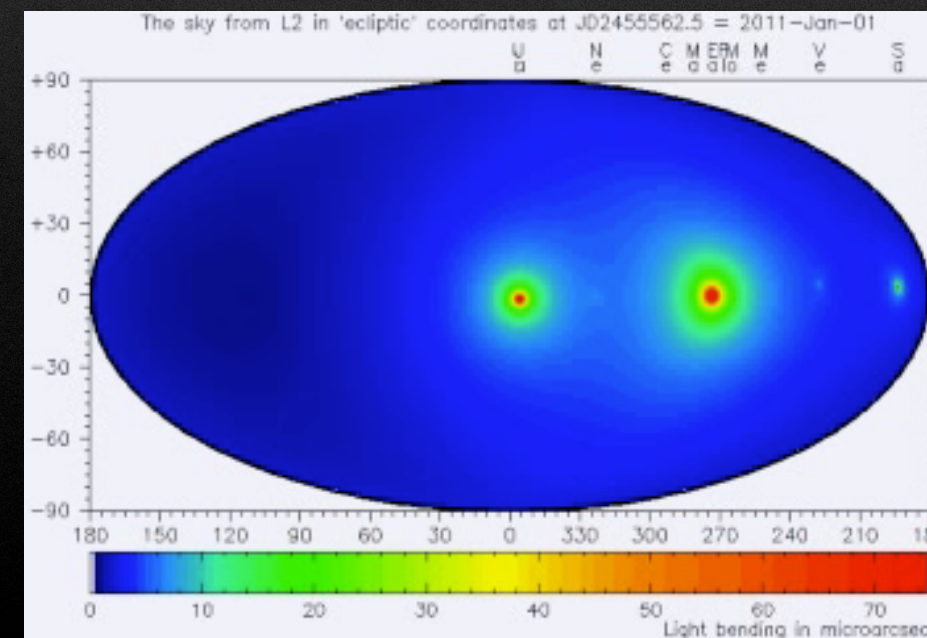
the trajectories of photons emitted by the stars
- null geodesics -
should be as fundamental as the equation of stellar evolution!



$$g_{00} = -1 + \frac{2}{c^2} w(t, \mathbf{x}) - \frac{2}{c^4} w^2(t, \mathbf{x}),$$

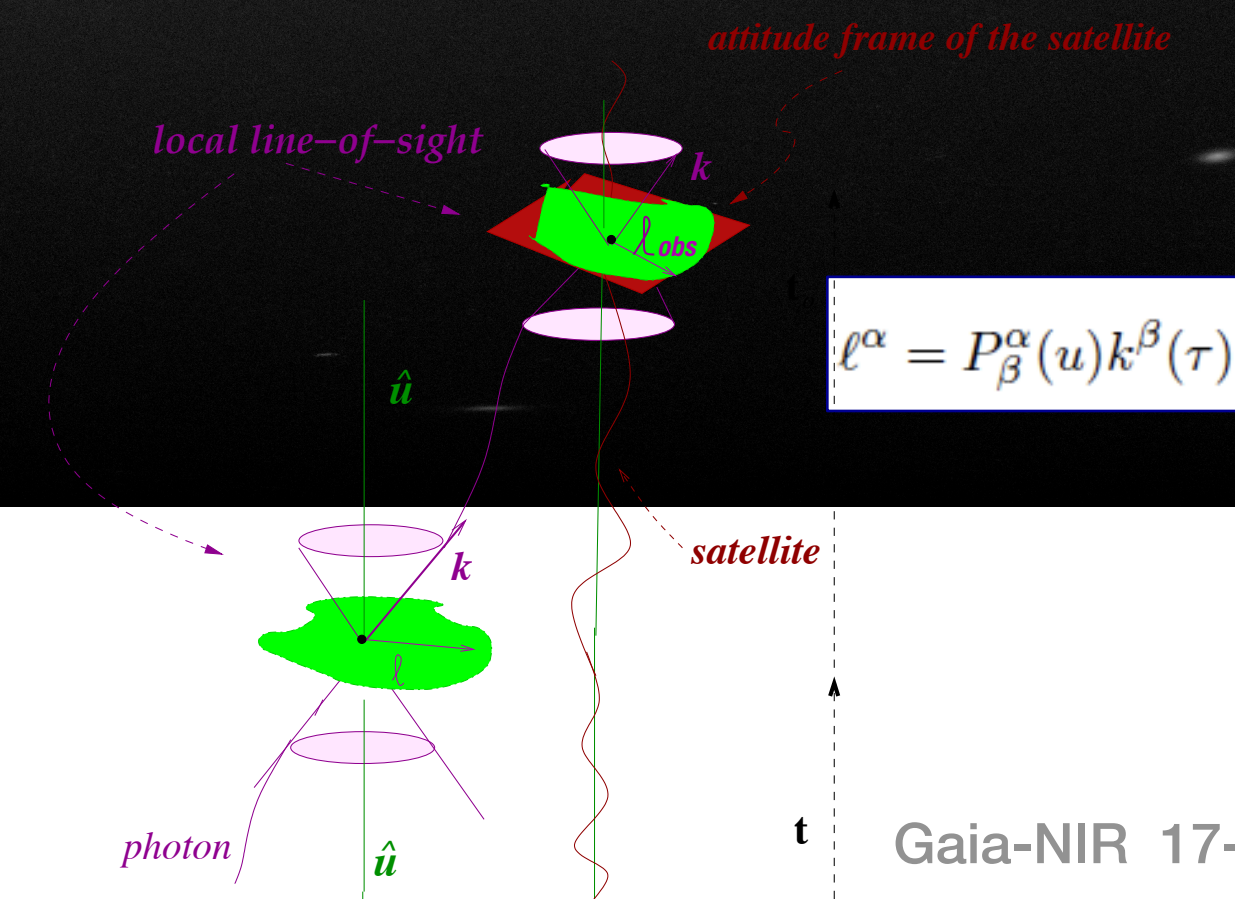
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M.Crosta. "Astrometry in the 21st century. From Hipparchus to Einstein". In: La Rivista del Nuovo Cimento 42 (2019) and refer. therein

M. Crosta et al. "General relativistic observable for gravitational astrometry in the context of the Gaia mission and beyond". Phys. Rev. D 96 (2017), p. 104030



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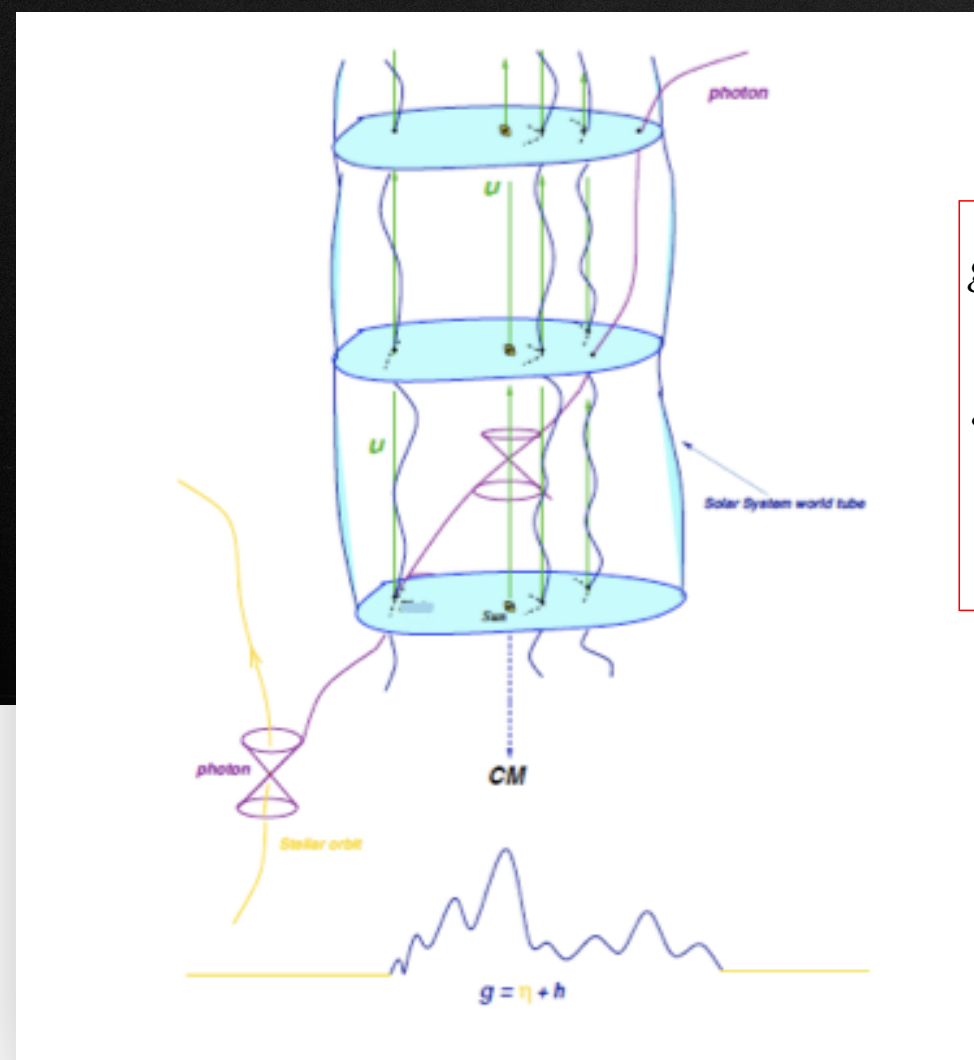
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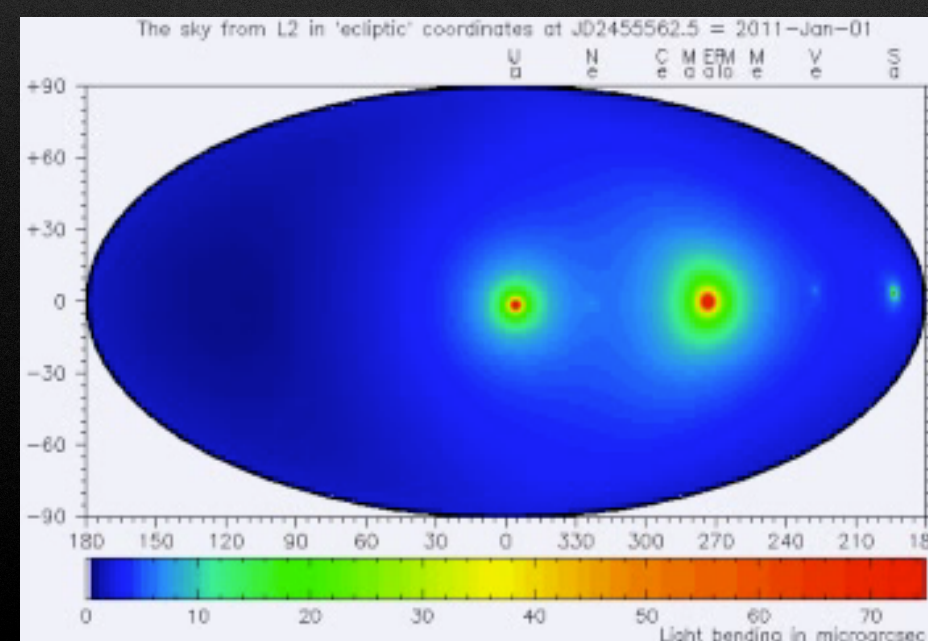
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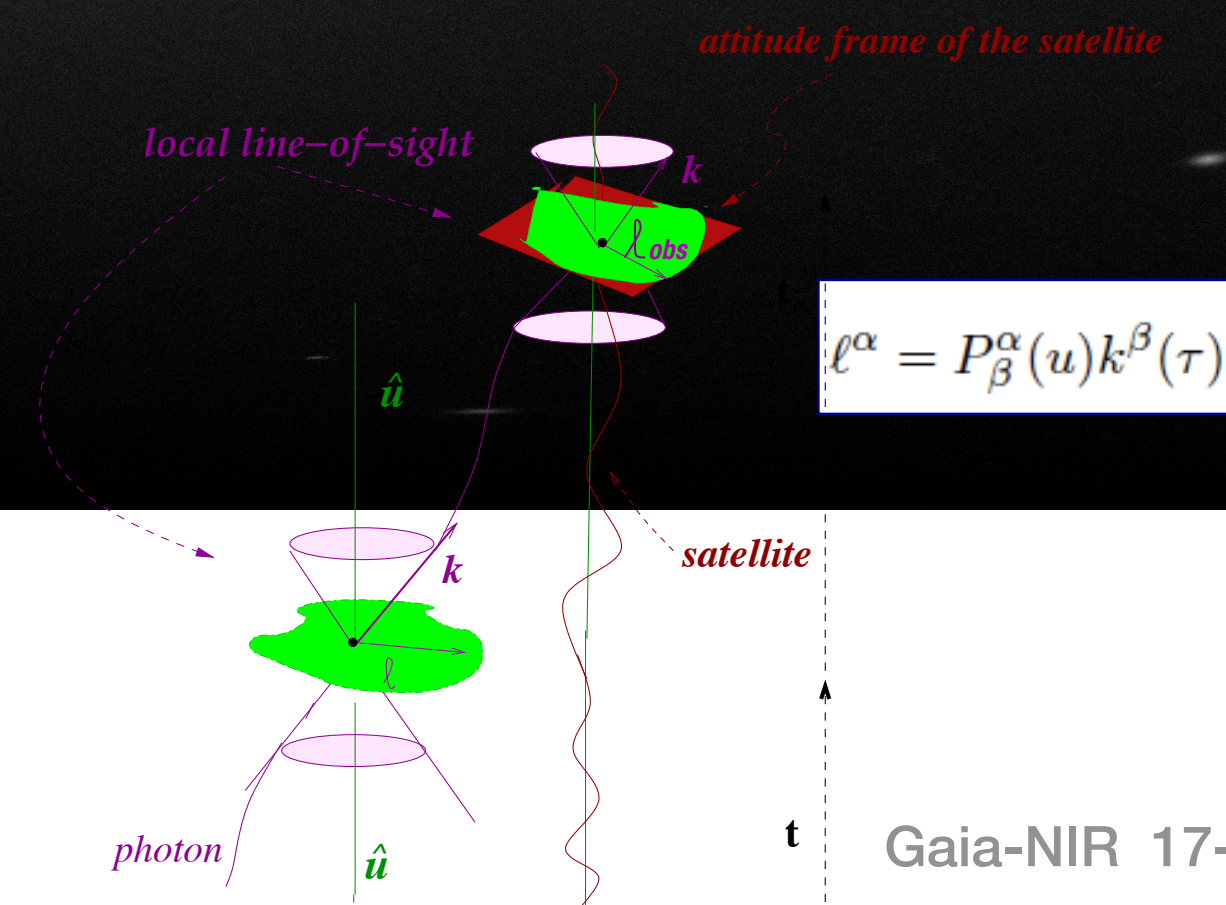
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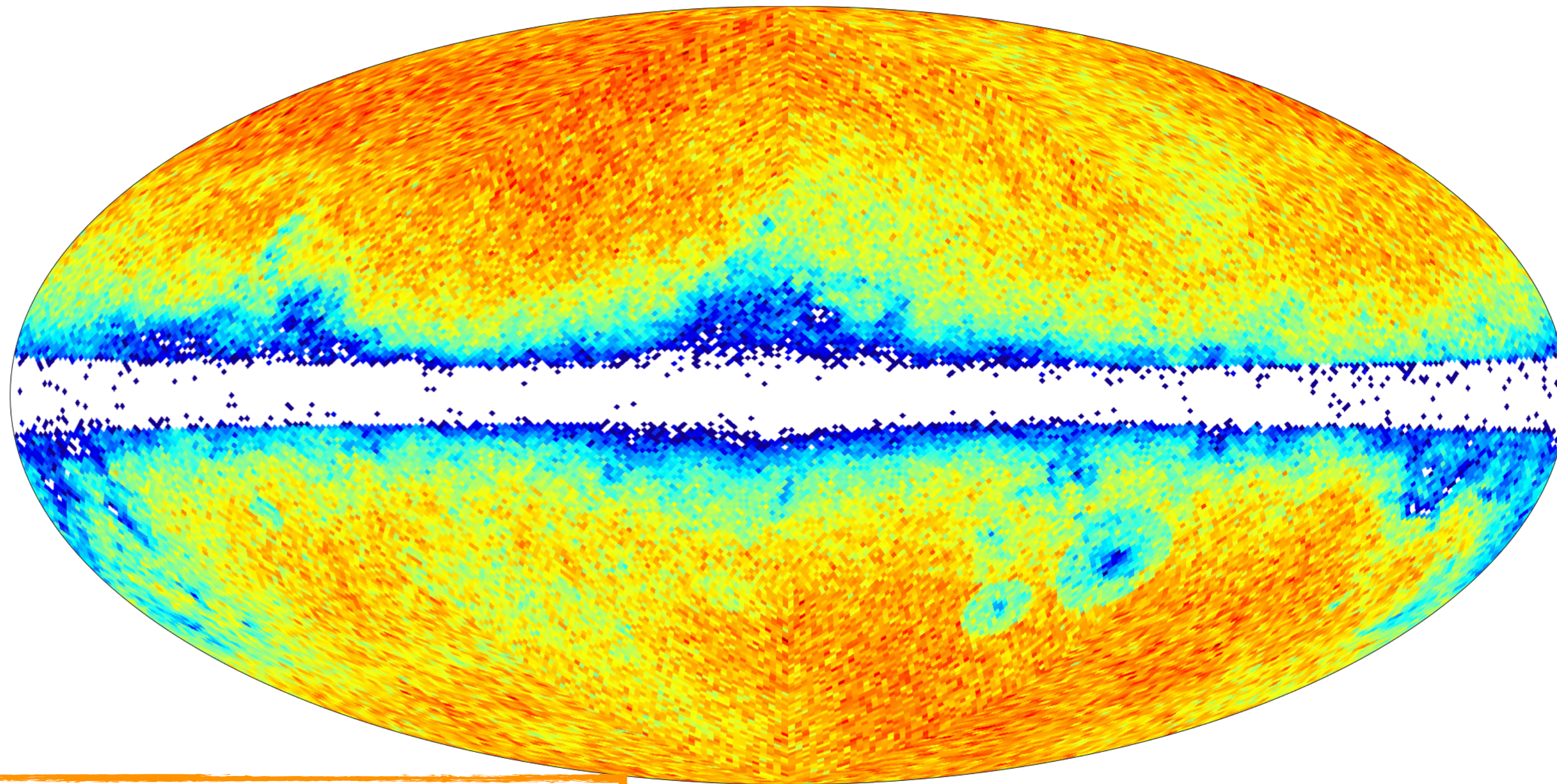
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The (Celestial) Sphere Reduction/Reconstruction is Gaia's primary objective
first direct materialization of a dense absolute reference frame at visual bands
one of the most important fundamental physics task

quasi-inertial kinematically non-rotating global optical frame meeting the ICRS prescriptions/IAU recommendations

reference frame



Credits:
ESA/Gaia/DPAC

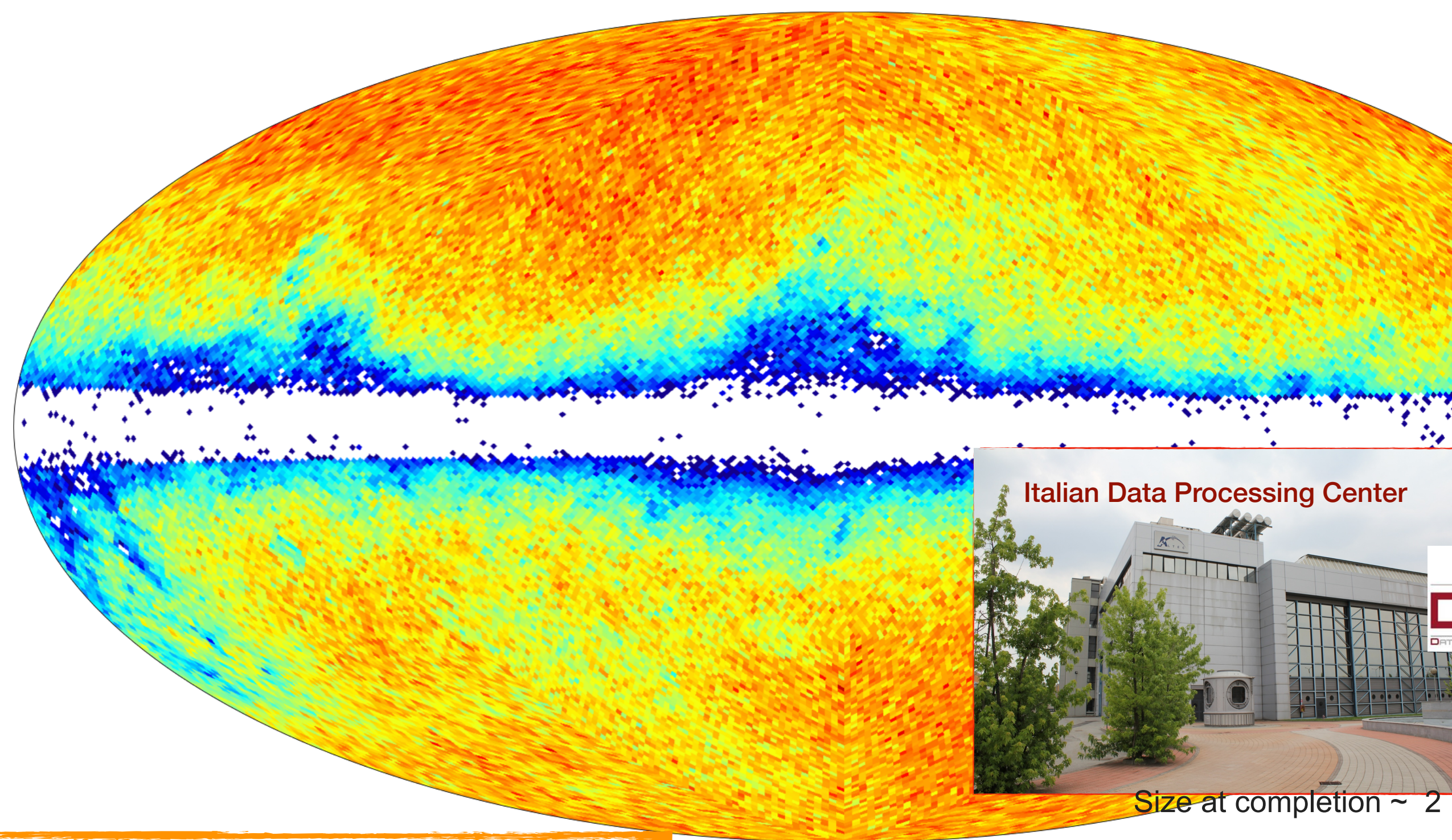
the Consortium constituted for the Gaia data reduction (DPAC) agreed to set up, respectively, two independent global sphere solutions: AGIS and **GSR**

2 independent GR models:
GREM (Gaia RELativistic Model)
RAMOD (Relativistic Astrometric MODEL)

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Gaia Data Processing and Analysis Consortium (DPAC)

Small external contributions from:
Algeria, Brazil, Chile, Israel, United States, European Southern Observatory

