Fitting and Comparing Galactic Foreground Models for Unbiased 21-cm Cosmology

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Image credit: JWST NIR-cam

2 Main Sources of Bias and Error in 21-cm Signal Extraction

1. Overlap between FG-space and Signal-space

- Inevitable as these vector spaces are not, in general, orthogonal.
- Main focus of pipelines.
- Decrease overlap by utilizing LST-dependence of FG and polarization.

THIS WORK

2. Inadequate FG Models

- Models which cannot fit the FG down to the noise level IN THE SIGNAL will introduce ADDITIONAL bias and error, regardless of (1).
- Requires testing the ability of FG models to fit realistic FG-only spectra.
- Less attention, if any, in the literature.

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The Flowchart of Despair

Unmodelled FG + Overlap with Signal Model → Biased Signal Extraction

↓

Increase number of FG parameters/complexity

↓

Complex, HD Parameter FG + Overlap with Signal Model → Large Signal Errors

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The FG Test

STEP ONE → STEP TWO → STEP THREE

- Generate hyper-realistic mock spectra (NO 21-cm SIGNAL)

- Fit the spectra with seven different commonly employed FG models

- Compare which model(s) produces the best fits and remove all FG power down to the noise level

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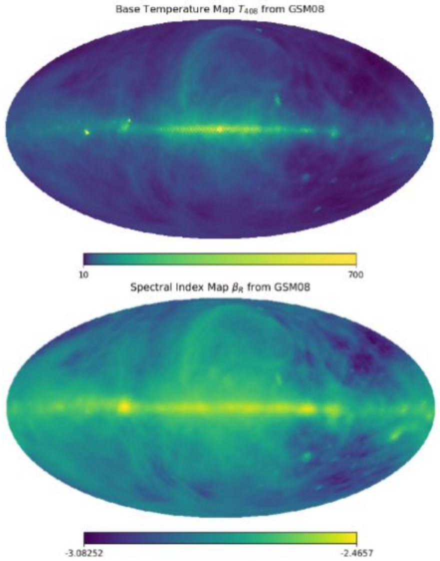
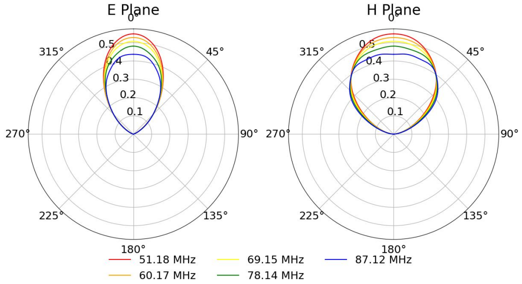
Foreground Tests: Mock Spectra Simulations

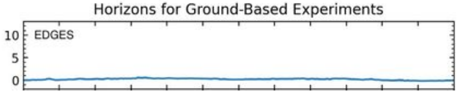
Figures from Hibbard et al. 2023, under review



Intrinsic FG



Beam Simulation,
51-87 MHz band



Horizon, pointing

$$f_n(t - t_i) = \frac{1}{2N_n\delta t}\sum_{i=1}^{N_n} I(|t - t_i| < \delta t)$$

