Analytic solution of Chemical Evolution Models with contribution of Type Ia SNe in the era of Gaia DR3

P. A. Palicio
Observatoire de la Côte d'Azur, CNRS

Chemical Evolution Models (IRA)

- The **Instantaneous Recycling Approximation**:
  - IMF does not depend on time
  - Gas is well mixed at any time (IMA)
  - Stars with m<1M_{sun} live forever, m>1M_{sun} die immediately
  - Not good for iron produced by Type Ia SNe.

$$\frac{d\sigma_X(t)}{dt} = -\alpha\,\sigma_X(t) + \langle y_X \rangle (1-R)\,\psi(t)$$

Density of X-element (unknown)

Constants

SFR (known)

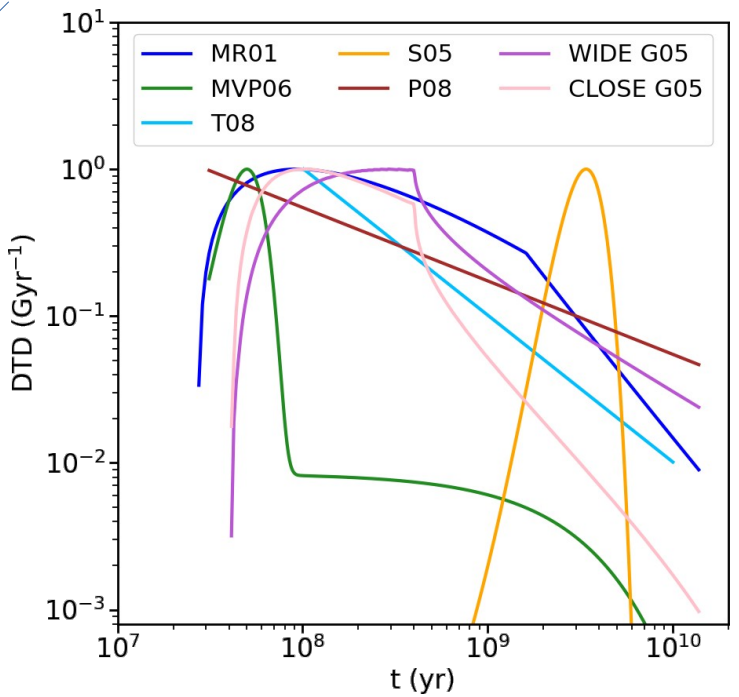
Chemical Evolution Models: beyond IRA

$$\frac{d\sigma_X(t)}{dt} = \overbrace{-\alpha\,\sigma_X(t) + \langle y_X \rangle\,(1-R)\,\psi(t)}^{\text{IRA terms}} + \overbrace{\langle m_{X,\mathrm{Ia}} \rangle\,\mathcal{R}_{\mathrm{Ia}}(t)}^{\text{Type Ia SNe term}}$$

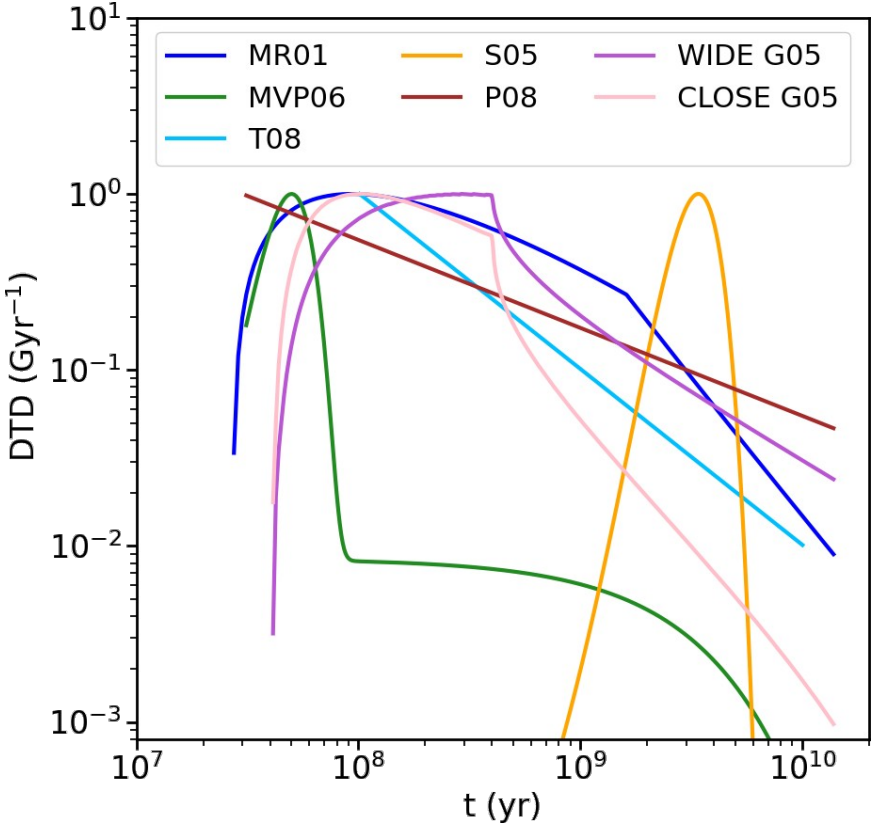
$$\mathcal{R}_{\mathrm{Ia}}(t) = C_{\mathrm{Ia}} \int_{\tau_1}^{\min(t,\tau_2)} \mathrm{DTD}_{\mathrm{Ia}}(\tau)\,\psi(t-\tau)\,d\tau$$

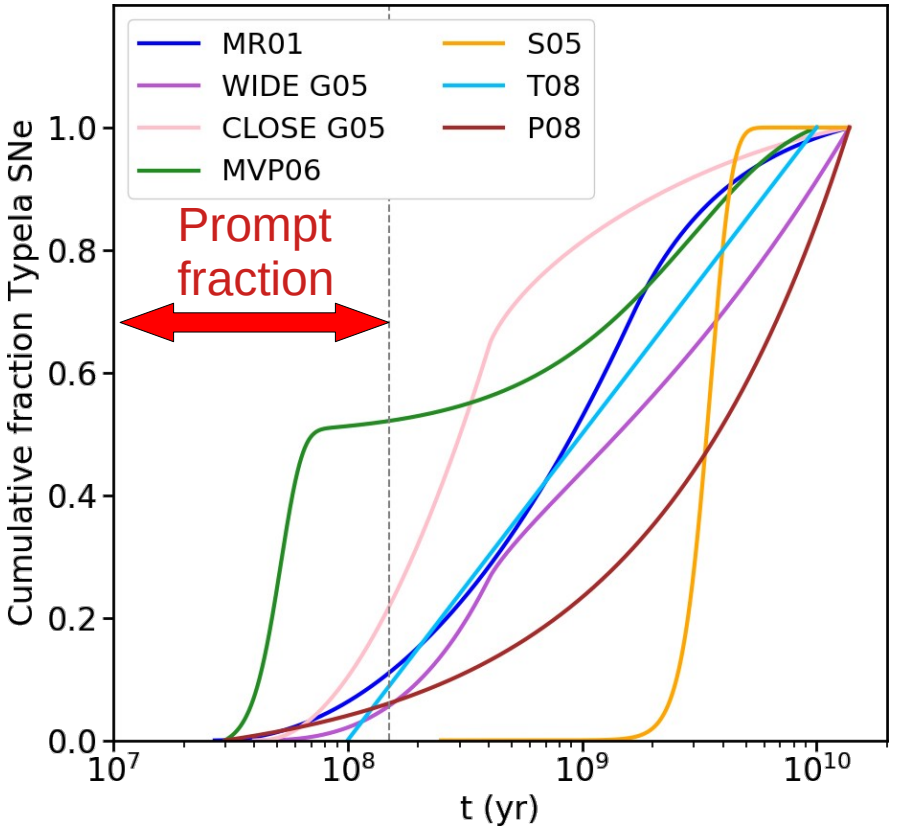
DTD: Delay-time distribution

ψ(t): Star formation rate



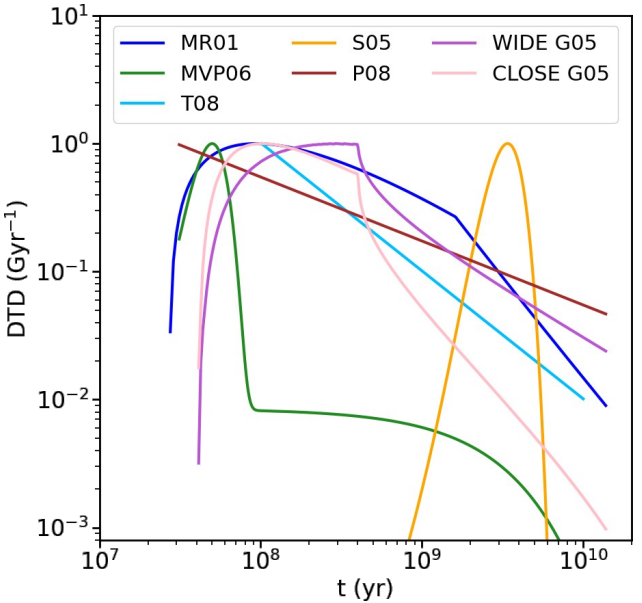
The Delay-Time Distribution formalism (DTD)





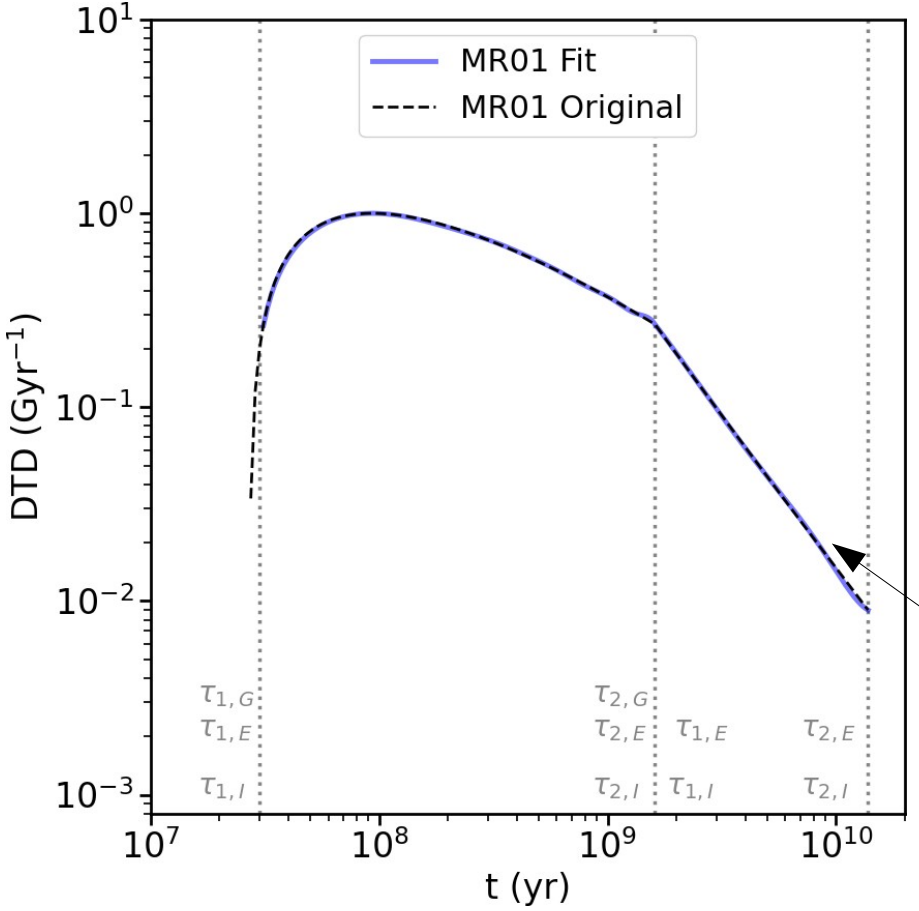
The Delay-Time Distribution formulation

Abbrev.	Reference	Scenario	Formulation	Integration
MR01	Matteucci & Recchi (2001)	Single Degenerate	Complex	Indirect
WIDE G05	Greggio (2005)	Double Deg.	Complex	Indirect
CLOSE G05	Greggio (2005)	Double Deg.	Complex	Indirect
MVP06	Mannucci et al. (2006)	Empirical	Exponential +Gaussian	Direct*
T08	Totani et al. (2008)	Empirical	1/t	Direct
P08	Pritchet et al. (2008)	Empirical	1/sqrt(t)	!
S05	Strolger et al. (2005)	Empirical	Gaussian	Direct



*Partially solved in the literature Weinberg et al. 2017, Pantoni et al. 2019, Lapi et al. 2020.