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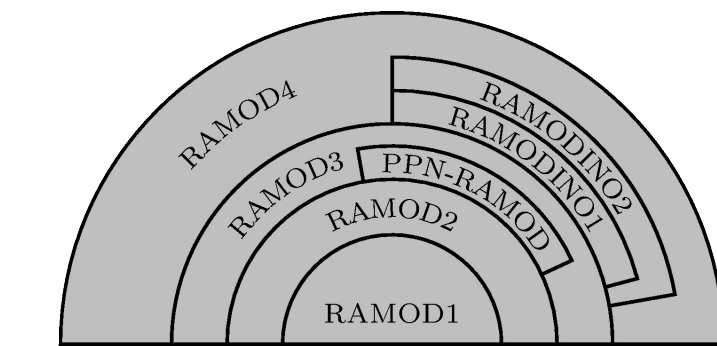
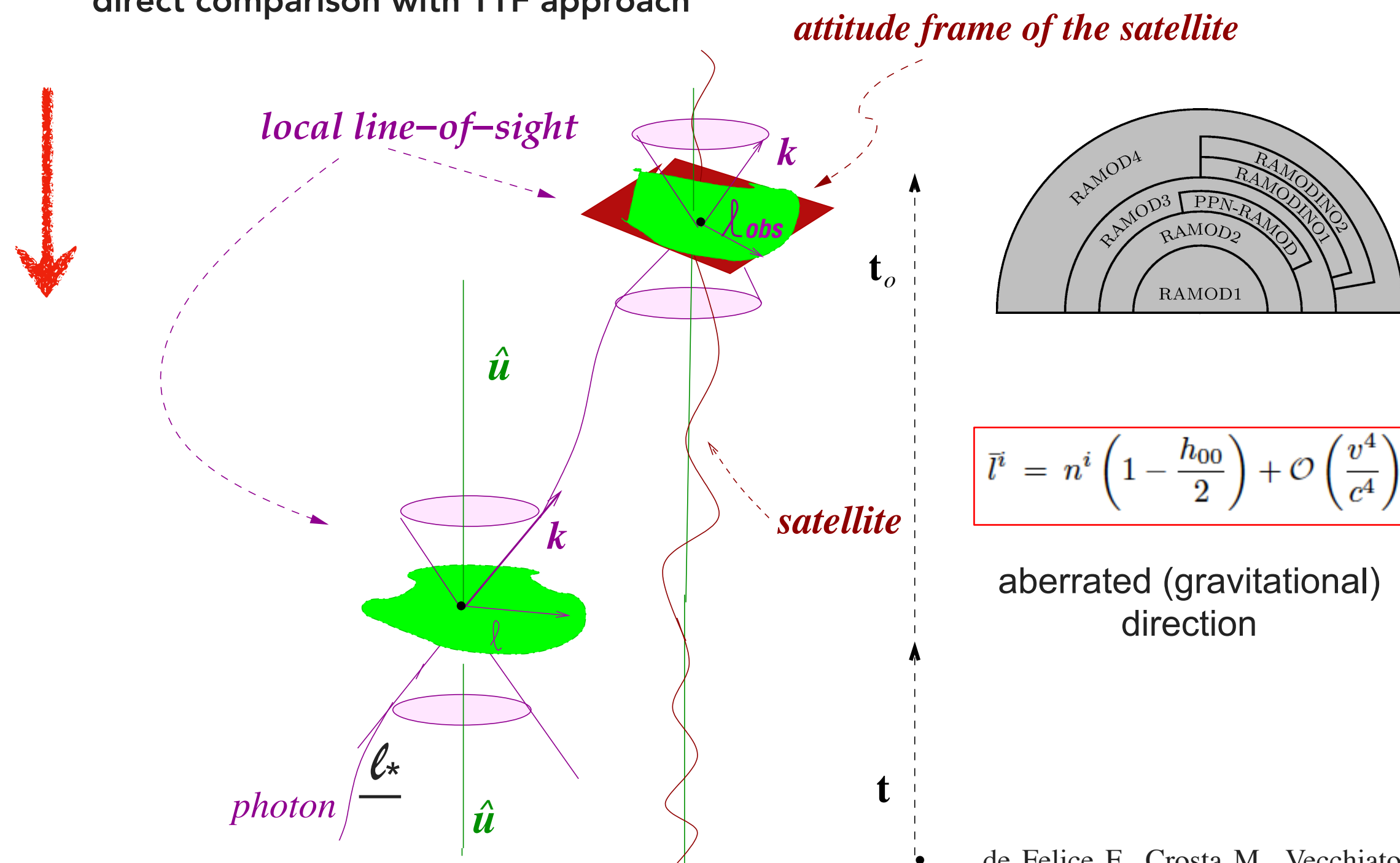
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RAMOD is a framework of general relativistic astrometric models with increasing intrinsic accuracy, adapted to many different observer's settings, **interfacing numerical and analytical relativity**

fully based on algorithms in General Relativity (GR) -> *no a priori approximations, top-down approach*

simultaneous observations in a curved space-time -> *GR protocol measurements*

direct comparison with TTF approach



$$\bar{l}^i = n^i \left(1 - \frac{h_{00}}{2} \right) + \mathcal{O} \left(\frac{v^4}{c^4} \right)$$

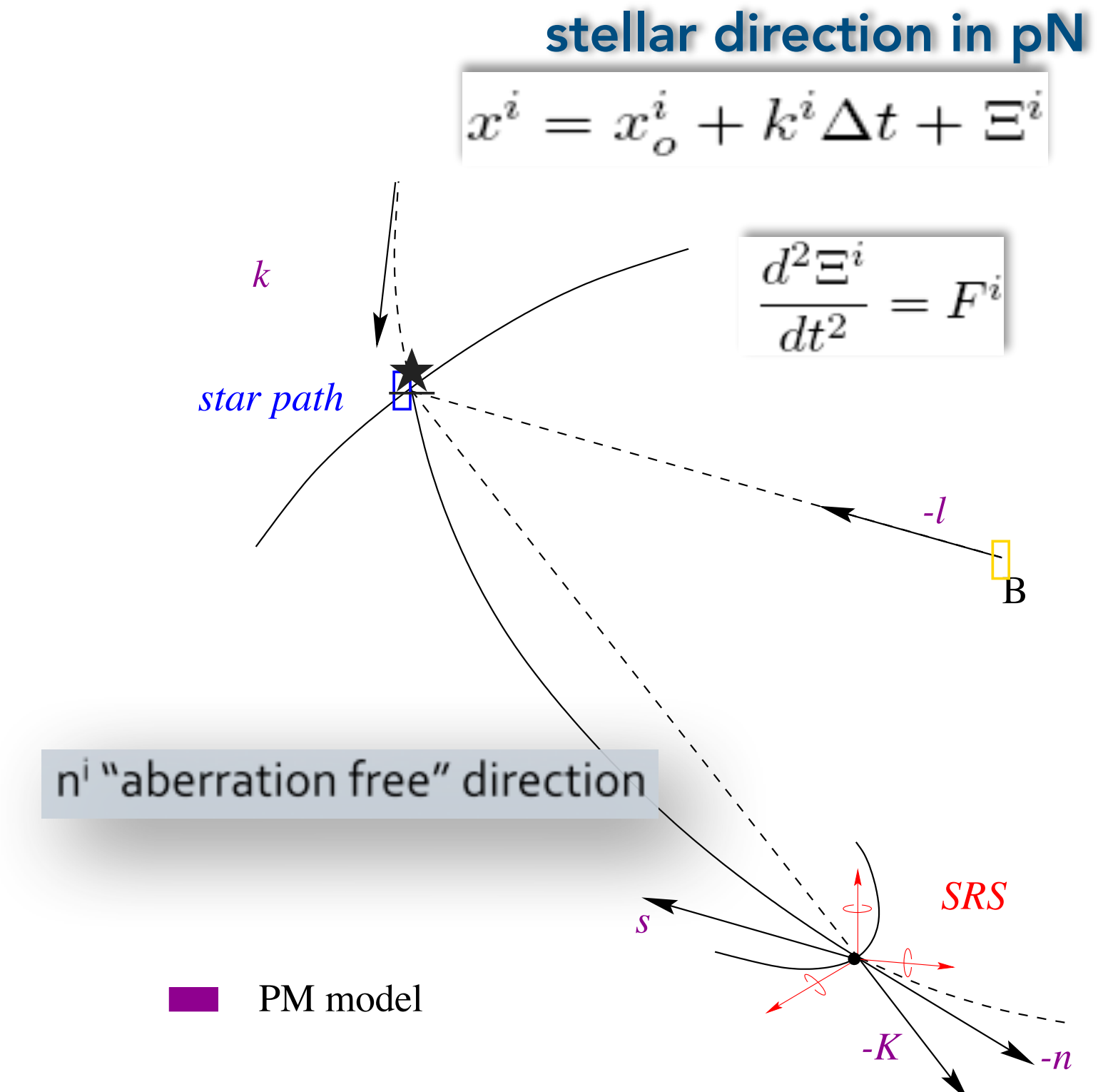
aberrated (gravitational) direction

$$\underline{x_*} = F(x_{\text{obs}}, g_{\mu\nu}, \ell_{\text{obs}}, E_a, \dots)$$

- de Felice F., Crosta M., Vecchiato A. and Lattanzi M. G., *Astrophys. J.*, 607 (2004) 580
- Crosta M., Geralico A., Lattanzi M. G. and Vecchiato A., *Phys. Rev. D*, 96 (2107) 104030.
- S. Bertone et al., 2014 *Class. Quantum Grav.* 31 015021

GREM,

baselined for the Astrometric Global Iterative Solution for Gaia (AGIS), based on post-Newtonian approximations



- Klioner S. A., *Astron. Astrophys.*, 404 (2003) 783.

GREM observed direction converts into a coordinate one via several steps, which separate the effects of the aberration, the gravitational deflection, the parallax, and proper motion -> **bottom-up approach**

Gaia&GaiaNIR ASTROGRAWANT

ASTROmetric GRAvitational Wave ANTenna

- New concept stemming from our general relativistic models developed for Gaia
- AstroGraWAnt is based on **close pairs of point-like sources as natural antenna "arms"** to record the very tiny variations in their angular separations induced by passing gravitational waves (GWs): all-differential formulation of the astrometric observable

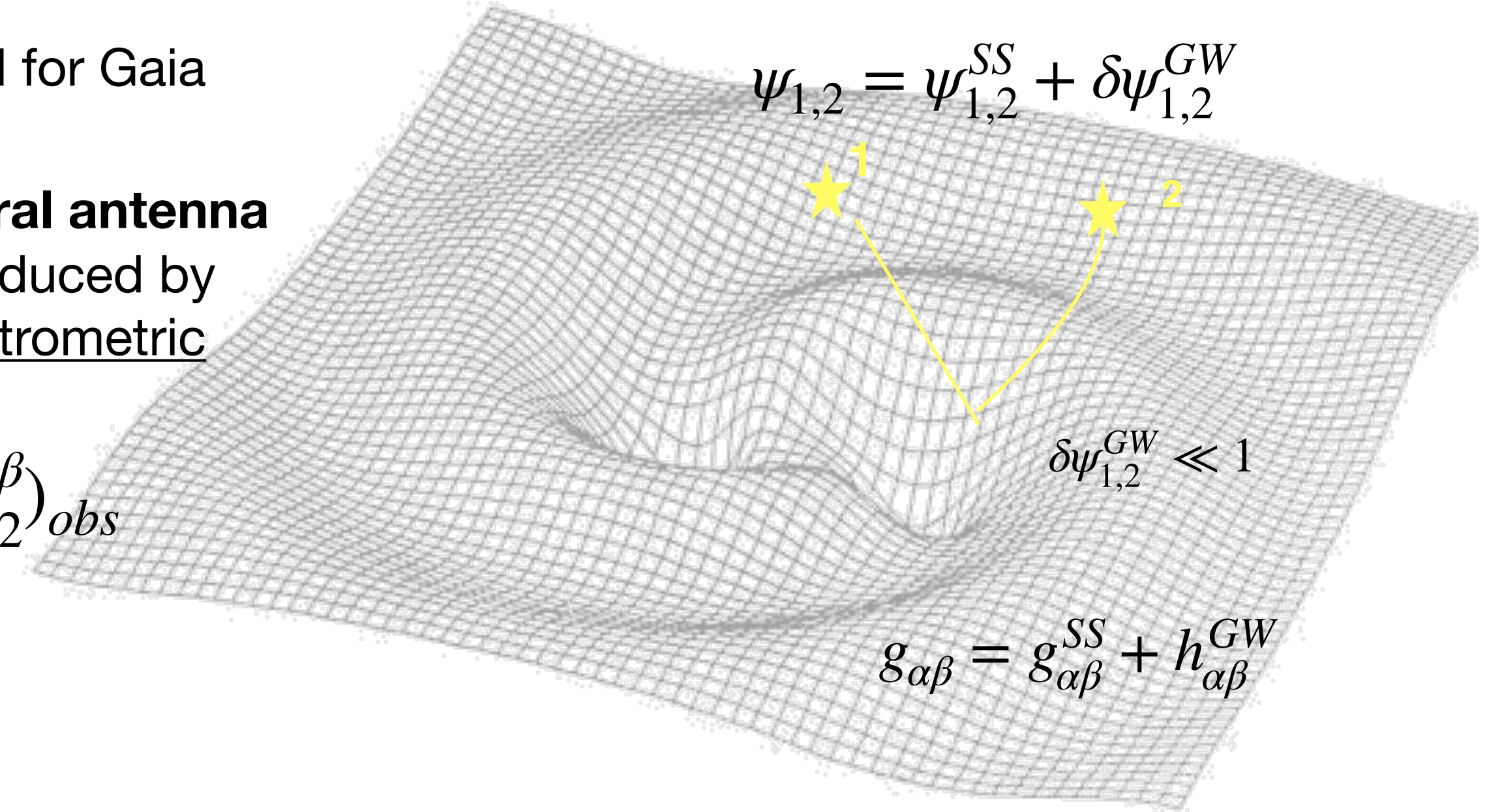
M.Crosta, Rivista del Nuovo Cimento 42, 10 (2019)

$$\cos \psi_{1,2} = g_{\alpha\beta}(\bar{\ell}_1^\alpha \bar{\ell}_2^\beta)_{obs}$$

$$\psi_{1,2} = \psi_{1,2}^{SS} + \delta\psi_{1,2}^{GW}$$

$$\delta\psi_{1,2}^{GW} \ll 1$$

$$g_{\alpha\beta} = g_{\alpha\beta}^{SS} + h_{\alpha\beta}^{GW}$$

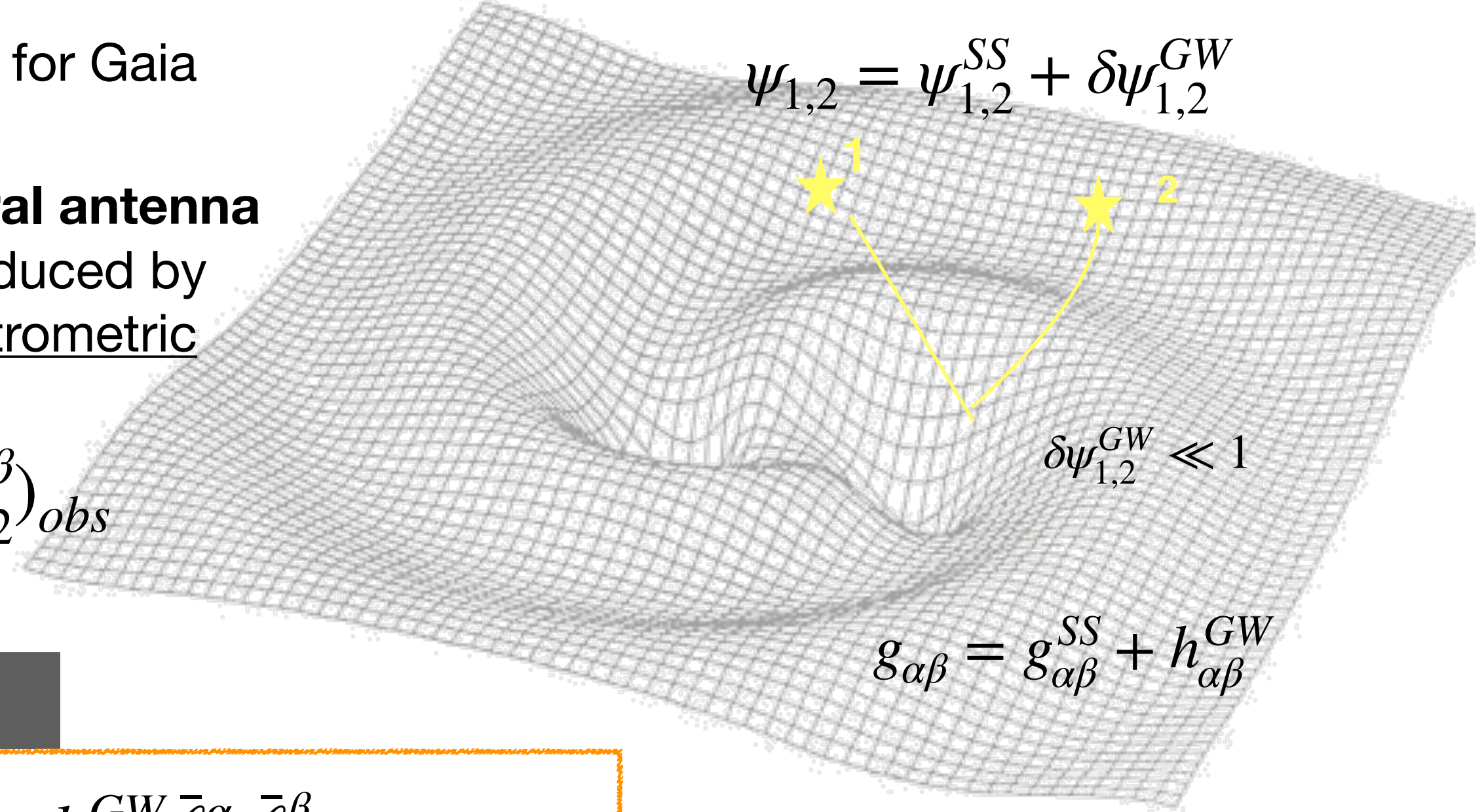


M.Crosta, M.G. Lattanzi, C. Leponcin-Lafitte, M. Gai, Q. Zhaoxiang, A.Vecchiato, *On the principle of Astrometric Gravitational Wave Antenna*, 2021 under review process, <https://arxiv.org/pdf/2203.12760.pdf>

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$$\cos \psi_{1,2} = g_{\alpha\beta}(\bar{\ell}_1^\alpha \bar{\ell}_2^\beta)_{obs}$$

New GW observable

$$\delta \hat{\psi}_{1,2}^{GW}(t_i) = - \frac{\eta_{\alpha\beta}(\bar{\ell}_{10}^\alpha \delta \ell_2^\beta + \bar{\ell}_{20}^\alpha \delta \ell_1^\beta)_{obs} + h_{\alpha\beta}^{GW} \bar{\ell}_{10}^\alpha \bar{\ell}_{20}^\beta}{\sin(\psi_{1,2}^{SS})} + O(h^2)$$

unperturbed star direction

Telescope optical resolution

GW Strain

$$\frac{d\delta \ell^i}{d\sigma} \approx -\frac{1}{2} \bar{\ell}_0^j \bar{\ell}_0^k (2h_{ij,k}^{GW} - h_{jk,i}^{GW}) - \bar{\ell}_0^j h_{ij,0}^{GW} + \frac{\bar{\ell}_0^j \bar{\ell}_0^k \bar{\ell}_0^i}{2} h_{jk,0}^{GW}$$

extra-shift due to a passing GW

M.Crosta, M.G. Lattanzi, C. Leponcin-Lafitte, M. Gai, Q. Zhaoxiang, A.Vecchiato, *On the principle of Astrometric Gravitational Wave Antenna*, 2021 under review process, <https://arxiv.org/pdf/2203.12760.pdf>

The New Observable:

Three lines-of-sight for GW discrimination and full characterization (including 'phase closure', i.e. direction)

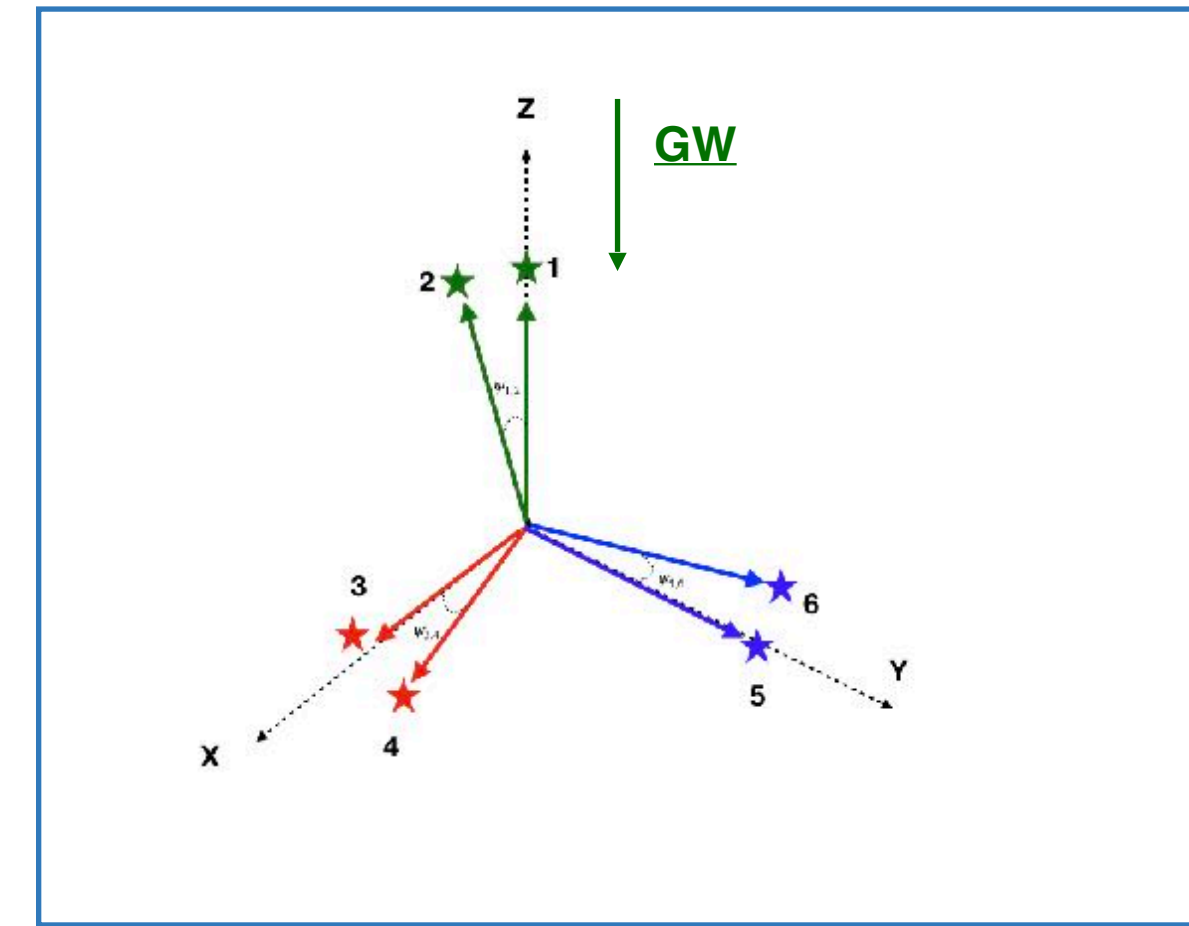
Avantages in using close pairs of stars:

- implementing multiple (at least 3) line-of-sights within a **relatively compact configuration**
- the 3 perturbed angles in the three orthogonal directions are directly linked to the GW strain **-> source direction!**
- **the GW observability is AMPLIFIED** through a factor depending on the angle between the unperturbed directions to star-like objects that acts as a "signal amplifier" for the GW detection, **limited only by the resolving power of the optics**
- mitigation of high perturbative terms; large number of null geodesics, link the properties of a GW source with extensive statistic; avoid satellite's attitude