Discrete time-sampling

$$\sigma_n^2 = \frac{T_n^2(\nu)}{D_w},$$

Radiometer noise

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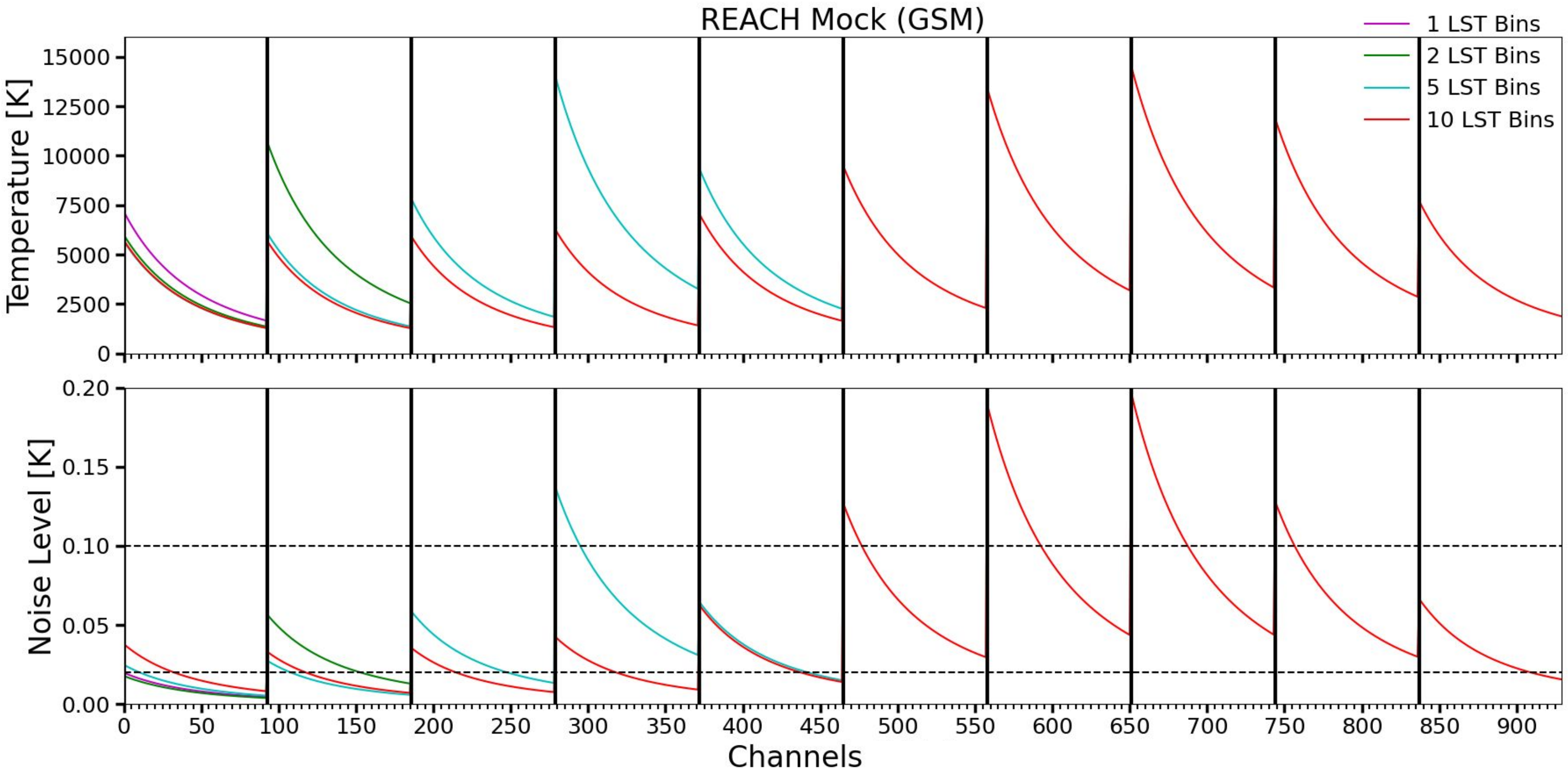
Mock Spectra

Figures from Hibbard et al. 2023, under review



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Phenomenological models

- Add more terms to increase complexity
- Good at modelling high-order ripple effects (unknowns)
- Difficult to include time-dependence, other Stokes parameters
- Unclear if they capture all kinds of systematics effectively without forward-modelling.

Forward Models

- Require simulations of the beam, intrinsic FG, local environment, etc.
- Can be computationally expensive, requiring physics simulations and models of all effects
- Can include time dependence, other Stokes parameters
- Easy to model and include other “physical” effects
- Break degeneracies with signal model

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Phenomenological

Polynomial

$$\left(\frac{\nu}{\nu_0}\right)^{-2.5}\sum_{k=0}^{N_{py}} a_k \left[\ln\frac{\nu}{\nu_0}\right]^k \quad (10)$$

$$\left(\frac{\nu}{\nu_0}\right)^{-2.5}\sum_{k=0}^{N_{py}} a_k \left(\frac{\nu}{\nu_0}\right)^k \quad (11)$$

$$\left(\frac{\nu}{\nu_0}\right)^{-2.5}\sum_{k=0}^{2} a_k \left(\ln\frac{\nu}{\nu_0}\right)^k + a_3\left(\frac{\nu}{\nu_0}\right)^{-4.5} + a_4\left(\frac{\nu}{\nu_0}\right)^{-2} \quad (12)$$

(Pritchard and Loeb 2012, Bernardi et al. 2016, Mozdzen et al. 2016, Bowman et al. 2018, Singh et al. 2021, etc.)

- Polynomials, modulated by a constant power-law representing the "average" galactic spectral index.
- 5-6 terms, traditionally.

