Disruption of star clusters in Newtonian and MOND gravity

Pierfrancesco Di Cintio[1,2,3,4]

[1]ISC-CNR,[2]Universitá di Firenze, [3]INFN-Firenze, [4]INAF-Arcetri

From star clusters to field populations: survived, destroyed and migrated clusters, Villa il Gioiello, Arcetri 23-11-2023

Context

- The disruption of a star cluster or satellite inside the parent galaxy involves, tidal effects, phase-mixing (collisionless) and dynamical friction (collisional)
- Different theories of gravity have degeneracy with respect to kinematics but, in principle, different dynamics.
- How differently does the dynamical friction on a cluster sinking into a galactic potential in MOND differs from its Newtonian counterpart?

MOND, basics

- Modified Newtonian Dynamics (MOND, Milgrom 1983) has been proposed as an alternative to the dark matter problem.
- In the Lagrangian formulation (Bekenstein & Milgrom 1984) it amounts to the modification of the Poisson equation

$$\nabla \cdot \left[\mu \left(\frac{||\nabla \Phi||}{a_0} \right) \nabla \Phi \right] = 4\pi G \rho_* .$$

- $a_0 \approx 10^{-8}$cm s^{-2} is a scale acceleration and $\mu(x)$ is the MOND interpolating (monotonic) function known only by its asymptotic limits

$$\mu(x) \sim \begin{cases} 1, & x \gg 1, \quad \mathrm{Newtonian regime} \\ x, & x \ll 1, \quad \mathrm{deep-MOND regime.} \end{cases}$$

usually it assumed $\mu(x) = x/\sqrt{x^2+1}$

MOND, equivalent Newtonian system

Any baryonic mass density ρ_* can be taken out from Poisson and non-linear Poisson (AquaL) equations obtaining the relation

$$\mu\left(\frac{||\mathrm{g}_M||}{a_0}\right)\mathrm{g}_M = \mathrm{g}_N + \mathrm{S}$$

where $\mathrm{S} \equiv \nabla \times \mathrm{h}(\rho)$ is a density-dependent solenoidal field that is null in spherical systems. In those cases the equivalent Newtonian system has an effective halo given by an isolated spherical system one has

$$\rho_{DM} = (4\pi G)^{-1}\nabla \cdot (\mathrm{g}_M - \mathrm{g}_N).$$

Quasi-Linear formulation of MOND (QuMOND)

Milgrom (2010) introduced a quasi-linear formulation of MOND where

$$\Delta\Phi = \nabla \cdot [\nu(||\nabla\Phi_N||/a_0 \nabla\Phi_N)]$$

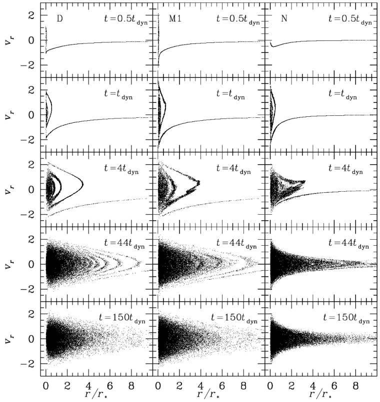
where Φ_N is the Newtonian potential generated by ρ_*,
$\nu(y) = 1/\mu(x)$ and $x\mu(x) = y$.
The MOND potential Φ enters the **linear** Poisson equation for the density

$$\tilde{\rho} = -\frac{1}{4\pi G}\nabla \cdot [\nu(g_N/a_0)g_N],$$

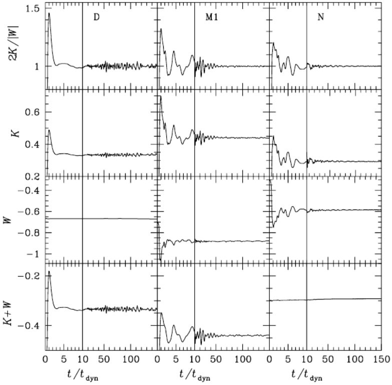
that allows to recover the "phantom" DM density as $\rho_{DM} = \tilde{\rho} - \rho_*$

Collisionless relaxation in MOND

Numerical evidences that in MOND and Newtonian gravity (Nipoti et al. 2007a,b,c,2011) collective relaxation process work differently



Collisionless relaxation in MOND



Violent relaxation and phase-mixing are less effective in MOND.
Non-linearity or long-range behaviour?

