The Sun's differential rotation is controlled by baroclinically-unstable high-latitude inertial modes

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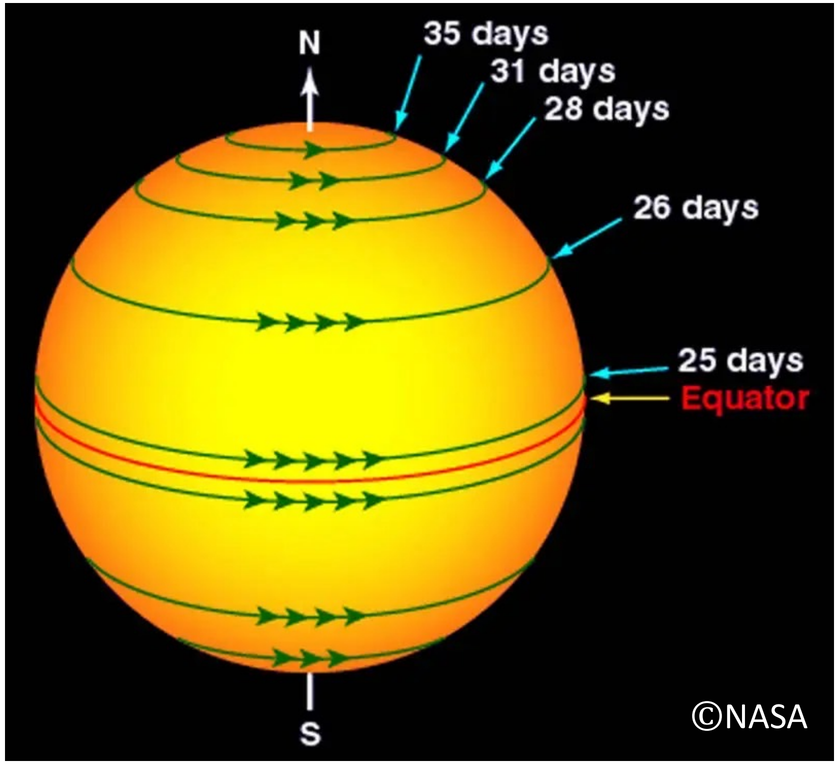
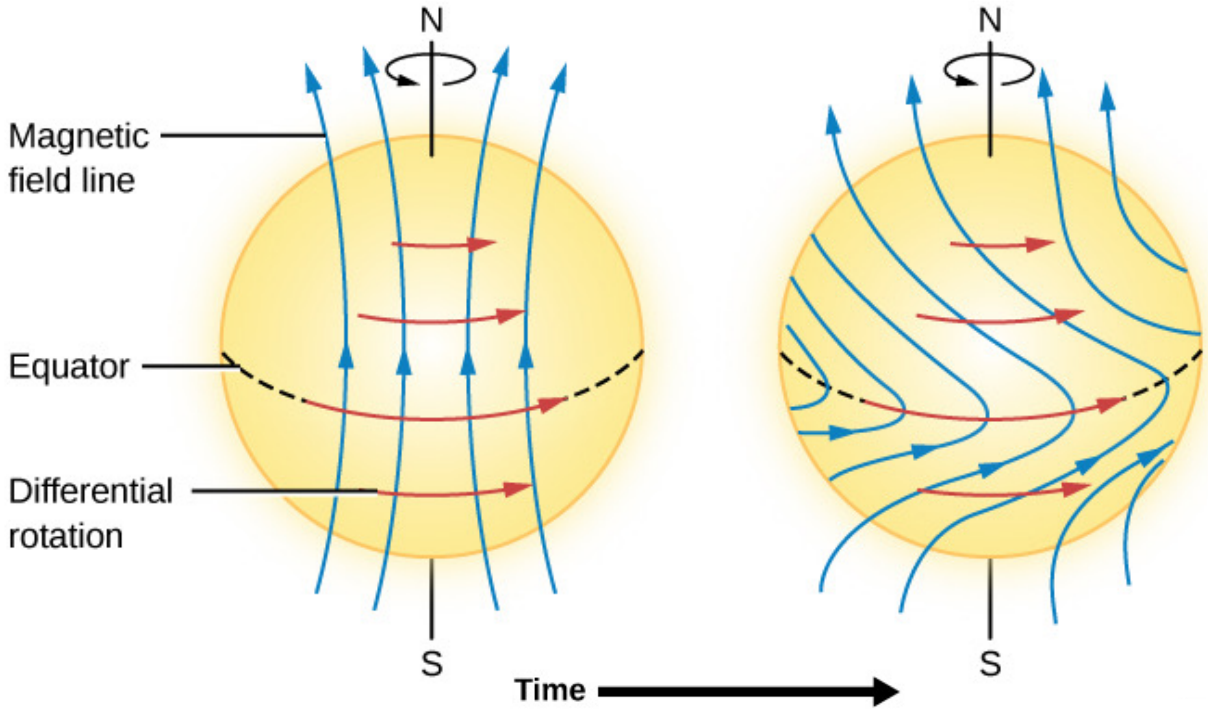
The Sun's differential rotation

- The Sun's outer 30% is called **convection zone**
- The Sun's convection zone is known to rotate differentially, i.e., the **equator rotates faster** ($P \approx$24 days) whereas the **poles rotate slower** ($P \approx$35 days)
- This differential rotation is believed to **play a crucial role in sustaining the Sun's** magnetic activity via Ω-effect





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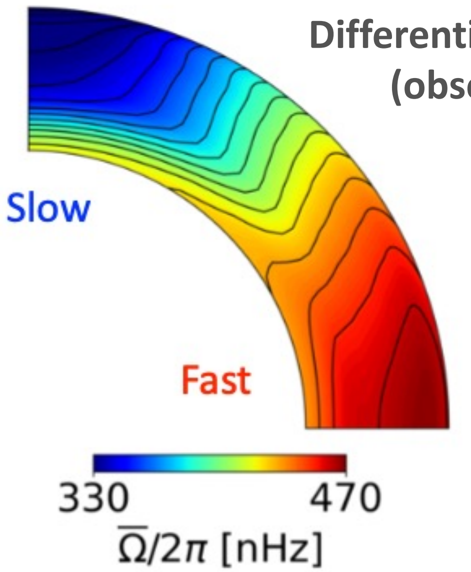
Thermal wind balance: Role of the latitudinal entropy gradient

