The Role of the Third Dredge-up and Mass Loss in Shaping the Initial–Final Mass Relation of White Dwarfs

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Initial–Final Mass Relation of White Dwarfs

Kink at $M_{ini} \sim 1.6 – 2.1\ M_\odot$

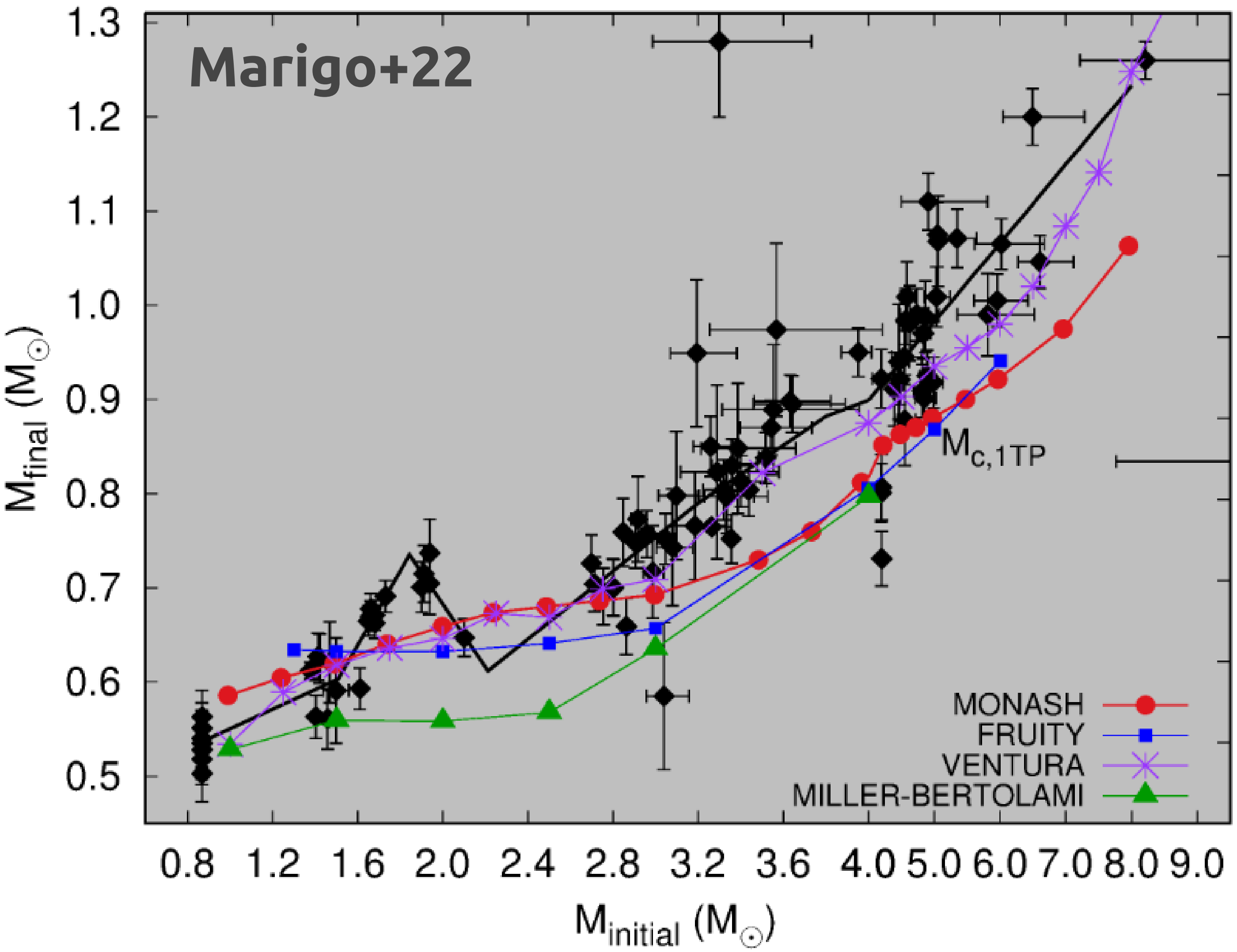
IFMR in open clusters at solar metallicity is not monotonic (Marigo+20).

Why do models give such IFMR?

The shape depends on the dredge-up history and how the surface reacts to such enrichment.

Missing ingredients

It is important to include a carbon-excess dependent wind and opacities. Lifetime is critical!



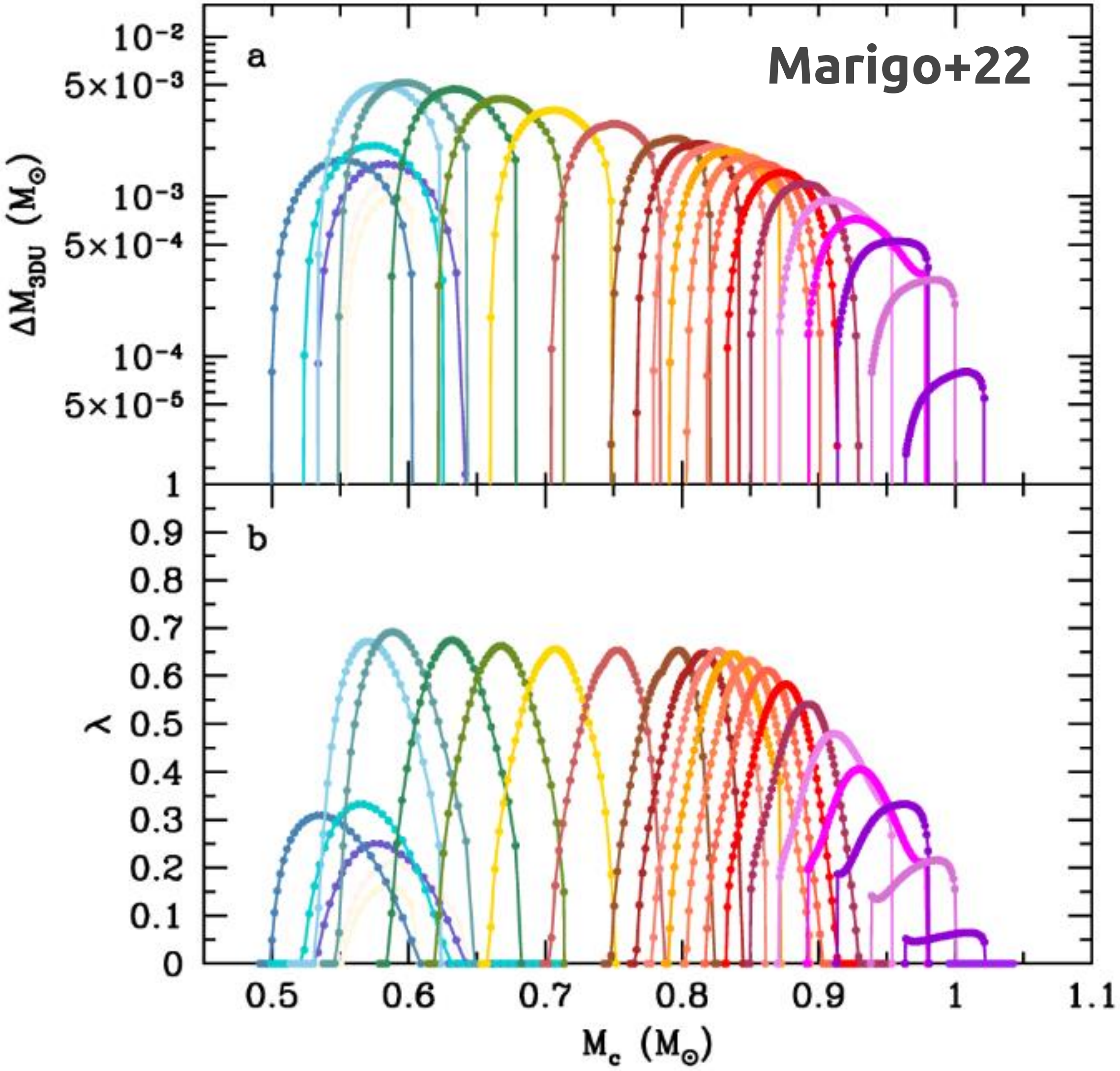
Initial–Final Mass Relation of White Dwarfs

IFMR provides **calibration for 3rd dredge-up efficiency** (Marigo+20)

$$\lambda = \frac{\Delta M_{\rm dup}}{\Delta M_{\rm core}}$$

Goal:

Use full stellar evolution calculations to understand the **extra-mixing** and the coupling with mass loss.



