From Vlasov-Poisson to Schrödinger-Poisson

Approaching dark matter simulation with Quantum Computers

Luca Cappelli PhD, (*UniTs – INAF - ICSC Spoke 10*)
luca.cappelli@inaf.it

Supervised by Giuseppe Murante (*INAF*) &

Stefano Borgani (*INAF - UniTs*)

From a previous collaboration with Ivano Tavernelli &

Francesco Tacchino (*IBM Research Zurich*).

Introducing the problem

Not quantum computer efficient

Vlasov Eq.

- Distribution function

$$f(\boldsymbol{x}, \boldsymbol{v}, t)$$

- Collisionless particles
- Self-interacting potential

$$\begin{cases} \frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} - V \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0 \\ \nabla^2 V = 4\pi G(\rho - \rho^*) \end{cases}$$

$\mathcal{M}$ →

← $\mathcal{M}^{-1}$, $\hbar/m \to 0$

Schrödinger-Poisson Eq.

- Quantum field $\psi(\boldsymbol{x}, t)$
- $\rho \to |\psi|^2$
- Non-linear Schrödinger
- Efficient on quantum computing

$$\begin{cases} i\frac{\partial \psi}{\partial t} = -\frac{\hbar}{2m}\nabla^2\psi + \frac{m}{\hbar}V\,\psi \\ \nabla^2 V = |\psi|^2 - 1 \end{cases}$$

[1] P.Mocz et al. , Physical Review D 97, 083519 (2018).

«Quantum» problems

$$\begin{cases} i\partial_t \Psi = -\frac{\hbar}{2m}\nabla^2\Psi + \frac{m}{\hbar} V[\Psi]\,\Psi \\ \nabla^2 V = |\Psi|^2 - 1 \end{cases}$$