Dynamical friction, classical

The classical Chandrasekhar (1943) dynamical friction coefficient for a test mass m_T is given by

$$\eta(v) = 4\pi G^2 \rho_*(m_T + m_*) \ln \Lambda \frac{\Psi(v)}{v^3},$$

with velocity volume function given as:

$$\Psi(v) = 4\pi \int_0^v f(v')v'^2 \mathrm{d}v',$$

for a 2-component model (stars + DM) is proportional to $(M_T + < m >)(\rho_* + \rho_{DM})$

Dynamical friction, MOND

A naive dMOND estimation of t_{2b} gives (Ciotti & Binney 2004):

$$\frac{t_{2b}}{t_{cross}} = \frac{v_{typ}^4 r_0^2 N}{2G^2 M^2}$$

implying a dynamical friction coefficient of the order of $1/t_{cross}$. A more refined treatment yields instead

$$\frac{t_{2b}^N}{t_{2b}^N} = \frac{\sqrt{2}}{(1+R)^2}$$

from which it descends that

$$\eta_M = (1+R)\eta_N/\sqrt{2}$$

where $R = M_{DM}/M_*$ is the ratio of the dark matter in the equivalent Newtonian system and the baryonic mass.
MOND dynamical friction is stronger than its (baryon only) counterpart.

Dynamical friction, QuMOND

- In QuMOND, for $\nu(y) = 1 + 1/\sqrt{y}$ a point source produces a gravitational acceleration $g_Q \approx Gm/r^2 + Gm/rr_0$, where $r_0 = \sqrt{G(m+M)/a_0}$ is the usual MOND radius.
- Following the standard Chandrasekhar derivation the QuMOND DF coefficient becomes (Di Cintio & Re 2023)

$$\eta(v) = 4\pi G^2 \rho_*(m_T + m_*)\left(\ln\Lambda + \frac{2b_{max}}{r_0} + \frac{b_{max}^2}{r_0^2}\right)\frac{\Psi(v)}{v^3},$$

- All stars are in the dMOND regime as $r_0 \ll R$, however when $a_0 \lesssim |\nabla\Phi_T|$ even weak interactions are in Newtonian regime (external field effect)

Model

The satellite star cluster sinking into the parent galaxy is modeled with:

- Unperturbed time independent galactic potential (spherical γmodel) in Newton or MOND

$$\rho_i(r) = \frac{3-\gamma}{4\pi} \frac{M_i r_{c,i}}{r^{\gamma}(r + r_{c,i})^{4-\gamma}}$$

- Self consistent potential for star cluster
- Analytical dynamical friction on satellite particles + noise

Single particle dynamics

The motion particles belonging to the cluster with self-consistent Φ_c under the effect of galactic potential Φ_{gal} and its discreteness effects is then

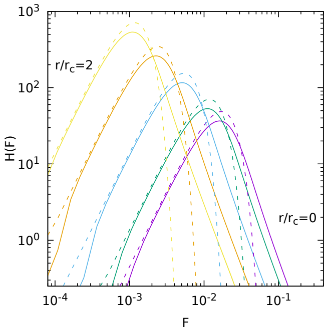
$$\frac{d^2\mathrm{r}}{dt^2} = -\nabla\Phi_{tot}(\mathrm{r}) - \eta\frac{d\mathrm{r}}{dt} + F_W$$

where η is either due to baryons only (in MOND) or to baryons and DM (in Newton) and the fluctuating force F_W is a fluctuating force per unit mass.

