Spatio-Temporal correlations in earthquakes and solar flares: self-organized criticality beyond the sand pile model

ICTP Trieste 09/02/23
Dynamical complexity in astrophysical contexts

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

Outline

Seismic-occurrence has been recognized as the paradigmatic example of self-organized criticality, since from the origin of the theory

Catastrophic events «must not be considered as anomalous, since could be attributed to abnormal circumstances, but they are intrinsic to the dynamics, the same dynamics that produces small, ordinary events»

"I do not intend to chronicle the suffering it has inflicted on men, nor to provide a list of cities razed to the ground or inhabitants buried under rubble……
Such a narrative would be moving and, perhaps, touching the heart, might even have an uplifting effect. I, however, entrust this kind of narrative to more experienced hands.
I will describe here only the work of nature, the surprising natural circumstances that accompanied the terrible event and their causes."

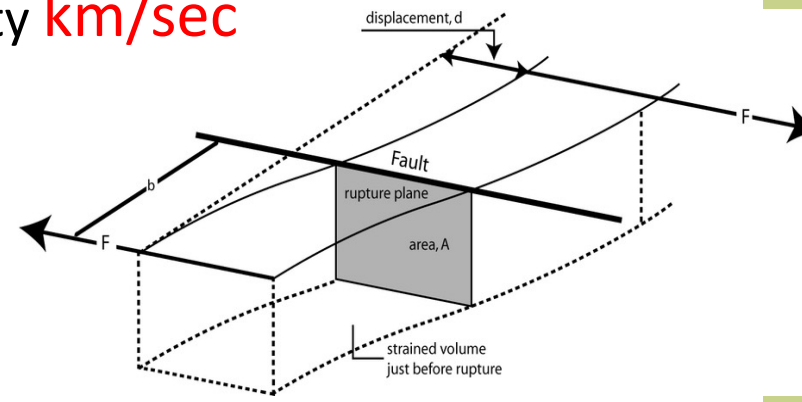
I. Kant, *Allgemein Naturgeschichte und Theorie des Himmels",1755*

Outline

Seismic-occurrence has been recognized as the paradigmatic example of self-organized criticality, since from the origin of the theory

Catastrophic events «must not be considered as anomalous, since could be attributed to abnormal circumstances, but they are intrinsic to the dynamics, the same dynamics that produces small, ordinary events»

- Separation of time scales
  - Strain accumulation rate cm/year
  - Strain propagation velocity km/sec
- Gutenberg-Richter law
- Burridge-Knopoff- Spring-block model



Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

Outline

- Gutenberg-Richter law

  Burridge-Knopoff- Spring-block model

- Non trivial spatio temporal patterns in seismic occurence

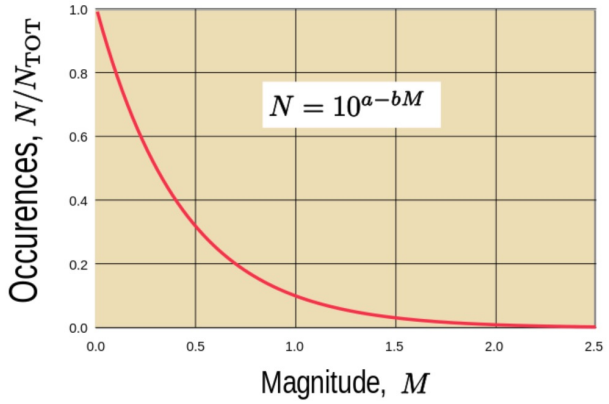
  Similarity and differences with similar patterns in solar flare occuence

- Epidemic models for seismic and solar flare occurrence

- Generalization of the sand-pile model for realistic temporal correlations

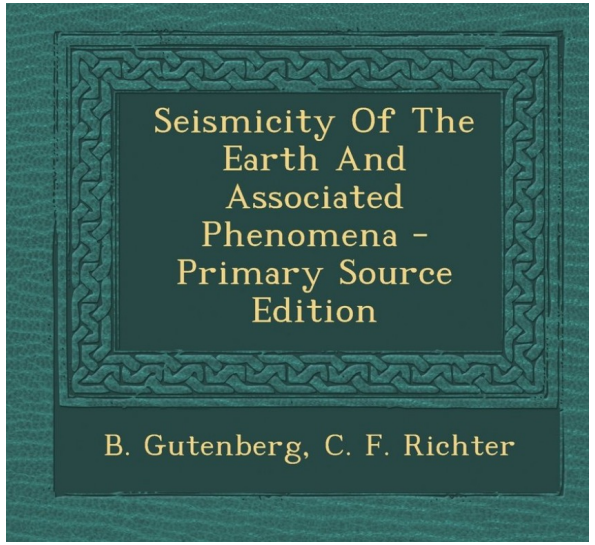
Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

1956 Gutenberg–Richter law



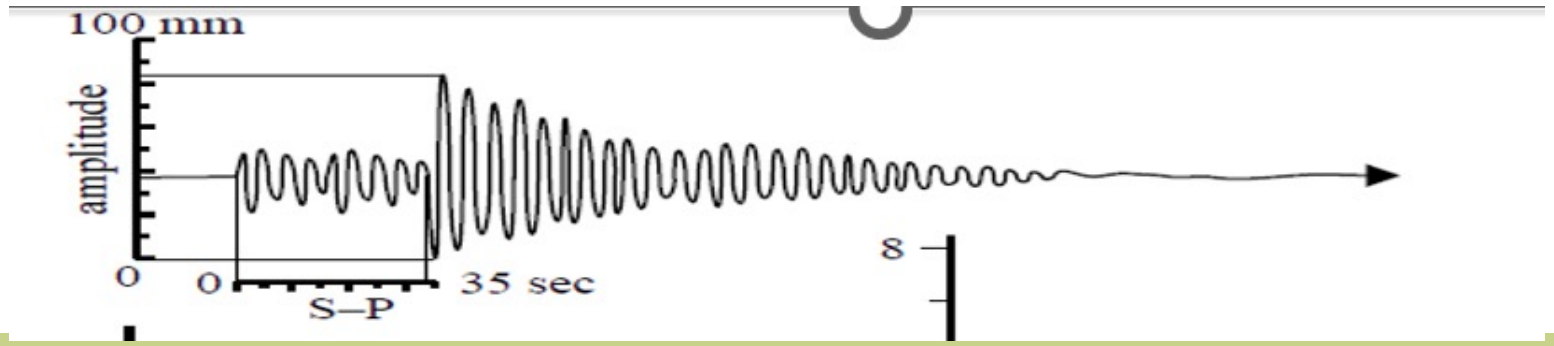
Beno Gutenberg

Charles Francis Richter ...



$$P(m) = k10^{-bm}$$

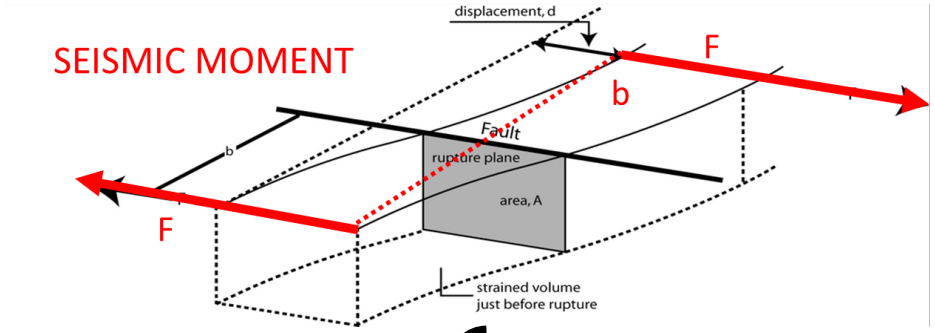
b very importat parameter



Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

1979

Moment magnitude



Hiroo Kanamori |

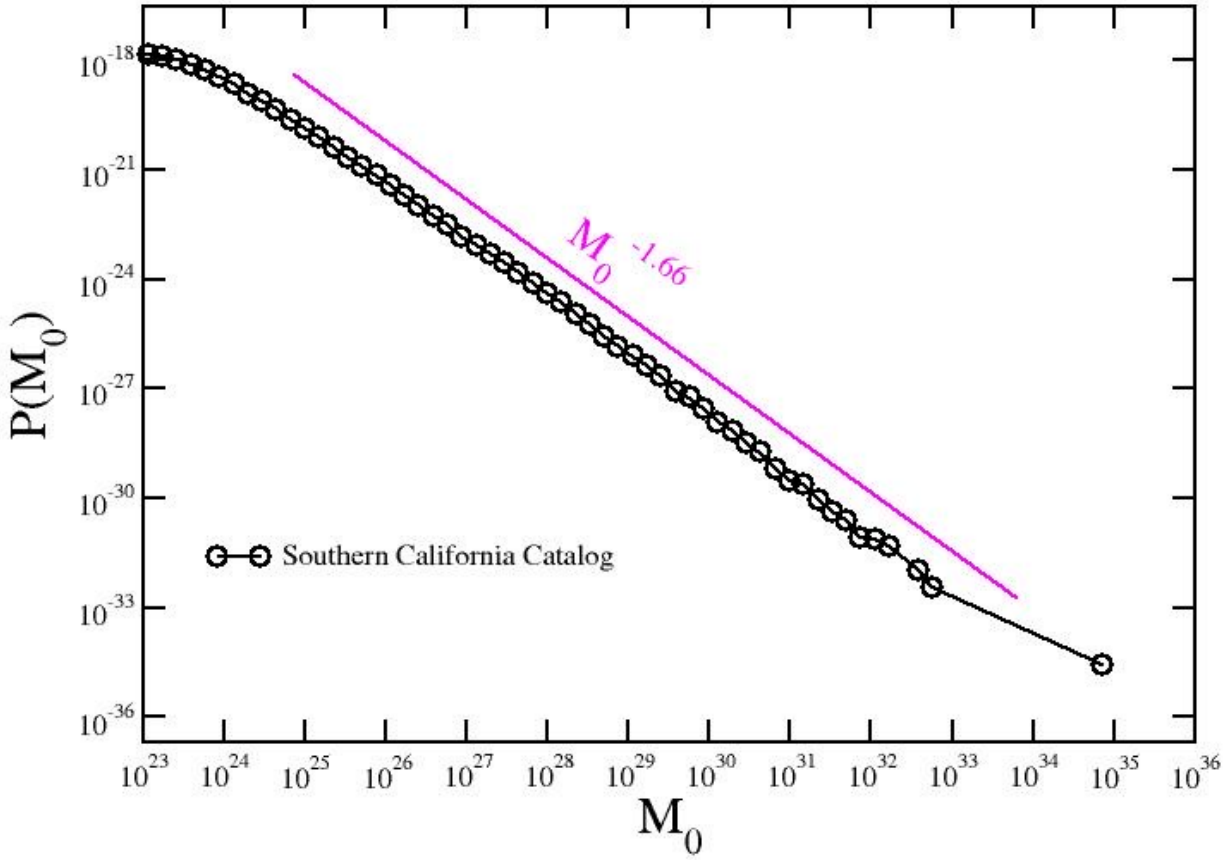
$M_0 = \mu A d$

- μ Lamè constant (shear modulus)
- A fractured area
- d average displacement

$$m = \frac{2}{3}\log_{10}(M_0) - 10.7$$

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

Power law behavior of size distribution (Gutenberg-Richter law)



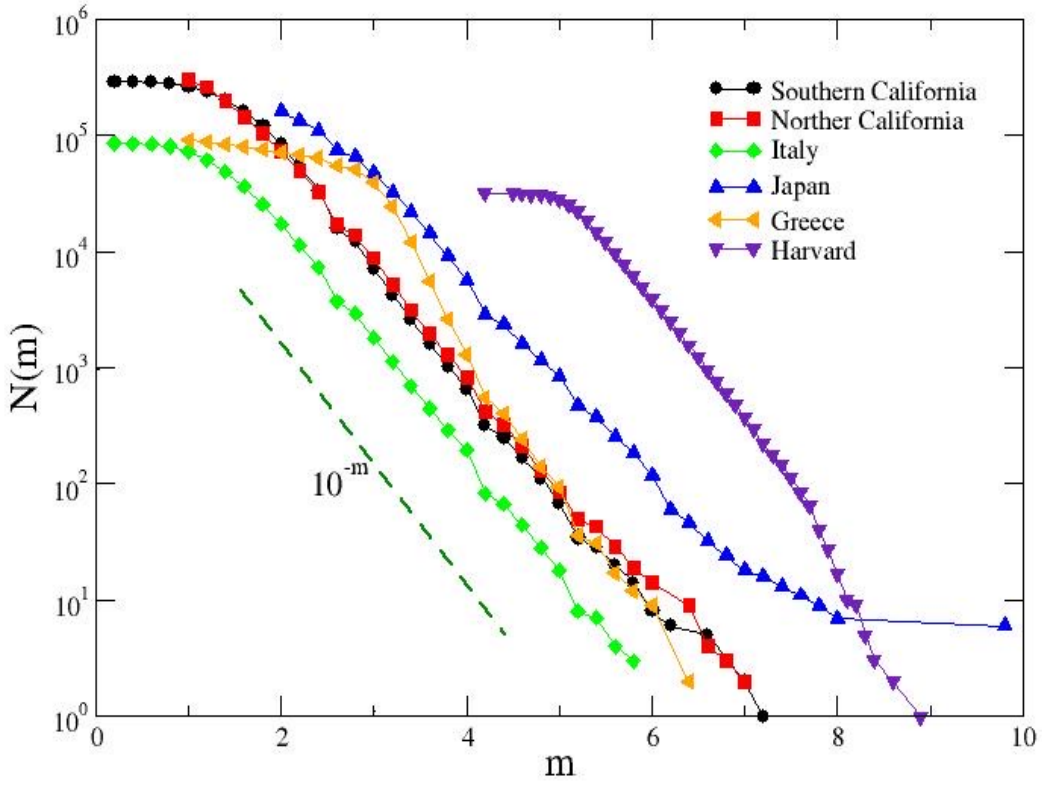
$$P(m) = k10^{-bm}$$

$$m = \frac{2}{3}\log_{10}(M_0) - 10.7$$

$$\approx$$

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

Power law behavior of size distribution (Gutenberg-Richter law)
Universality of the b-value



