Improved reconstruction of solar magnetic fields through spatio-temporal regularisation

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The ever-lasting problem of low S/N

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- Filtering of Q,U&V (makes them inconsistent with Stokes I)

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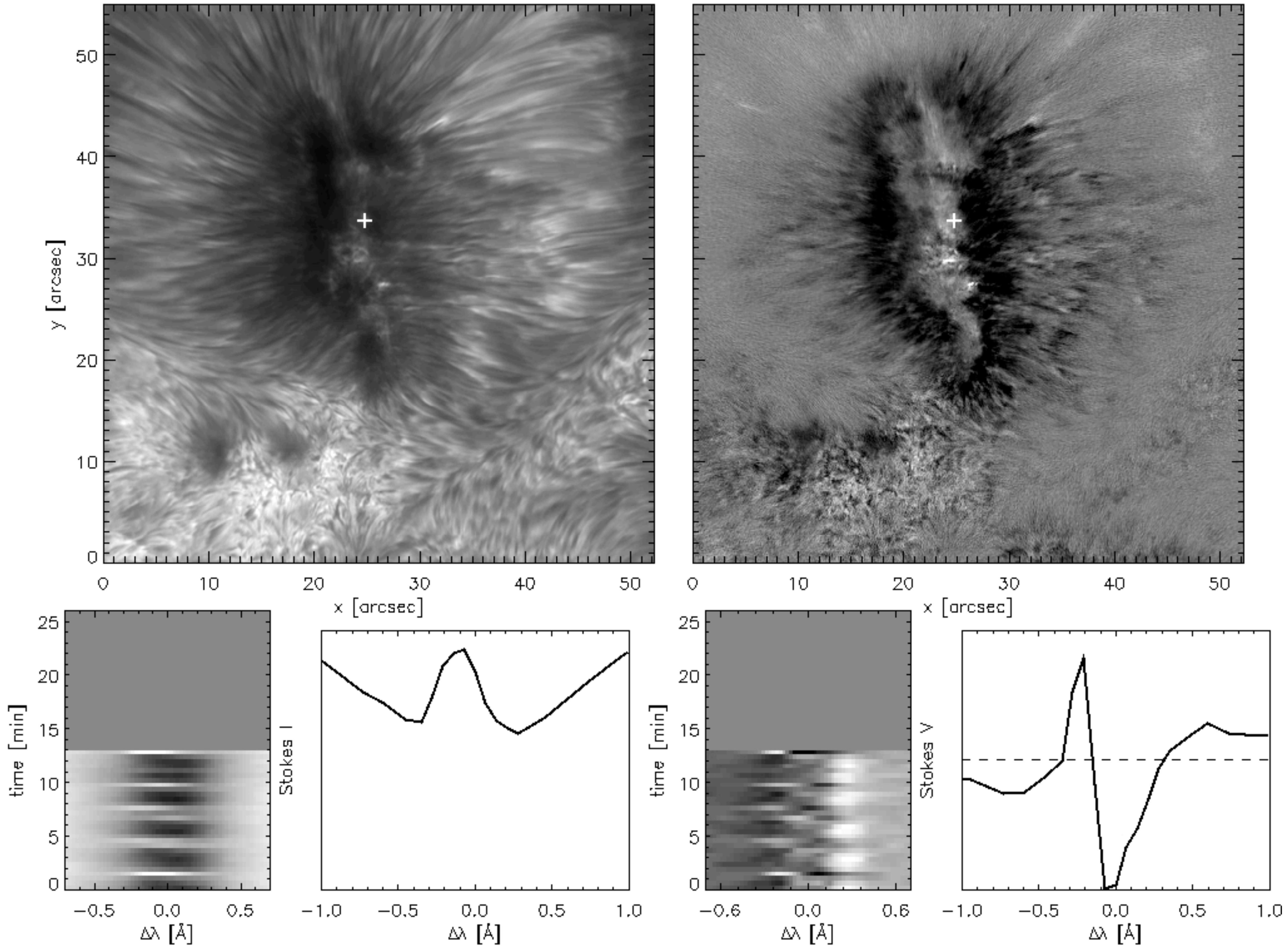

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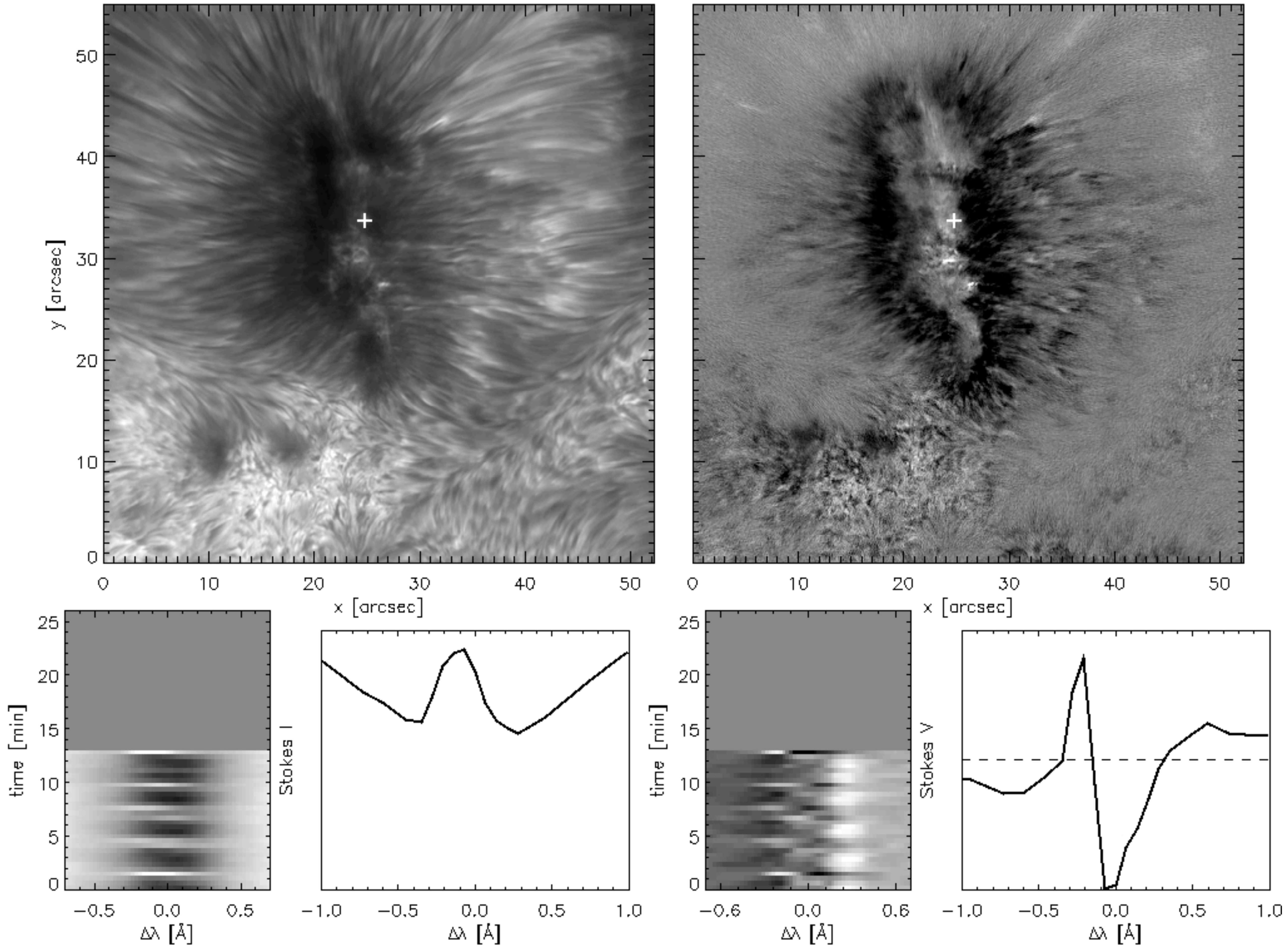

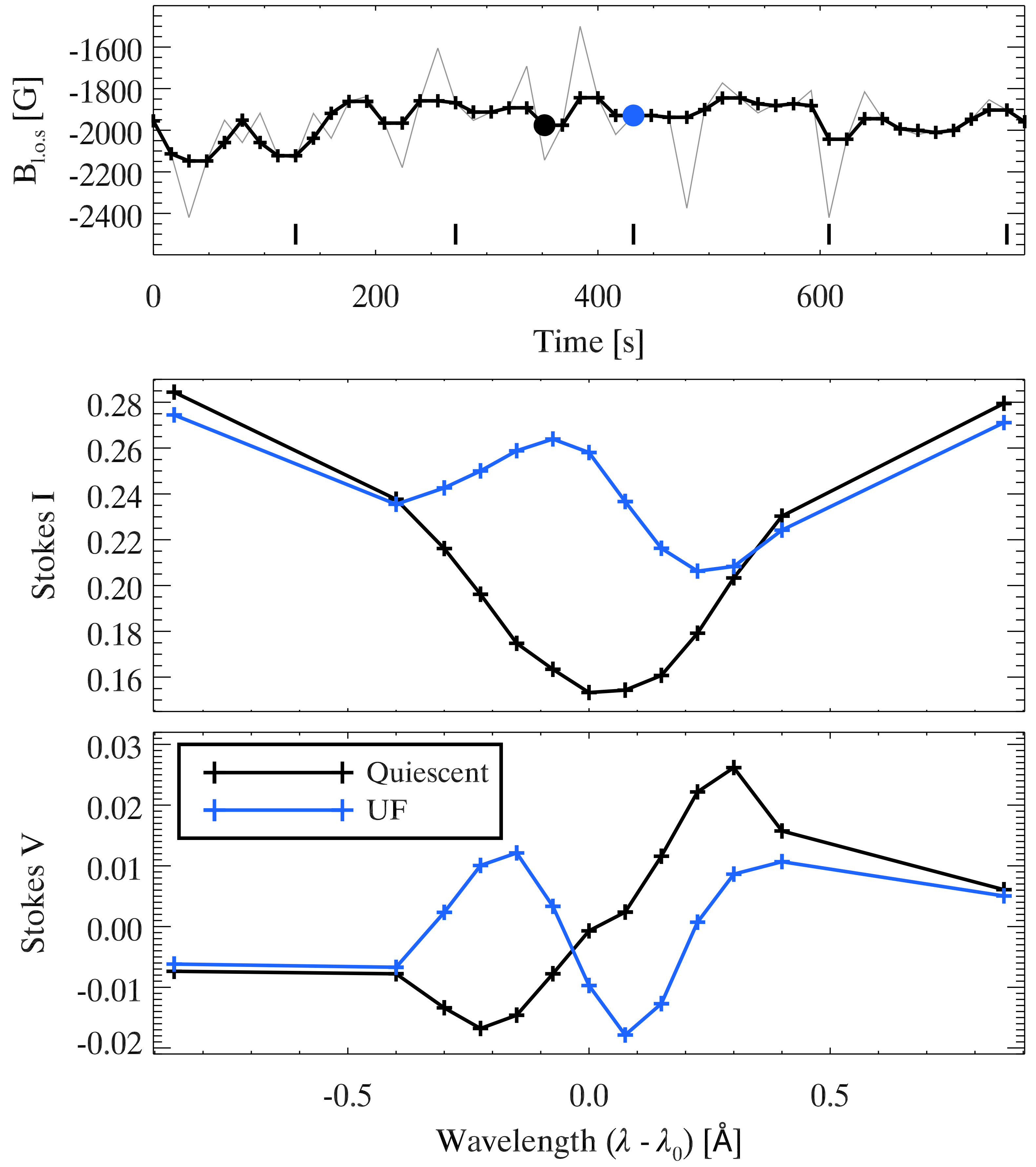

de la Cruz Rodríguez et al. (2013)

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$$\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{o_i - s_i(\boldsymbol{x})}{\sigma_i} \right)^2 + \sum_{p=1}^{M} \alpha_p \Gamma(\boldsymbol{x})^2$$

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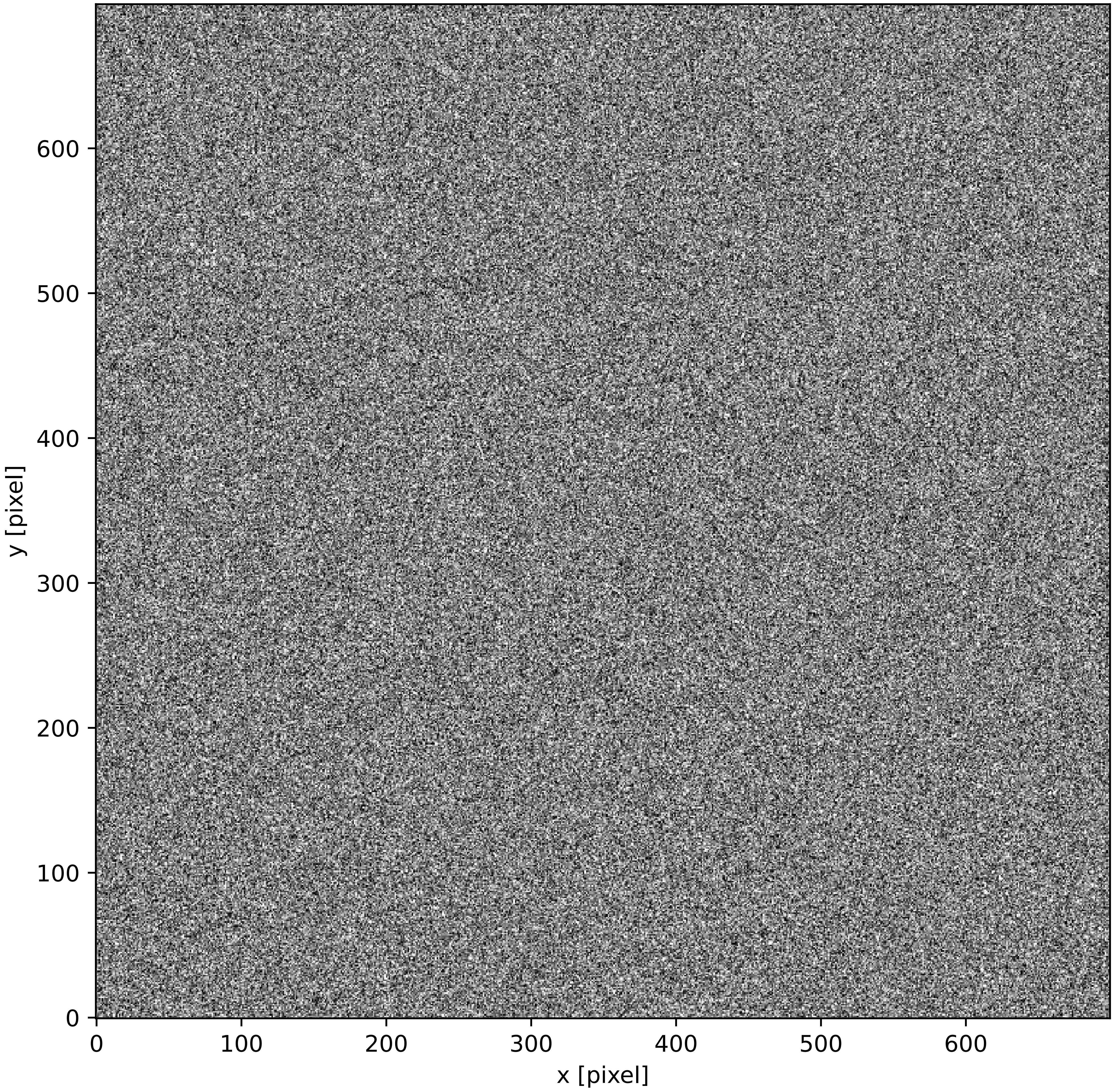
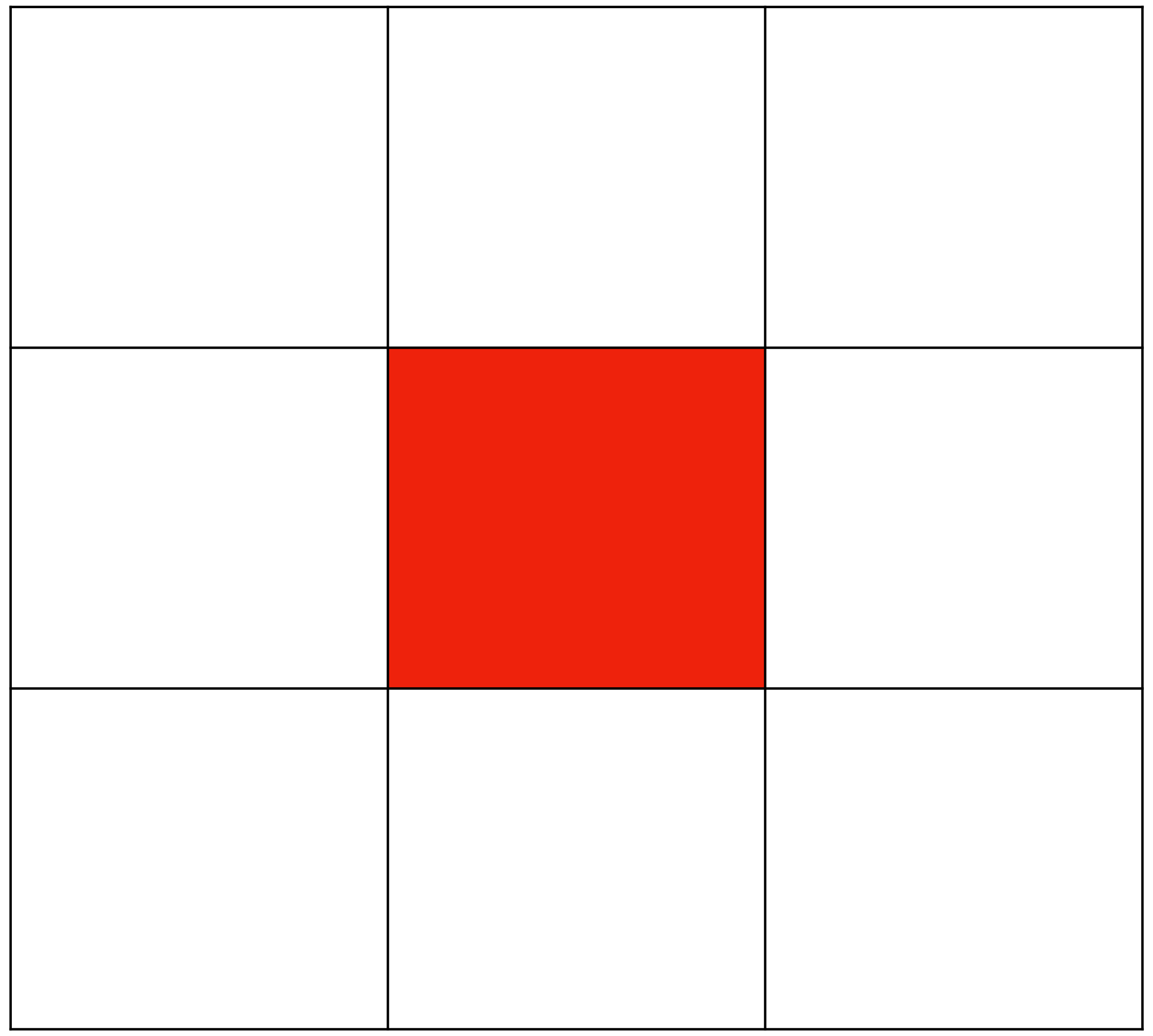