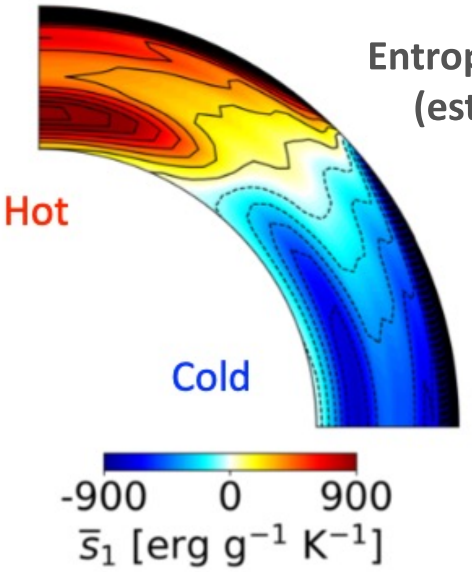
- Internal angular velocity profile (**differential rotation**) of the Sun is precisely measured by **global helioseismology** [e.g., Schou et al. 1998]
- The solar differential rotation does not follow the **Taylor-Proudman's theorem** which predicts constant rotation rates on cylinder
- To break the Taylor-Proudman's constraint, **latitudinal entropy difference** is believed to exist in the Sun's convection zone



Thermal wind balance approx.

$$r\sin\theta \frac{\partial \Omega^2}{\partial z} \approx \frac{g}{r c_p}\frac{\partial s_1}{\partial \theta}$$

Deviation from TP theorem — Latitudinal entropy gradient



- The estimate latitudinal entropy difference corresponds to **a latitudinal temperature difference of $\Delta_\theta T$ ($=T_{\text{pole}}-T_{\text{eq}}$) $\approx$ 5-10 K** [e.g., Miesch 2005], which is small and very difficult to measure [e.g., Kuhn et al. 1997, Rast et al. 2008]

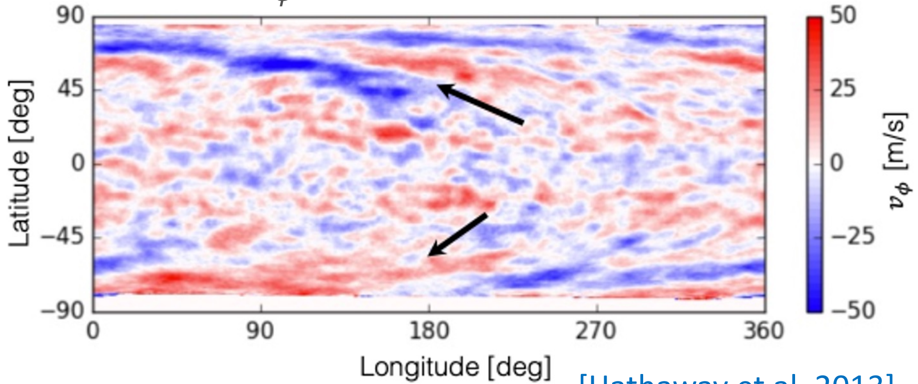
Polar spiral flows → *High-latitude inertial modes*

- Near the polar region of the Sun, **spiraling flow patterns** are observed [Hathaway et al. 2013, Bogart et al. 2015]
- Largely consist $m = 1$ component and have the velocity amplitude of $20 - 30\mathrm{m/s}$ at the surface
- Often **(mis)interpreted as the giant convection cells** from the deep interior that transport the angular momentum equatorward [Hathaway et al. 2013, Hathaway & Upton 2021]

⇩

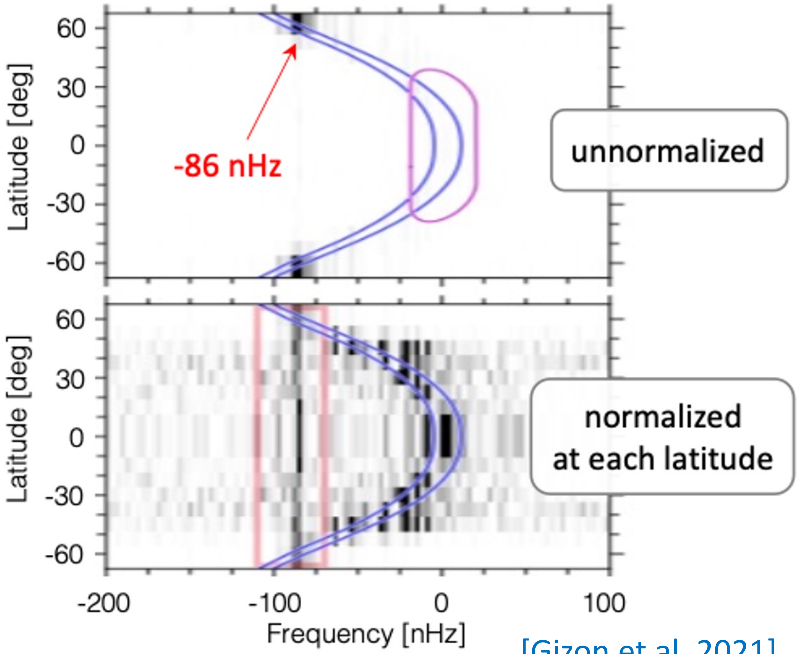
- Recently, Gizon et al. (2021) instead showed that these polar flows are **characterized by single frequency at all latitudes** (once the spectrum is normalized)
- Indicating that they are *global-scale* ***modes of inertial oscillation*** (where the restoring force is the Coriolis force)
- We call them **"high-latitude inertial modes"**

v_ϕ (SG tracking map)



[Hathaway et al. 2013]

