FastQSL: A Fast Computation Method for Quasi-separatrix Layers

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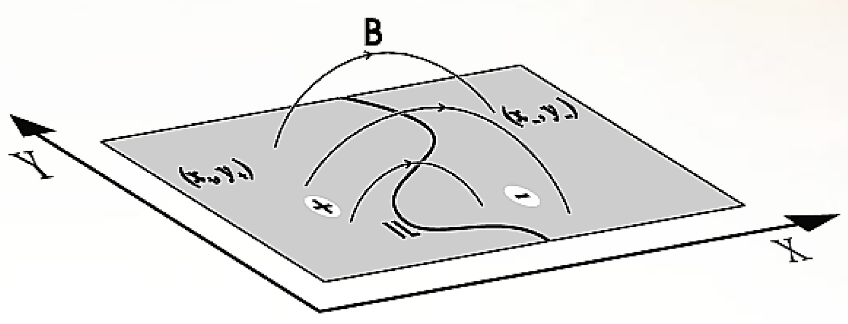
Squashing factor Q



$$(x_+, y_+) \rightarrow (x_-, y_-)$$

$$\underset{+-}{D} = \begin{pmatrix} \frac{\partial x_-}{\partial x_+} & \frac{\partial x_-}{\partial y_+} \\ \frac{\partial y_-}{\partial x_+} & \frac{\partial y_-}{\partial y_+} \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

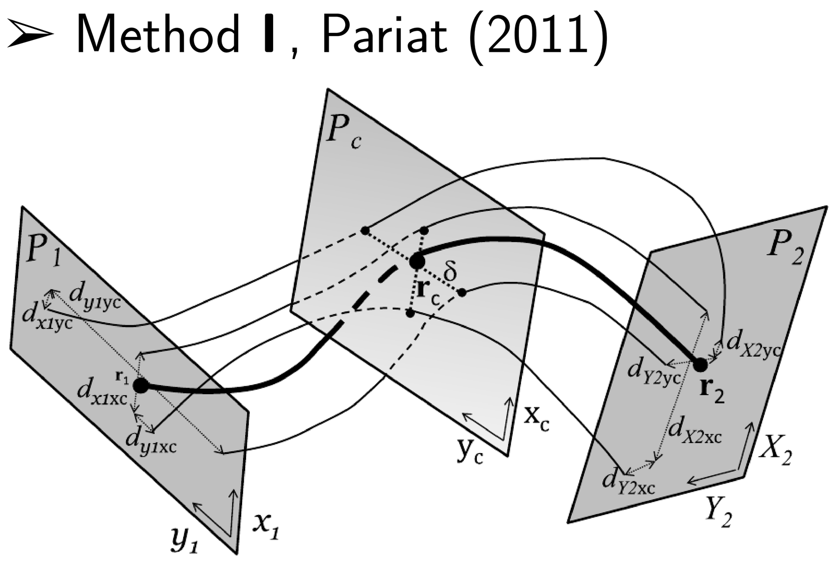
Titov (2002)

$$Q\,(x_+, y_+) = (a^2{+}b^2{+}c^2{+}d^2)\,/\,\left|\mathrm{Det}\,\underset{+-}{D}\right| = |\lambda_1/\lambda_2|{+}|\lambda_2/\lambda_1| = Q\,(x_-, y_-)$$

- $\boldsymbol{B}\cdot\nabla Q = 0$, Aulanier (2005). The value of Q is invariant along a field line
- $Q = \infty$: separatrix
- the theoretical minimum of Q is 2. If $Q \gg 2$: Quasi-separatrix layer (QSL), the preferential place to commence magnetic reconnetions