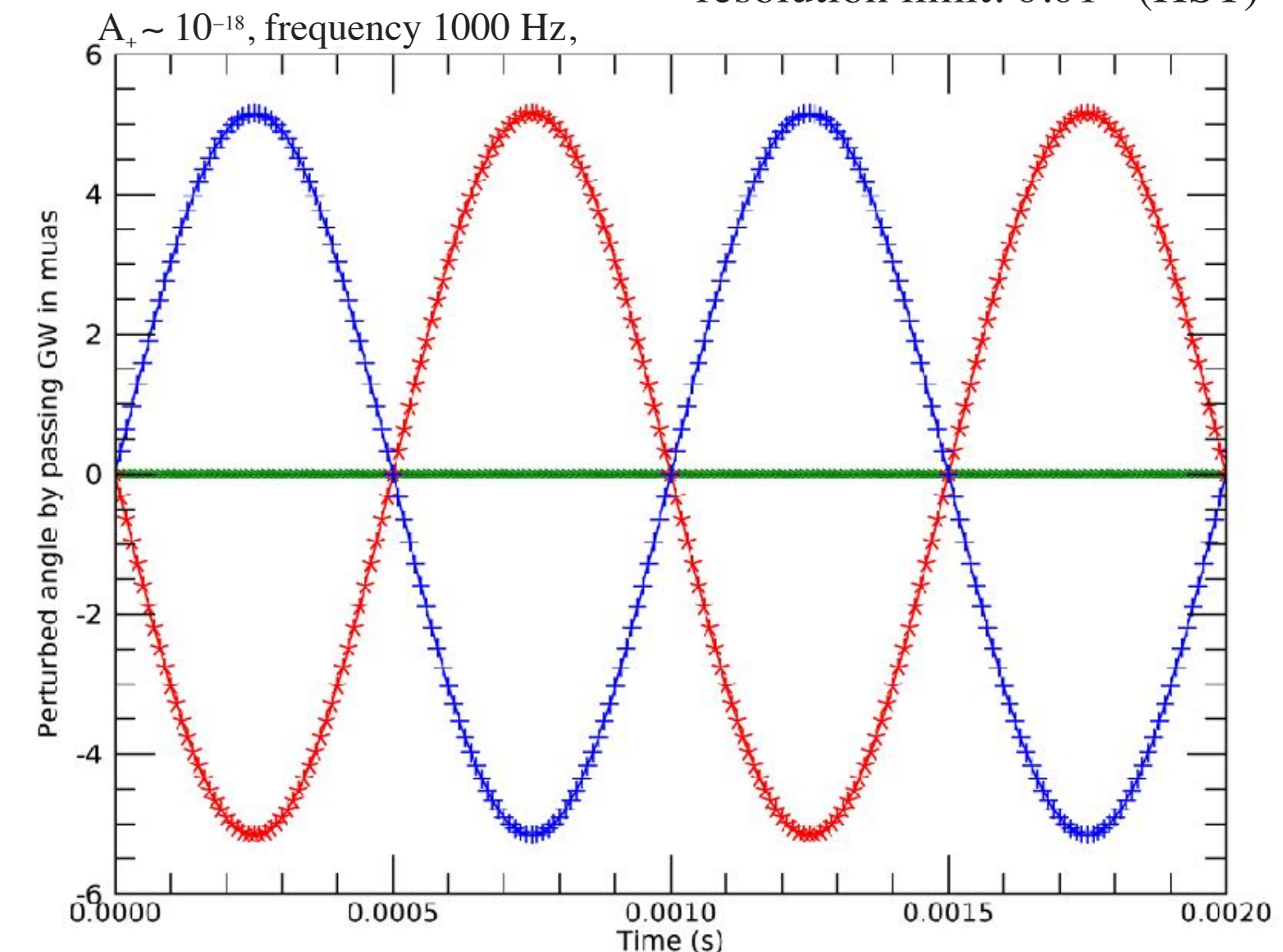
$$\delta\psi_{a,b}^{GW}(t) \sim \frac{h_{(obs)}^{GW}(t; a, b)}{\sin(\psi_{a,b}^{SS})}$$

with

$$\delta\psi^{GW} \ll \psi^{SS}$$



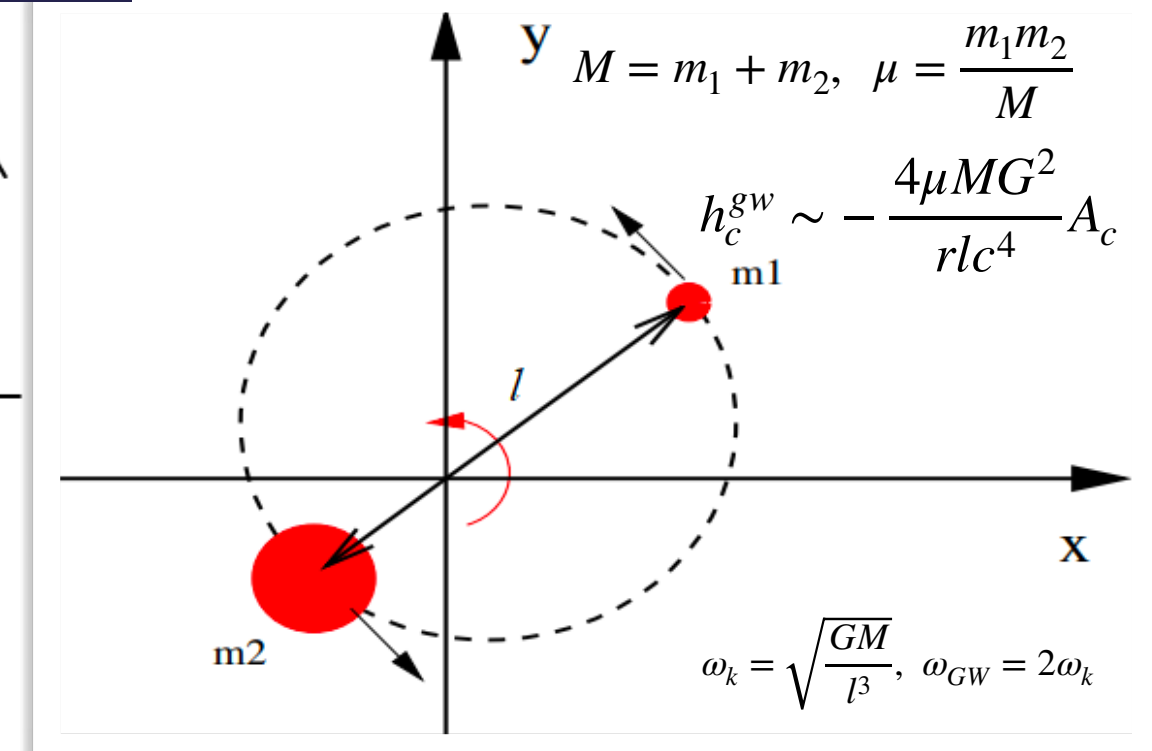
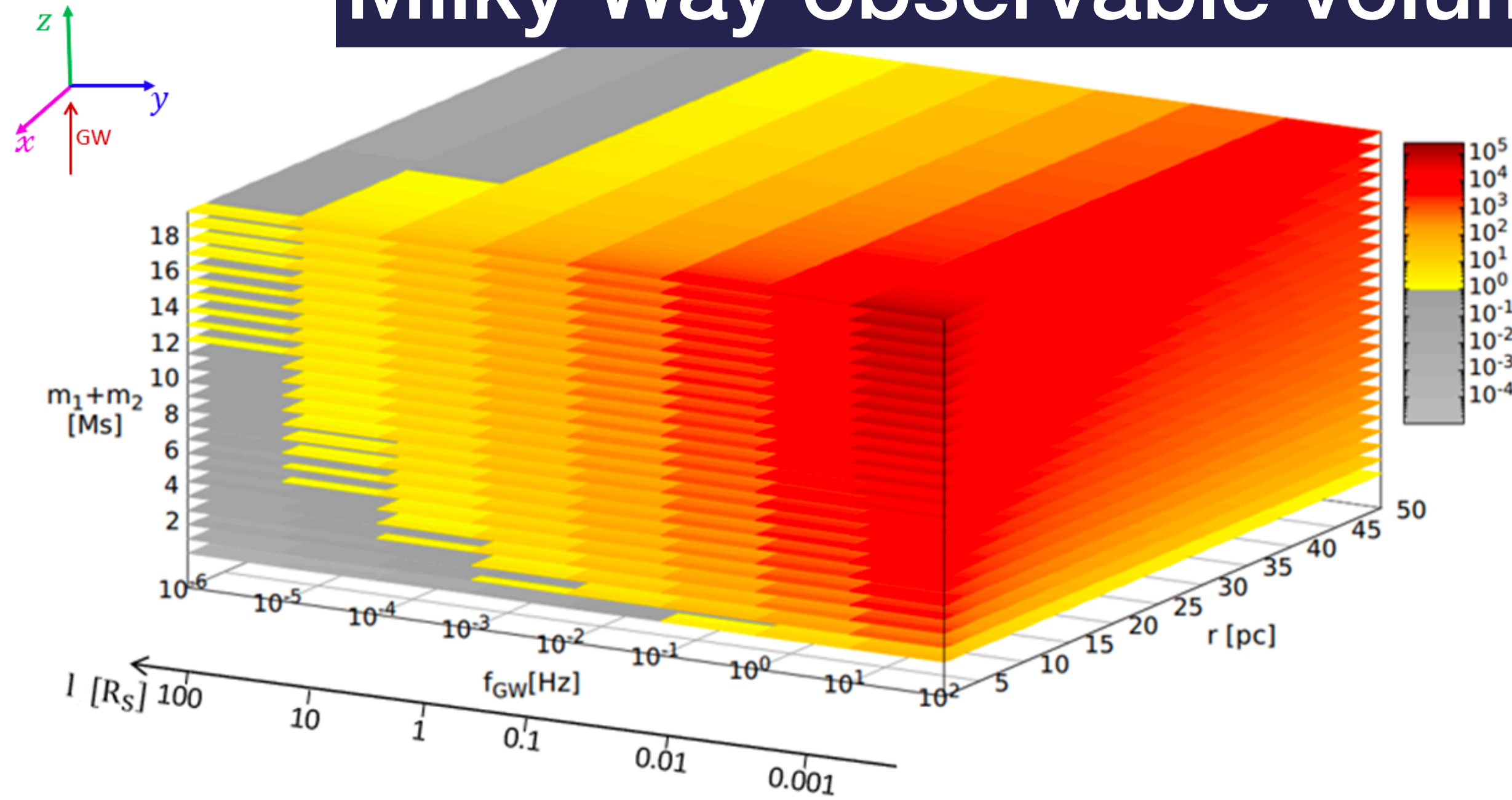
resolution limit. 0.01" (HST)



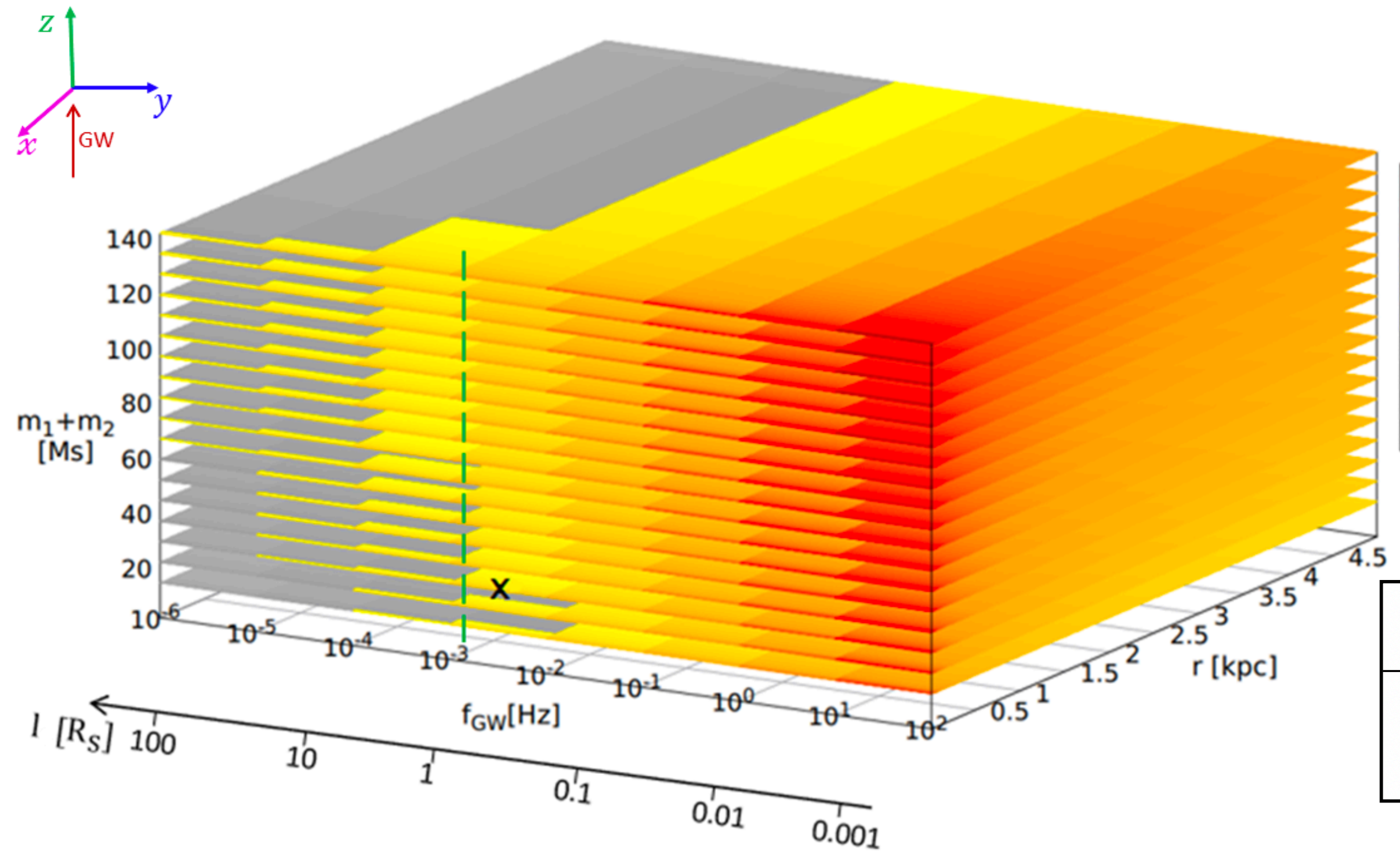
$max(\delta\psi_{1,2}^{GW})$ (μas)	$max(\delta\psi_{3,4}^{GW})$ (μas)	$max(\delta\psi_{5,6}^{GW})$ (μas)	A_+ (radians)	ψ_{i_0, j_0} (")
4.12×10^{-15}	5.12	5.12	10^{-18}	0.01
4.12×10^{-16}	51.57	51.57	10^{-18}	0.001

supernova, $f_{GW} = 10^3$ Hz

Milky Way observable volume



$\psi_{3,4(5,6)}^{SS} = 10\text{mas}$	m_1 (M_\odot)	m_2 (M_\odot)	l (R_\odot)	r (pc)	f_{GW} (Hz)	$\max(\delta\psi_{3,4})$ (μas)
PSR J1756-2251	1.341	1.230	1.18	1000	2.5×10^{-4}	$\sim 2 \times 10^{-3}$
PSR J1756-2251 _{hyp}	1.341	1.230	1.18	50 _{hyp}	2.5×10^{-4}	$\sim 5 \times 10^{-2}$
se $\psi_{3,4}^{SS} = 0.1 \text{ mas}$	1.341	1.230	1.18	50 _{hyp}	2.5×10^{-4}	$\Rightarrow 5$

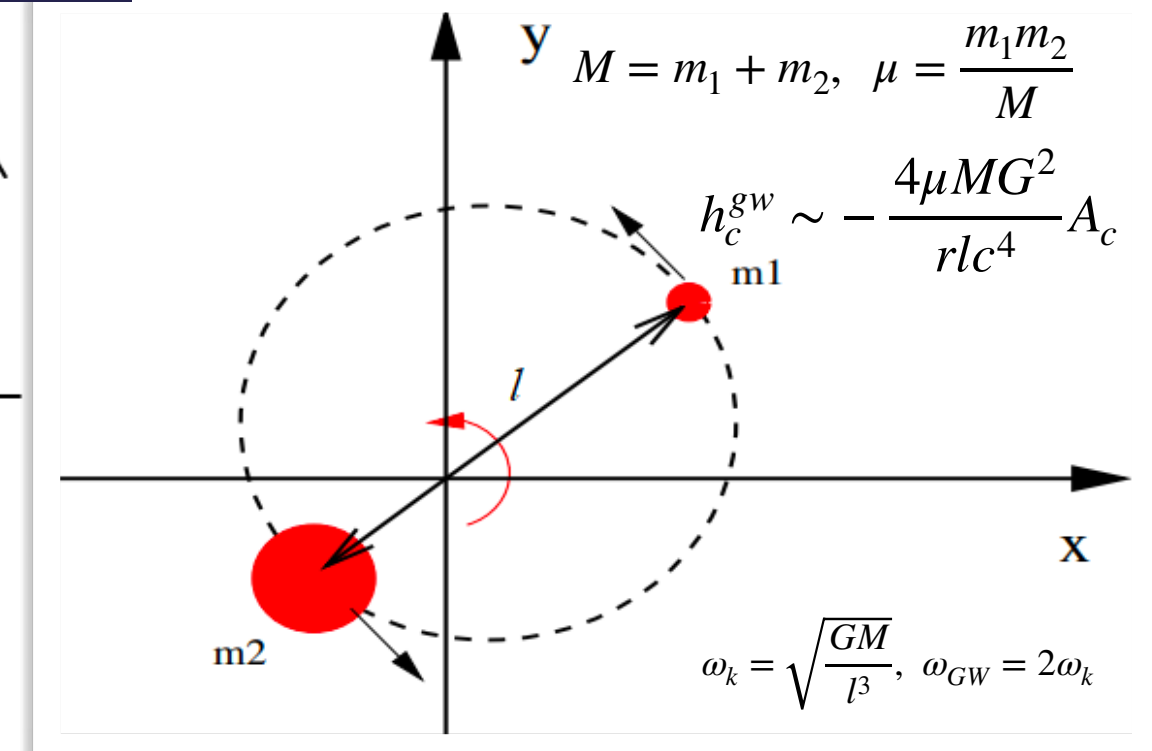
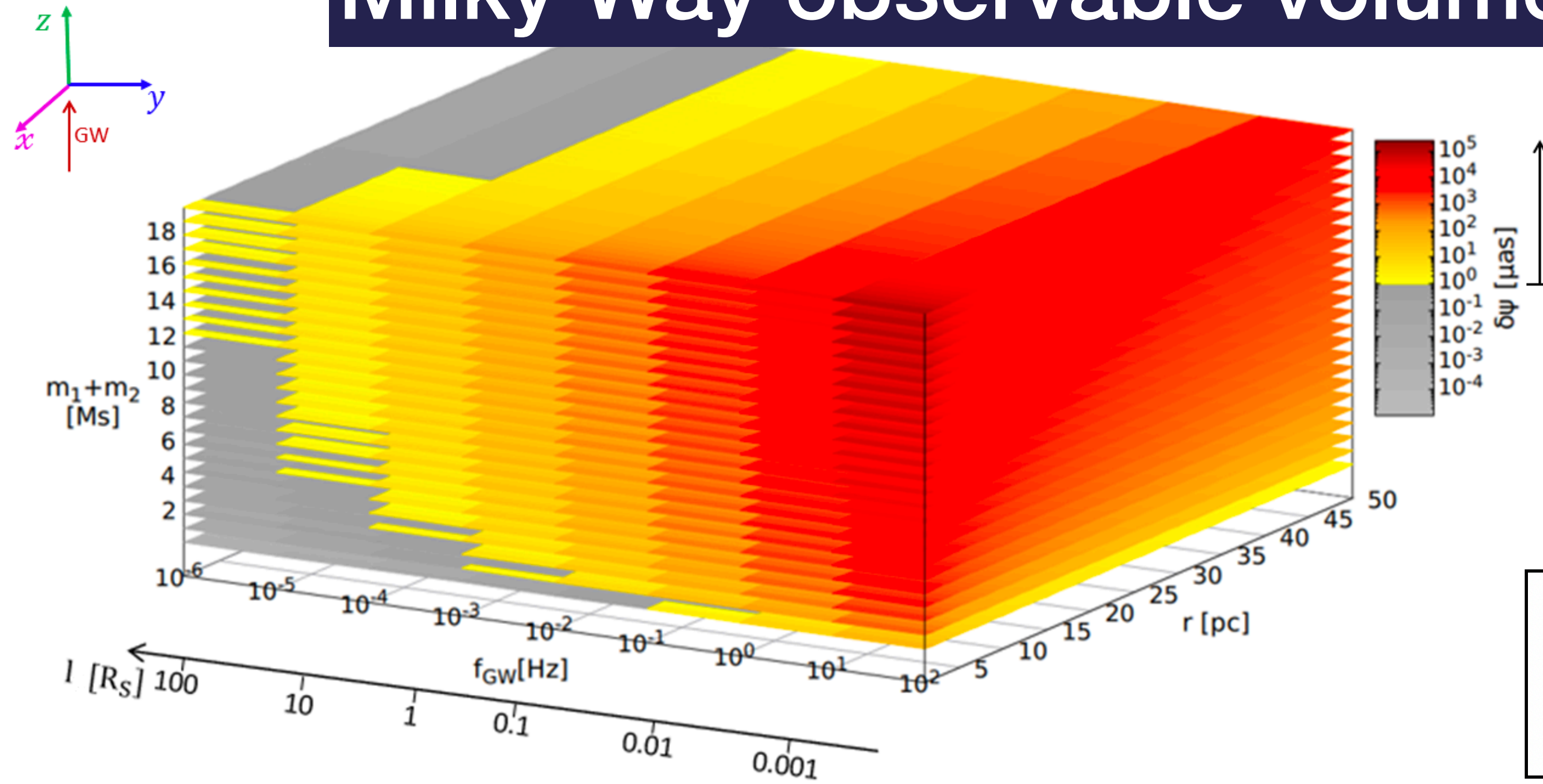


Spectral Density Function of the Astrometric Gravitational Wave Antenna I.

F. Santucci, M. Crosta, M.G. Lattanzi in submission

$\psi_{3,4(5,6)}^{SS} = 10\text{mas}$	m_1 (M_\odot)	m_2 (M_\odot)	l (R_\odot)	r (Mpc)	f_{GW} (Hz)	$\max(\delta\psi_{3,4})$ (μas)
GW 150914	36.5	30.8	~ 0	440	30 – 250	$\sim 4 \times 10^{-3}$
GW 150914 _{hyp}	36.5	30.8	47.8	0.005 _{hyp}	5×10^{-6}	$\sim 8 \times 10^{-3}$
oggetto X	20	15	1	100 pc	10^{-3}	~ 5

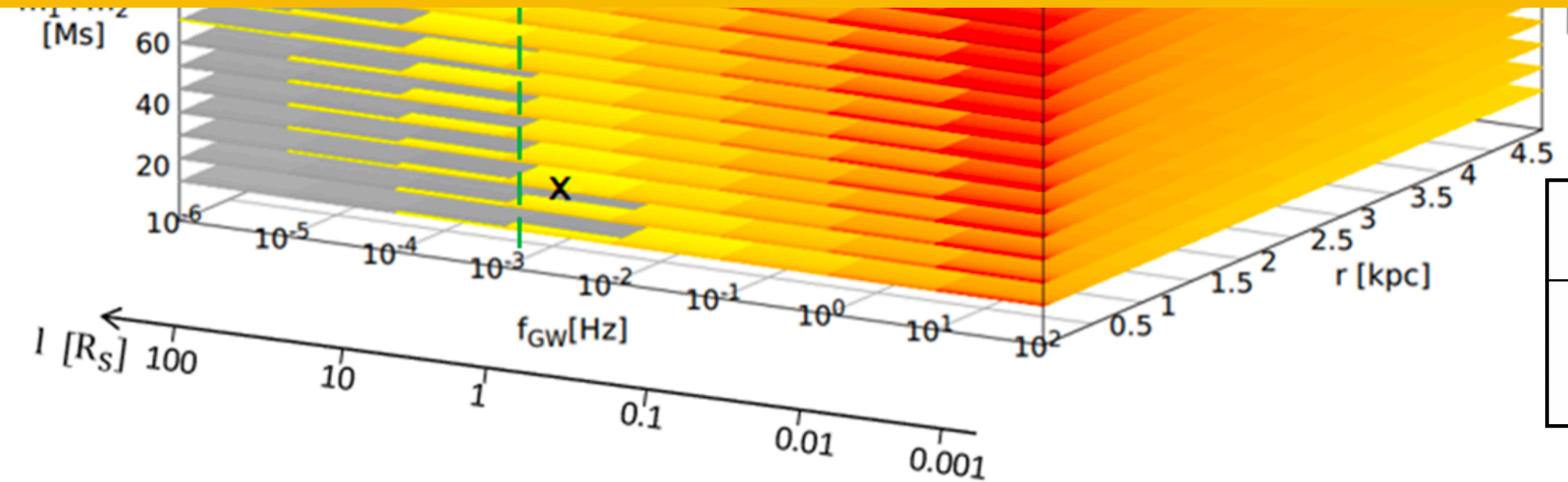
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AstroGraWAnt can pinpoint GW source direction to unprecedented, sub-arcsecond, precision as a game changer in multi-wavelength/multi-messenger identification and astrophysical characterization campaigns

-> GW sentinel to build statistic on event rate (“survey” of GW sources)