Power spectrum of $m = 1$ v_ϕ

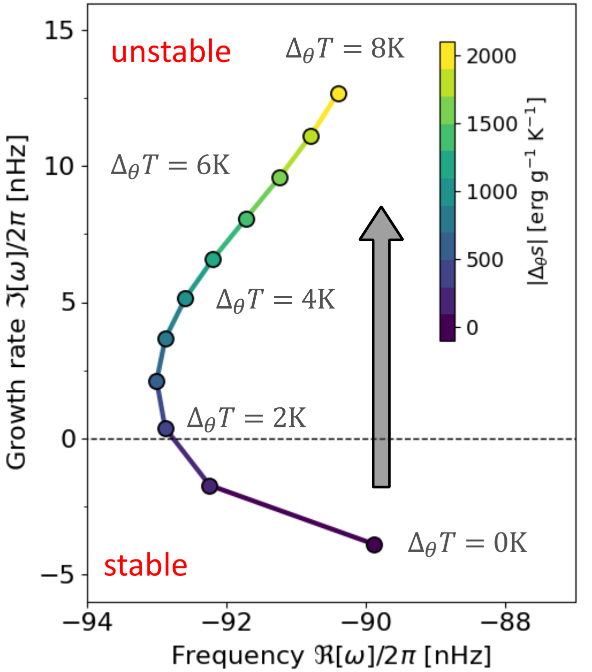


[Gizon et al. 2021]

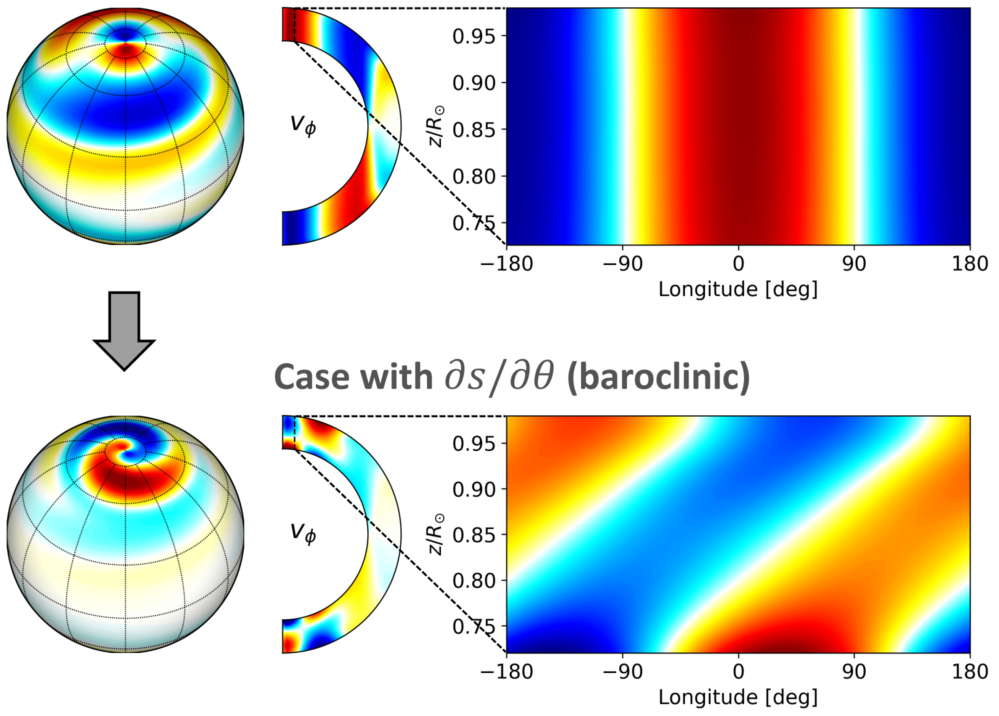
High-latitude inertial modes are baroclinically unstable

- Bekki et al. (2022a) carried out a **linear eigenmode analysis** to show that the **high-latitude modes become linearly unstable when a latitudinal entropy gradient $\partial s/\partial\theta$ exists, i.e., baroclinically unstable**
- **The observed spiral pattern** can be obtained only when a large $|\partial s/\partial\theta|$ is included
- The dispersion relation of the baroclinically-unstable modes agree well with the observations

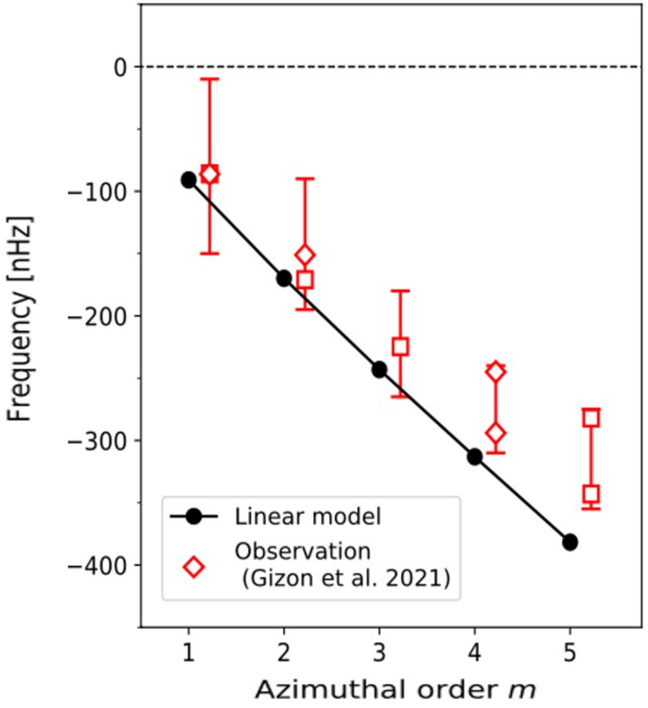
Linear eigenfrequency ($m = 1$)



Case without $\partial s/\partial\theta$ (non-baroclinic)



Dispersion relation



3D mean-field simulations of solar large-scale flows

- To study the **amplitudes of the high-latitude inertial modes**, we carry out a set of **mean-field** simulations of the solar large-scale flows in a **3D** spherical shell [Bekki & Cameron 2023, Bekki et al. 2024]
- **Small-scale convection is NOT solved** but parametrized and modelled (e.g., Λ-effect, turbulent diffusion)

mass:
$$\frac{\partial \rho_1}{\partial t} = -\frac{1}{\xi^2}\nabla\cdot(\rho_0 \boldsymbol{v}),$$

