

# Analytical 3x2pt Covariance

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this goes on every 2nd and 4th Friday  
of the month @ 15.00 CET

IST:NL leaders  
~ 20 people participating, ~ 7 really active

**Davide Sciotti & Marco Bonici et al.**

# General thoughts

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- the covariance will be computed from CLOE recipes but external to the code
- it will be computed once before the MCMC runs
- covariance setups need to be defined: window functions, masks, sky fractions, redshift distributions, non-linear reference models, scale-cuts or not scale-cuts, BNT transformation (**K. Benabed, F. Bernardeau, M. Delaire**, and action in **IST:L**) for the weak lensing part etc...



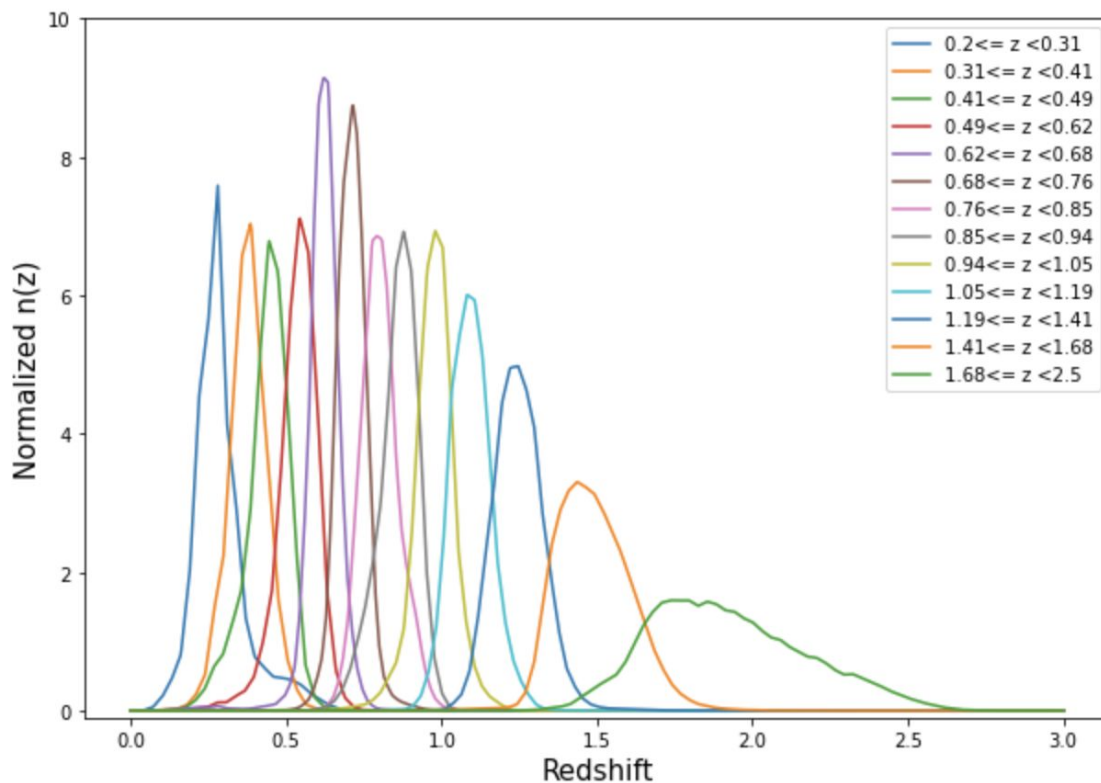
# Photometric Catalogue(s)

## From the F2 simulations:

After testing different types and number of tomographic bins, the following selection has been chosen as fiducial for both lenses and sources:

- 13 tomographic bins
- equipopulated bins (same number density of galaxies per bin)
- minimum photo-z value: 0.2
- maximum photo-z value: 2.5

24.3 galaxies / arcmin<sup>2</sup>



# Photometric Catalogue(s)

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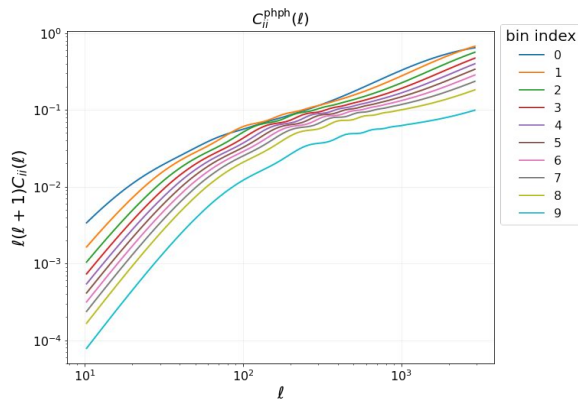
$$W_i^A(z) = b^A(z) \frac{H(z)}{c} n_i^A(z) \quad \text{Clustering}$$

$$W_i^\gamma(z) = \frac{3}{2} \left( \frac{H_0}{c} \right)^2 \Omega_m (1+z) \chi(z) \int_z^{z_{\max}} dz' n_i^{\text{ph}}(z') \left[ 1 - \frac{\chi(z)}{\chi(z')} \right] \quad \text{Weak Lensing (+IA components)}$$