Regularized Weak-Field approximation



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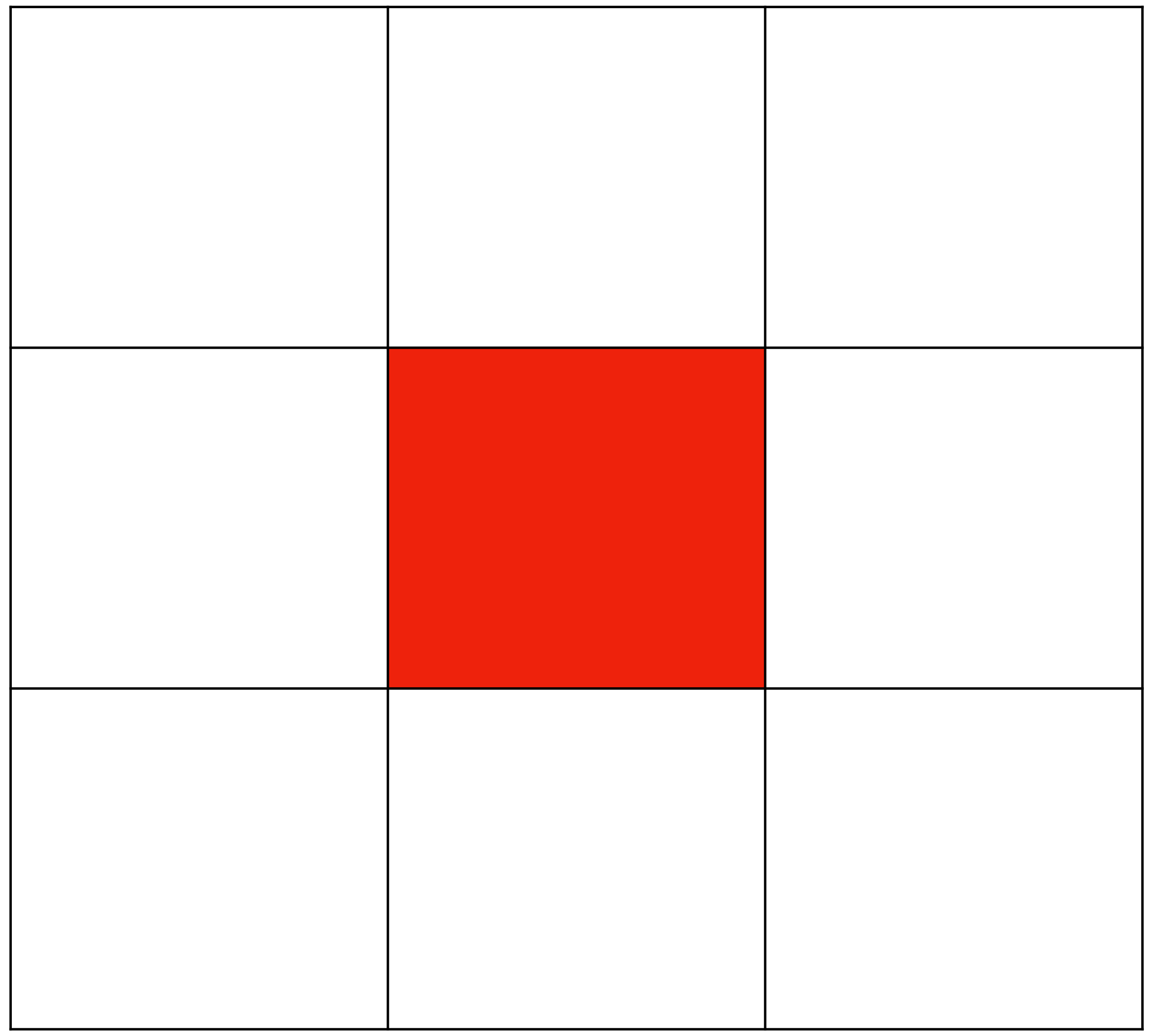

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$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{V_i^{(x,y,t)} - cB_\parallel^{(x,y,t)}\frac{\partial I_i^{(x,y,t)}}{\partial\lambda_i}}{\sigma_i}\right)^2$$

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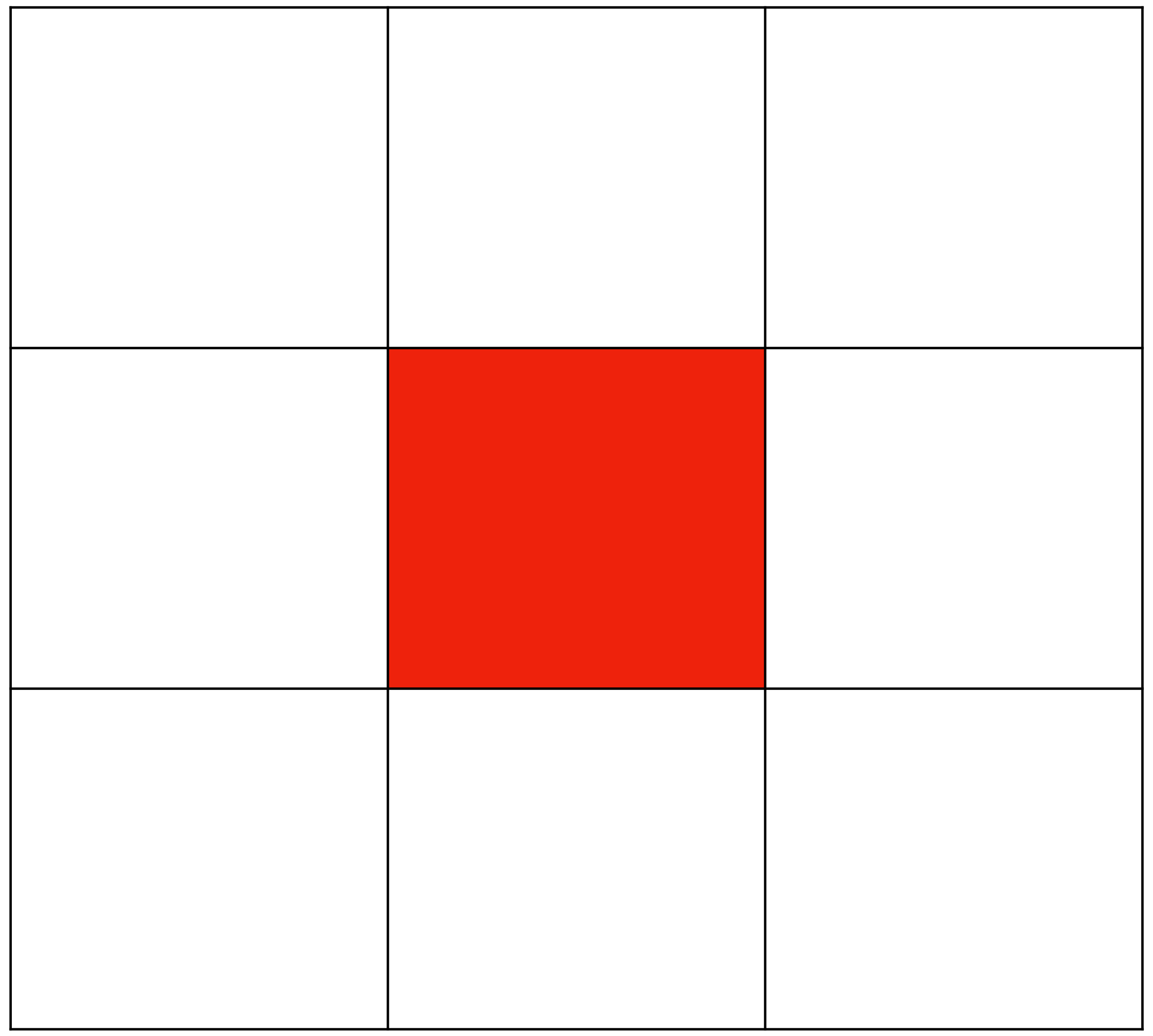

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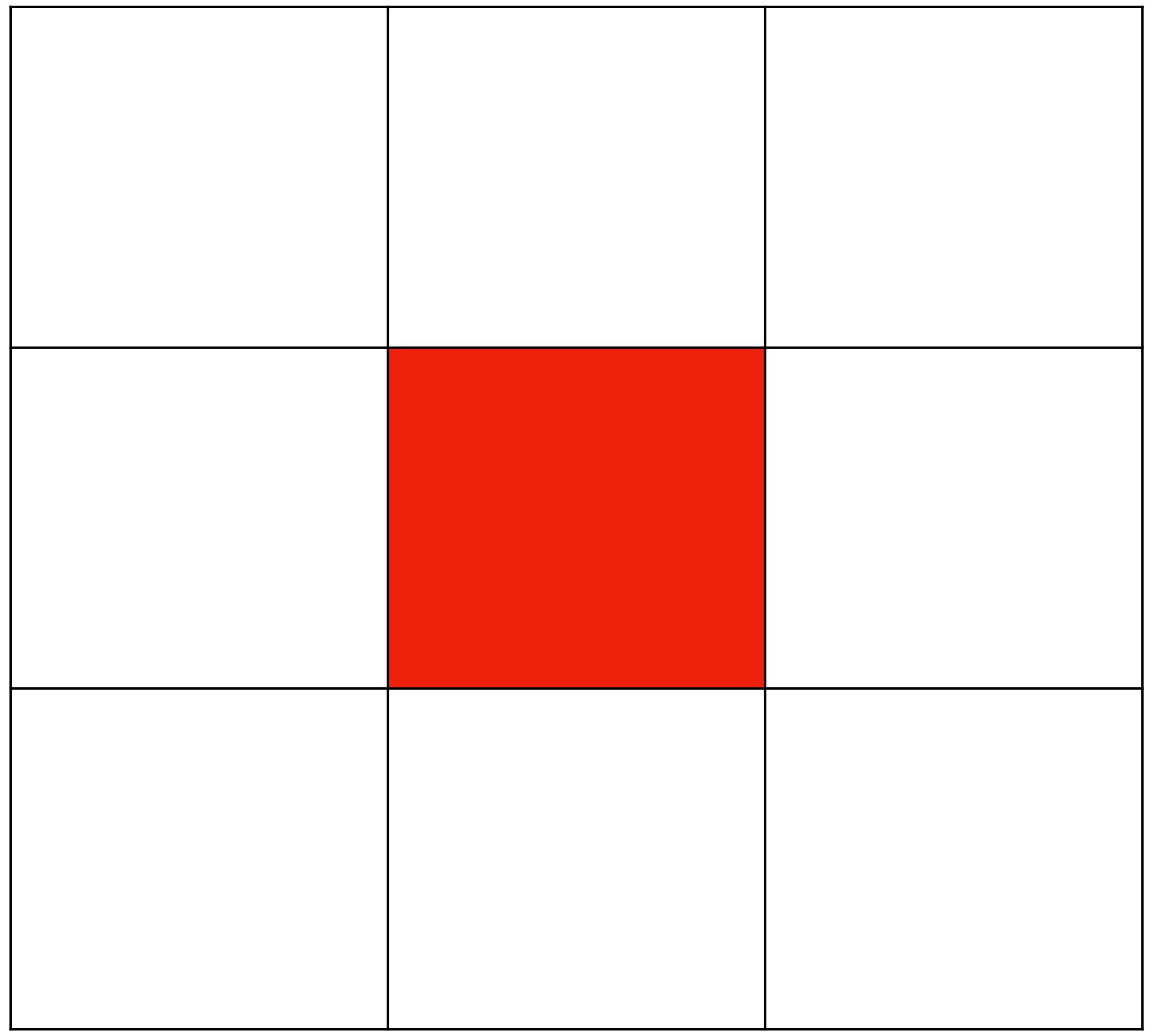

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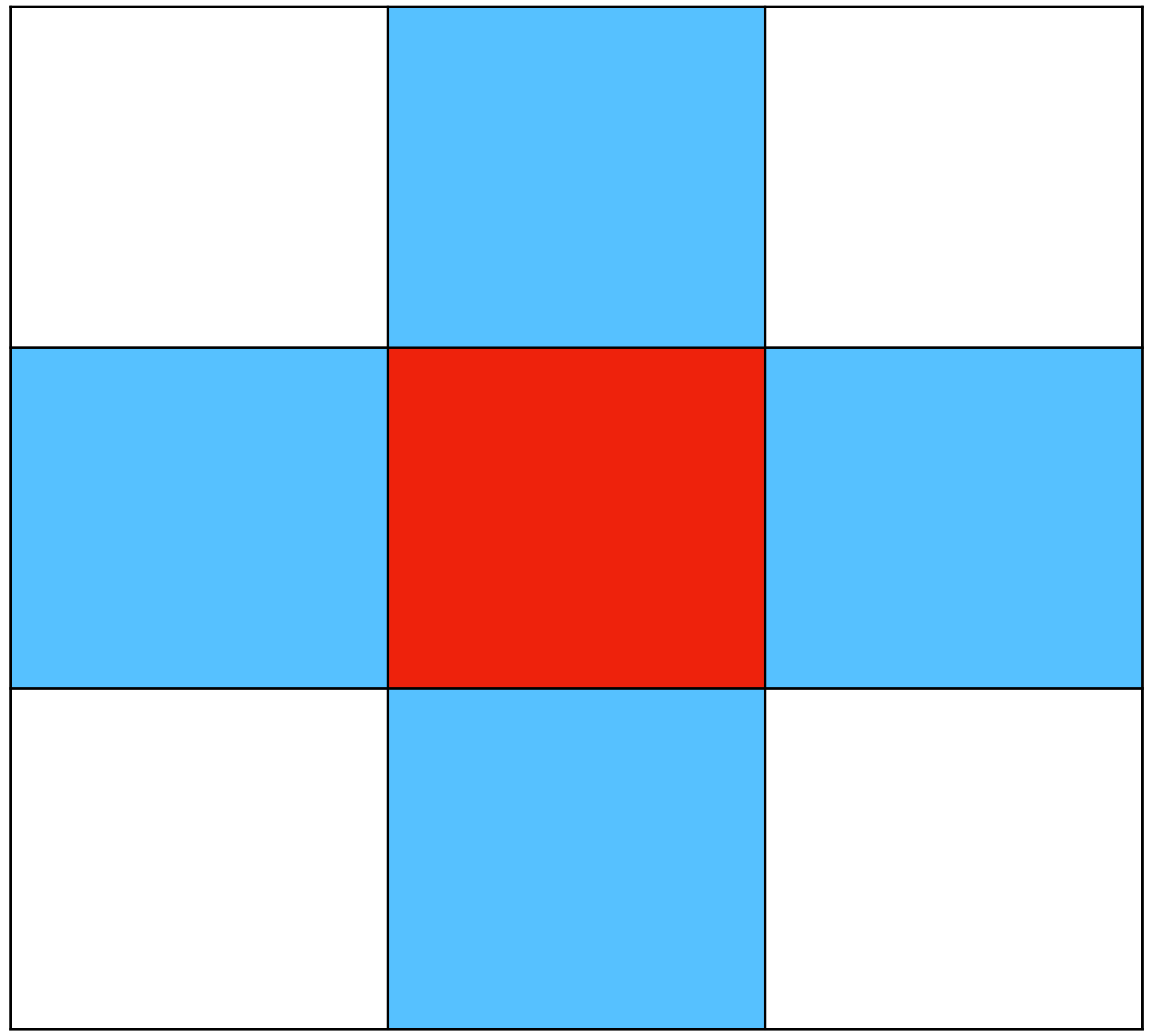