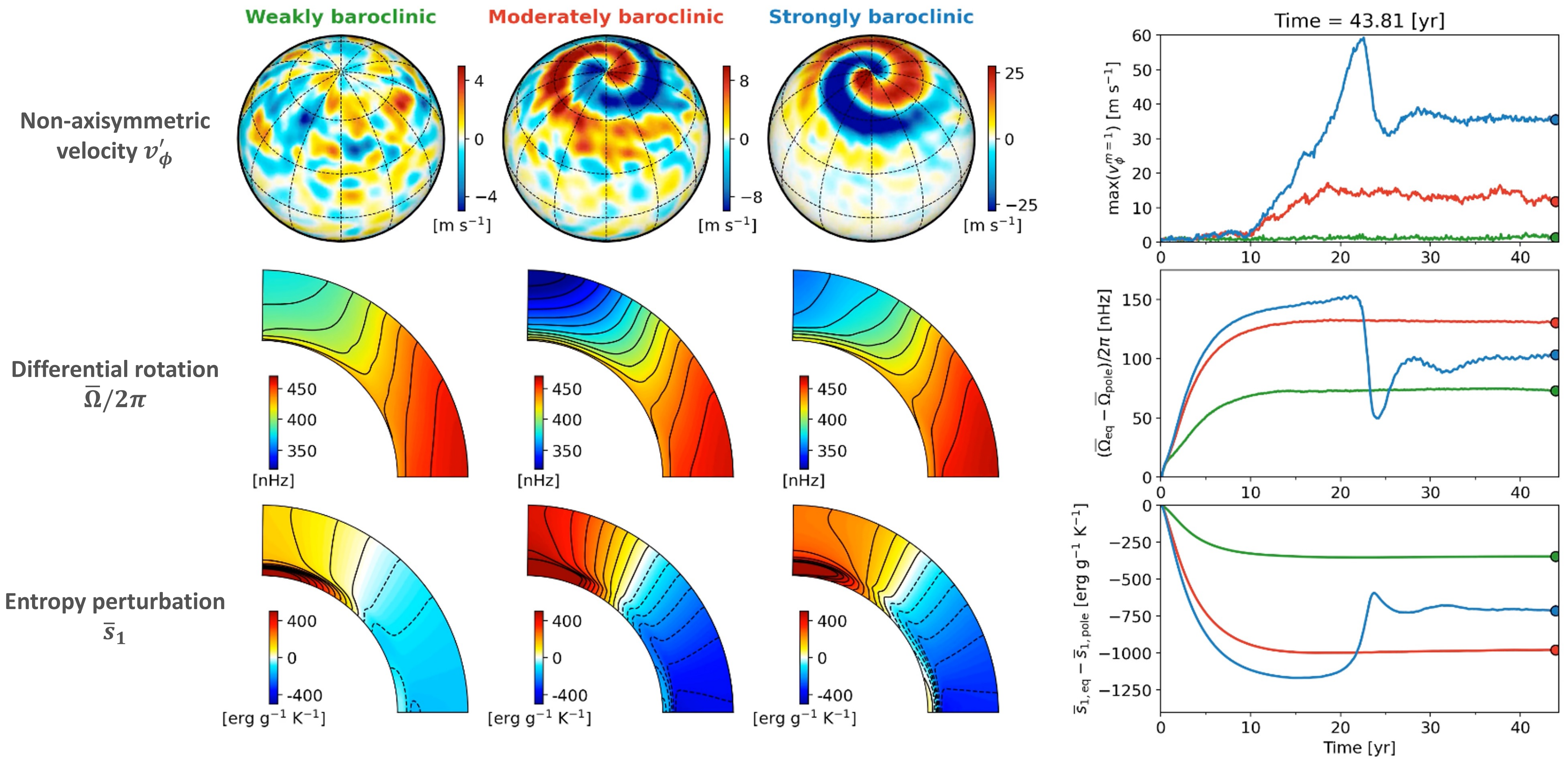
motion:
$$\frac{\partial \boldsymbol{v}}{\partial t} = -\boldsymbol{v}\cdot\nabla\boldsymbol{v} - \frac{\nabla p_1}{\rho_0} - \frac{\rho_1}{\rho_0} g \boldsymbol{e}_r + 2\boldsymbol{v}\times\Omega_0 \boldsymbol{e}_z + \frac{1}{\rho_0}\nabla\cdot\boldsymbol{\mathcal{R}},$$

entropy:
$$\frac{\partial s_1}{\partial t} = -\boldsymbol{v}\cdot\nabla s_1 + c_p\delta\frac{v_r}{H_p} + \frac{1}{\rho_0 T_0}\nabla\cdot(\rho_0 T_0 \kappa \nabla s_1) + \frac{1}{\rho_0 T_0}(\boldsymbol{\mathcal{R}}\cdot\nabla)\cdot\boldsymbol{v},$$

Reynolds stress:
$$\mathcal{R}_{ik} = \rho_0 \nu \left[\left(\mathcal{S}_{ik} - \frac{2}{3}\delta_{ik}\nabla\cdot\boldsymbol{v} \right) + \Lambda_{ik}\Omega_0 \right],$$

- **Differential rotation** and **meridional circulation** are driven by prescribed Λ–effect (following Rempel 2005's 2D model)
- **Base of the convection zone is assumed to be weakly subadiabatic** ($\delta < 0$) by which the latitudinal entropy gradient is generated via the interaction with the meridional circulation [Rempel 2005]
- **We vary the subadiabaticity at the base $\delta_0 (< 0)$ as a free parameter which controls the baroclinicity**