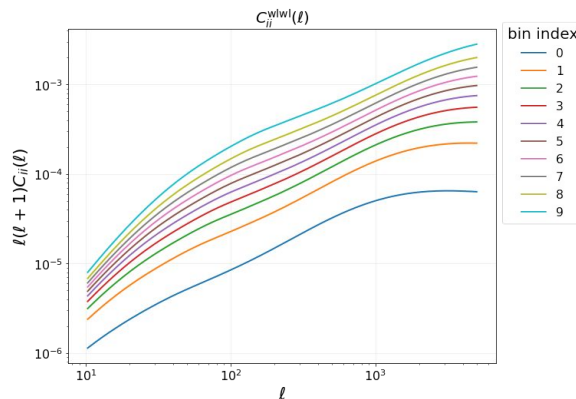
$$C_{ij}^{AB}(\ell) \simeq \frac{c}{H_0} \int_{z_{\min}}^{z_{\max}} dz \frac{W_i^A(z) W_j^B(z)}{E(z) \chi^2(z)} P_{\delta\delta} \left[ k = \frac{\ell + 1/2}{r(z)}, z \right] \quad \text{In the Limber approximation}$$

# Baseline recipes for 3x2pt

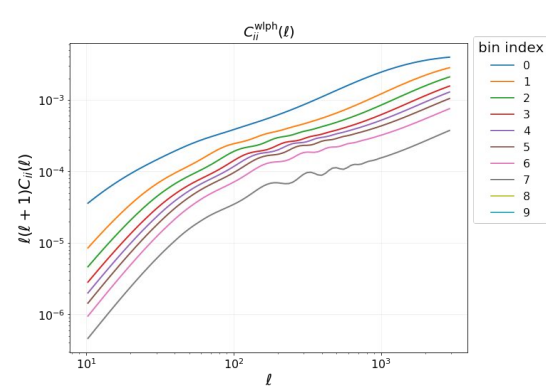
ph-ph



wl-wl

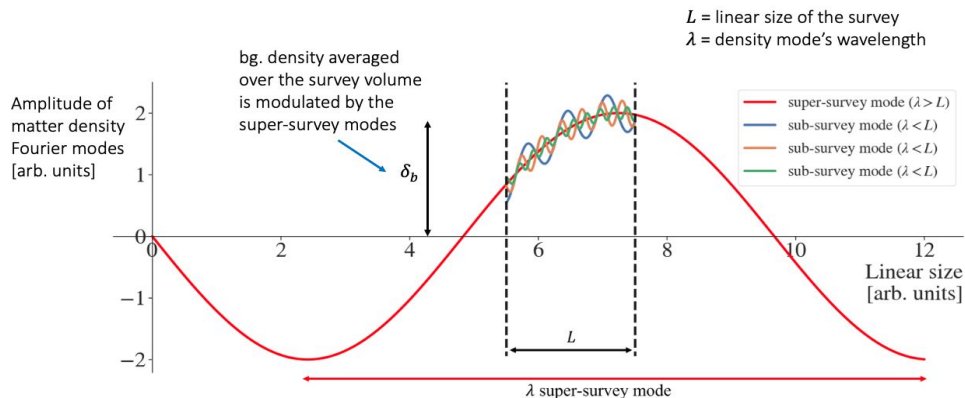


cross: ph-wl



	$l_{\min}$	$l_{\max}$	$n_{\text{bin}} (\log_{10})$
ph-ph	10	3000	200
wl-wl	10	5000	100
ph-wl	10	3000	100

# Why do we care about Super Sample Covariance?



Power spectrum : all scales probes react to  $\delta_b$

→ more important when more modes

All probes react → more important when more probes

Separate universe (e.g. Wagner et al. 2015) :  
can mimick  $\delta_b$  with a change of cosmology

# Current Status

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- **Gaussian:** [published package](#) for easy computation, saves a bit of ordering-related headaches
- **SSC:** three (and a half) codes available
  - PySSC:
    - Fast, good control over observables and responses
    - Slowly varying response, may be inaccurate for WL?
  - PyCCL:
    - Slower, less control over the observables (but still ok in general)
    - Dirac delta approximation for  $\sigma^2$ , suggested to be less accurate for GCph (**Lacasa+19**)
  - **Spaceborne**:
    - Slowest
    - No approximation (except full-sky, as with the others)
    - Custom-built, full control over recipe, observables and responses
  - CosmoLike:
    - Same recipe as PyCCL, excellent agreement found with input files of R. Upham
    - Runnable on a Docker image by Marco, least control over observables, tests ongoing
    - Already used in the literature
- **cNG:** ongoing tests with PyCCL



# SSC Covariance: $\sigma^2(z)$ [CosmoLike](#), [PyCCL](#)

$$\text{Cov}^{\text{SSC}}(C_{AB}^{ij}(l_1), C_{CD}^{kl}(l_2)) = \int d\chi \frac{q_A^i(\chi) q_B^j(\chi) q_C^k(\chi) q_D^l(\chi)}{\chi^4} \frac{\partial P_{AB}(l_1/\chi, z(\chi))}{\partial \delta_b} \frac{\partial P_{CD}(l_2/\chi, z(\chi))}{\partial \delta_b} \sigma_b(\Omega_s; z(\chi)),$$

with  $\sigma_b(\Omega_s, z(\chi))$  the variance of the background mode over the survey window,

$$\sigma_b(\Omega_s; z) = \int \frac{d^2 k_\perp}{(2\pi)^2} P_{\text{lin}}(k_\perp, z) |\tilde{W}_s(k_\perp, z)|^2 \approx \int \frac{d^2 k_\perp}{(2\pi)^2} P_{\text{lin}}(k_\perp, z) \left[ \frac{2J_1(k_\perp \chi(z) \theta_s)}{k_\perp \chi(z) \theta_s} \right]^2, \quad \text{Krause \& Eifler 2016}$$

(assumes cylindrical window function)

with  $q_A^i$  the weight functions.

Approximation used by DES and KiDS analyses. (KiDS: different pipeline, not public)

- Single redshift integral: approximates  $\sigma^2(z)$  as a Dirac delta at  $z_1 = z_2$
- If no mask is passed as input, assumes the 3D window function  $W(\vec{x})$  to be much wider in the radial direction than in the transverse direction.
- **Validity: expected OK for broad overlapping kernels (WL), but not for narrow independent kernels (GC)** - opposite as slowly varying approximation!