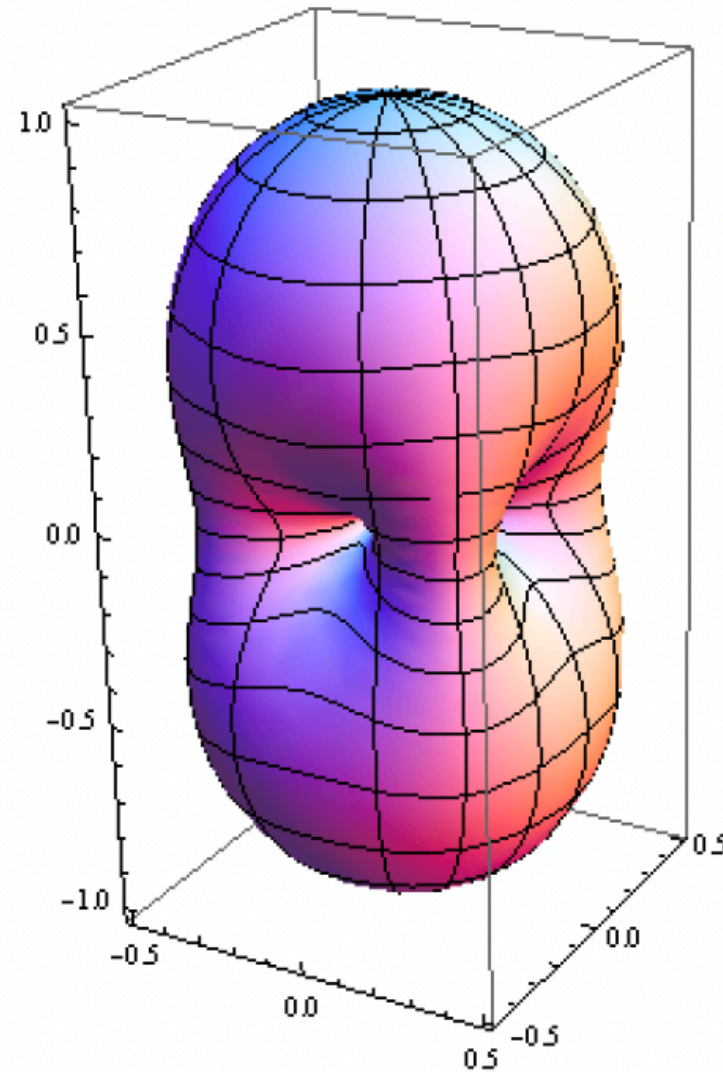
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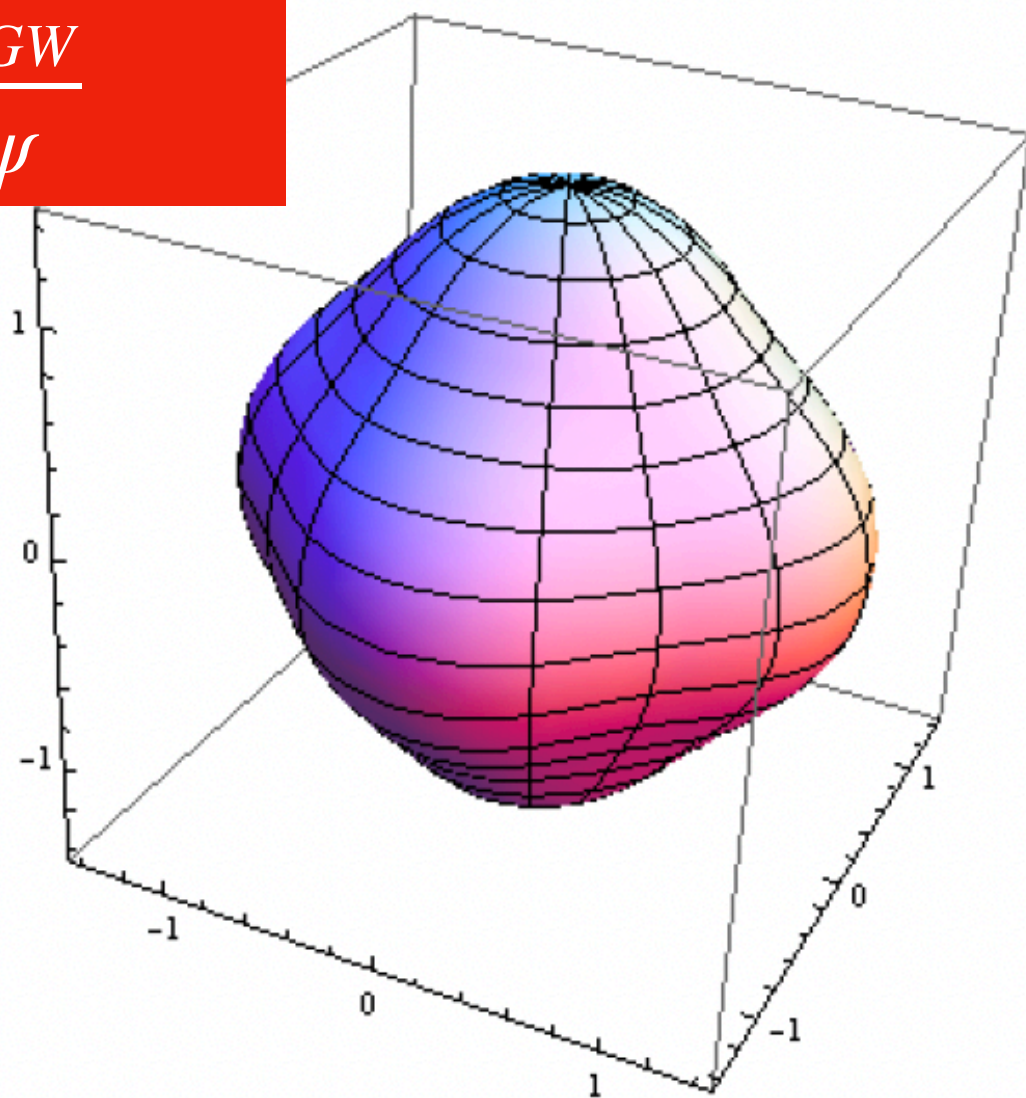
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AstroGraWAnt represents the “dual” analogue (angular versus linear arms) of the extant linear antennas

Interferometer
 $\delta L \propto h_{GW} L$



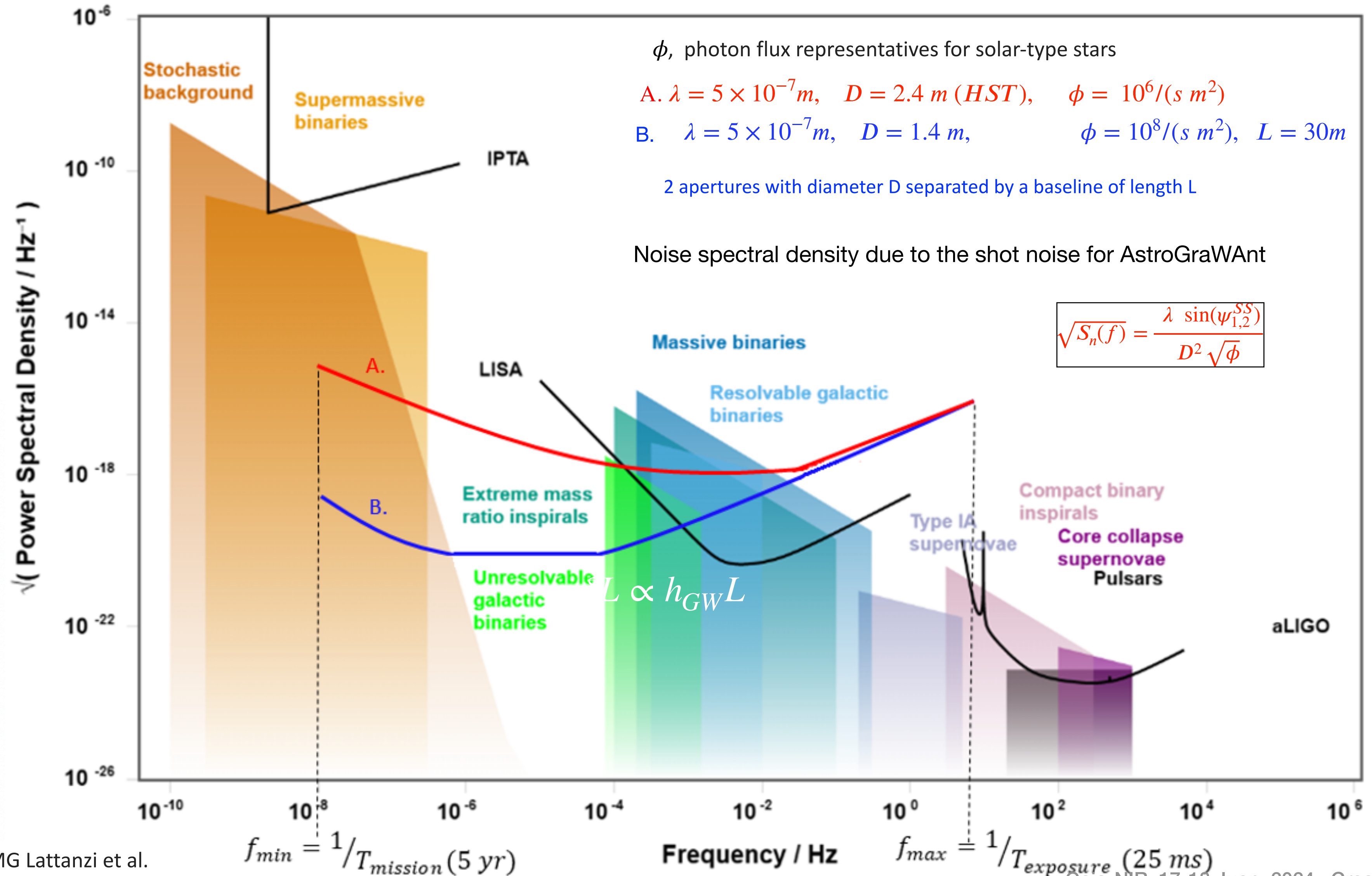
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F. Santucci, M.Crosta, MG Lattanzi et al.

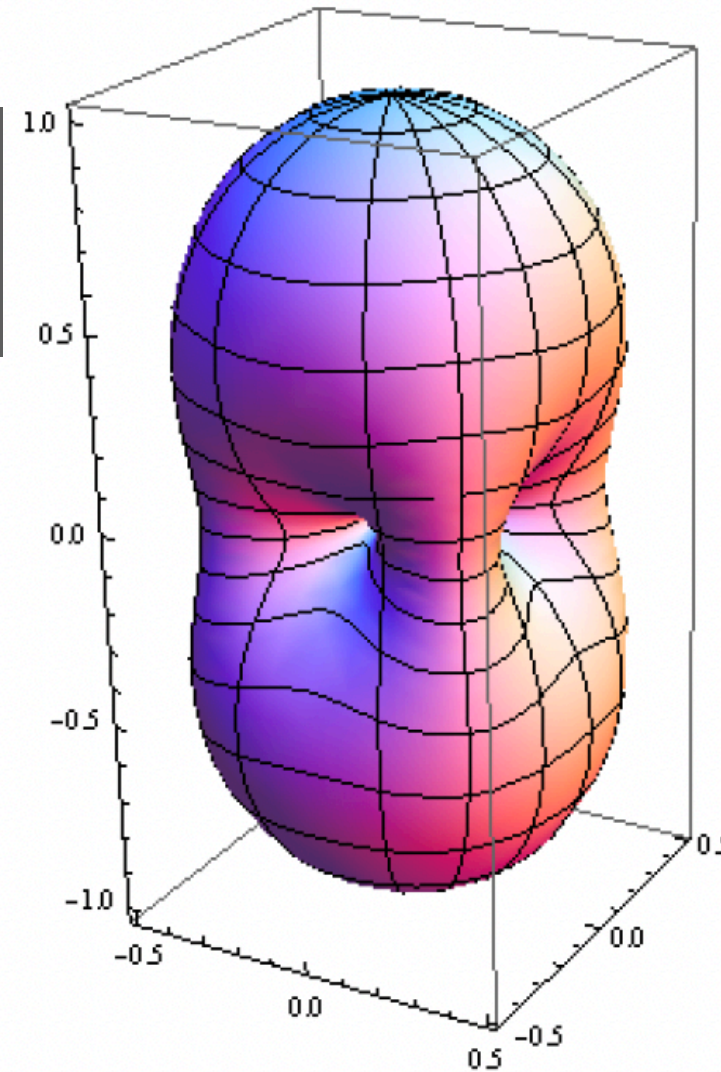
Shot noise gives the minimum of the sensitivity curve for the Astrometric GW Antenna

Output of real detector: $s_{out}(t) = h_{out}(t) + n_{out}(t)$

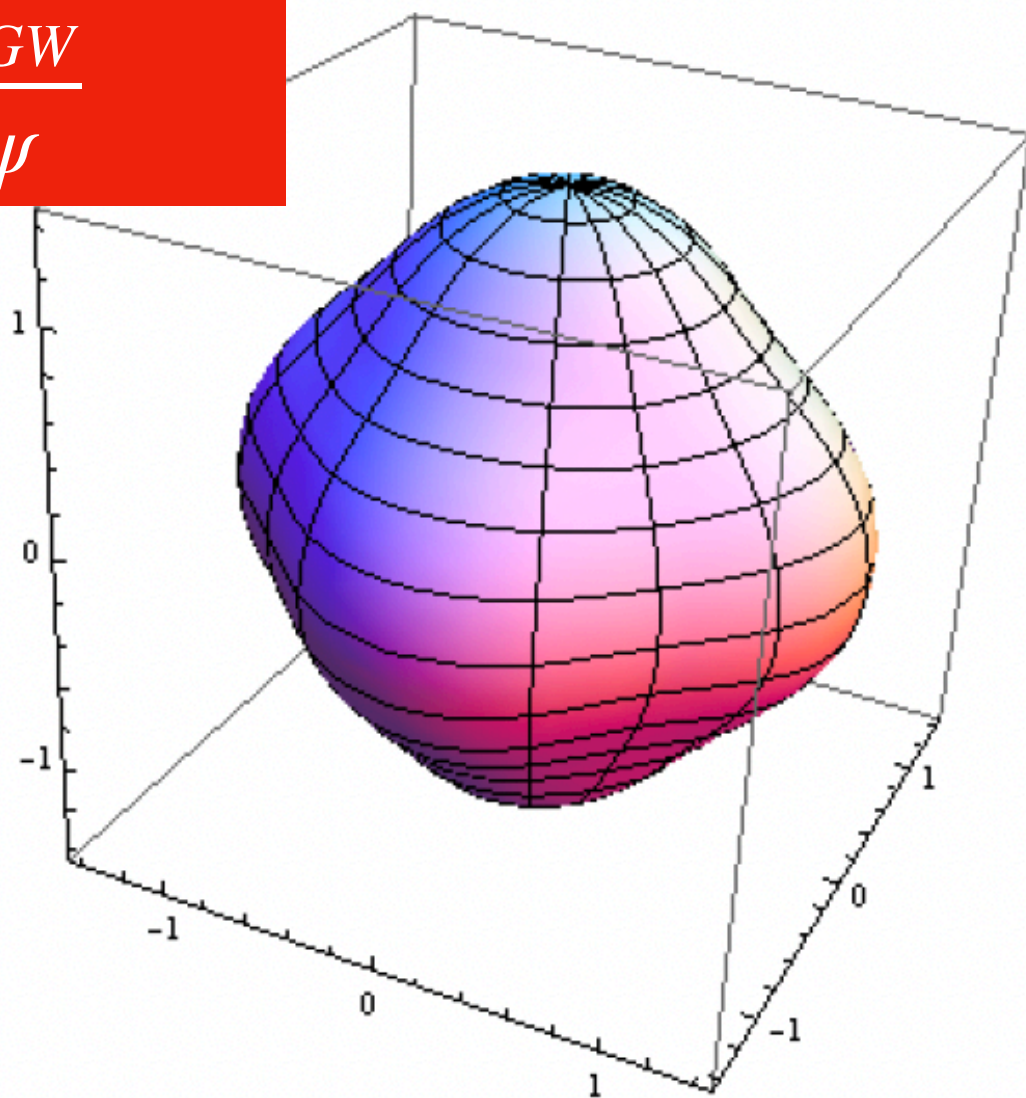


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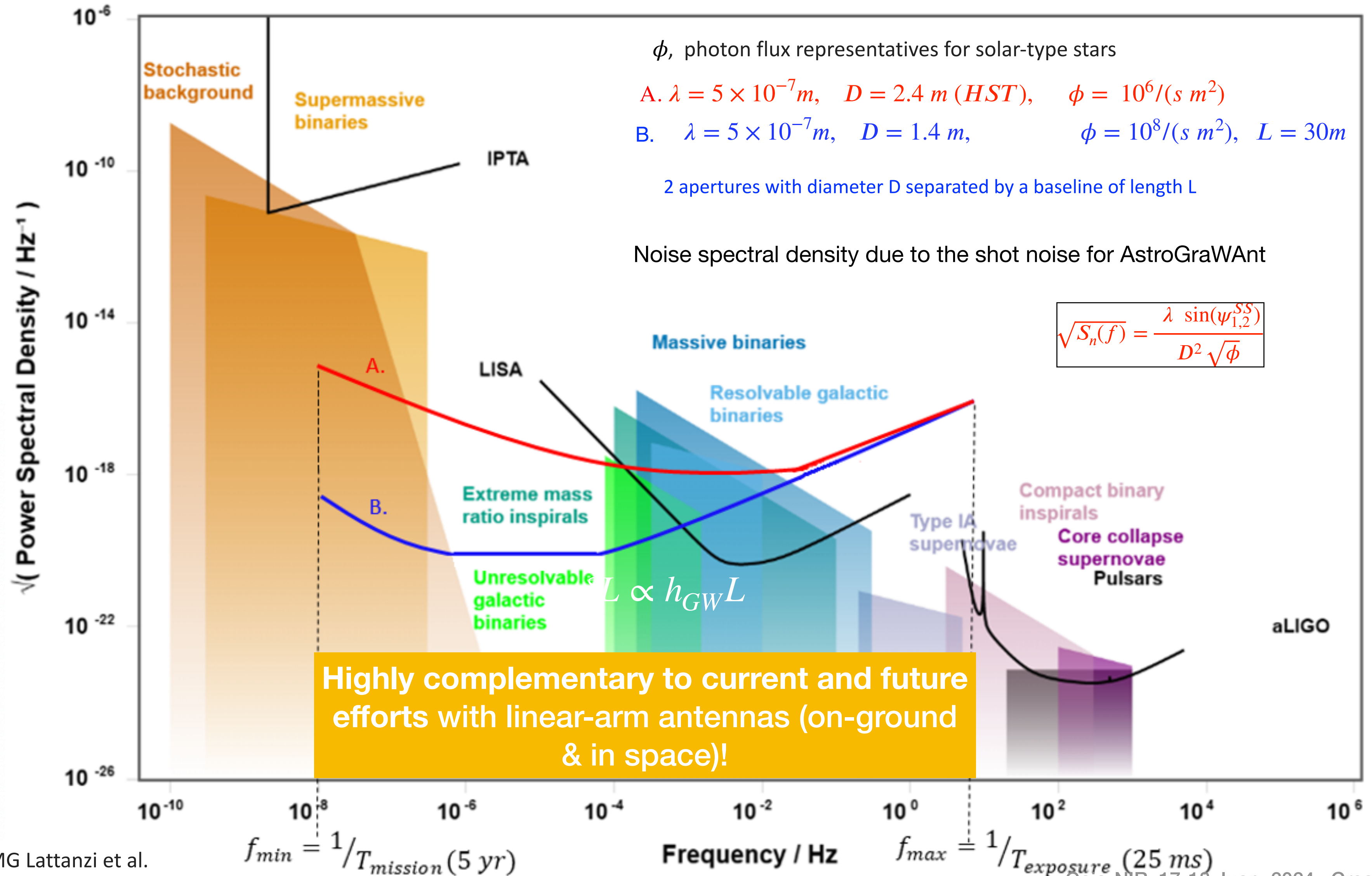
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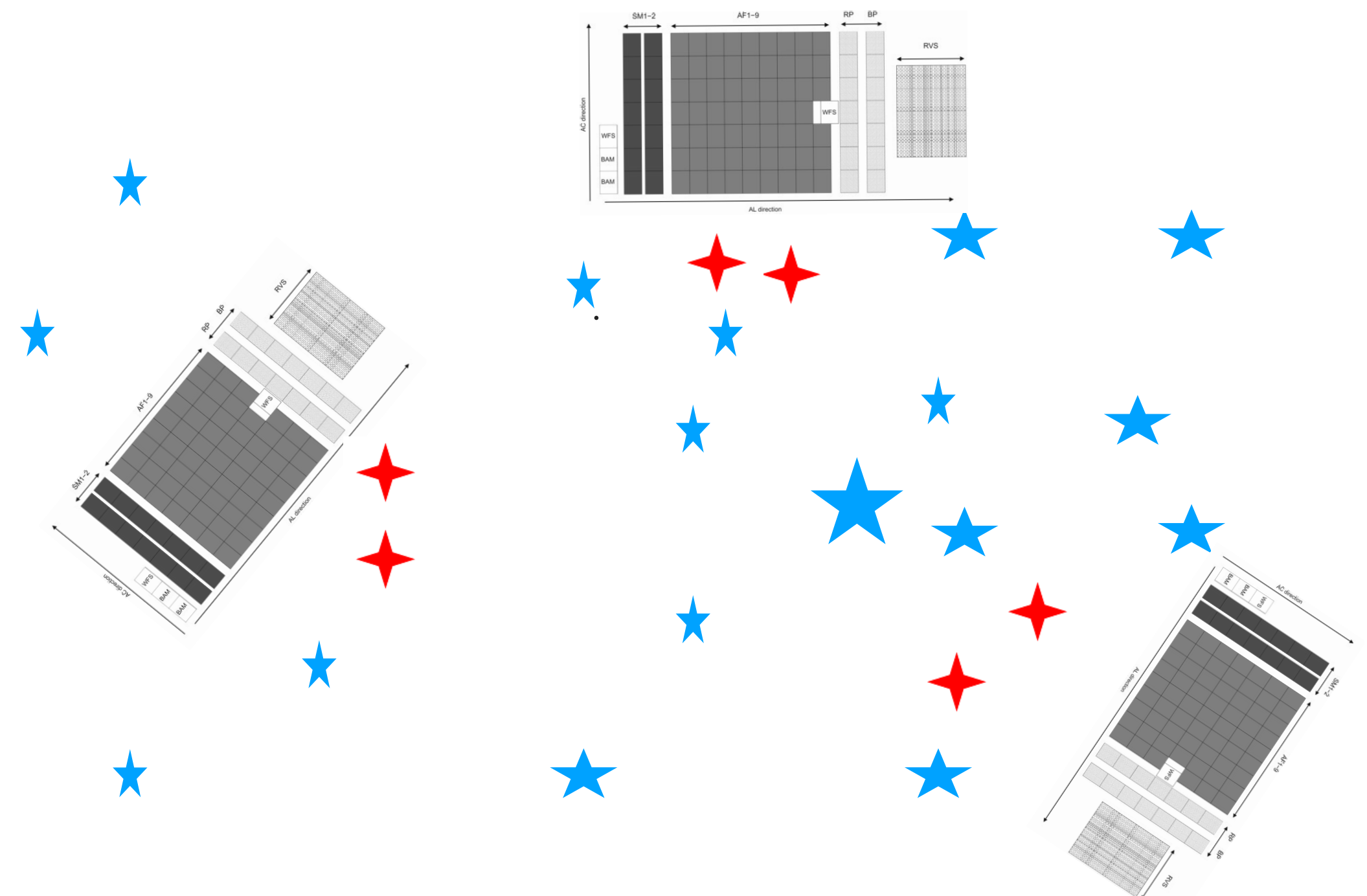
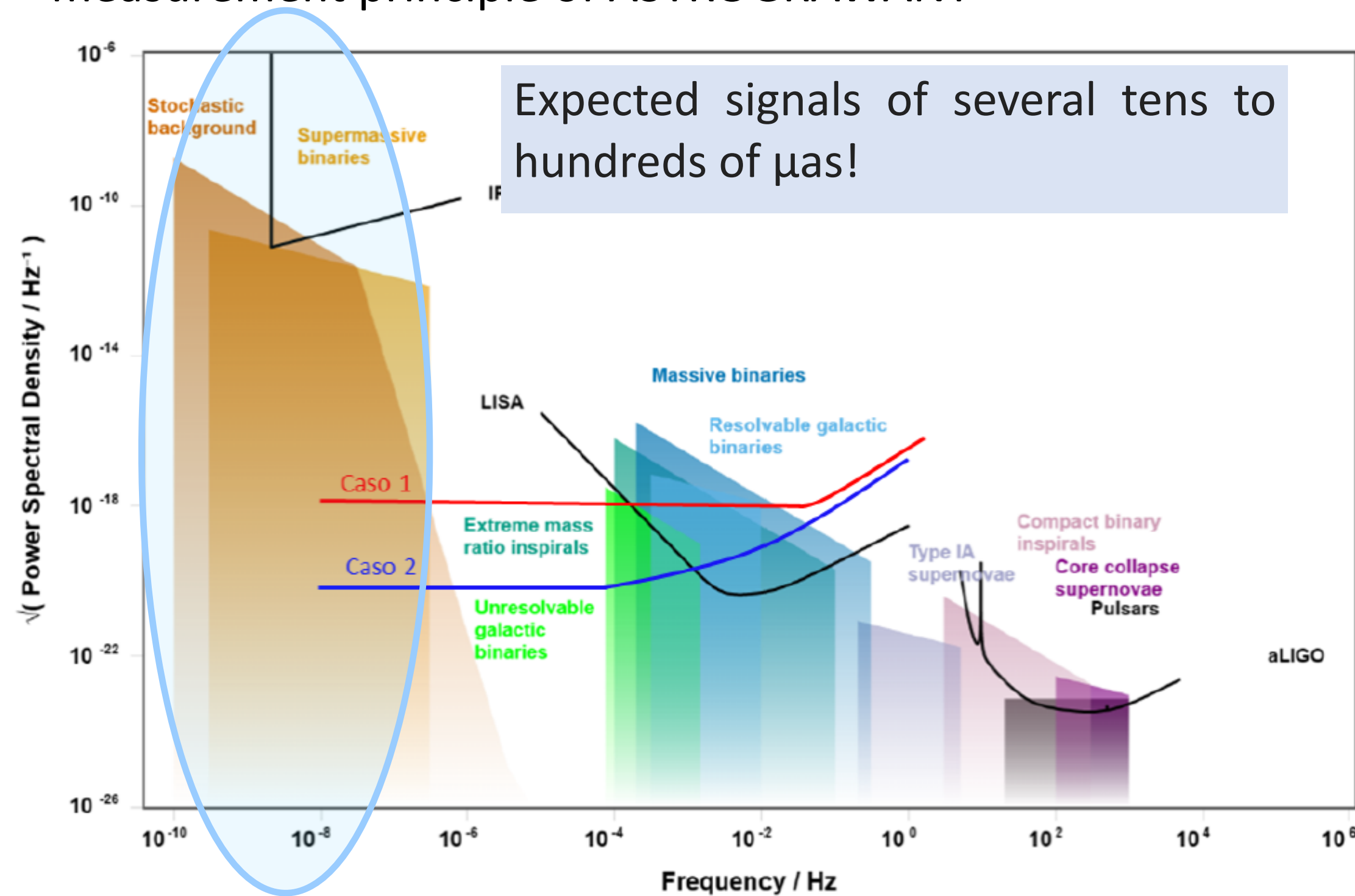


Perspectives for the GW astrometric detection with Gaia&GaiaNIR

- Gaia actual mission life time (after 3 extensions): 10.6 yrs, i.e., f_{lim} **3×10^{-9} Hz + Gaia-NIR**
Exploit long astrometric time series for millions of sources

Methods:

- Analyze through Vectorial Spherical Harmonics the proper motions of millions of QSOs -> GW induced common pattern on proper motions
- differential procedure** similar to that of GAREQ experiment with Gaia [Crosta & Mignard, 2006 CQG, Abbas et al., 2021, A&A] to monitor angular distances from close Gaia stellar pairs (i.e., observed during the same transits on focal plane) -> **digitally replicate** the measurement principle of ASTROGRAWANT



- Synergies with IPTA and ET (RU- INAF OATo “GW sources survey and signal sentinel”)**

Gravitational astrometry@Milky Way scale

A complete **GR picture** of MW to ensure a **coherent Local Cosmology laboratory** against which the role of gravity in shaping the constituents of our Galaxy can be fully tested

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To what extent the MW structure is dictated by GR?