$$P(m) = k10^{-bm}$$

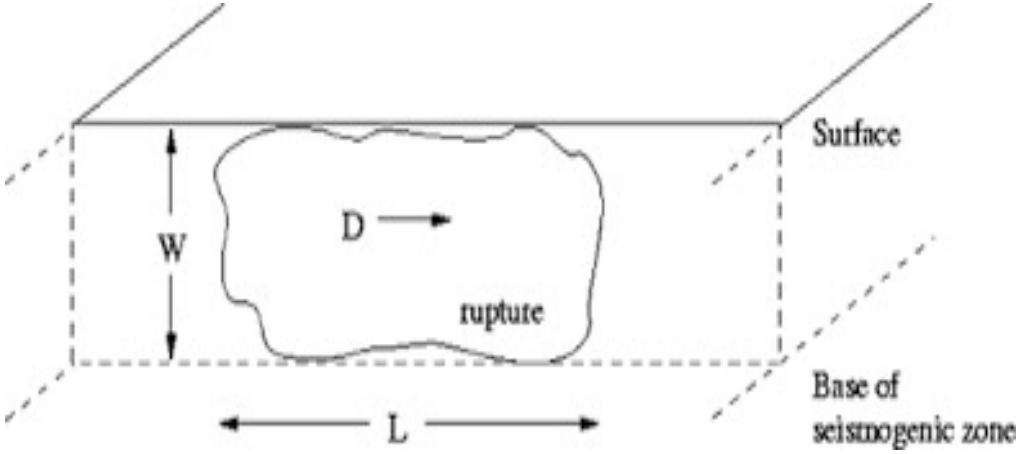
$$m = \frac{2}{3}\log_{10}(M_0) - 10.7$$

$$\approx$$

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

SCALING LAWS IN SEISMIC OCCURRENCE

In the geophysical community scaling laws are related to the standard definition of scale invariance: only one length scale in the process L



$$\begin{cases} W \sim L \\ D \sim L \\ A \sim L^2 \\ M_0 \sim L^3 \end{cases}$$

$$M_0 = \mu A d$$

$$M_0 \sim L^3 \sim A^{3/2}$$

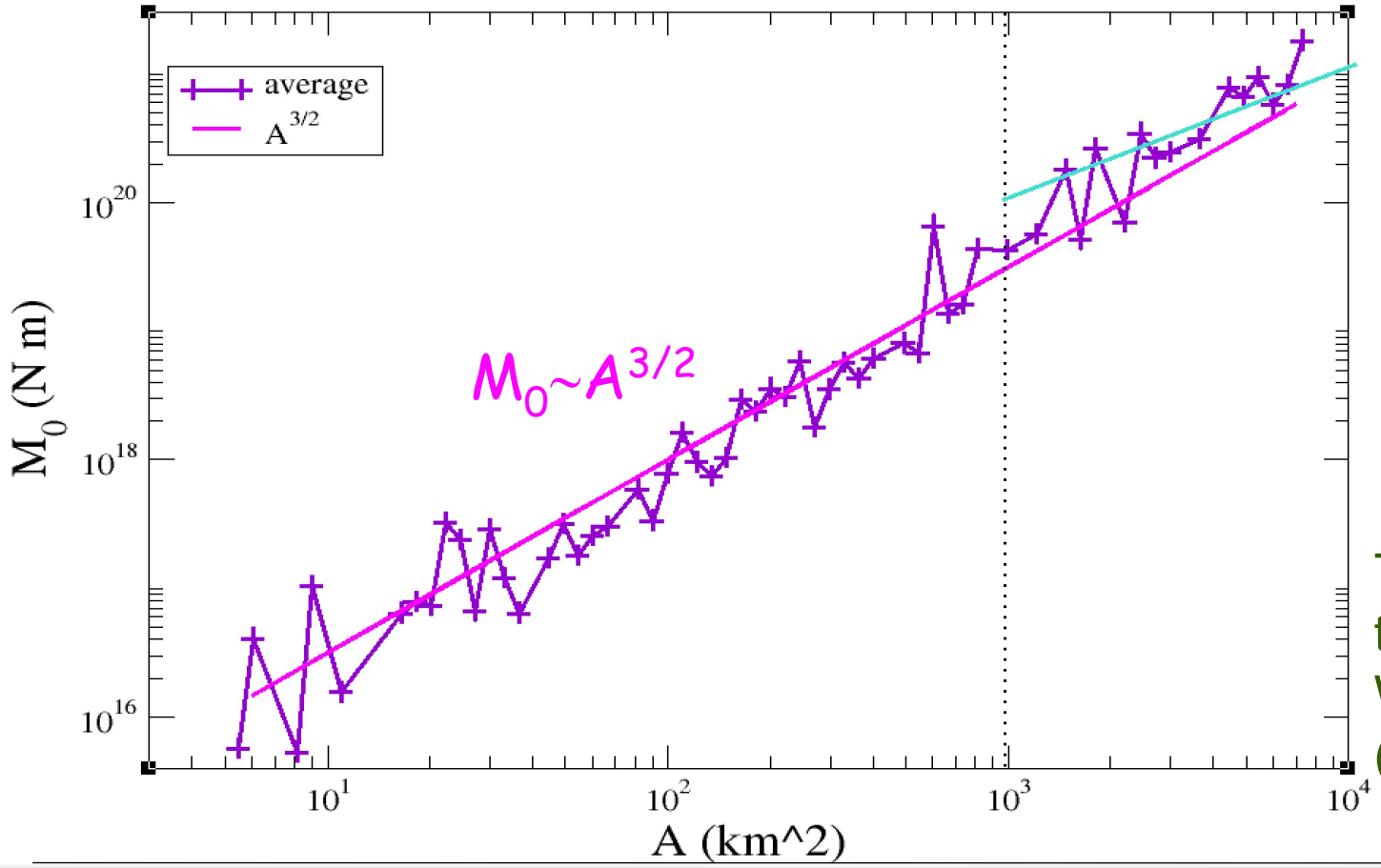
$$m = \frac{2}{3}\log_{10}(M_0) - 10.7$$

$$m \sim \gamma \log(A) \quad \gamma = 1$$

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

SCALING LAWS IN SEISMIC OCCURRENCE

A crossover is observed at large L



Different scaling for $M_0 > 10^{19}$ (m>7.5)

The crossover is usually attributed to the finiteness of the seismogenic layer: W cannot be larger than H so $W \sim L^0$ (SHOLTZ 1982)

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

1989

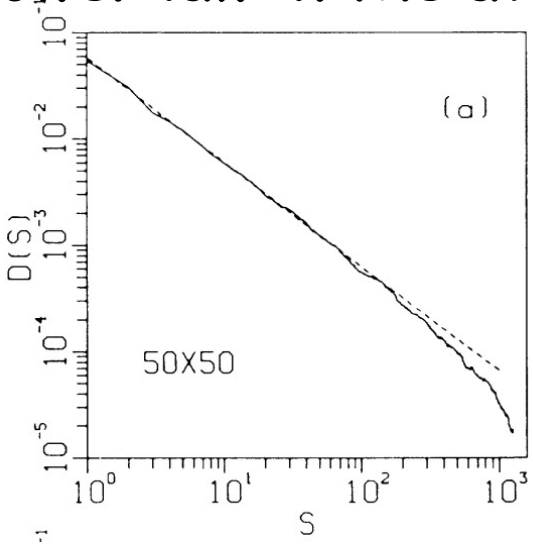
JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 94, NO. B11, PAGES 15,635–15,637, NOVEMBER 10, 1989

SELF-Organized Criticality

Earthquakes as a Self-Organized Critical Phenomenon

PER BAK AND CHAO TANG

Power law in the avalanche size distribution



Bak, Tang & Wiesenfeld 1987
Bak & Tang 1989

KEY INGREDIENTS:

time scale separation

fast avalanches (instantaneous)

slow drive velocity $V_0 \sim 0$

$$Z(i,j) \rightarrow Z(i,j) - 4$$

$$Z(i \pm 1, j) \rightarrow Z(i \pm 1, j) + 1$$

$$Z(i, j \pm 1) \rightarrow Z(i, j \pm 1) + 1$$

The model is actually very close to the generally accepted "block spring" picture of earthquakes [*Burridge and Knopoff*, 1967; *Mikumo and Miyatake*, 1978, 1979]. This is precisely why we believe that our results apply to earthquakes; we do not have to invoke a new and different local mechanism.

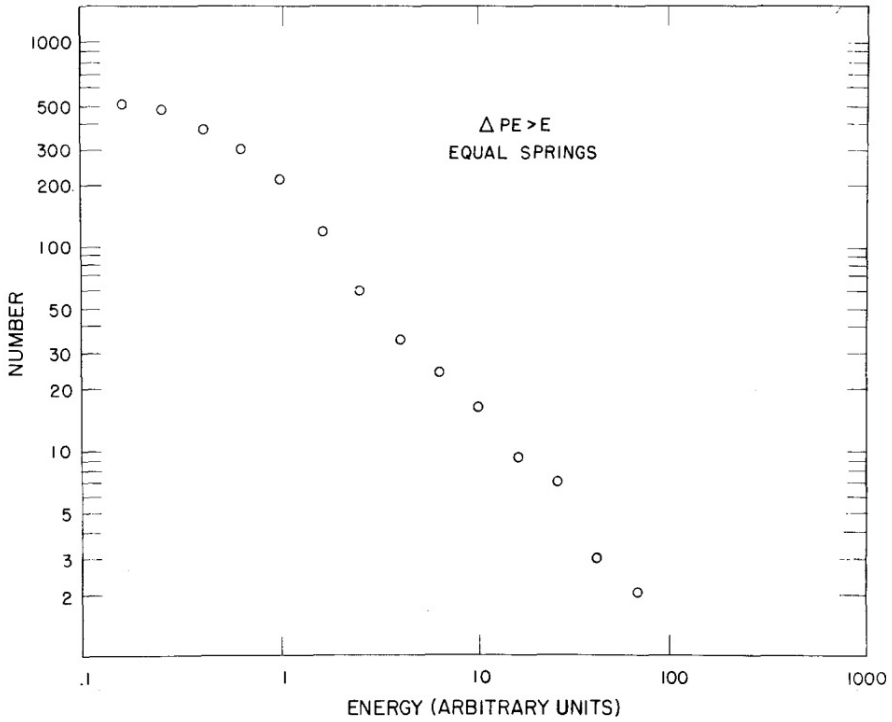
Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

1967

Bulletin of the Seismological Society of America. Vol. 57, No. 3, pp. 341–371. June, 1967

MODEL AND THEORETICAL SEISMICITY

By R. Burridge and L. Knopoff



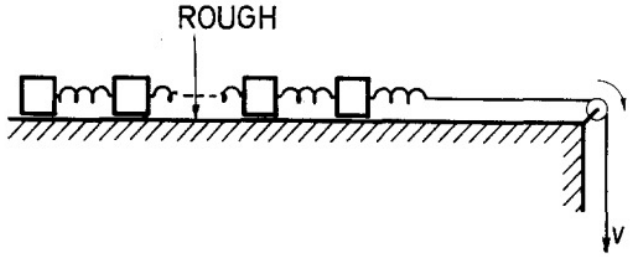


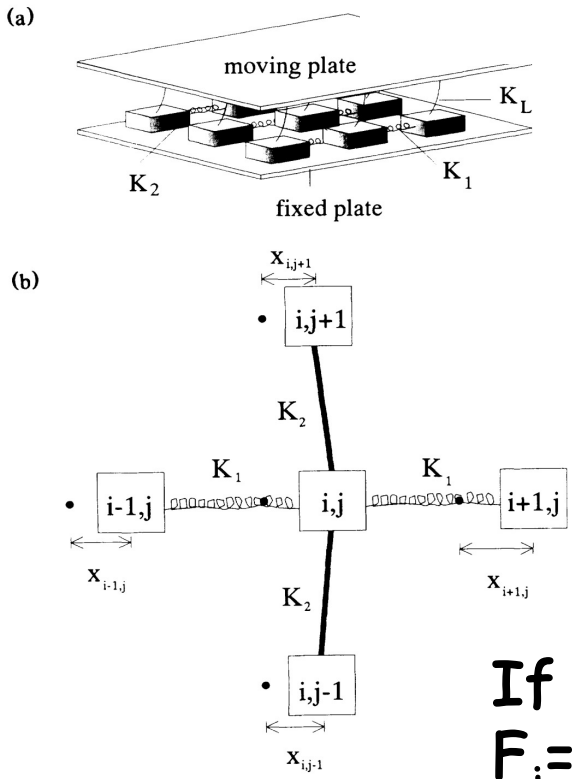
Fig. 3. Schematic diagram of the laboratory model.