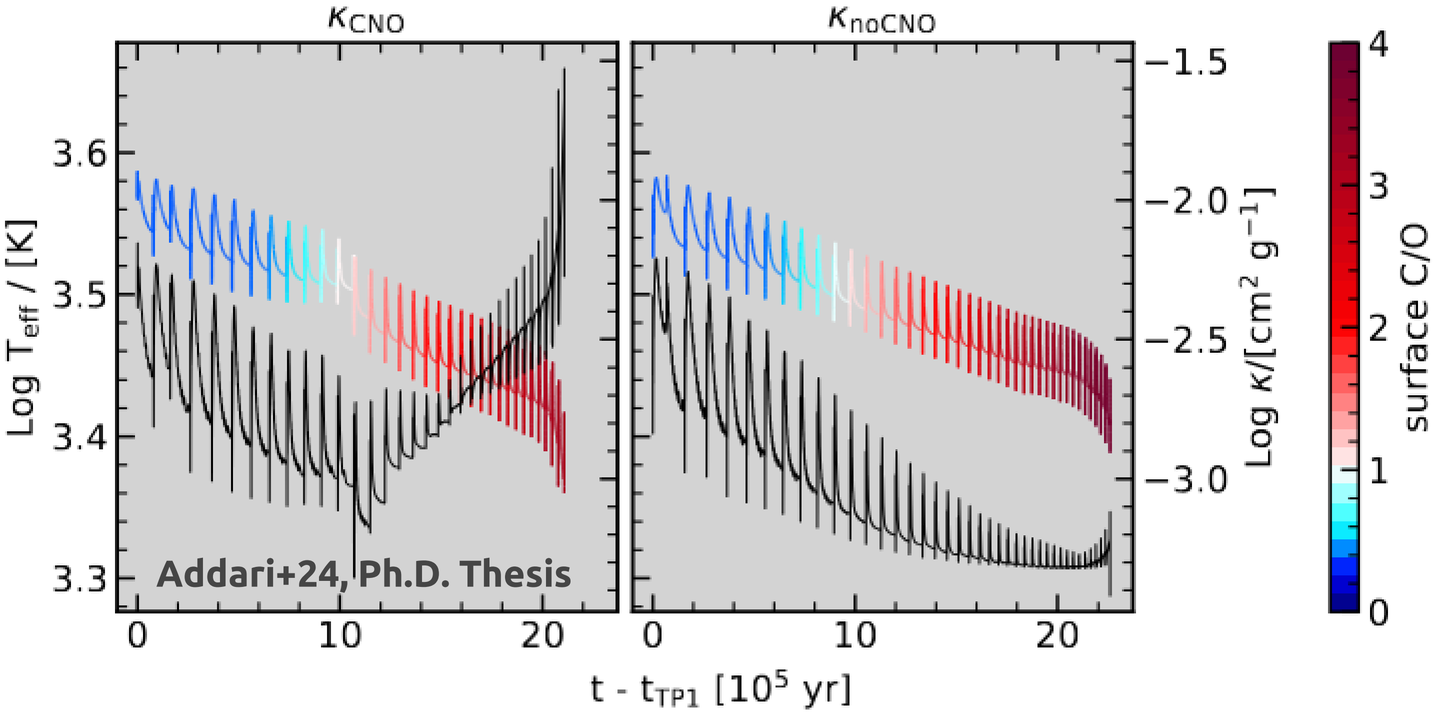
Numerical setup

PARSEC (Bressan+12, Chen+14, Fu+18, Costa+19, Nguyen+22)

COLIBRI (Marigo+13, Pastorelli+19, Pastorelli+20, Marigo+22)

Input physics additions

- ☐ Low-T AESOPUS opacities with variable C, N and O (Marigo+09, Marigo+22)
- ☐ Wind prescription as in Pastorelli+20 and Marigo+20, C – O dependent.
- ☐ Exponential overshooting (Freytag+96, Herwig+00)



Addari+24, Ph.D. Thesis

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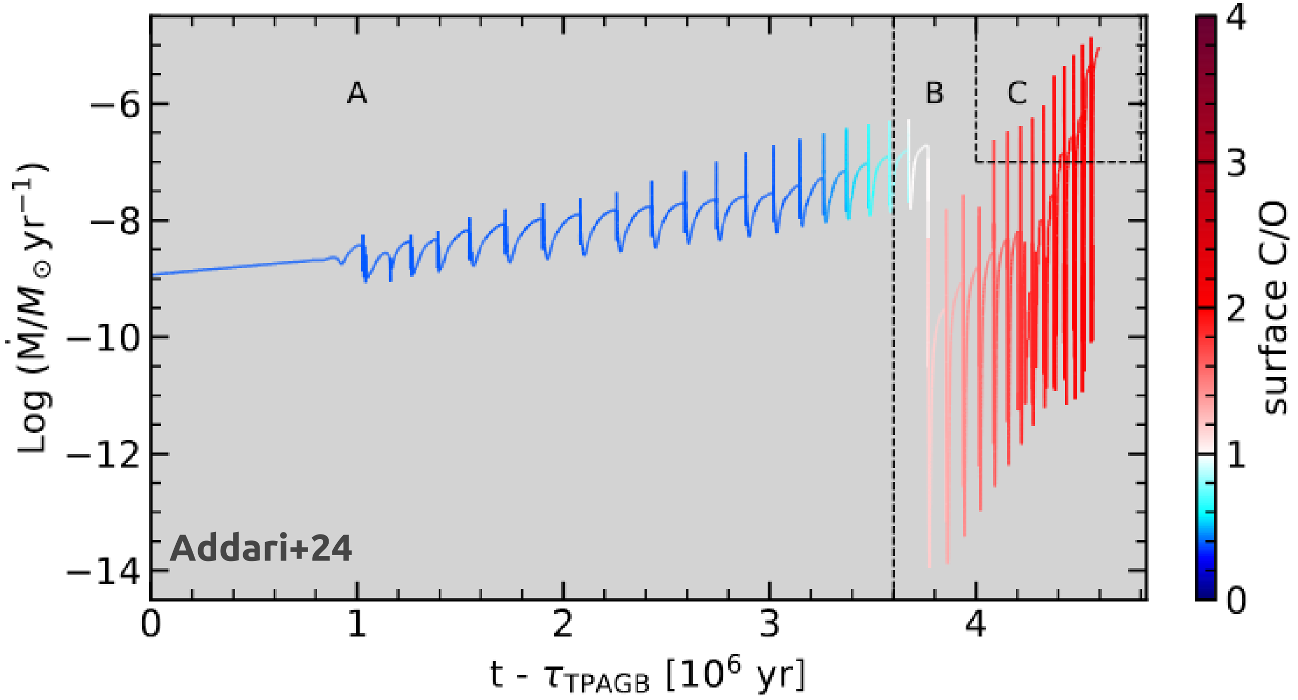

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$$D(r) = D_0 \exp\left(-2\frac{|r - r_0|}{\boxed{f_{\rm ov}} H_{\rm p}}\right)$$

$f_{\rm env}$

$f_{\rm pdcz}$

0.5 x f_{ov} (Choi+16)

$$r_0 = r_{\rm cnv} \pm f_{0,\rm ov} H_{\rm p}$$

Use IFMR to calibrate!

