Global coronal models driven with Alfvén and kink waves

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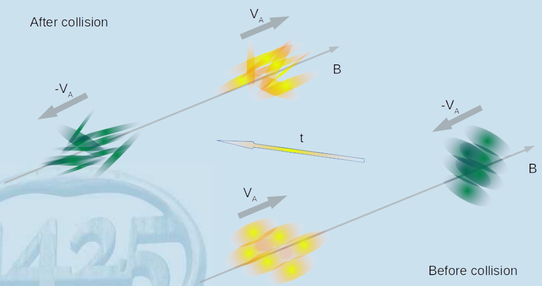
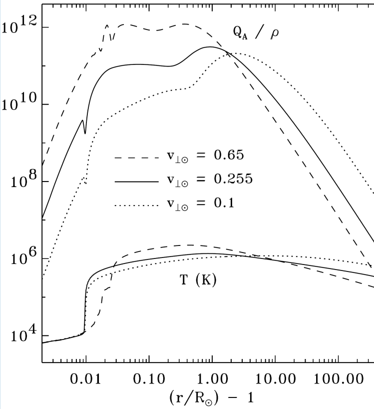
Alfvén wave heating models

Alfvén wave turbulence:
counterpropagating Alfvén waves collide and create turbulence
→ successful in heating solar atmosphere and wind





Van Doorsselaere et al. (2020) Cranmer et al. (2007)

AWSOM

Van der Holst et al. (2014): AWSOM = Alfvén Wave Solar Model

- Starts from MHD/2-fluids
- adds two extra equations for Alfvén wave energy $w_{\mathrm{A}}^{\pm}$: advection, reflection, cascade
- includes extra force due to Alfvén wave pressure P_{A}
- includes extra heating terms due to cascade $\Gamma_{\mp} w_{\pm} \sim \sqrt{w_{\mp}} w_{\pm}$

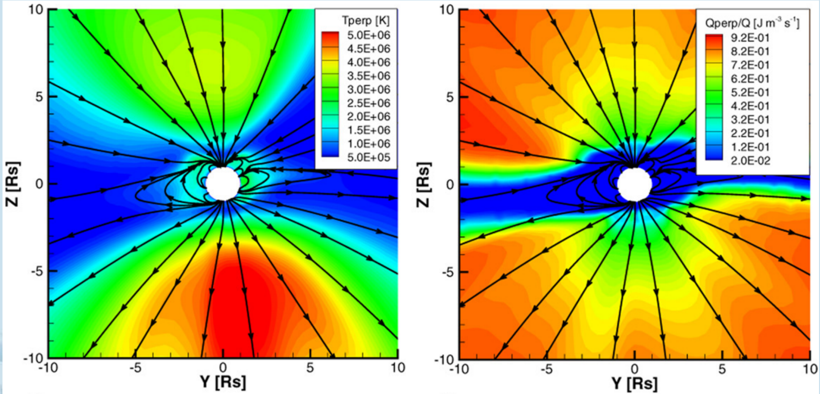
$$\frac{\partial}{\partial t}\left(\frac{P_i + P_e}{\gamma - 1} + \frac{\rho u^2}{2} + \frac{B^2}{2\mu_0} + w_+ + w_-\right) + \nabla \cdot \left[\left(\frac{\rho u^2}{2} + \frac{\gamma (P_i + P_e)}{\gamma - 1} + \frac{B^2}{\mu_0}\right)\mathbf{u} - \frac{\mathbf{B}(\mathbf{u}\cdot\mathbf{B})}{\mu_0}\right] + \nabla \cdot \left[\left(w_+ + w_- + P_A^{\mathrm{full}}\right)\mathbf{u} + (w_+ - w_-)\mathbf{V}_A\right] + Q_{\mathrm{noncons}} = -\nabla \cdot \mathbf{q}_e - Q_{\mathrm{rad}} - \rho \frac{GM_\odot}{r^3}\mathbf{r}\cdot\mathbf{u}, \tag{29}$$