For cluster counts, [Lacasa+2018](#) showed it works well in auto-redshift but not in cross-redshift

# SSC approximations : slowly varying response (PySSC)

If the response  $\frac{\partial \mathcal{O}}{\partial \delta_b}$  varies slowly with redshift compared to  $\sigma^2(z_1, z_2)$

$$\text{Cov}_{\text{SSC}} [\mathcal{O}(i, \alpha), \mathcal{O}(j, \beta)] = \frac{\partial \mathcal{O}(i, \alpha)}{\partial \delta_b} \frac{\partial \mathcal{O}(j, \beta)}{\partial \delta_b} S_{\alpha, \beta}$$

$\frac{\partial \mathcal{O}(i, \alpha)}{\partial \delta_b} \frac{\partial \mathcal{O}(j, \beta)}{\partial \delta_b} = R(i, \alpha) O(i, \alpha)$  (definition of R)

$\mathcal{O}(i, \alpha)$  cosmological probe: cluster counts,  $C_l$  of WL / GCp / XC / CMBX,  $P(k)$  of GCs, high order statistic...

$R(i, \alpha)$  probe response: from theory, ansatz or SU simulations.  
Depends on probe, redshift ( $\alpha$ ), scale / other param ( $i$ ).

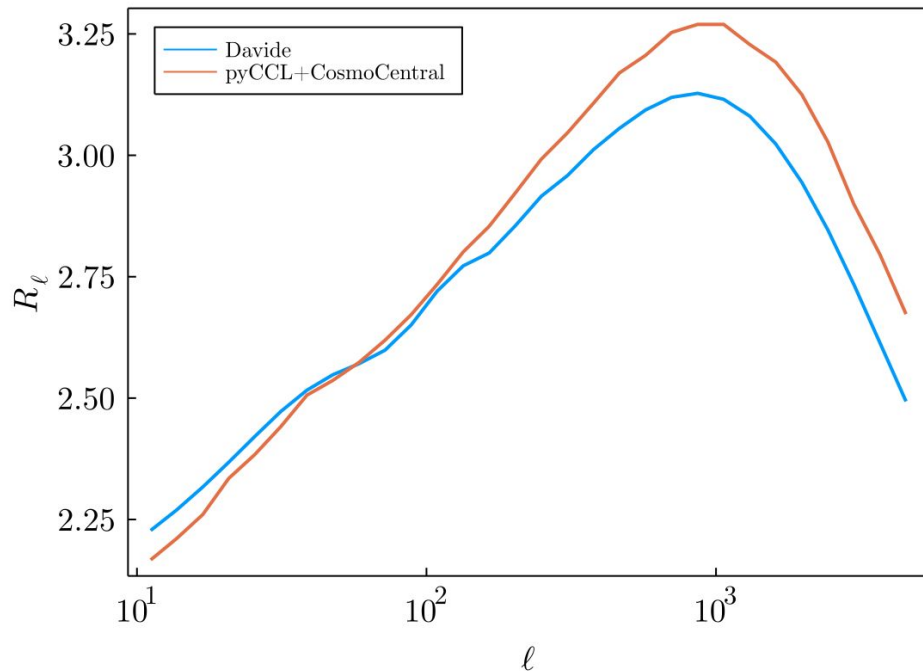
$S_{\alpha, \beta}$  covariance matrix of the  $\delta_b$  of each redshift bin  
Can be computed with PySSC ([Lacasa & Grain 2019](#))  
Sensitive to sky fraction / mask ([Gouyou Beauchamps et al. 2022](#)) - *fsky* rescaling works well for large survey areas (in general SSC does **not** scale as  $1/fsky$ )

**Output of PySSC**  
(essentially,  $\sigma^2(z_1, z_2)$   
term weighted by the  
kernels)

**Validity: expected OK for narrow independent kernels (GC),  
worse for broad overlapping kernels with low-z contribution (WL)**

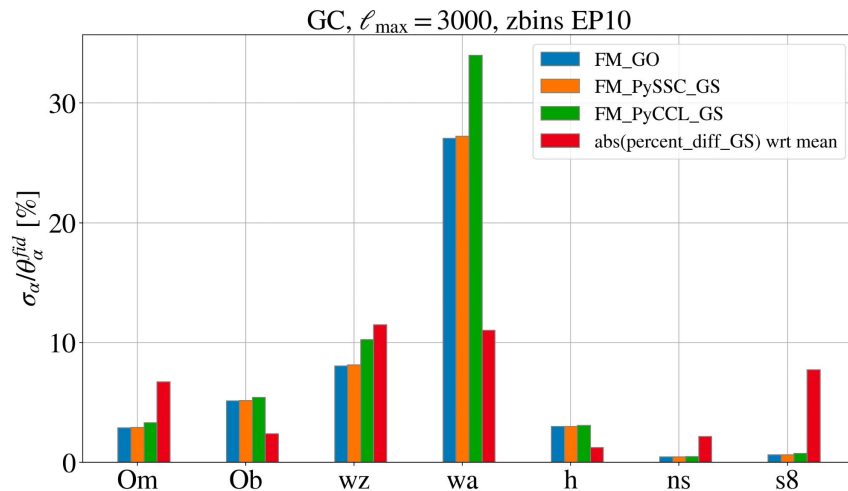
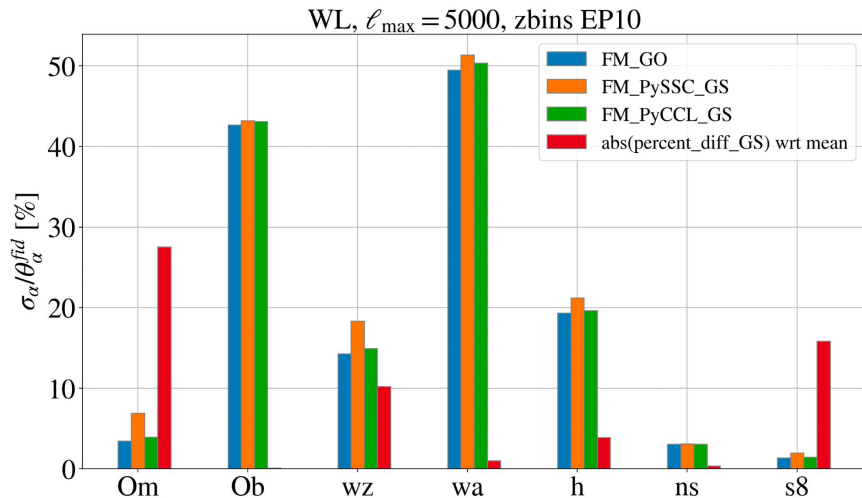
# Probe response