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gravitational potential or “relativistic effects” at the MW scale are usually “small”, then

✓ negligible..

✓ locally Newton approximation is retained valid at each point..

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 $(v_{Gal}/c)^3 \sim 0,57 \times 10^{-9} \text{ (rad)} \sim 120 \mu\text{as}$
the individual astrometric error is $< 100 \mu\text{as}$

$\epsilon \sim v^2/c^2 \sim GM/rc^2 \sim \text{mas accuracy}$

which requires determination of

g_{00} even terms in ϵ , lowest order $\epsilon^2 \sim \text{mas}$

g_{0j} odd terms in ϵ , lowest order $\epsilon^3 \sim \mu\text{-as}$

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Lesson from Gaia: For the Gaia-like observer the weak gravitational regime turns out to be "strong" when one has to perform high accurate measurements

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The small curvature limit in General Relativity may not coincide with the Newtonian regime

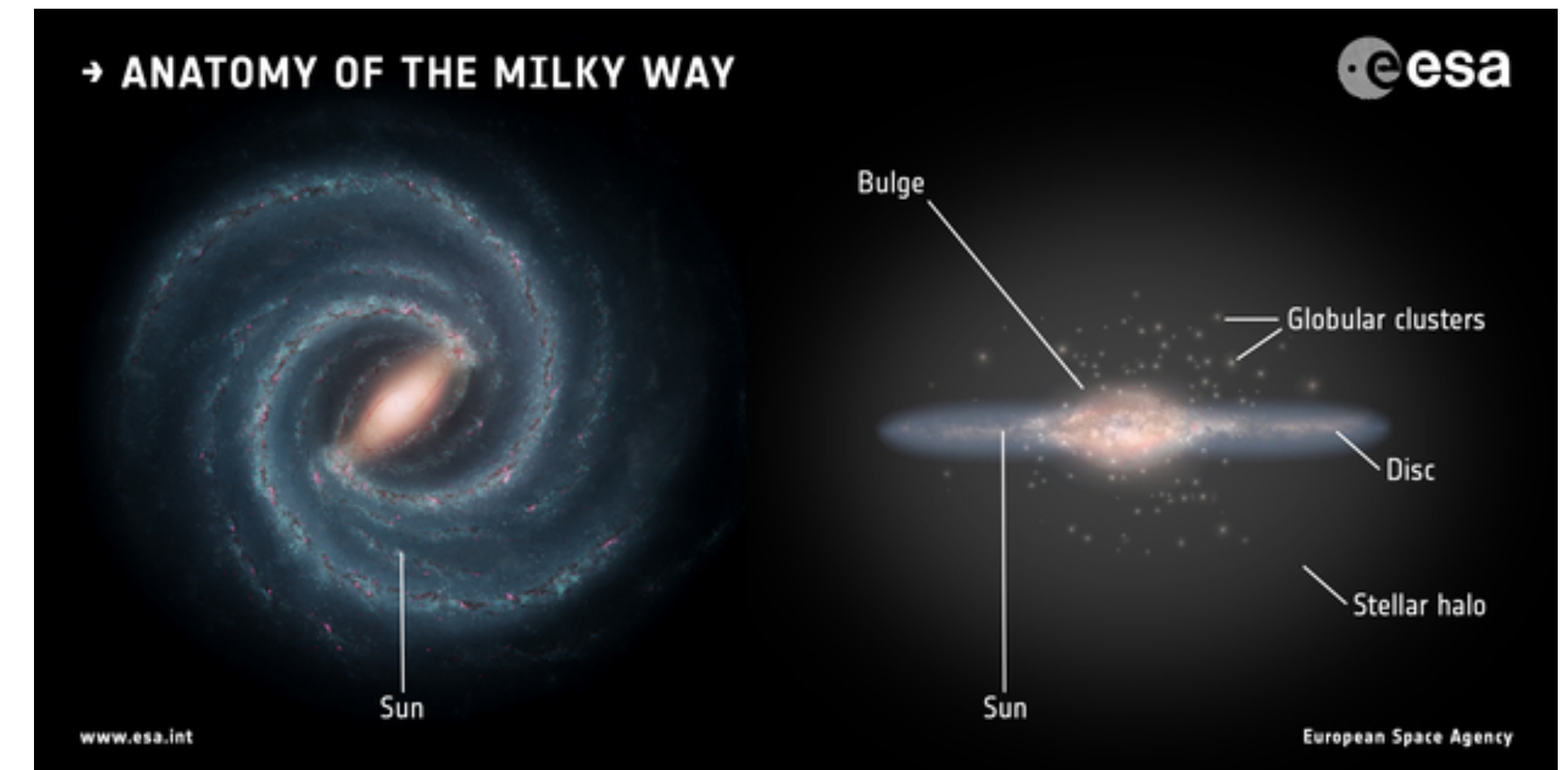
-> need to compare the GR model and the classical/(Lambda)-CDM model one for the MW

“Classic” Milky Way (MWC) model with Dark matter halo

- Newtonian limit applied for Galactic dynamics -> Poisson's equation

$$\nabla^2 \Phi_{tot} = 4\pi G(\rho_b + \rho_{td} + \rho_{Td} + \rho_h)$$

bulge + thin and thick discs + halo



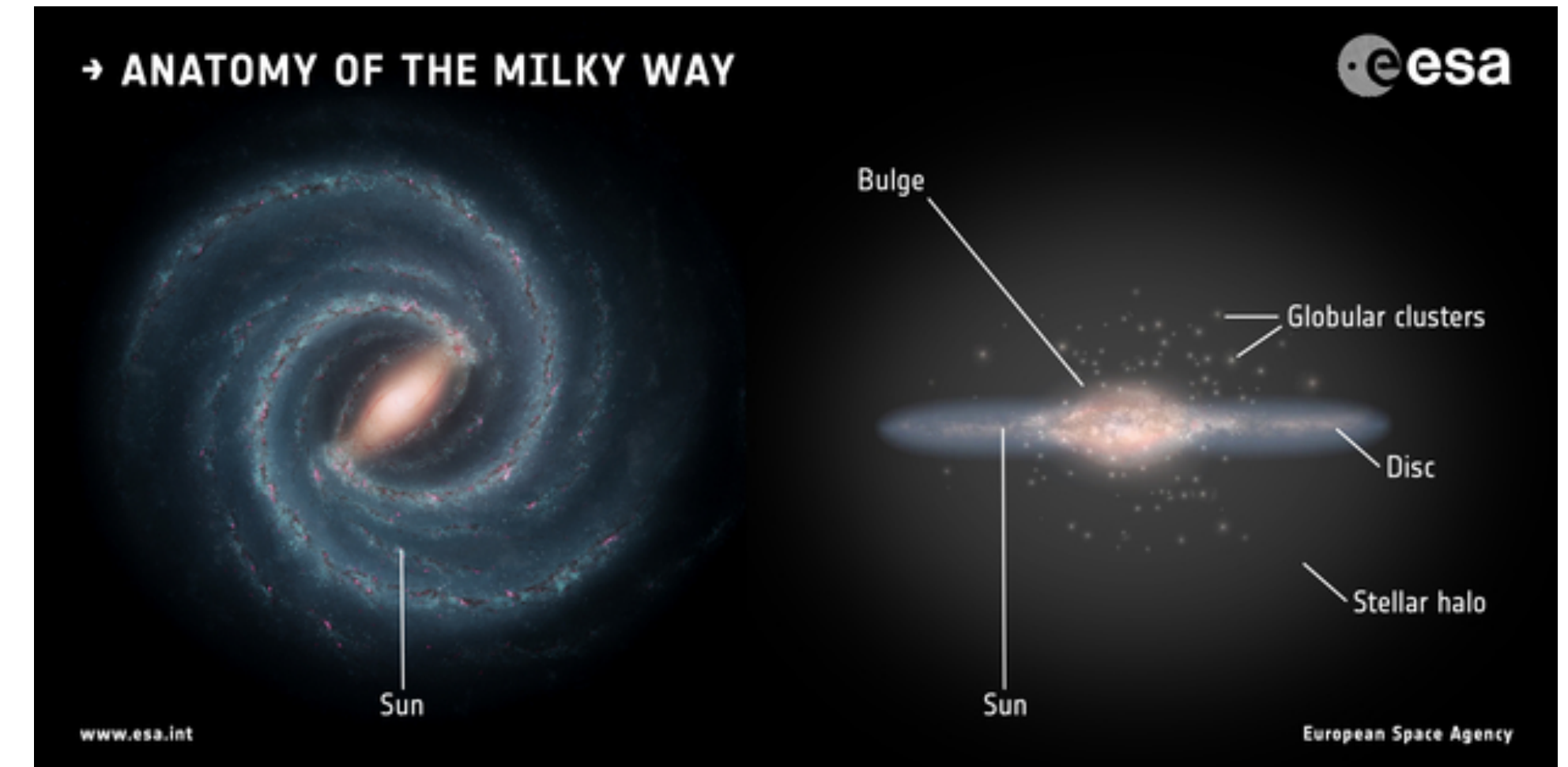
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$$\nabla^2 \Phi_{tot} = 4\pi G(\rho_b + \rho_{td} + \rho_{Td} + \rho_h) \quad \rightarrow \quad V_c^2 = R \left(d\Phi_{tot}/dR \right)$$

bulge + thin and thick discs + halo

MWC velocity profile



Parameters: M_b , M_{td} , M_{Td} , a_{td} , a_{Td} , b_b , b_d , ρ_0^{halo} and A_h corresponding to the bulge mass, the masses and the scale lengths/heights of the thin and thick discs, the halo scale density, and the halo radial scale + g_0

“Classic” Milky Way (MWC) model with Dark matter halo

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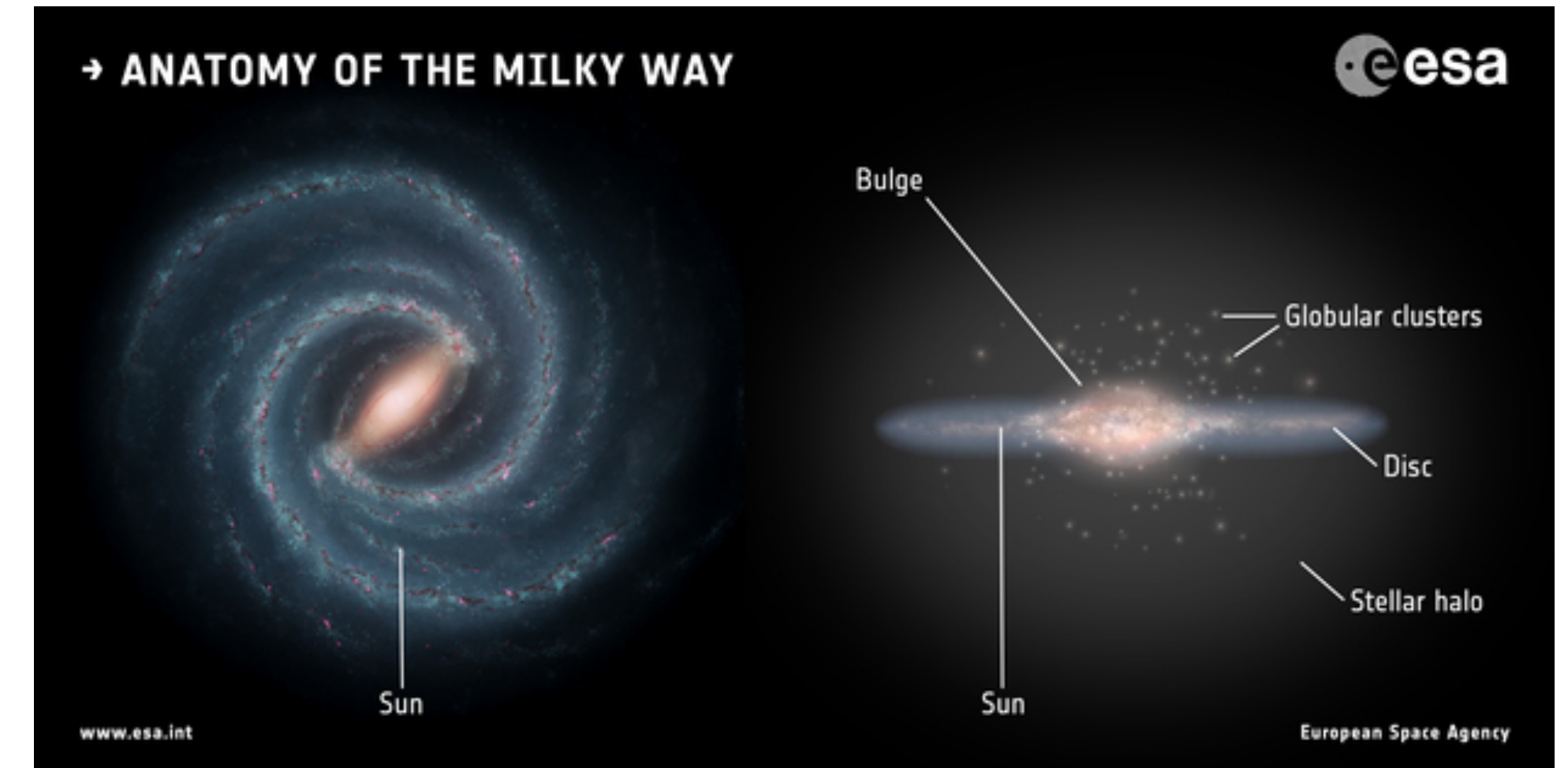
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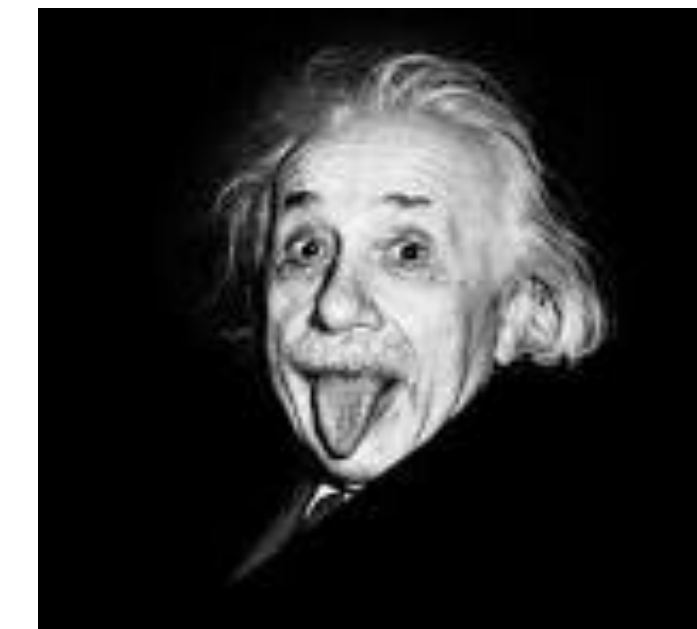
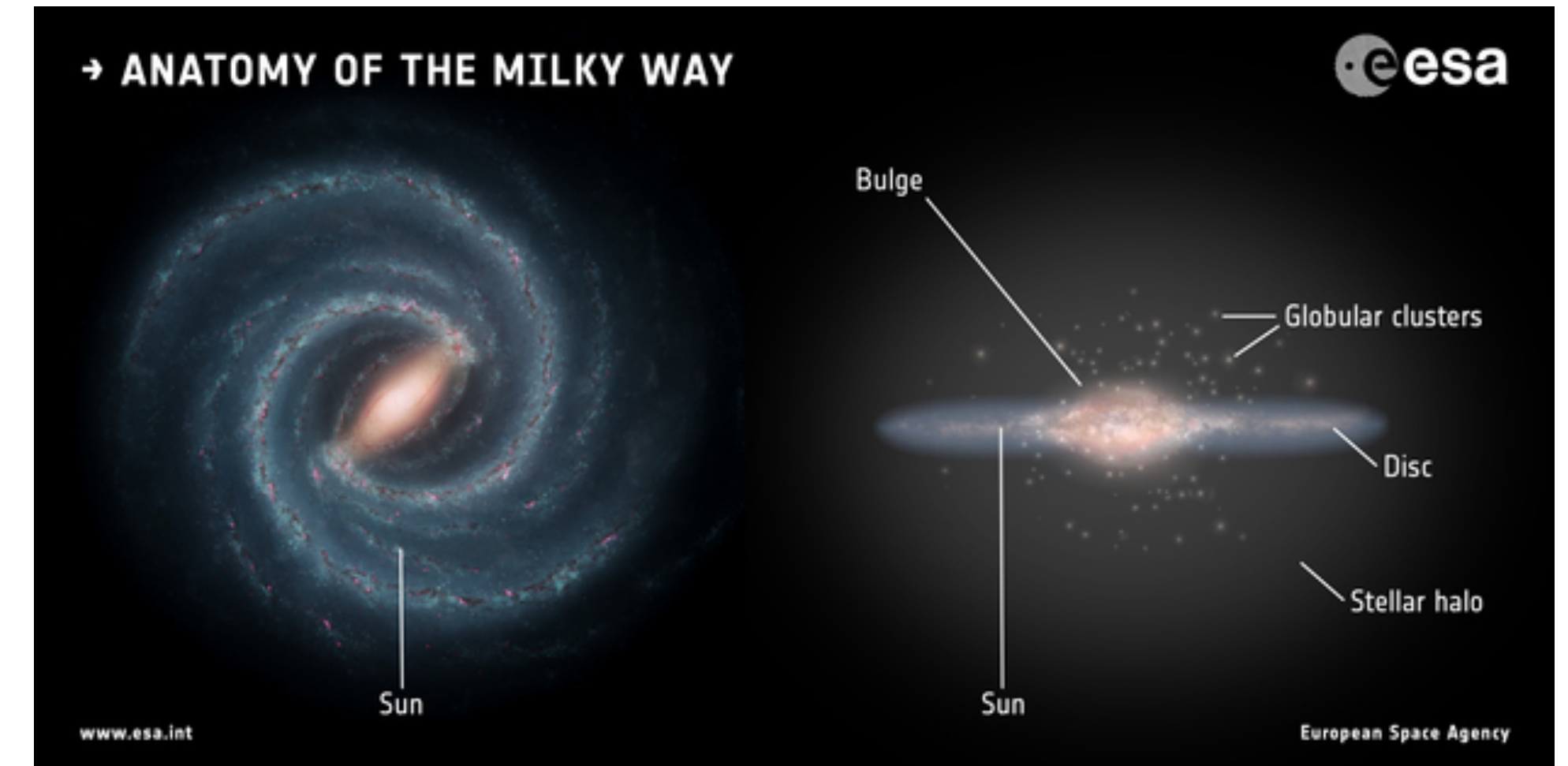
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Einstein’s equations are very difficult to solve analytically and Galaxy is a multi-structured object making it even the more difficult to detail a metric for the whole Galaxy



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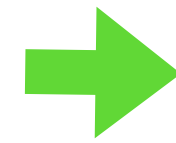
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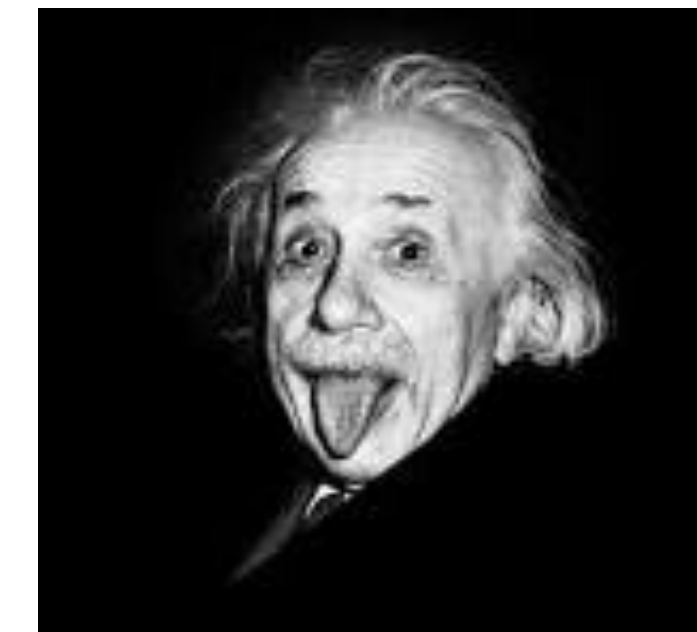
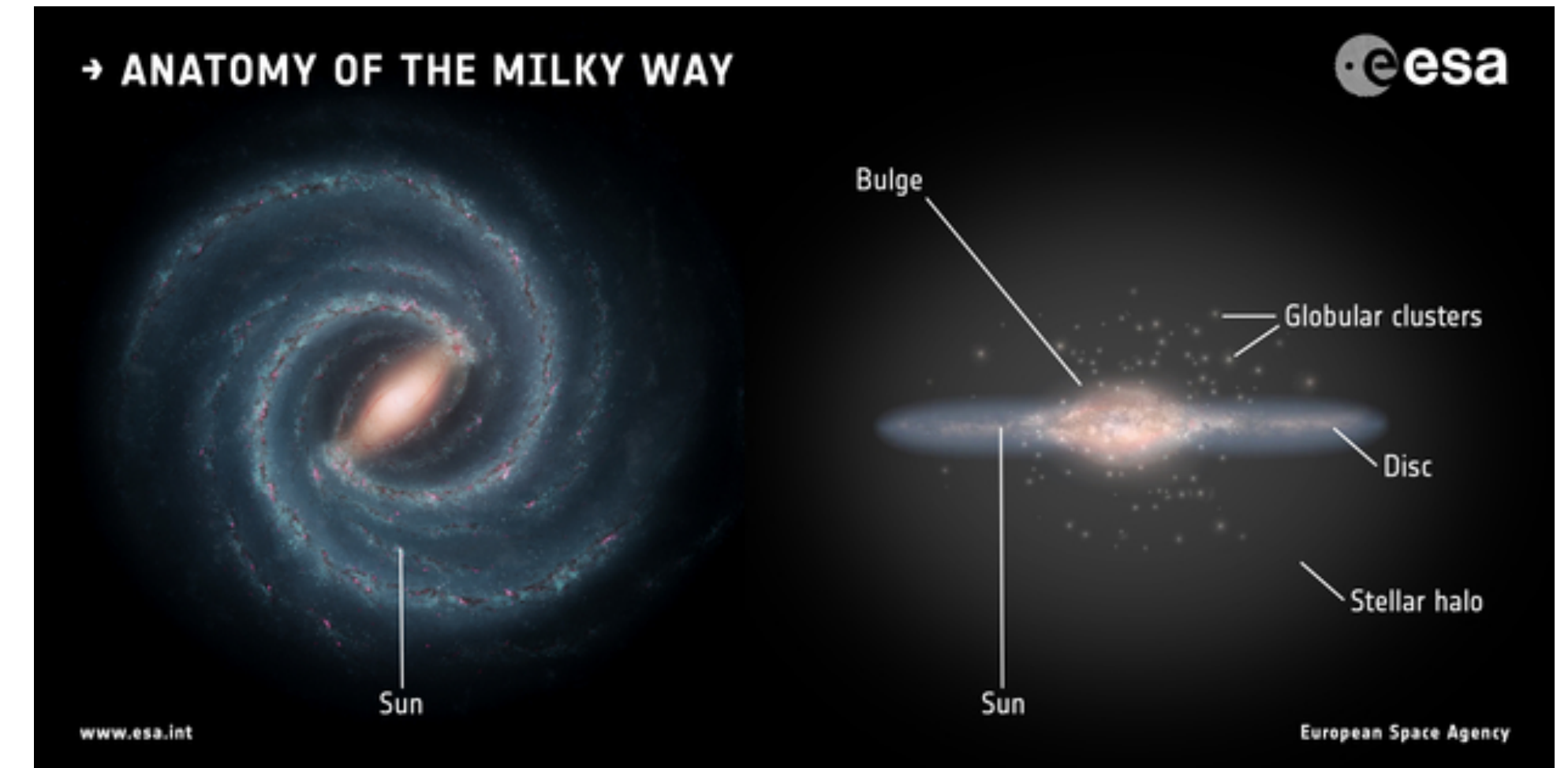
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2. Reflection symmetry (around the galactic plane)