Complex DTDs? No problem!



Solution: Use functions you can integrate: Gaussians, exponentials and 1/t-like functions.

$$\sum_{i=1}^{N_G} A_{G,i} \exp\left(-\frac{(t-\tau_i')^2}{2\sigma_i'^2}\right) + \sum_{i=1}^{N_E} A_{E,i} \exp\left(\frac{-t}{\tau_{D,i}}\right) + \sum_{i=1}^{N_I} A_{I,i} \frac{\tau_I}{t-\tau_0}$$

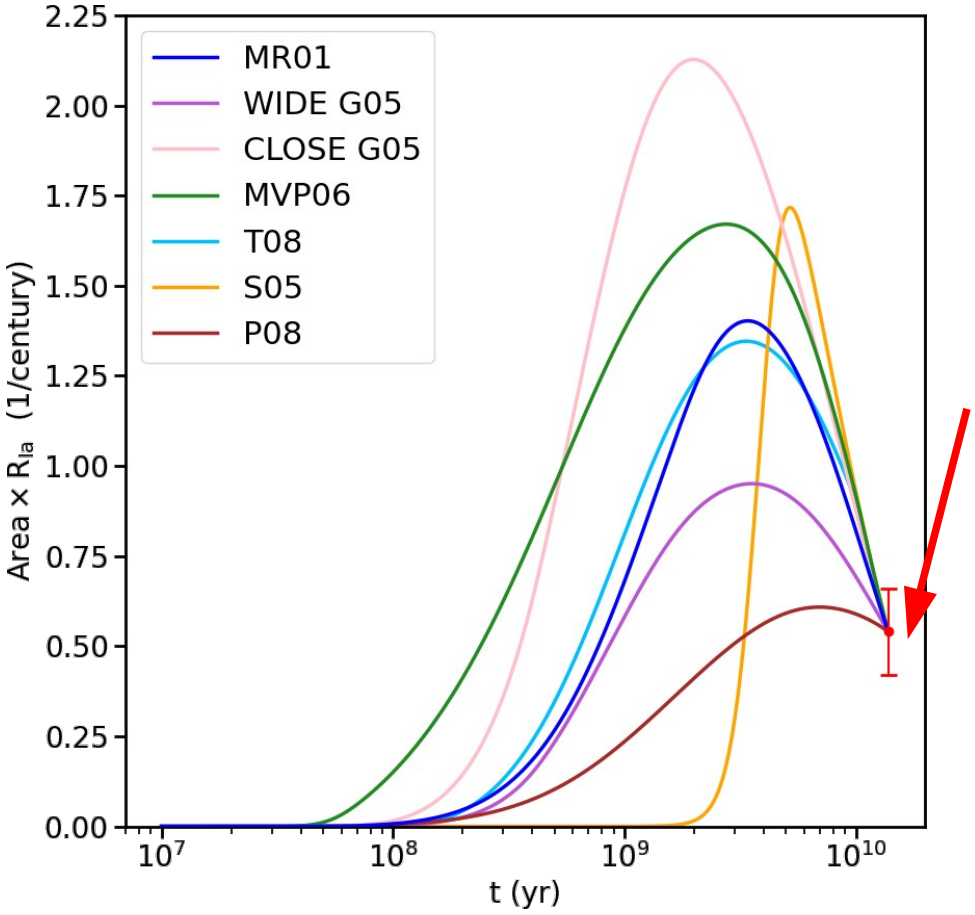
Restricted Least-Squares fitting (accounts for the continuity, smoothness, peaks… of the original DTD).

There are two curves here!

1-infall predicted Type Ia SN rates

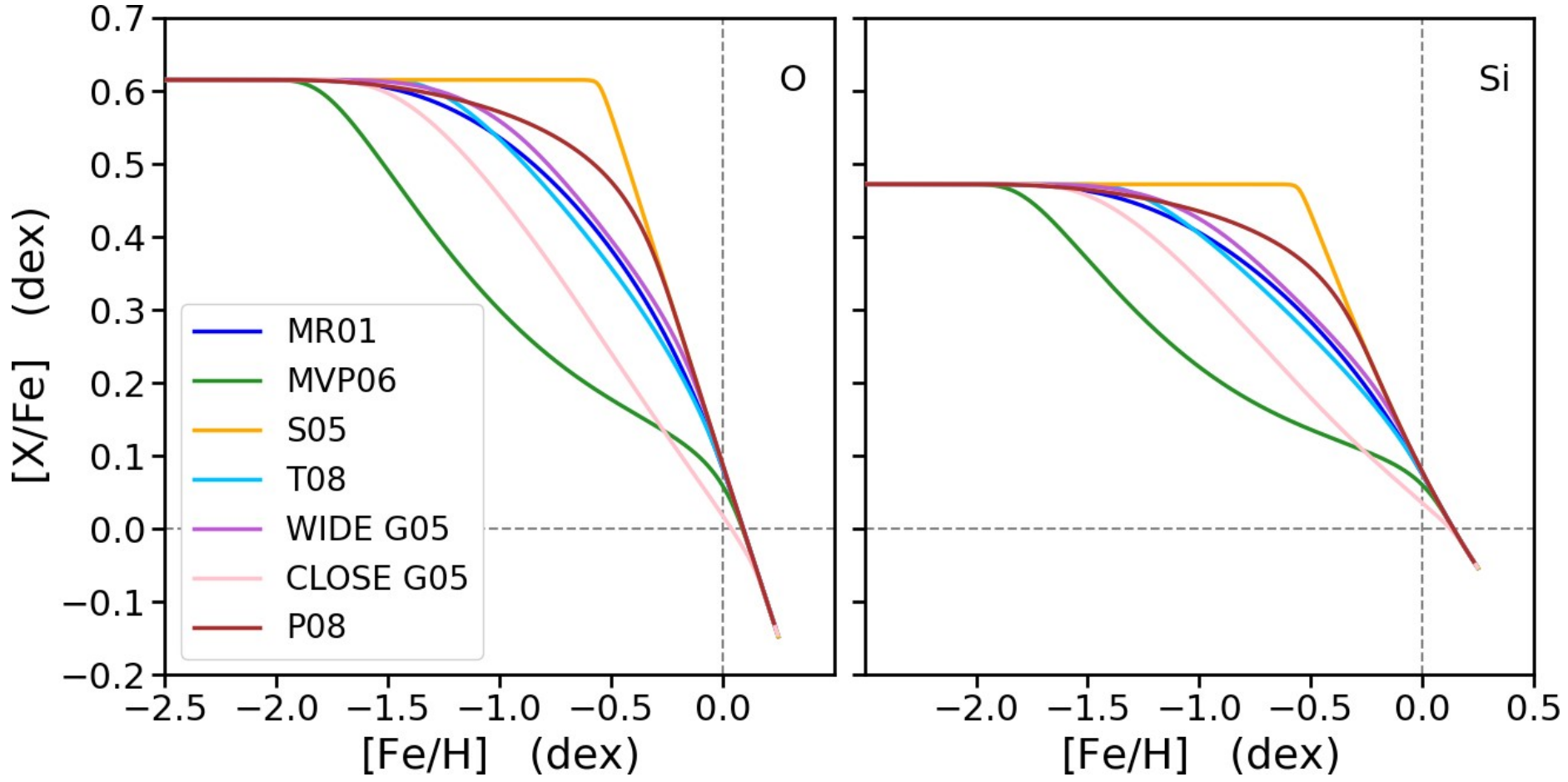
- Model parameters and infall of **Vincenzo et al. (2017)**
- [Si/Fe] and [O/Fe] vs [Fe/H] in 3<R<20 kpc.
- $[X/\mathrm{Fe}] = \log_{10}\left(\frac{\sigma_X}{\sigma_{\mathrm{Fe}}}\right) - \mathrm{SV}^X_{\mathrm{Fe}}$
- **Exponential infall** gives SFR(t) of the form:

$$\psi(t) \sim e^{-t/\tau} - e^{-\alpha t}$$

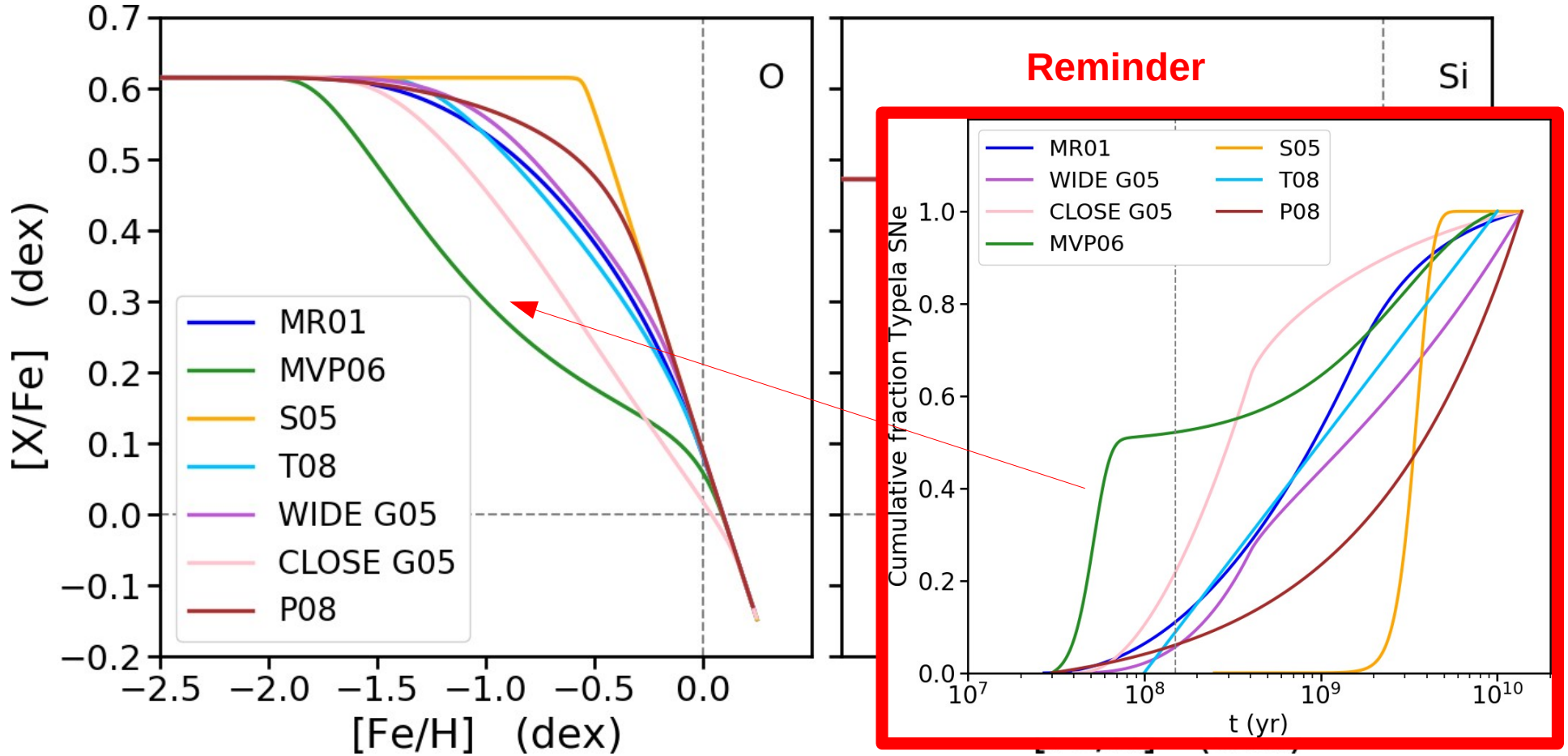


Present-day SN Type Ia rate (**Li et al. 2011**)