1989

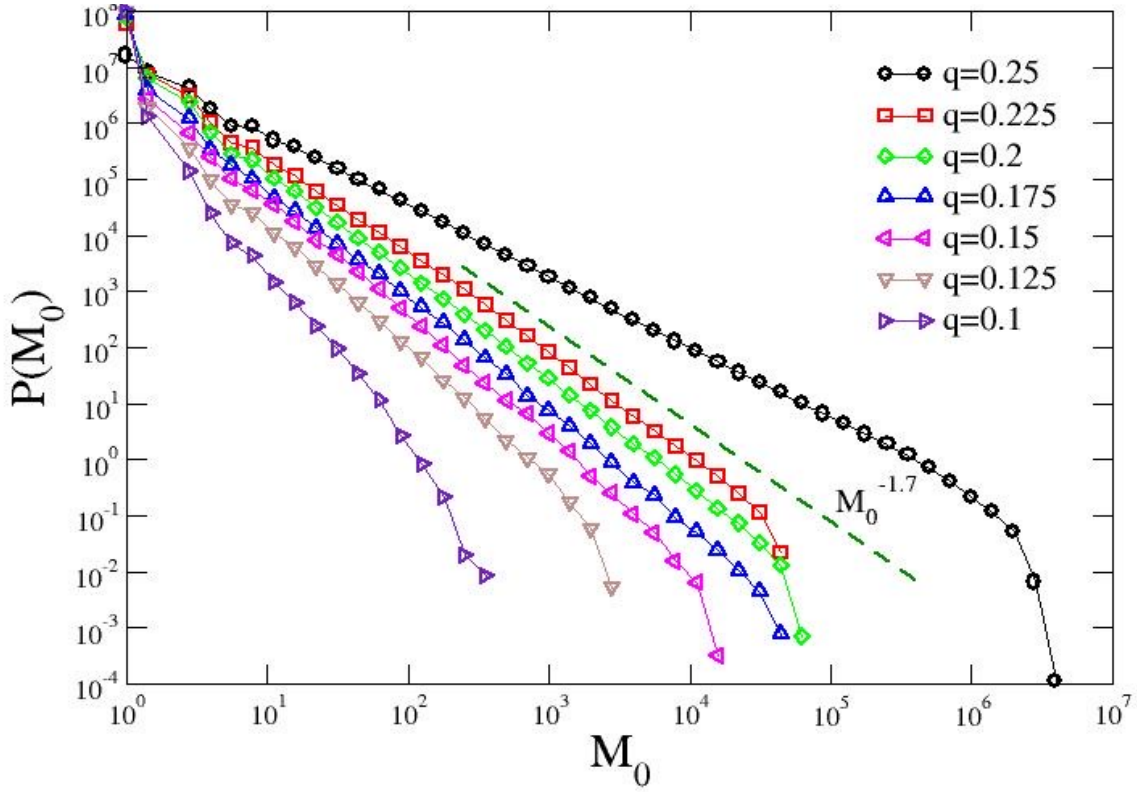
VOLUME 68, NUMBER 8 PHYSICAL REVIEW LETTERS 24 FEBRUARY 1992

Self-Organized Criticality in a Continuous, Nonconservative Cellular Automaton Modeling Earthquakes

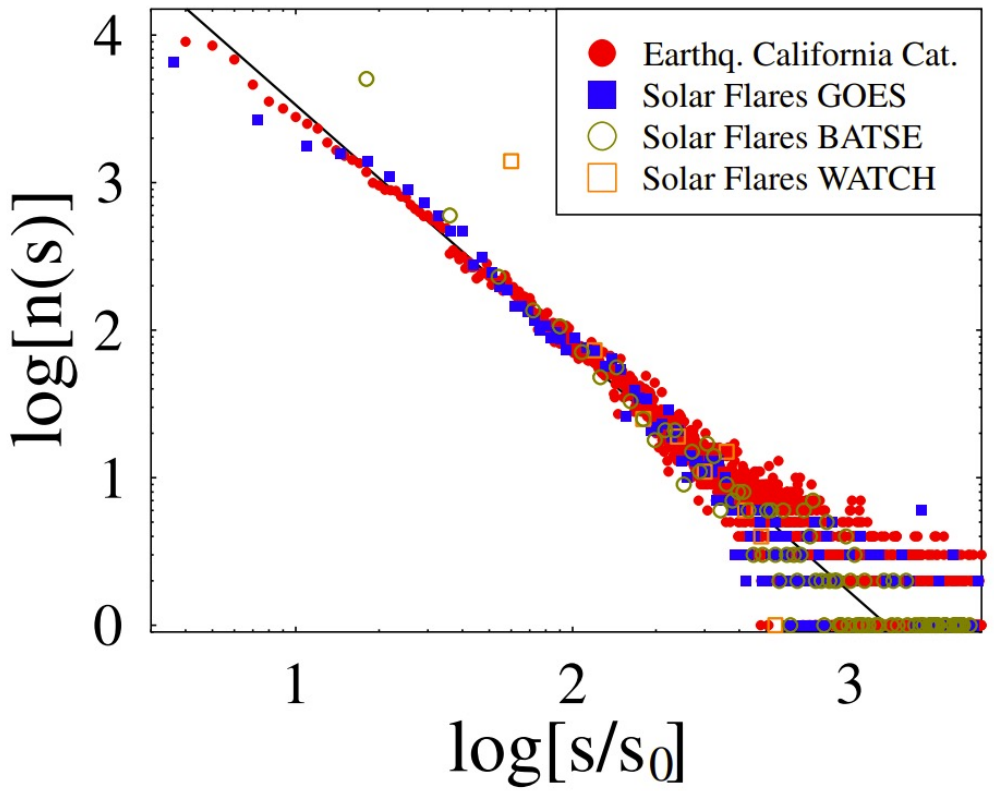
Zeev Olami, Hans Jacob S. Feder, and Kim Christensen [a]
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973
(Received 19 August 1991)



If $F_i > F_{th}$, $F_i = 0$
$F_j = F_j + qF_i$
with $q = (k_1 + k_0)/4k_1$

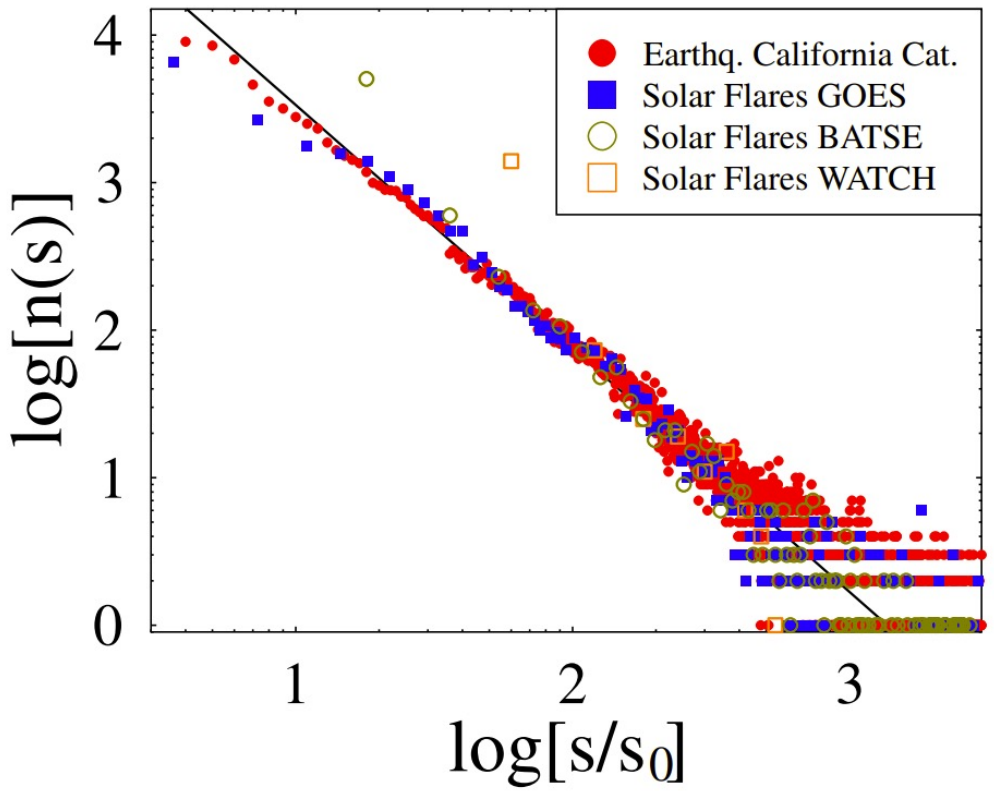


Similar scaling law in Solar Flare occurrence



Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

Similar scaling law in Solar Flare occurrence



debates 11 March 1999

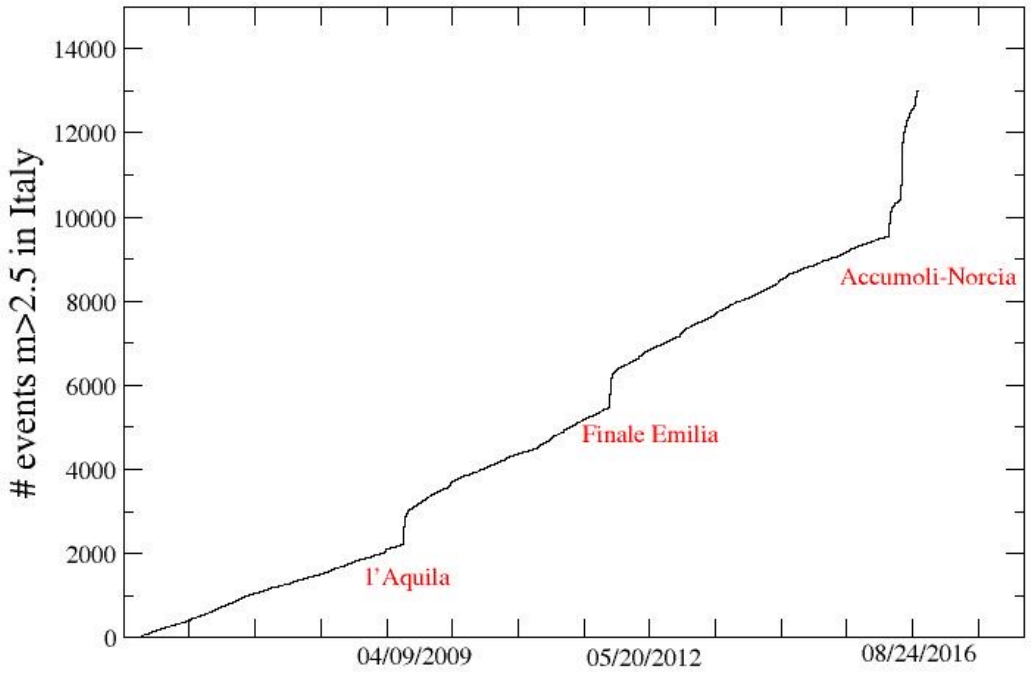
Earthquake prediction is difficult but not impossible

LEON KNOPOFF

Statistics of rare events

The small number of events means that again we need a physics-based theory of the precursory process to amplify the meager data. In the area of physics, another blind alley was followed. The beguiling attractiveness of the illusion of scale-independence of the G-R law suggested that the model of self-organized criticality (SOC), which also yielded scale-independent distributions, might be appropriate. (The logic is evidently faulty: if mammals have four legs, and tables have four legs, it does not follow that tables are mammals, or the reverse.) The model of SOC permits a hierarchical development of large events out of the nonlinear interaction of smaller events, at rates in relation to their sizes, and culminating in the largest event. However, there are several important arguments against the applicability of SOC to the earthquake problem.

Temporal Clustering

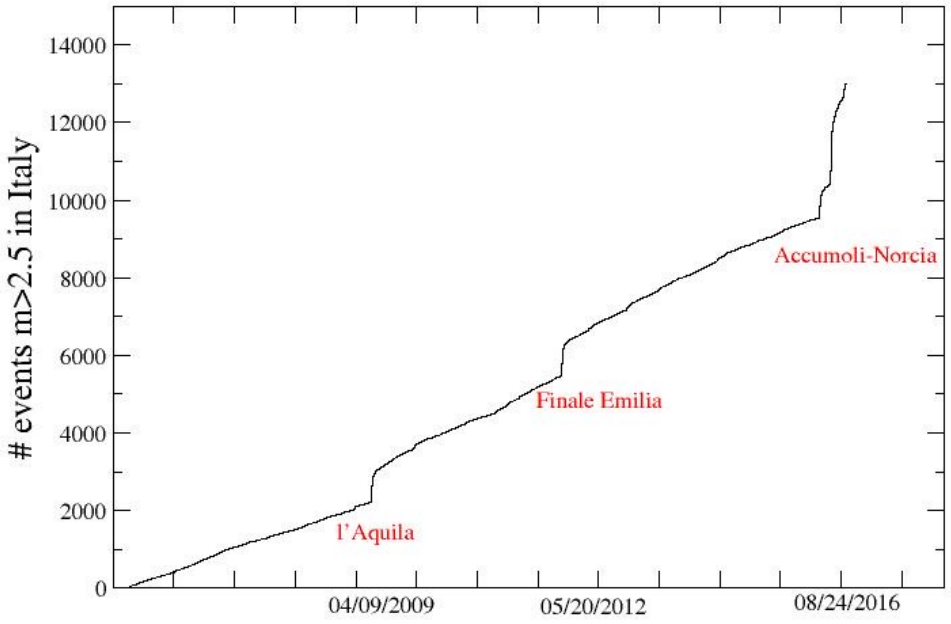


Mainshock:The largest event in a seismic sequence

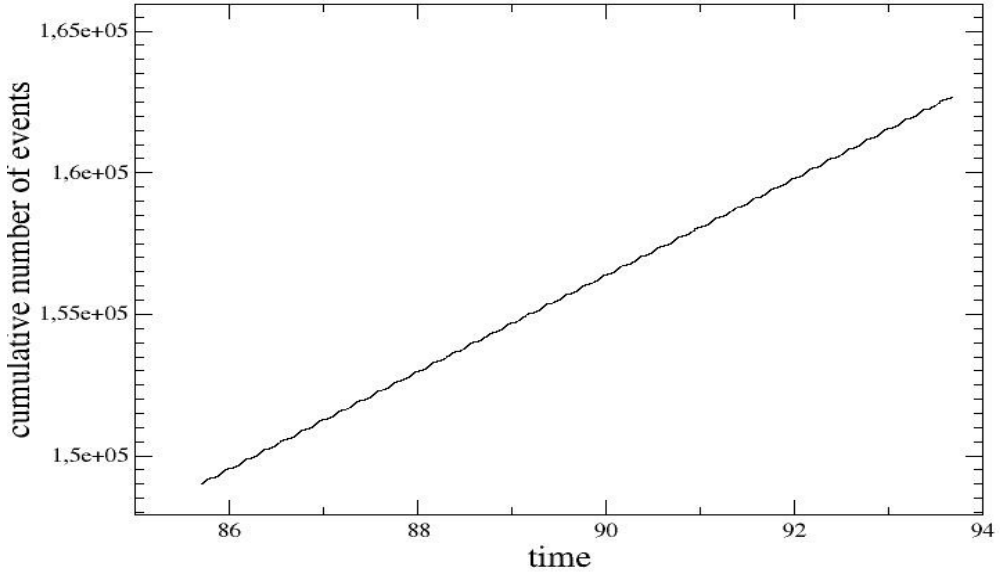
Aftershocks the earthquakes that follow the largest shock of an earthquake sequence. They are typically smaller than the mainshock and occur at few km from the mainshock epicenter

The majority of events in seismic catalogs are aftershocks!