3. Masses inside a large portion of the Galaxy interact only gravitationally and reside far from

the central bulge region/ Disc is an equilibrium configuration of a pressure-less rotating perfect fluid (GR dust)



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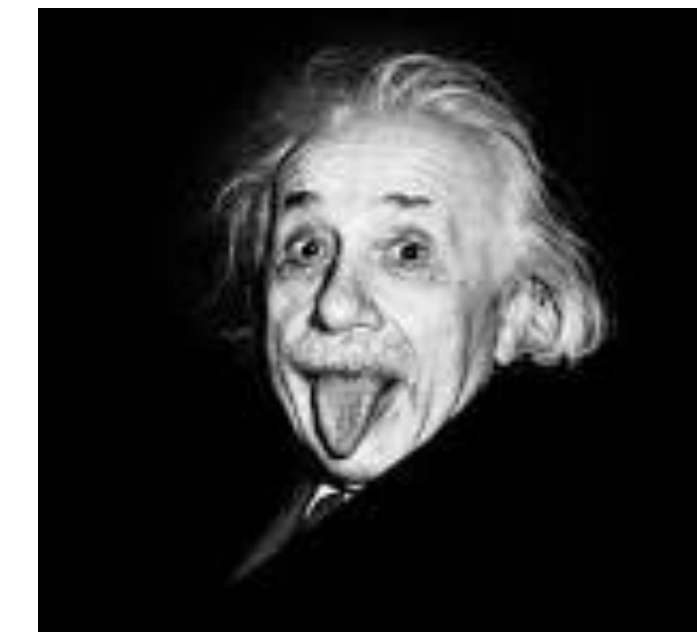
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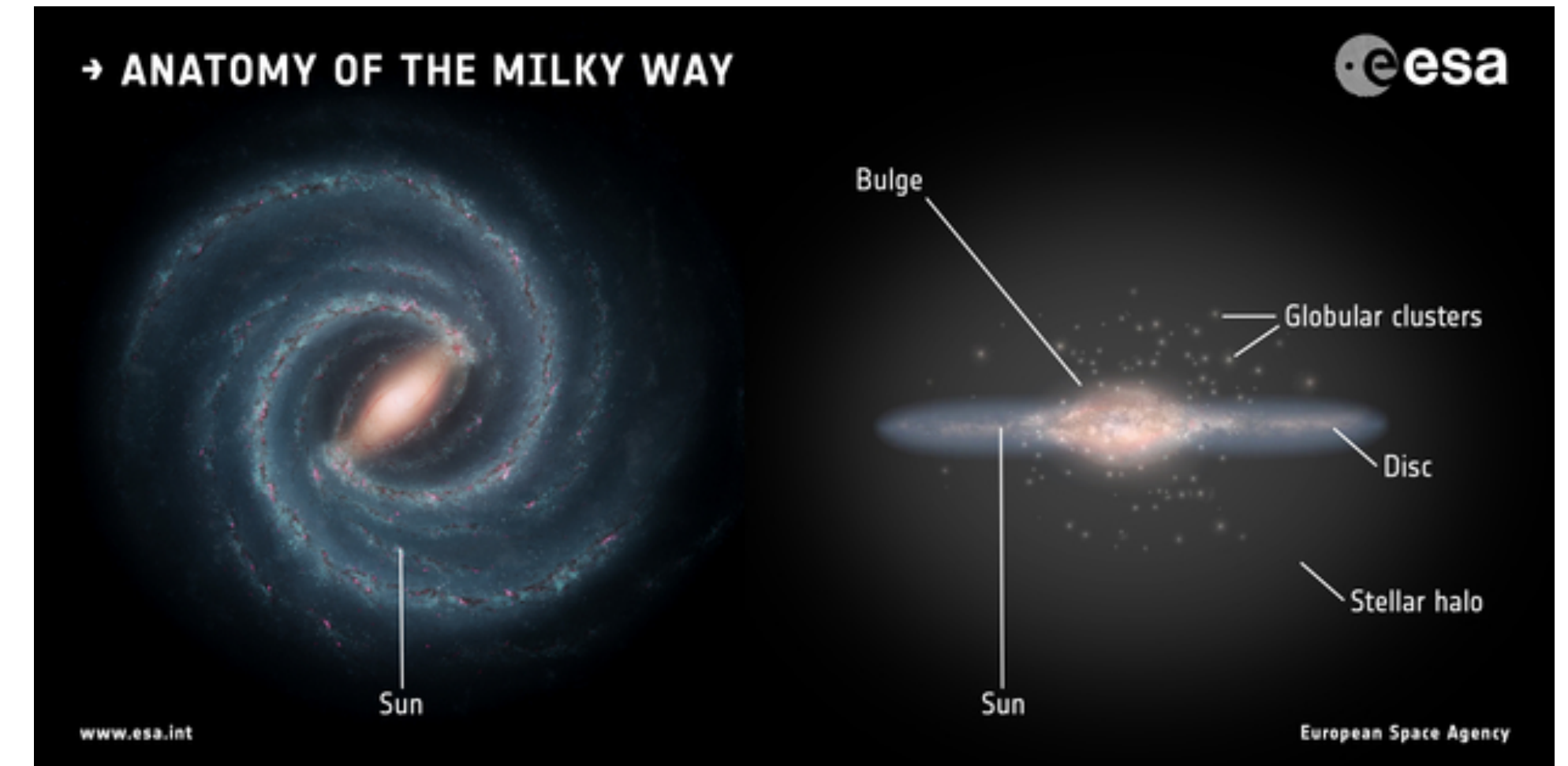
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$$G_{\mu\nu} = kT_{\mu\nu}$$

↓ Einstein field Eq.s

Set of differential equations for velocities **and** density



MCMC fit to the Gaia DR3 data - Classical (MWC), MOND and GR (BG) RC

Ansatz: the MW rotation curve is geometry driven?

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DR3 sample:

complete Gaia
astrometric dataset

3 bands (G, BP, RP)

parallaxes good to 20%

radial velocity with better
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uncertainties

719143 young disc stars
within $|z| < 1$ kpc
and up to $R = 19$
kpc

241'918 OBA stars, 475'520 RGB
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radial cut at 4.5 kpc
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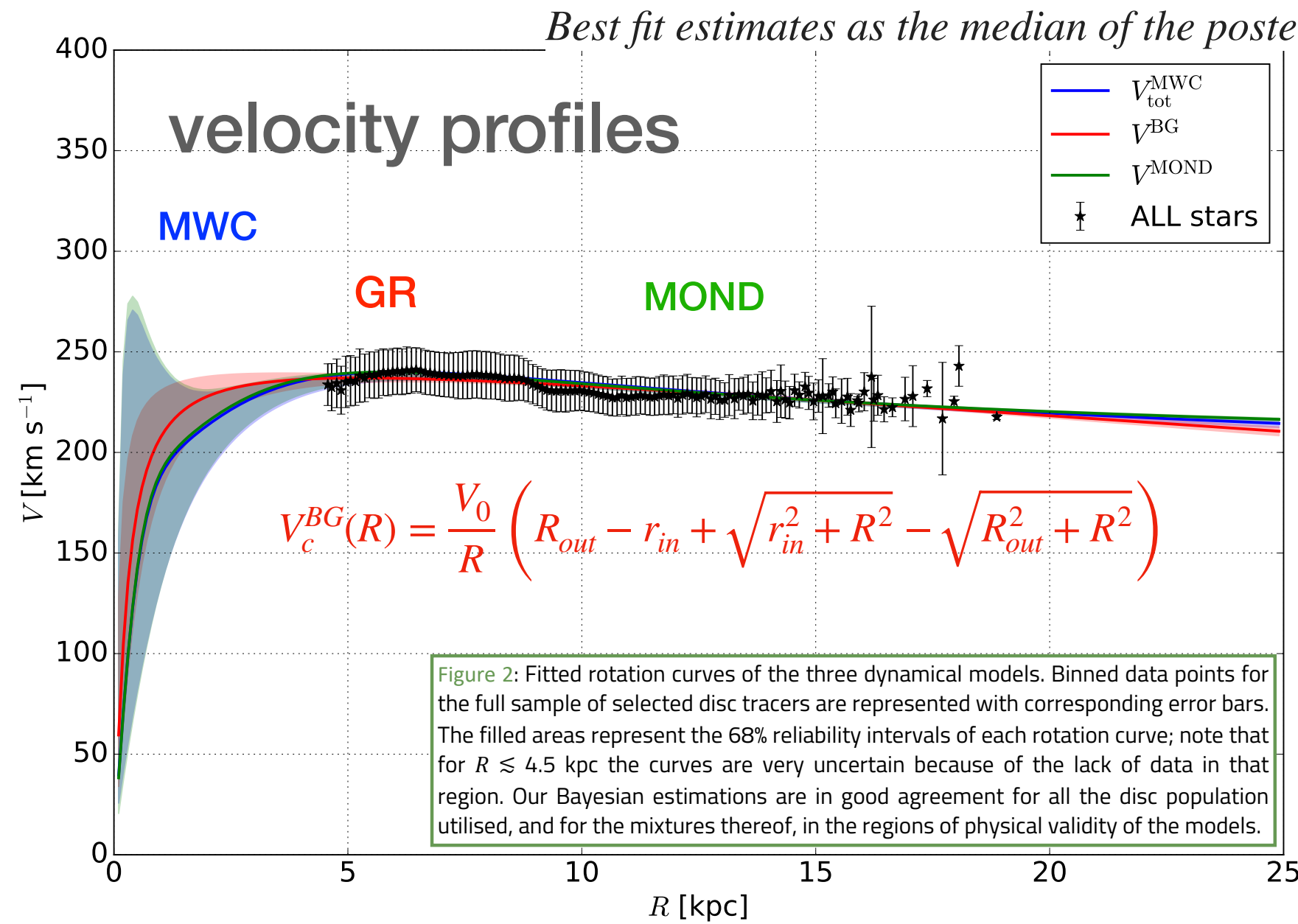
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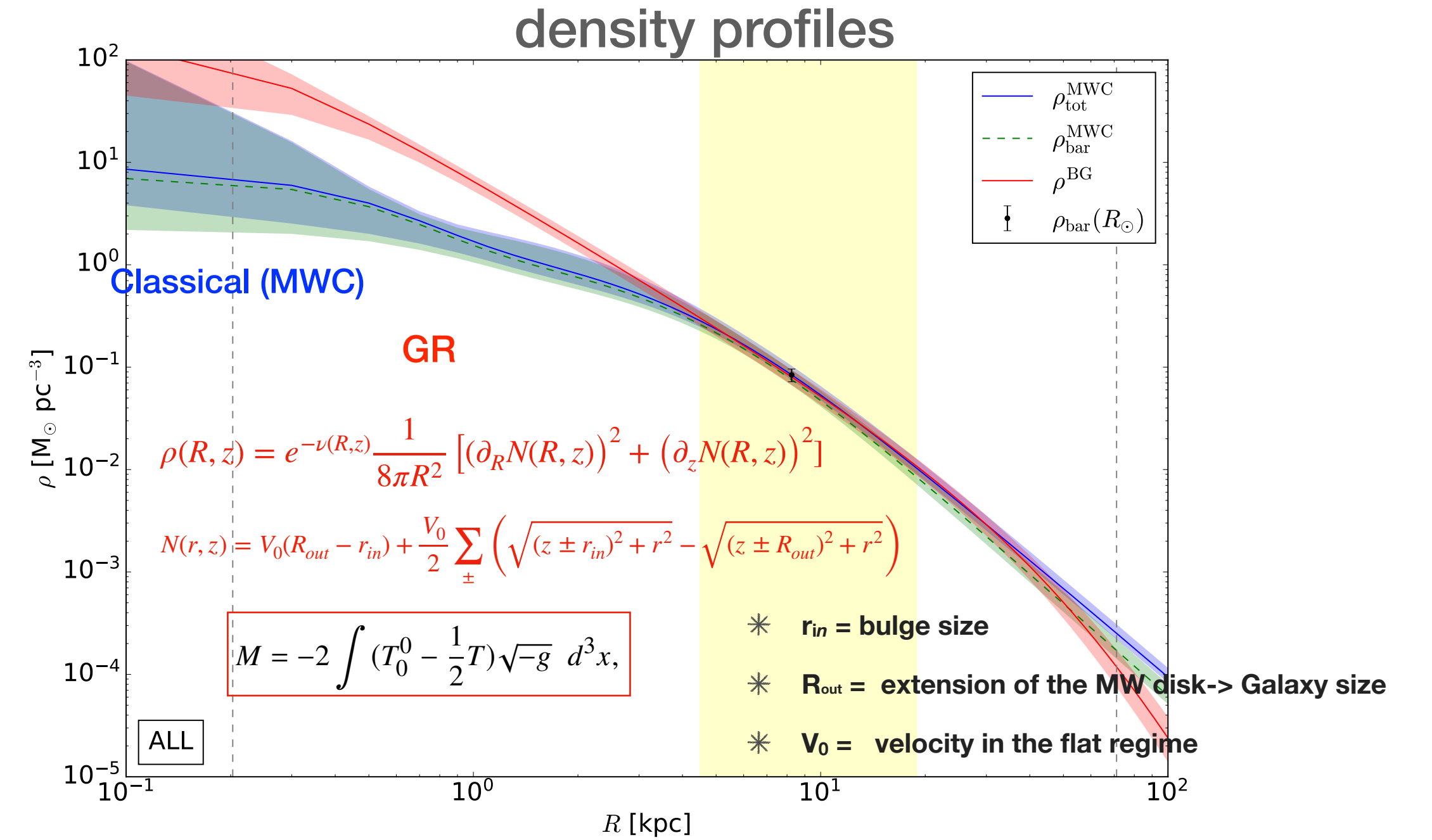
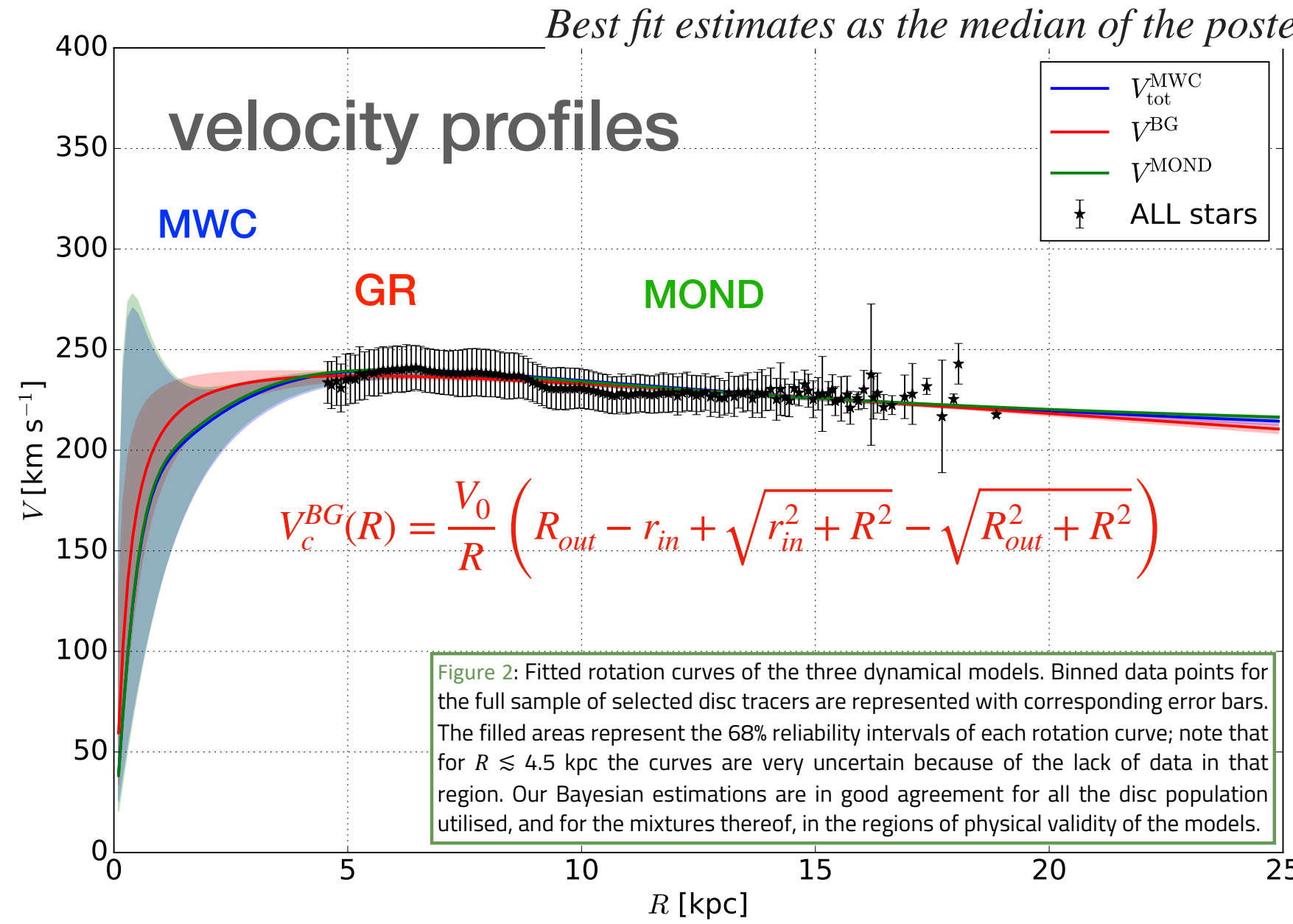
- * r_{in} = bulge size
- * R_{out} = extension of the MW disk -> Galaxy size
- * V_0 = velocity in the flat regime