TP-AGB modeling is costly

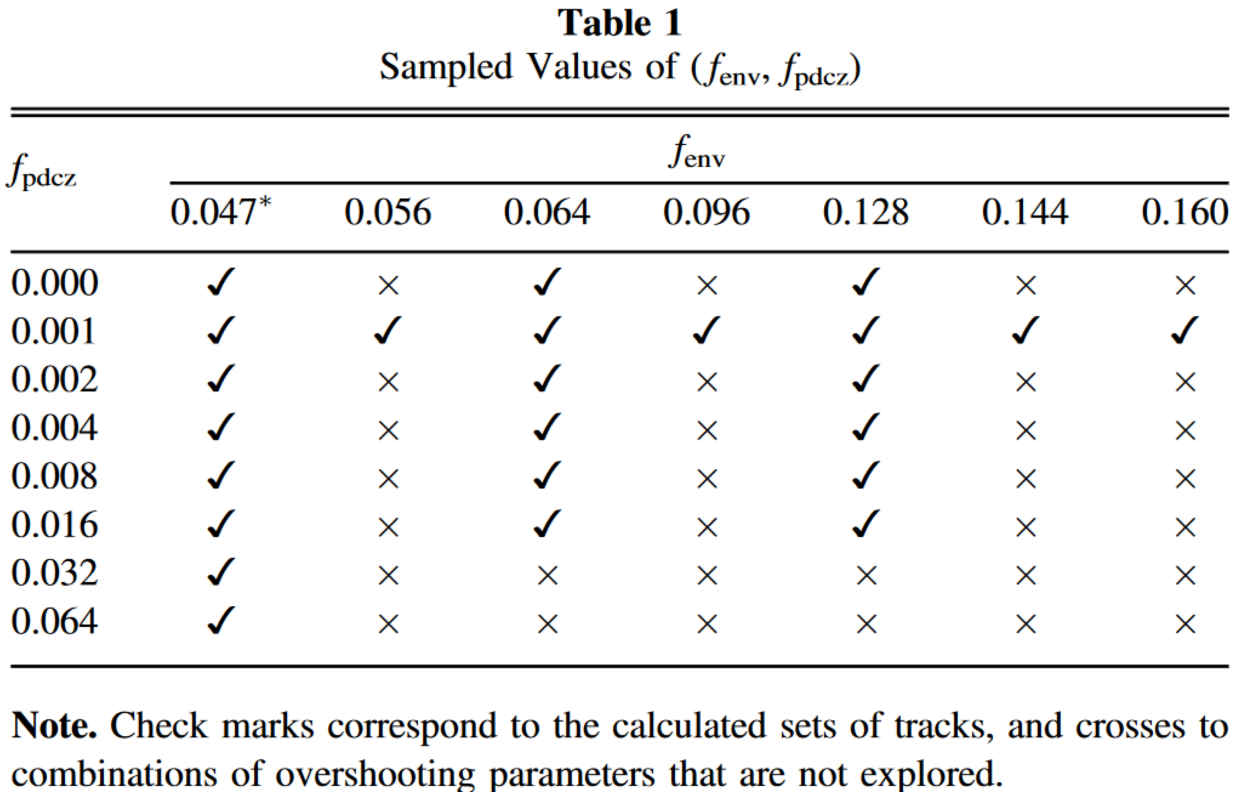


Table 1
Sampled Values of ($f_{\rm env}$, $f_{\rm pdcz}$)

$f_{\rm pdcz}$	$f_{\rm env}$						
	0.047*	0.056	0.064	0.096	0.128	0.144	0.160
0.000	✓	×	✓	×	✓	×	×
0.001	✓	✓	✓	✓	✓	✓	✓
0.002	✓	×	✓	×	✓	×	×
0.004	✓	×	✓	×	✓	×	×
0.008	✓	×	✓	×	✓	×	×
0.016	✓	×	✓	×	✓	×	×
0.032	✓	×	×	×	×	×	×
0.064	✓	×	×	×	×	×	×

Note. Check marks correspond to the calculated sets of tracks, and crosses to combinations of overshooting parameters that are not explored.

Some numbers

> 400 TP-AGB tracks in 24 sets

>12k complete pulse cycles

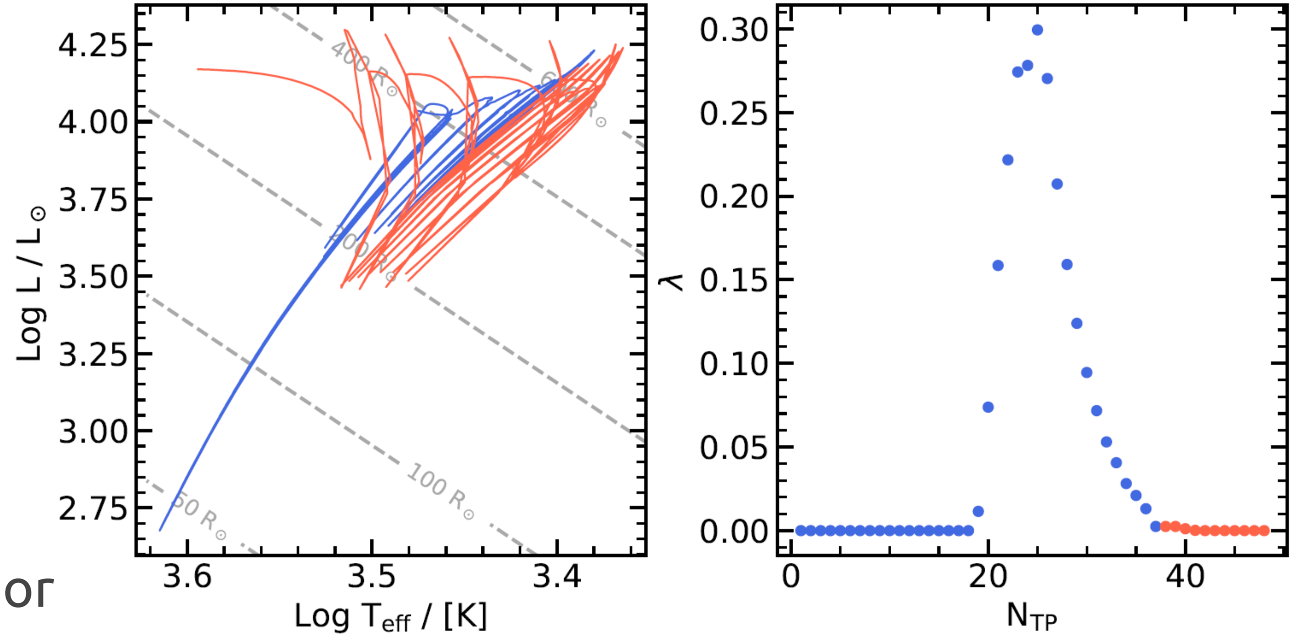
> 9 yr of CPU time, would have been **x 3** without shell shifting!

> 2 TB of history and structure files

Why do TP-AGB models struggle?

As many others, **PARSEC** tracks end few pulses before the end (T_{eff} < 2500 K).

COLIBRI can reliably complete them!



☐ **Larger mass-loss rate** brings the track to the end.

☐ No reliable solution with **smaller/larger Δt** or **coarser/finer meshgrid**.

