UNIVERSITY OF CAMBRIDGE

Directivity modeling for global 21 cm experiments

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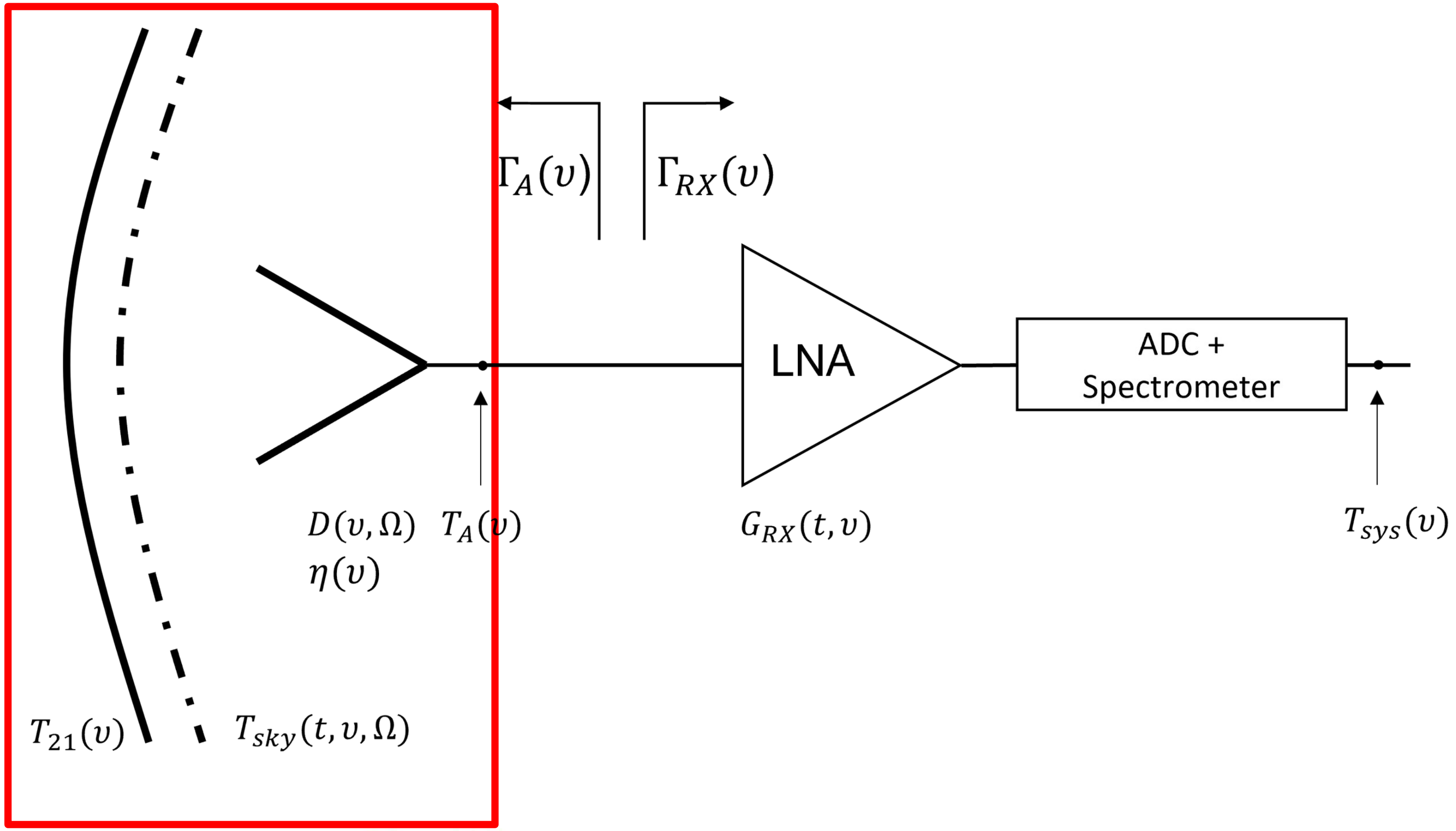
Antenna front end (Antenna temperature)



$$T_{\mathrm{A}}(\nu,t)=\frac{1}{4\pi}\int_{\Omega}D(\nu,\Omega)\eta(\nu)(T_{\mathrm{sky}}(t,\nu,\Omega)+T_{21}(\nu))d\Omega.$$

‘Back of the envelope’ uncertainty

$$T_A(\nu,t) = \frac{1}{4\pi}\int_\Omega D(\nu,\Omega)\eta(\nu)(T_{sky}(t,\nu,\Omega)+T_{21}(\nu))d\Omega.$$

- Uncertainty in $T_A \sim T_{sky}$ * uncertainty in D
  - T_{sky} between 1000 K and 12000 K
  - D between 0 and 10
  - $T_{21} < 500$ mK
- So we want uncertainty in D better than 1 in 1000

A more quantified version

- Want one number to quantify the difference between two patterns ΔD
  - Vary over both frequency and space
- Dealing with small numbers so will take in dB

$$\Delta \mathbf{D}(\nu) = \langle |\mathbf{D}_{\rm obs}(\nu, \Omega) - \tilde{\mathbf{D}}_{\rm obs}(\nu, \Omega)| \rangle_\Omega$$