Temporal Clustering

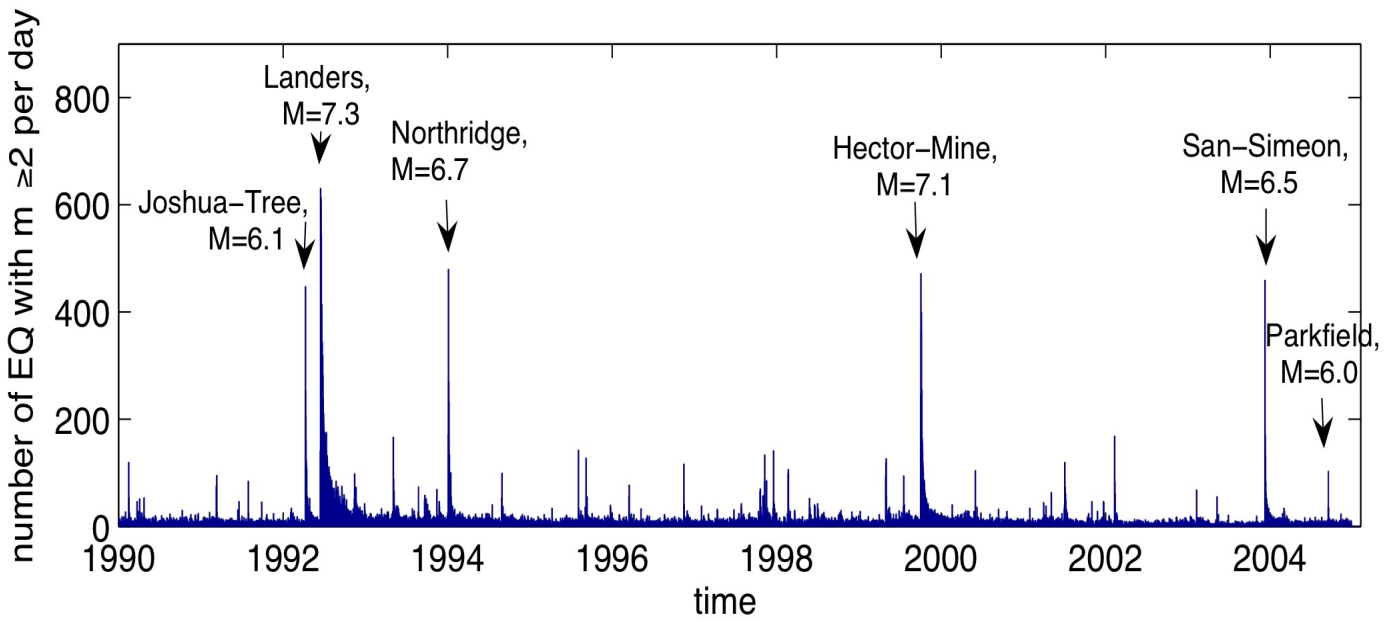


N0 temporal clustering (NO aftershocks) in the Sand_pile model



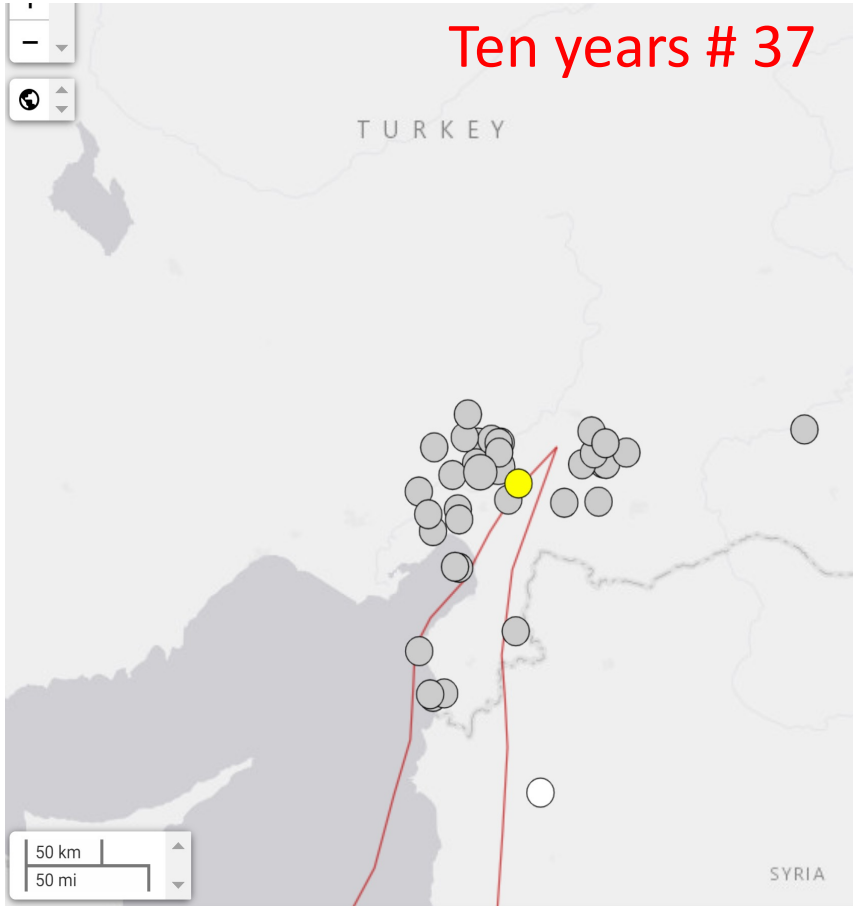
Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

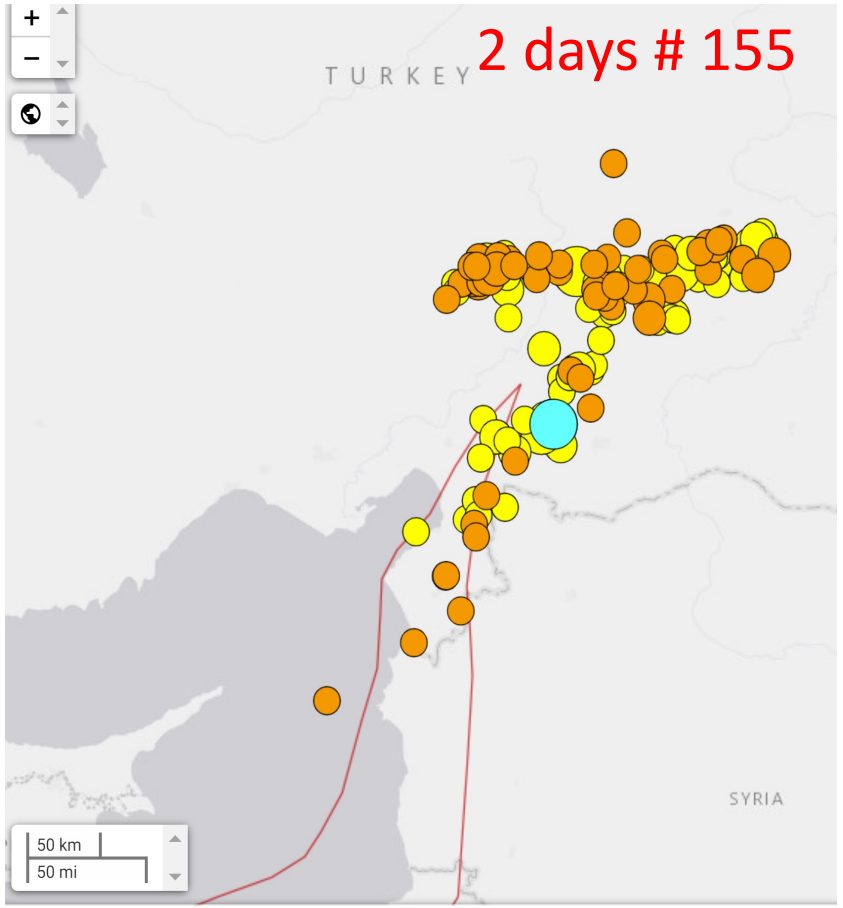
Temporal Clustering



The majority of events in seismic catalogs are aftershocks!

05-02-2023 Earthquake





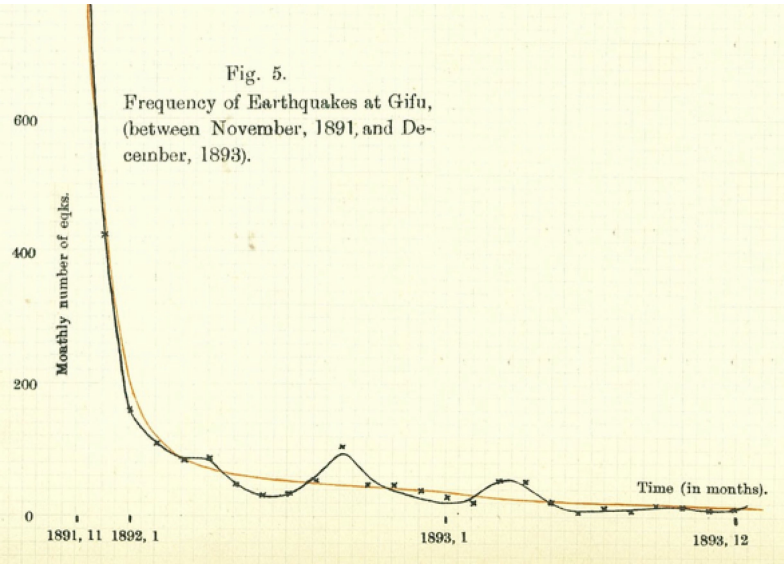
Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

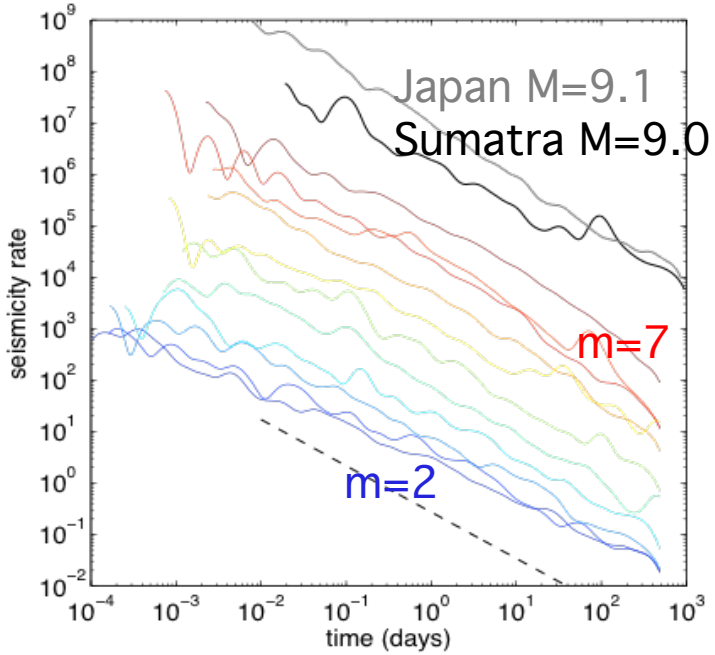
1889 The Omori law

Fusakichi Omori in 1889 published his work on the aftershocks of earthquakes, in which he stated that aftershock frequency decreases by roughly the reciprocal of time after the main shock.

$$n(t) = \frac{k}{t}$$

Fusakichi Omori





The modified version of Omori's law, now commonly used, was proposed by Utsu in 1961, with typical values of p [0.75:1.5].