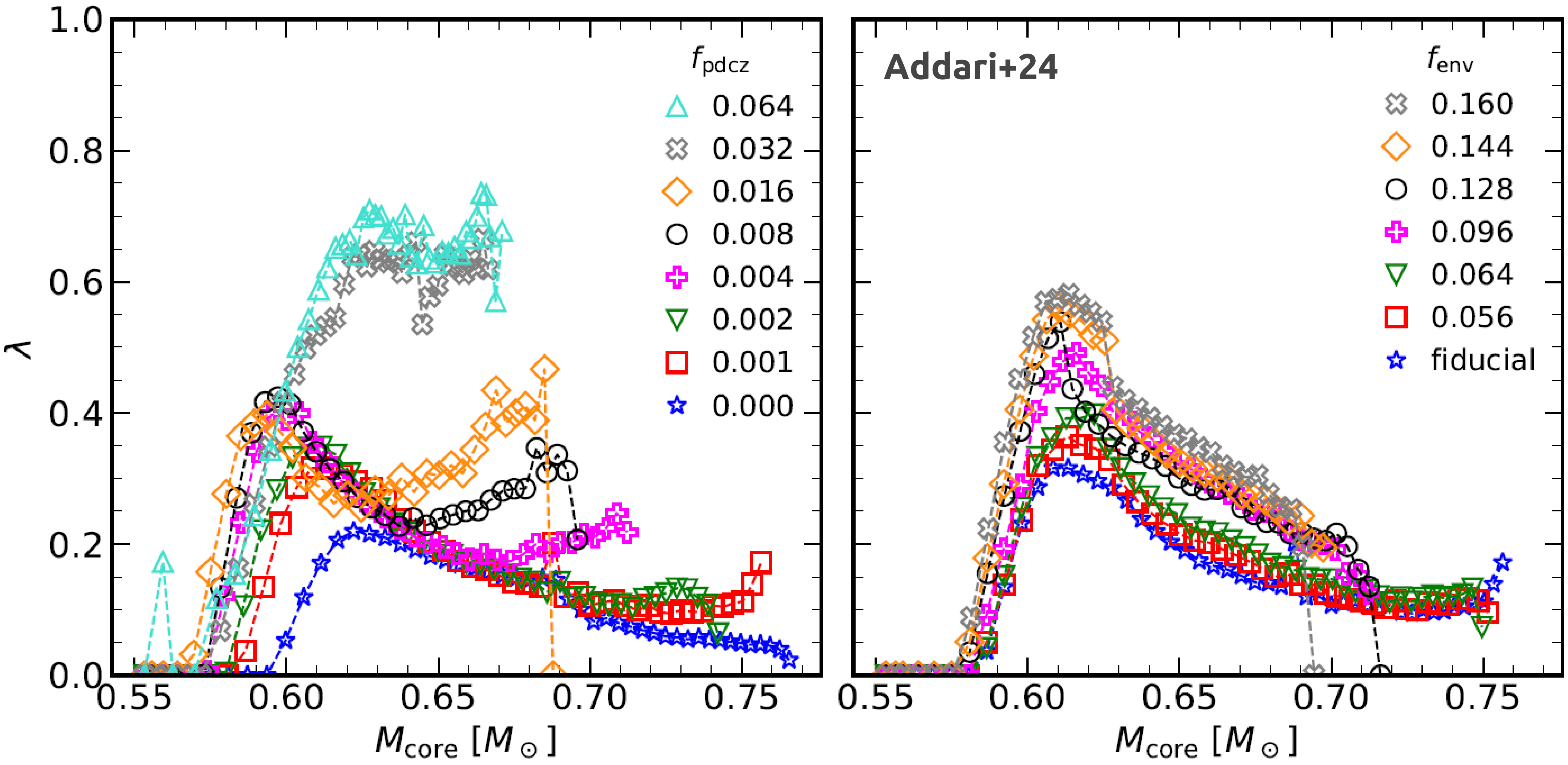
☐ **No hydrogen recombination** (Wagenhuber and Weiss 1994, Miller-Bertolami 2016)

☐ Matching **core** conditions with the **slowly evolving envelope**?

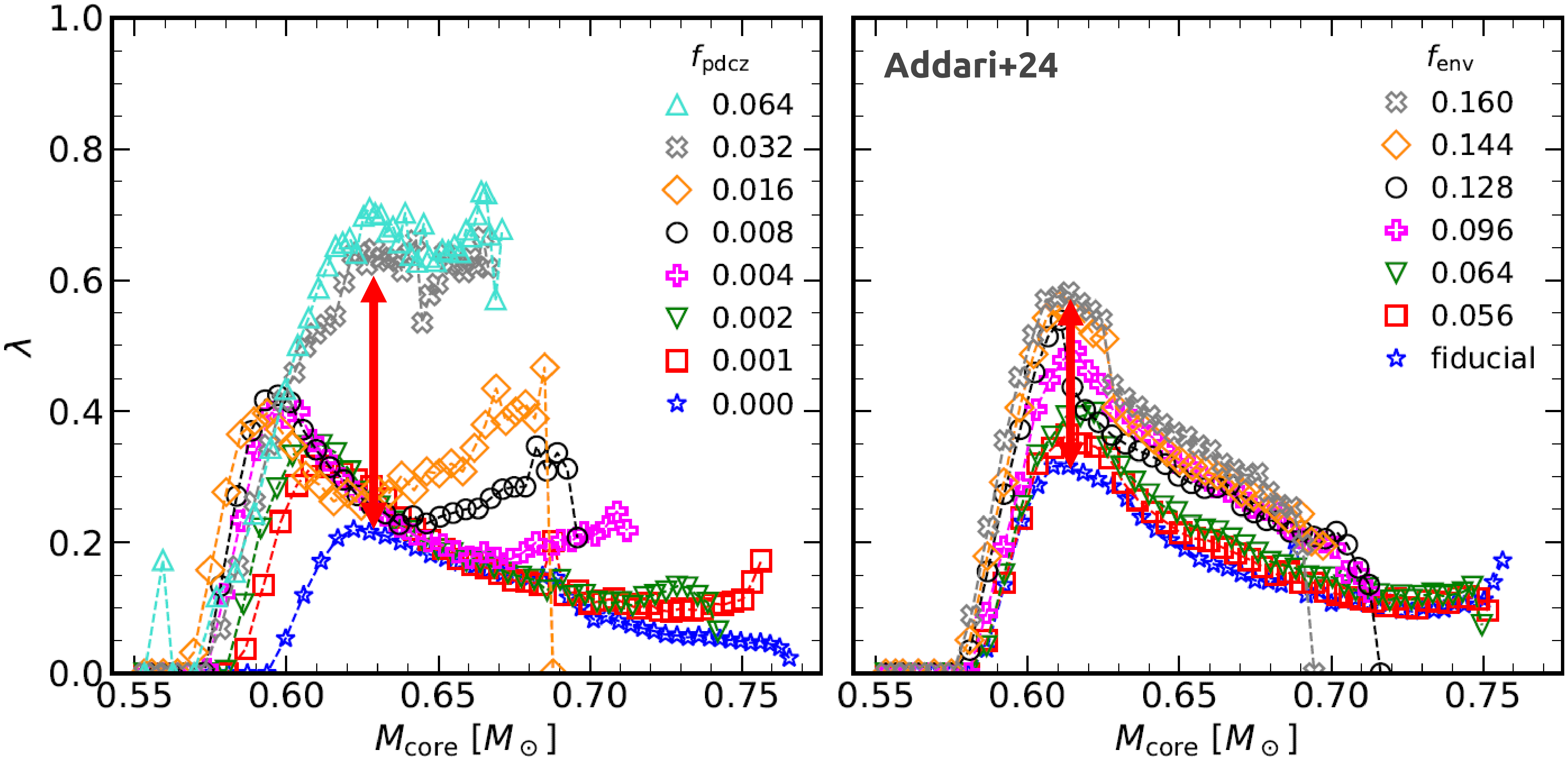
☐ Independent on **non-simultaneous / simultaneous solver** (Addari+20 Master's Thesis)

Now results!