- *Nonlinear* dynamics of Ψ

  vs

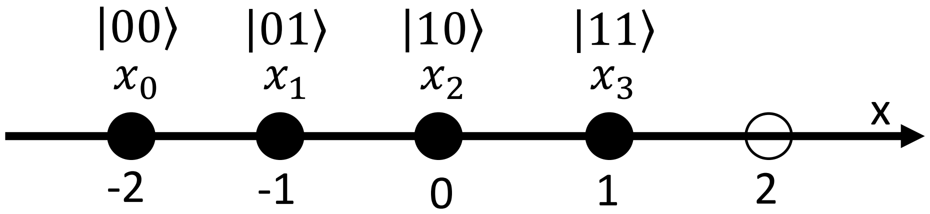
  linear Quantum Computing

$$\langle\psi|\psi\rangle = \sum_k |\psi_k|^2 = 1$$

- Self consistent potential

Discretization and encoding

1D: N = 4; L = 4



- Classical discretization – Physical wavefunction

$$\Psi_k \simeq \Psi(x_k)$$

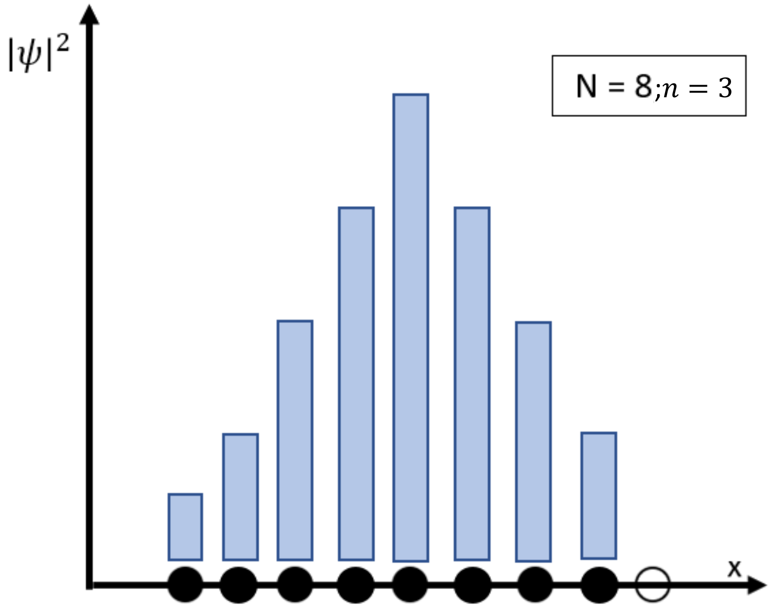
- Generic 2-qubits quantum state

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

Generic First quantization encoding

$$|\psi\,\rangle = \sum_{j=0}^{N-1} \psi_j |bin(j)\rangle$$

Logarithmic encoding: $n = \log_2 N$



Solving the self-consistency problem-*The algorithm*

Initial state preparation
$|\psi(\boldsymbol{\theta}_0)\rangle = U(\boldsymbol{\theta}_0)|\mathbf{0}\rangle$

Solution of Poisson Eq.

- Minimization of
  $$min_{\boldsymbol{\phi}} \left|\left| \boldsymbol{\nabla}^2 V(\boldsymbol{\phi}(t)) - |\Psi(\boldsymbol{\theta}(t))|^2 + \mathbf{1} \right|\right|^2$$
- $\rightarrow |V(\boldsymbol{\phi}(t))\rangle = \phi_V(t)\tilde{V}(\boldsymbol{\phi}(t))$