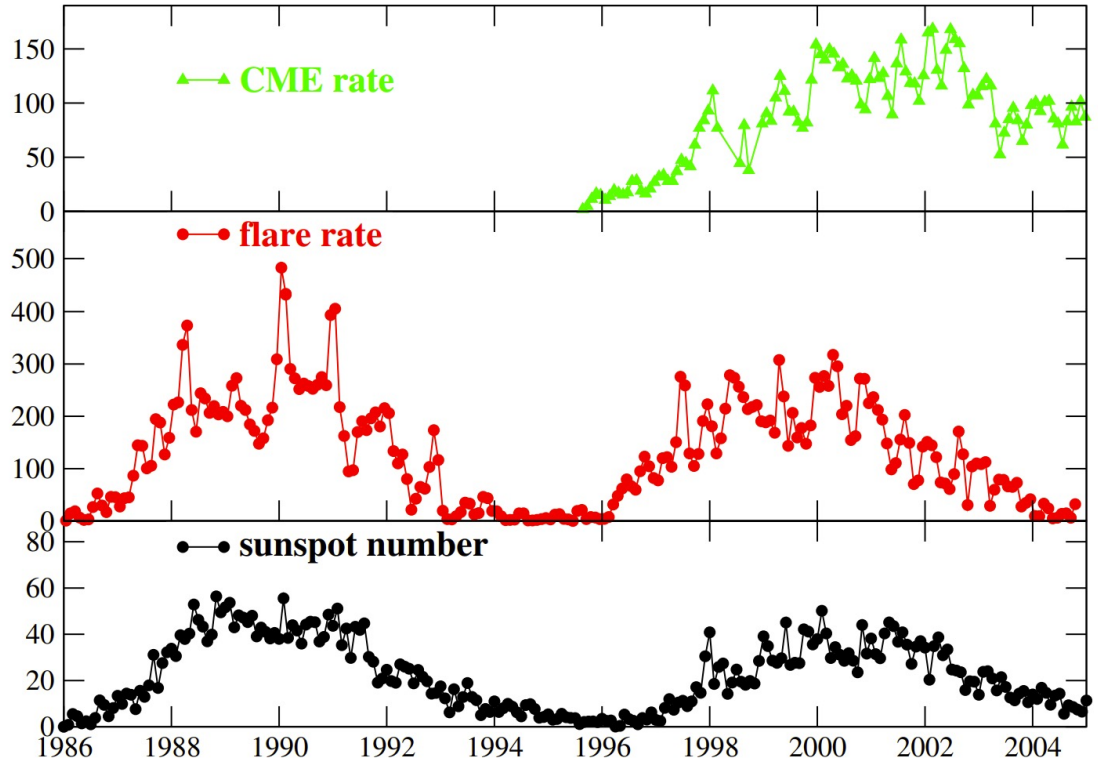
$$n(t) = \frac{k}{(t+c)^p}$$

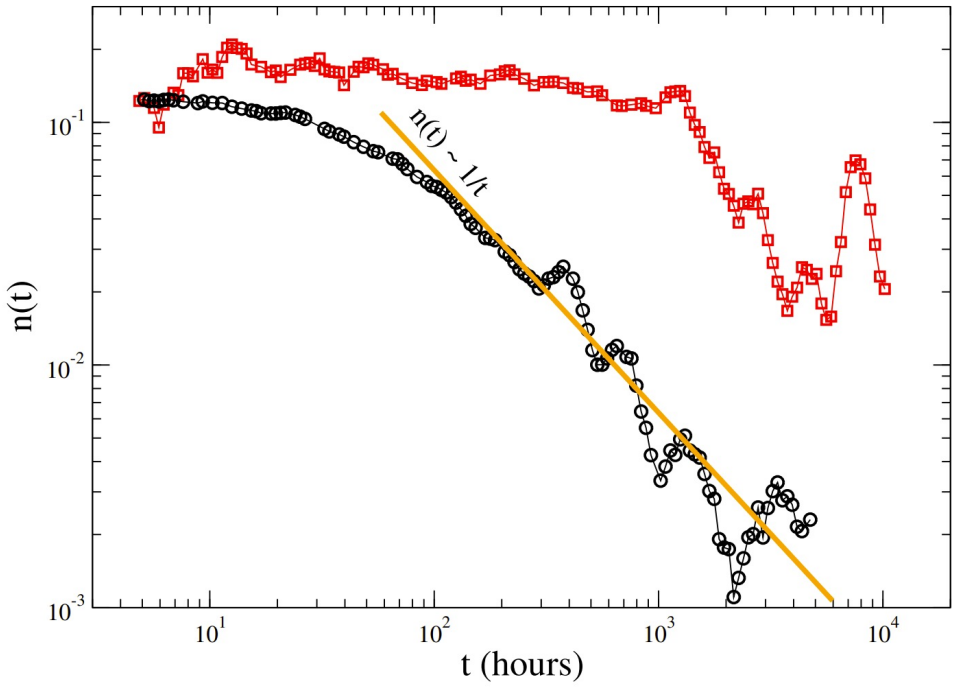
Mainshocks: largest event of a sequence

Aftershocks: its following earthquakes within a given space-time distance

Similar scaling law in Solar Flare occurrence

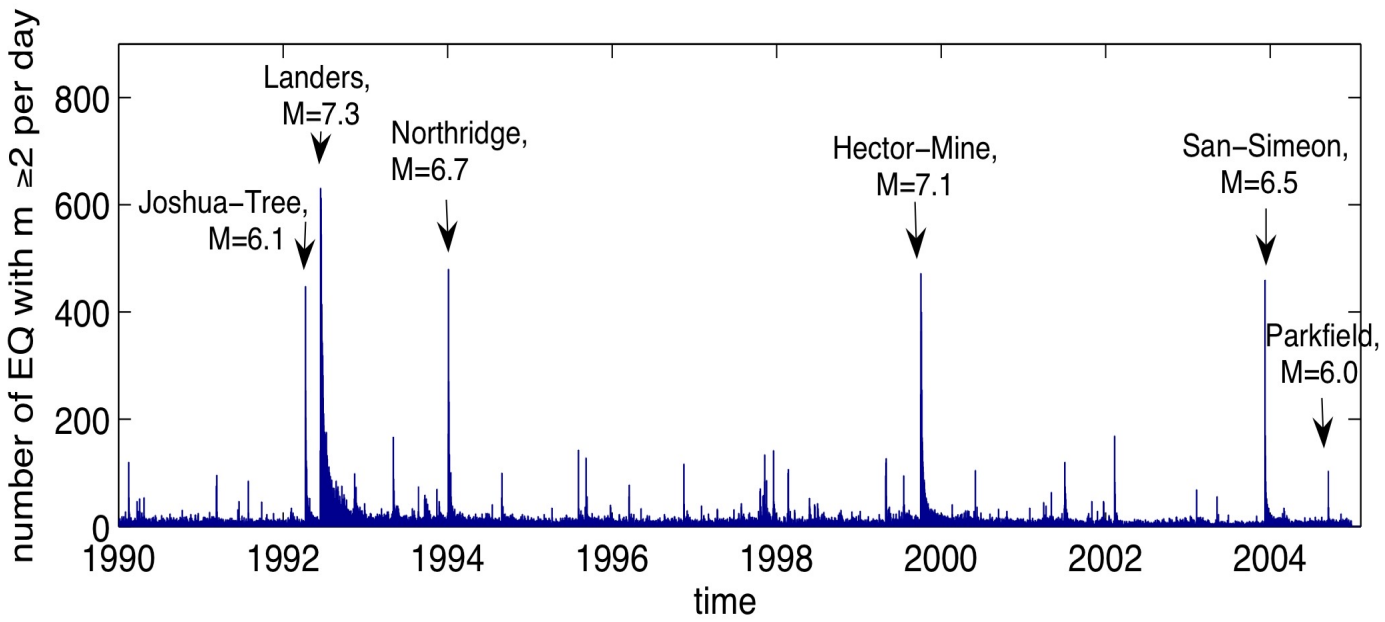
$$n(t) = \frac{\mathcal{N}(t, e_{th})}{\sum_{i=1}^{L} \Theta(t - t_{i+1})}$$





Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

Temporal Clustering

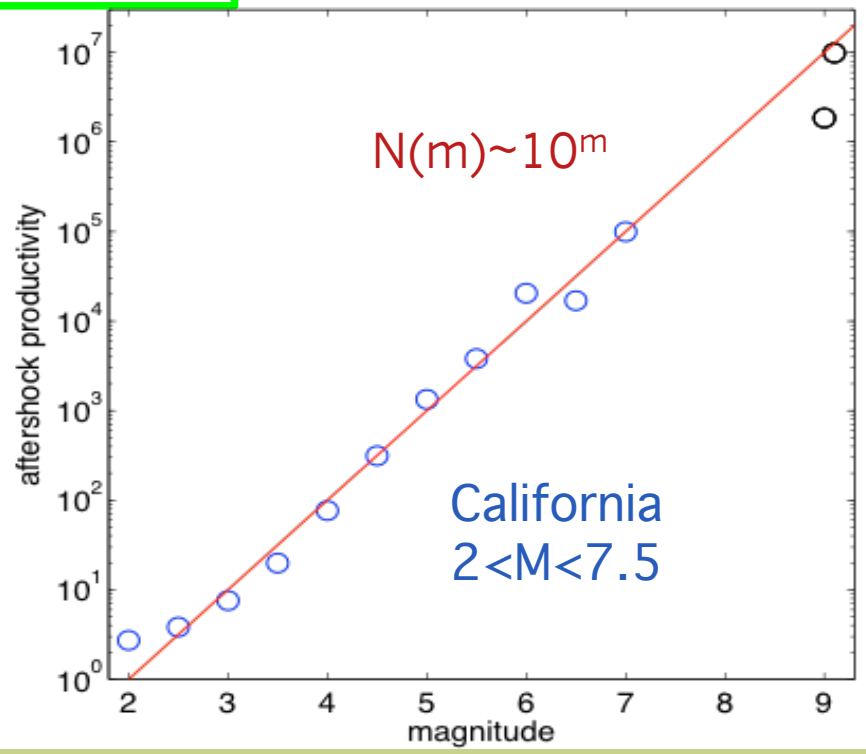


The majority of events in seismic catalogs are aftershocks!

Productivity law

$$N_{aftershocks} \approx 10^{\alpha m} \quad m \approx 2/3 \log_{10}(M_0) \Rightarrow N_{aftershocks} \approx M_0^{1+2/3\alpha} \quad \alpha \approx 1$$

In 1970 Utsu observed that the number of aftershocks grows exponentially with the mainshock magnitude

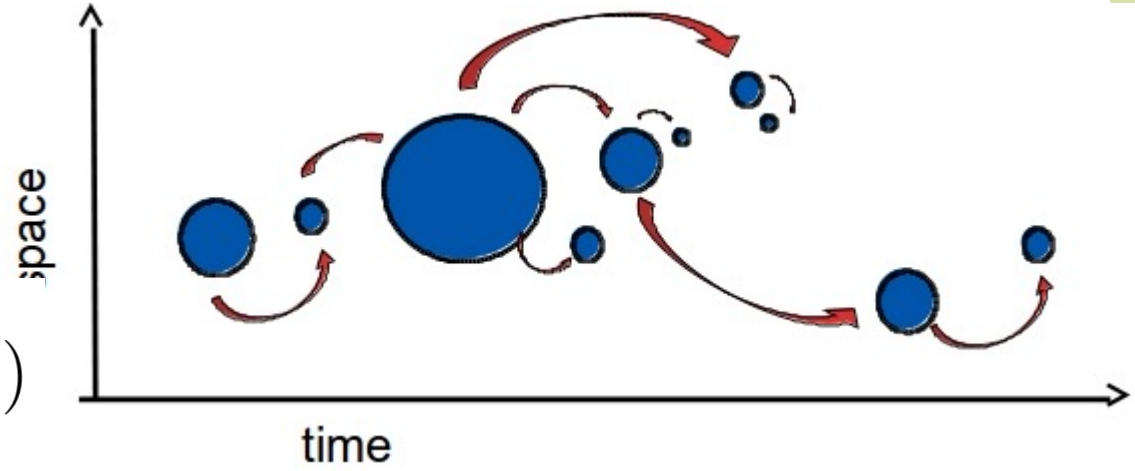


Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

POINT-PROCESS models for seismic forecasting

An Earthquakes can trigger more than one daughter earthquake:

EPIDEMIC TYPE AFTERSHOCK SEQUENCE (ETAS) model



Conditional Probability to have an earthquake of aagnitude M at time t

$$\Psi(M,t) = \sum_{i:t_i<t} \rho(M,t|M_i,t_i) + \mu P(M)$$

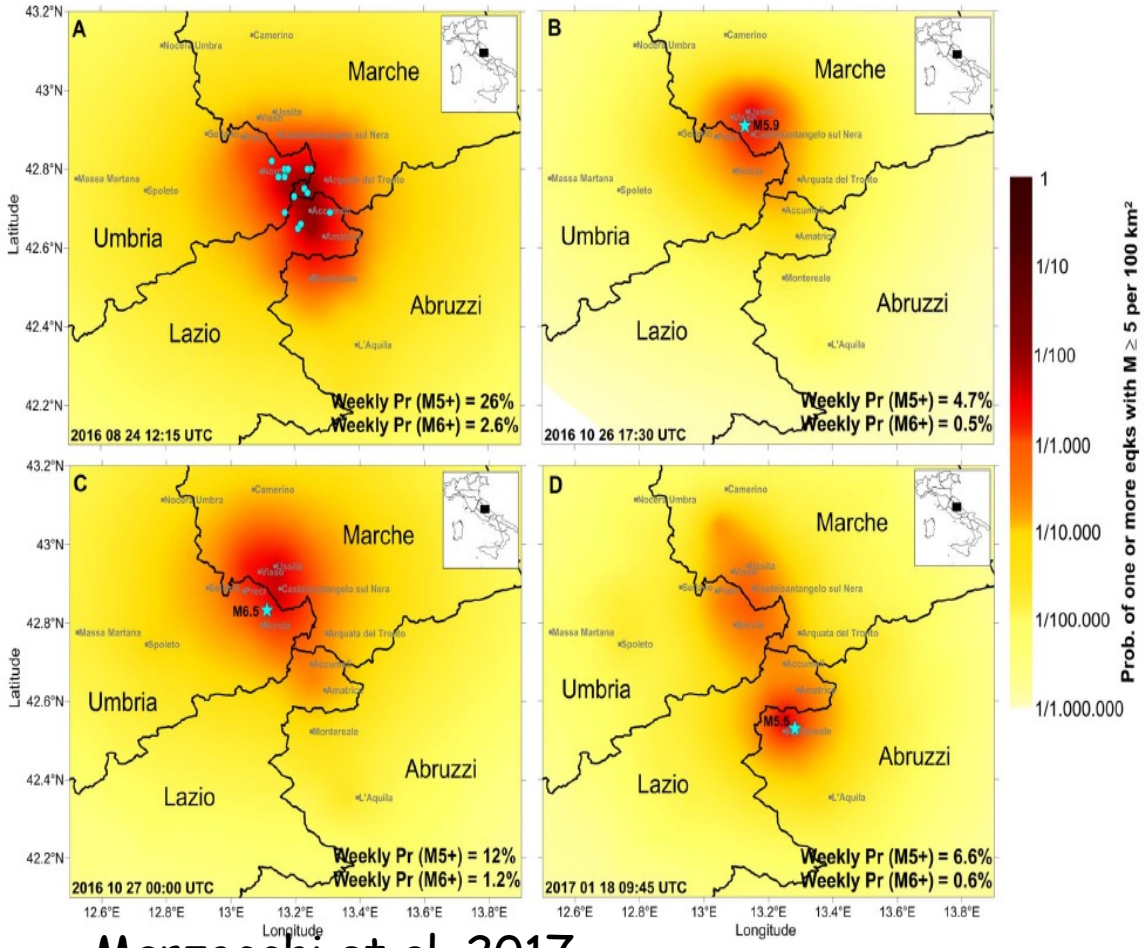
The sum extends over all previous earthquakes

$$\rho(M,t|M_i,t_i) = K10^{\alpha M_i}\left[\frac{(t-t_i)}{c}+1\right]^{-p} P(M)$$