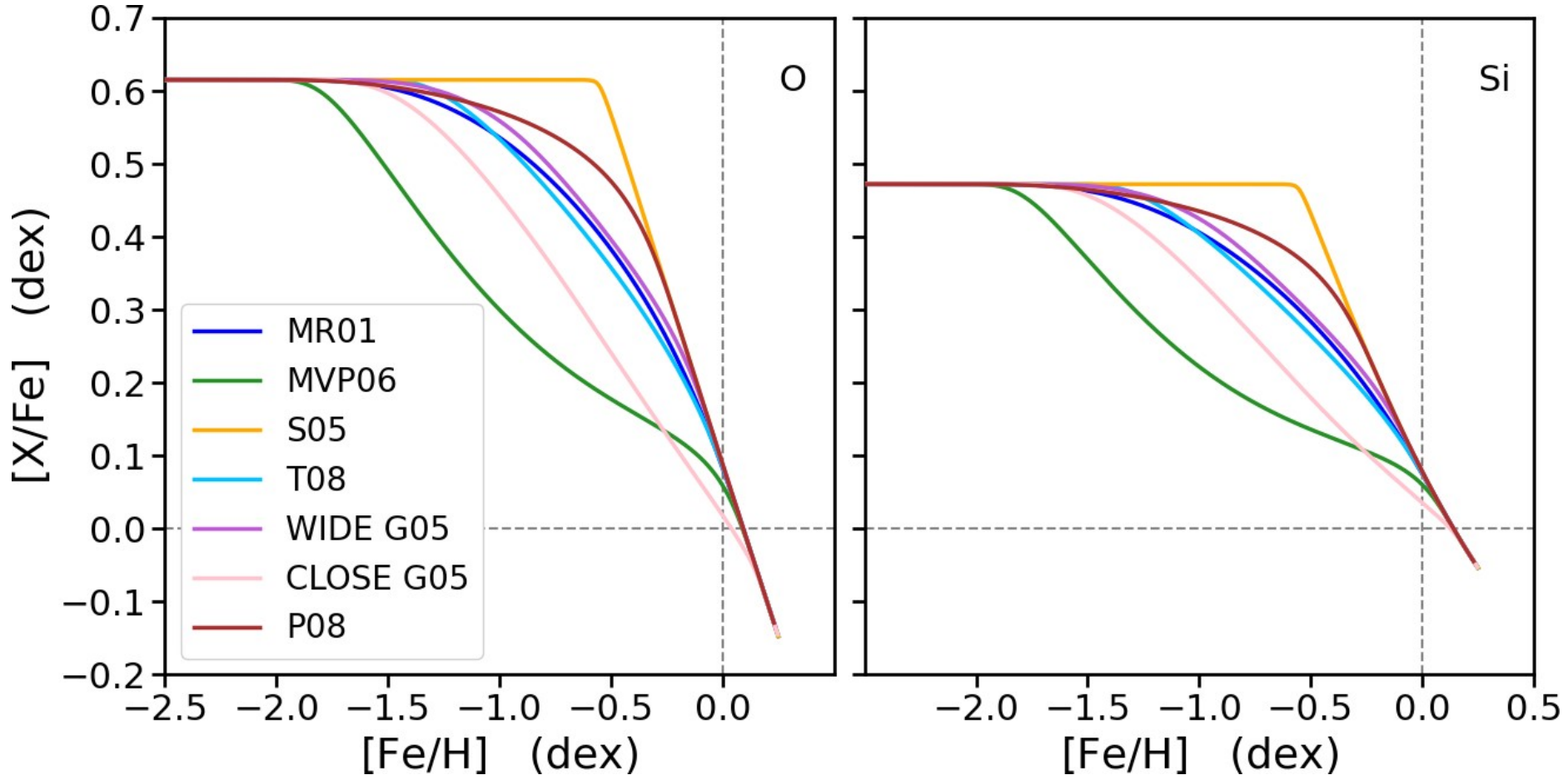
1-infall Chemical Evolution Model



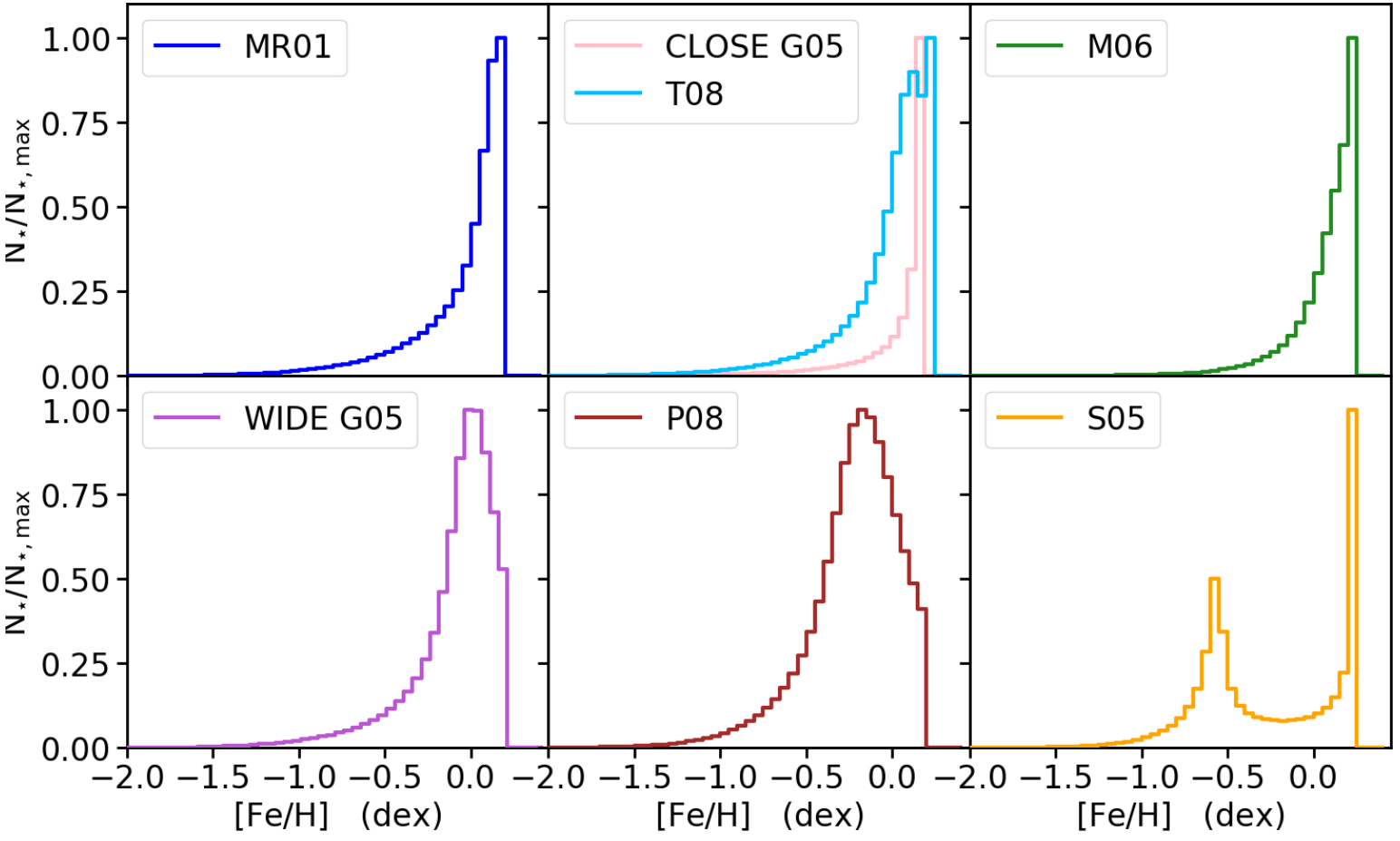
1-infall Chemical Evolution Model



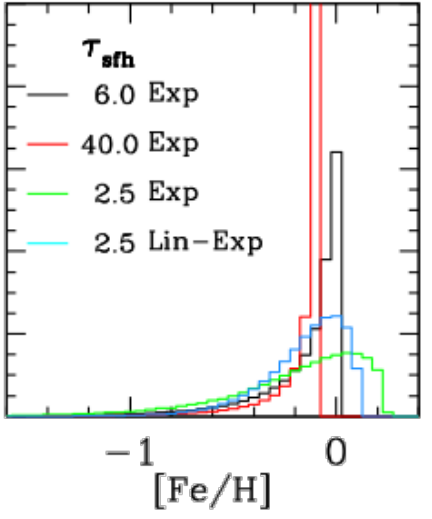
1-infall Chemical Evolution Model



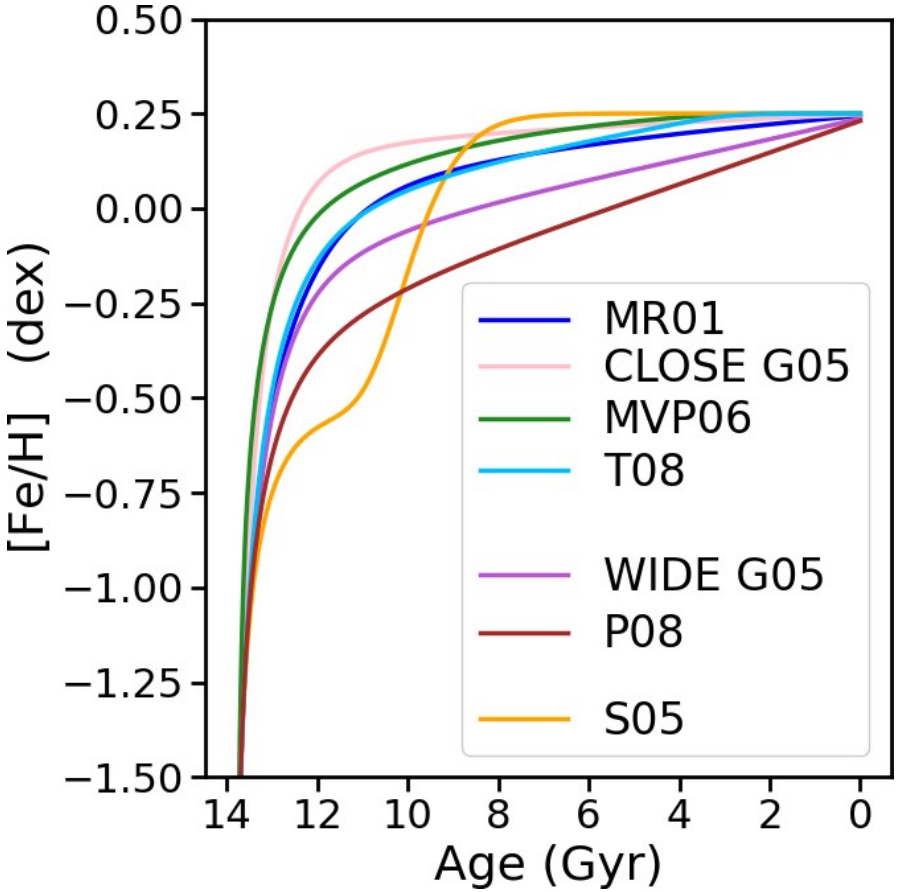
1-infall Metallicity distribution functions

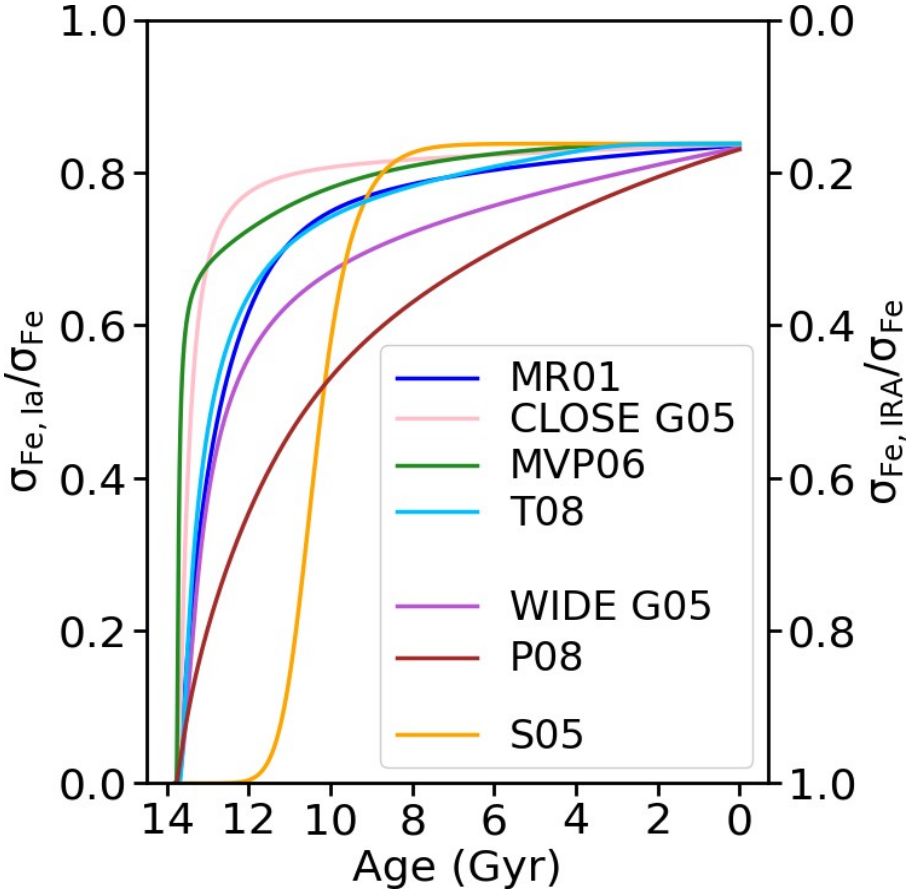


Similar shape as **Weinberg et al. (2017),** who considered an exponential DTD

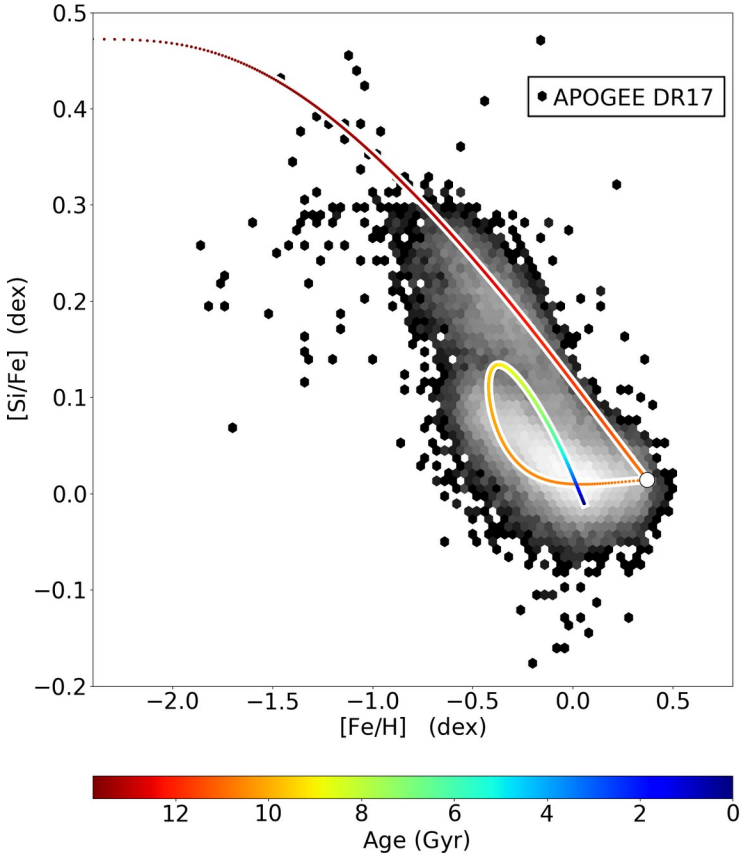


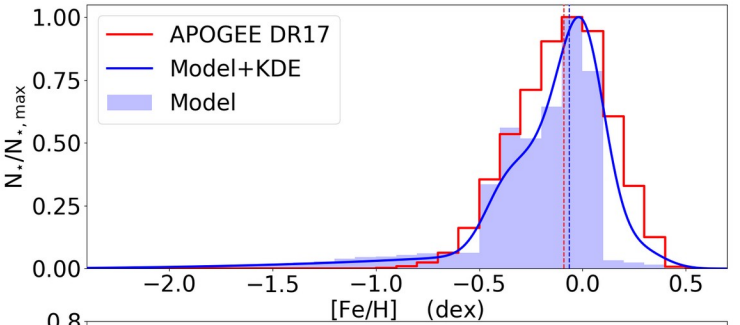
1-infall model: age-metallicity relation