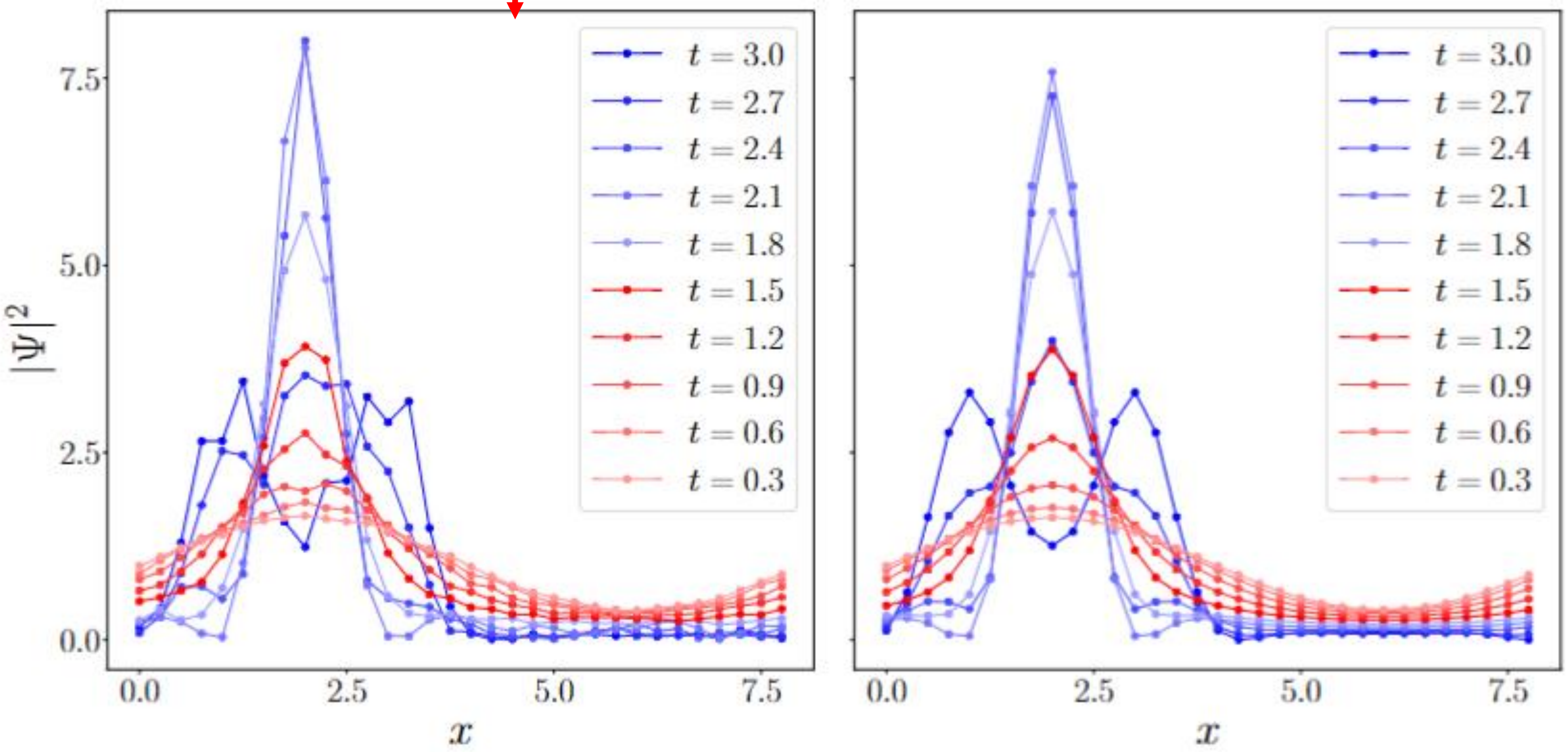
Time evolution

- Quantum Measurement of :
  $$\mathrm{M}_{k,l} = \Re\left\{\langle \partial_{\theta_k}\Psi \mid \partial_{\theta_l}\Psi\rangle - \langle \partial_{\theta_k}\Psi \mid \Psi\rangle \langle \Psi \mid \partial_{\theta_l}\Psi\rangle\right\}$$
  $$\mathrm{B}_k = \Im\left\{\langle \partial_{\theta_k}\Psi|\mathcal{H}|\Psi\rangle - \langle \partial_{\theta_k}\Psi \mid \Psi\rangle \langle \Psi|\mathcal{H}|\Psi\rangle\right\}.$$

- Classically solve $\mathrm{M}\,\dot{\boldsymbol{\theta}} = \mathrm{B}$
- Integrate (Euler method)
  $\boldsymbol{\theta}(t+\delta t) = \dot{\boldsymbol{\theta}}\delta t$

	D_ψ	D_V	M_p	N_t	r_c	ϵ	$\mathcal{F}$
4-qubits	4	4	32	$6 \cdot 10^2$	10^{-7}	10^{-3}	0.976
5-qubits	5	6	50	$9 \cdot 10^3$	10^{-8}	10^{-4}	0.944
5-qubits	5	6	50	$2 \cdot 10^4$	10^{-8}	10^{-4}	0.960
5-qubits	6	6	60	$6 \cdot 10^3$	10^{-8}	10^{-4}	0.956