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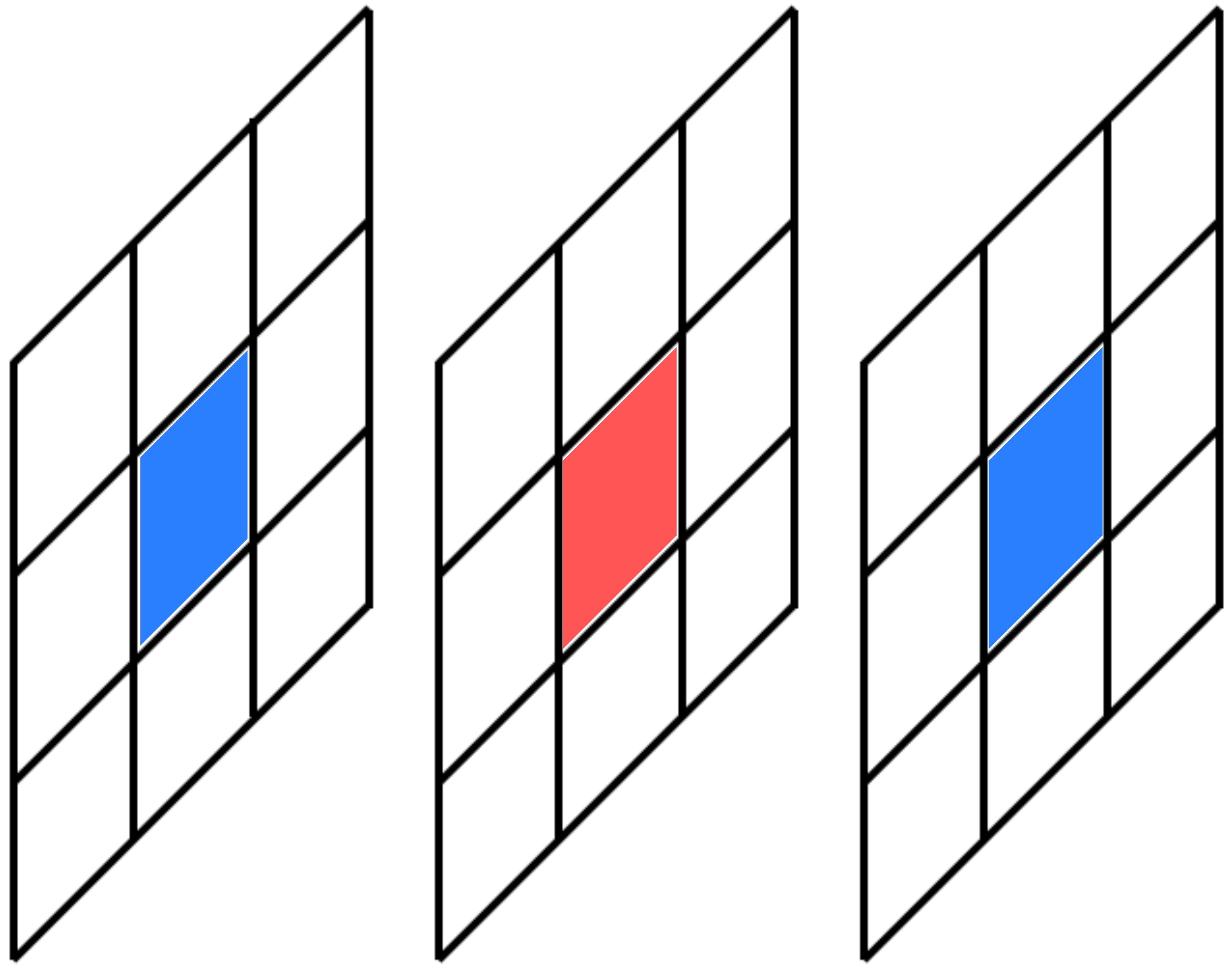

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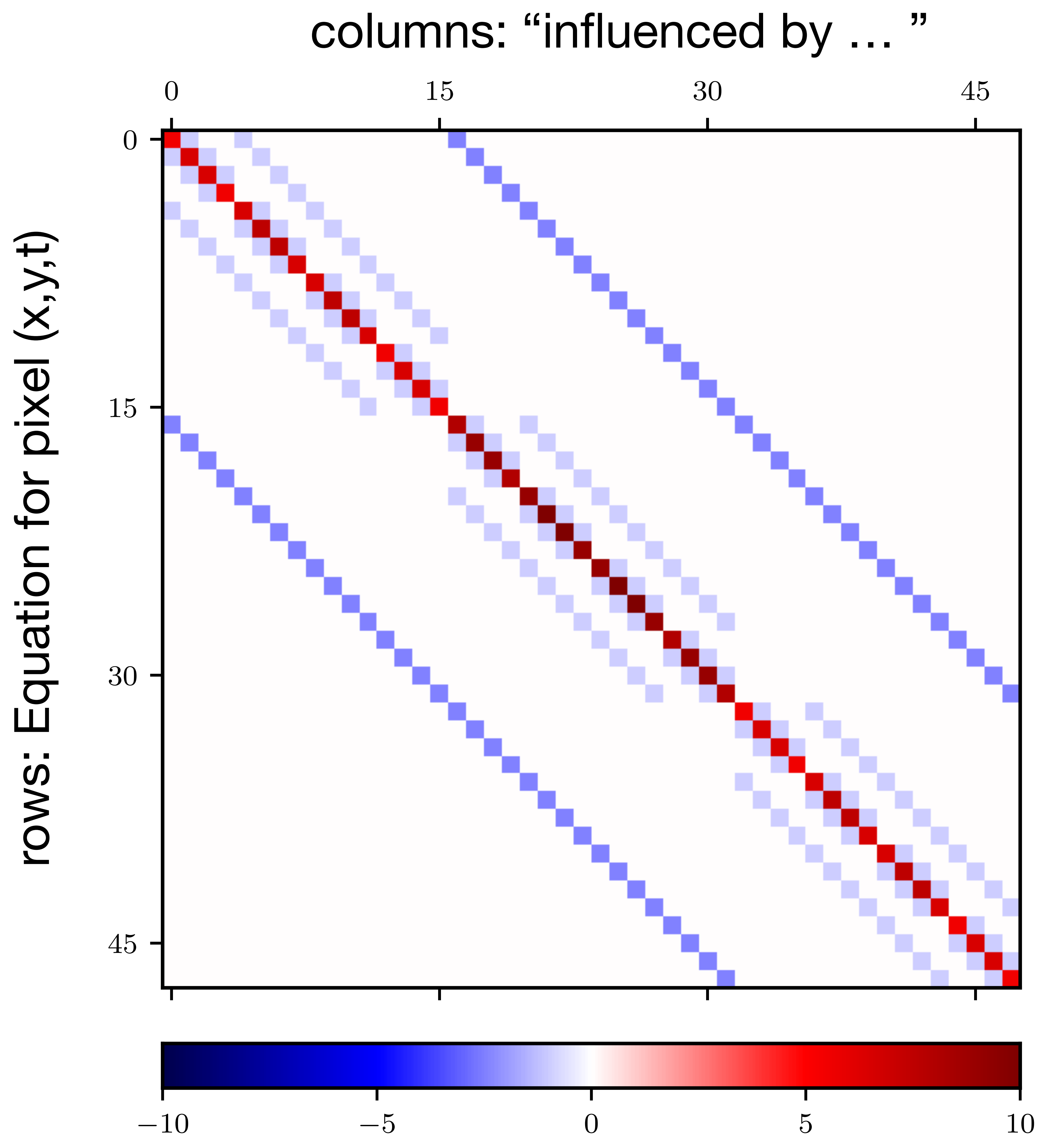

$$n_x = 4,\ n_y = 4,\ n_t = 3$$

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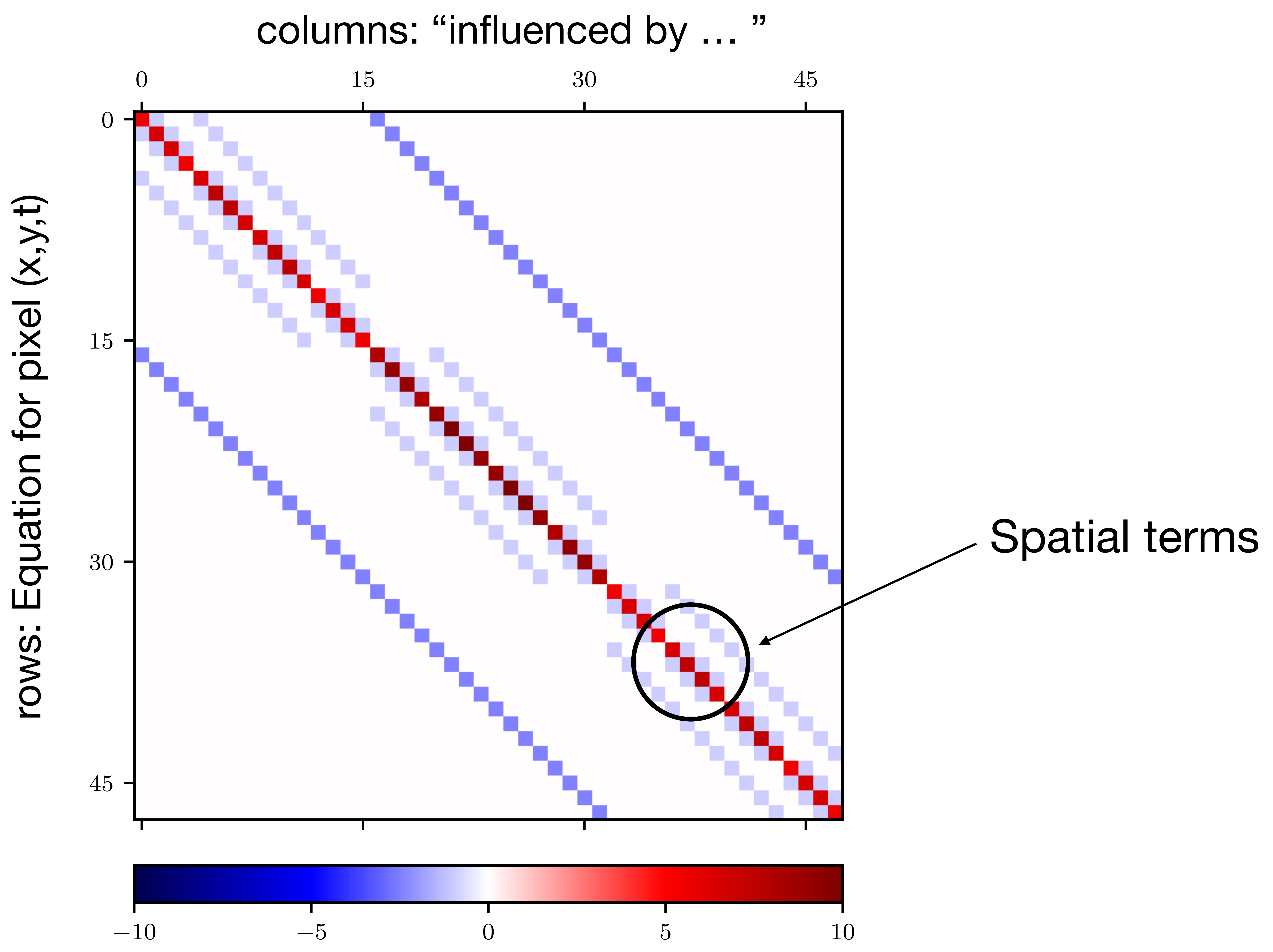

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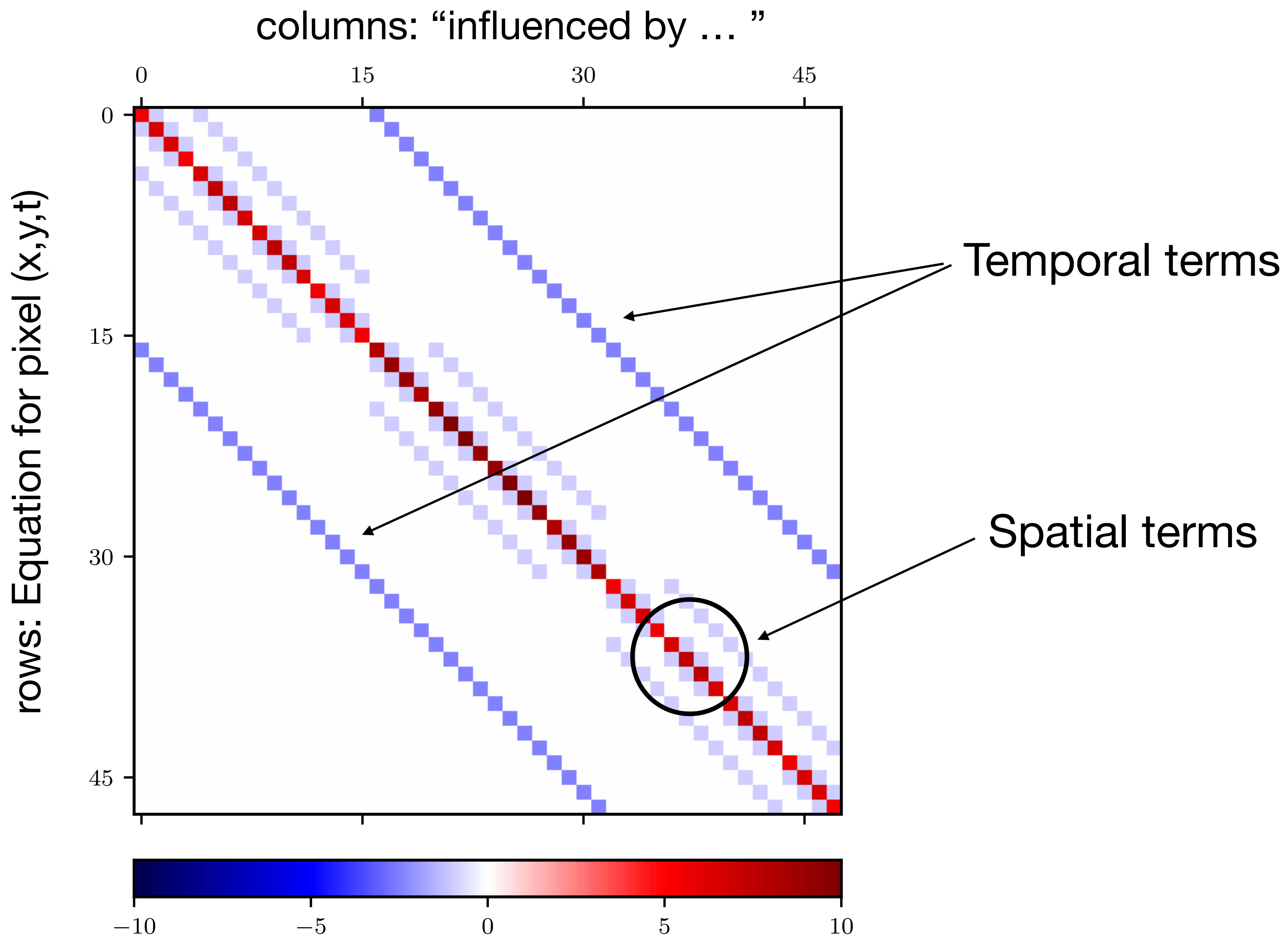