- for $M_T \sim m$: $\eta; F \neq 0$
- for $m \gg M_T$ $\eta = 0$ and $F \neq 0$
- for $M_T \gg m$: $\eta \neq 0$ and $F = 0$

We assume two different distributions of local random kicks. 3D Gaussian and Holtsmark (1911) distribution (Chandrasekhar & von Neumann 1942,1943):

$$H(F) = \frac{2}{\pi F} \int_0^\infty \exp\left[-\alpha(\xi/F)^{3/2}\right] \xi \sin(\xi) \mathrm{d}\xi; \quad \alpha = \frac{4}{15}(2\pi G m_*)^{3/2} n_*$$



Fat tailed distribution. For large forces (small mean inter-particle distance) $\tilde{H}(F) \sim 2\pi n_* (G m_*)^{3/2} F^{-5/2}$

Numerical methods

Stochastic ODEs, like Langevin equations, are not an easy computational task. We adopt the robust quasi-symplectic Mannella (2004) scheme:

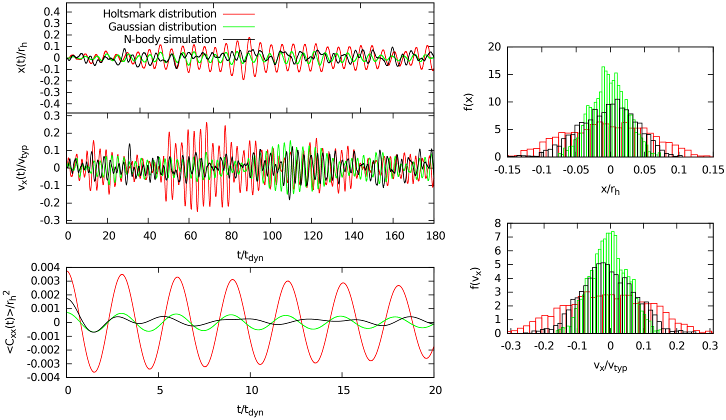
$$
\begin{aligned}
x(t+\Delta t/2) &= x(t) + \frac{\Delta t}{2} v(t) \\
v(t+\Delta t) &= c_2 \left[c_1 v(t) + \Delta t \nabla \Phi(x') + d_1 \tilde{F}(x') \right] \\
x(t+\Delta t) &= x(t+\Delta t/2) + \frac{\Delta t}{2} v(t+\Delta t).
\end{aligned}
$$

where:

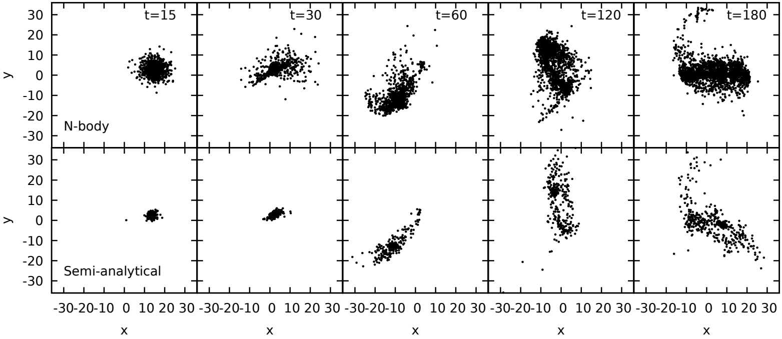
$$
c_1 = 1 - \frac{\eta \Delta t}{2}; \quad c_2 = \frac{1}{1+\eta\Delta t/2}; \quad d_1 = \sqrt{2\zeta\eta\Delta t}.
$$

For $\eta;\zeta = 0$ becomes the standard second order *leapfrog*.

Tests: N-Body vs semi-analytical



Numerical simulations



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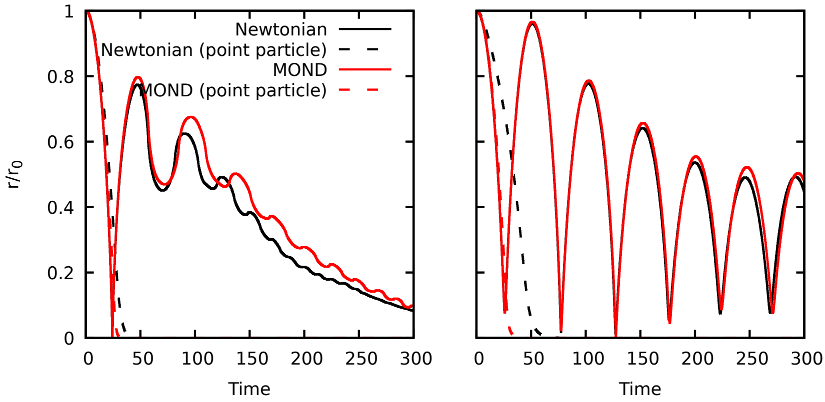