Key approximation:
Factorization between space, time and magnitude

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

The ETAS model for the 2016 Amatrice-Norcia sequence



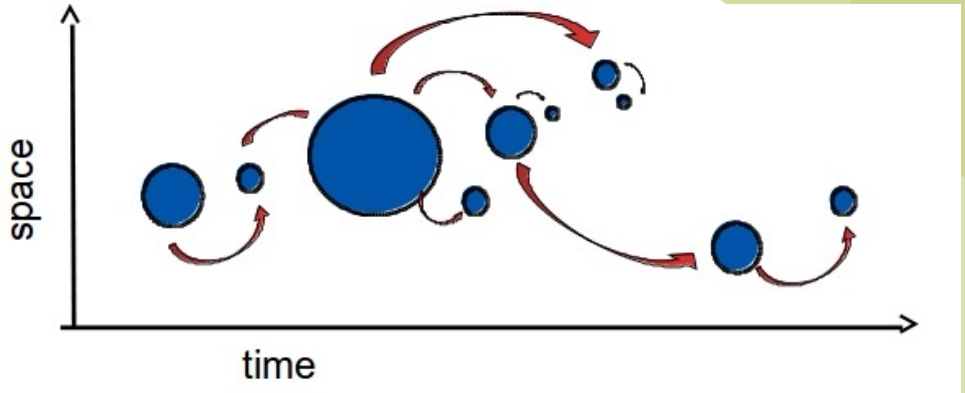
Marzocchi et al. 2017

ETAS is usually considered the best available forceasting instrument

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

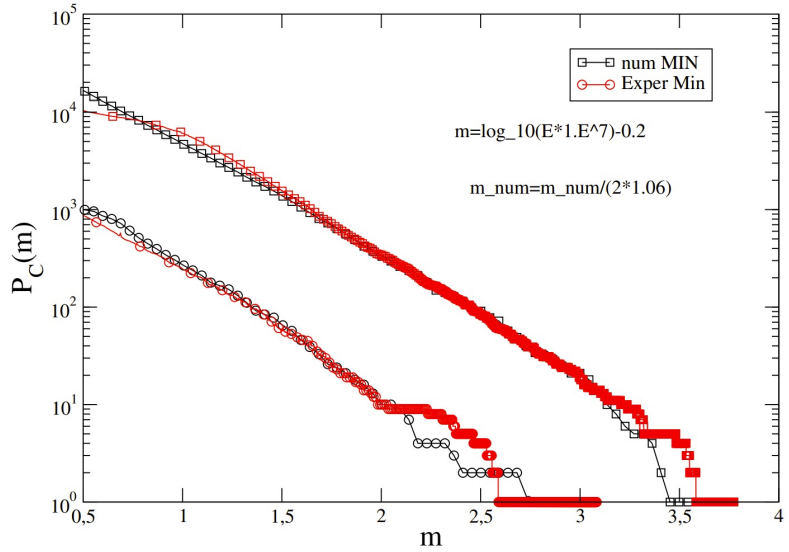
POINT-PROCESS models for solar flares

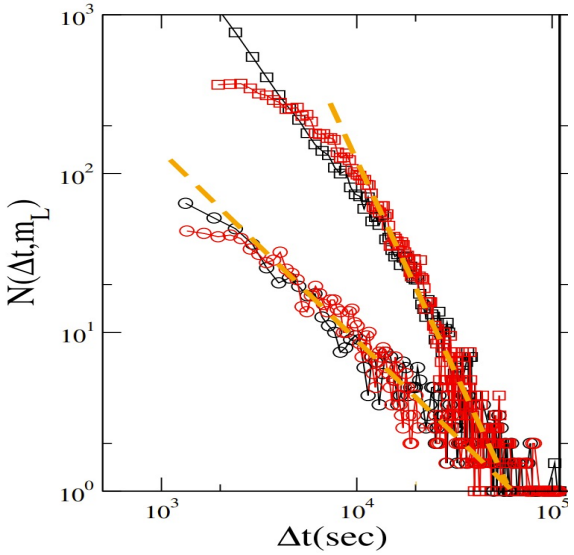
$$\Psi(M,t) = \sum_{i:t_i<t} \rho(M,t|M_i,t_i) + \mu P(M)$$



$$\mu(t) = A\cos(2\pi t/T)$$

$$\rho(M,t|M_i,t_i) = \frac{k}{t-t_i}$$



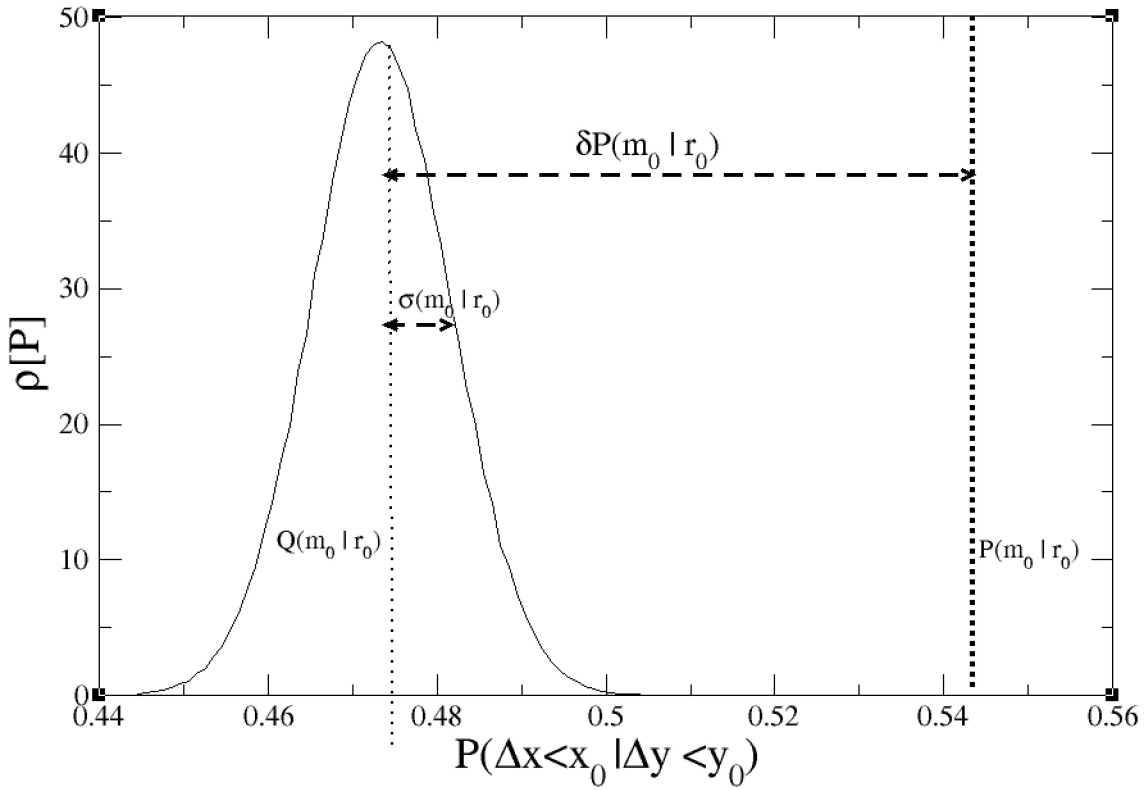


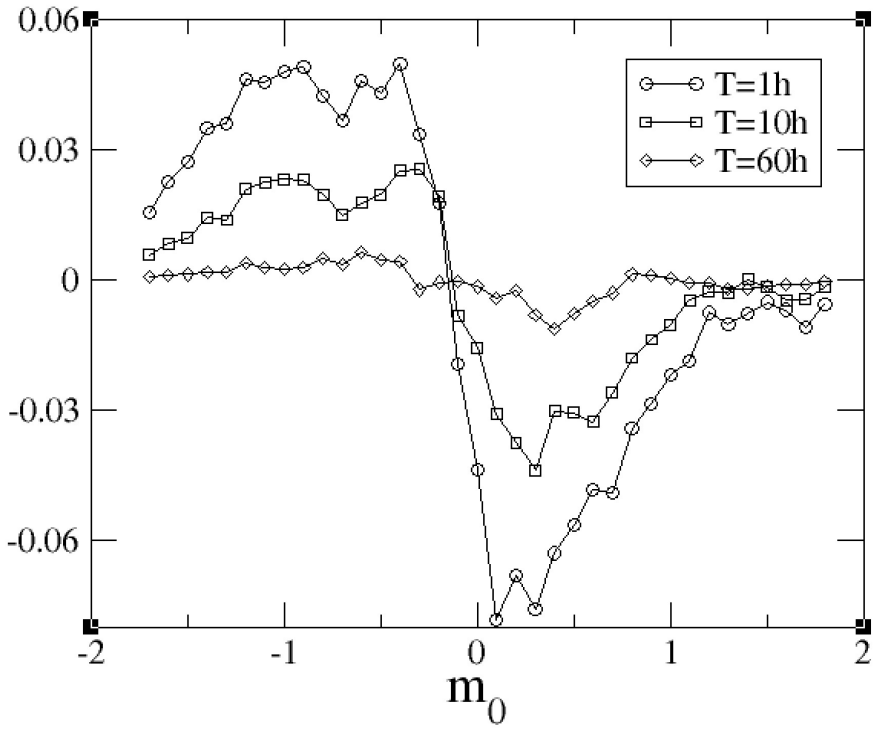
Key approximation:
Factorization between space, time and magnitude

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

TIME-MAGNITUDE Correlation

$$dP(M_{i+1} = M_i + m_0 | t_{i+1} - t_i < T)$$
$$= P(M_{i+1} = M_i + m_0 | t_{i+1} - t_i < T) - P(M_{k*} = M_{i+1} + m_0 | t_{i+1} - t_i < T)$$



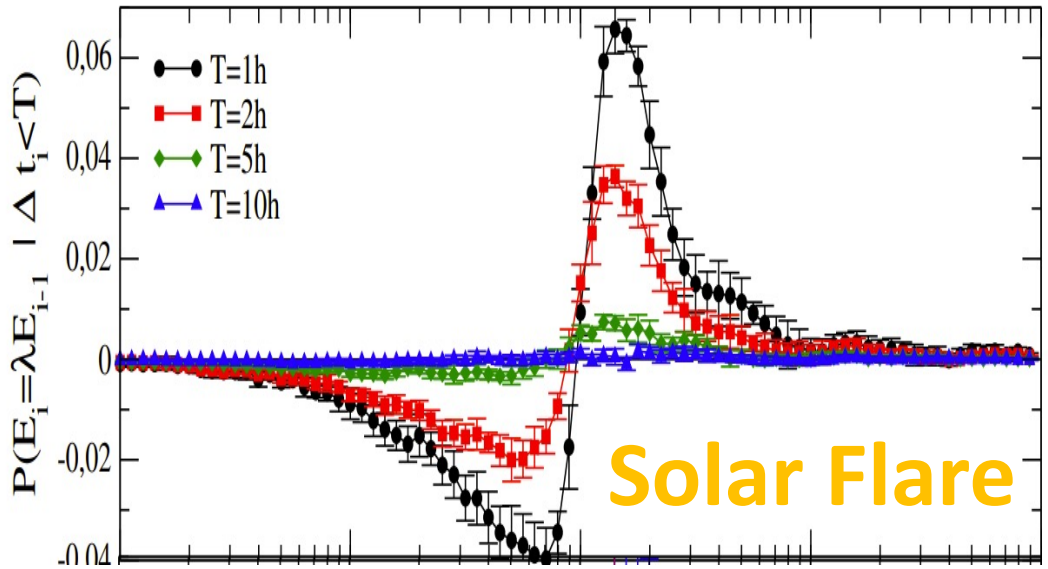


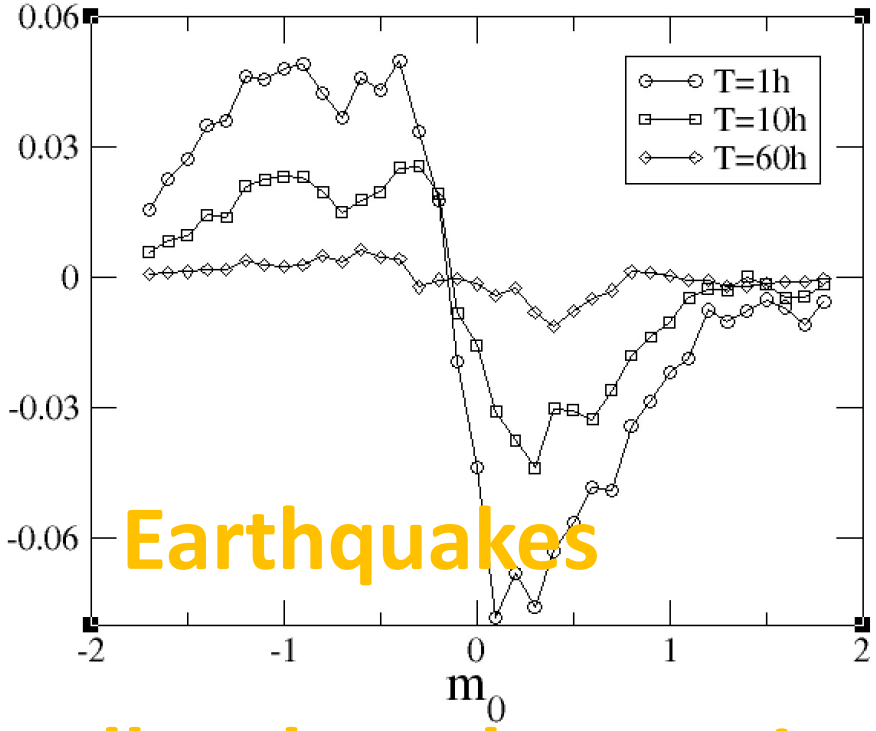
The next earthquake is about -1 smaller than the previous one

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

TIME-MAGNITUDE Correlation

$$dP(M_{i+1} = M_i + m_0 | t_{i+1} - t_i < T)$$
$$= P(M_{i+1} = M_i + m_0 | t_{i+1} - t_i < T) - P(M_{k*} = M_{i+1} + m_0 | t_{i+1} - t_i < T)$$