- SU probe response vs probe response from PyCCL (thanks to M. Bonici)
- Good overall agreement
- Next step: gauge the effect of the different recipes by plugging PyCCL responses into PySSC (spoiler alert: small impact on the final constraints)



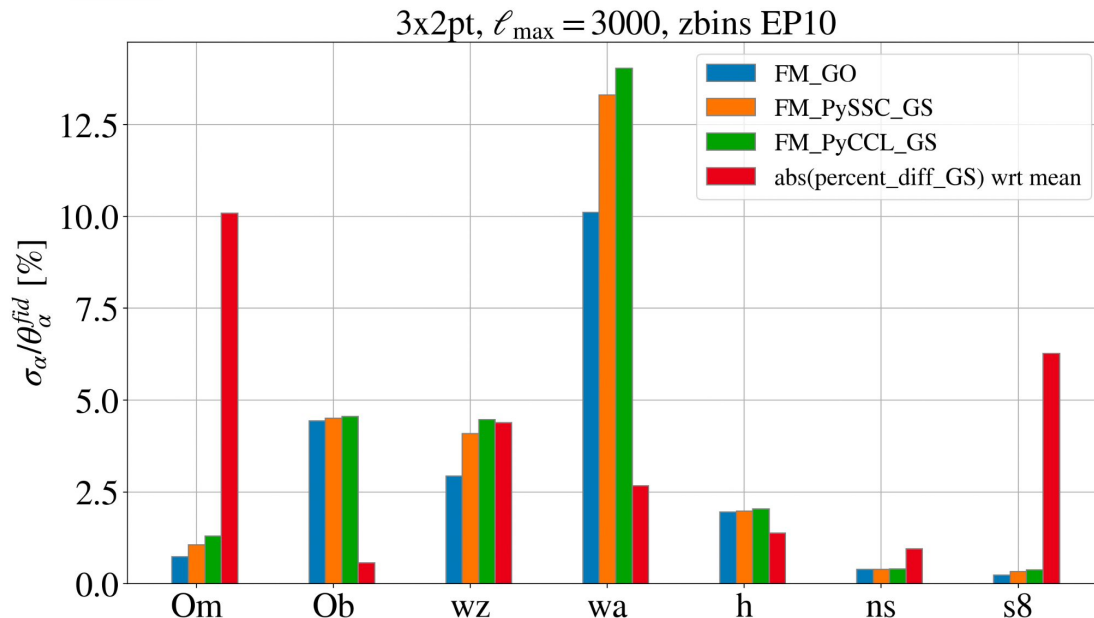
# PySSC vs PyCCL: FM constraints, WL, GCph

- Same pipeline with SSC matrix from PySSC or PyCCL
- Good agreement found, except on the most impacted parameters for WL
- Difference between GS constraints wrt the *mean* shown in the plot
- Not the best metric for agreement between codes?
  - Direct comparison of the SSC covariance trickier, shows large discrepancies