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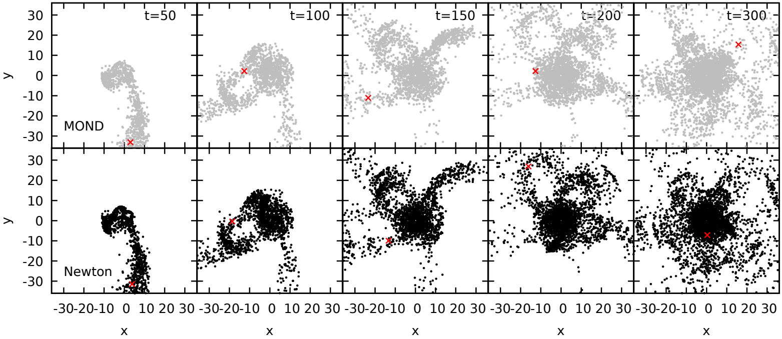

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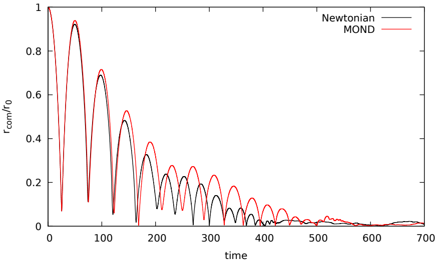

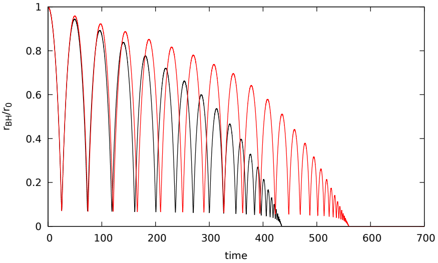

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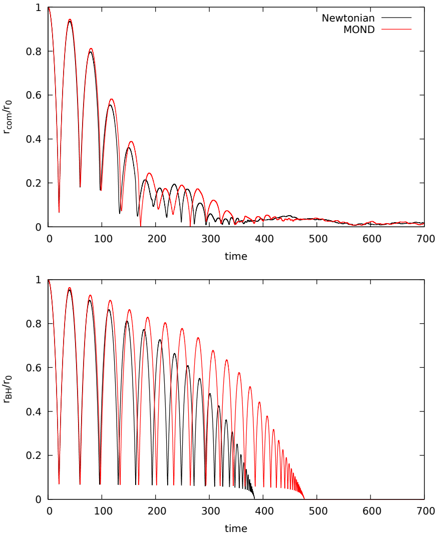
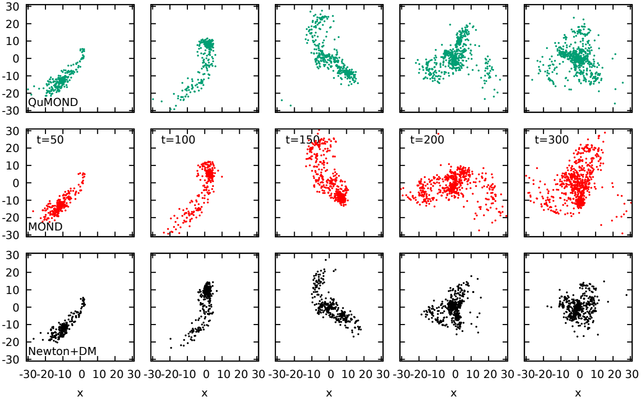
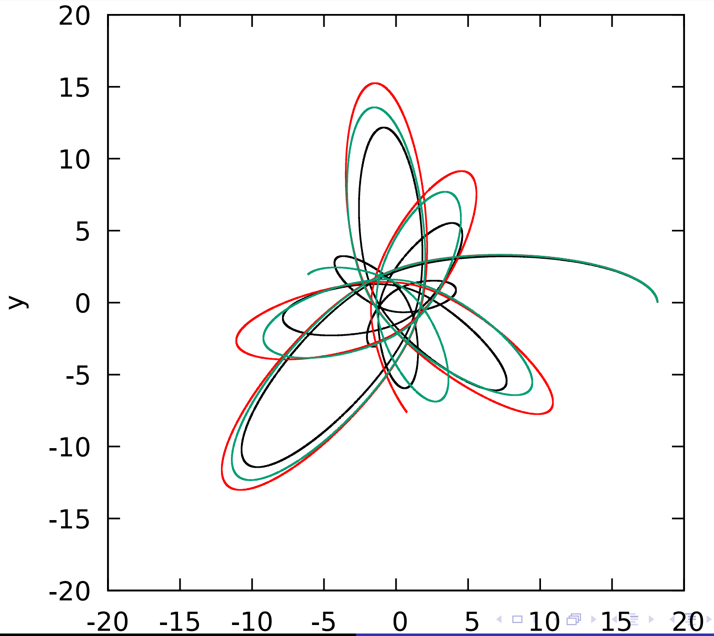