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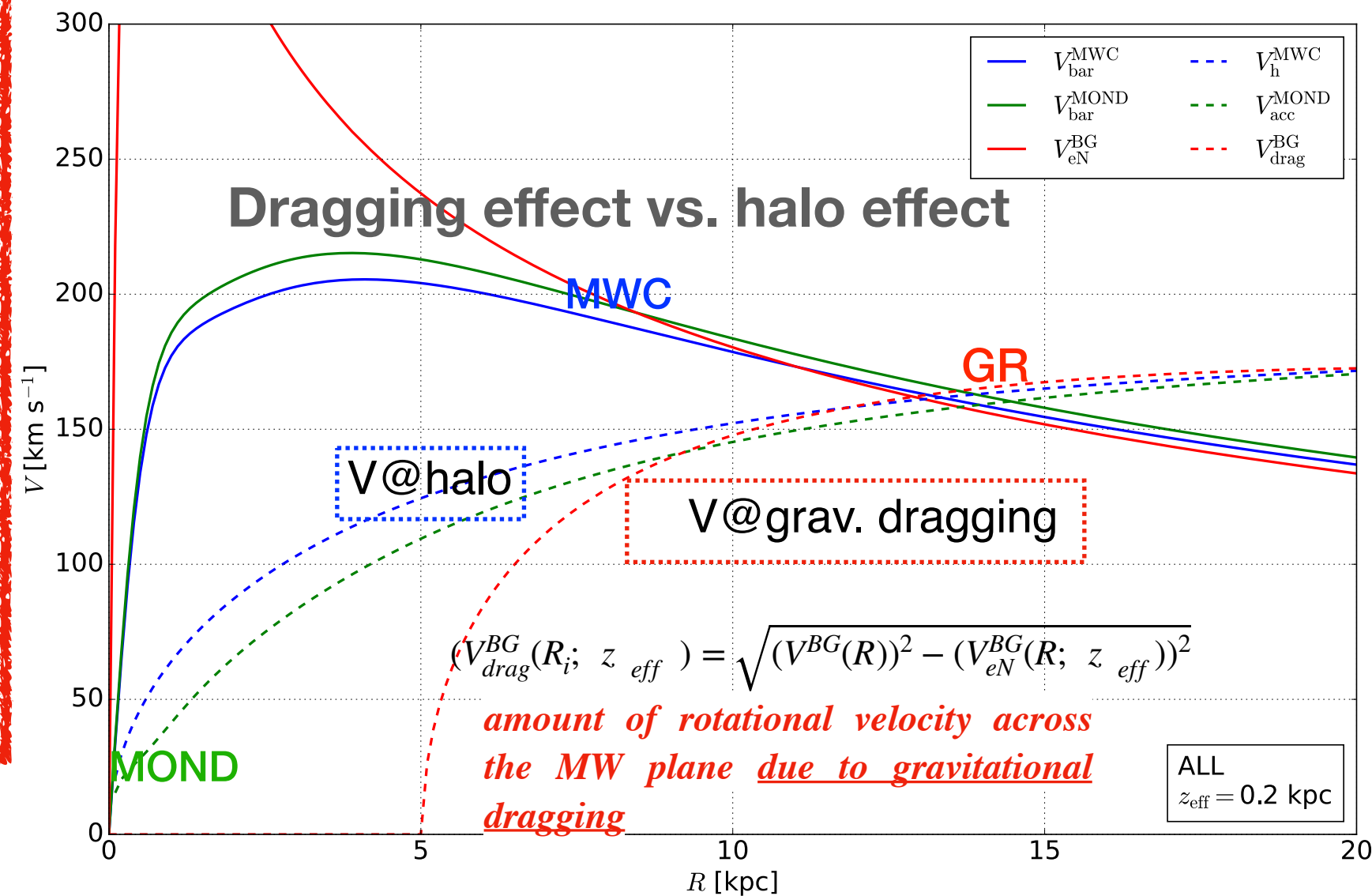
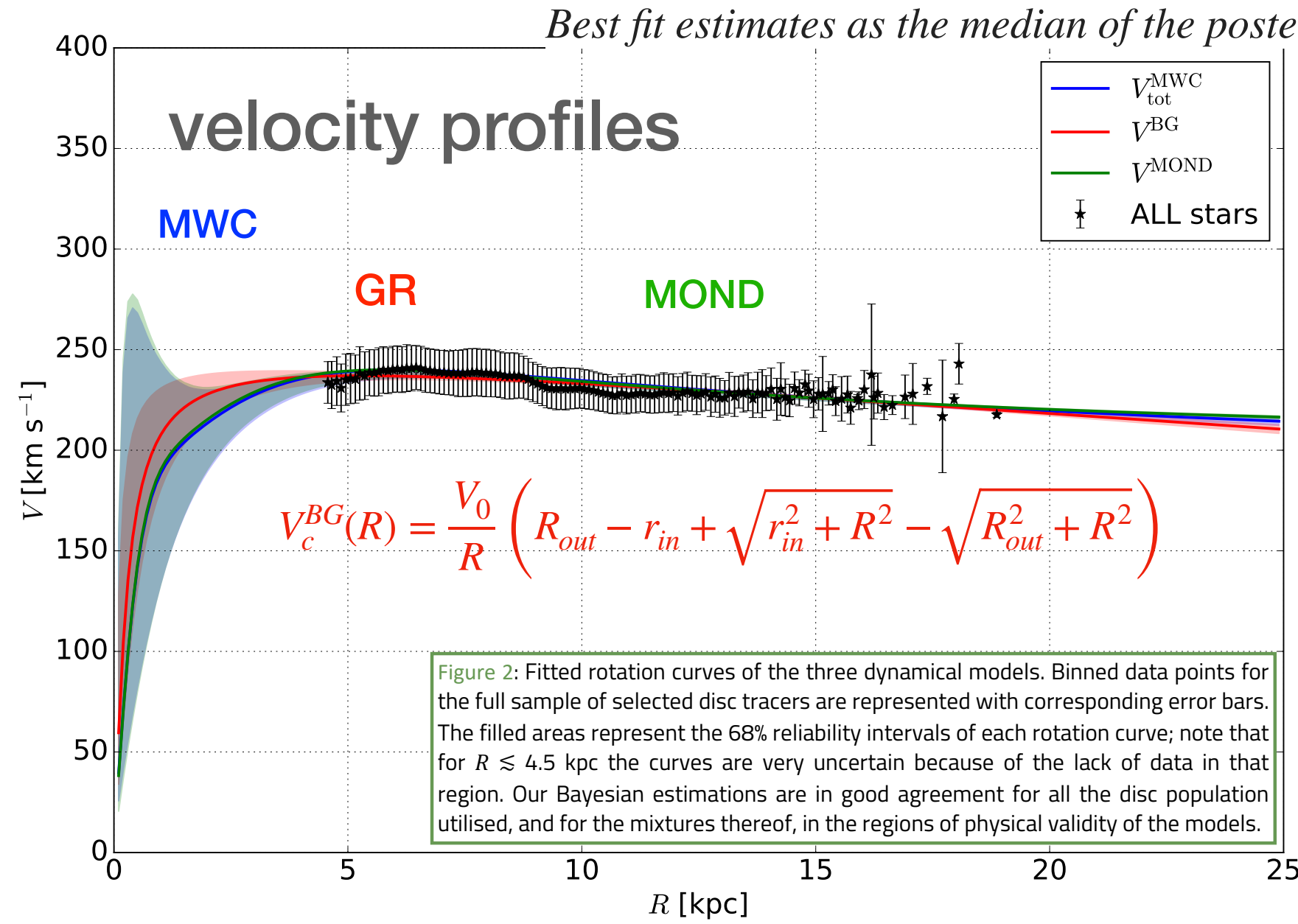
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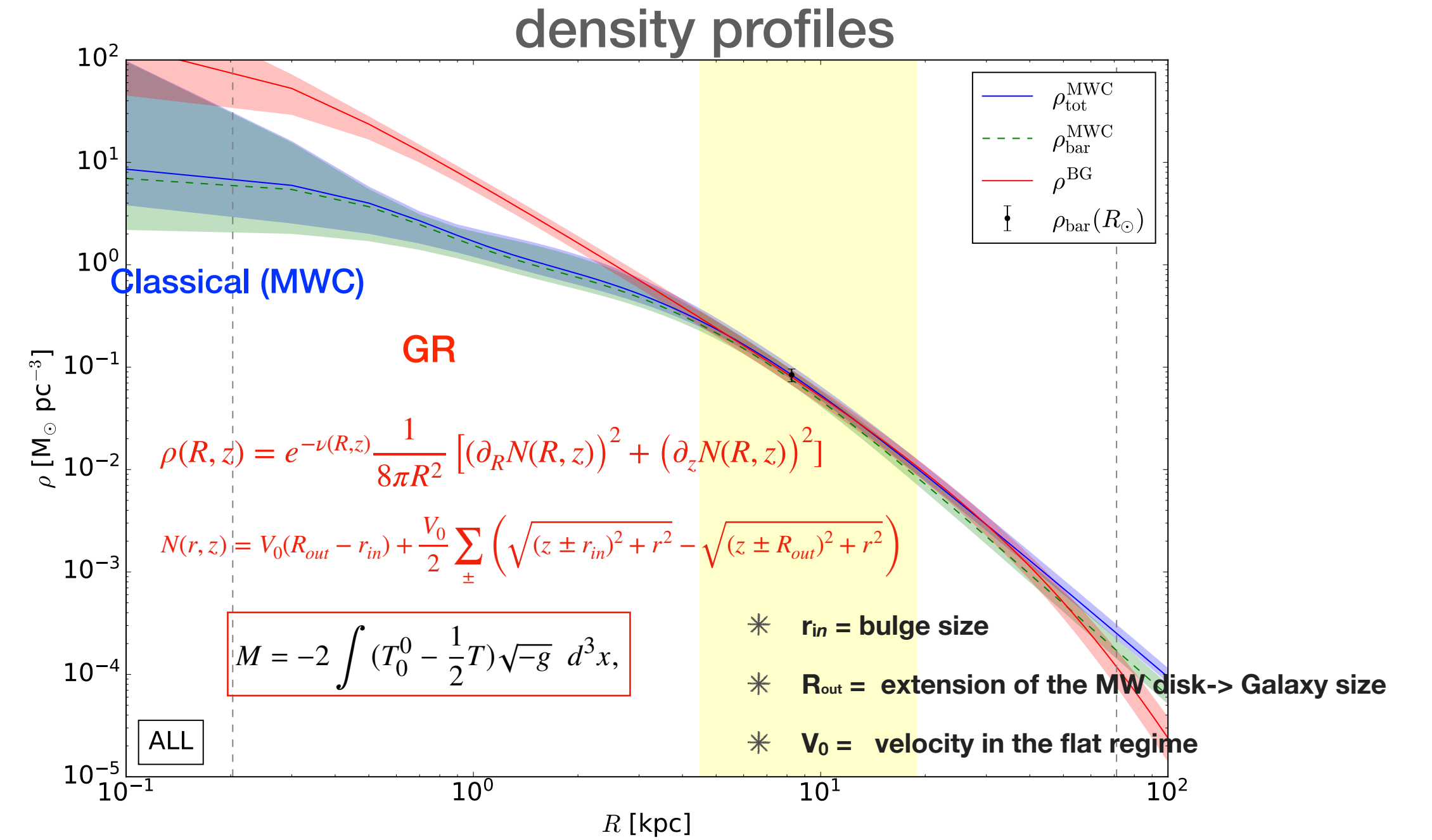
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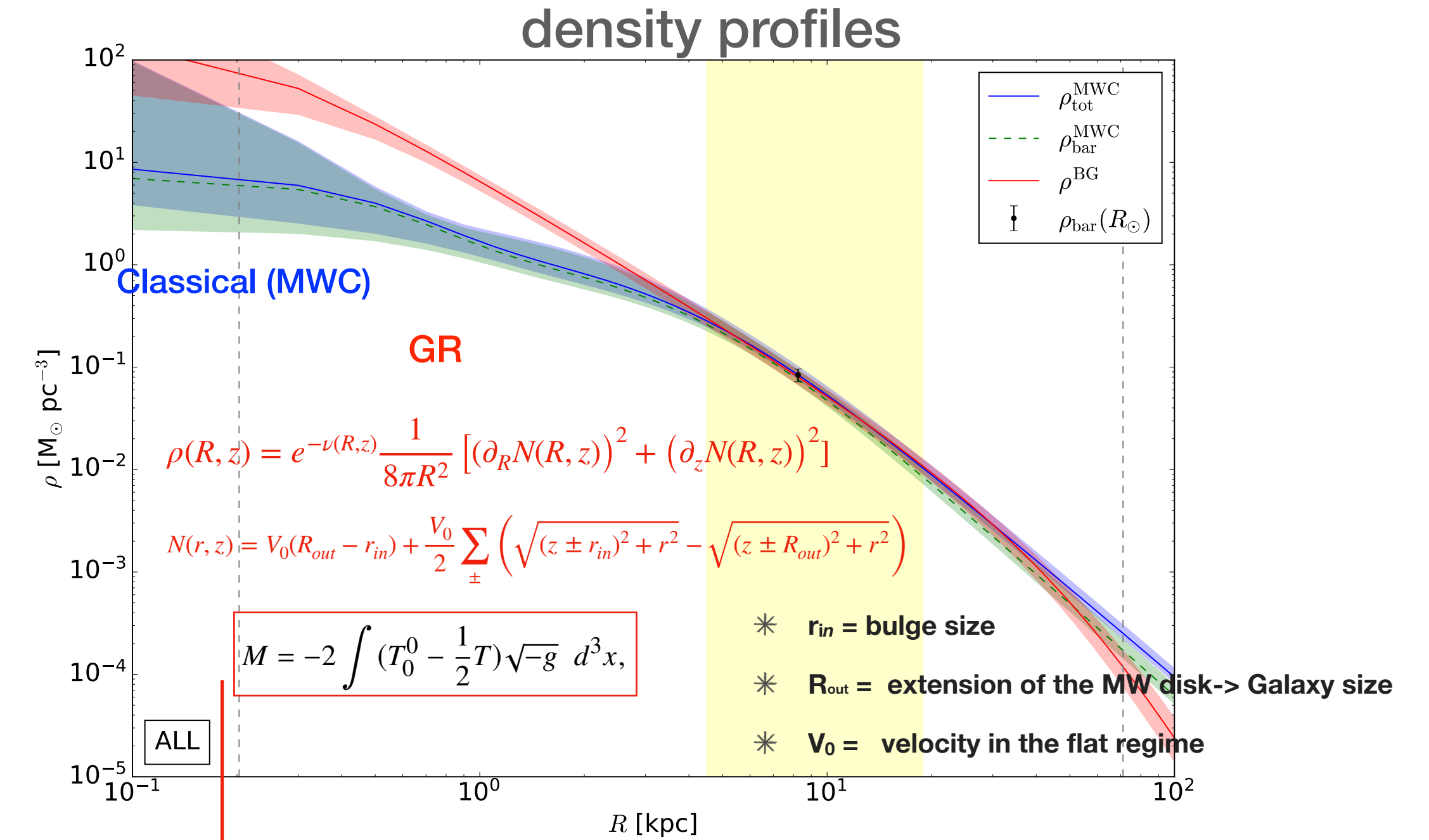
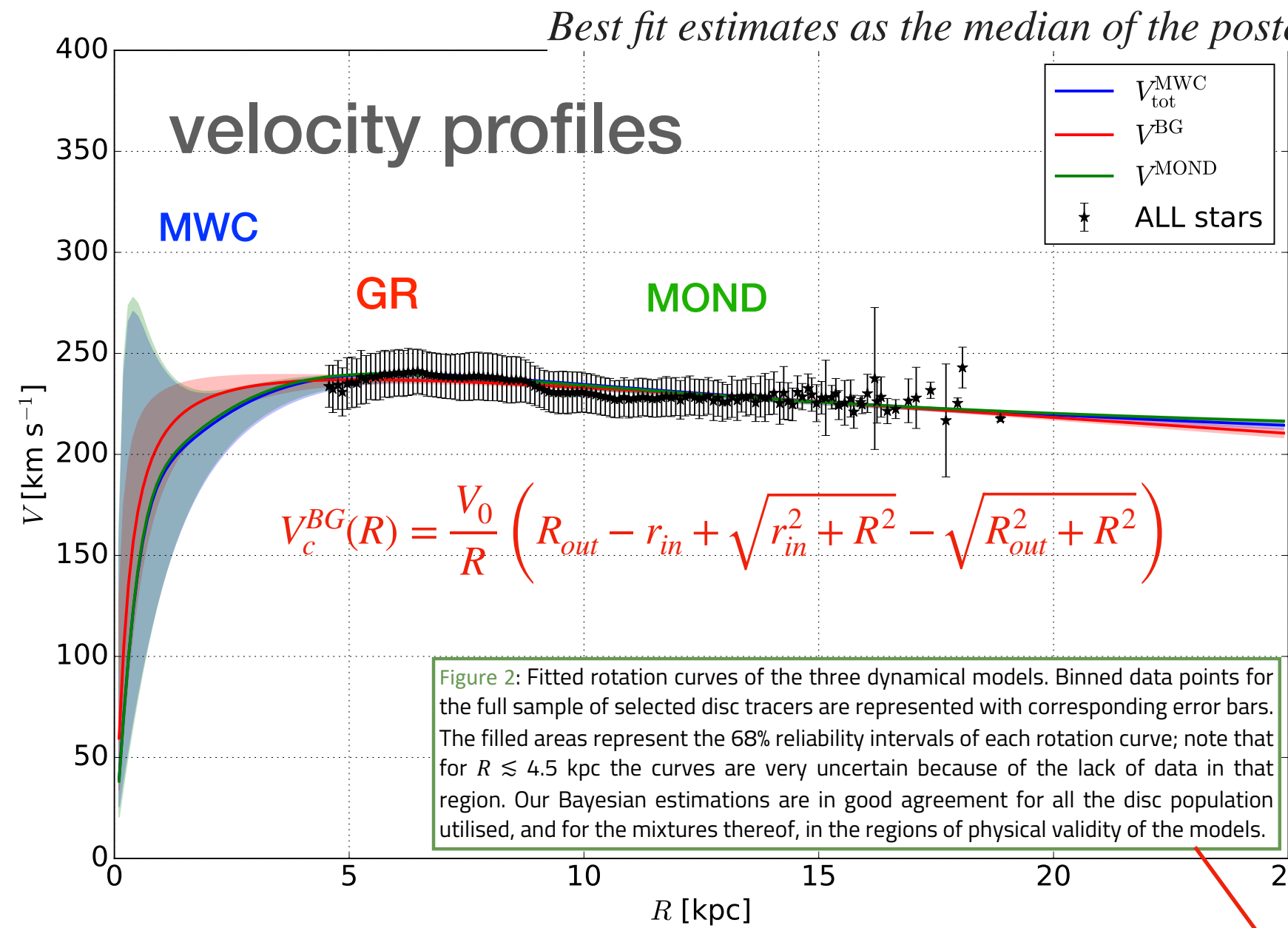
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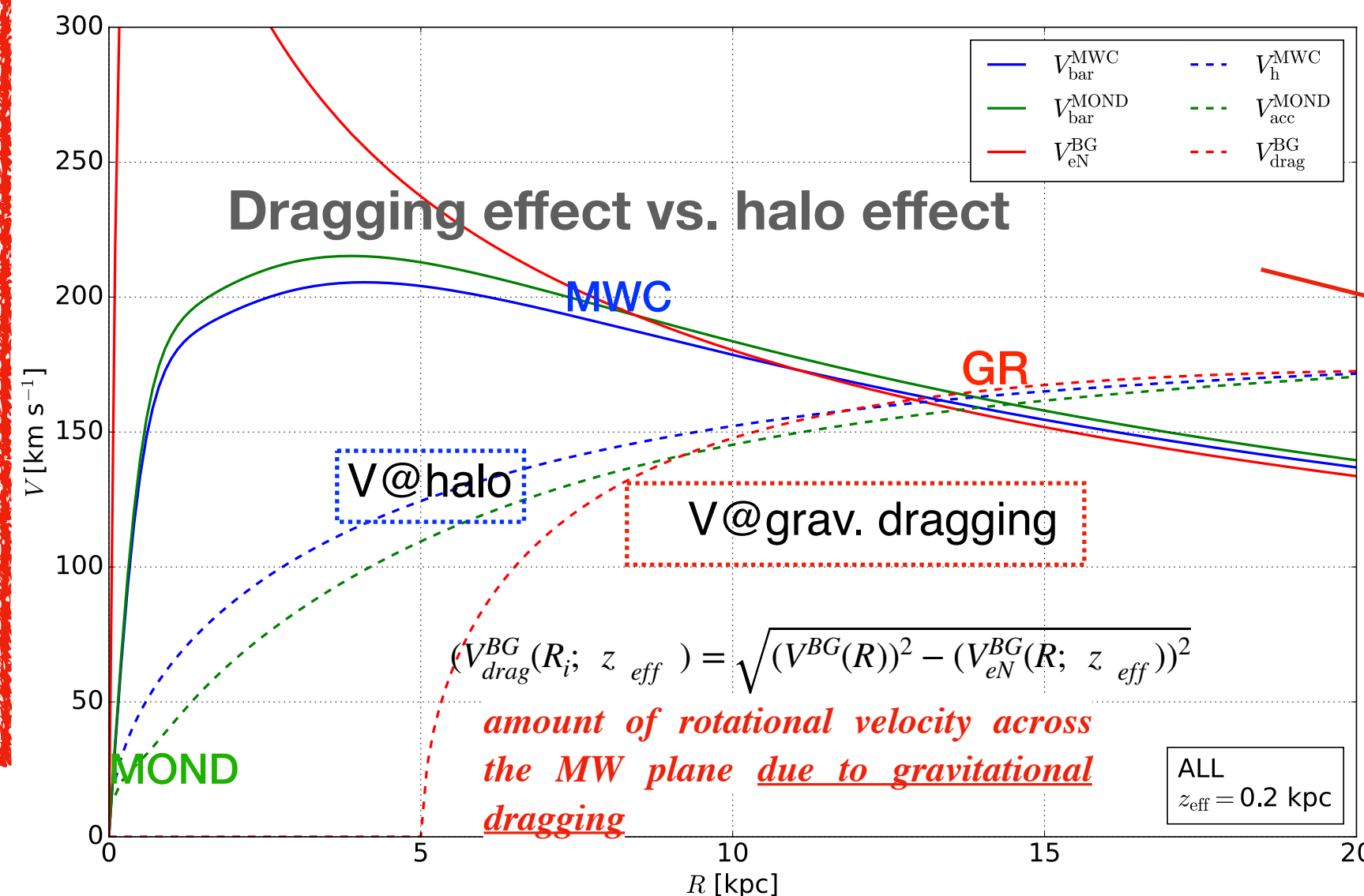
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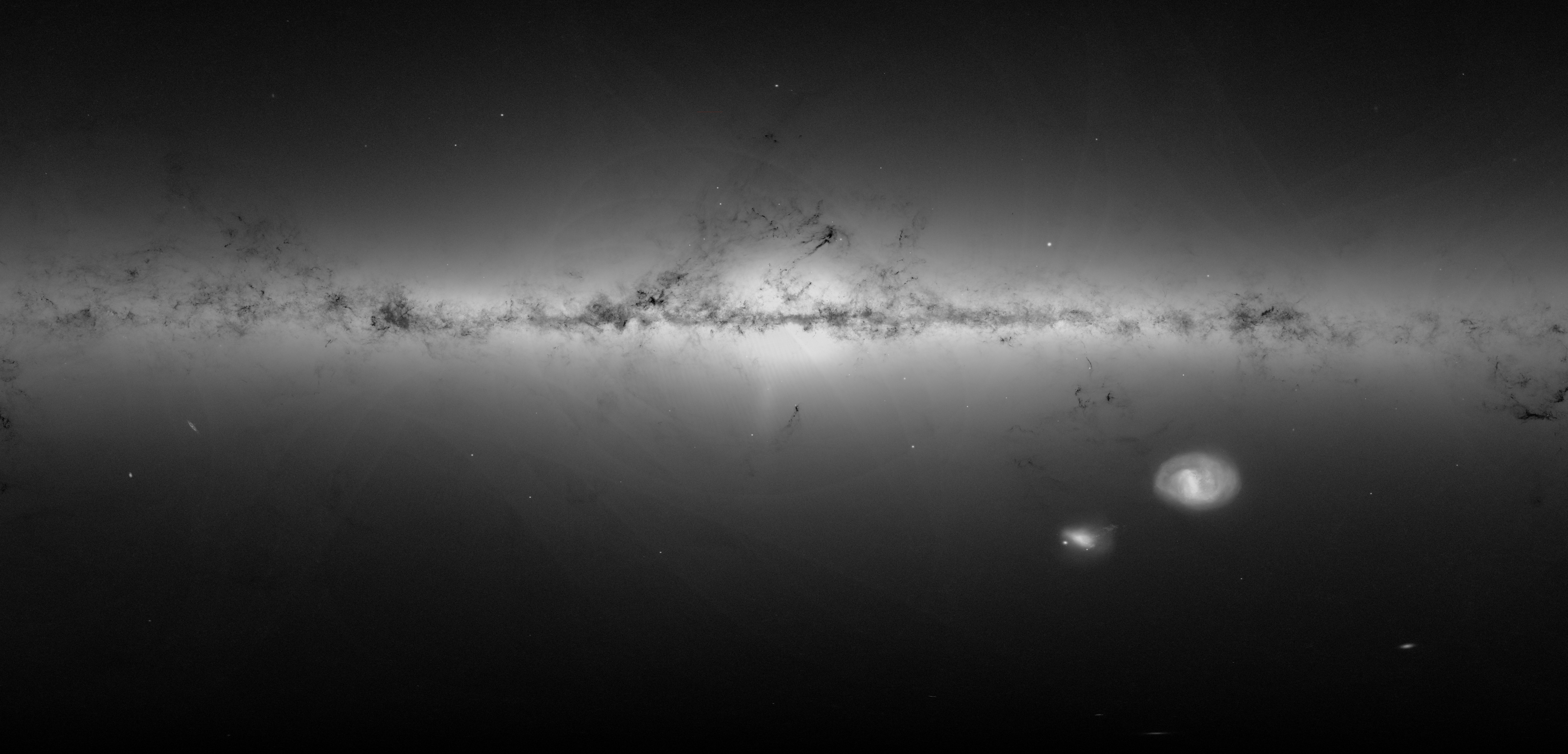


This again favourably points to the fact that a gravitational dragging-like effect could sustain a flat rotation curve

Crosta M., Giammaria M., Lattanzi M. G., Poggio E., MNRAS (2020)

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2. Regions around the bulge and the bar need relativistic hydrodynamics, where equilibrium conditions are not possible



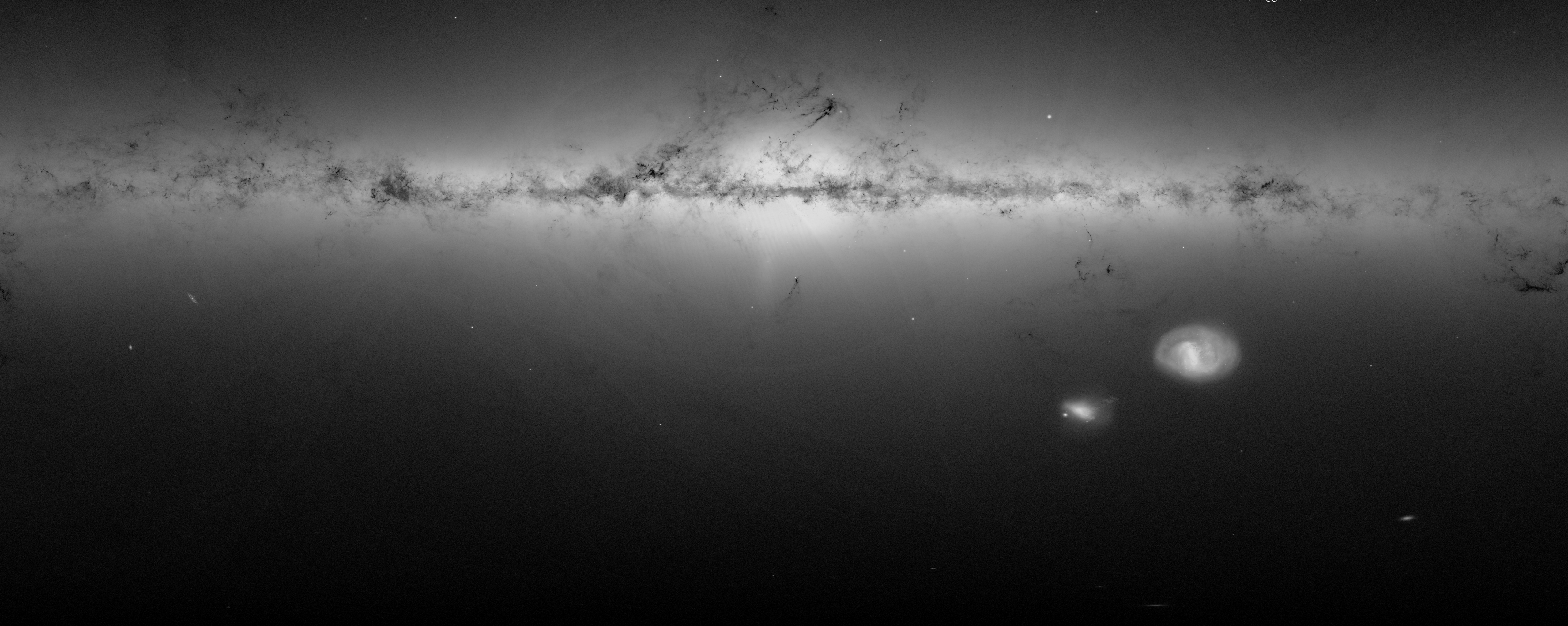
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β coordinate angular velocity
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Gravitational dragging working at disc scale?