$$Q_{\mathrm{noncons}} = \frac{\rho}{2}\left[\mathbf{z}_+ \cdot (\mathbf{z}_- \cdot \nabla)\mathbf{u} + \mathbf{z}_- \cdot (\mathbf{z}_+ \cdot \nabla)\mathbf{u} + \frac{\mathbf{z}_- \cdot (\mathbf{z}_+ \cdot \nabla)\mathbf{B} - \mathbf{z}_+ \cdot (\mathbf{z}_- \cdot \nabla)\mathbf{B}}{\sqrt{\mu_0 \rho}}\right]. \tag{30}$$

$$\frac{\partial w_\pm}{\partial t} + \nabla \cdot [(\mathbf{u} \pm \mathbf{V}_A) w_\pm] + \frac{w_\pm}{2}(\nabla \cdot \mathbf{u}) = \mp \mathcal{R}\sqrt{w_- w_+} - \Gamma_\pm w_\pm, \tag{36}$$

AWSOM

Van der Holst et al. (2014)

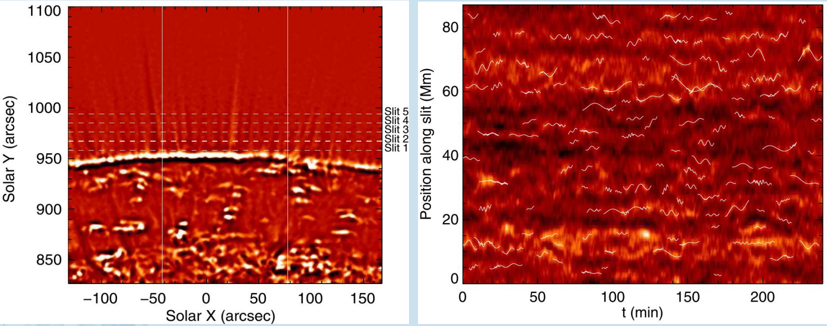


Van der Holst (private communication): problems in open-field regions, not enough driving of wind

Downs (private communication): insufficient heating in low corona

Transverse waves in coronal plumes

Transverse kink waves present in (magnetically open) coronal plumes:
(also source of fast solar wind)



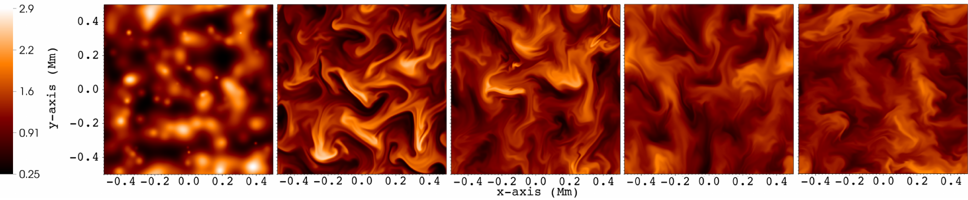
Thurgood et al. (2014)

See also Tomczyk et al. (2007), Morton et al. (2015, 2016, 2019)

Uniturbulence in inhomogeneous corona

Magyar et al. (2017): 250 Gaussian density enhancements ("plumes"), drive with varying, transverse polarisation. No reflected waves.

Overview



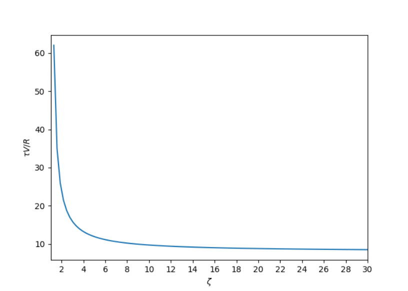
Non-linear damping of kink waves:
uniturbulence (= turbulence from unidirectional waves)

Uniturbulence damping

Van Doorsselaere et al. (2020): non-linear damping of propagating kink waves

$$\tau = \sqrt{5\pi}\frac{R}{V}\frac{2(\zeta+1)}{|\zeta-1|} = \sqrt{5\pi}\frac{P}{2\pi a}\frac{2(\zeta+1)}{|\zeta-1|}.$$

with density contrast $\zeta = \rho_{\rm i}/\rho_{\rm e}$, velocity amplitude V, maximal displacement $\eta = aR$, period P, radius R