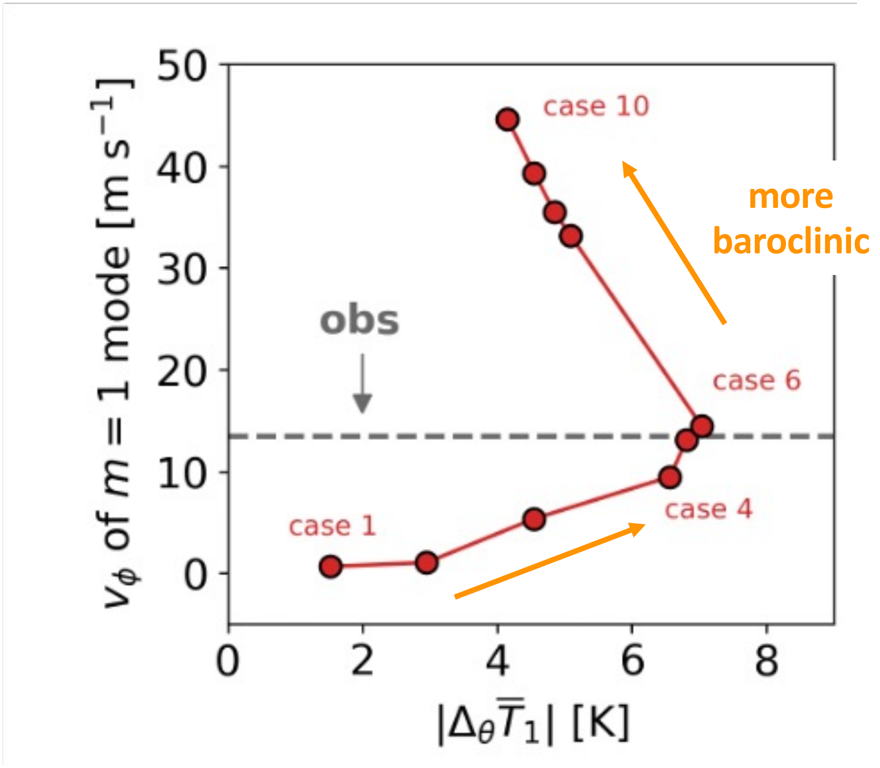
Nonlinear evolution of baroclinically-unstable modes

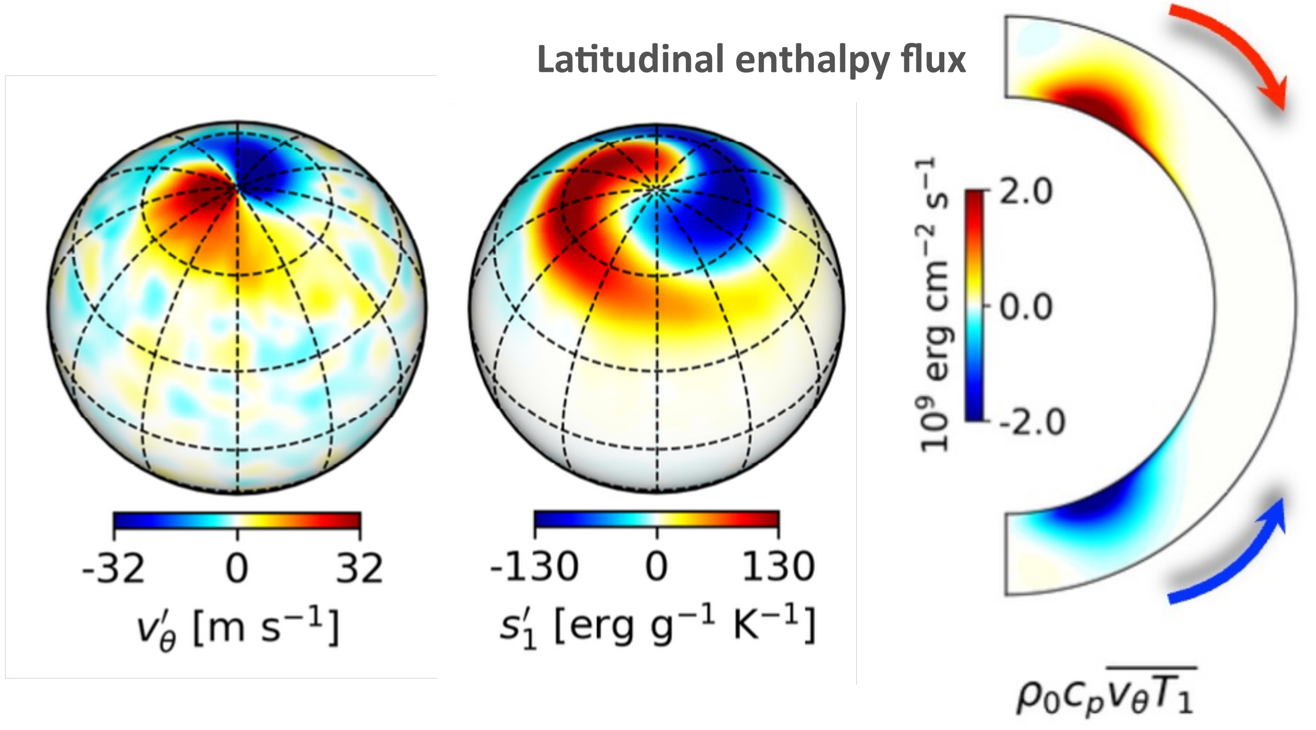


Nonlinear saturation: Equatorward heat transport by modes

- We find that both **amplitudes of baroclinic modes and $\Delta_\theta T$ initially increase** with increasing baroclinicity
- However, when the mode amplitudes exceed a threshold value, they start to give **a significant negative feedback**,

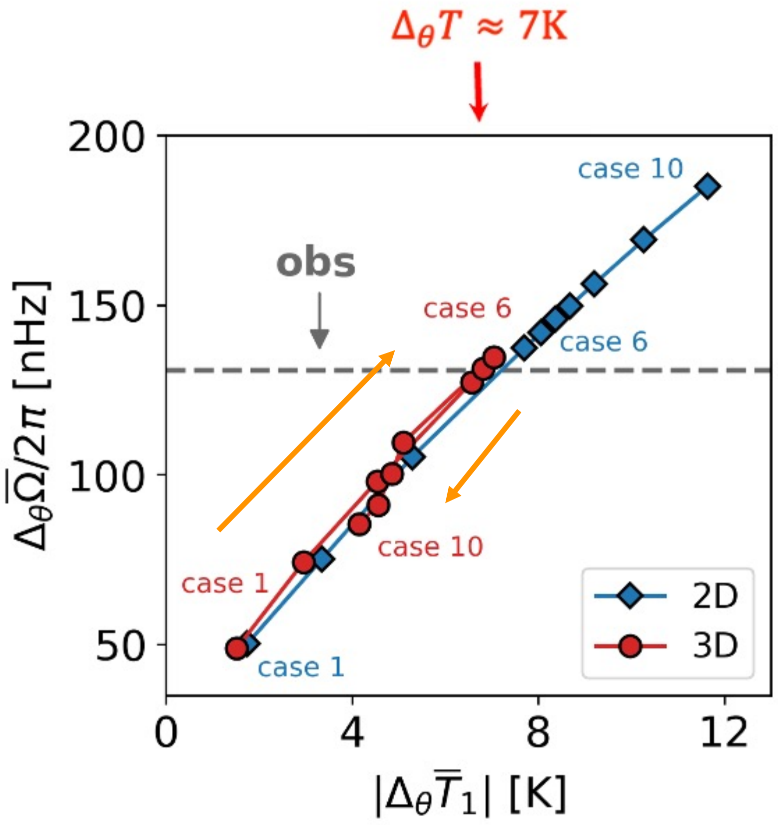
  i.e., **$\Delta_\theta T$ decreases** as baroclinic forcing increases and mode amplitudes increase
- This is because of **the equatorward heat transport** by baroclinic modes, which reduces $\Delta_\theta T$ (nonlinear saturation)
- **The observed mode amplitudes imply $\Delta_\theta T \approx$ 7K in the middle convection zone** (observational evidence of the thermal wind balance)