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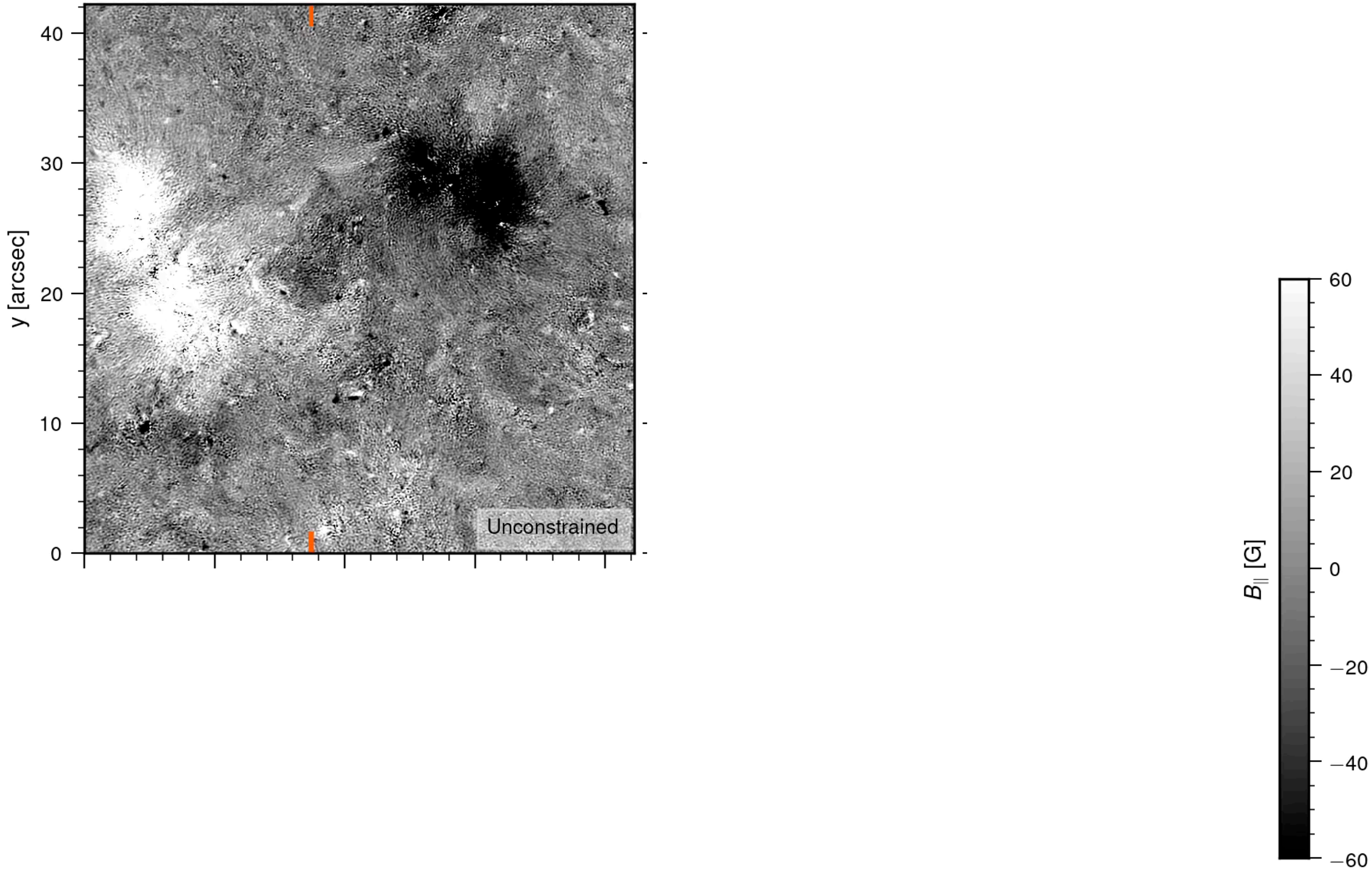

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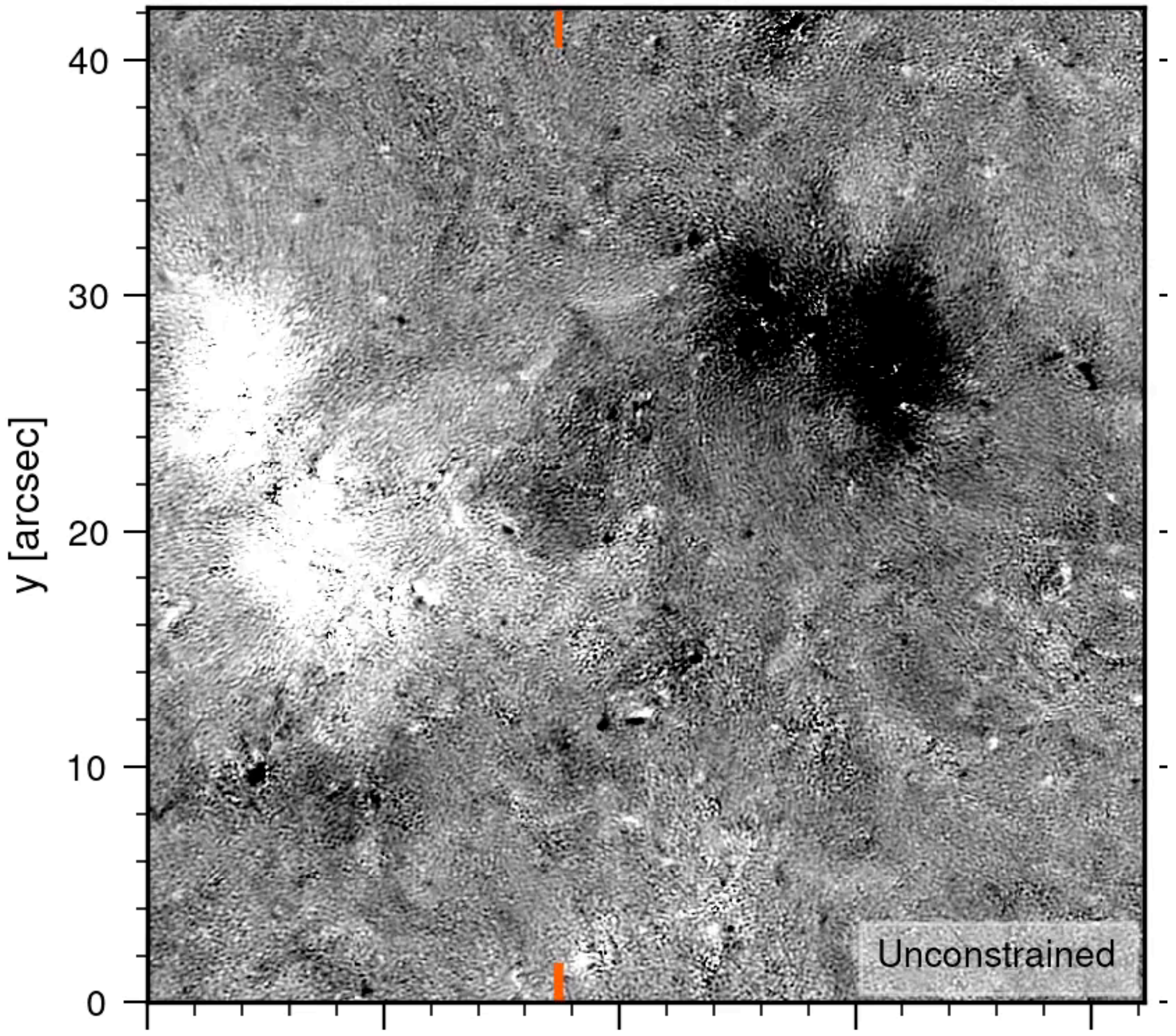

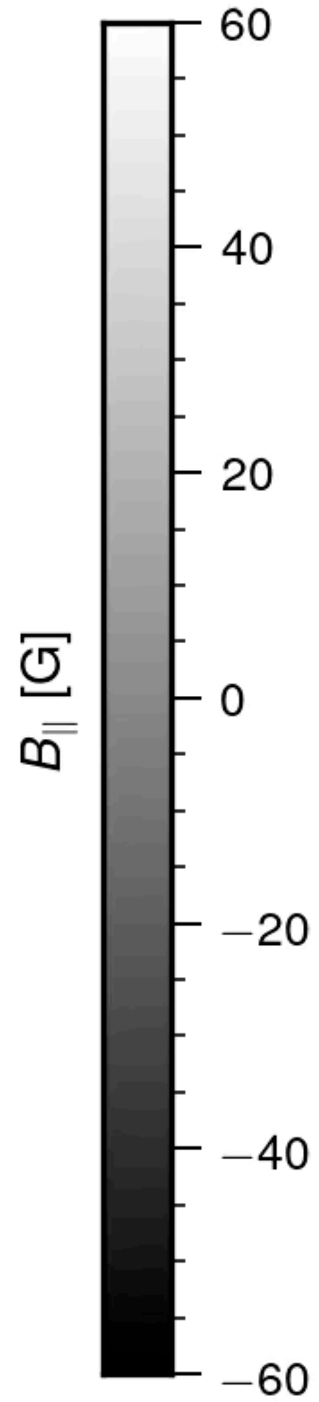

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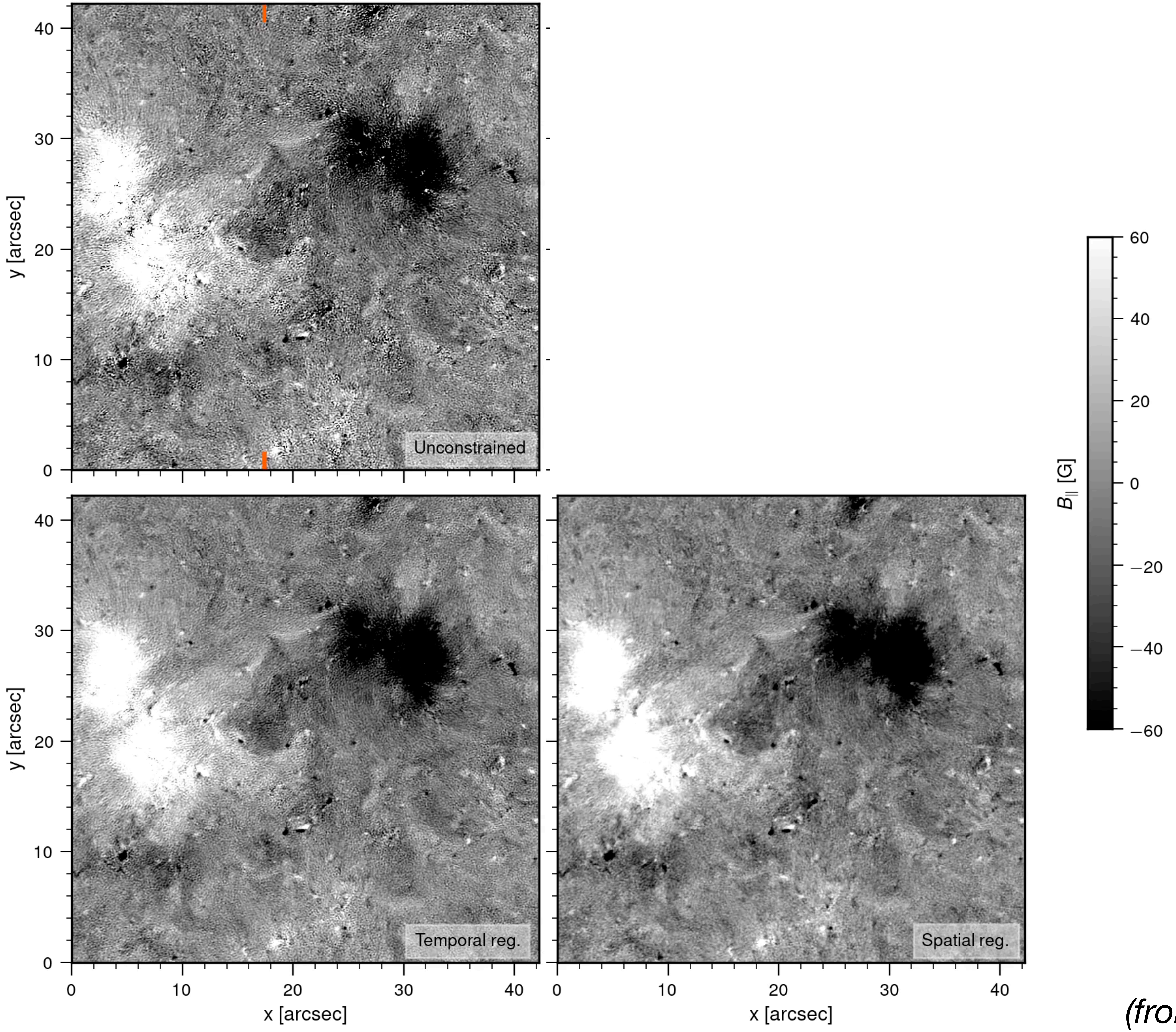

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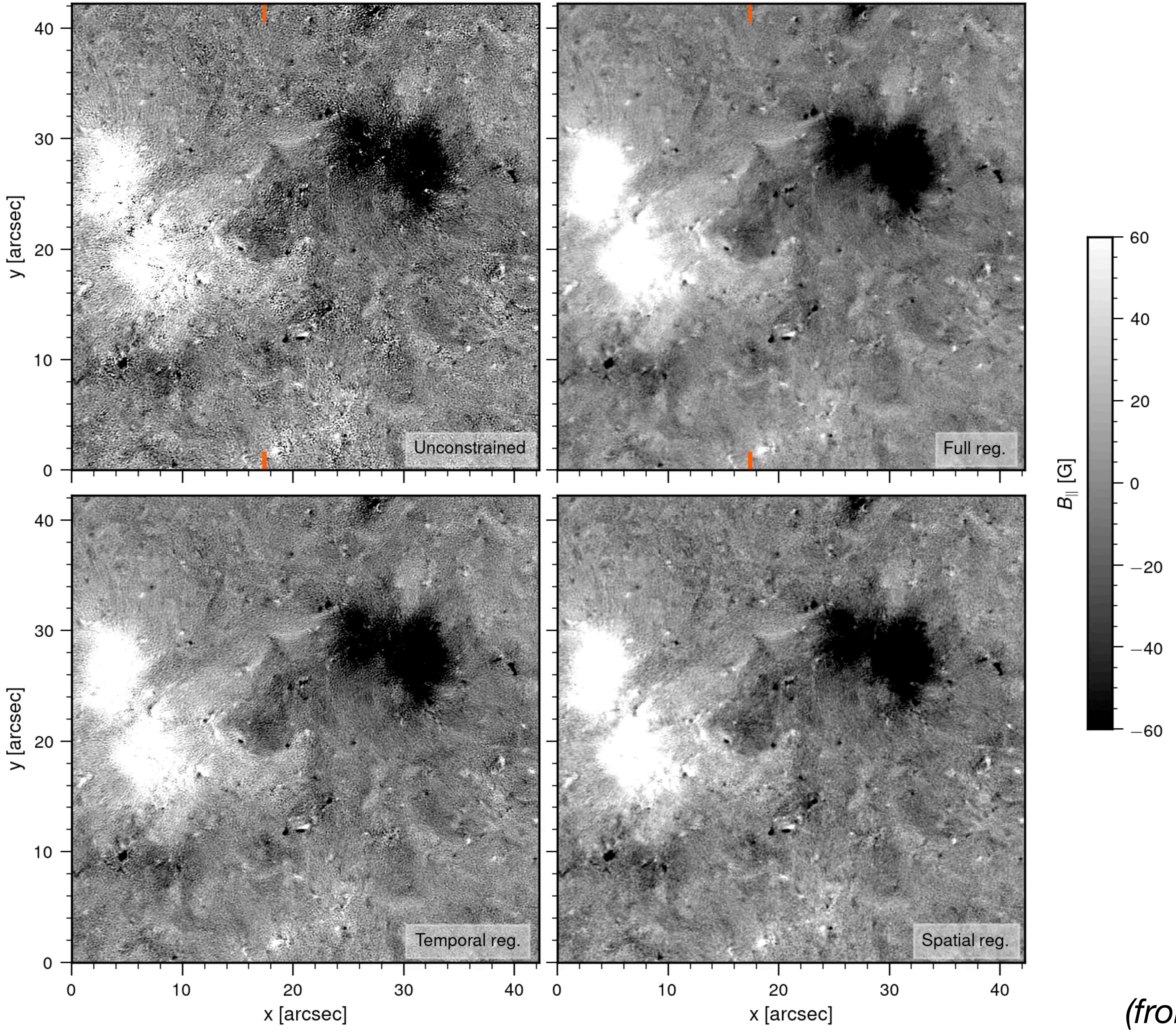