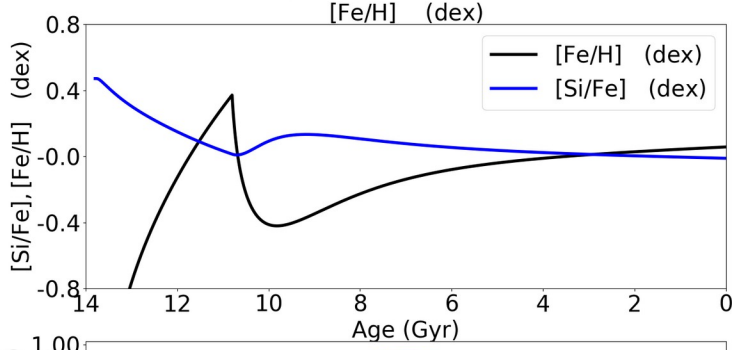


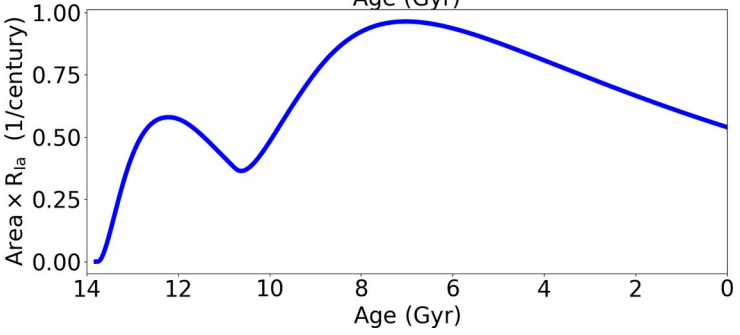


2-infall model: chemical evolution



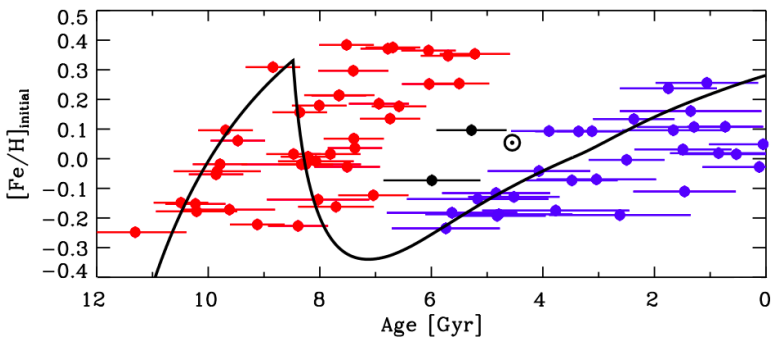


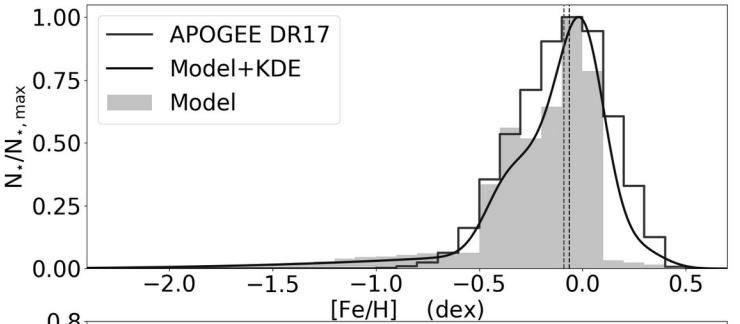


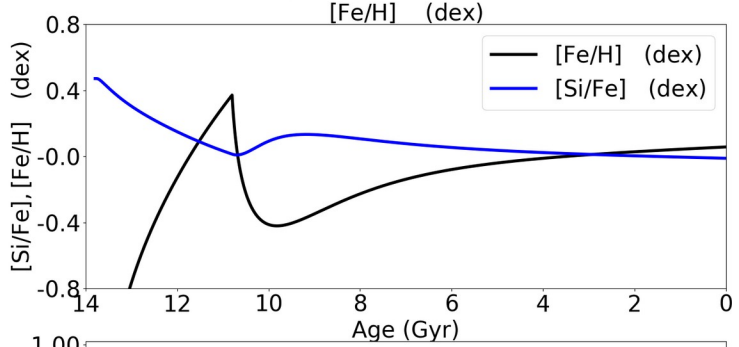


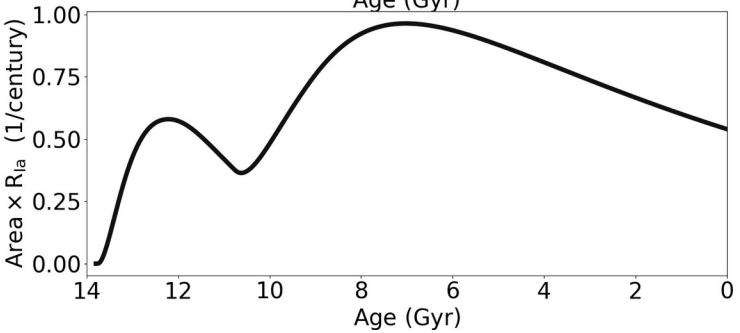
2-infall model: chemical evolution

Nissen et al. 2020

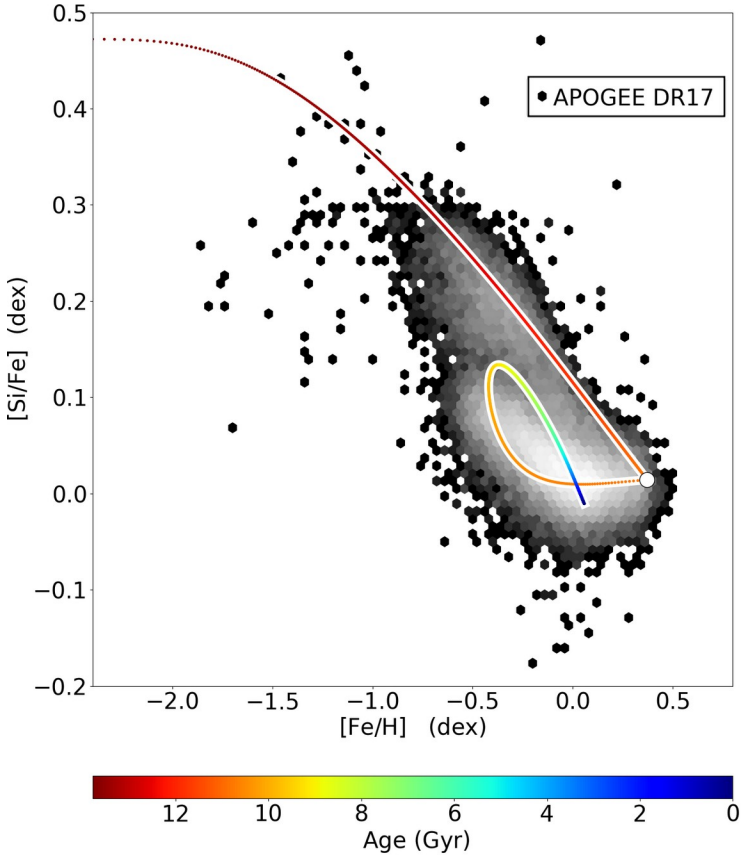


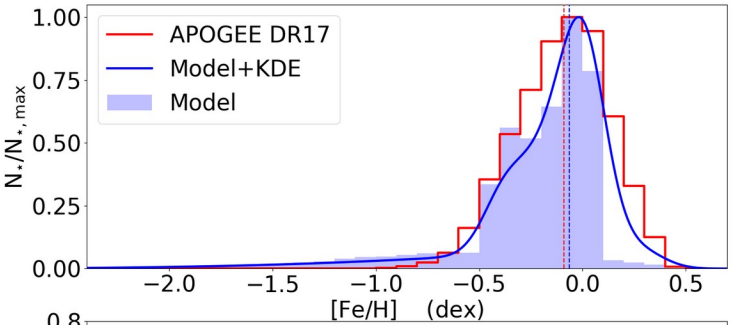