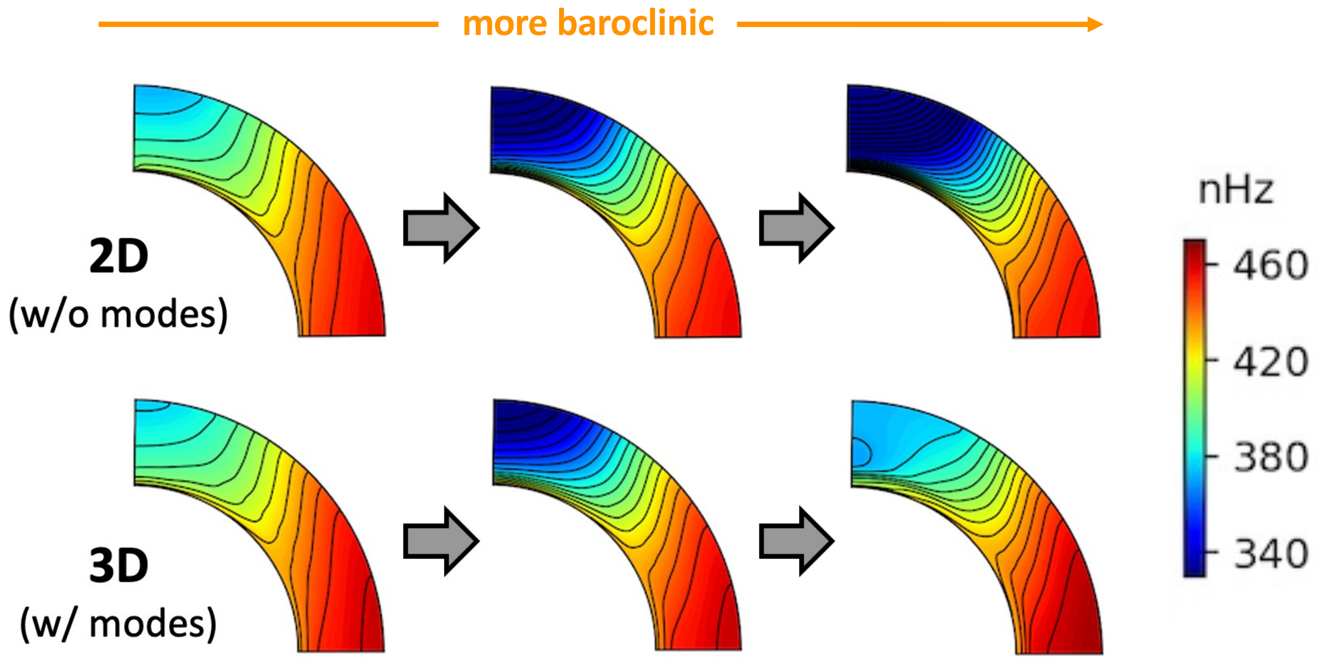




Significant impact on differential rotation amplitudes

- In **2D axisymmetric model (where the baroclinic modes are excluded)**, latitudinal differential rotation amplitudes $\Delta_\theta\Omega$ can be increased by changing the model parameters
- In realistic 3D model (where the non-axisymmetric modes play a role), **$\Delta_\theta\Omega$ is limited by baroclinically unstable modes**
- **The solar observation implies that the Sun's differential rotation likely reaches its possible maximum value**





Role of baroclinicity on angular momentum balance

Angular momentum eq.

$$\rho_0 \frac{\partial \mathcal{L}}{\partial t} = -\nabla \cdot \left(\boldsymbol{F}^{\mathrm{MC}} + \boldsymbol{F}^{\mathrm{mode}} + \boldsymbol{F}^{\Lambda} + \boldsymbol{F}^{\mathrm{vis}} \right),$$

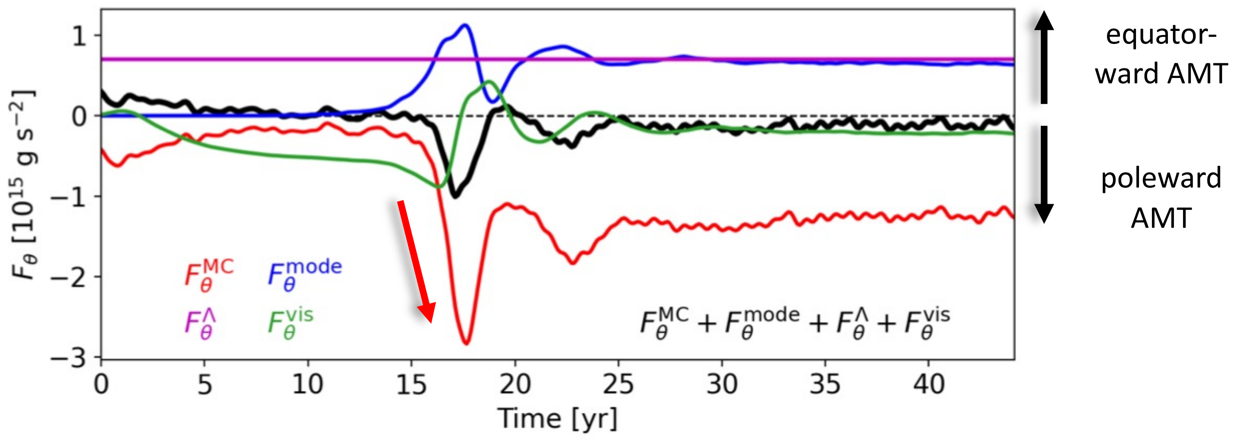
$$\boldsymbol{F}^{\mathrm{MC}} = \rho_0 \overline{\boldsymbol{v}}_{\mathrm{m}} \mathcal{L},$$ Meridional circulation