Maximally-Smooth

$$\frac{d^m \mathcal{M}_{MDP}}{d\nu^m} \leq 0$$

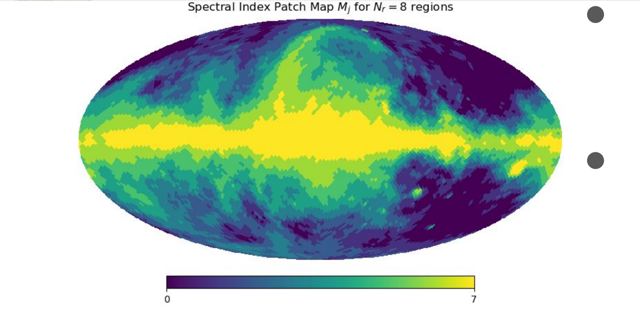
$$\frac{d^m \log_{10}(\mathcal{M}_{MLLP})}{d\log_{10}(\nu)^m} \leq 0$$

(Rao et al. 2017, Bevins et al. 2021, Singh et al. 2021)

- Polynomials with derivatives set to have no inflections or "ripples."
- Supposed to account for smoothly varying foregrounds without accidentally picking up any signal power.

Forward-Models

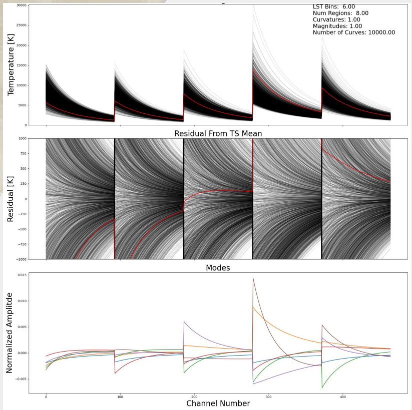
Nonlinear



(Anstey et al. 2021, etc., REACH Collab.)

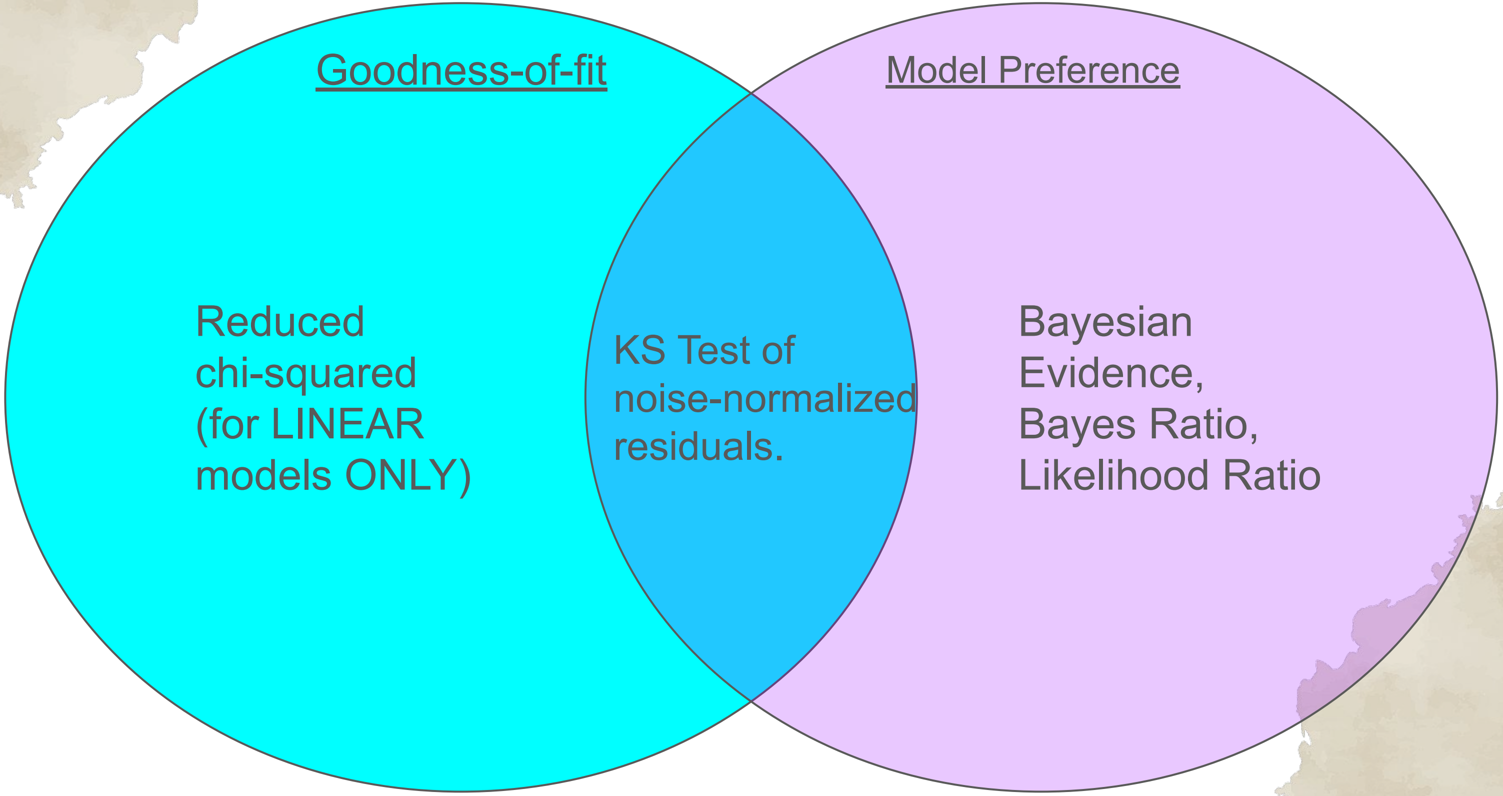
- Split sky into N_r equal-perce ntile regions
- Assign index, magnitude, and curvature to each region