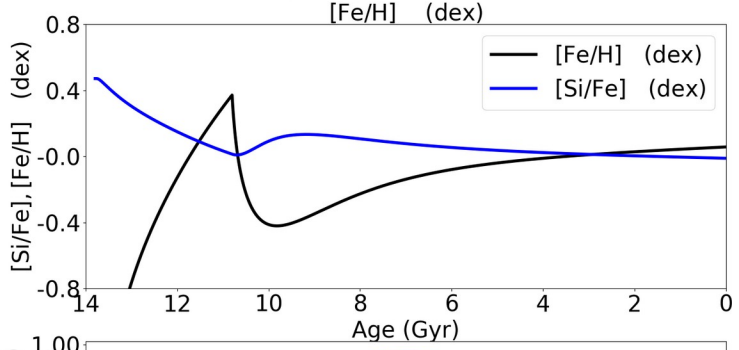


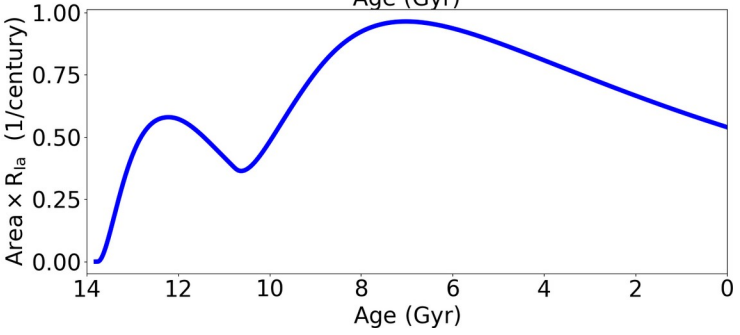


2-infall model: chemical evolution

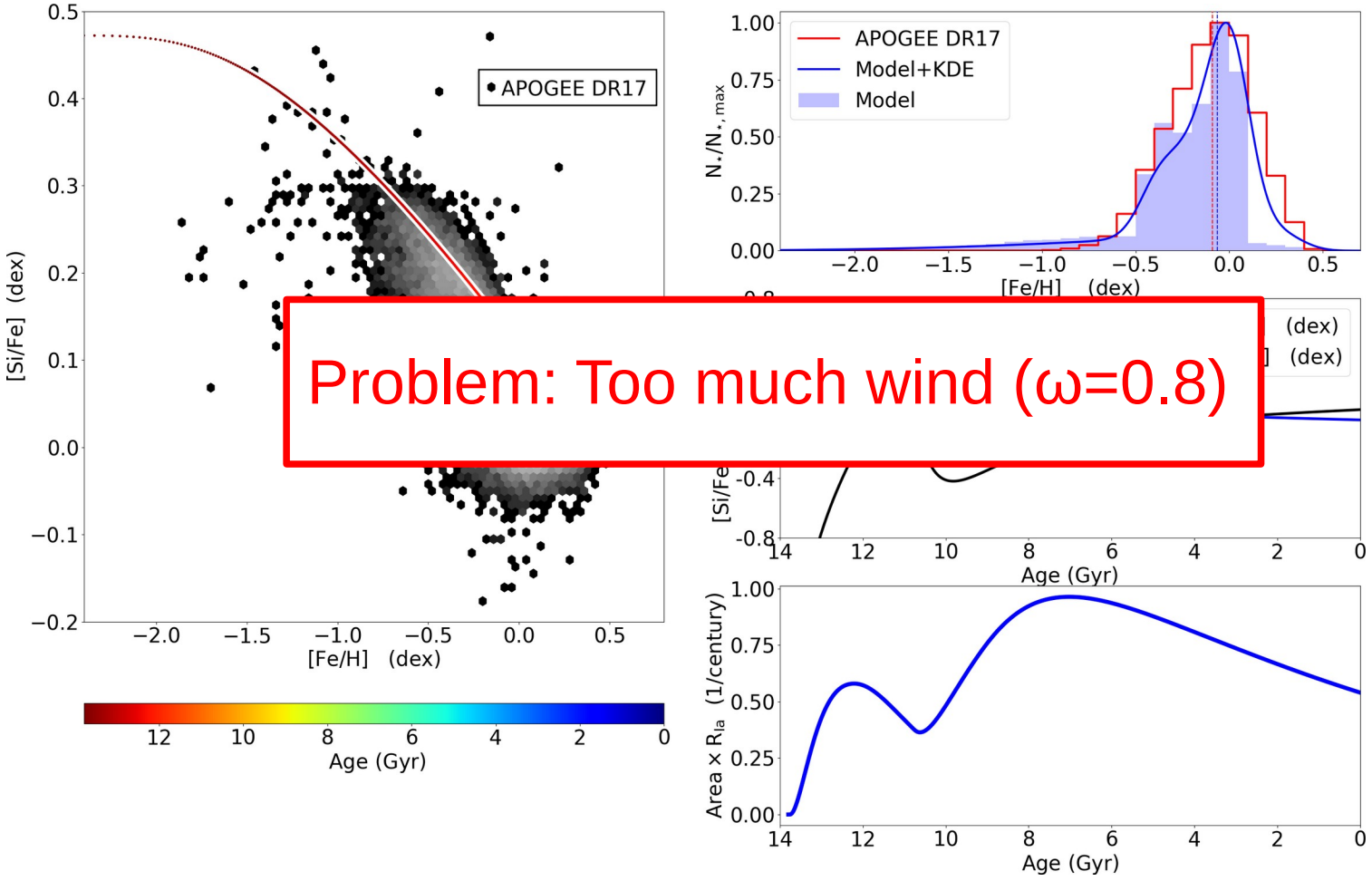








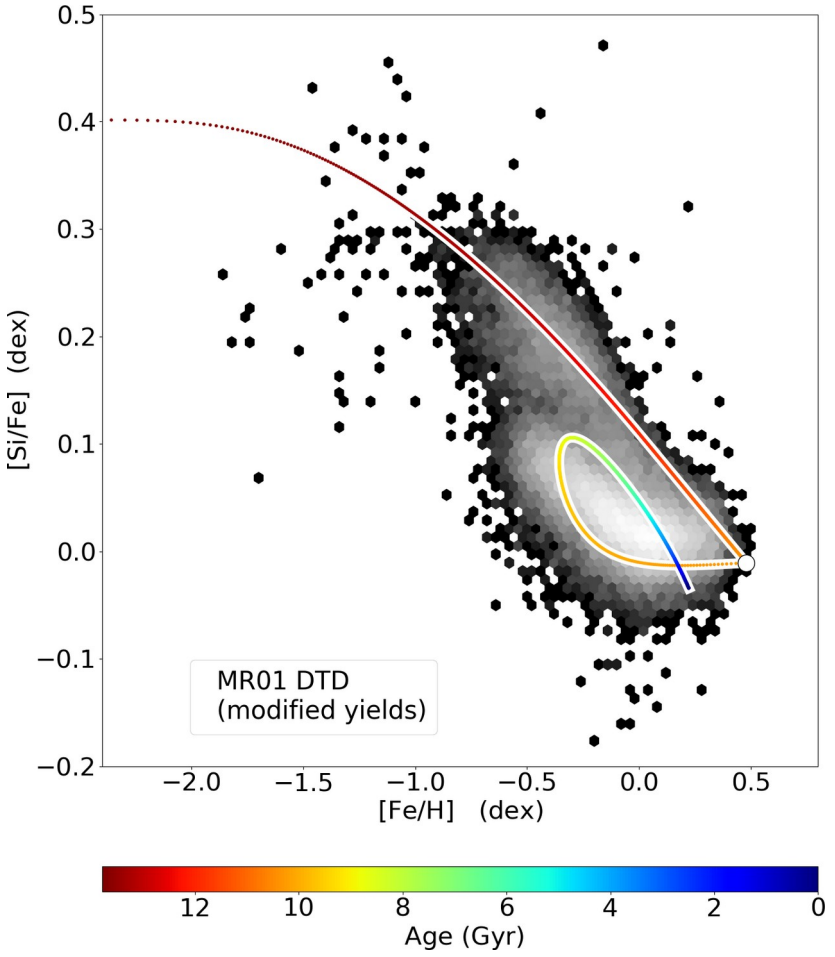
2-infall model: chemical evolution



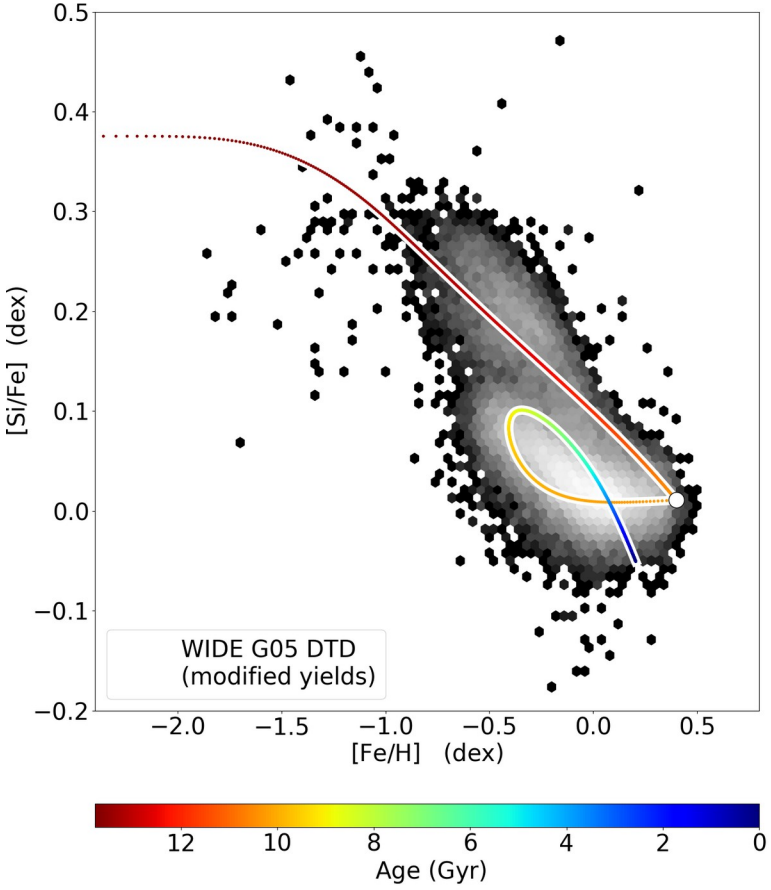
Problem: Too much wind (ω=0.8)

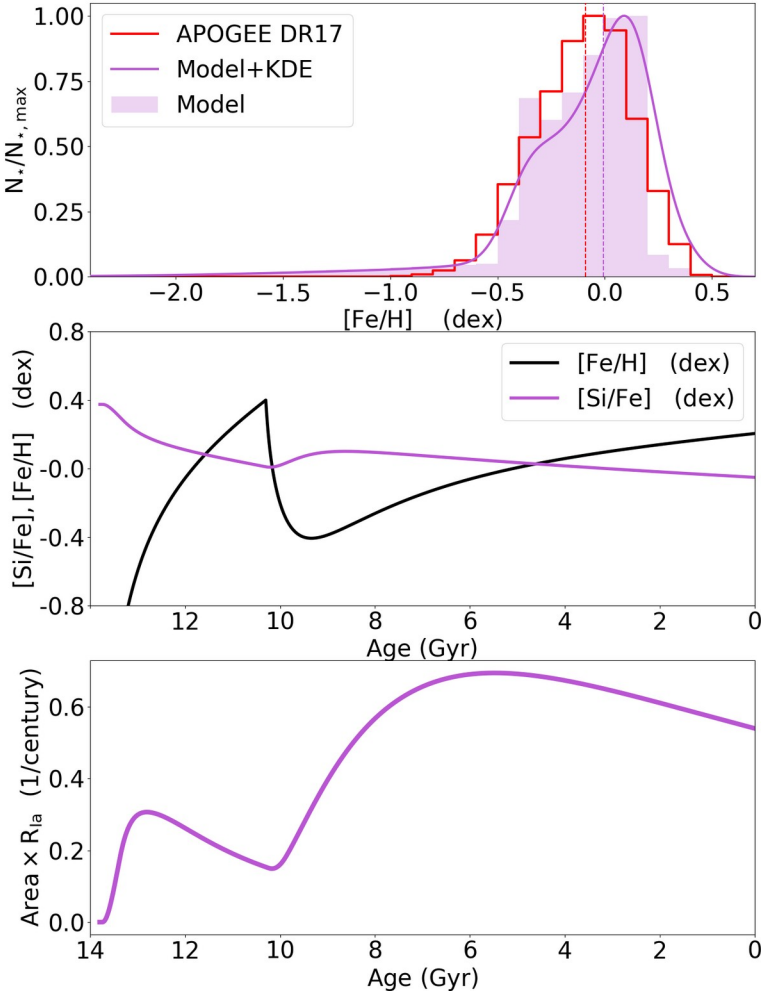
2-infall model: chemical evolution (modified yields)

