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Moment closure problem: equation discovery and deep learning techniques applied to kinetic plasma simulations

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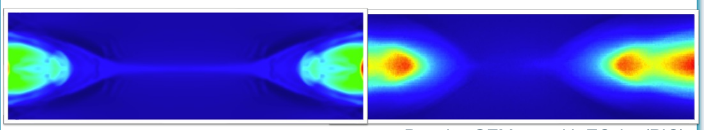
Introduction

- A major unresolved issue for magnetic reconnection: **the multi-scale interactions with mesoscale and global scale dynamics** [1]
- We address this issue by seeking **fluid closures** that capture kinetic effects using Deep Learning (DL)
- The goal is to find **functional relationship between higher and lower order moments** to close the hierarchy. To be implemented in COOLFluiD.



e Density, GEM run with COOLFluiD — e Density, GEM run with ECsim (PIC)

Methods

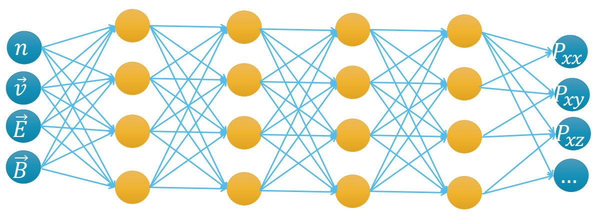
Collisionless: $$\frac{\partial f_s}{\partial t} + v_i \frac{\partial f_s}{\partial x_i} + \frac{q_s}{m_s}\left(E_i + \varepsilon_{abi} v_a B_b\right)\frac{\partial f_s}{\partial v_i} = 0$$

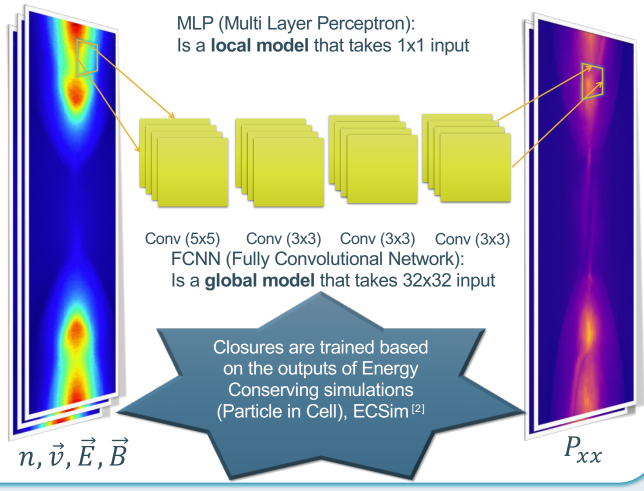
Moments of the VDF: $$\int d^3v f_s \underbrace{v_l v_m v_n \ldots}_{k\text{-times}} = \mu^{(k)}_{s;l,m,n\ldots}$$

$$\frac{\partial \mu^{(k)}_{s;l,m,n\ldots i}}{\partial t} = -\frac{\partial \mu^{(k+1)}_{s;l,m,n\ldots,j}}{\partial x_j} + \frac{q_s}{m_s}\sum_{\substack{\text{cyclic perm.}\\ \text{of free indices}}} \mu^{(k-1)}_{s;l,m,n\ldots} E_i + \varepsilon_{ipq}\mu^{(k)}_{s;l,m,n\ldots,p} B_q$$

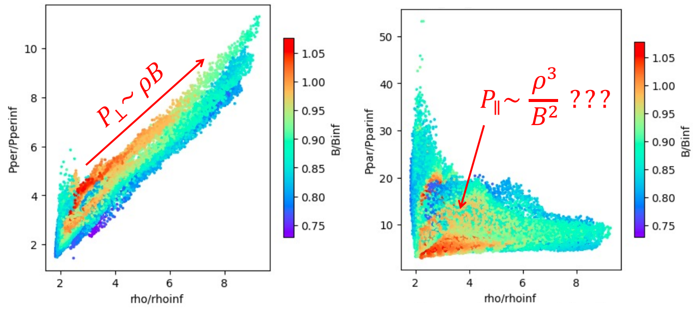
$\mu^{(1)}_s = n_s \mathbf{u}_s$: particle flux density

$\mu^{(2)}_s m_s = \mathbb{P}_s$ stress tensor — Energy flux tensor $\mu^{(2)}_s \frac{m_s}{2} = \mathbb{Q}_s$





Results

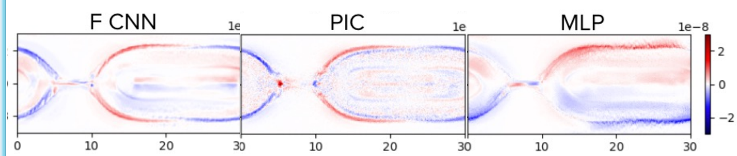


Perpendicular and parallel pressure during the nonlinear stage of reconnection

● New run ○ Old run

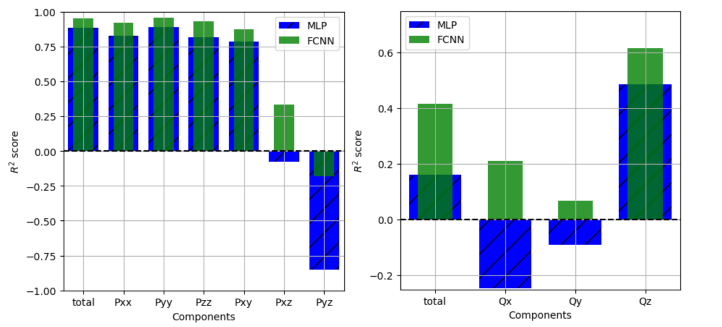
B_z/B_x	0	0.05	0.1	0.2	0.3	0.4	0.6	1	2	3
	test	train	train	train	train	train	train	train	train	train

A table with the choice of guide fields for each **ECsim** [2] (Particle in Cell) simulation



MLP vs FCNN reconstruction of heat flux x-component on a hold-out test run.

Coefficient of determination: $$R^2 = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \bar{y})^2}$$



Side-by-side comparison of the total determination score (1 = perfect reconstruction, less than 0 = poor) for MLP and FCNN for pressure tensor and heat flux vector components

Conclusions

- We report significant improvements compared to local Multi-Layer Perceptron (MLP) driven closure[1]
- This is achieved by exploiting **global information** using **Fully Convolutional (FCNN) architecture**.

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[1] Nakamura, R. et al, arXiv July 12, **2024**
[2] Lapenta, G. et al, J. of Comp. Phys, 334:349–366, **2017**
[3] Laperre, B. et al, Phys. Plasmas **2022**, 29 (3), 032706

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