Linear

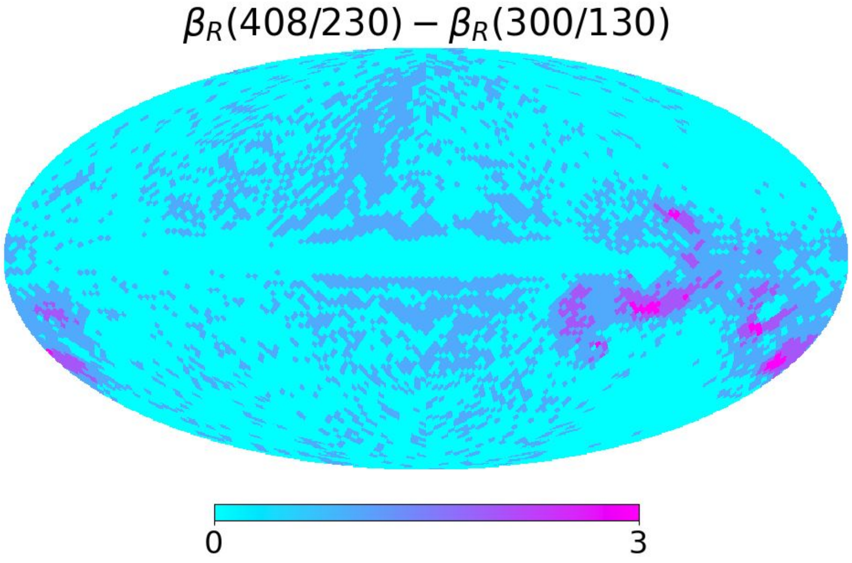


- Generate ten thousand example spectra using nonlinear forward-model and large parameter ranges
- Eigenmodes (spectra) from SVD.
- "...ought to rebrand as SBI, because that's what it is..." - W. Handley

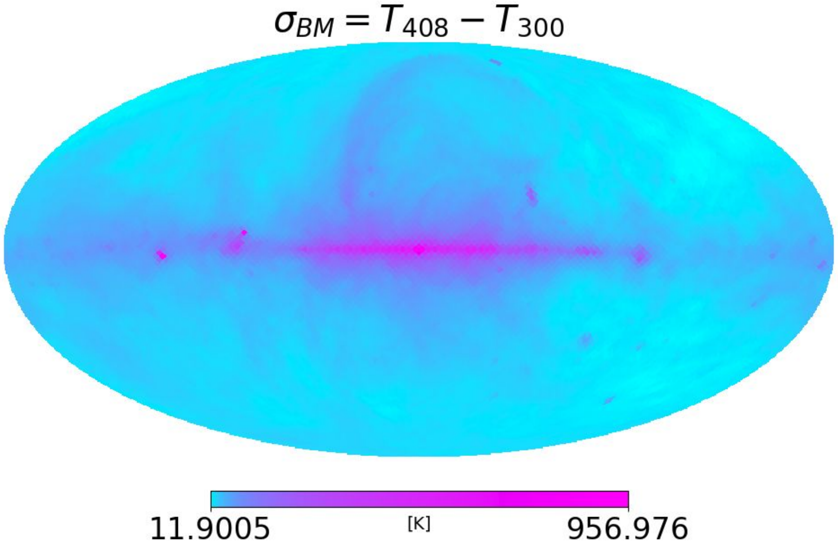
(Pylinex paper series, Tauscher et al. 2018, etc.)