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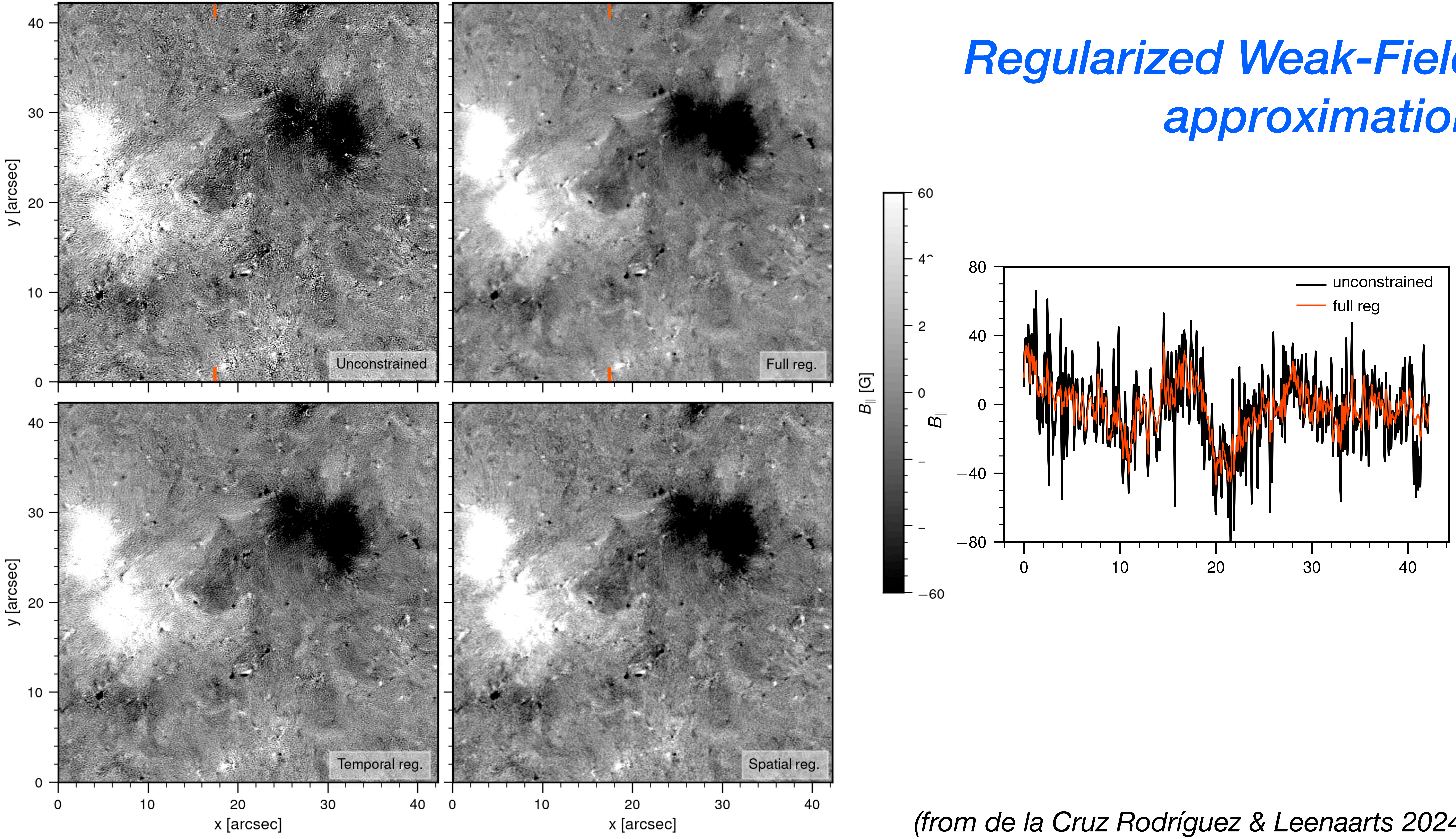

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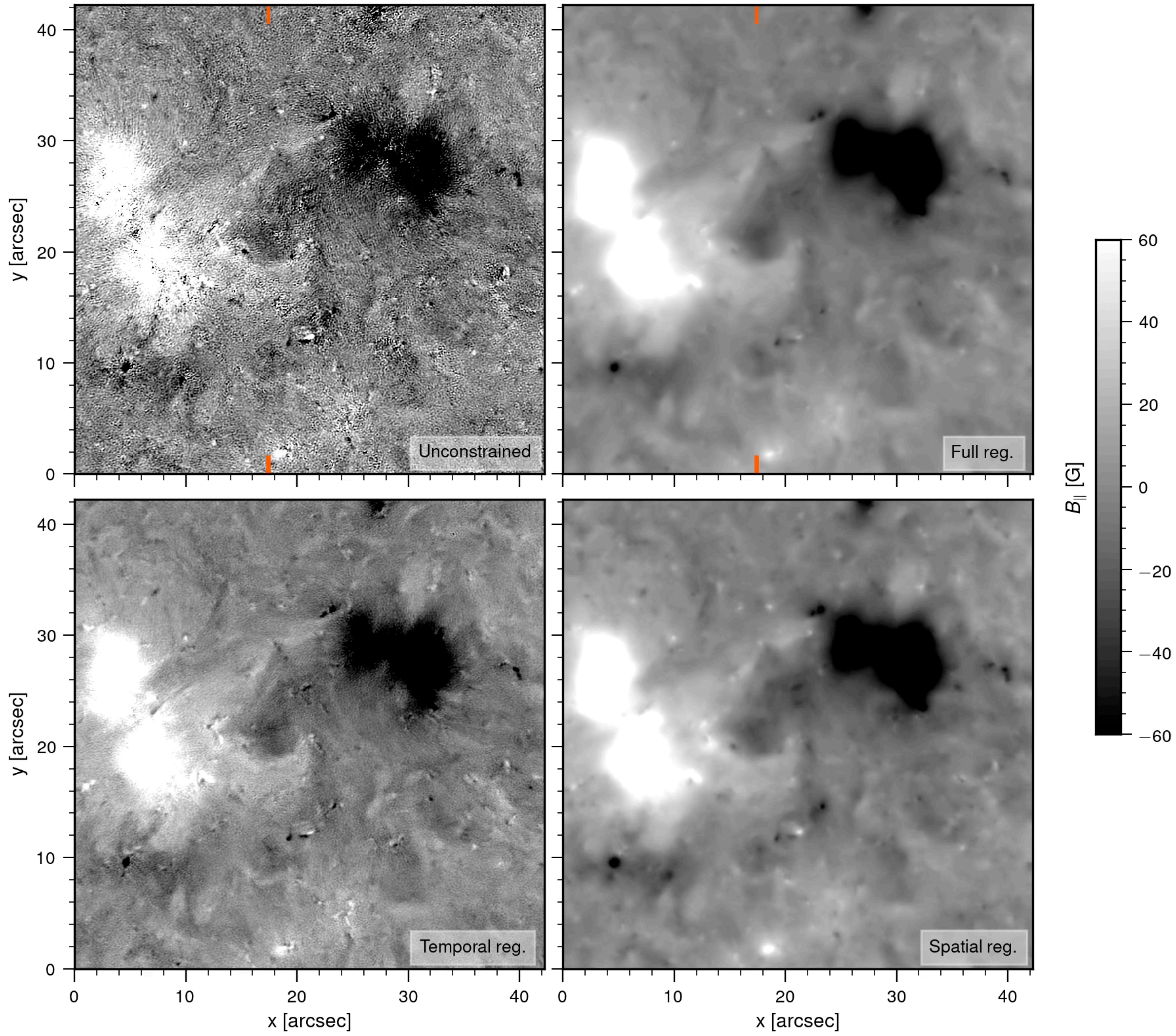

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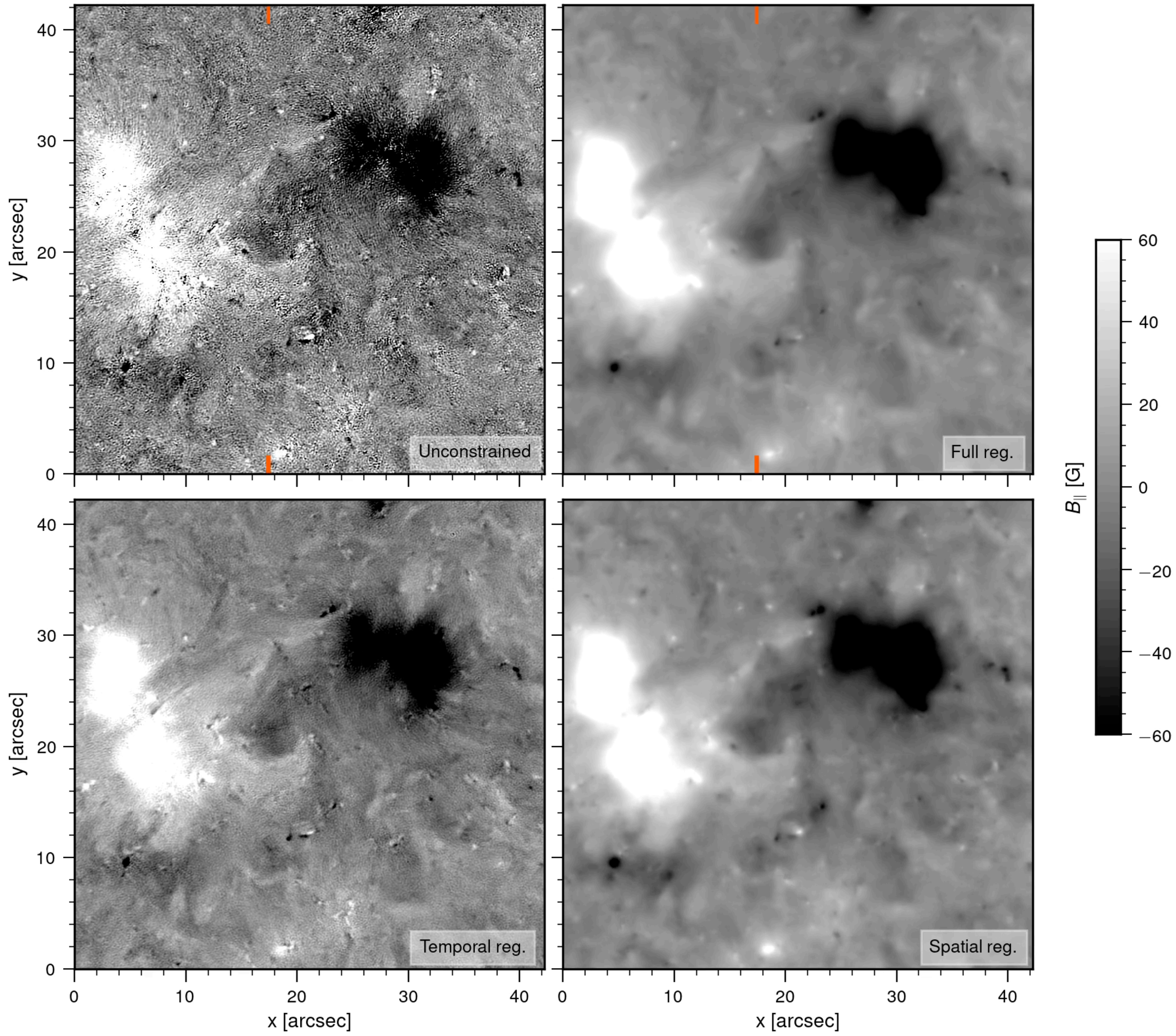

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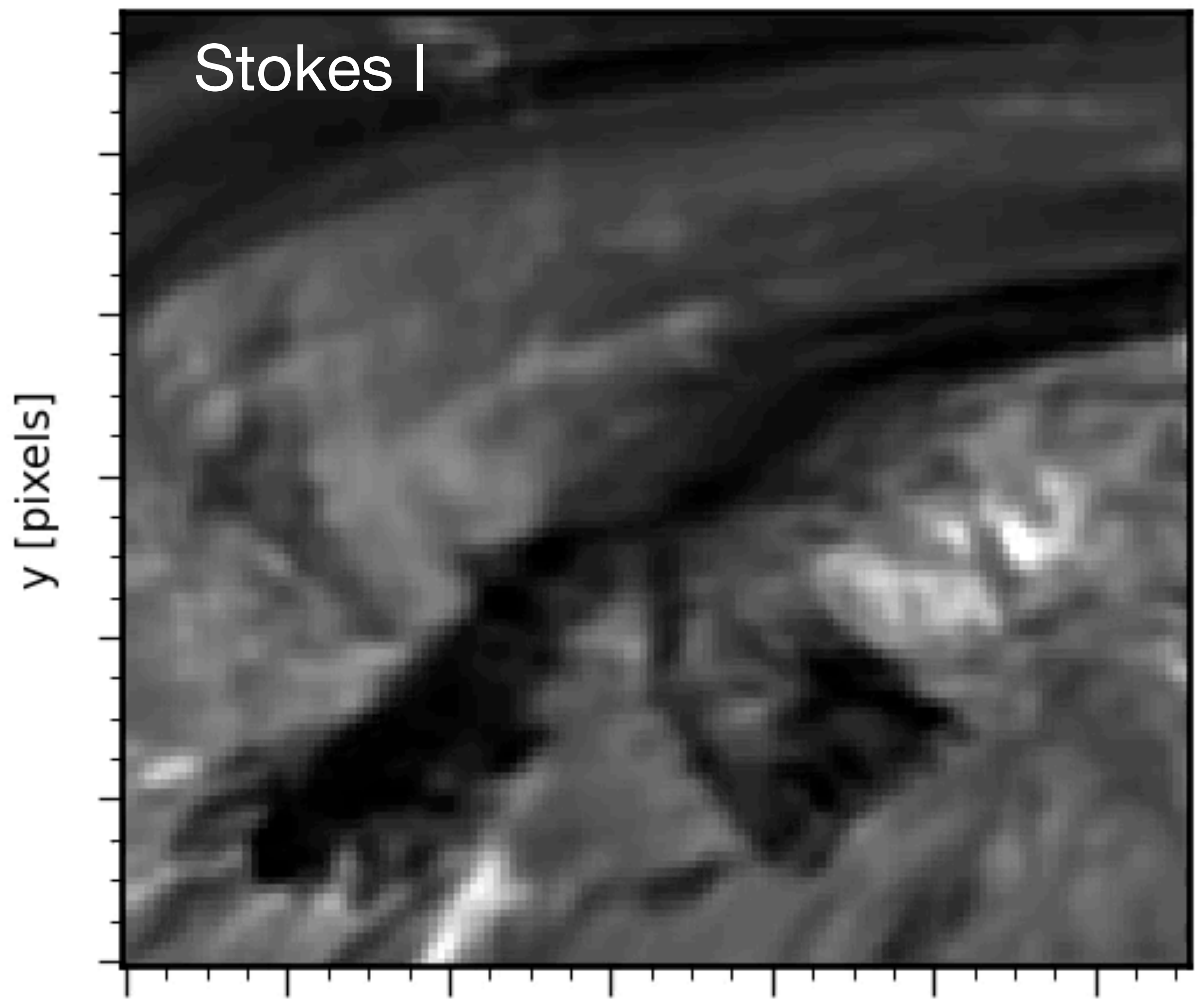
A test of the weak-field approximation applied to MiHi $H\alpha$ data