Now ω=0.2

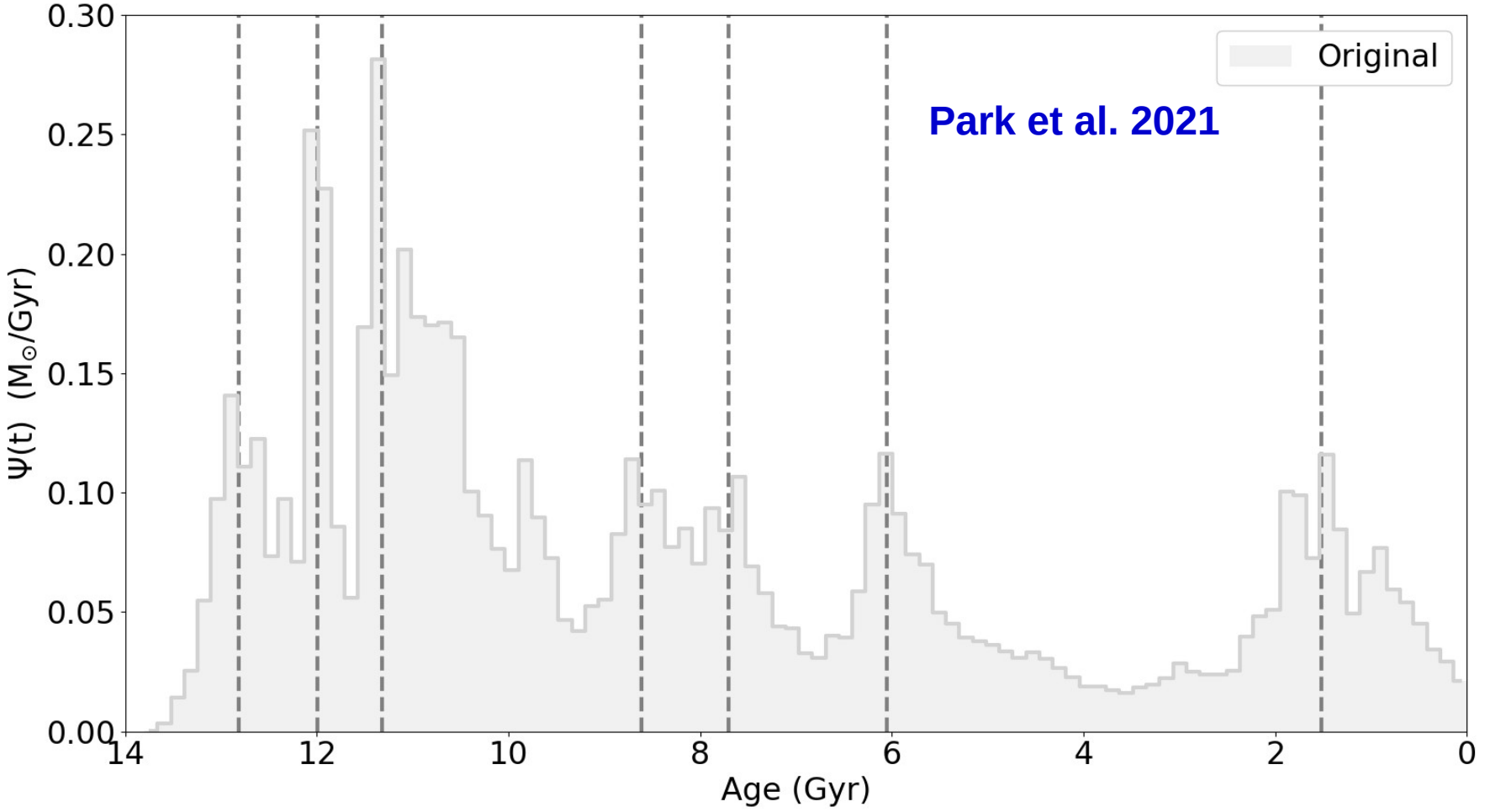


2-infall model: chemical evolution



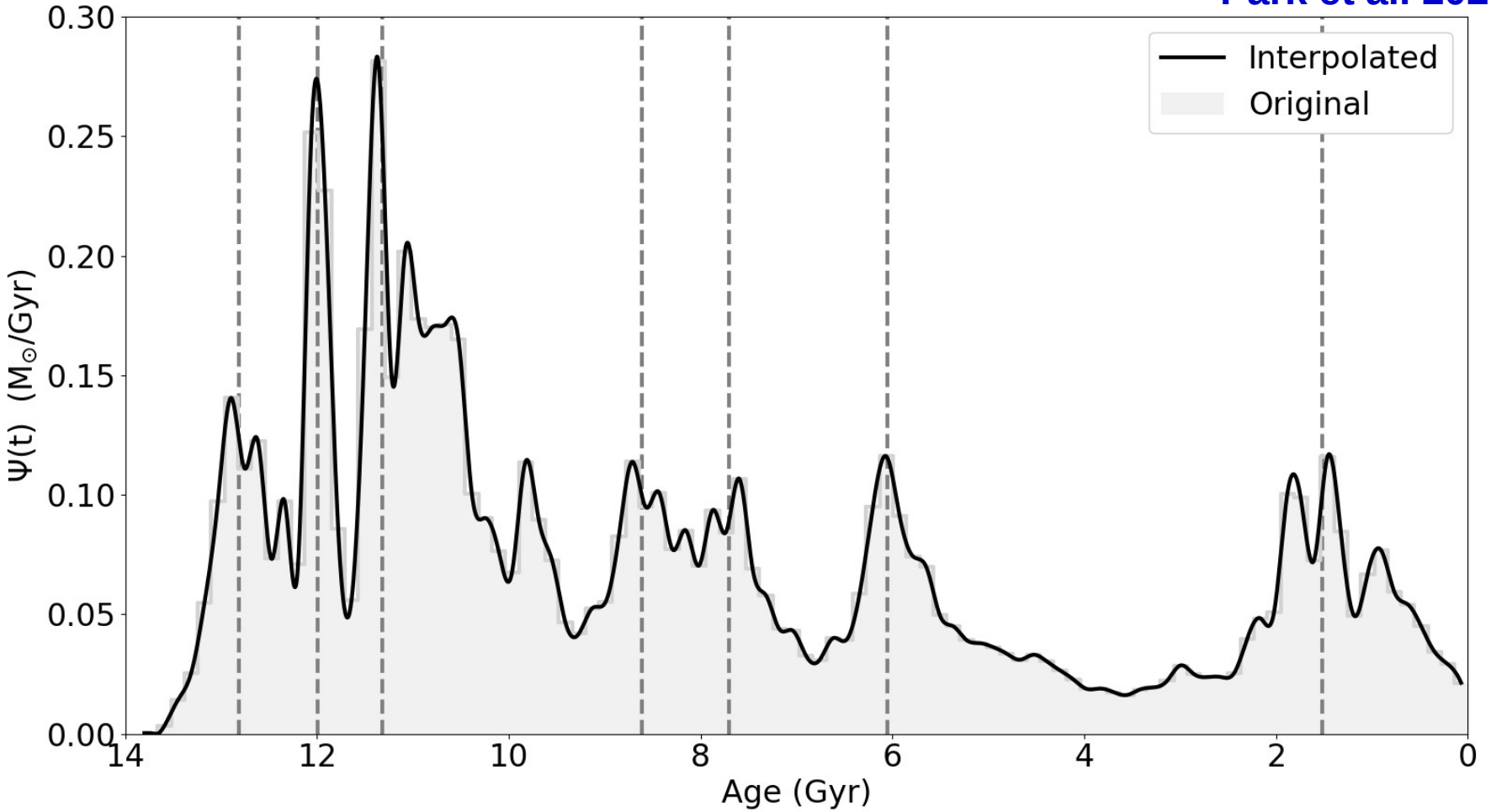


Chemical modeling of simulated galaxies



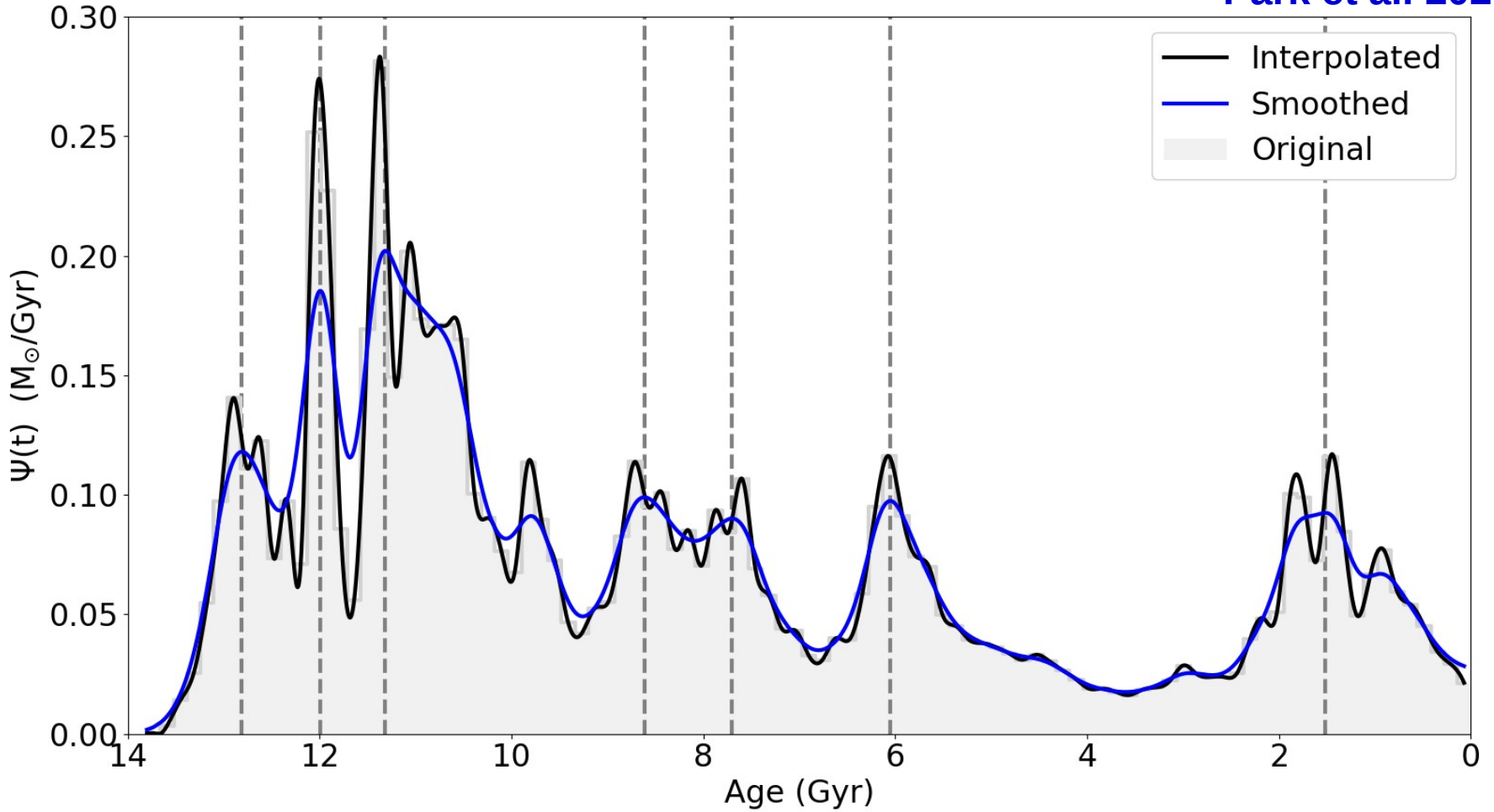
Chemical modeling of simulated galaxies

Park et al. 2021



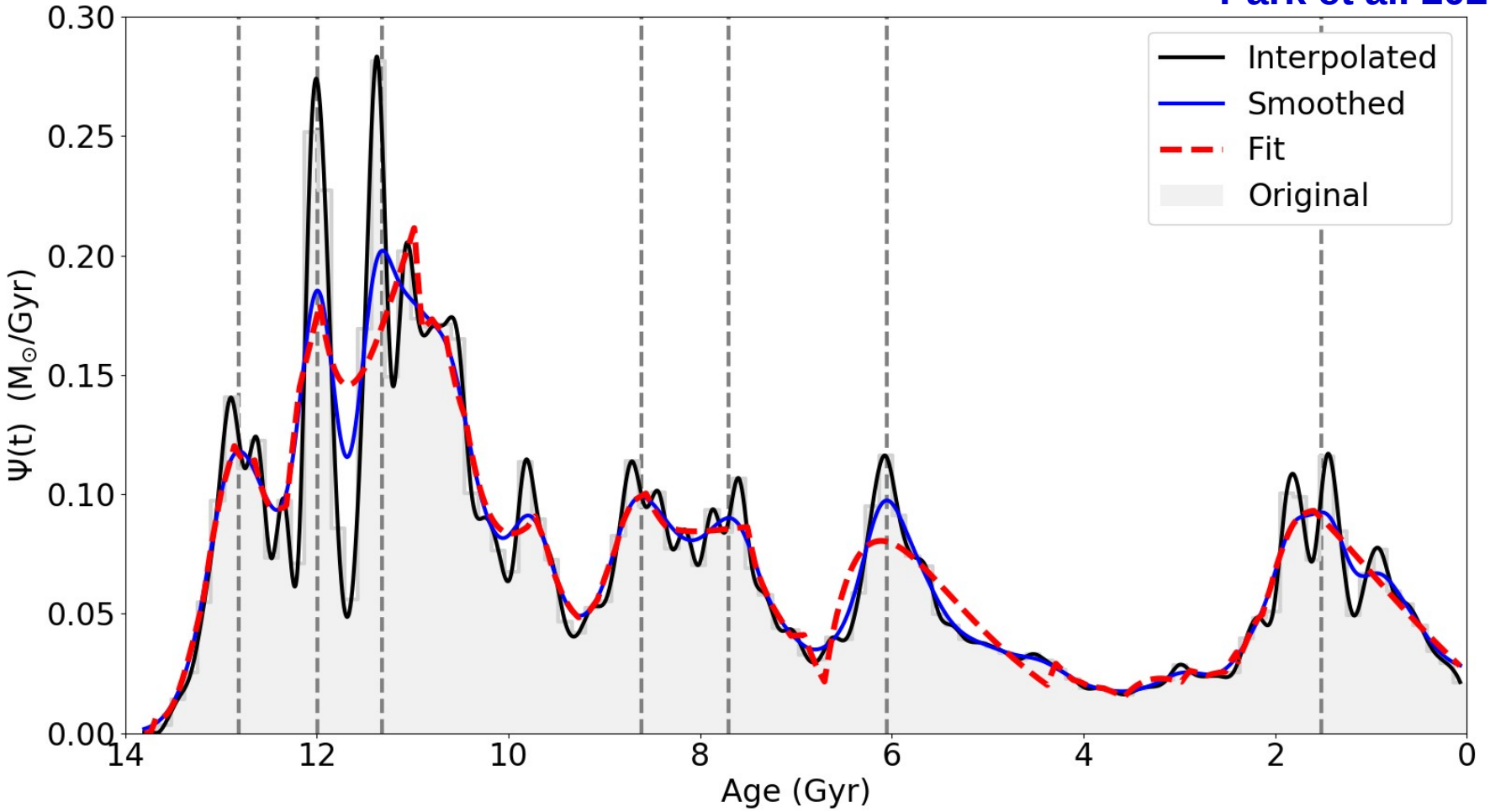
Chemical modeling of simulated galaxies

Park et al. 2021

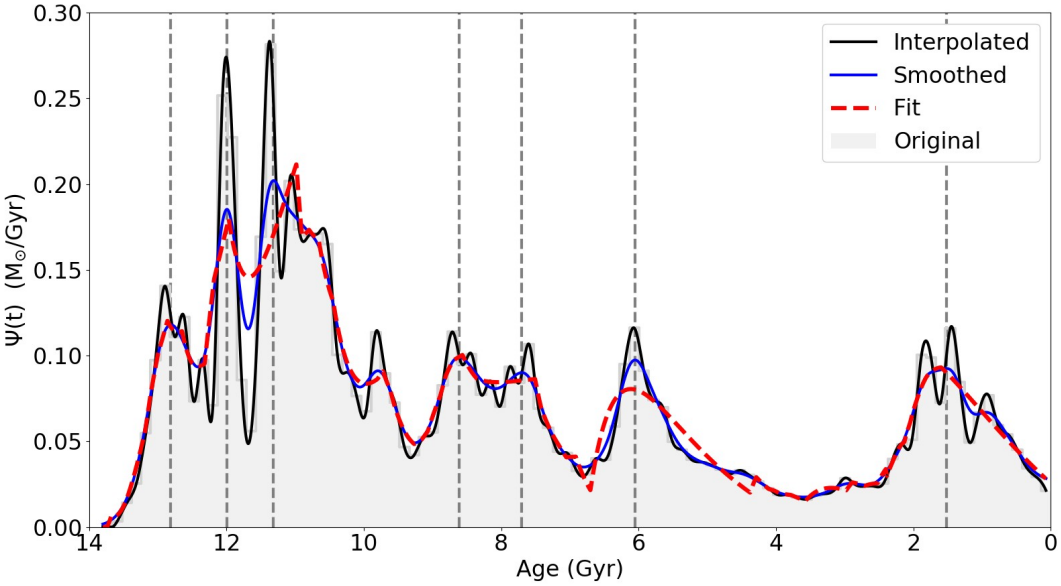


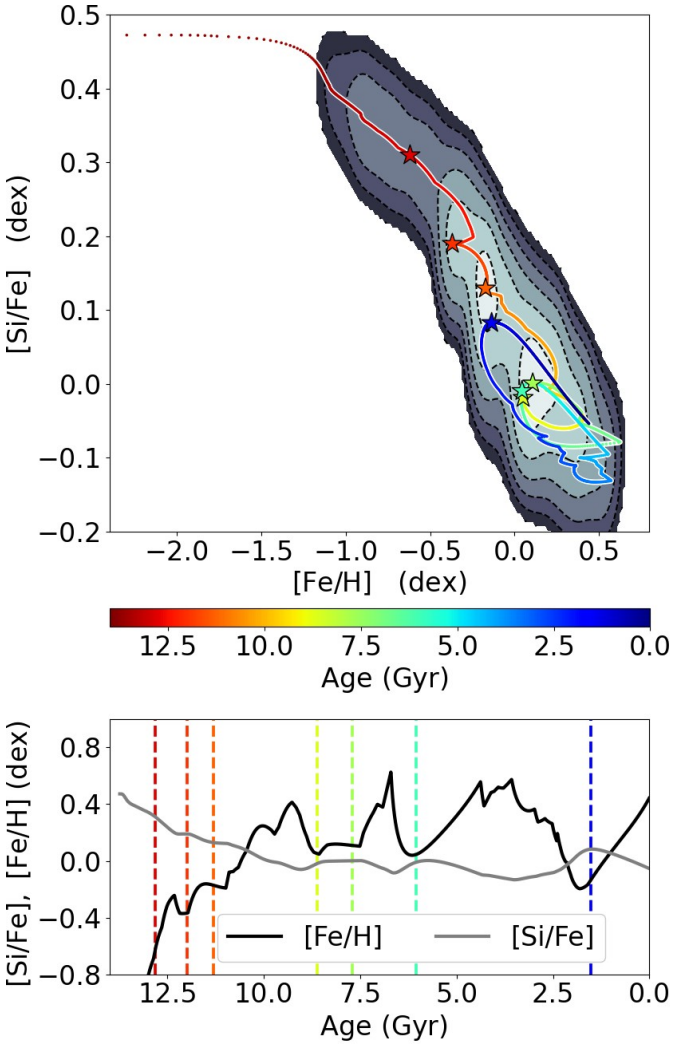
Chemical modeling of simulated galaxies

Park et al. 2021

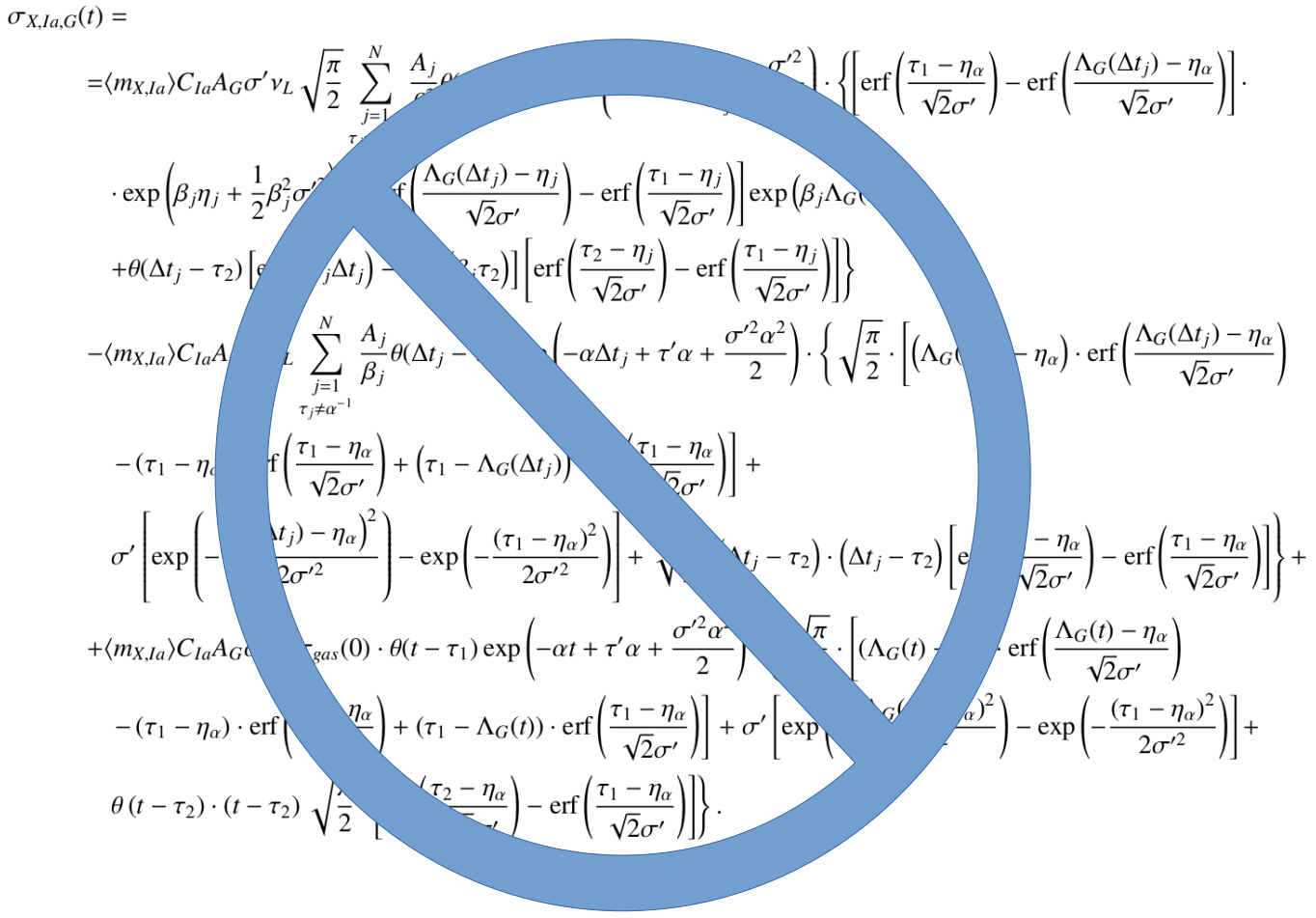


Chemical modeling of simulated galaxies



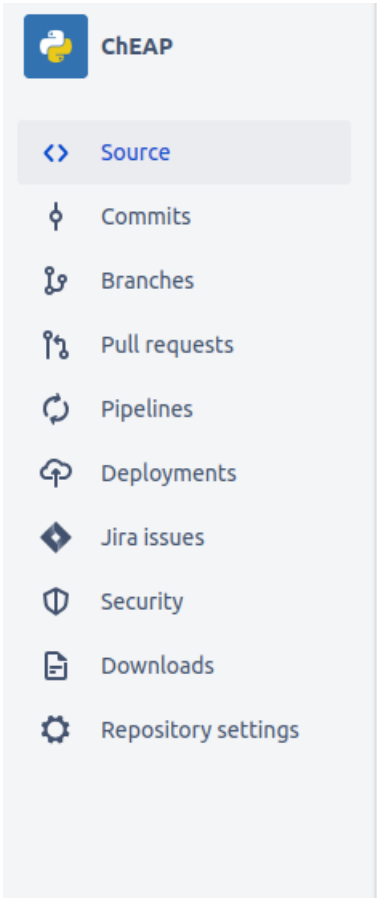


The solution (one third)



- Too long to be evaluated term by term.
- Available in Appendix B of **Palicio et al. (2023b)**
- Distributed as a python library

ChEAP



https://bitbucket.org/pedroap/cheap/src/master/

README.md

ChEAP: Chemical Evolution Analytic Package

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The **ChEAP (Chemical Evolution Analytic Package)** code implements the analytic solution to the Chemical Evolution Model with TypeIa SNe presented in Palicio et al. (accepted., https://arxiv.org/abs/2304.00042, hereafter P23). The functions required to compute the solution are contained in the `CheapTools.py` file, which should be imported as a Python library. We include also the `RandomTester.py` file to illustrate, with a random-parameter chemical evolution model, the accuracy of our analytic solution compared to the numerical integration.

ChEAP is a Python code created by P.A. Palicio and included in the paper "Analytic solution of Chemical Evolution Models with Type Ia SNe" (P23). If you make use of ChEAP in your work, please consider including the proper citation to this paper. For any question about ChEAP, please do not hesitate to contact the author at ***pedro.alonso-palicio(at)oca.eu***

1. The Chemical Evolution equation