Goodness-of-fit
Model Preference
Reduced chi-squared (for LINEAR models ONLY)
KS Test of noise-normalized residuals.
Bayesian Evidence, Bayes Ratio, Likelihood Ratio

Testing Model Spatial Inputs (Maps)



Spatial “Error” in input spectral index patch map (PM).



Spatial “Error” in input base temperature map (BM).

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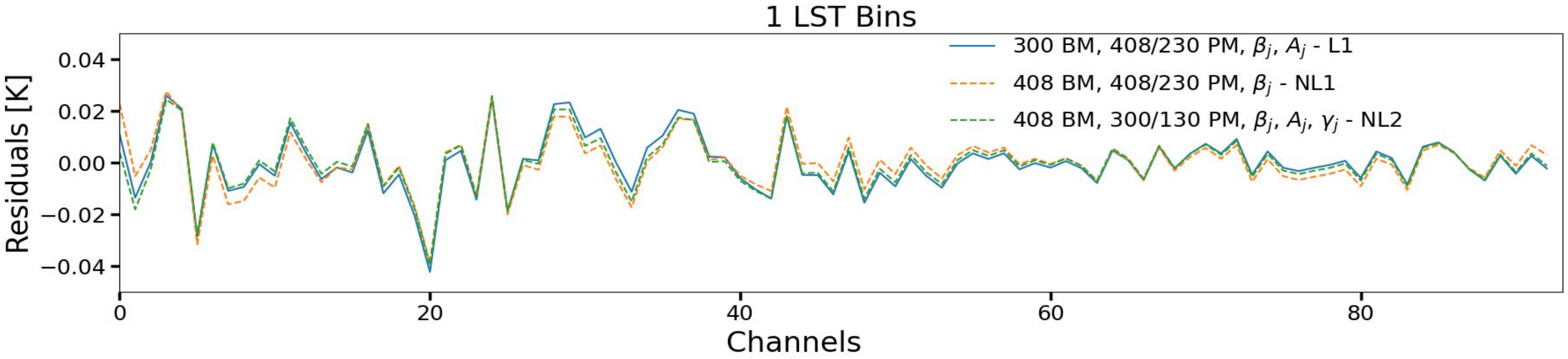
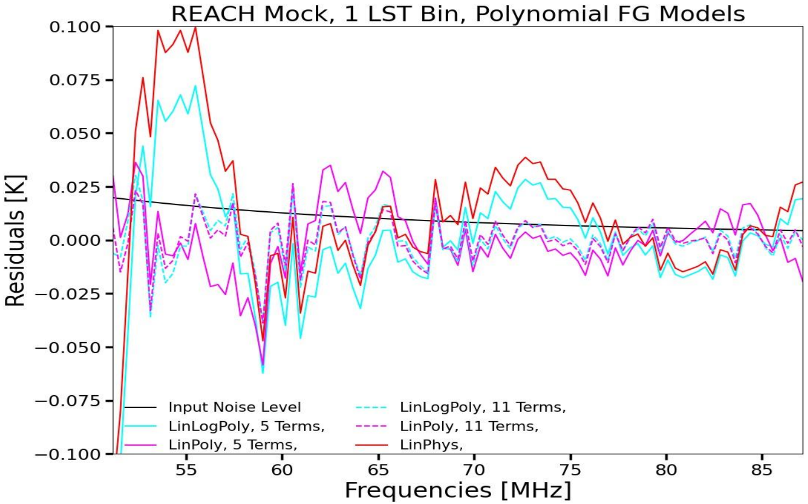
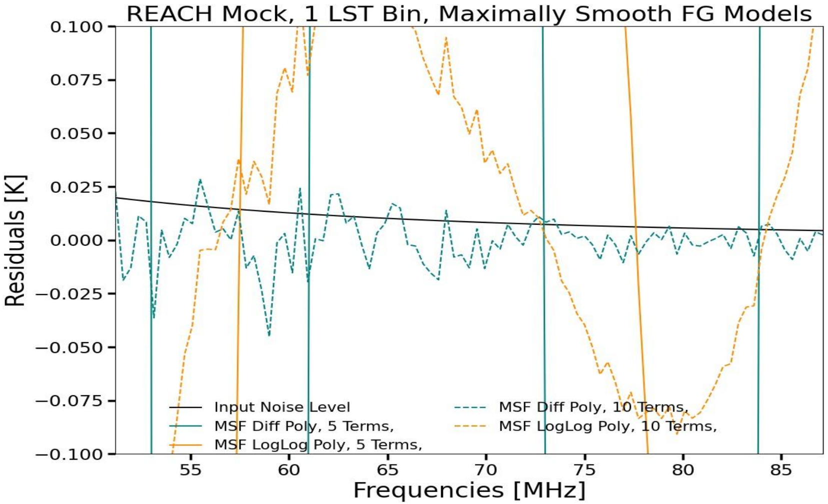
Comparison - 1 LST Bin