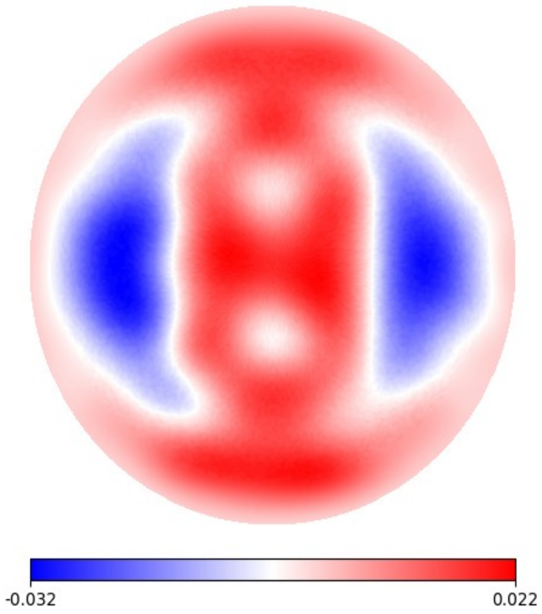
$$\Delta D = \langle \Delta \mathbf{D}(\nu) \rangle_\nu .$$

$$\Delta D \mathrm{dB} = 10 \log_{10} \Delta D$$

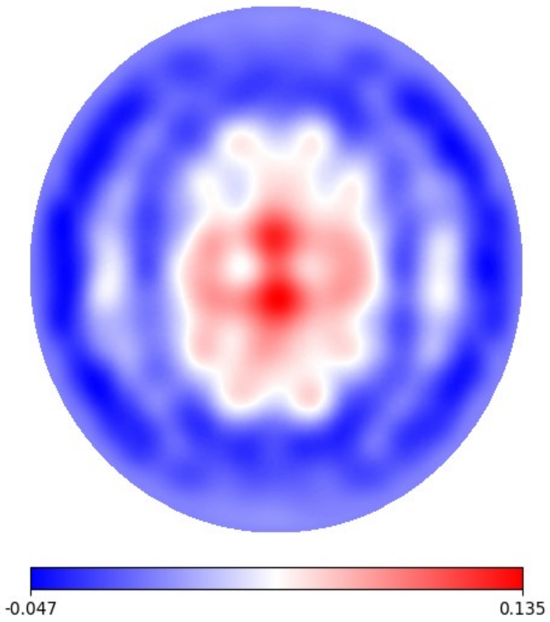
Quantifying directivity uncertainty (Example)

- Subtraction between two directivity patterns.
  - Vary over both frequency and space