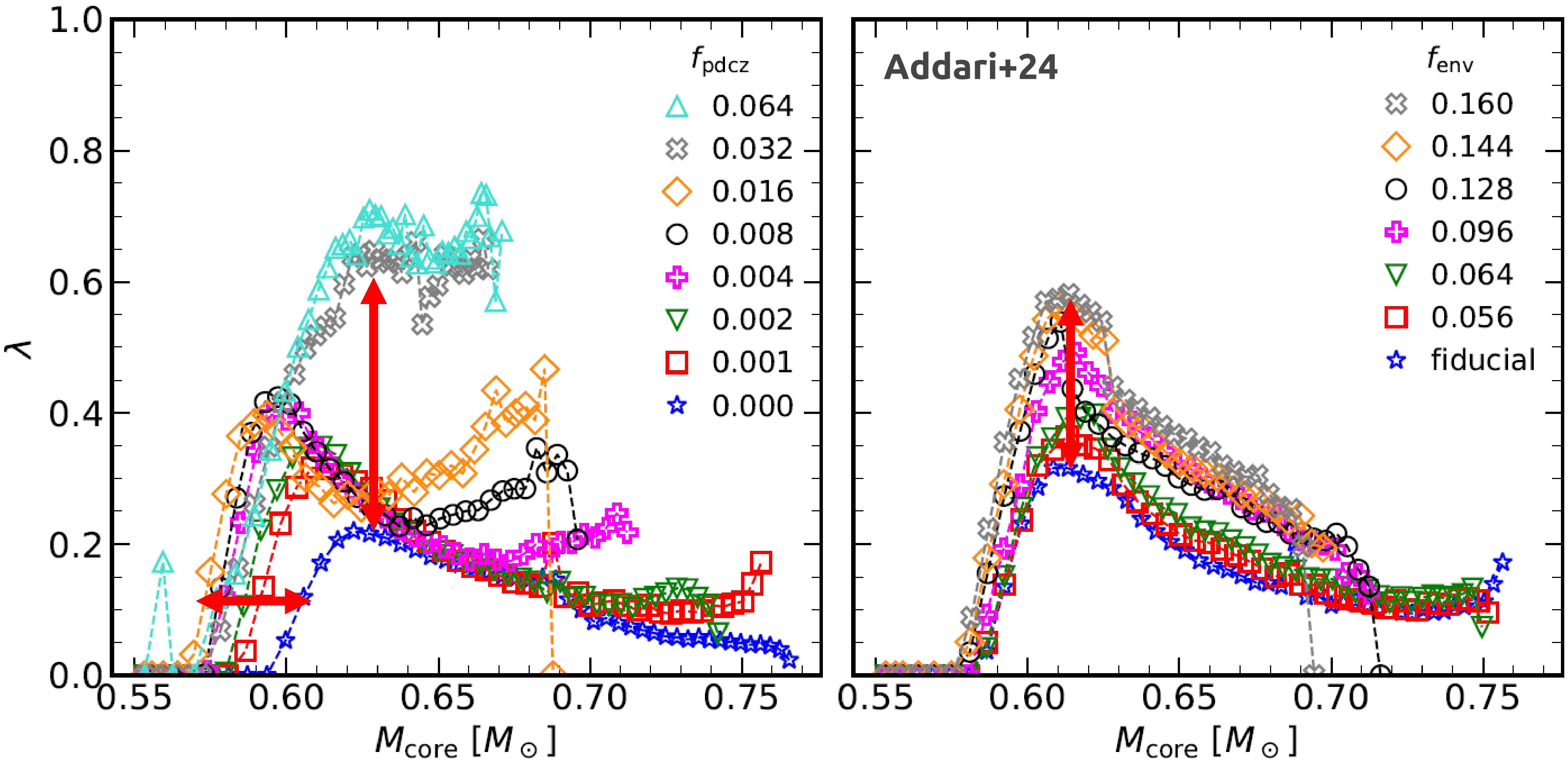
TDU efficiency



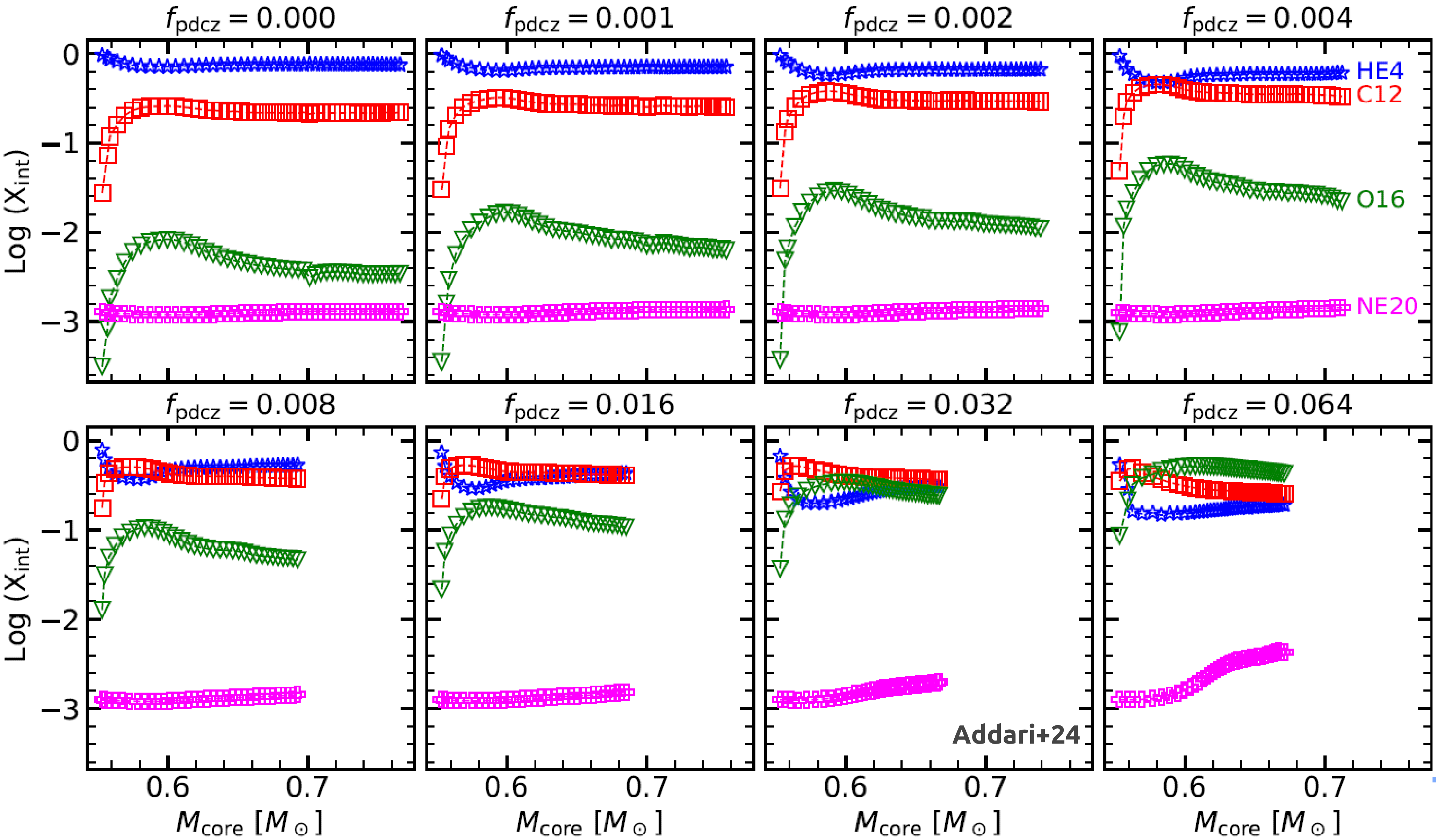
TDU efficiency



TDU efficiency

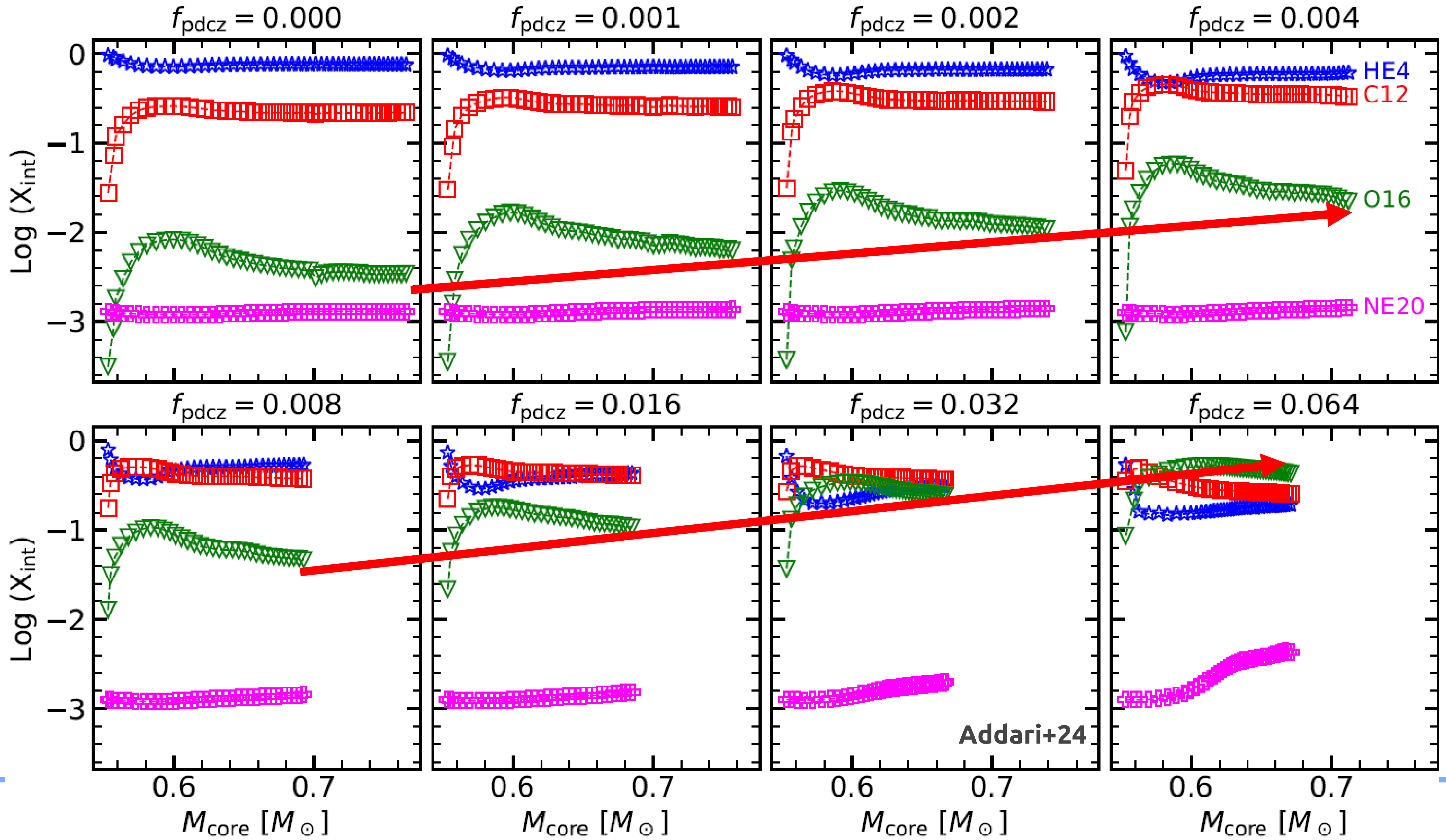


Intershell composition