In this section, we provide a brief overview of the equations we solved analytically. This is not intended to be an exhaustive description of the chemical equation, but rather a general introduction to justify the notation used in the following sections, which focus more on the solution itself. Furthermore, due to the limitations of the markup language, we cannot illustrate these equations with clear notation. Thus, we refer interested readers to the main paper of this code (P23), as well as to the review by Matteucci (2021) and Section 2 of Vincenzo et al. (2017), which motivated this work.

Some notes about ChEAP

- Does not require rare python libraries (just the usual numpy, scipy, etc.)
- Does not require installation, just copy the script in your working directory.
- All the solution verification tests succeed so far→ the few numerical discrepancies found can be attributed to numeric effects that do not invalidate our solution.
- Some overflows reported when the infall timescale is small → computation limit. We propose the usage of higher precision floats, or just handle with the analytic solution.
- Read the *Readme* file found in the repository.
- The model, ans so its solution, assumes constant SFE.
- For problems, suggestions, help… do not hesitate to contact me (email address in Palicio et al. 2023b)

Conclusions & Future work

- We have computed the **analytic solution** of chemical evolution models with **Type Ia SNe** contribution for **different of DTDs**., distributed in the **ChEAP** python code. This constitutes a useful tool for interpret the observations.
- Different scenarios for the DTD can explain the APOGEE DR17 data, but we cannot conclude which one is better. **More constraints are needed** (ages, multiple elements, independent determination of the infall parameters...)
- An **excess of super-solar metallicity** stars is observed w.r.t. APOGEE DR17.
- We presented a preliminary work for **modelling the chemistry of simulated galaxies** generated without chemical information.

- Model the chem. Data of large surveys, especially GSP-Spec Gaia DR3.
- Analytic solutions for **more DTDs** can be computed (and so I did).
- Modifications of the chem. Equations, while having analytic solution, to account for **radial migration** seem plausible.
- Multi-zone implementation: Can we model the effect of the **spiral arms**?