Figures from Hibbard et al. 2023, under review







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1 LST Bin–Statistics
Gold - Pass KS Test, Gray - Fail

Tables from Hibbard et al. 2023, under review

Forward Models

	Model		Linear							Nonlinear					
LSTs	Input	θ_j	N_r	χ^2_{red}	σ	$\ln Z$	N_x	p_{ks}	BF	N_r	χ^2_{red}	$\ln Z$	N_θ	p_{ks}	BF
	IDEAL	β_j	4	0.96	1	-615.71	7	0.94	–	4	1.02	-81.05	4	0.99	NL1
		β_j, A_j	3	1.14	1	-625.24	6	0.97	–	4	1.02	-105.78	8	0.97	–
		β_j, A_j, γ_j	2	1.14	1	-628.66	6	0.90	–	4	1.18	-119.07	12	0.88	–

Polynomial

Model	N_{py}	χ^2_{red}	σ	RMS (mK)	p_{ks}
LinLogPoly	5	6.2	34	31	9.3e-6
	6	1.44	3	13	0.40
	11	1.11	1	11	0.94
LinPoly	5	2.85	12	16	0.04
	6	1.60	4	14	0.33
	11	1.1	1	10	0.95
LinPhys	7	9.04	53	36	1.87e-11

MSF

Model	N_{MSF}	χ^2_{red}	RMS (mK)	p_{ks}
Diff Poly	5	5e5	6e3	1.4e-24
	6	2.81e4	1.36e3	1.56e-21
	10	1.28	12	0.76
	15	1.41	11	0.58
LogLog Poly	5	2.9e4	2.1e4	8.9e-27
	6	3.1e4	2.2e3	8.9e-27
	10	110	103	1.4e-17
	15	3.92	23	0.008

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1 LST Bin - Incorrect Map Inputs?

1	300 BM	β_j	4	0.96	1	-614.26	7	0.93	–	4	1754	-7.8e4	4	1.36e-24	–
		β_j, A_j	3	1.10	1	-623.23	6	0.99	L1	4	1.02	-107.39	8	0.98	–
		β_j, A_j, γ_j	2	1.12	1	-626.91	6	0.95	–	4	1.10	-98.22	12	0.93	–
	(300/130) PM	β_j	4	0.99	1	-613.8	5	0.998	–	4	3.9e10	-1.74e12	4	~ 0	–
		β_j, A_j	3	1.06	1	-618.83	8	0.998	–	4	1.03	-112.26	8	0.94	–
		β_j, A_j, γ_j	2	1.09	1	-623.53	6	0.88	–	4	1.09	-112.71	12	0.999	NL2

Tables from Hibbard et al. 2023, under review

- BM → Wrong Input Base Map to Mock spectra
- PM → Wrong Input Patch Map to Mock spectra

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Comparison - Multiple LST Bins

2	IDEAL	β_j	5	1.00	1	-1129.88	9	0.68	–	8	1.13	-154.24	8	0.81	–
		β_j, A_j	4	1.02	1	-1160.86	13	0.81	–	8	1.03	-335.72	16	0.83	NL3
		β_j, A_j, γ_j	15	1.03	1	-1192.70	18	0.80	–	8	1.53	-307.33	24	0.09	–
	300 BM	β_j	5	1.03	1	-1130.74	13	0.51	–	8	7.83e4	-6.96e6	8	5.6e-45	–
		β_j, A_j	4	1.02	1	-1155.56	13	0.89	L2	8	1.15	-364.17	16	0.82	–
		β_j, A_j, γ_j	4	1.08	1	-1189.71	14	0.78	–	8	1.27	-234.77	24	0.50	–
	(300/130) PM	β_j	6	1.13	1	-1134.67	21	0.71	–	8	2.1e10	-1.82e12	8	~ 0	–
		β_j, A_j	4	1.00	1	-1142.86	13	0.97	L3	8	3.43	-500.73	16	0.002	–
		β_j, A_j, γ_j	4	1.04	1	-1162.65	14	0.96	–	8	1.34	-220.24	24	0.26	–

Tables from Hibbard et al. 2023, under review

- Nonlinear model performs well for Ideal case, but begins to break down.