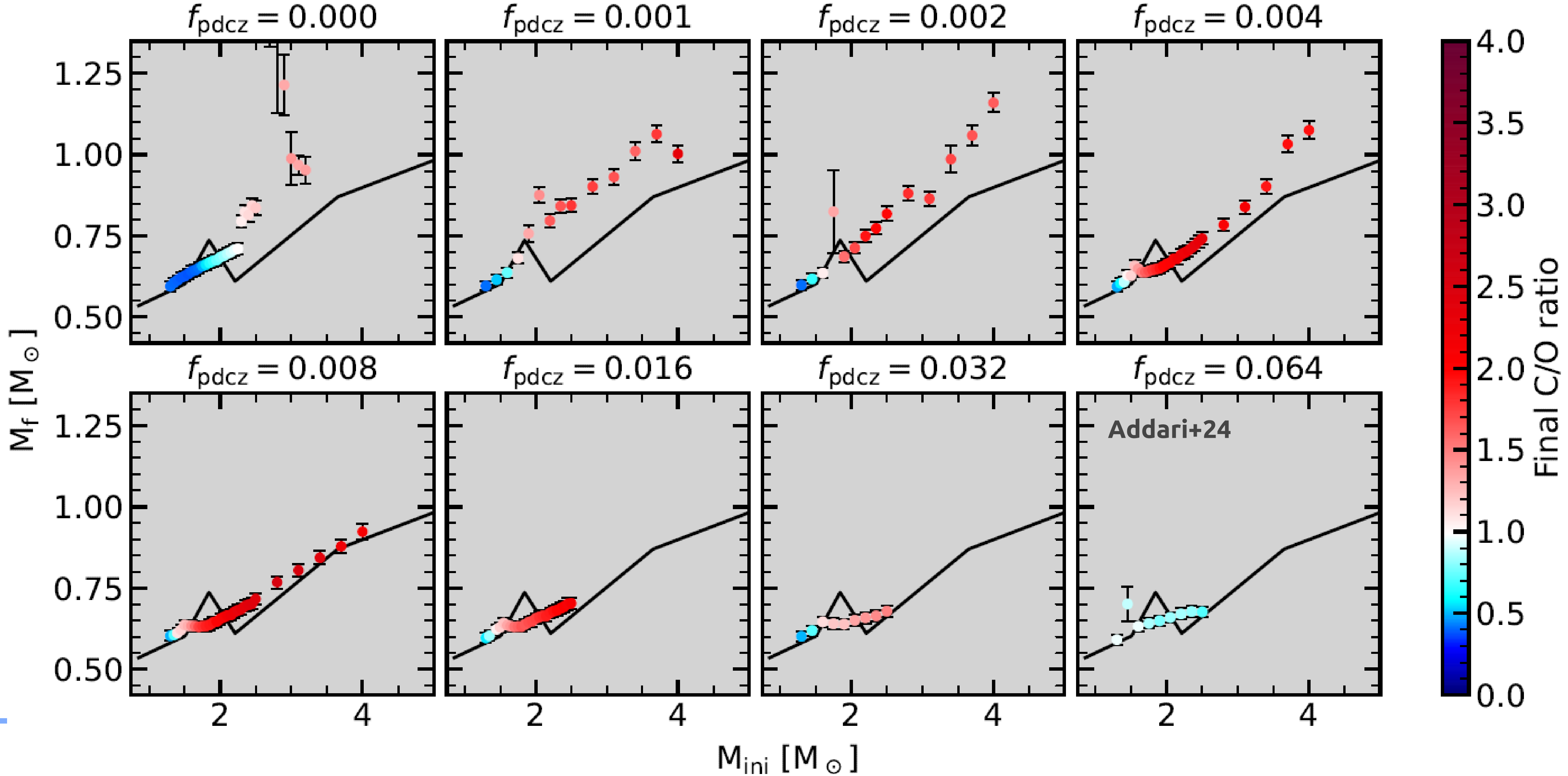


Addari+24

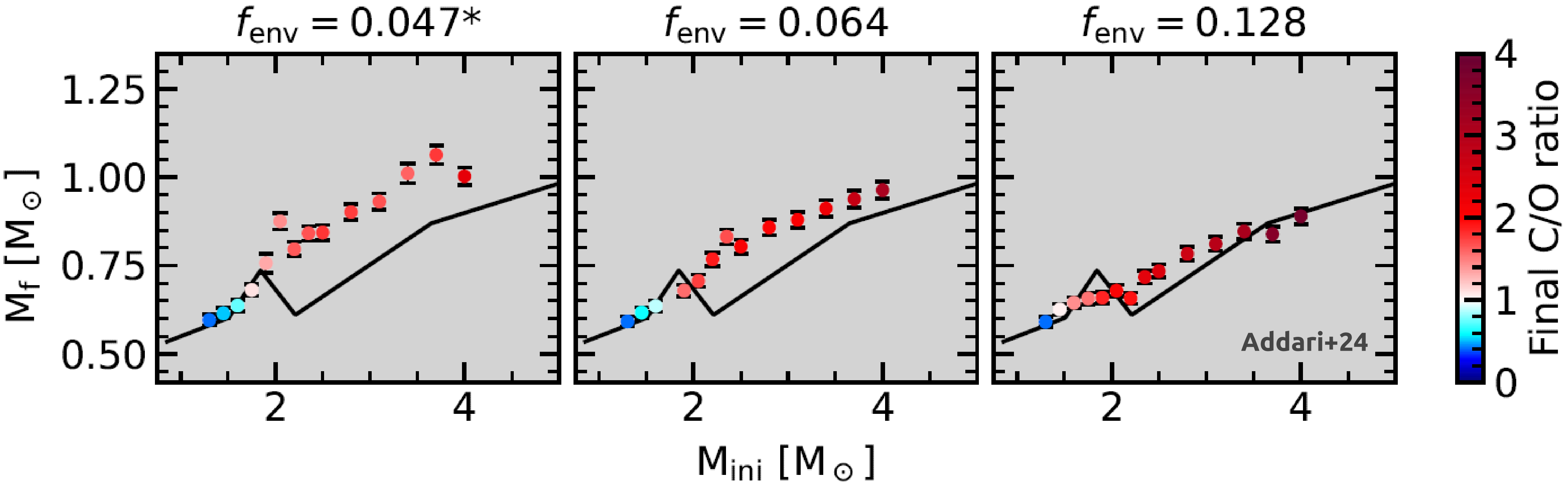
Intershell composition



Shape of the IFMR – fixed f_{env}



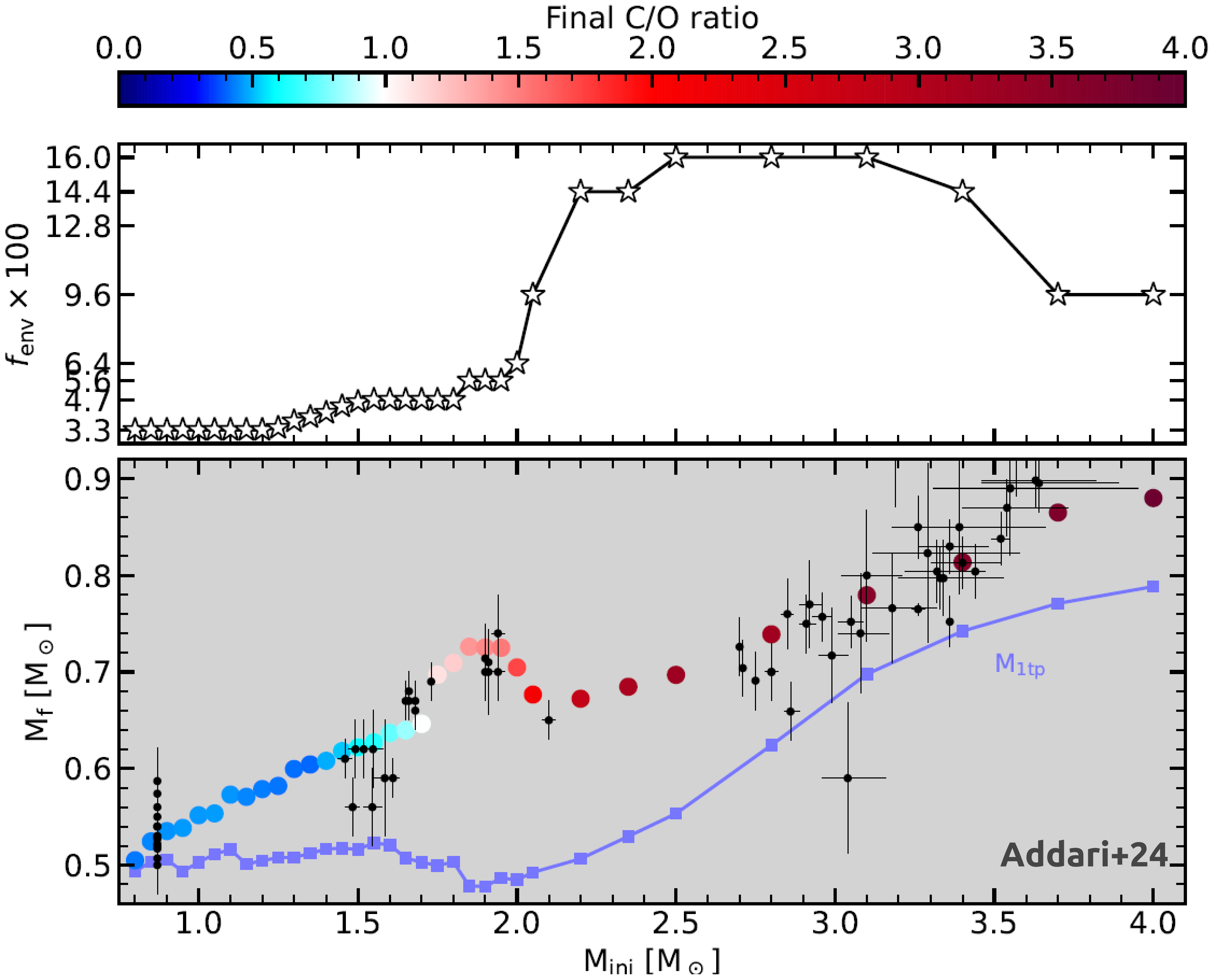