$$\boldsymbol{F}^{\mathrm{mode}} = \rho_0 r \sin\theta \overline{\boldsymbol{v}'_{\mathrm{m}} v'_{\phi}},$$ Reynolds stress (modes)

$$\boldsymbol{F}^{\Lambda} = -\rho_0 r \sin\theta \nu \boldsymbol{\Lambda} \Omega_0,$$ Λ-effect (prescribed)

$$\boldsymbol{F}^{\mathrm{vis}} = -\rho_0 r^2 \sin^2\theta \nu \nabla \overline{\Omega},$$ Viscous diffusion

Temporal evolution of angular momentum fluxes

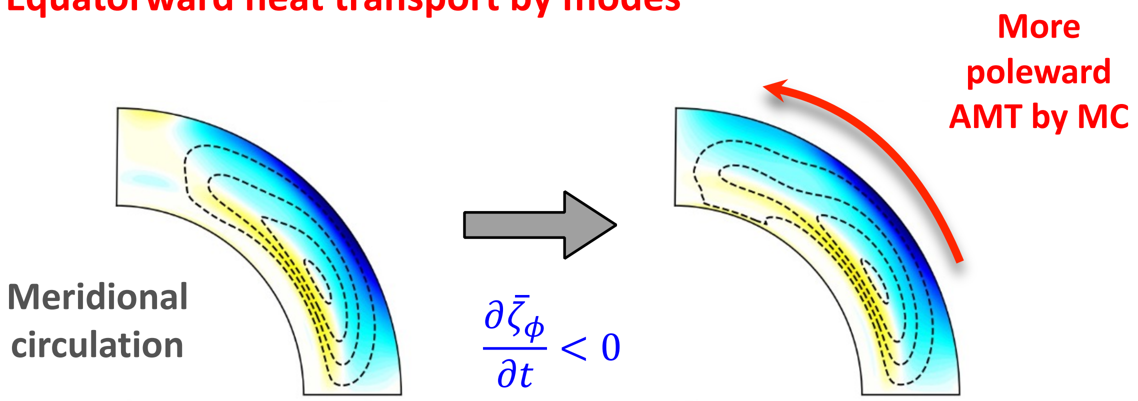


- Reduction of $\Delta_\theta \Omega$ is dominantly caused by the **poleward angular momentum flux by meridional flow** F_θ^{MC}
- This is caused by the **change in the baroclinic torque** ⇐ **Equatorward heat transport by modes**

Meridional vorticity eq.

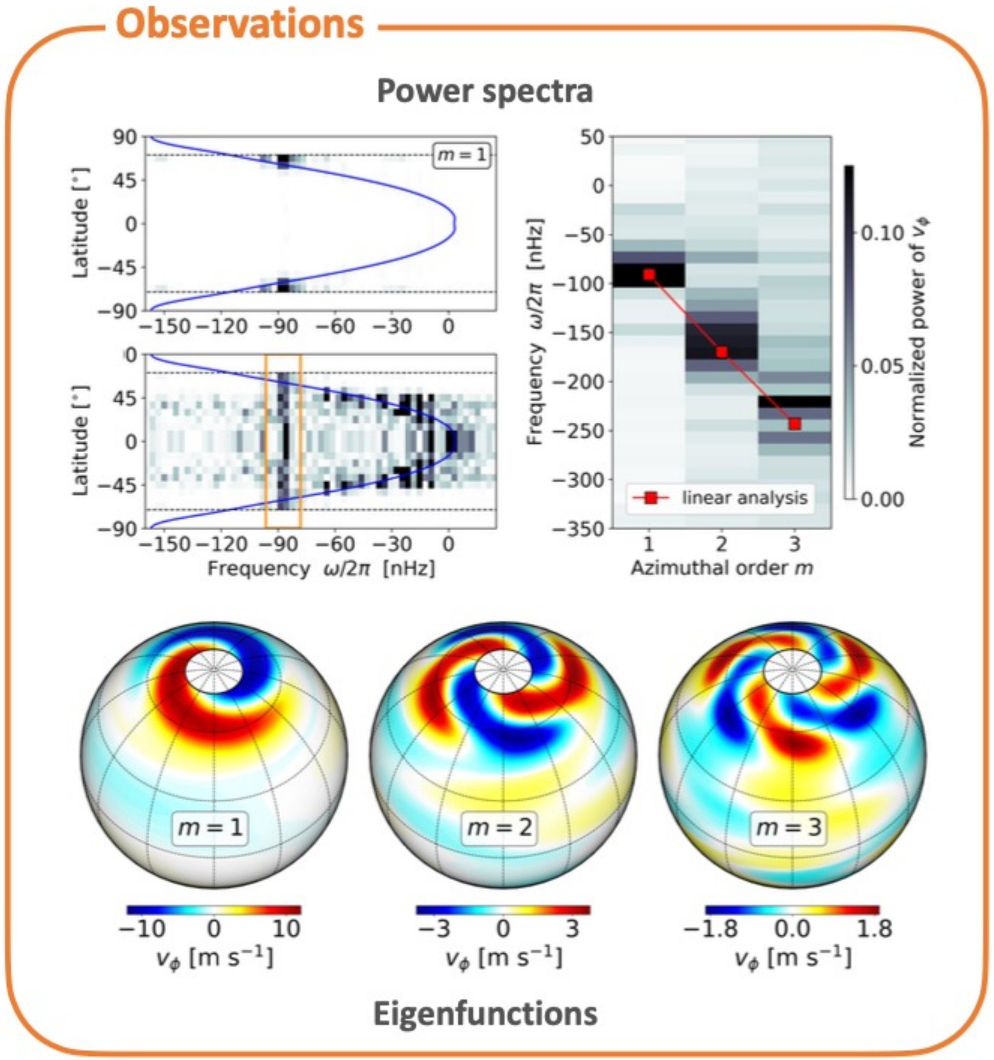
$$\frac{\partial \bar{\zeta}_\phi}{\partial t} = 2r \sin\theta\, \Omega_0 \frac{\partial \Omega_1}{\partial z} - \frac{g}{r c_p} \frac{\partial \bar{s}_1}{\partial \theta} + [\ldots]$$