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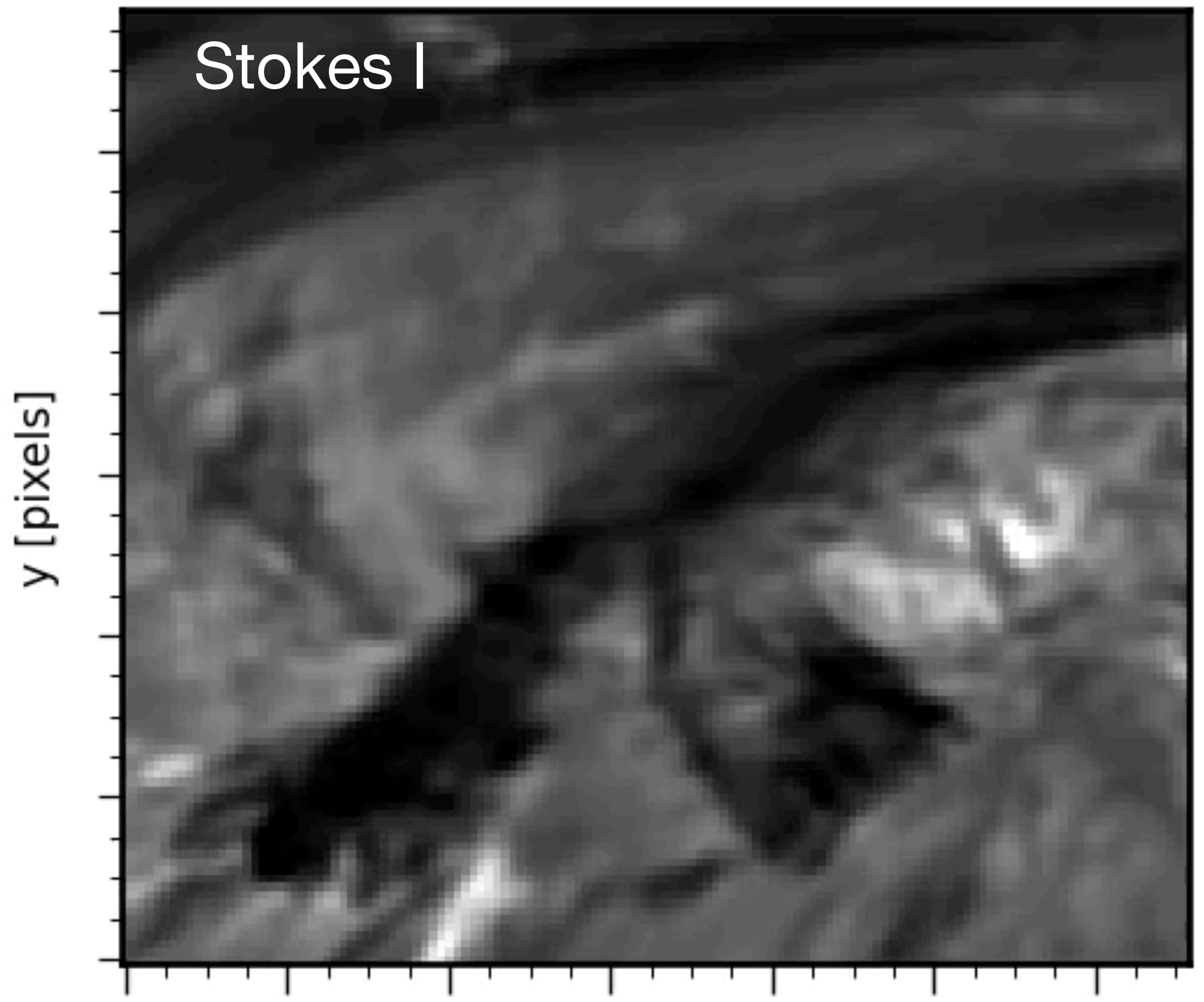

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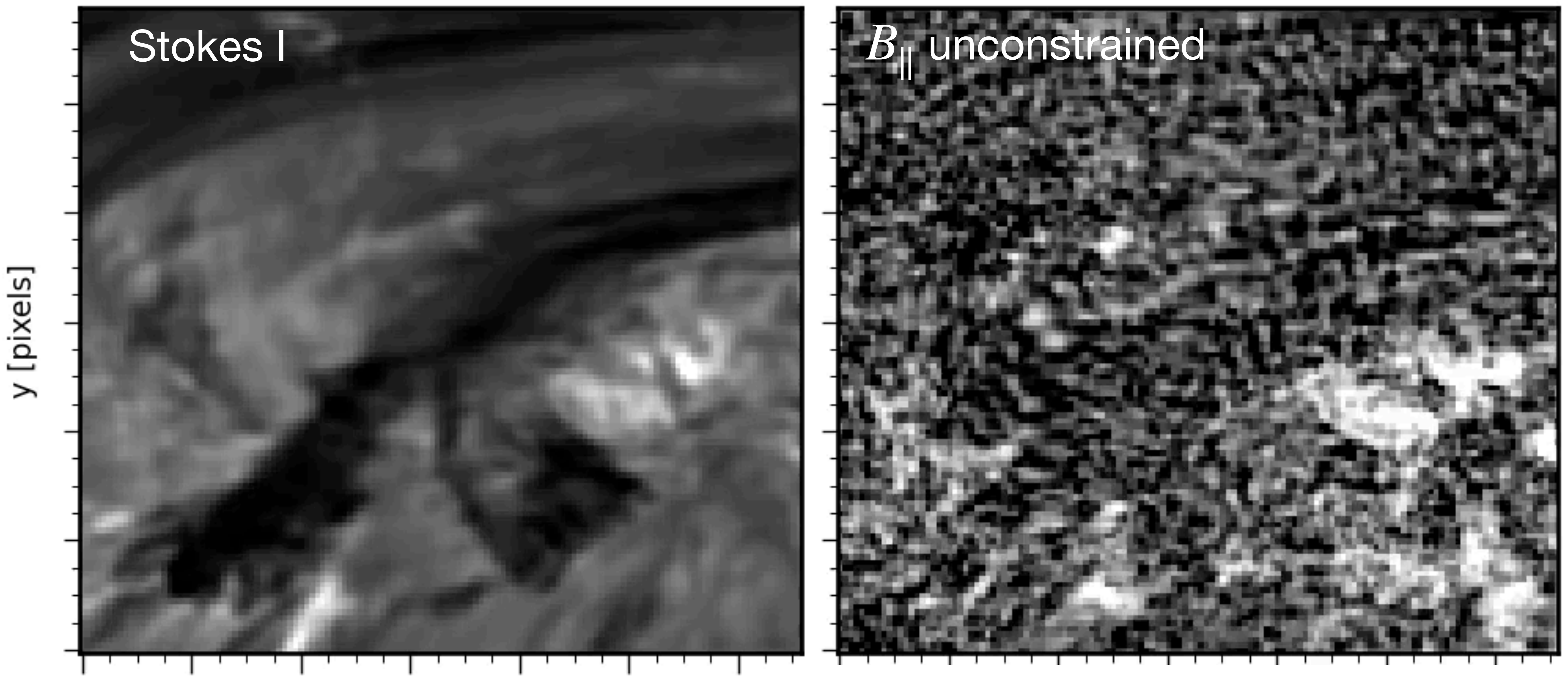

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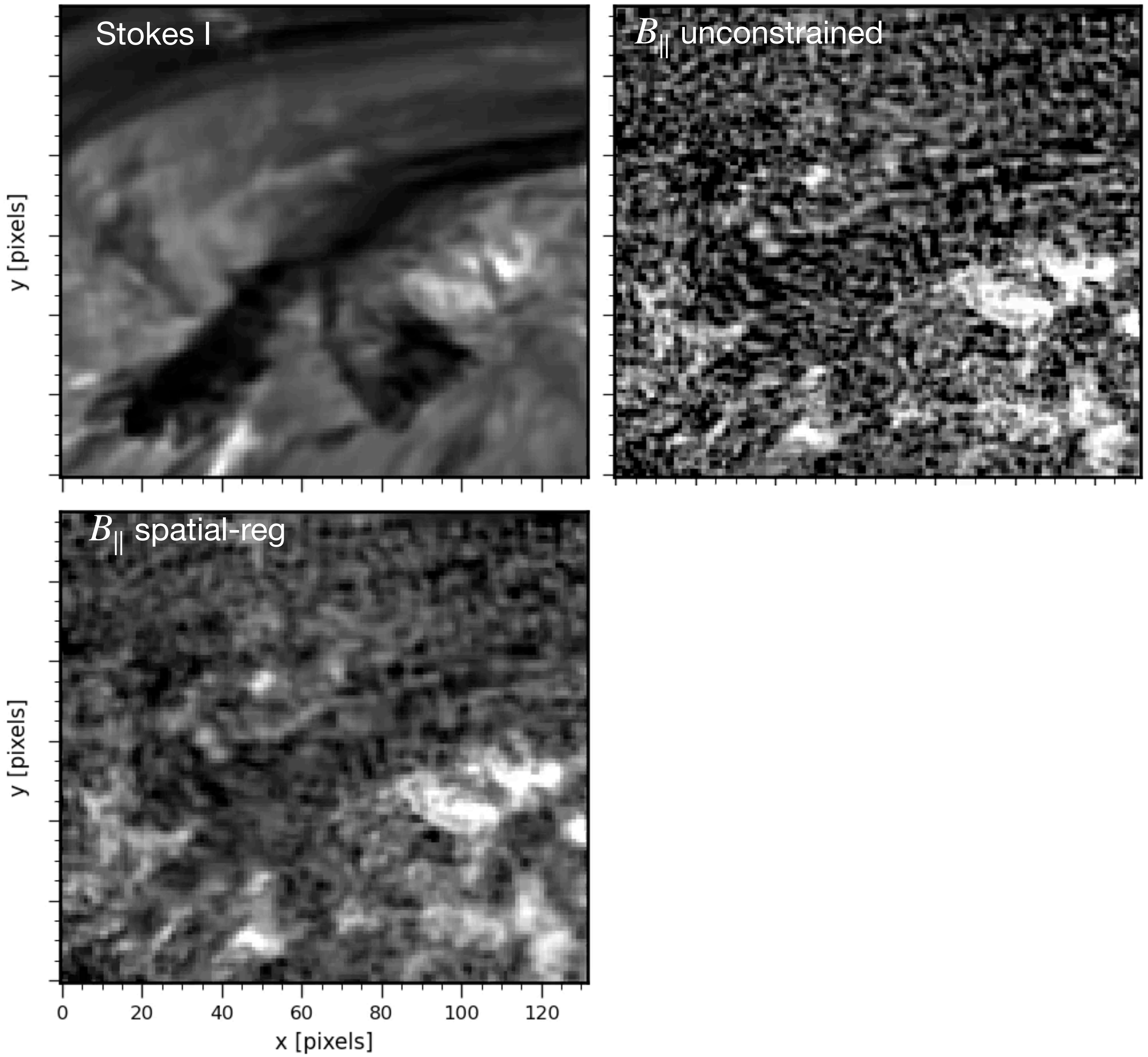

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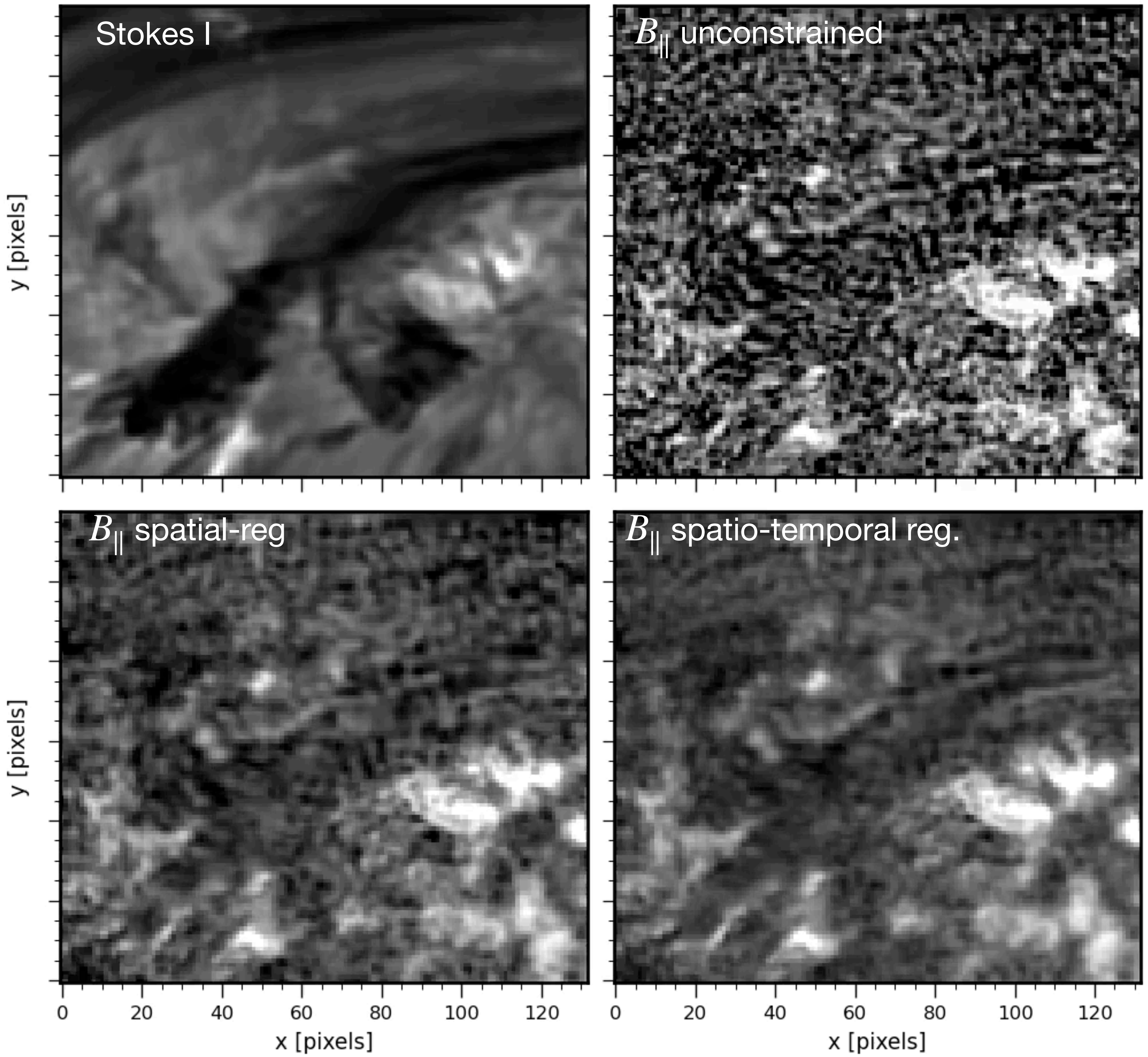

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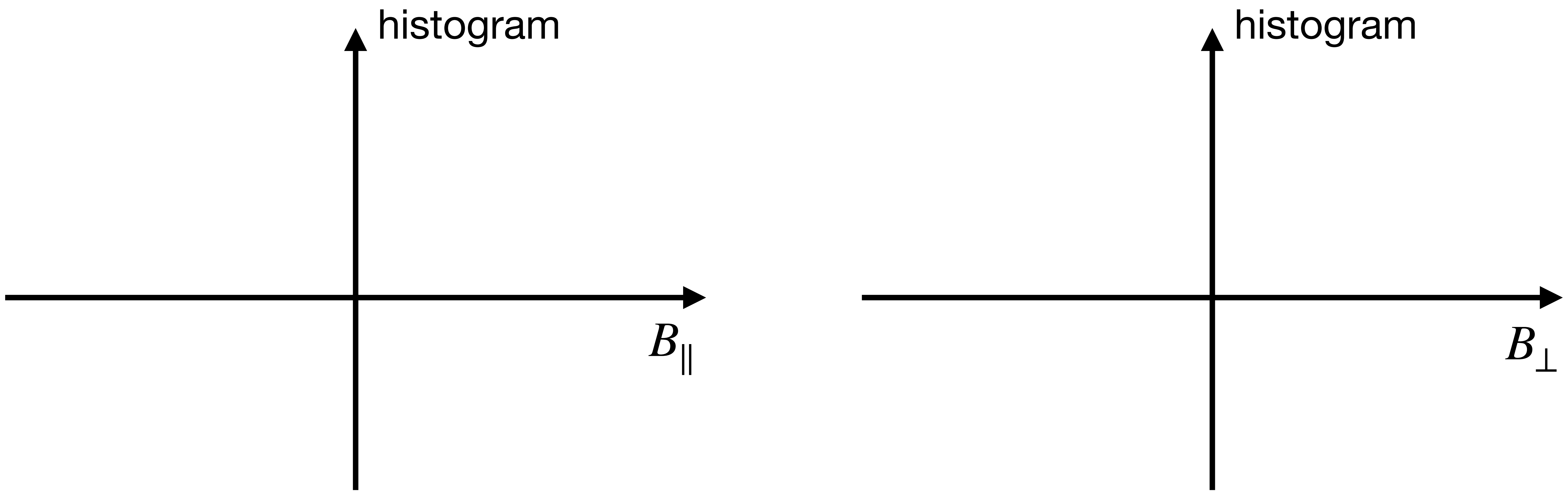