Shape of the IFMR – fixed $f_{\rm pdcz}$



Final IFMR!

Take-away:

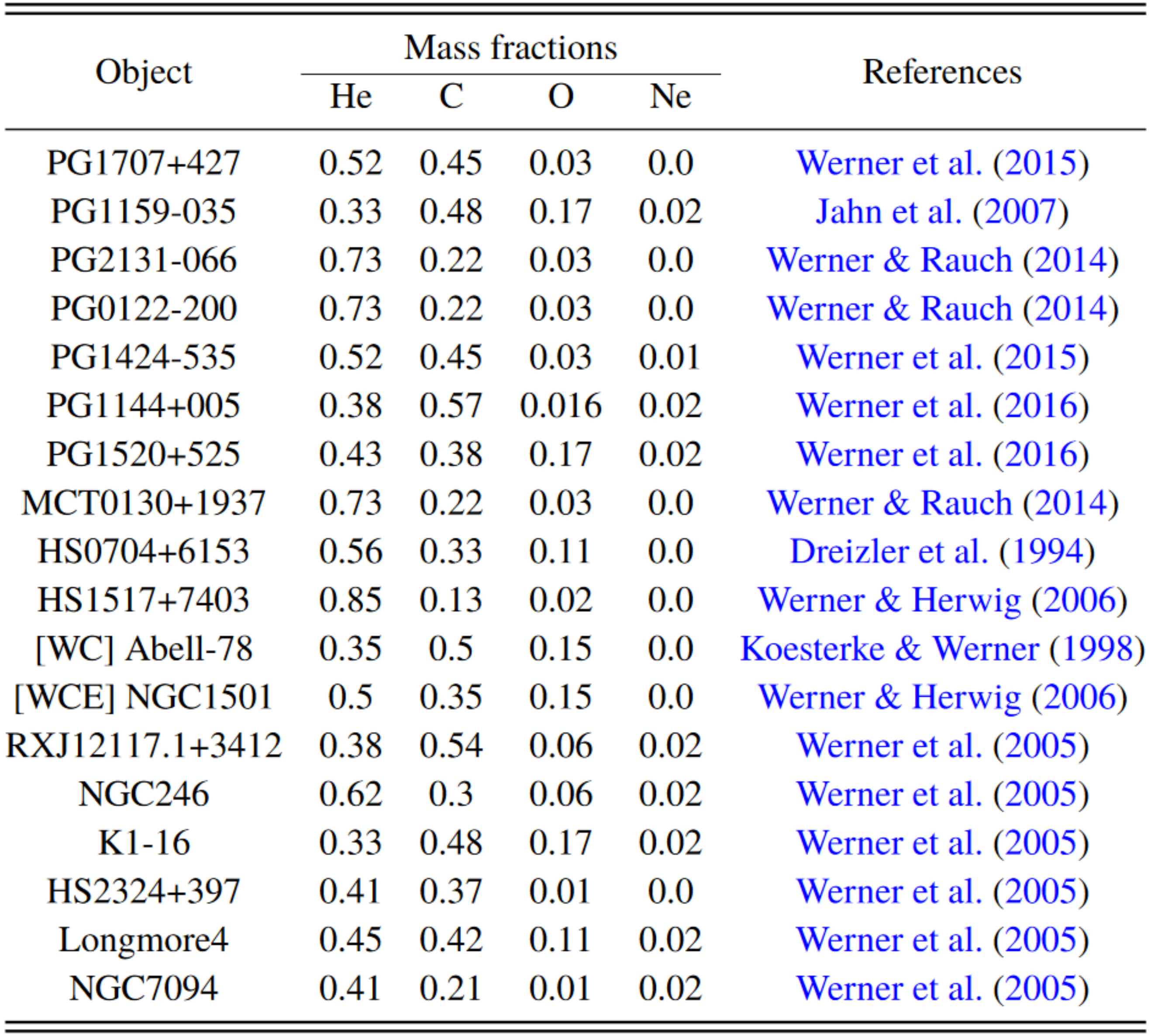
- ☐ No kink without f_{pdcz}
- ☐ No kink without increasing f_{env}
- ☐ C-O excess sets lifetime