Quantum-VTE Results

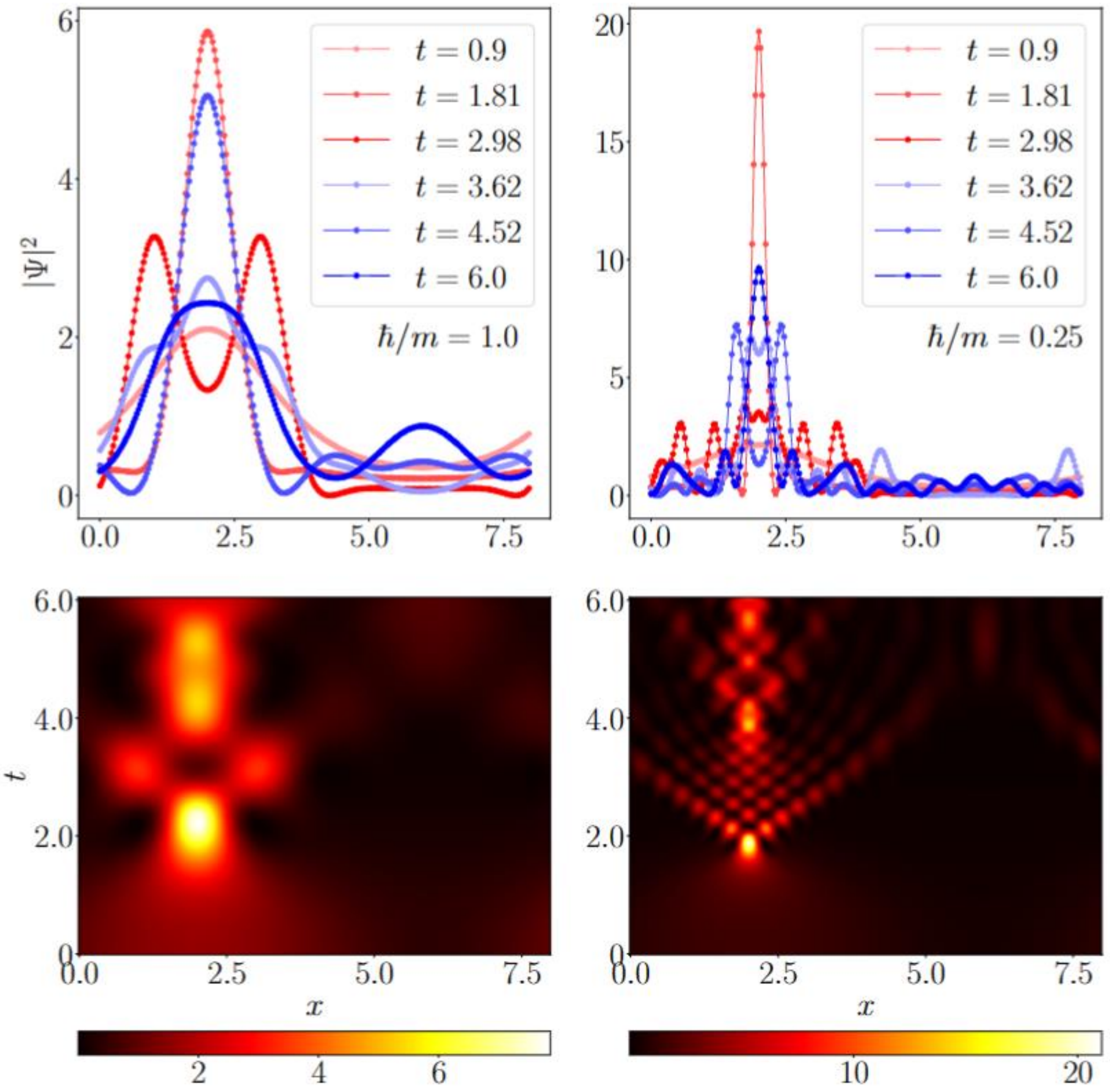


Dynamical Scale

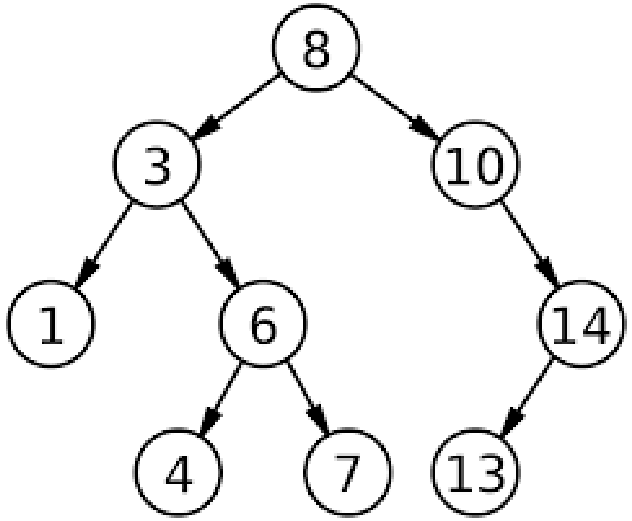
(with classical simulations)

$$i\partial_t \Psi = \left(-\frac{\lambda}{2}\nabla^2 + \frac{1}{\lambda}V\right)\Psi$$

$$\nabla^2 V = |\Psi|^2 - 1$$



Quantum Tree-Search for Tree method



Improvements

New Algorithms

Thanks for the attention