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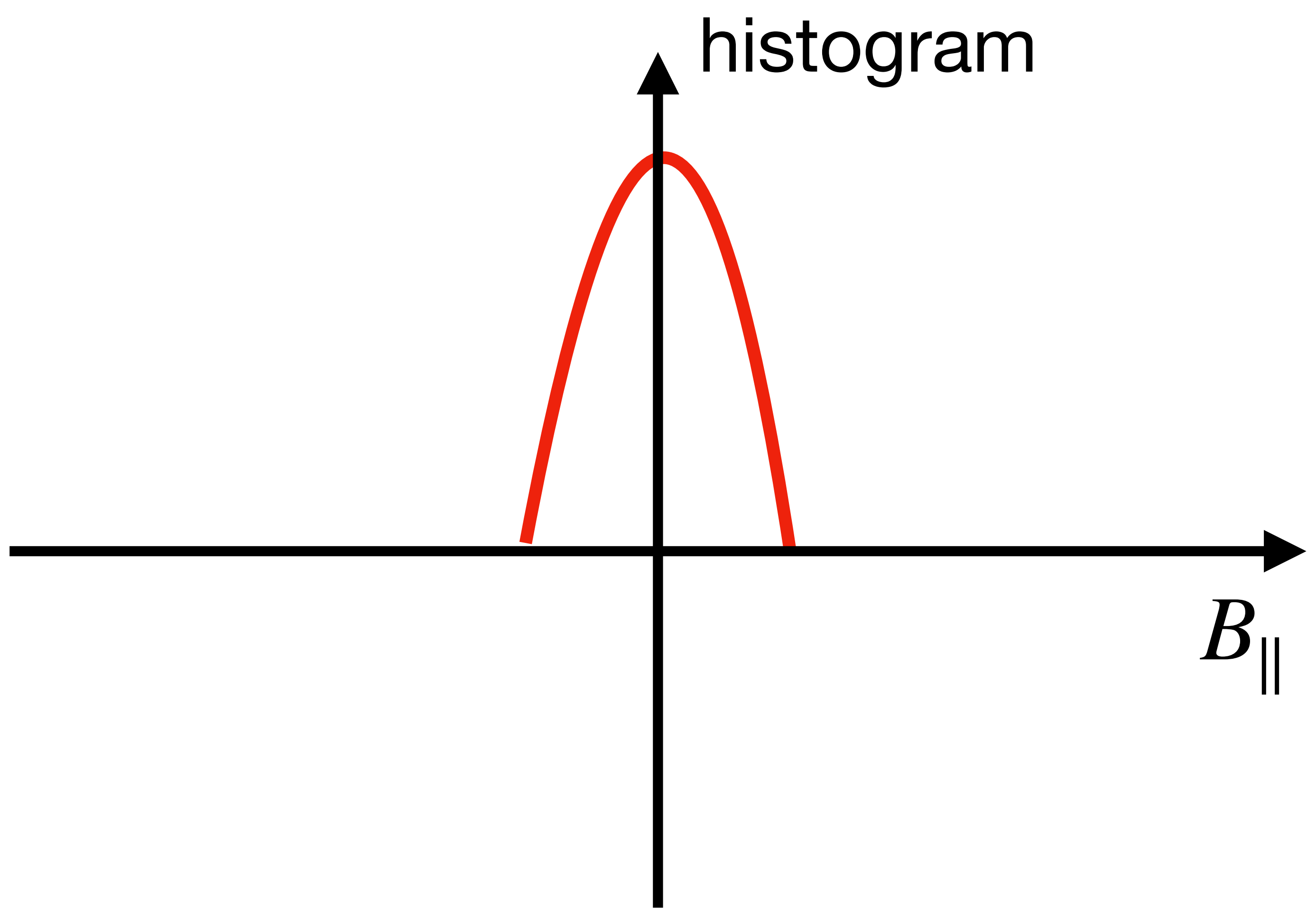

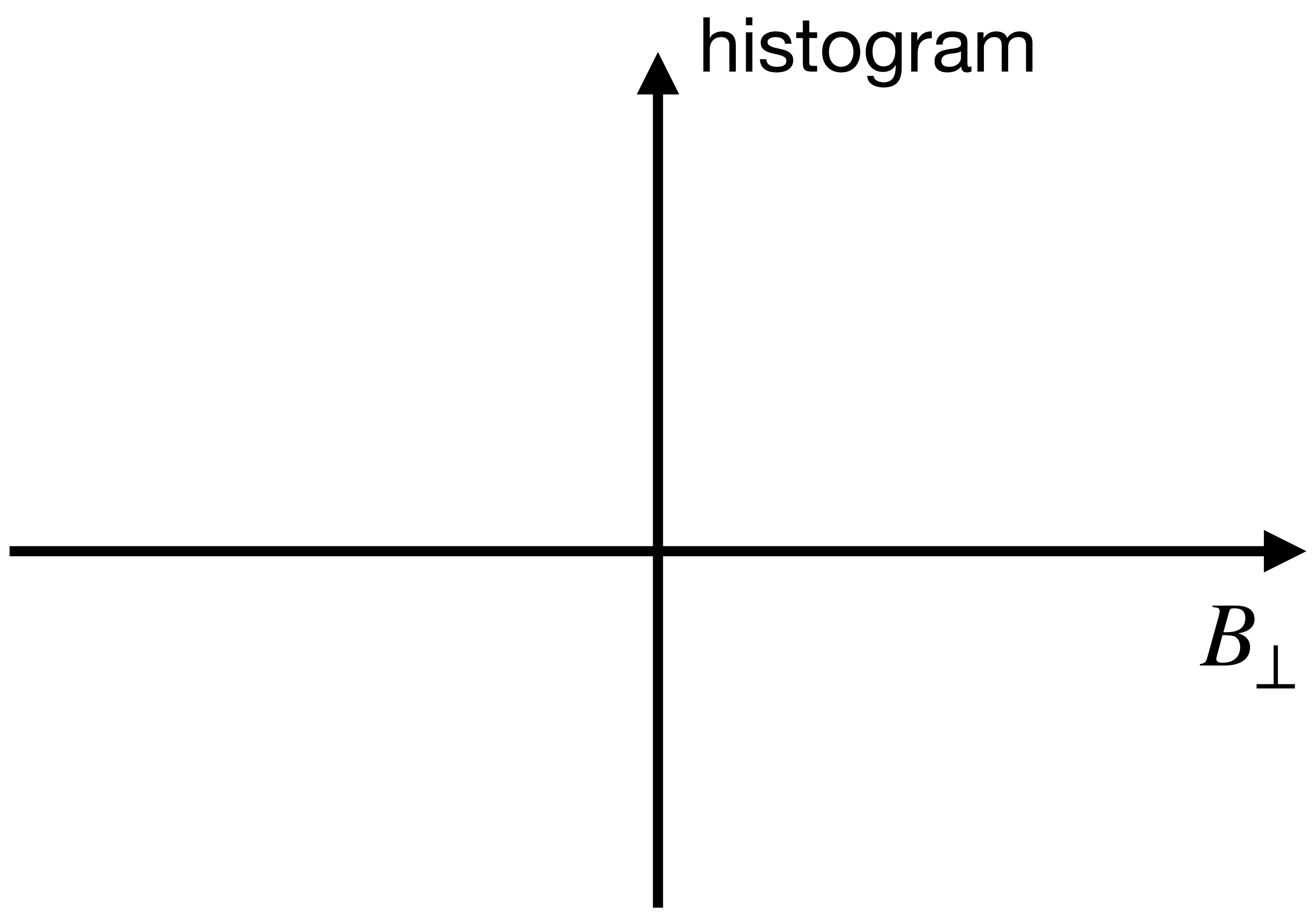

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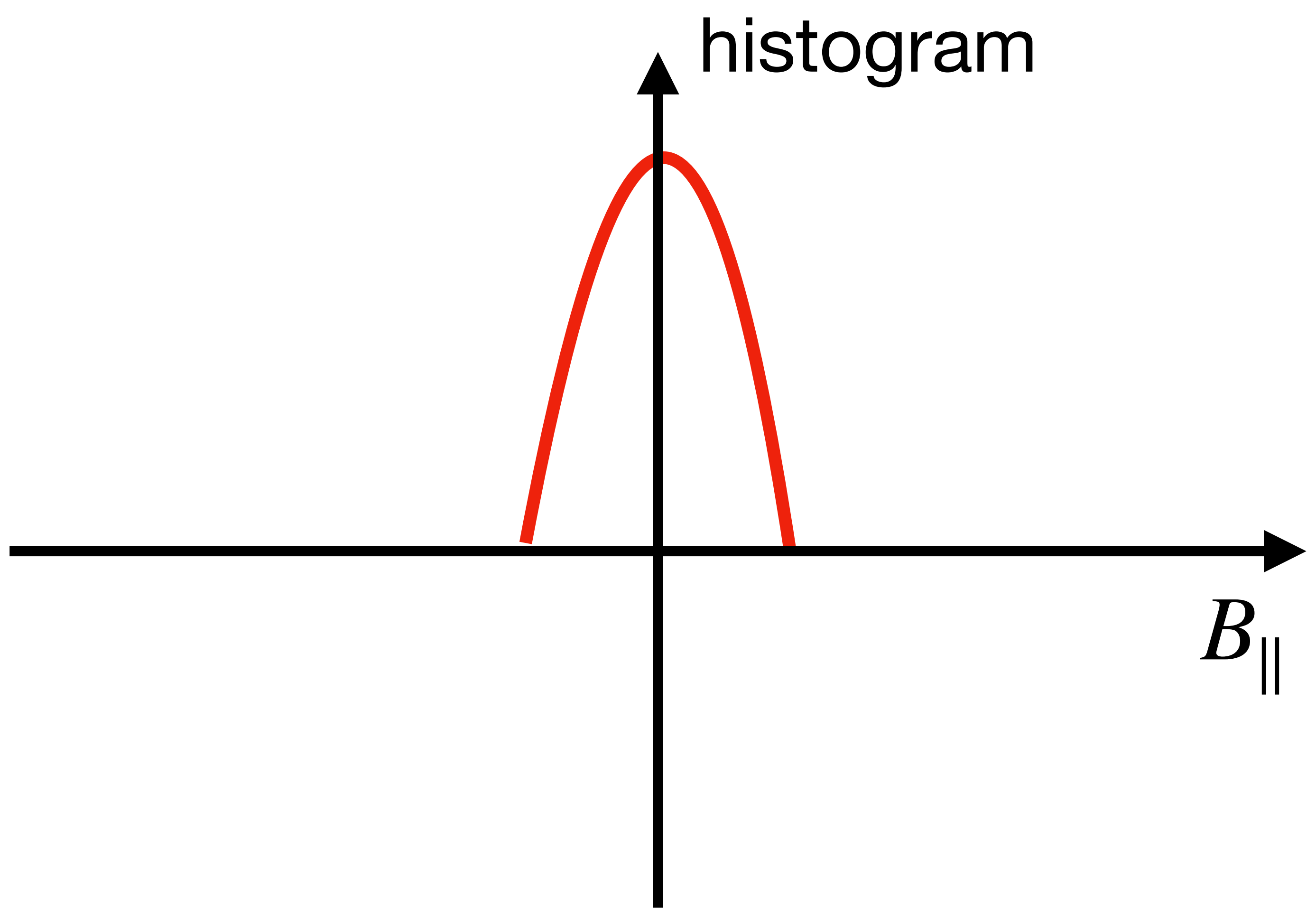

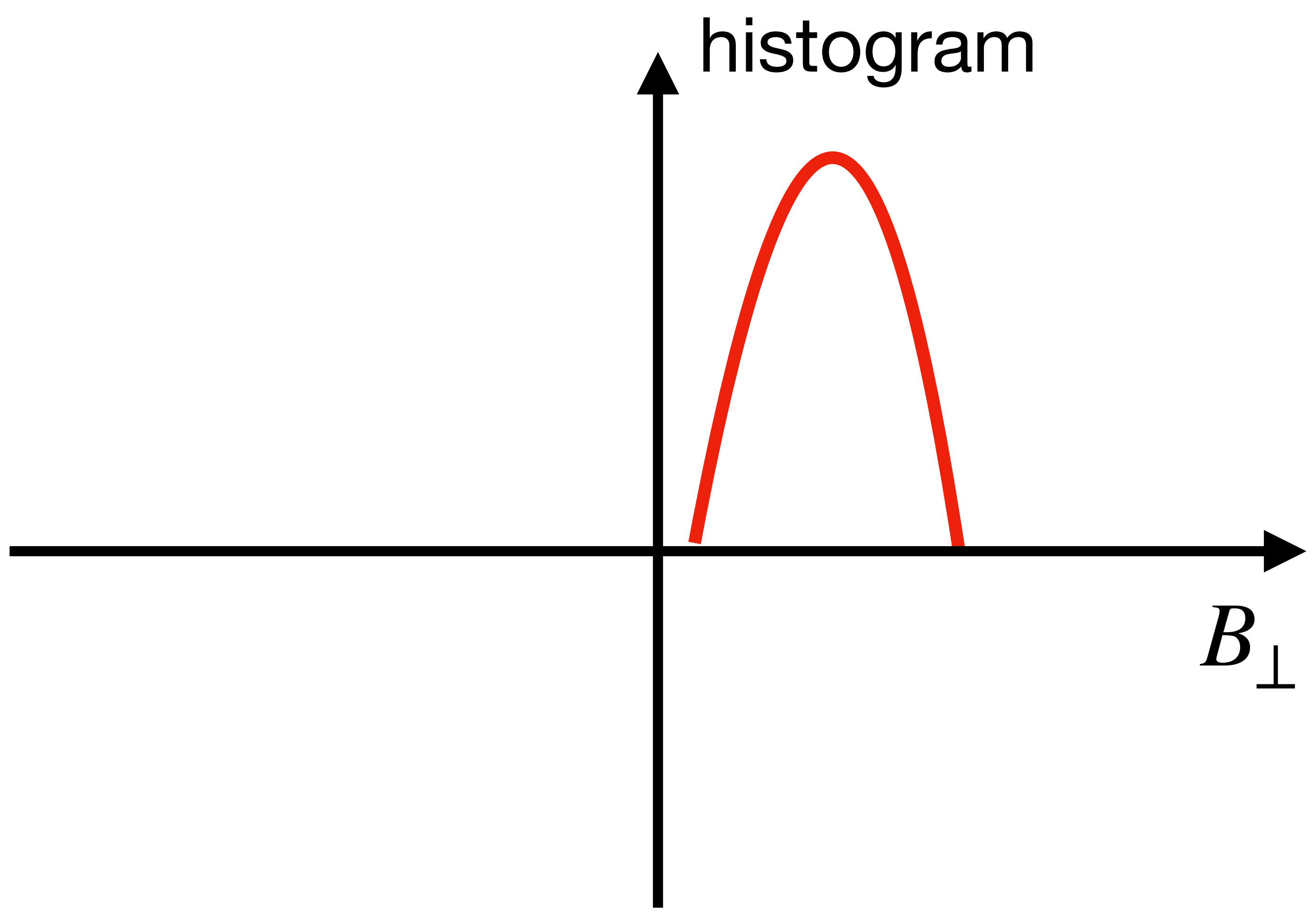

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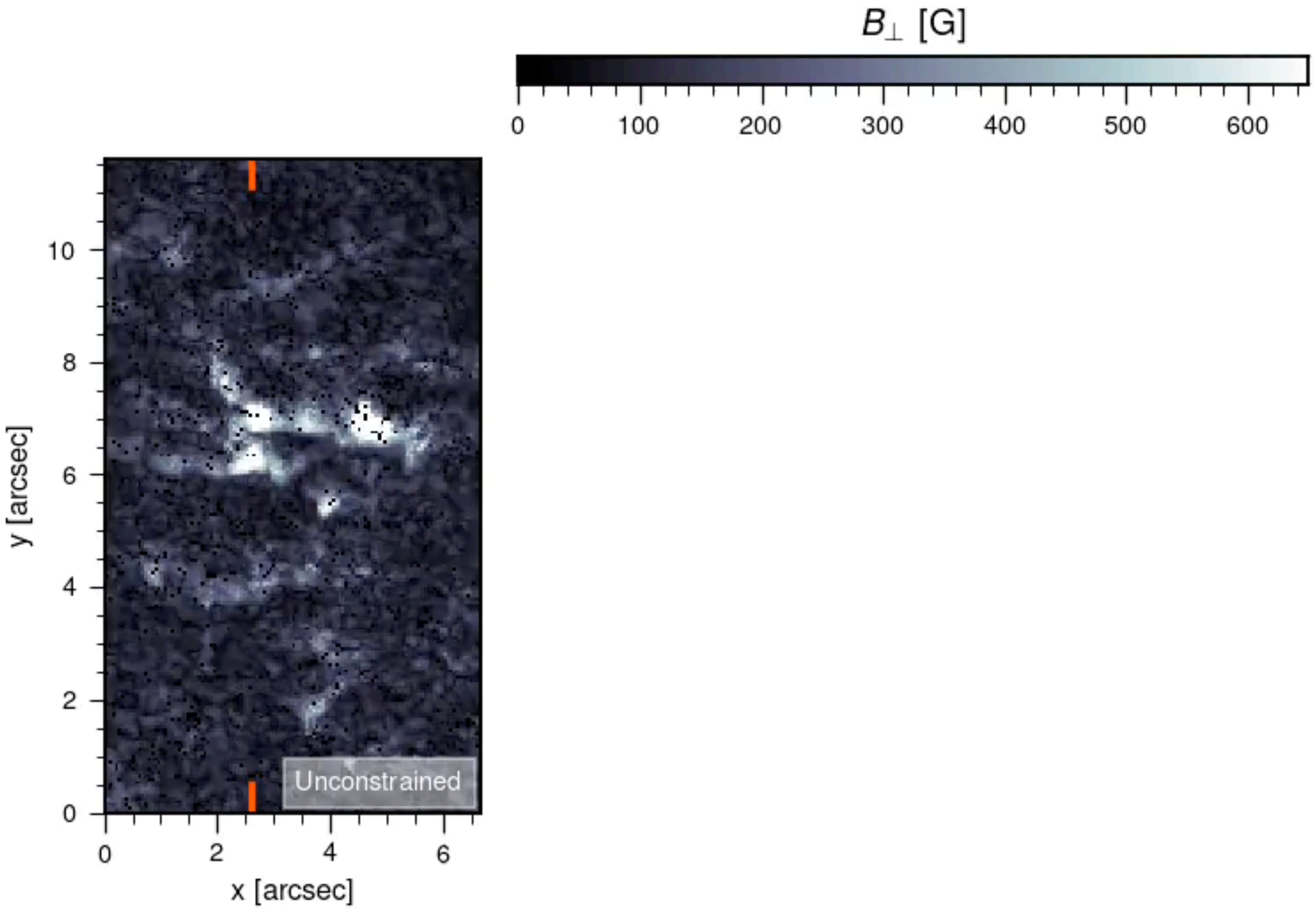