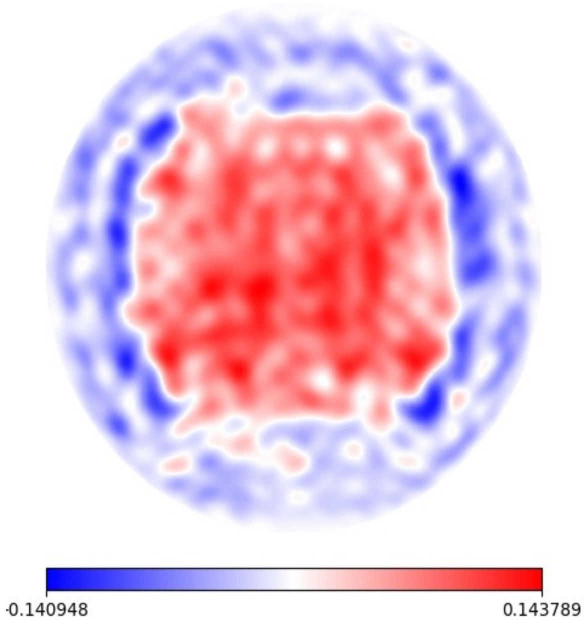
50 MHz



90 MHz

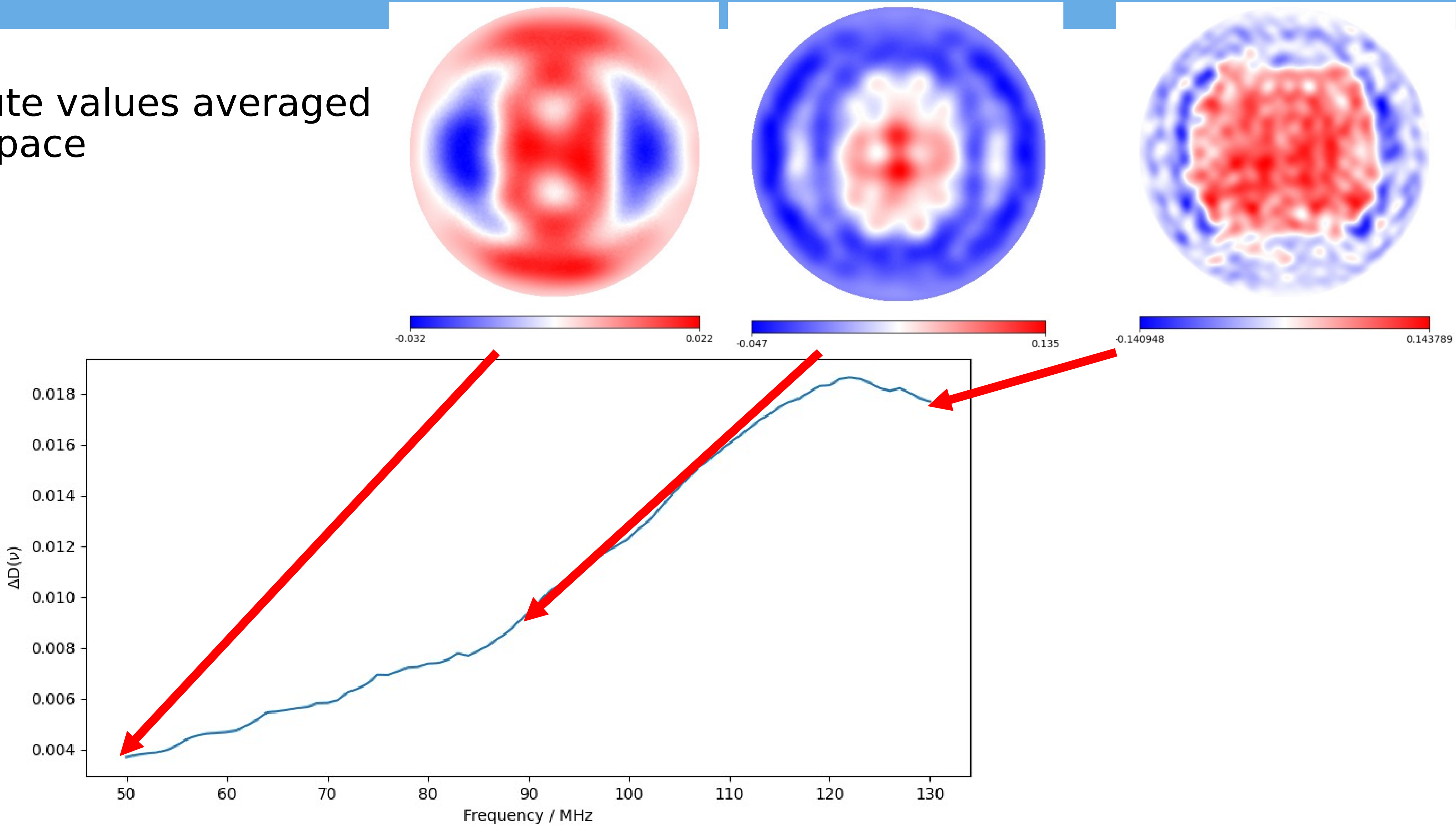


140 MHz



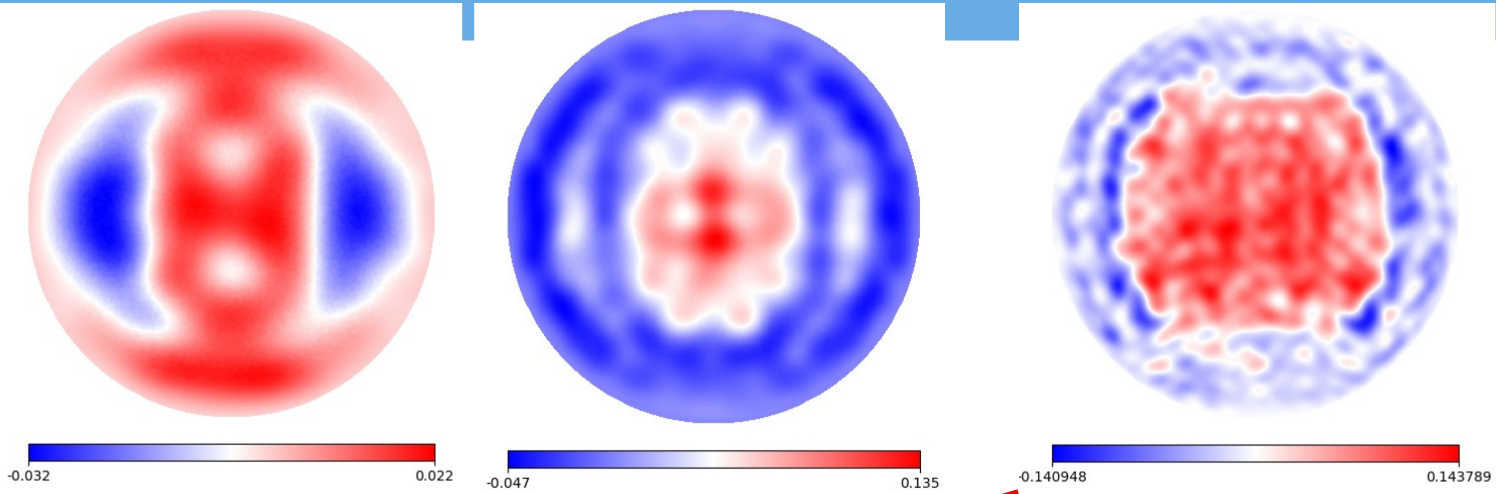
Quantifying directivity uncertainty (Example)