# PySSC vs PyCCL: FM constraints, 3x2pt

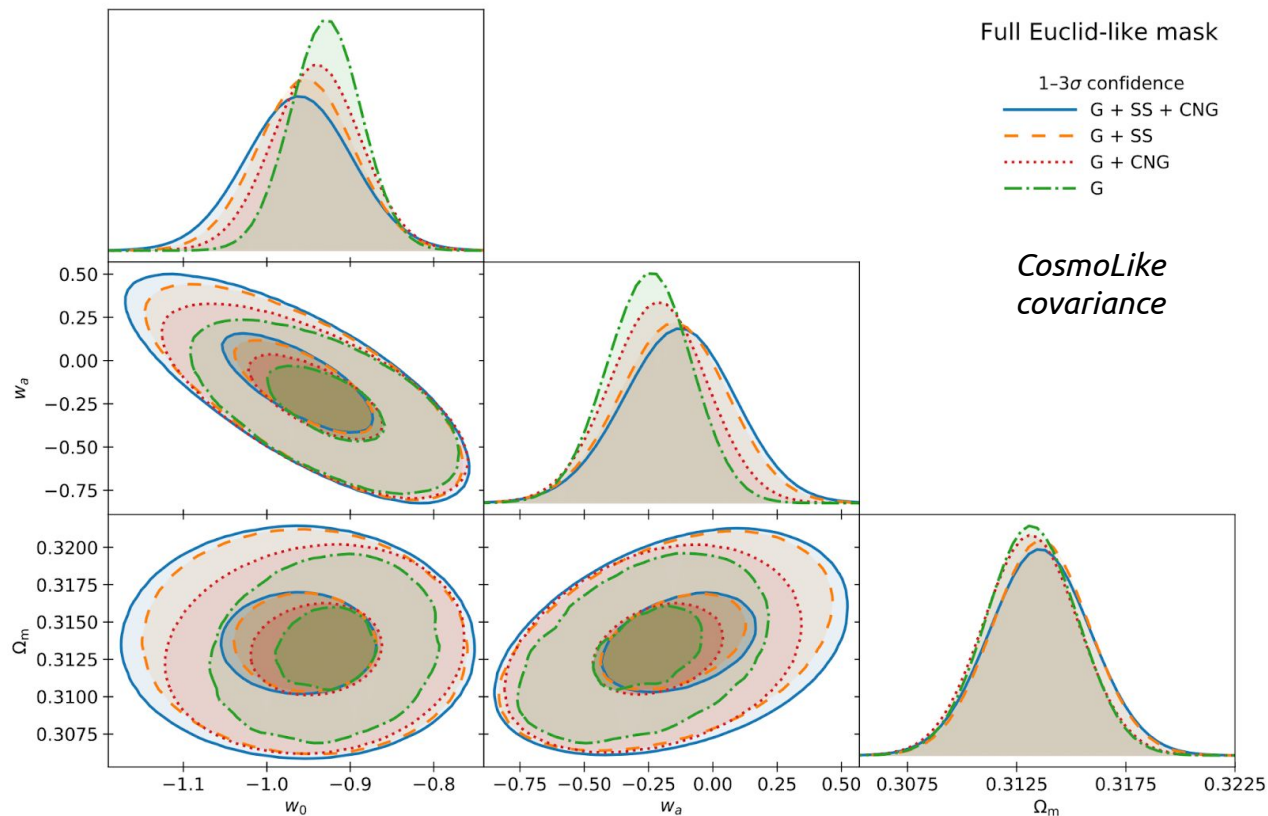
- Good agreement, 10% max discrepancy
- PyCCL more pessimistic (than PySSC) for GCph, more optimistic for WL



# Shear pseudo-Cl covariance

Importance of non-Gaussian terms:

- Mock parameter constraints including different contributions to covariance

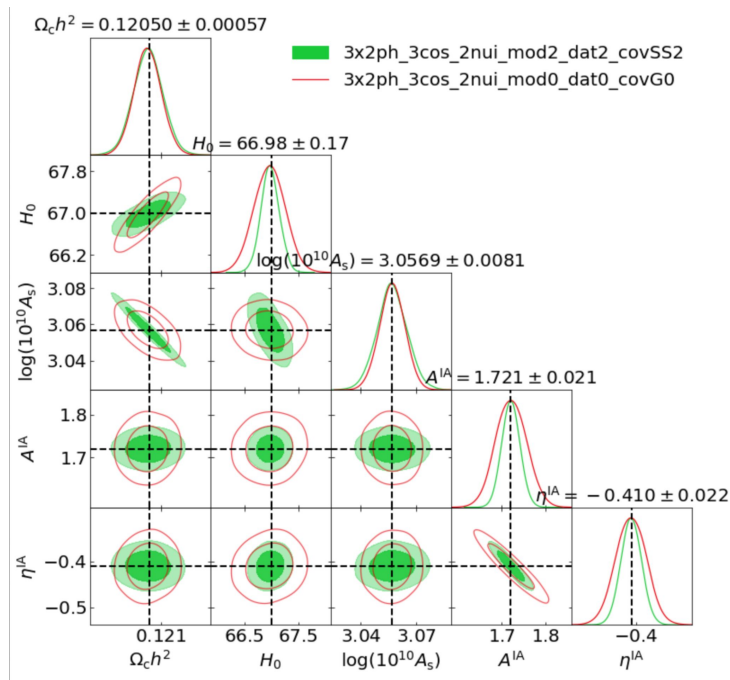


# MCMC analyses

Improvement in constraints using non-linear modelling

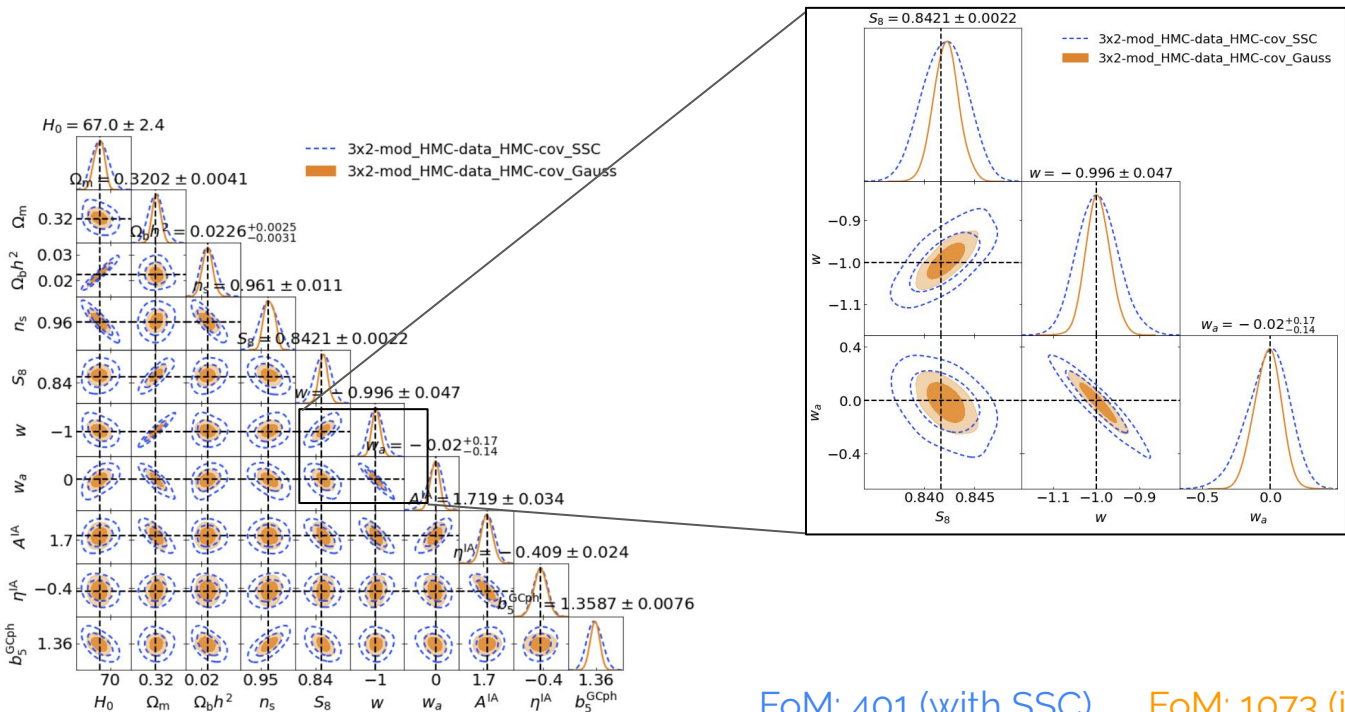
3x2pt photometric

- **Red** lines: Linear model and linear data vector, Gaussian diagonal covariance
- **Green** solid: Nonlinear model HMcode, with same model for data vector and covariance. DM-only. SSC + Gaussian covariance.



# MCMC analyses

Impact of SSC on parameter constraints



3x2pt photometric data: HMCode  
 model: HMCode  
 covariance: HMCode

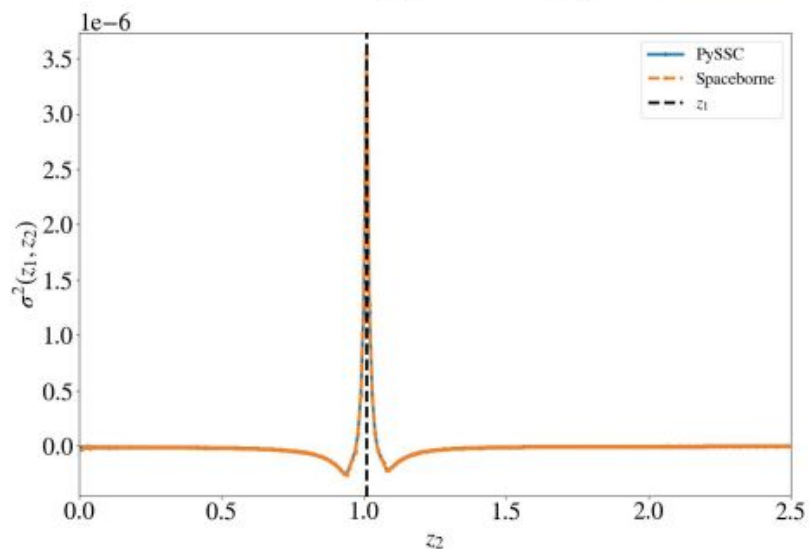
- **Blue dashed** lines: SSC+Gaussian covariance
- **Orange solid**: Gaussian covariance.




# Spaceborne SSC

- Possible test of  $\sigma^2$  against latest version of PySSC, excellent agreement ✓
- Responses from SU simulations, good match against PyCCL (extracted by Marco) ✓
- Including magnification bias (for latest SPV3 forecasts) creates some issues with PySSC and PyCCL, work in progress to bring these up to speed

$$\text{Cov}_{\text{SSC}} [C_{ij}^{AB}(\ell), C_{kl}^{CD}(\ell')] = \int dV_1 dV_2 W_i^A(z_1) W_j^B(z_1) W_k^C(z_2) W_l^D(z_2) \times \frac{\partial P_{AB}(k_\ell, z_1)}{\partial \delta_b} \frac{\partial P_{CD}(k_{\ell'}, z_2)}{\partial \delta_b} \sigma^2(z_1, z_2)$$