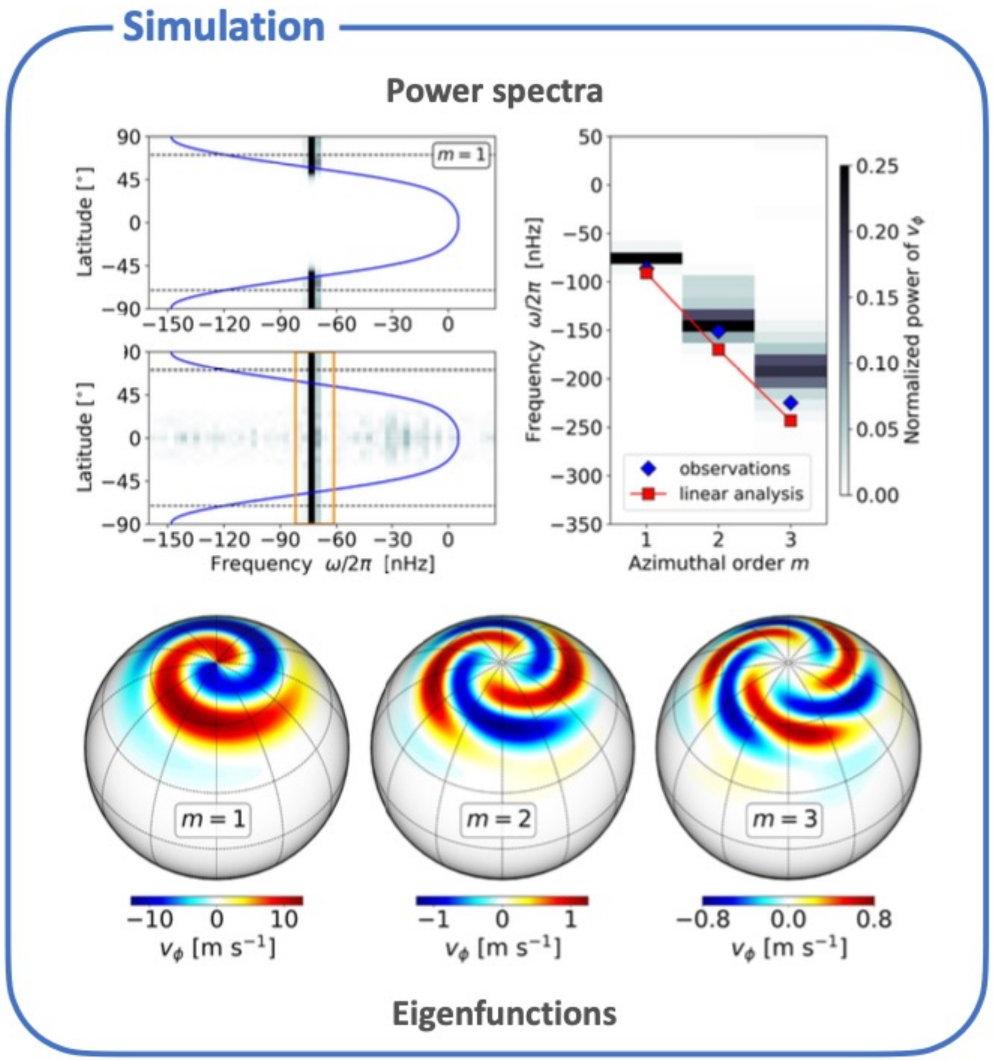
Baroclinic torque



Various observed properties of high-latitude inertial modes explained

- Our 3D mean-field model with $\Delta_\theta T \approx 7$K nicely reproduces many observed properties of the high-latitude inertial modes





Summary

➢ Differential rotation of the Sun

- The **non-Taylor-Proudman differential rotation** is believed to be sustained by **thermal wind balance**
- A small **latitudinal temperature difference $\Delta_\theta T$ ($=T_{\text{pole}}-T_{\text{eq}}$)** is expected to exist in the Sun but difficult to measure

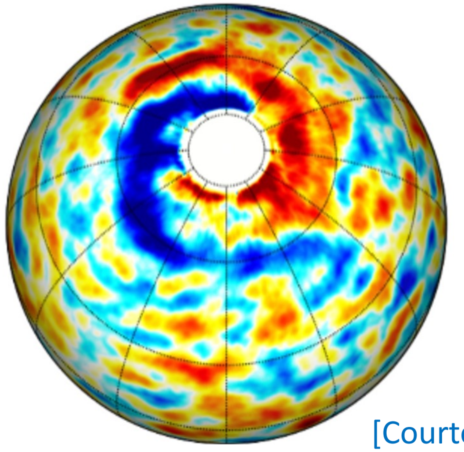
➢ High-latitude inertial modes (polar spiral flows)

- The high-latitude inertial modes have the largest velocity amplitudes [Gizon et al. 2021]
- They are **baroclinically unstable** even in the presence of such a small $\Delta_\theta T$ [Bekki et al. 2022]

➢ Nonlinear saturation of high-latitude modes [Bekki et al. 2024]

- The high-latitude modes saturate by **transporting heat equatorward** and reducing $\Delta_\theta T$ in CZ
- They **control the latitudinal differential rotation** by regulating the baroclinicity
- The observed amplitudes of baroclinic modes can be used to infer **$\Delta_\theta T \approx$ 7 K**
- Our study implies that the Sun's differential rotation is close to its **maximum possible value**

Observation (SG tracking)



[Courtesy: D. Hathaway]

Mean-field sim.