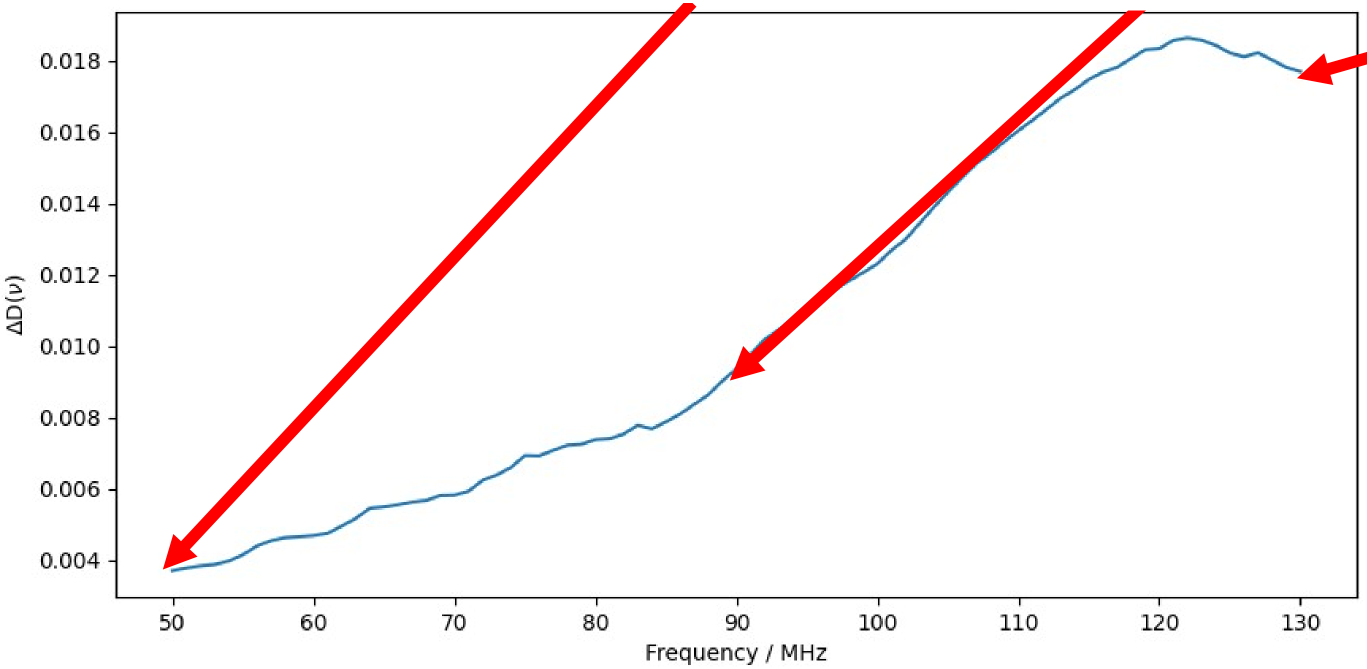
- Absolute values averaged over space



Quantifying directivity uncertainty (Example)

- Averaged over frequency for a single number





Averaged = 0.0106 difference

ΔD = -20 dB

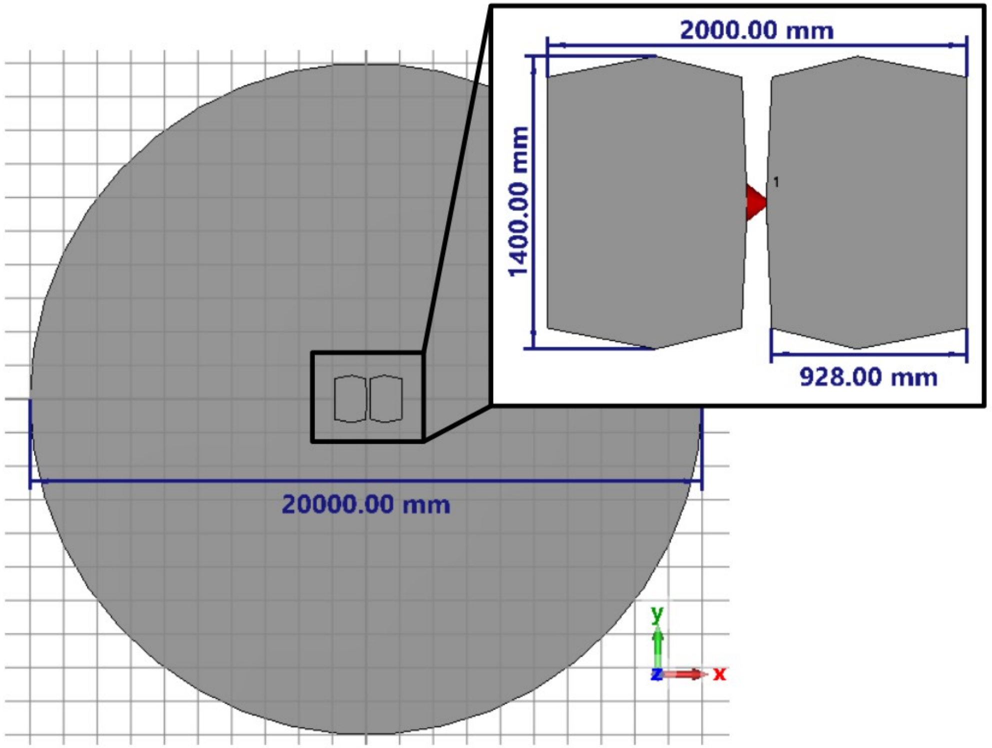
Sources of directivity uncertainty in the computational model

- Not modeled physical objects
  - Far surroundings, approximation of highly complex areas, soil (details of)
- Construction tolerances
  - How accurately is the construction compared to the model
- Wear of the instrument causing differences
  - Objects moving over time, defects forming
- Computational uncertainties
  - A computer can only solve the equations so accurately

Controllable modeling of directivity

- Using a singular value decomposition
  - Can be carried out on either the directivity itself or the E fields
- Allows for highly predictable accuracy
  - Compared to physical parameter variation which is harder to control
- Requires a comparatively low number of coefficients compared to spherical harmonics or similar