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Comparison - Multiple LST Bins

10	IDEAL	β_j	14	1.08	2	-4833.53	36	0.80	–	9	477.92	-2.20e5	9	1.4e-198	–
		β_j, A_j	13	2.58	33	-6783.17	46	1.91e-8	–	9	64.25	-2.9e4	18	7.1e-148	–
		β_j, A_j, γ_j	19	1.04	1	-5059.1	58	0.82	L6	9	27.6	-1.29e4	27	2.5e-107	–
	300 BM	β_j	12	1.40	8	-5004.78	39	0.03	–	9	76973	-3.54e7	9	2.4e-252	–
		β_j, A_j	13	2.39	29	-6696.14	46	1.3e-7	–	9	32.01	-1.48e4	18	1.7e-122	–
		β_j, A_j, γ_j	19	1.07	2	-5183.84	56	0.94	L7	9	19.66	-9134.45	27	8.6e-80	–
	(300/130) PM	β_j	17	2.34	28	-6408.78	54	2.7e-9	–	9	4.0e9	-1.84e12	9	~ 0	–
		β_j, A_j	20	2.02	5	-6060.94	73	1.0e-4	–	9	41717	-1.90e7	18	2.0e-251	–
		β_j, A_j, γ_j	20	1.34	7	-5104.81	79	0.12	L8	9	18109	-8.18e6	27	6.6e-223	–

Tables from Hibbard et al. 2023, under review

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Conclusions

- **Inadequate FG Models are a significant source of bias and error in 21-cm signal extractions.**
- For 1 LST bin, the nonlinear forward-model is preferred (slightly)
  - Linear forward-model also works
  - Polynomials and MSFs require >5 parameters, at least, and some don't work at all.
- For multiple LST bin fits, linear forward-model is highly preferred
- KS-test is a robust way of measuring goodness-of-fit and model preference.

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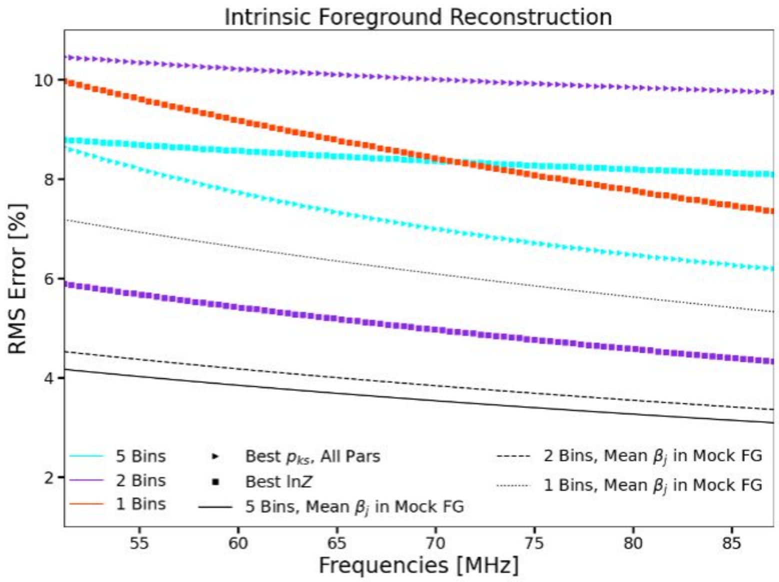
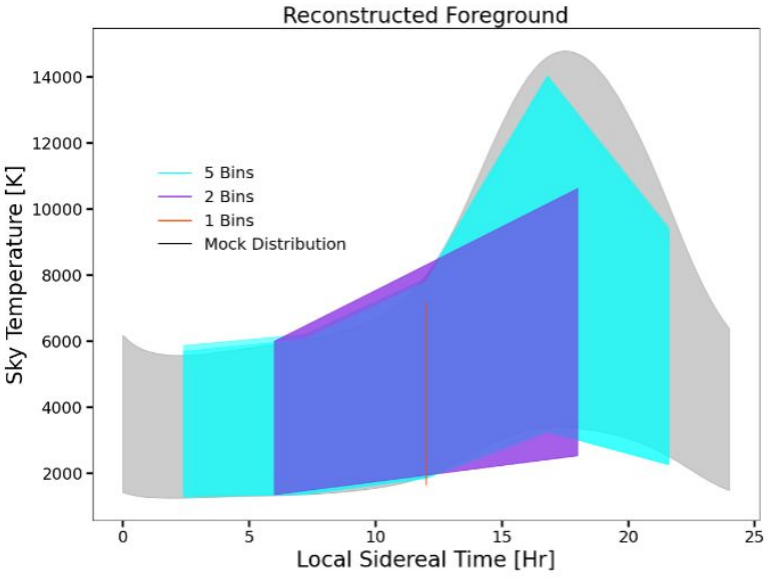
Nonlinear FG Reconstruction?