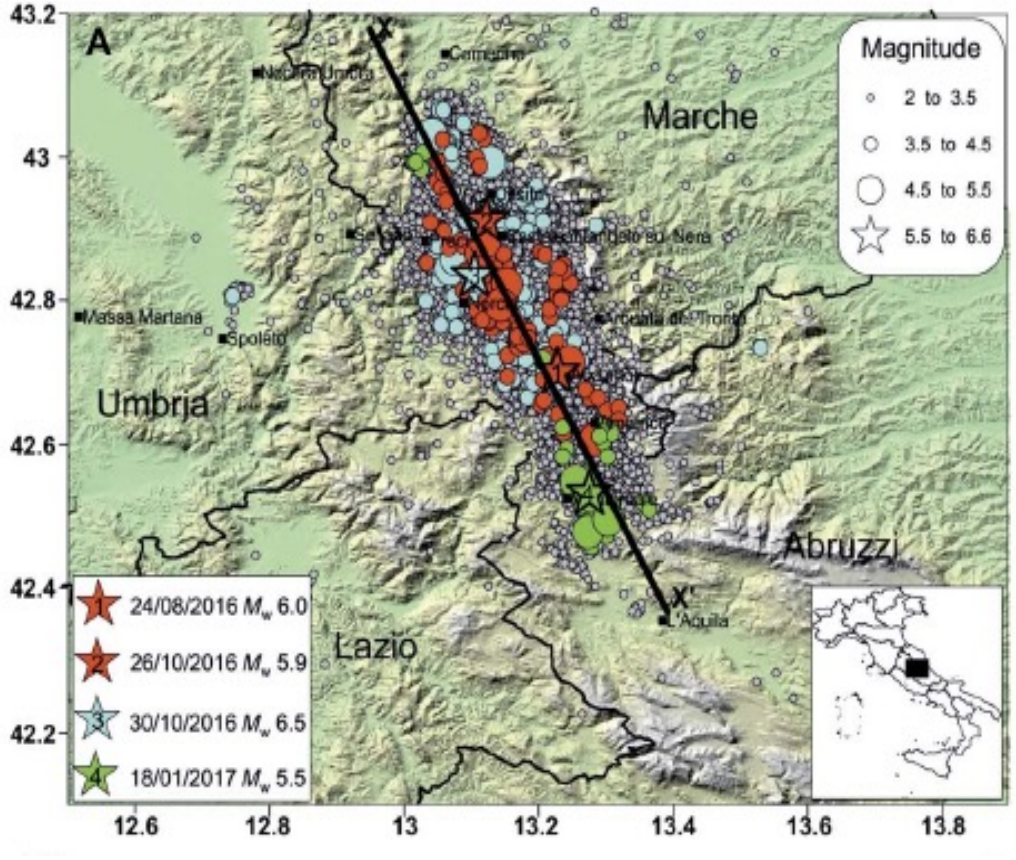


The next earthquake is about -1 smaller than the previous one

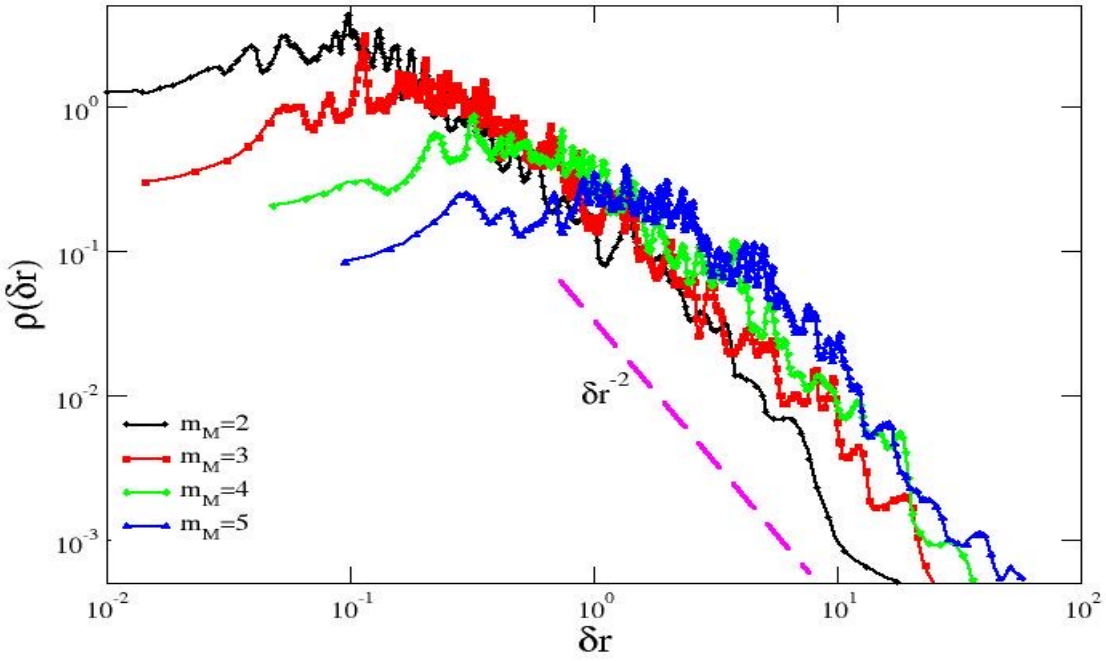
The next flare is on average larger than the previous one

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

SPATIAL clustering



$$n(\delta r) = \frac{k}{(\delta r)^{\gamma}}$$



Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

MISSING INGREDIENTS

debates 11 March 1999

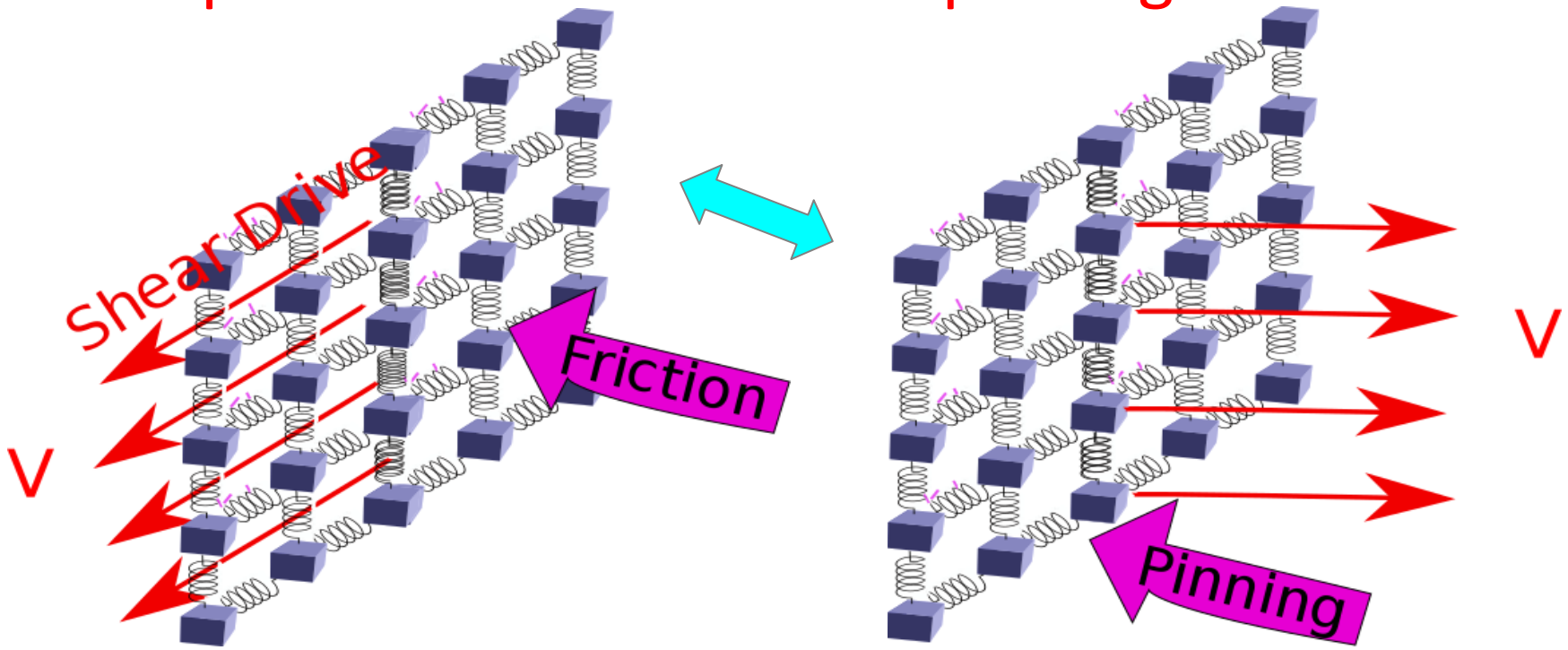
Earthquake prediction is difficult but not impossible

LEON KNOPOFF

their sizes, and culminating in the largest event. However, there are several important arguments against the applicability of SOC to the earthquake problem.

1. Faults and fault systems are inhomogeneous: we have already noted the presence of several scale sizes.

The Map on elastic interface depinning



Friction force ⟷ Pinning force

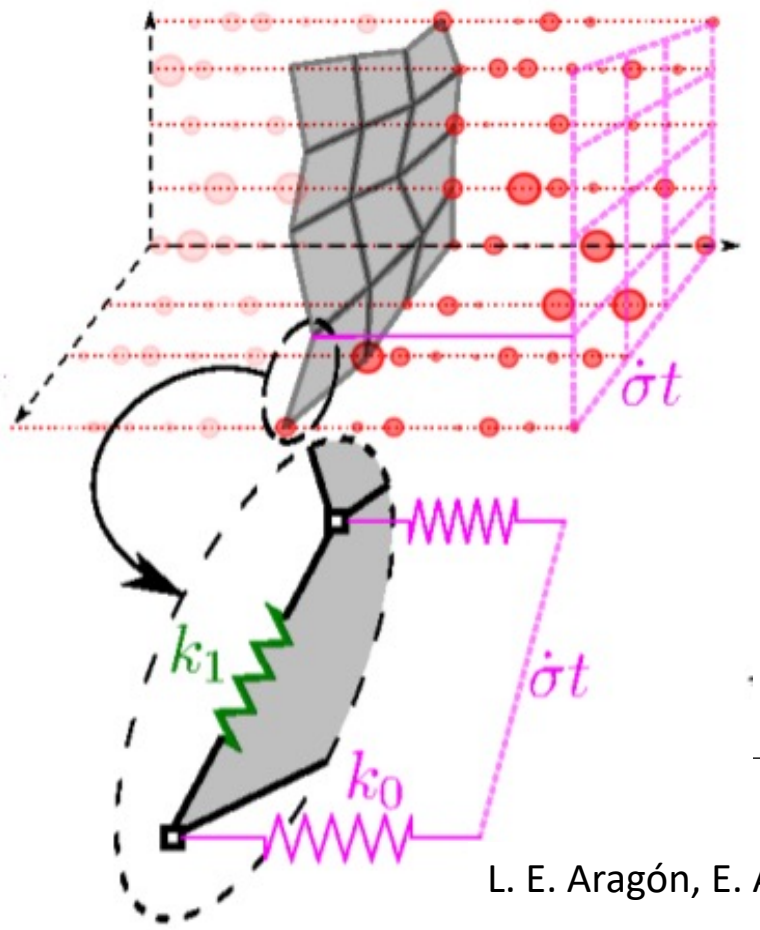
⟷ Drive perpendicular to the interface

⟷ Impurities in the sample

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

The Map on elastic interface depinning

quenched Edwards-Wilkinson (qEW) model



Red dots represent pinning centers

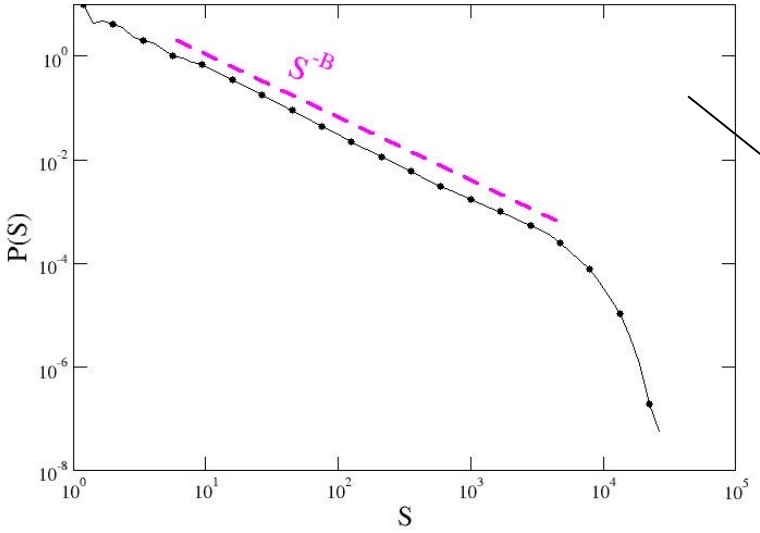
heterogeneous in space and in depth

L. E. Aragón, E. A. Jagla, and A. Rosso, Phys. Rev. E (2012)

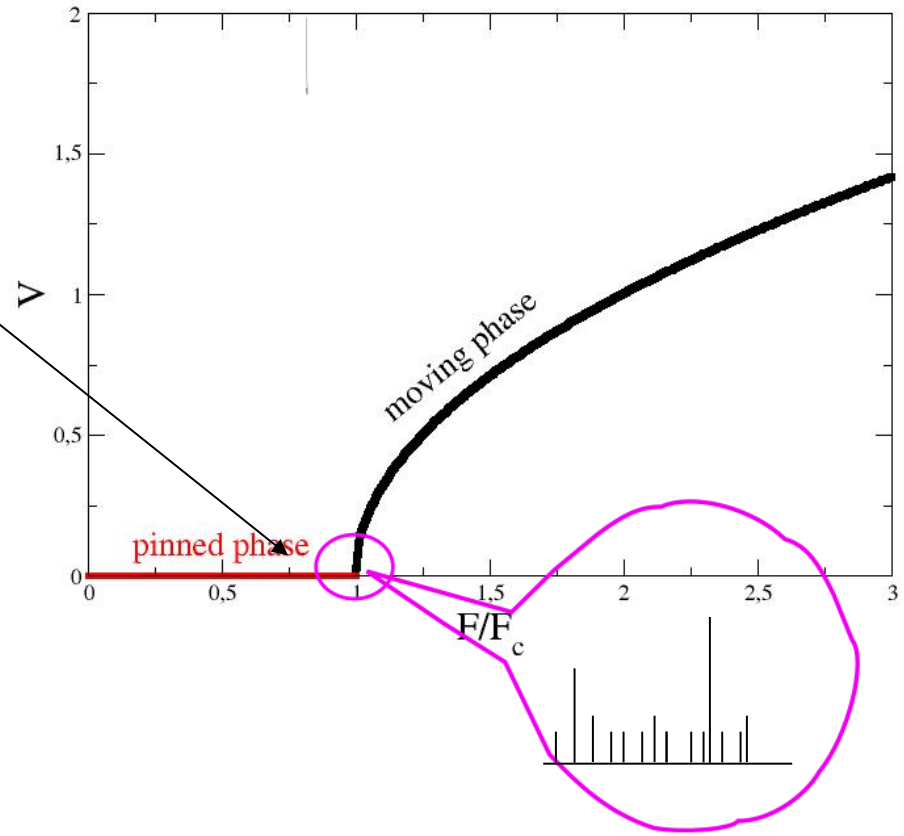
Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