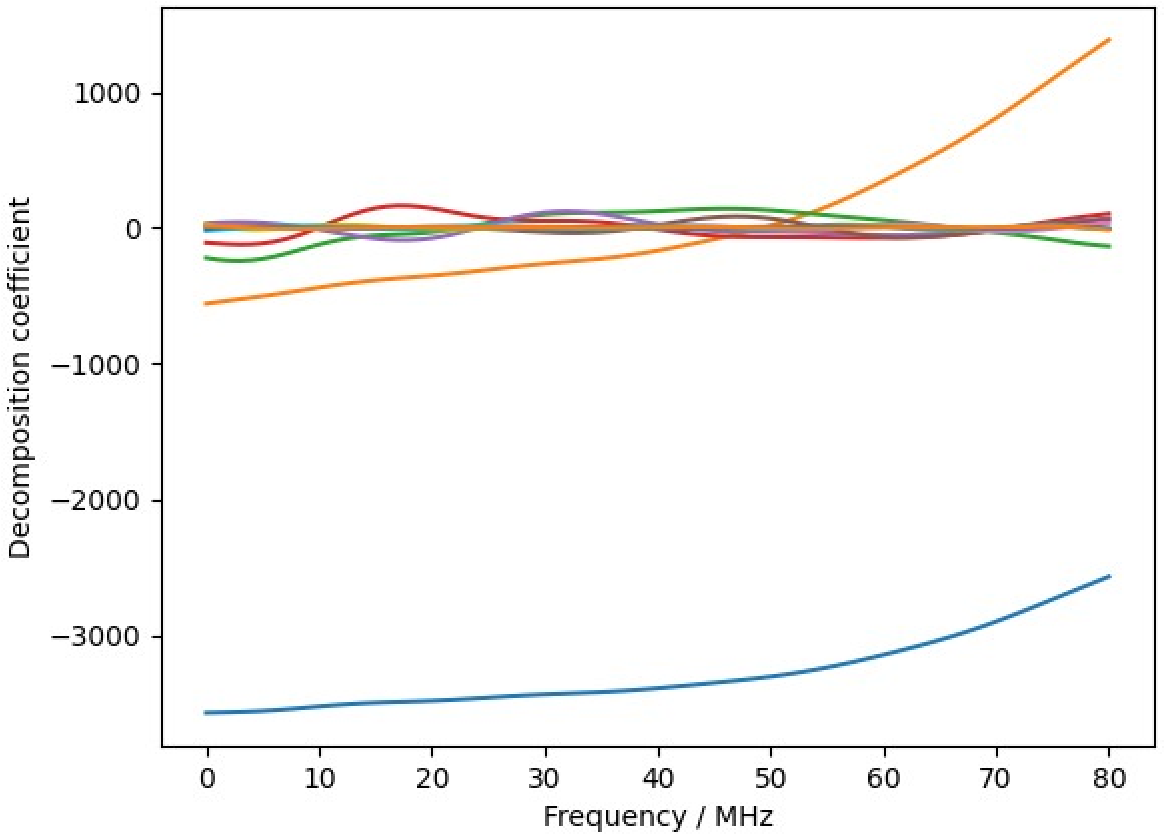
$$D = U \Sigma V^{\dagger}$$



Frequency and amplitude information

Frequency information weights

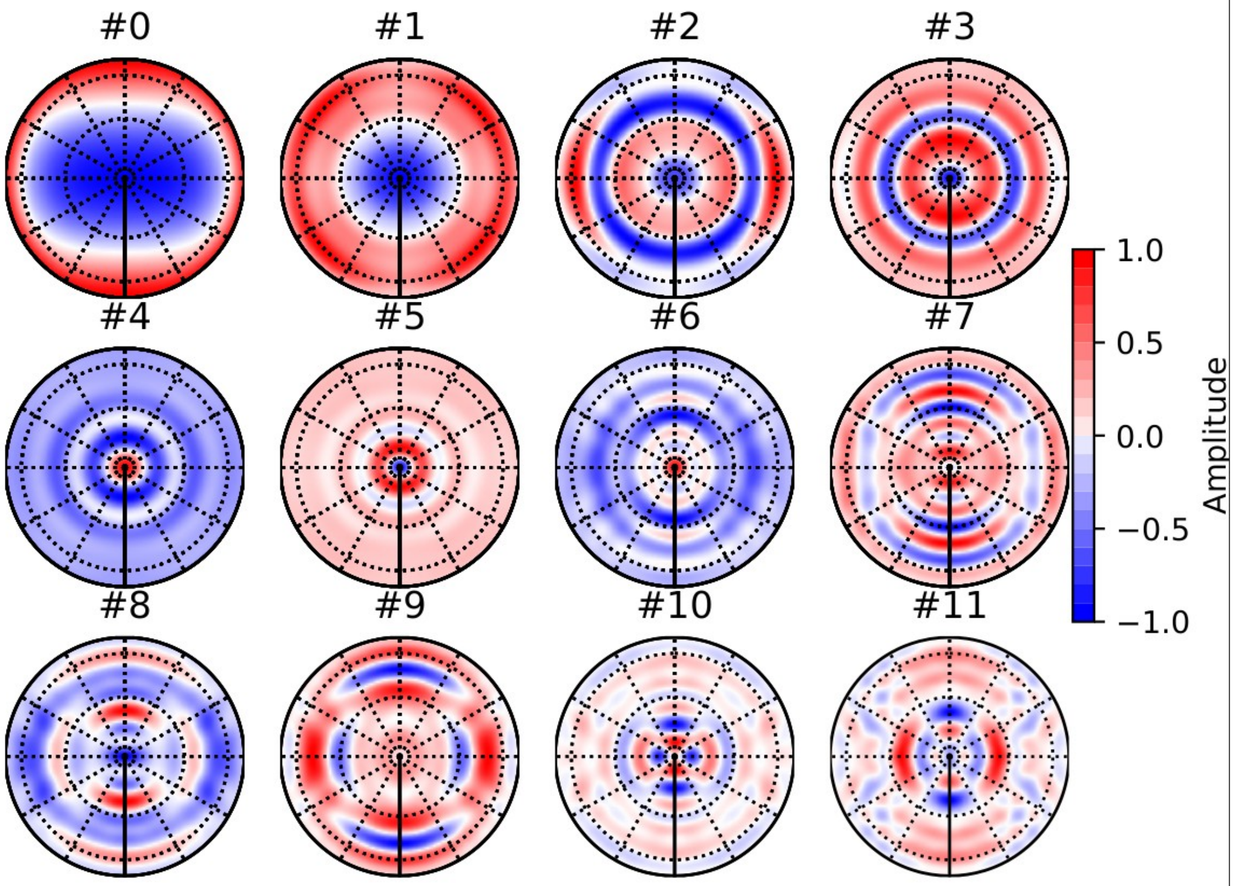
$$D = U\,\Sigma\,V^{\dagger}$$



Spacial information

Spacially dependent basis functions

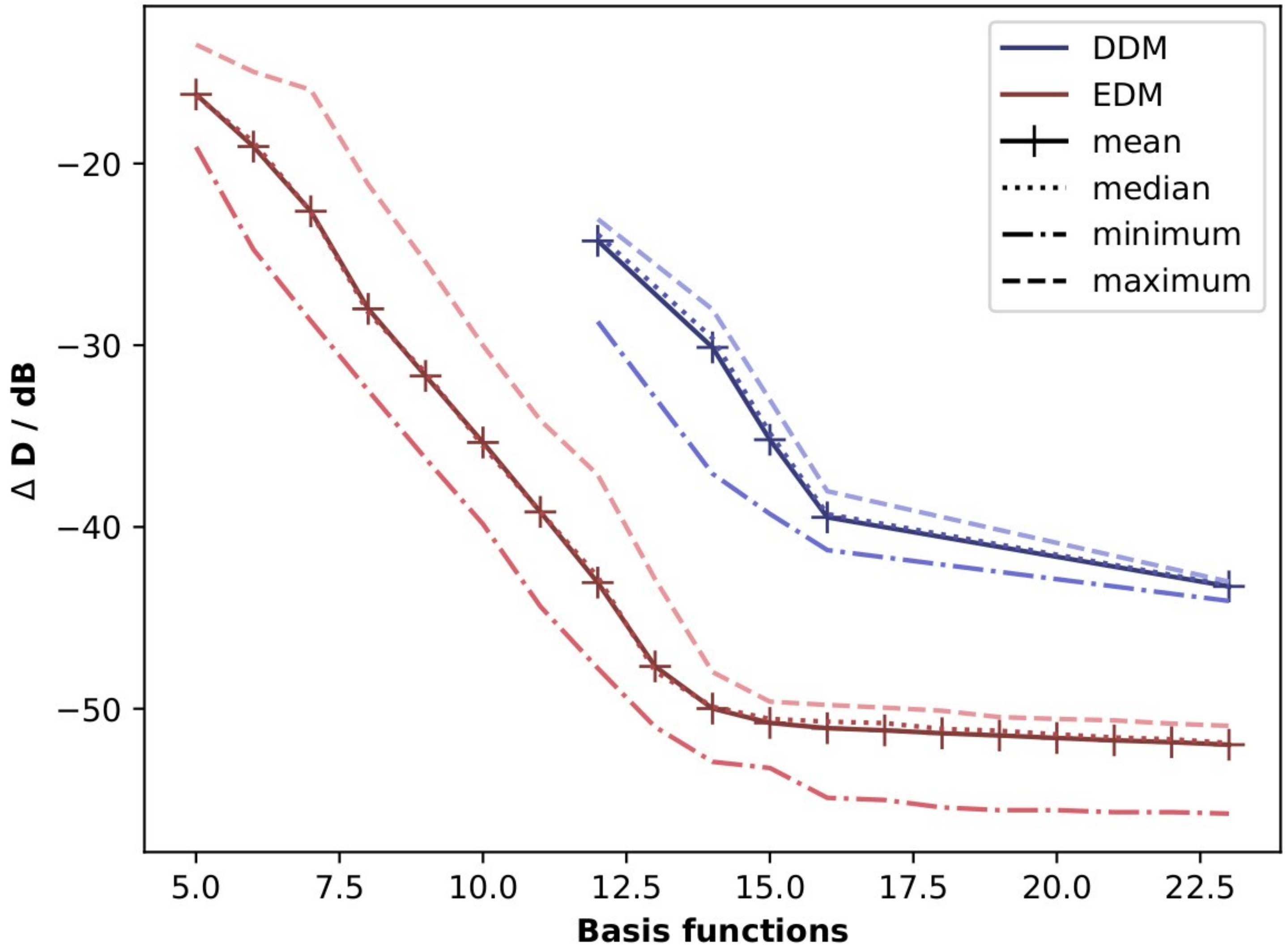
$$D = U \, \Sigma \, \boxed{V^\dagger}$$



Coefficients vs accuracy

- The accuracy of fits at various levels for the two different methods using,
  - Electric field
  - Directivity

- Both producing this type of linear improvement followed by a flat line (expected from SVD contributions)