$$\zeta^{\hat{\phi}}(r, z) \propto g_{0\phi}$$

of the co-rotating
star as seen by an
asymptotic observer
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Different from the IAU metric!

a gravitational dragging "DM-like" effect driving the Galaxy velocity rotation curve could imply that geometry - unseen but perceived as manifestation of gravity according to Einstein's equation - is responsible of the flatness at large Galactic radii

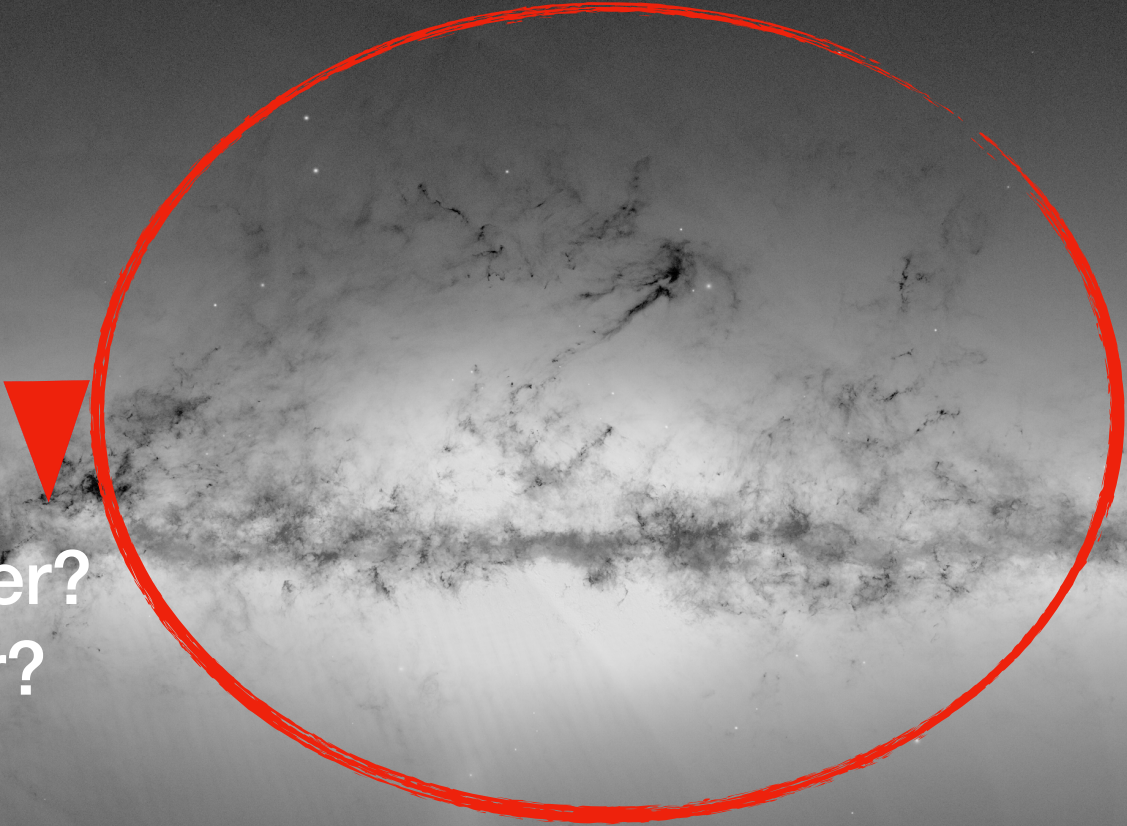
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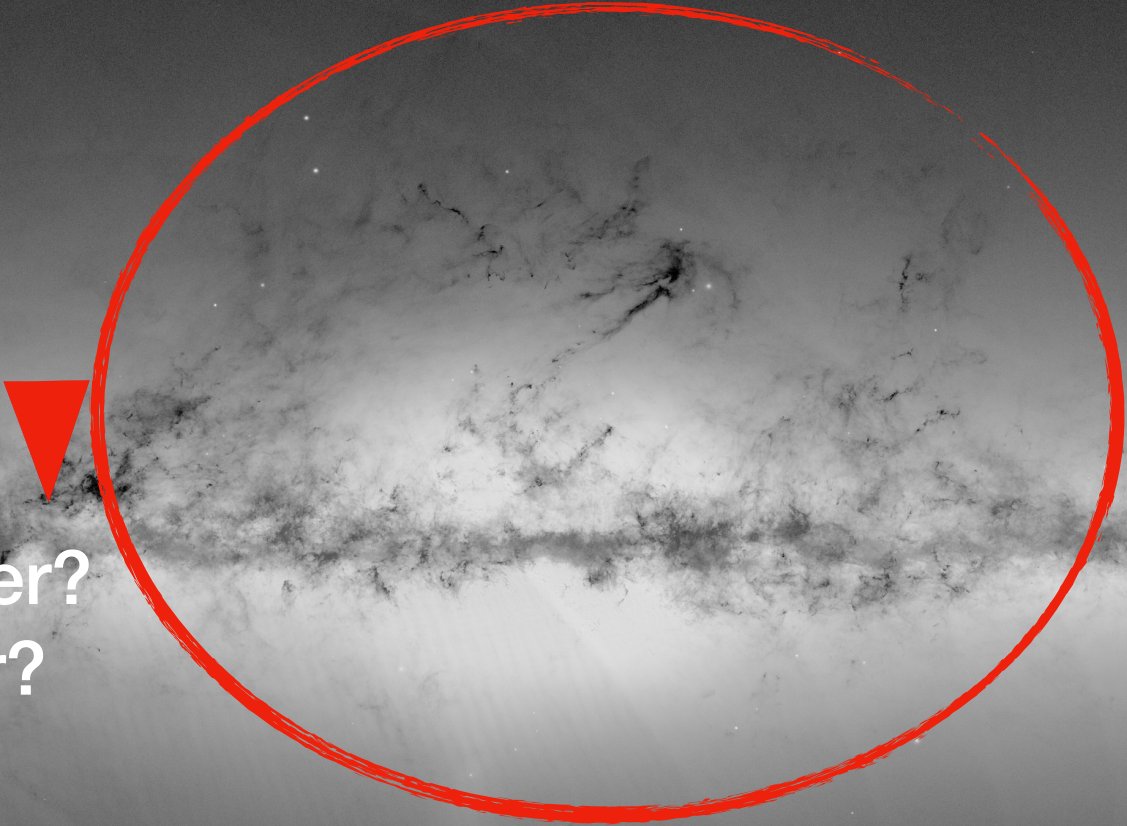
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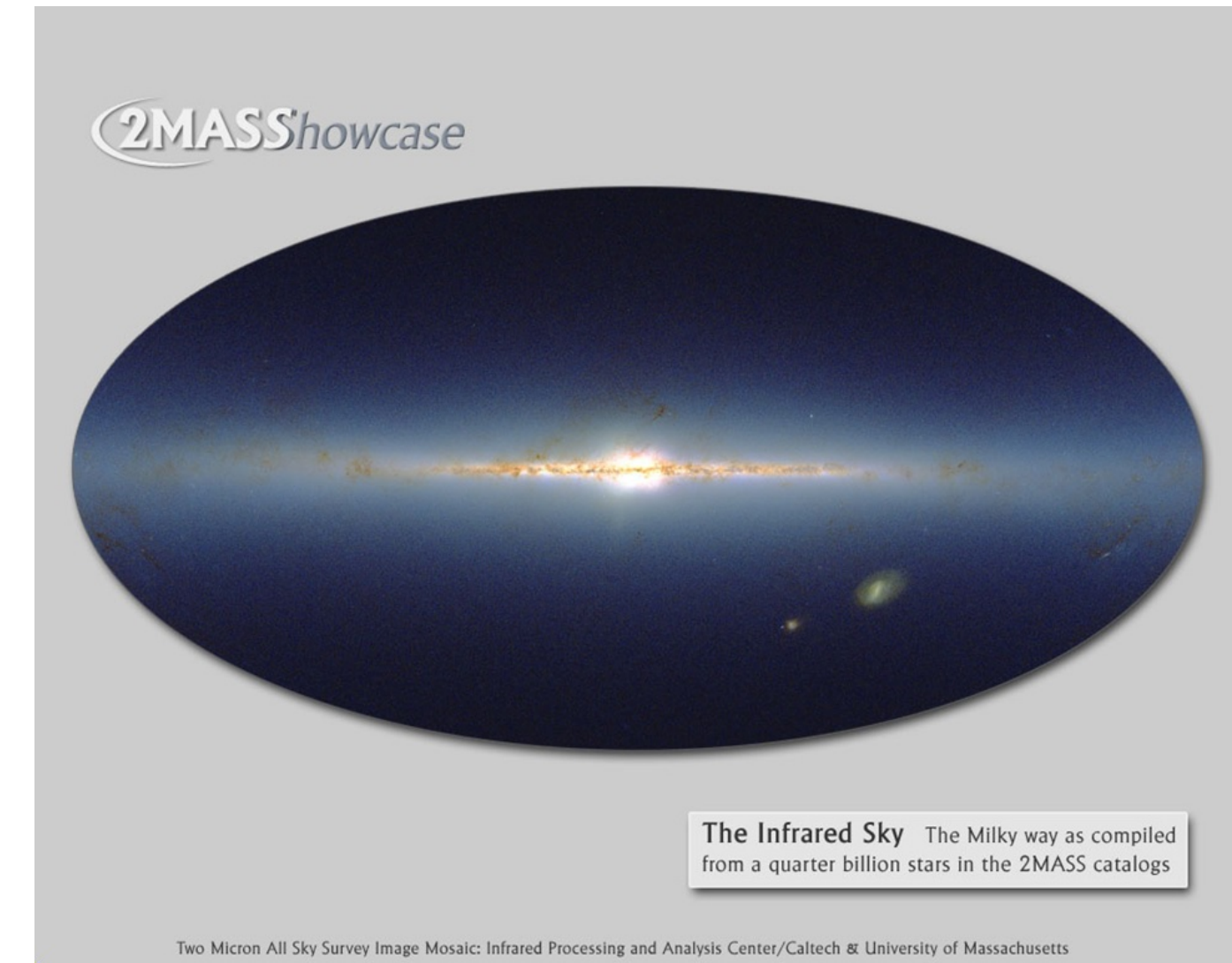
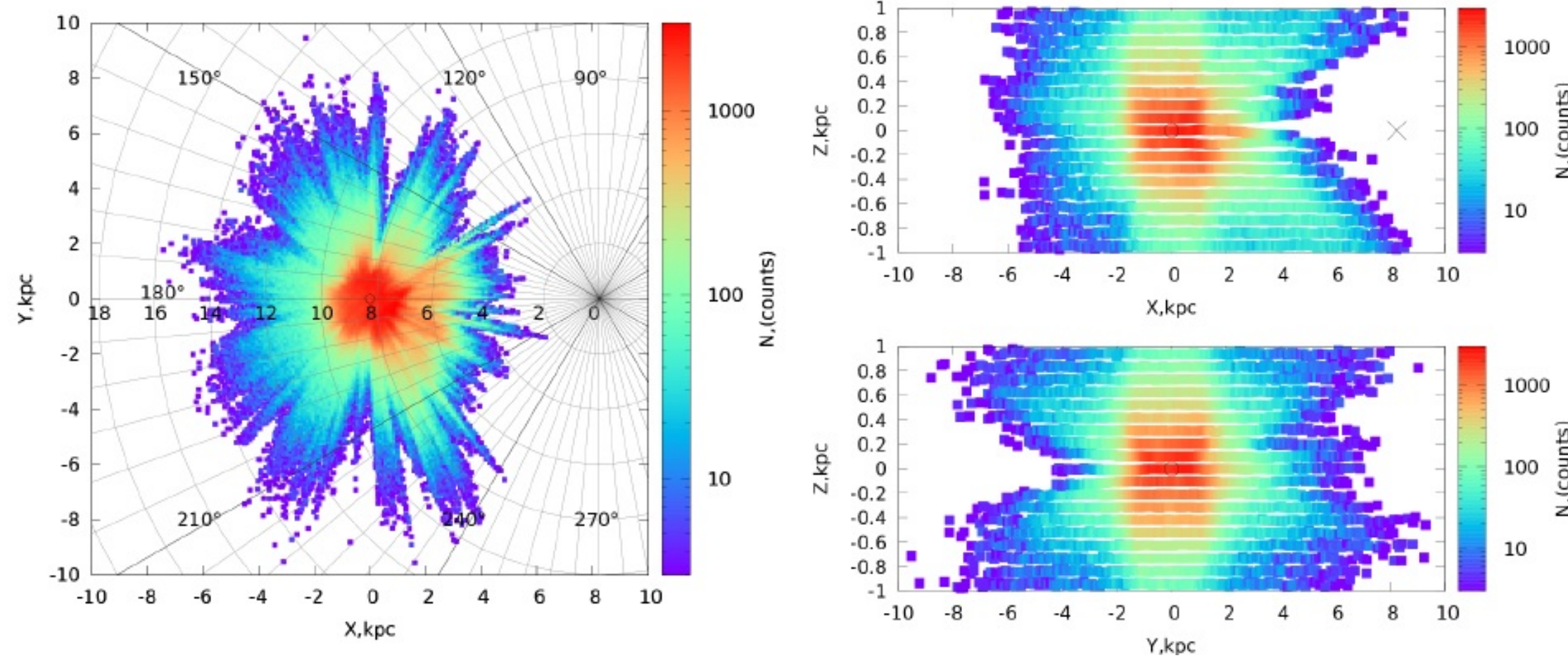
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Peering into hidden parts is utmost fundamental to establish boundary matching conditions between internal/
external Einstein's solutions

**new solutions & new observables (i.e. metric solutions to describe the evolution of a multistructured Galaxy,
avoiding unphysical global solutions)**

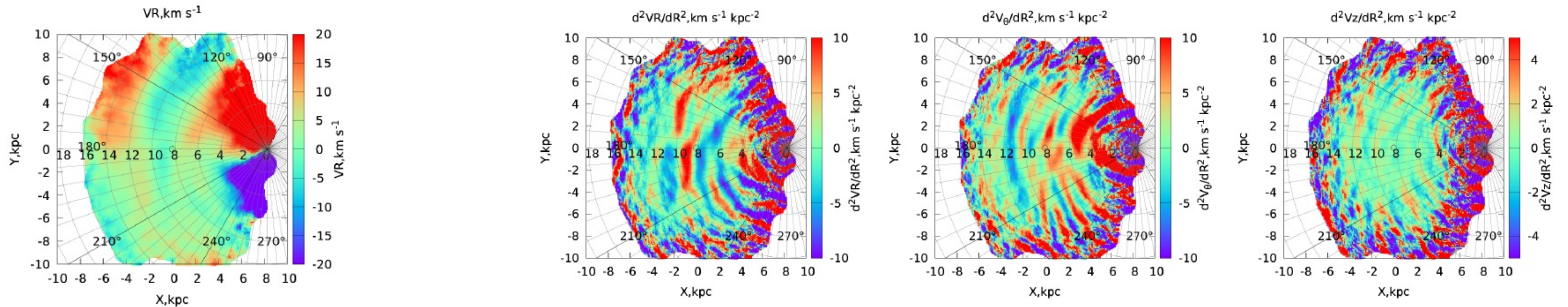
Gaia-Nir will peer through the dust of the MW to create a dense sampling of the phase-space to further test the bulge, bar, bar-disc interface and spiral arms

Distribution of 18 million high luminosity stars (i.e., young OB, giants and subgiants) from Gaia DR3



A new kinematic model of the Galaxy: analysis of the stellar velocity field from Gaia D3, Akhmetov et al. 2024, under review process

Kinematic analysis of the Galaxy with Gaia DR3 using a Taylor decomposition of the velocity field up to second order -> maps of the velocity components and of their partial derivatives with respect to Galactocentric coordinates within 10 kpc of the Sun reveal complex substructures



evidence of waps and non-axisymmetric bar features of the Galaxy

Second order partial derivatives of the stellar velocity field allows us to determine the values of the vertical gradient of the Galaxy azimuthal, radial and vertical velocities-> spiral arms

✓ Extend the MW “geometries” to other galaxies:., the “geometries” of the Galaxy can play a reference role for other galaxies, just like the Sun for stellar models

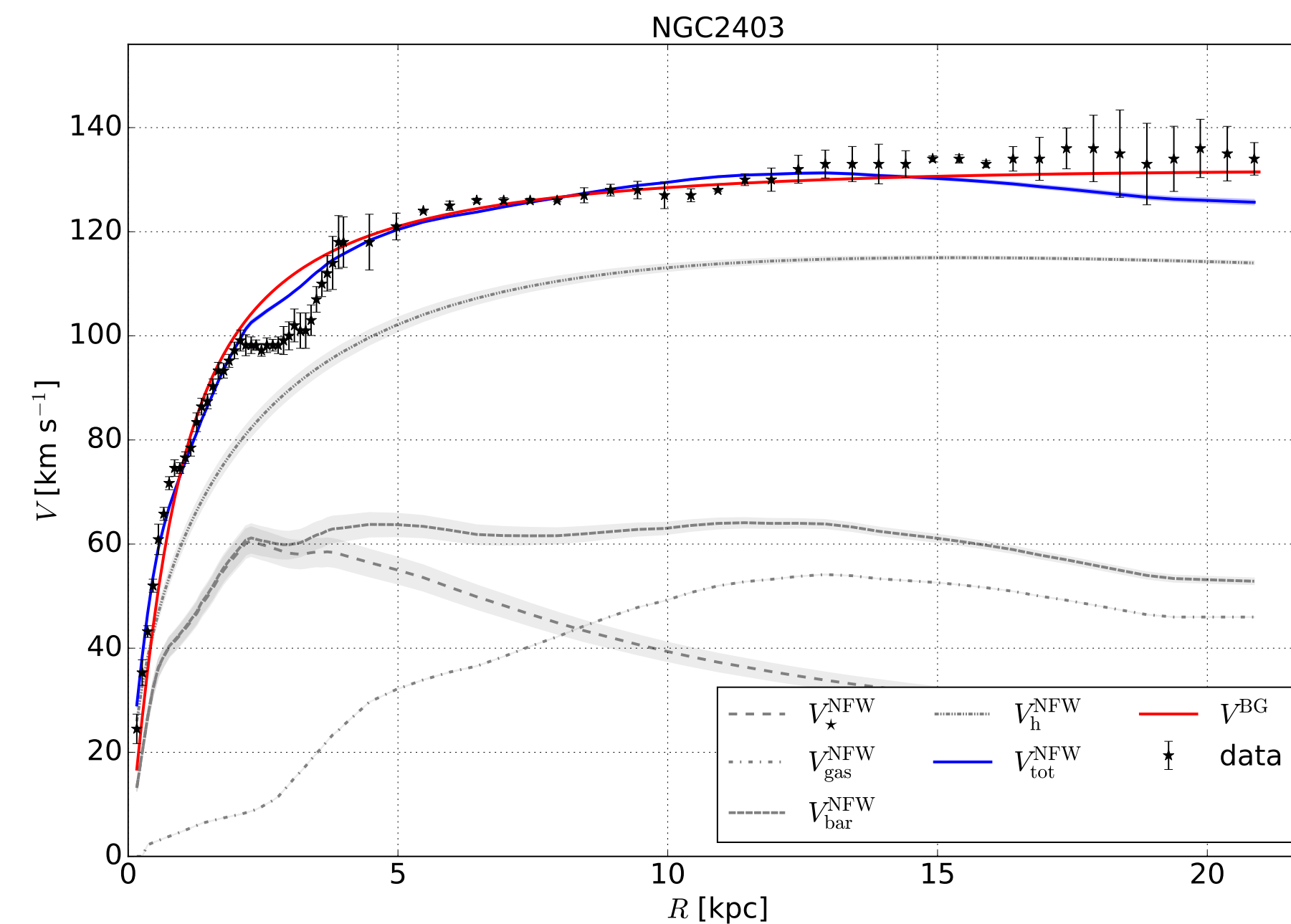
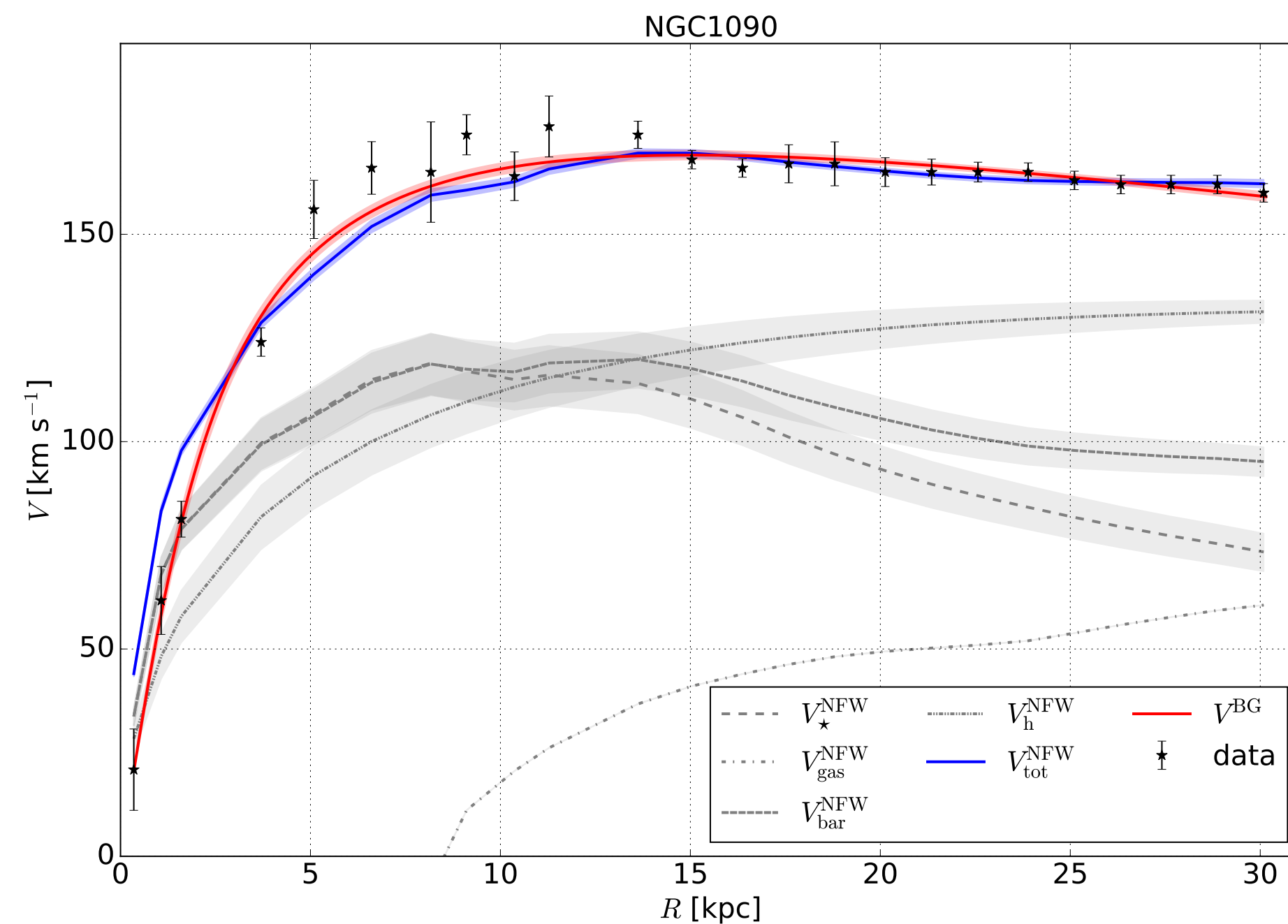
MCMC fit to external Galaxies

Velocity profiles (SPARC data)

Classical (MWC)

GR (BG)

Best fit estimates as the median of the posteriors and their 1σ level credible interval



Summary

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**From Relativistic Astrometry to Gravitational Astrometry:
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 - ☑ *Cosmological gravitational waves*
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 - ☑ *GW signatures from white dwarfs, stellar BHs and/or lensing effects*
 - ☑ *better definition of the MW rotation curves*
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 - ☑ *GW signatures from white dwarfs, stellar BHs and/or lensing effects*
 - ☑ *better definition of the MW rotation curves*
 - ☑ *what dark matter is and how is it distributed, how the Milky Way was formed and how has it been impacted by mergers and collisions?*

Many thanks for your attention!