# Spaceborne SSC: computing the integral

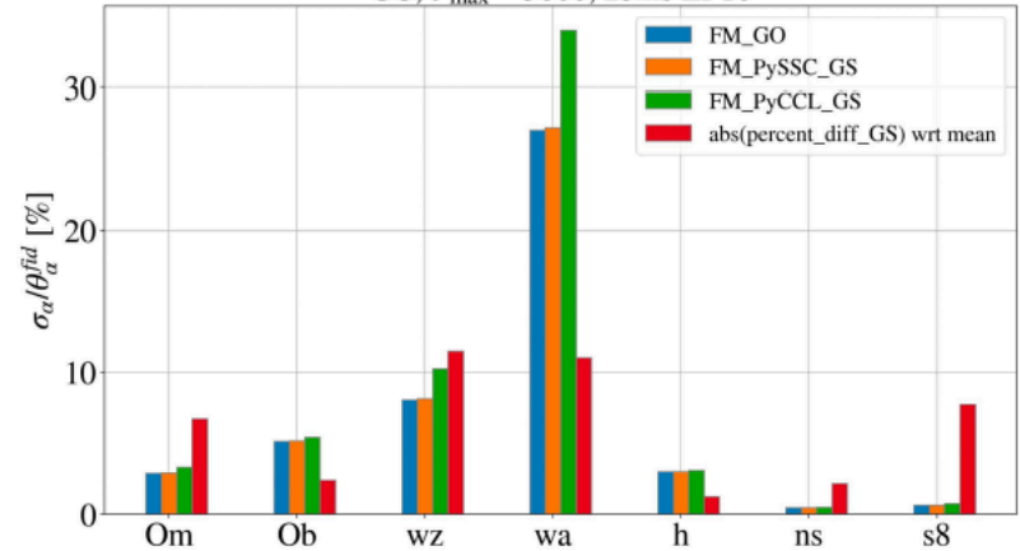
- Previous analysis: proof of concept with 2  $\ell$  bins, integral feasible in Python
- Full matrix (32  $\ell$  bins, 13 z bins) **much more** demanding:
  - Developed a Tensorflow version for GPU/TPU, good speed but memory still seems to be the bottleneck
  - Julia version by Marco, needs to be run on a cluster (10-ish hours for full matrix) 
  - Full control over recipe and approximations

```
67838032 Sep 19 04:43 cov_SSC_GGGG_4D_nbl32_ellmax5000_zbins13_zsteps2899_k1overMpc_conventionPySSC.npy
125984848 Sep 19 03:52 cov_SSC_GGGL_4D_nbl32_ellmax5000_zbins13_zsteps2899_k1overMpc_conventionPySSC.npy
67838032 Sep 19 02:21 cov_SSC_GGLL_4D_nbl32_ellmax5000_zbins13_zsteps2899_k1overMpc_conventionPySSC.npy
125984848 Sep 19 01:30 cov_SSC_GLGG_4D_nbl32_ellmax5000_zbins13_zsteps2899_k1overMpc_conventionPySSC.npy
233971792 Sep 18 23:55 cov_SSC_GLGL_4D_nbl32_ellmax5000_zbins13_zsteps2899_k1overMpc_conventionPySSC.npy
125984848 Sep 18 21:06 cov_SSC_GLLL_4D_nbl32_ellmax5000_zbins13_zsteps2899_k1overMpc_conventionPySSC.npy
67838032 Sep 18 19:31 cov_SSC_LGGG_4D_nbl32_ellmax5000_zbins13_zsteps2899_k1overMpc_conventionPySSC.npy
125984848 Sep 18 18:40 cov_SSC_LLGG_4D_nbl32_ellmax5000_zbins13_zsteps2899_k1overMpc_conventionPySSC.npy
67838032 Sep 18 17:09 cov_SSC_LLLG_4D_nbl32_ellmax5000_zbins13_zsteps2899_k1overMpc_conventionPySSC.npy
```

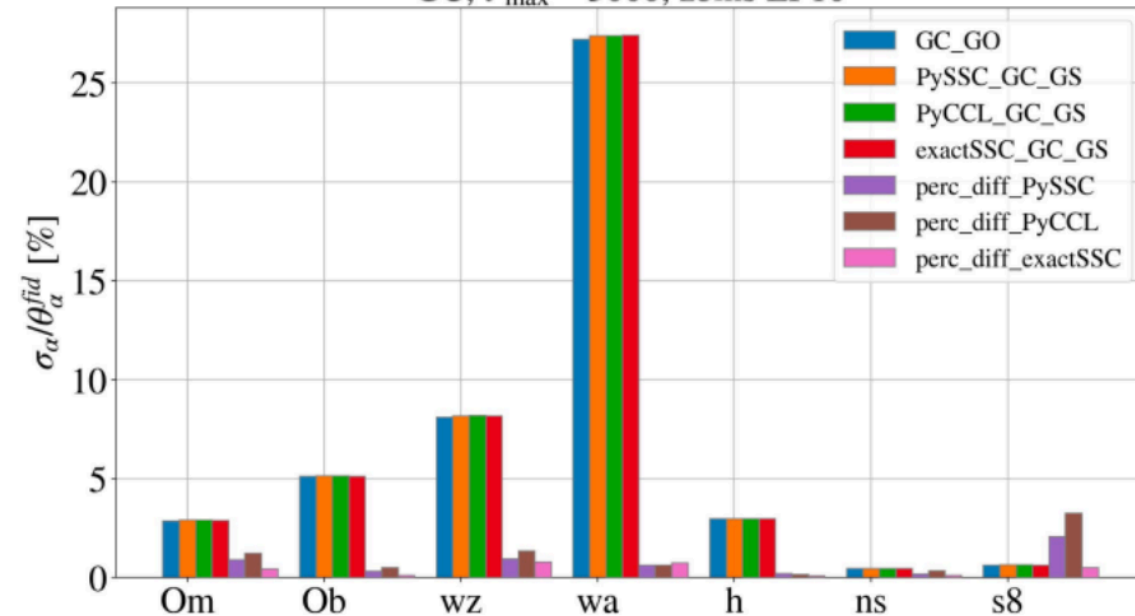
# Results GCph

- Low impact in general for GCph
- PyCCL approximation ( $\sigma^2$  approximated as a Dirac Delta) seems to be good for GCph
- More accurate PyCCL GCph ✓
- Good match with Spaceborne ✓