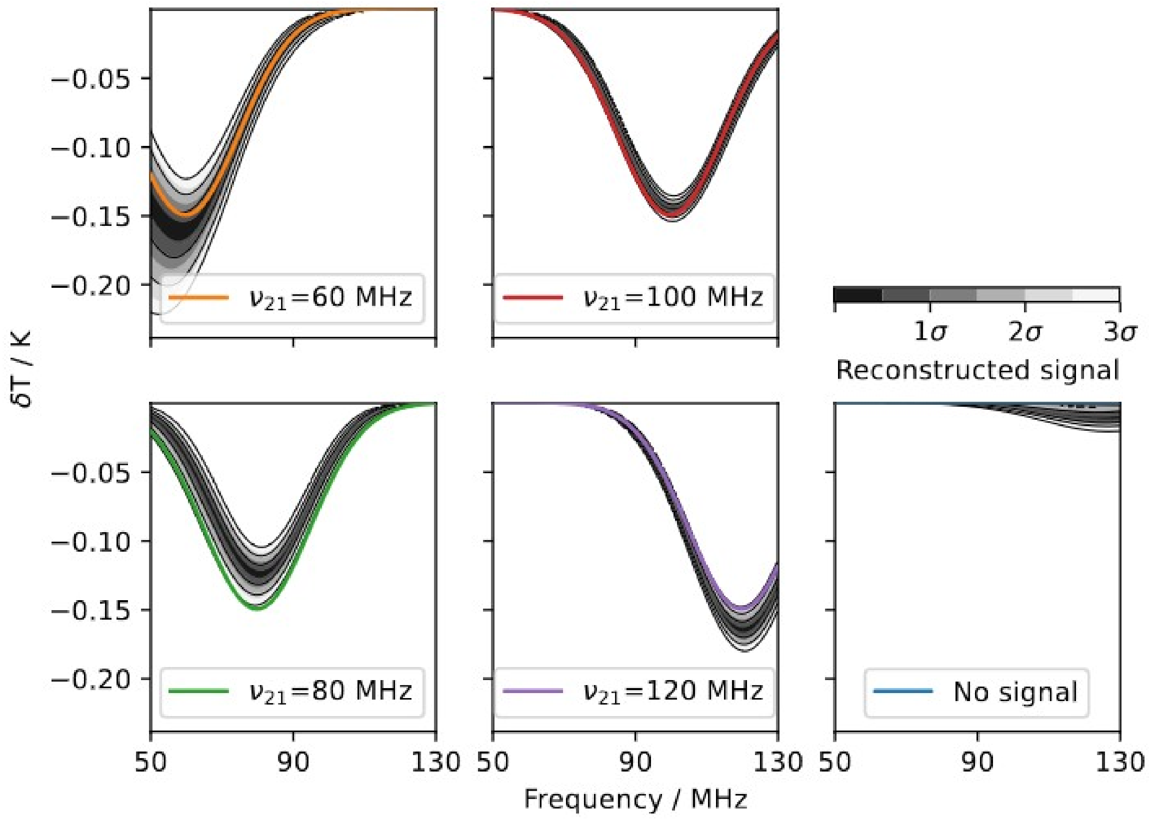


How to test accuracy?

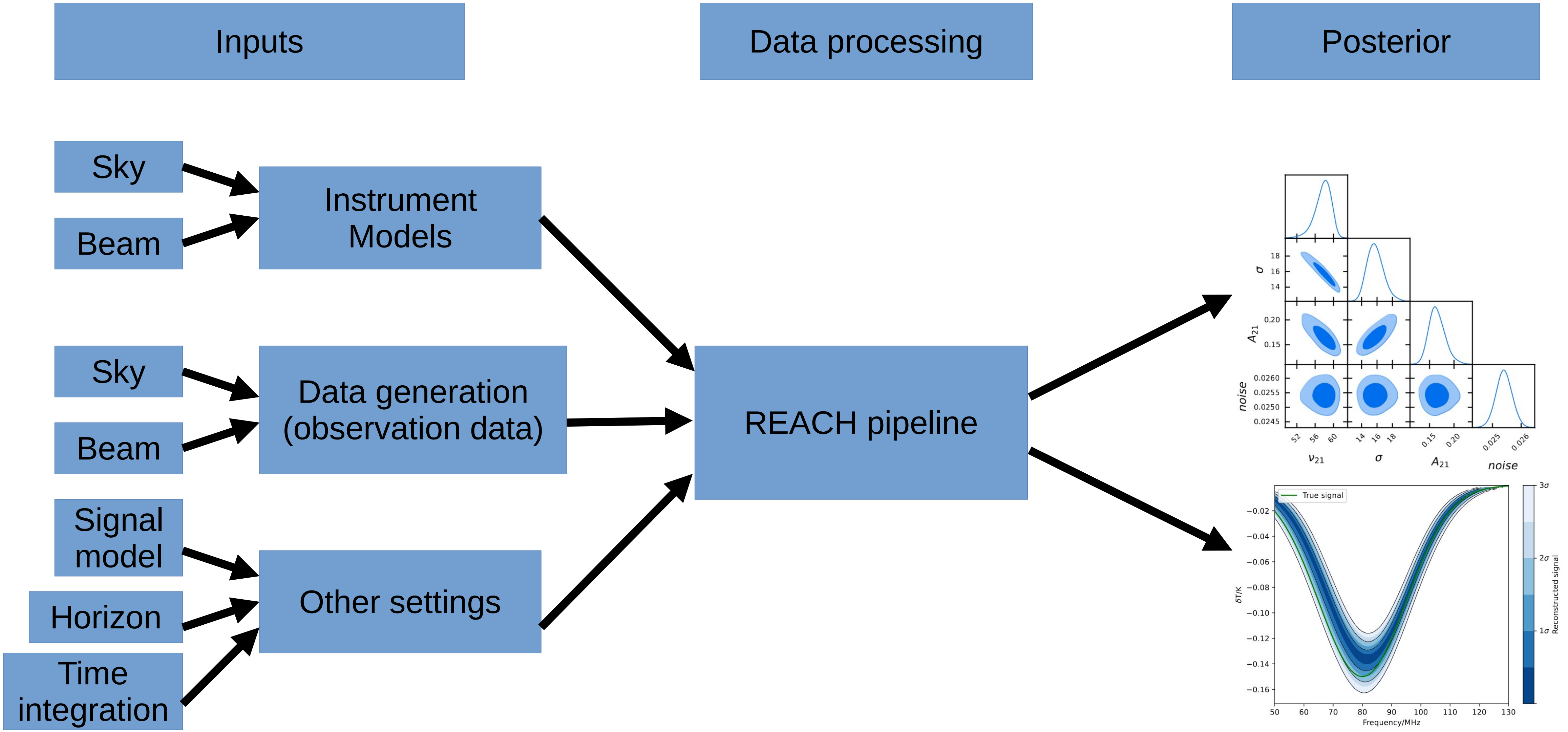
- Use REACH pipeline

- Use two different directivity patterns,
  - One for data generation
  - Second for refitting

- Calculate RMSE between original and refitted signals



REACH pipeline (A very rough flow chart)



REACH pipeline (A very rough flow chart)

Inputs

Data processing

Posterior

Sky

Beam

Instrument Models

Sky

Beam

Data generation (observation data)