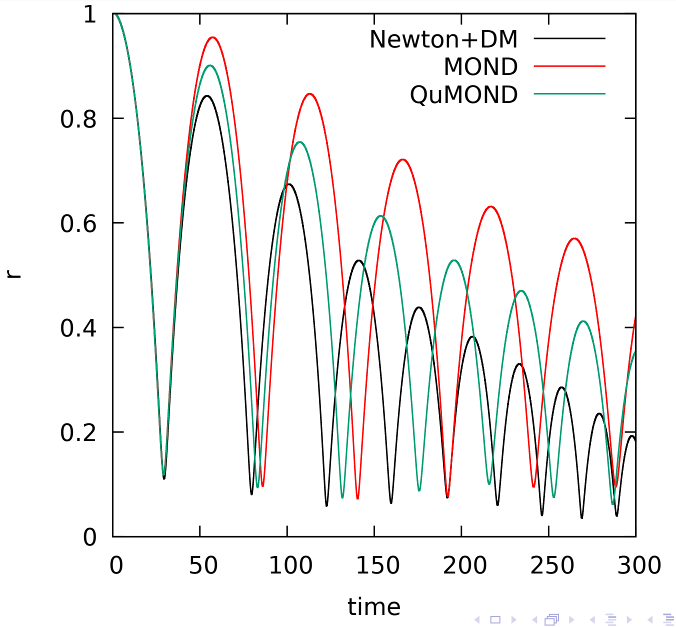
Numerical simulations: MOND vs QuMOND



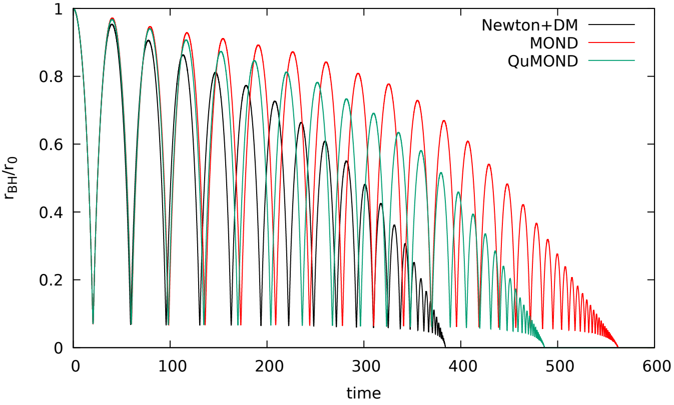
Numerical simulations: MOND vs QuMOND



Numerical simulations: MOND vs QuMOND



Numerical simulations: MOND vs QuMOND



Conclusion

- Hybrid Langevin-Nbody scheme are in certain regimes a valid alternative to pure (collisionless) $N-$body
- A stronger MOND dynamical friction partially restores the lack of DM
- Different MOND theories might behave differently with respect to collisional processes

THANK YOU FOR THE ATTENTION!

- M. Pasquato & P. Di Cintio Astronomy & Astrophysics, Volume 640, id.A79 (2020)
- F. Re & P. Di Cintio Astronomy & Astrophysics, Volume 678, id.A110 (2024)
- P. Di Cintio, F. Re & C. Chiari to be submitted soon (2023)
- P. Di Cintio, M. Pasquato & S. Sartorello to be submitted (early 2024)