GC,  $\ell_{\max} = 3000$ , zbins EP10



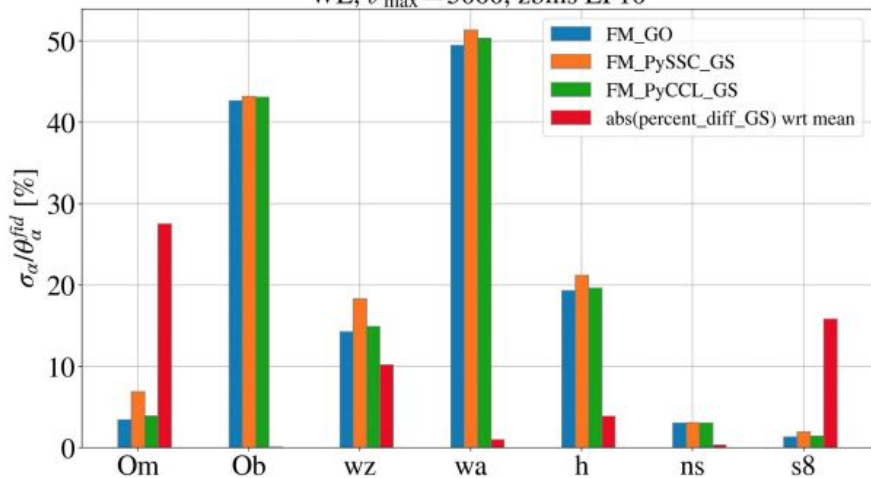
GC,  $\ell_{\max} = 3000$ , zbins EP10



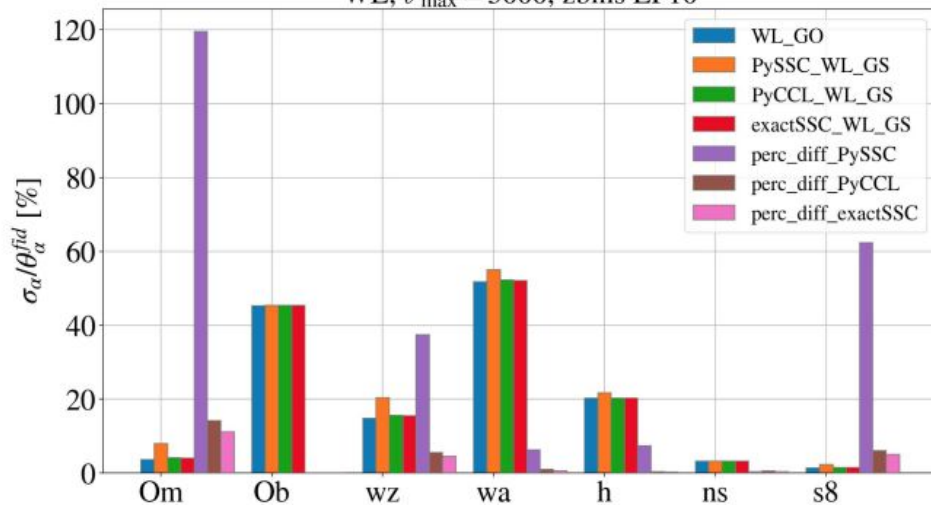
# Results WL

- (previously seen) mismatch between PySSC and other codes for most affected parameters
- Good match between PyCCL and exact SSC
- Found variability of results with varying precision parameter in PySSC
- Impact is now too small?

WL,  $\ell_{\max} = 5000$ , zbins EP10



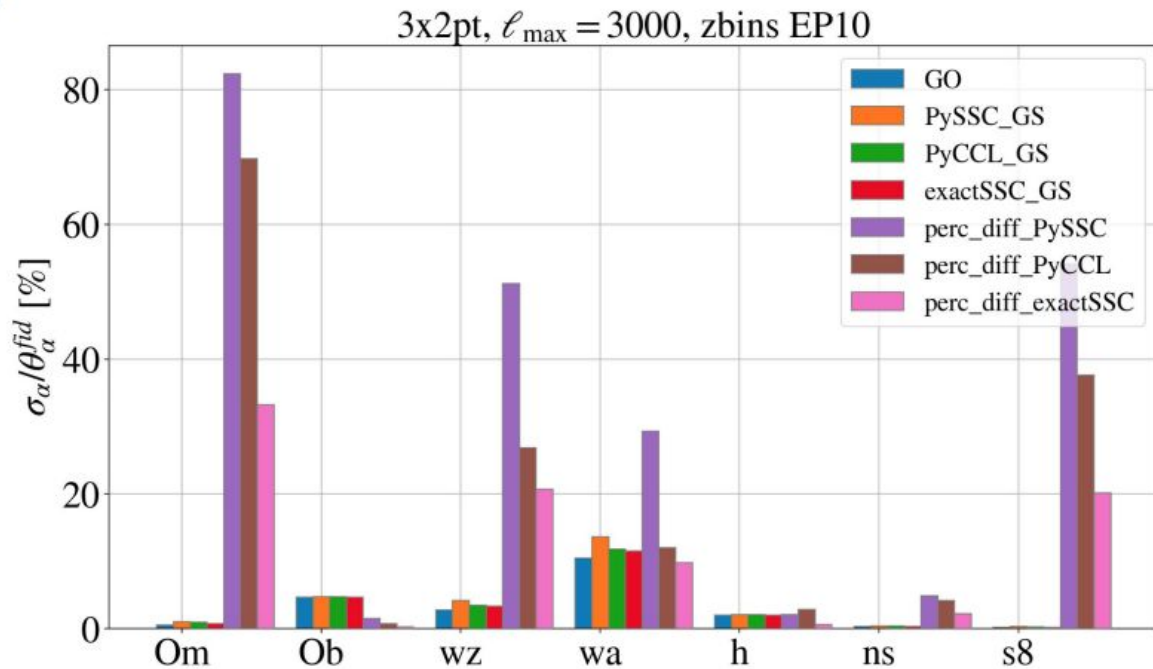
WL,  $\ell_{\max} = 3000$ , zbins EP10



# Results 3x2pt

- Largest spread among the 3 codes: issues in the XC term?

	FoM degradation [%]
PySSC	49
PyCCL	41
Spaceborne	26



# Next Steps and Future prospects

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- *Better understand PySSC WL mismatch and 3x2pt discrepancy*
- Finish building independent PyCCL pipeline
- **Finish implementing the cNG term**
- **Publish code on GitLab**
- Migrate to PyCCL v3
- *Improve integration routine in the Julia SSC integral  
(from trapz to simps), to reduce a bit number of steps*
- **Compare results against simulations!**
- **run MCMC with NL models in CLOE**