Examples:

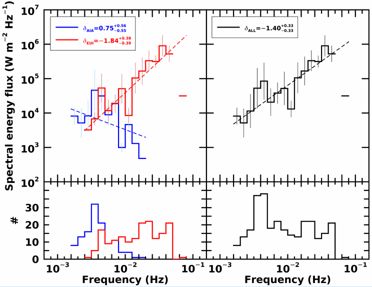
- Pant et al. (2019): velocity amplitude $V = 22\mathrm{km/s}$, radius $R = 250\mathrm{km}$ $\rightarrow \tau \sim 180\mathrm{s}$ ($\zeta = 3$)
- plumes with radius $R = 1\mathrm{Mm}$ and driver amplitude $V = 4\mathrm{km/s}$: $\rightarrow \tau \sim 3960\mathrm{s}$ ($\zeta = 3$)

Heating by kink waves

Lim et al. (2023, 2024, see poster):

EUI statistical study of kink waves

$\rightarrow$ lot of energy in high frequency ($\sim 10^4$ W/m^2)



Can kink waves heat the corona, in conjunction with Alfvén waves?

Extend AWSOM, including kink wave heating and uniturbulence

$\rightarrow$ UAWSOM (Uniturbulence and Alfvén Wave Solar Model)

$\rightarrow$ GOAL: derive energy evolution equation for kink waves

Q-variables

Van Doorsselaere et al. (2024): Introduce new Q-variables

$$\vec{Q}^{\pm} = \vec{V} \pm \alpha\vec{B},$$

($\vec{V}$ is velocity, $\vec{B}$ is magnetic field, α is parameter)

→ variation on Elsässer variables

($\vec{Z} = \vec{V} \pm \vec{B}/\sqrt{\mu\rho}$, i.e. $\alpha = 1/\sqrt{\mu\rho}$)

→ $\alpha\vec{B}$ is phase speed of wave

Allows to track other waves than Alfvén waves

e.g. kink waves in inhomogeneous plumes

Split between propagating up or down

→ Generalisation of: characteristics, Riemann invariants, Frieman-Rotenberg, Elsässer variables

MHD with Q

Van Doorsselaere et al. (2024): rewrite MHD with Q-variables $\left(\Delta\alpha^2 = \alpha^2 - \frac{1}{\mu\rho}\right)$

$$
\begin{aligned}
\frac{D^\mp}{Dt}\vec{Q}^\pm \mp \left(\frac{\vec{Q}^+ - \vec{Q}^-}{4}\right)\frac{D^\mp}{Dt}\ln\rho\alpha^2 = \\
- v_s^2\nabla\ln\rho - \frac{1}{8}\left(1 - \frac{\Delta\alpha^2}{\alpha^2}\right)\nabla(\vec{Q}^+ - \vec{Q}^-)^2 \\
+\frac{1}{4}\left(1 - \frac{\Delta\alpha^2}{\alpha^2}\right)(\vec{Q}^+ - \vec{Q}^-)^2\nabla\ln\alpha - \frac{1}{4}\frac{\Delta\alpha^2}{\alpha^2}(\vec{Q}^+ - \vec{Q}^-)\cdot\nabla(\vec{Q}^+ - \vec{Q}^-) \\
+\frac{1}{4}\frac{\Delta\alpha^2}{\alpha^2}(\vec{Q}^+ - \vec{Q}^-)\nabla\cdot(\vec{Q}^+ - \vec{Q}^-) \mp \left(\frac{\vec{Q}^+ - \vec{Q}^-}{8}\right)\nabla\cdot(3\vec{Q}^\pm - \vec{Q}^\mp) \\
+\left(\frac{\vec{Q}^+ - \vec{Q}^-}{4}\right)\left(\left(\frac{\vec{Q}^+ - \vec{Q}^-}{2}\right)\cdot\nabla\ln\rho\alpha^2\right)
\end{aligned}
$$