(a) RMS Error of the Best-fit Nonlinear Foreground Models Compared to the Input Intrinsic Foreground.



(b) Reconstructed foreground as a function of LST.

Figure from Hibbard et al. 2023, in prep

Even with the corruption of the beam, we can still recover Intrinsic FG's with less than ~10 % error across the band.

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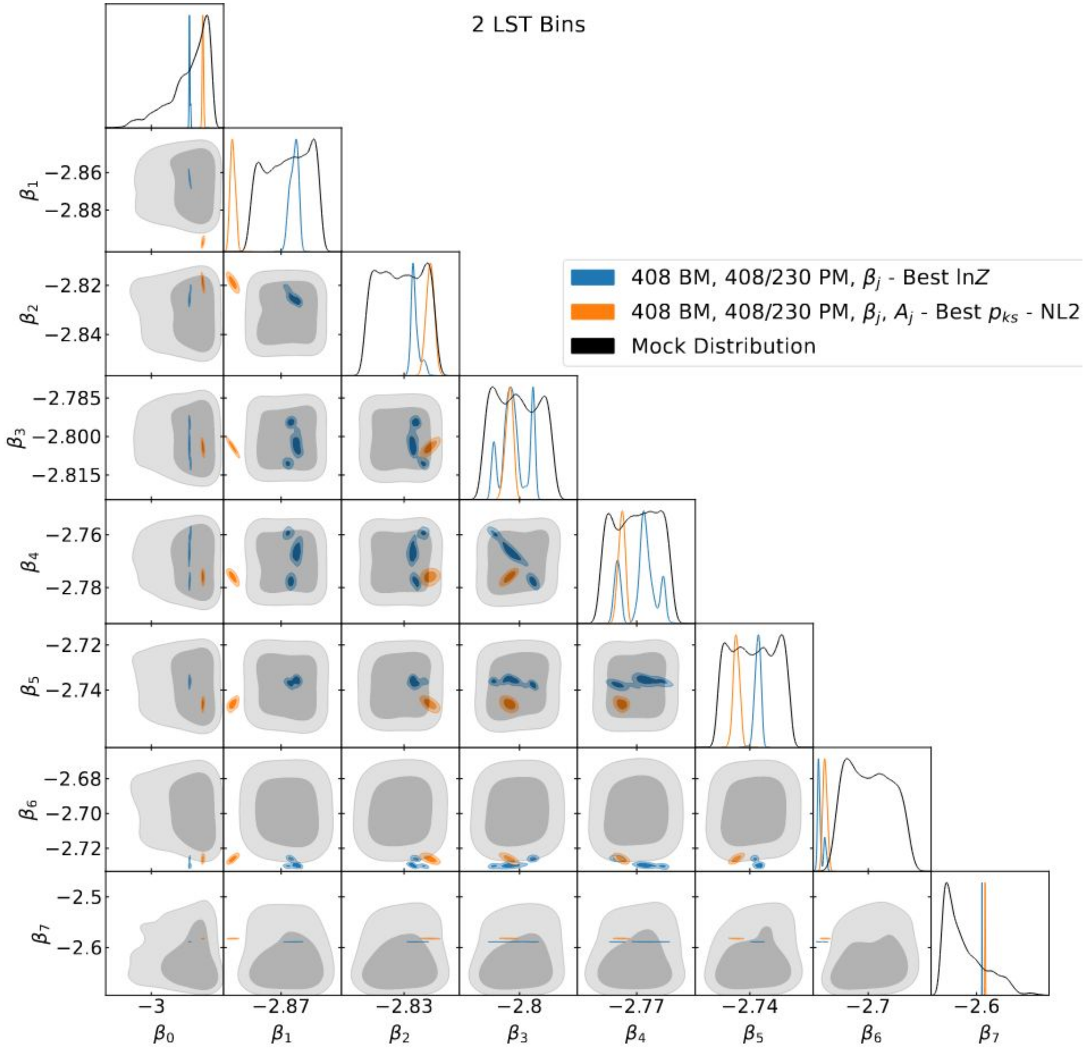

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