Numerical implementations

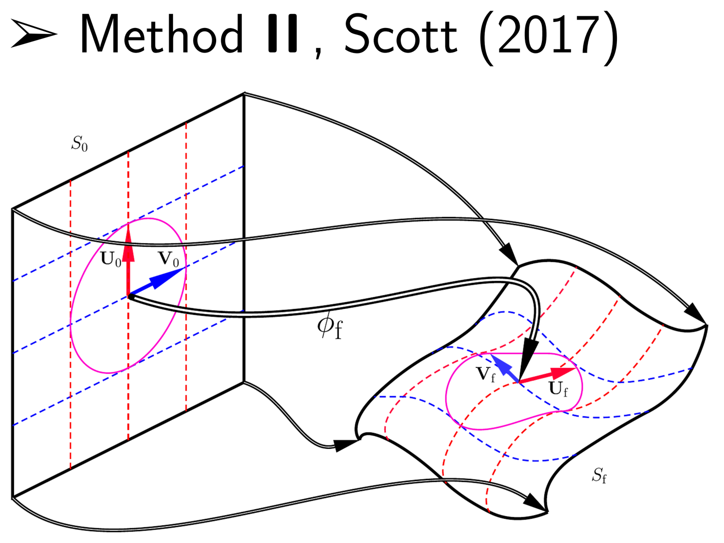
➢ Method **I**, Pariat (2011)



$$\underset{12}{D} = \begin{pmatrix} \frac{\partial x_2}{\partial x_c} & \frac{\partial x_2}{\partial y_c} \\ \frac{\partial y_2}{\partial x_c} & \frac{\partial y_2}{\partial y_c} \end{pmatrix} \begin{pmatrix} \frac{\partial x_c}{\partial x_1} & \frac{\partial x_c}{\partial y_1} \\ \frac{\partial y_c}{\partial x_1} & \frac{\partial y_c}{\partial y_1} \end{pmatrix}$$

Tracing 4 field lines from 4 start points, Q can be given by central differences

➢ Method **II**, Scott (2017)



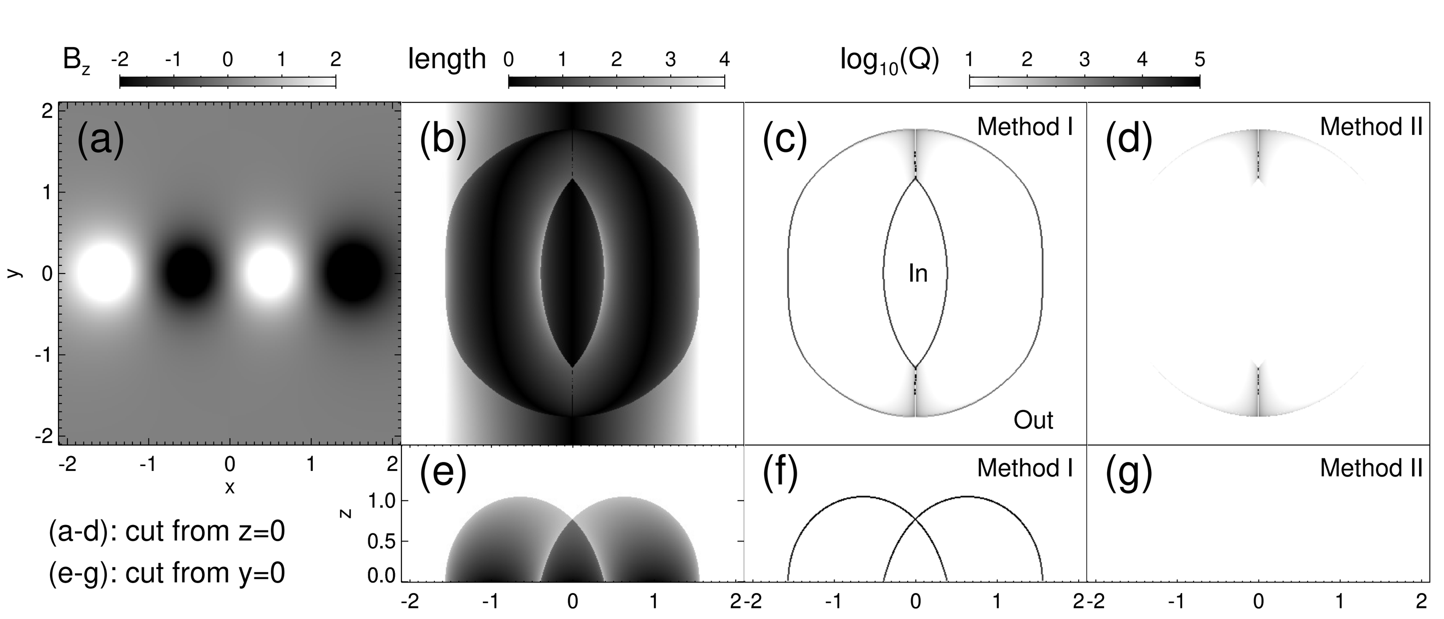
$$\frac{\mathrm{d}\{\boldsymbol{r}, \boldsymbol{U}, \boldsymbol{V}\}}{\mathrm{d}s} = \{\frac{\boldsymbol{B}}{B}, \boldsymbol{U}\cdot\nabla\frac{\boldsymbol{B}}{B}, \boldsymbol{V}\cdot\nabla\frac{\boldsymbol{B}}{B}\}$$