Signal model

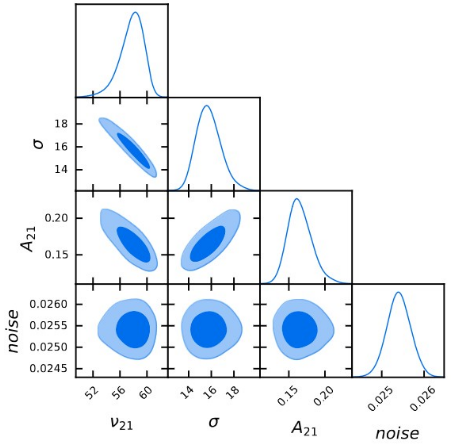
Horizon

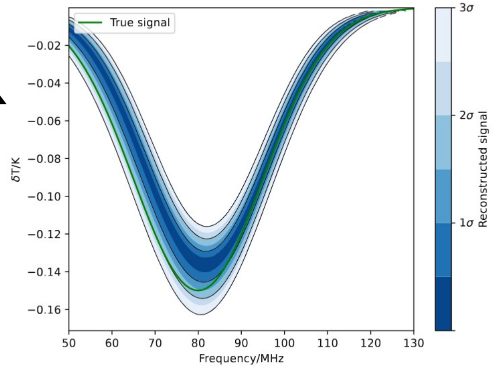
Time integration

Etc.

Other settings

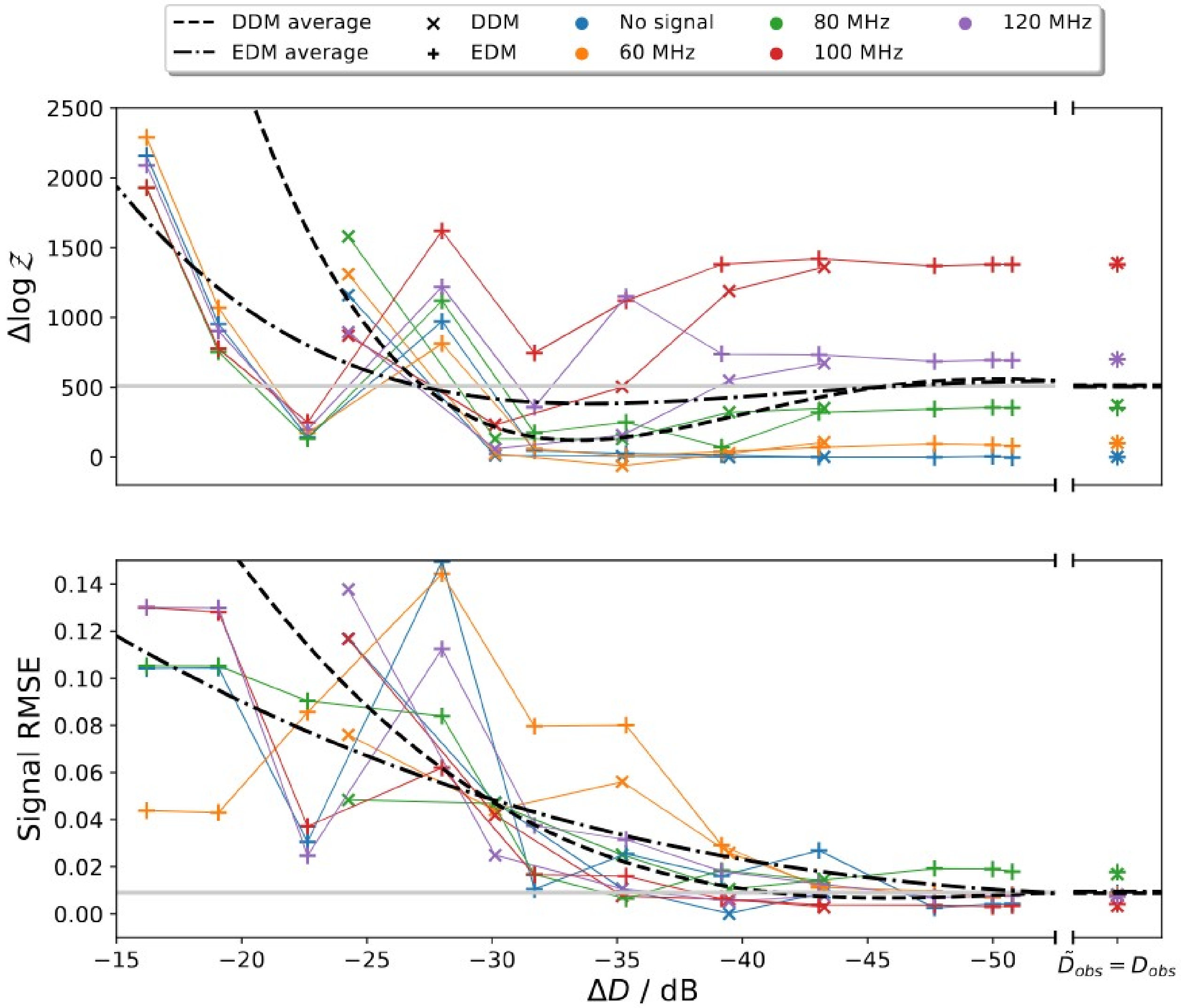
REACH pipeline





Sources of directivity uncertainty in the computational model

- Refitting results for the 5 signals
- At high directivity uncertainty high change of a false detection
- At around -40 dB fits produce similar results to using perfect knowledge



Summary

- Constructed a single value metric for the difference between directivity patterns ΔD

- Able to build a controllable difference in directivity patterns using an SVD breakdown on beam patterns

- Compared varying the observing beam and the fitting beam within the REACH pipeline

- Found that a value better than -35 dB is required for a confident detection, and significantly worse risks a false detection

Possible solutions to the uncertainty problem

- Making as complete and accurate computational model at possible for the instrument
- Fitting for some components of the directivity within the data analysis
- Choosing observation times to avoid the hot sky
- Fitting the difference as a time varying systematic