Full MHD! Contains all phenomena! Split waves between up and down

Towards UAWSOM

UAWSOM: Uniturbulence and AWSOM
$\rightarrow$ make equation for kink wave heating

- Linearise Q-MHD equation
- incompressible $\delta\rho = 0$
- only perp. components $\delta\vec{Q}_\perp^\pm$
- Scalar multiplication with $\rho_0 \delta\vec{Q}_\perp^\pm/2$

$\rightarrow$ energy density kink $W_\mathrm{k}^\pm = \frac{\rho_0}{4}\langle (Q^\pm)^2\rangle$

- Average over cross-section

$$\frac{\partial W_\mathrm{k}^\pm}{\partial t} + \nabla\cdot(\vec{Q}_\mathrm{k}^\mp W_\mathrm{k}^\pm) + \frac{W_\mathrm{k}^\pm}{2}\nabla\cdot\vec{V} = -\frac{1}{L_{\perp,\mathrm{VD}}}\frac{1}{\sqrt{\rho_\mathrm{e}}}(W_\mathrm{k}^\pm)^{3/2}$$

Perpendicular length $L_{\perp,\mathrm{VD}}$ scales with R, ζ and filling factor f

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Compare with Alfvén wave:

$$\mathrm{RHS}_{\mathrm{Alfvén}} = -\frac{1}{L_{\perp,\mathrm{A}}}\sqrt{\frac{W_{\mathrm{A}}^{\mp}}{\rho_0}}W_{\mathrm{A}}^{\pm}$$

$$\frac{\partial W_{\mathrm{k}}^{\pm}}{\partial t} + \nabla\cdot(\vec{Q}_{\mathrm{k}}^{\mp}W_{\mathrm{k}}^{\pm}) + \frac{W_{\mathrm{k}}^{\pm}}{2}\nabla\cdot\vec{V} = -\frac{1}{L_{\perp,\mathrm{VD}}}\frac{1}{\sqrt{\rho_{\mathrm{e}}}}(W_{\mathrm{k}}^{\pm})^{3/2}$$

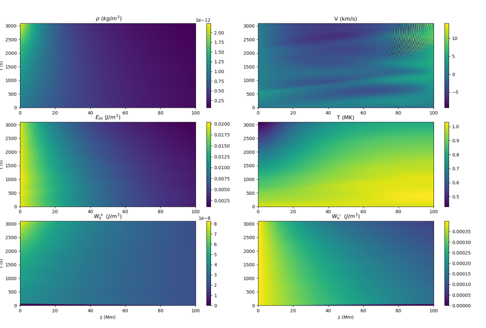
Perpendicular length $L_{\perp,\mathrm{VD}}$ scales with R, ζ and filling factor f

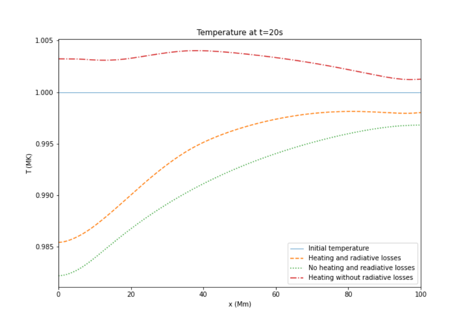
Implementation in 1D

fipy → python package for PDE: e.g. continuity equation

```
eq_continuity = (fipy.TransientTerm(var=dens) ==
- fipy.ConvectionTerm(coeff=v*[[1.]], var=dens))
```

include radiation & conduction





Heating low down in atmosphere. Compensating radiative losses?

Conclusions

- Kink waves in coronal holes, damp and heat plasma
- New Q-variables: co-propagating with waves $\vec{Q}^{\pm} = \vec{V} \pm \alpha \vec{B}$
  - Generalisation of Elsässer variables
  - Generalisation of Frieman-Rotenberg eqn. for flowing plasma
  - Generalisation of characteristics & Riemann invariants
- Q-variables split any wave in propagation direction
- Extend AWSOM to include kink wave heating: heat low down in coronal holes
- 1D implementation ready, 3D needed