$$\tilde{\boldsymbol{U}}_f = \boldsymbol{U} - \frac{\boldsymbol{U}\cdot\boldsymbol{n}}{\boldsymbol{B}\cdot\boldsymbol{n}}\boldsymbol{B}\Big|_{S_f}$$

$$Q = \frac{\tilde{\boldsymbol{U}}_f^2\tilde{\boldsymbol{V}}_b^2 + \tilde{\boldsymbol{U}}_b^2\tilde{\boldsymbol{V}}_f^2 - 2(\tilde{\boldsymbol{U}}_f\cdot\tilde{\boldsymbol{V}}_f)(\tilde{\boldsymbol{U}}_b\cdot\tilde{\boldsymbol{V}}_b)}{(B_{n,0})^2/(B_{n,f}\,B_{n,b})}$$

Method **II** have a better accuracy

Failed capturing of separatrix by Method II



If the distribution of Q likes the Dirac Delta fuction, then the values of Q on grids are small still, and the result from Method **II** can't present the full separatrix

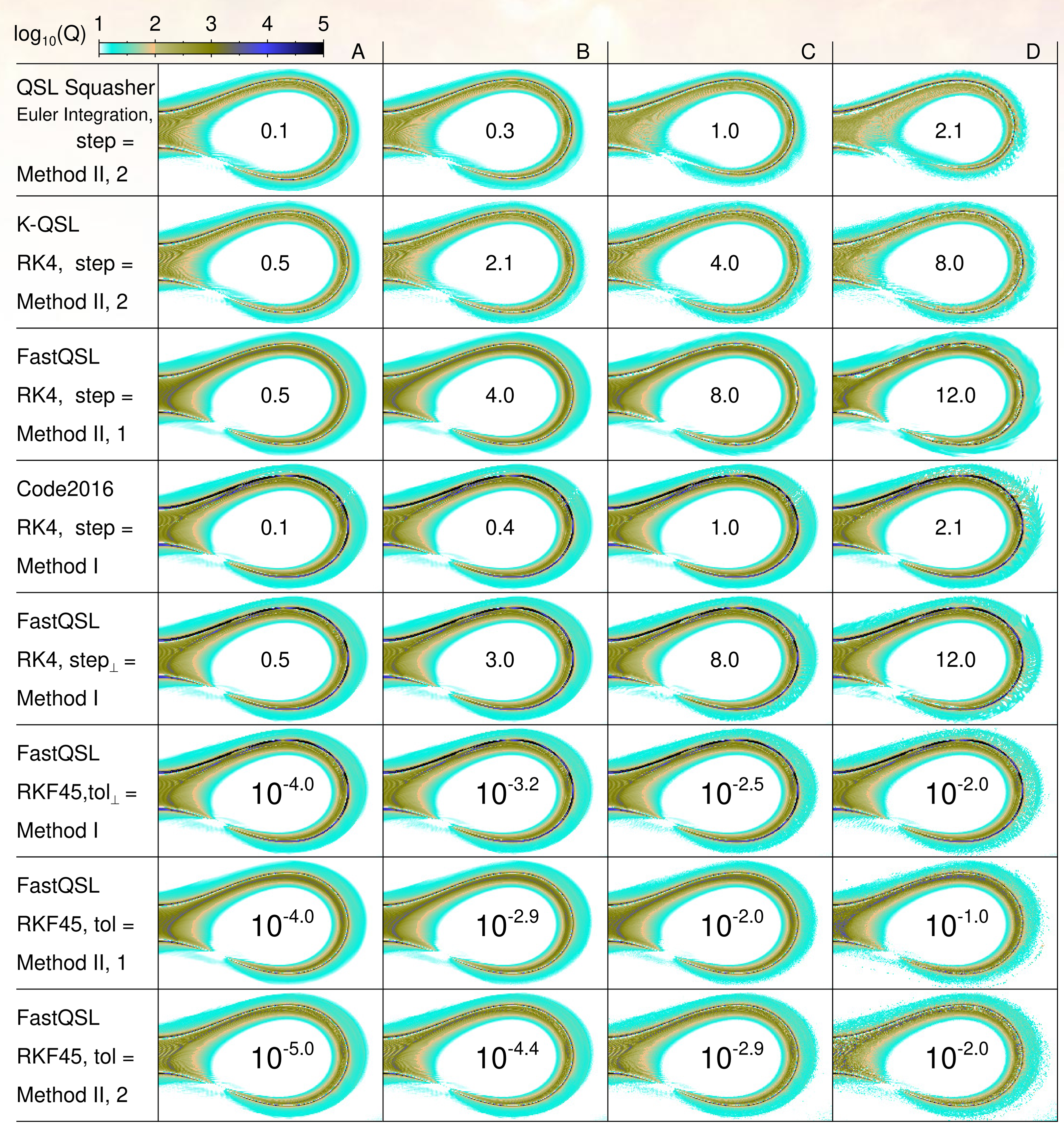
Published codes for QSL-calculation

QSL Squasher	`https://bitbucket.org/tassev/qsl_squasher/src/hg/`
K-QSL	`https://github.com/Kai-E-Yang/QSL`
UFiT	`https://github.com/Valentin-Aslanyan/UFiT`
Code2016	`http://staff.ustc.edu.cn/~rliu/qfactor.html`
FastQSL	`https://github.com/el2718/FastQSL` `https://github.com/Pjer-zhang/FastQSL`

Features of FastQSL

- supports uniformed and stretched grids
- uses 3/8-rule RK4, RKF45 to trace magnetic field lines
- supports the methods of Pariat (2011) and Scott (2017)