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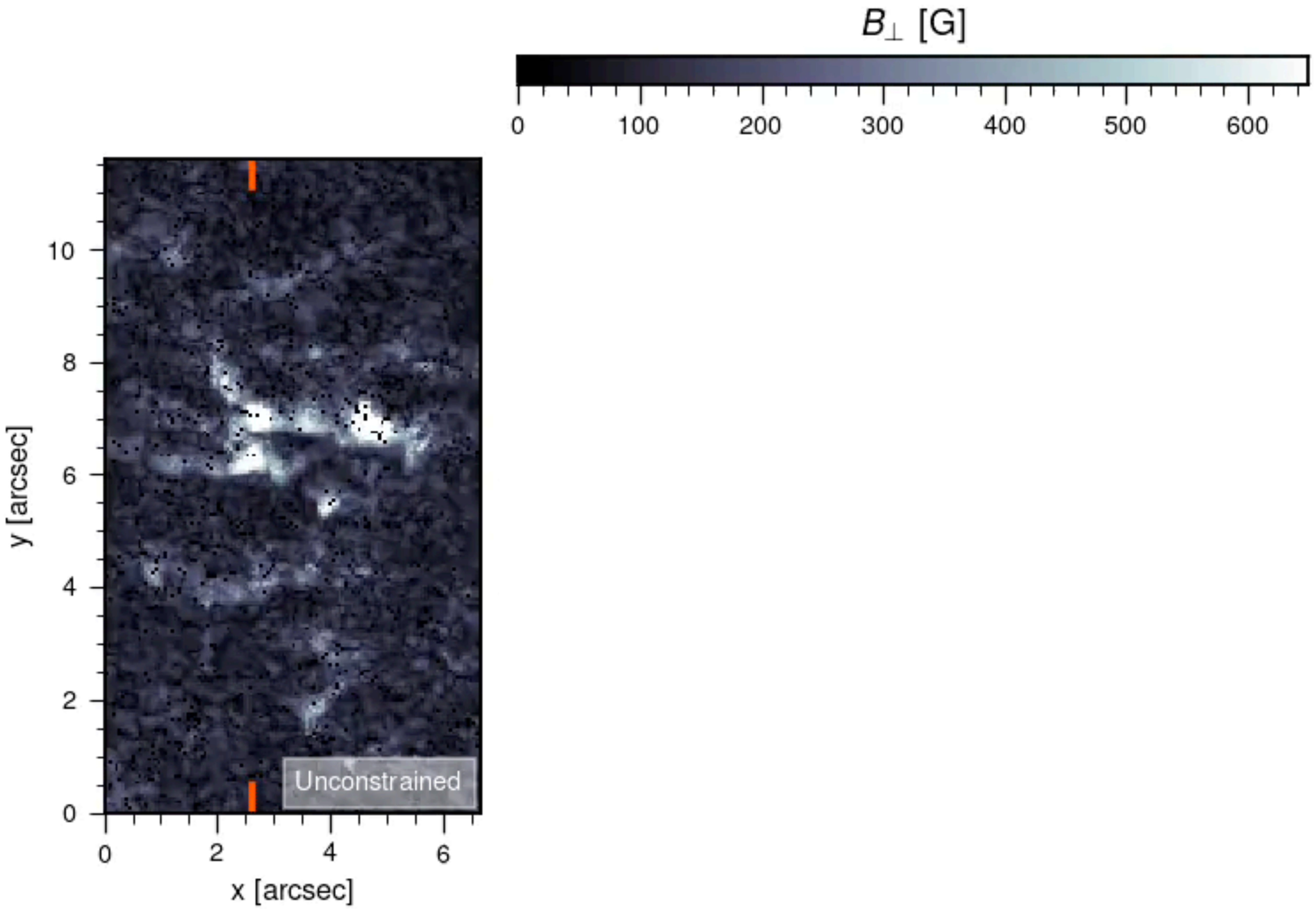

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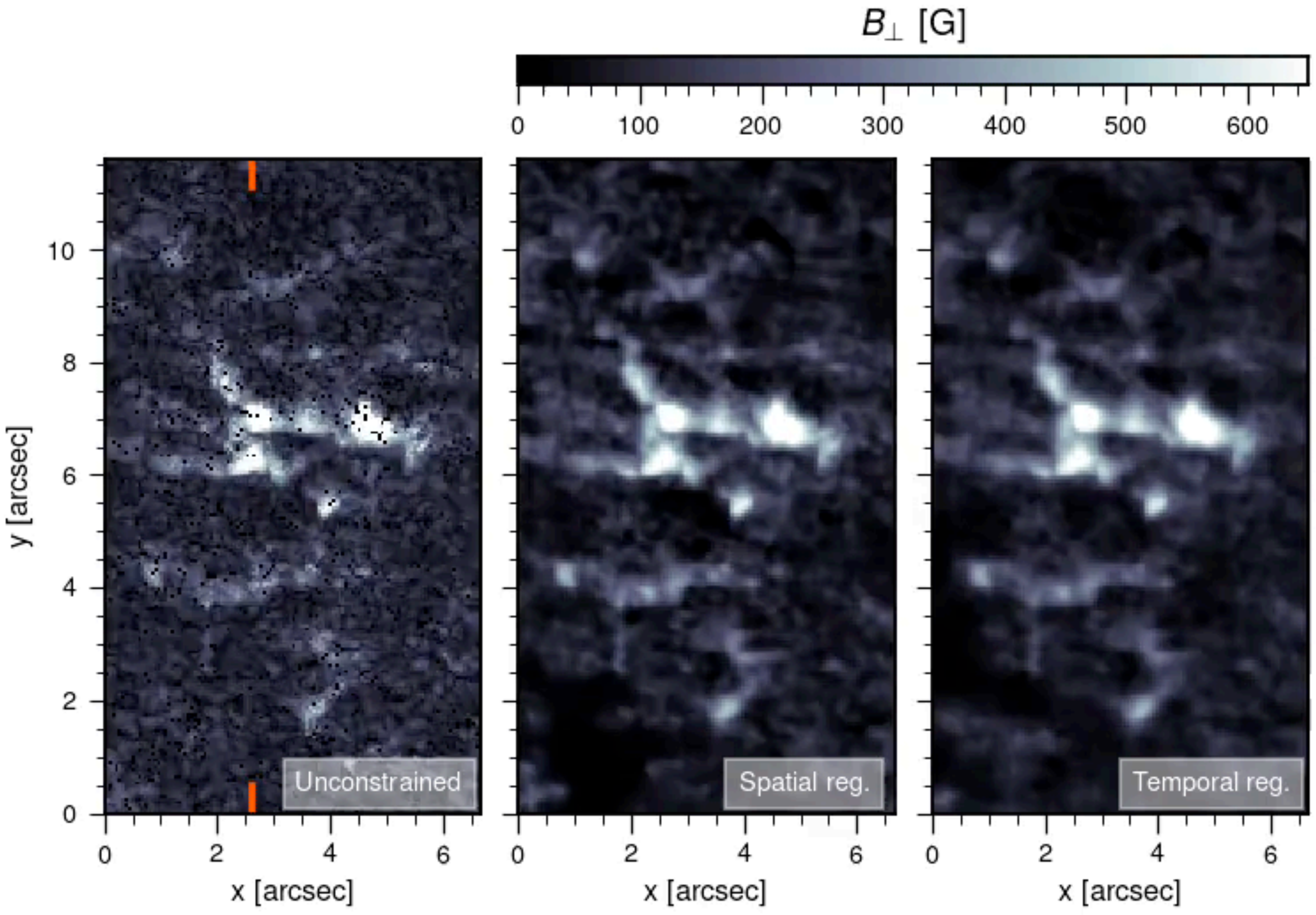

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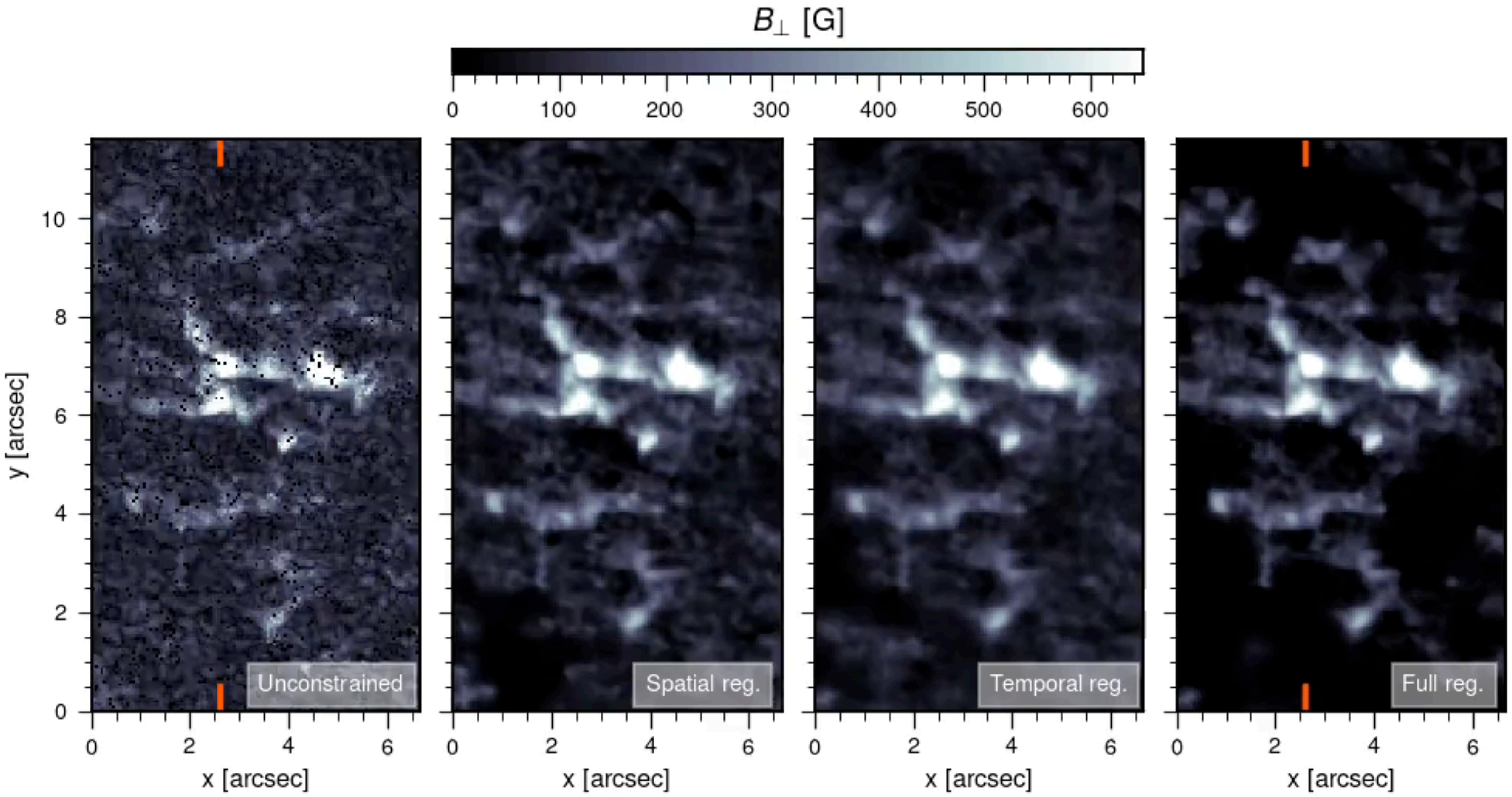

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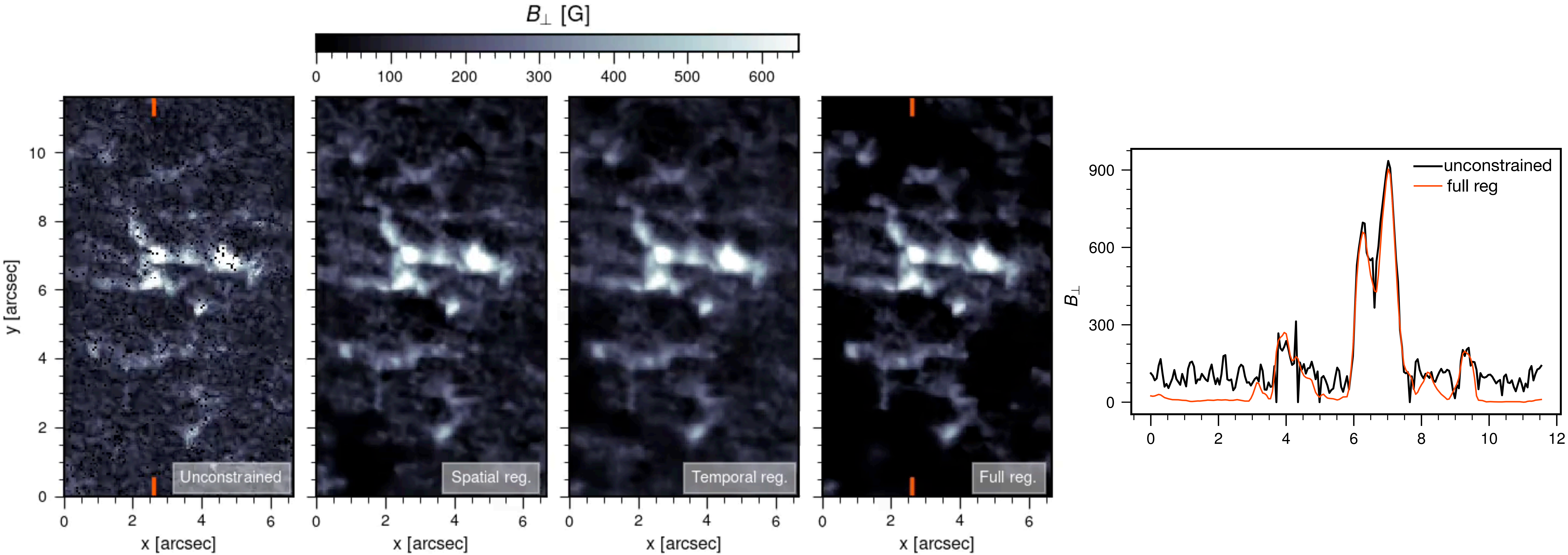

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Codes publicly available with commented examples:
https://github.com/jaimedelacruz/fullReg_wfa
https://github.com/jaimedelacruz/pyMilne

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