Intershell abundances

PG1159 and [WC]-type CSPNe

They show intershell abundances of on the last TP (which one?)



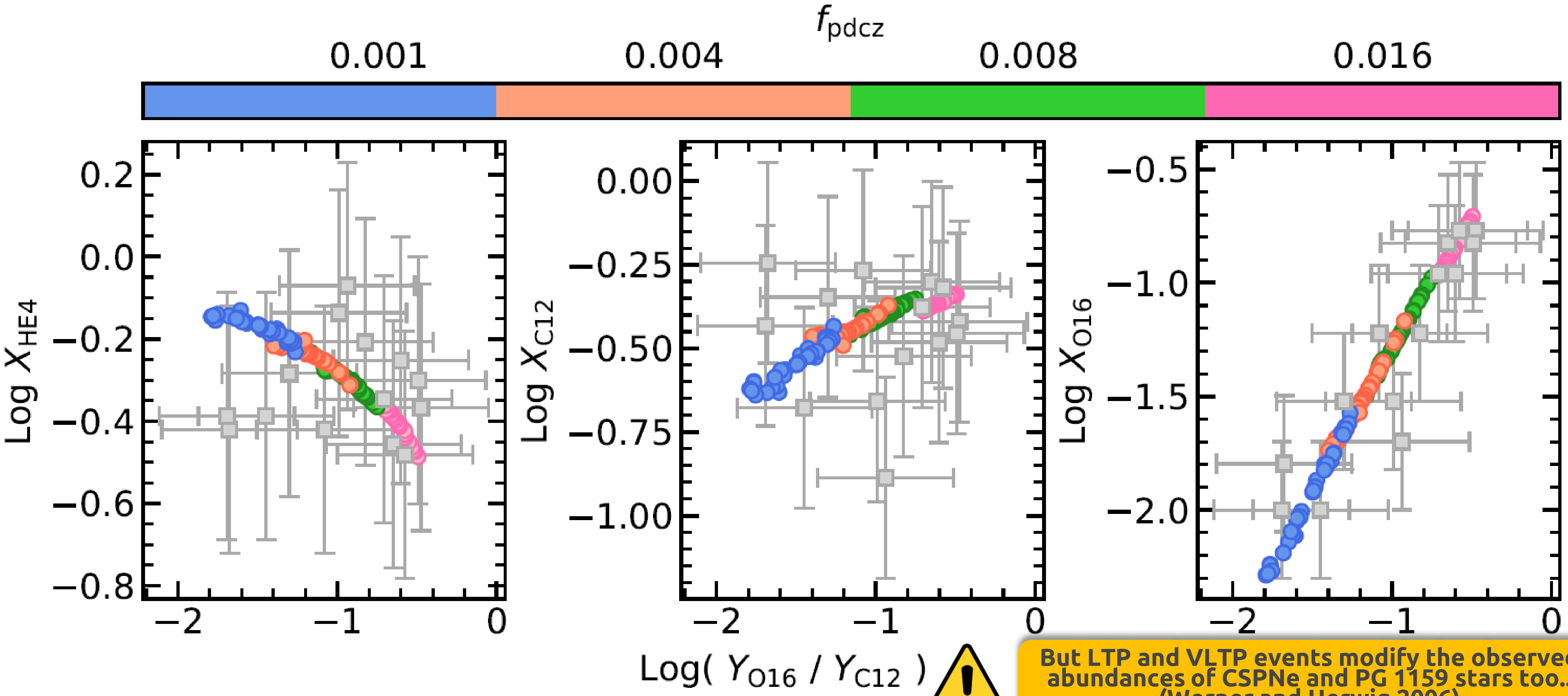
Object	Mass fractions				References
	He	C	O	Ne	
PG1707+427	0.52	0.45	0.03	0.0	Werner et al. (2015)
PG1159-035	0.33	0.48	0.17	0.02	Jahn et al. (2007)
PG2131-066	0.73	0.22	0.03	0.0	Werner & Rauch (2014)
PG0122-200	0.73	0.22	0.03	0.0	Werner & Rauch (2014)
PG1424-535	0.52	0.45	0.03	0.01	Werner et al. (2015)
PG1144+005	0.38	0.57	0.016	0.02	Werner et al. (2016)
PG1520+525	0.43	0.38	0.17	0.02	Werner et al. (2016)
MCT0130+1937	0.73	0.22	0.03	0.0	Werner & Rauch (2014)
HS0704+6153	0.56	0.33	0.11	0.0	Dreizler et al. (1994)
HS1517+7403	0.85	0.13	0.02	0.0	Werner & Herwig (2006)
[WC] Abell-78	0.35	0.5	0.15	0.0	Koesterke & Werner (1998)
[WCE] NGC1501	0.5	0.35	0.15	0.0	Werner & Herwig (2006)
RXJ12117.1+3412	0.38	0.54	0.06	0.02	Werner et al. (2005)
NGC246	0.62	0.3	0.06	0.02	Werner et al. (2005)
K1-16	0.33	0.48	0.17	0.02	Werner et al. (2005)
HS2324+397	0.41	0.37	0.01	0.0	Werner et al. (2005)
Longmore4	0.45	0.42	0.11	0.02	Werner et al. (2005)
NGC7094	0.41	0.21	0.01	0.02	Werner et al. (2005)

Typical errors are of 0.3 – 0.5 dex (see references)

$f_{\rm pdcz}$ ~ 0.008 – 0.016

Herwig 2000, Wagtaff+20, Addari+20

Intershell abundances



But LTP and VLTP events modify the observed abundances of CSPNe and PG 1159 stars too..
(Werner and Herwig 2006)

Addari+24, ApJ 964, 51

Summary

❑ C-O is critical in **setting the lifetime** and **letting the core mass grow** over the usual monotonic IFMR.

❑ To bring the needed amount of carbon, f_{env} **must increase with the initial mass** and a **minimum amount of** f_{pdcz} is needed. The value of f_{env} roughly matches the one needed to get blue loops width in more massive stars (Tang+2014)

❑ The value f_{pdcz} is not enough to cover the whole range of intershell abundances, which however may be **severely altered by LTP and VLTP** and have **large error bars**.