- supports Linux, MacOS, and Windows
- outputs: Q, twist number, field line length, ratio of B_n at two footpoints, and coordinates of mapped footpoints

Benchmark



* no result from UFiT here

Compared with column A, the images in column B with the marginal value should not show any recognizable difference. Therefore column B presents **marginal ground truth image**s of Q-map.

Code	Processor	Compiler	Method	Tracing scheme	Parameter	Performance
QSL Squasher	CPU	OpenCL/clang	**II**, 2	Euler integration	step $= 0.3$	29 kQ s^{-1}
	GPU					189 kQ s^{-1}
K-QSL	CPU	gfortran		classic RK4	step $= 2.1$	27 kQ s^{-1}
Code2016		ifort	**I**		step $= 0.4$	191 kQ s^{-1}
FastQSL		gfortran		3/8-rule RK4	step$_\perp = 3.0$	749 kQ s^{-1}
		ifort				854 kQ s^{-1}
			II, 1		step $= 4.0$	907 kQ s^{-1}
				RKF45	tol $= 10^{-2.9}$	1.11 MQ s^{-1}
			I		tol$_\perp = 10^{-3.2}$	1.43 MQ s^{-1}
	GPU	CUDA/C				4.53 MQ s^{-1}
			II, 2		tol $= 10^{-4.4}$	1.13 MQ s^{-1}

* **Parameter** is taken from the values in column B

* on Intel core i9 10900K, RTX 3070 OC

* no result from UFiT here

Spherical extension of FastQSL is coming

For a PFSS extrapolation from HMI magnetogram, the spherical extension of FastQSL achieves a global Q-map (1347×713) in 5 seconds.