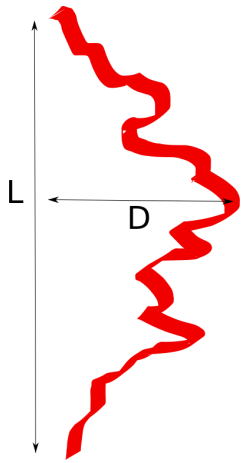
The depinning transition under a costant force

Intermittent behavior: avalanches



≈





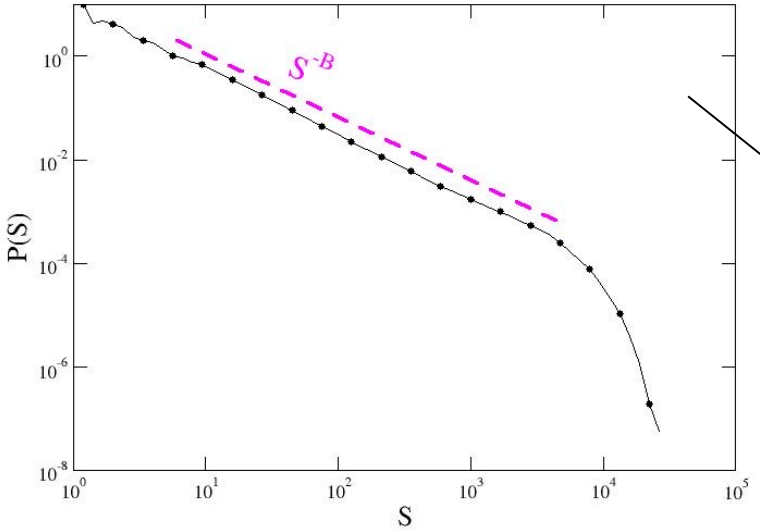
$B=2-2/(D+\xi)$

$M_0 \sim L^{D+\xi}$

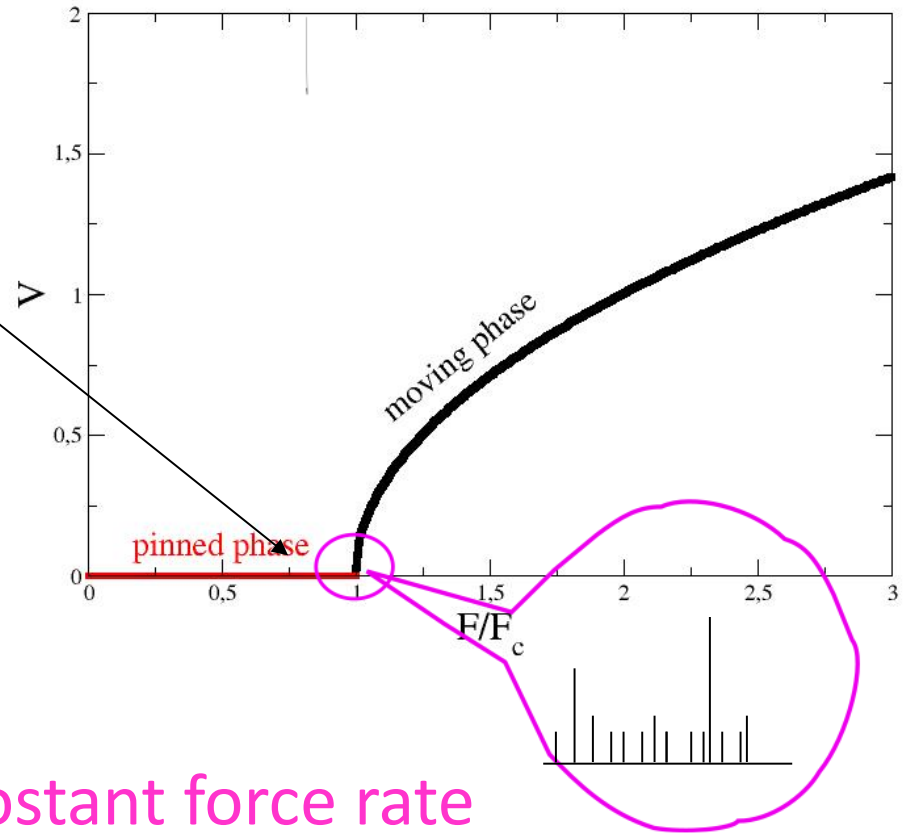
Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

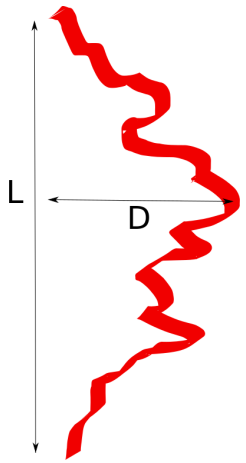
The depinning transition under a costant force

Intermittent behavior: avalanches



≈





$B=2-2/(D+\xi)$

$M_0 \sim L^{D+\xi}$

SOC: no costant force but costant force rate

$$v \to 0 \Rightarrow F \to F_C$$

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

The Map on elastic interface depinning

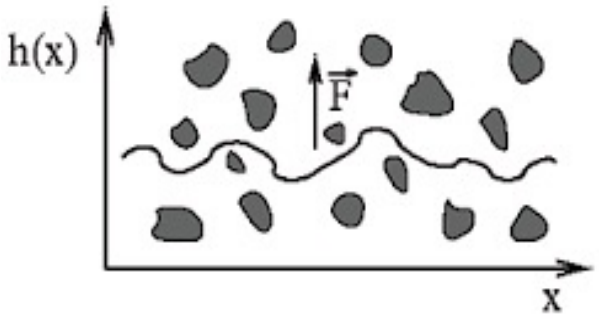
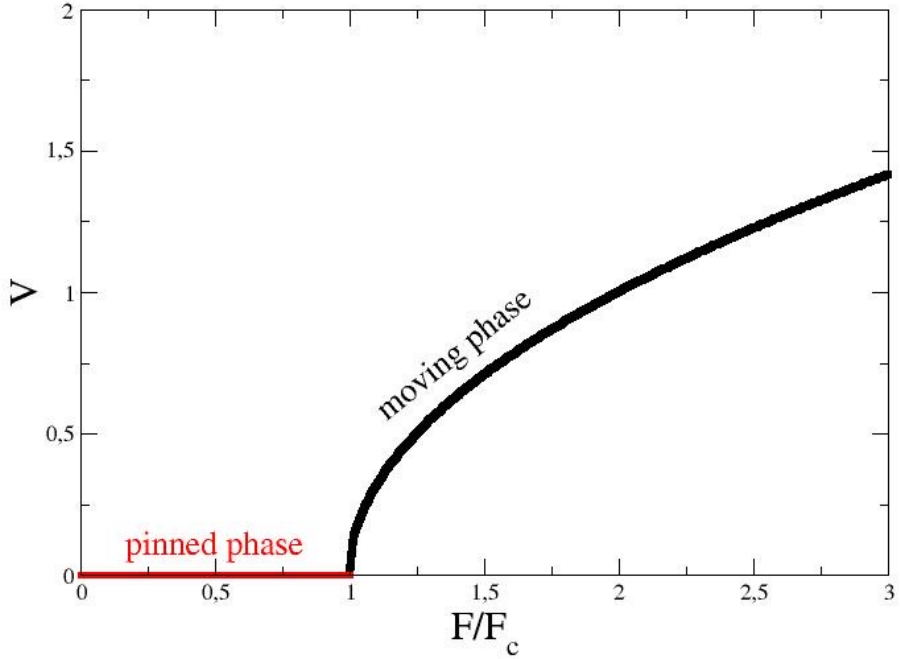


Figure 4. Schematic representation of a driven interface in a disordered media.



Different exponents of those of earthquake occurrence!!

Non uniform force $F=K(V_0 t - x(t))$, when $V_0 \sim 0$ $F \sim F_C$

Always close to a critical point

$B=2-2/(D+\xi)$

$M_0 \sim L^{D+\xi}$

$\xi=0.75$

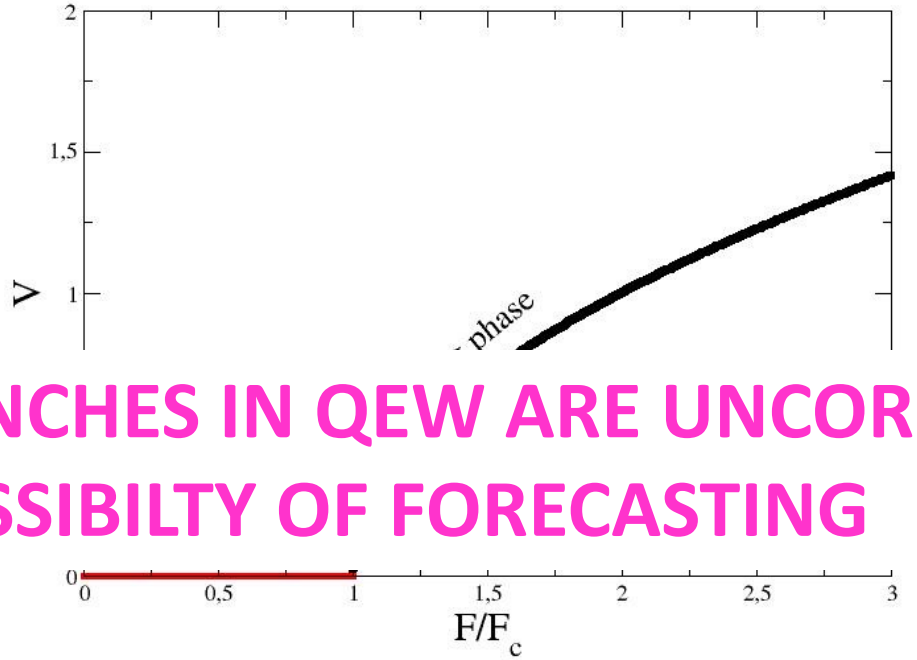
$B=1.27 < 1.7$

$M_0 \sim L^{2.75} < L^3$

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

The Map on elastic interface depinning

Figure 4. Schematic representation of a driv
dered media.



Different exponents of
rthquake
e!!

AVALANCHES IN QEW ARE UNCORRALETED

NO POSSIBILTY OF FORECASTING

Non uniform force $F=K(V_0 t - x(t))$, when $V_0 \sim 0$ $F \sim F_C$

Always close to a critical point

$B=2-2/(D+\xi)$

$M_0 \sim L^{D+\xi}$

$\xi=0.75$

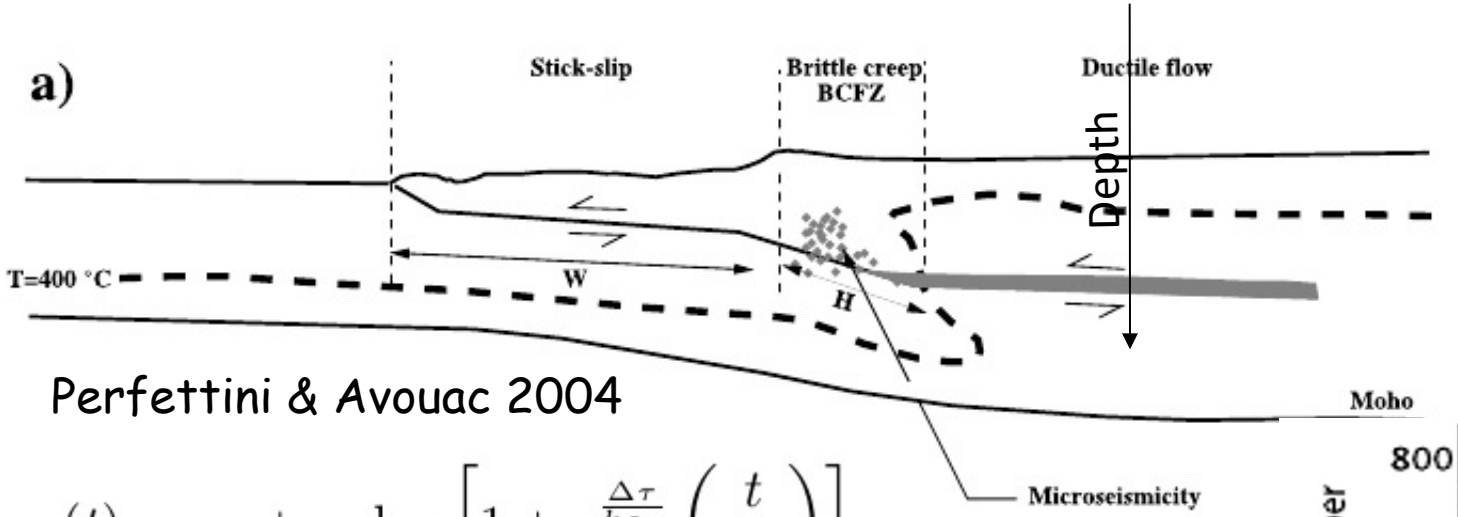
$B=1.27 < 1.7$

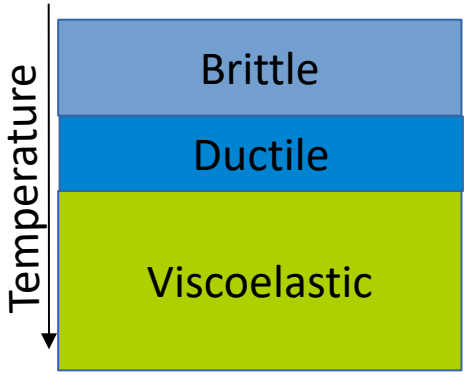
$M_0 \sim L^{2.75} < L^3$

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

The origin of aftershocks: Afterslip

Coupling with the ductile layer



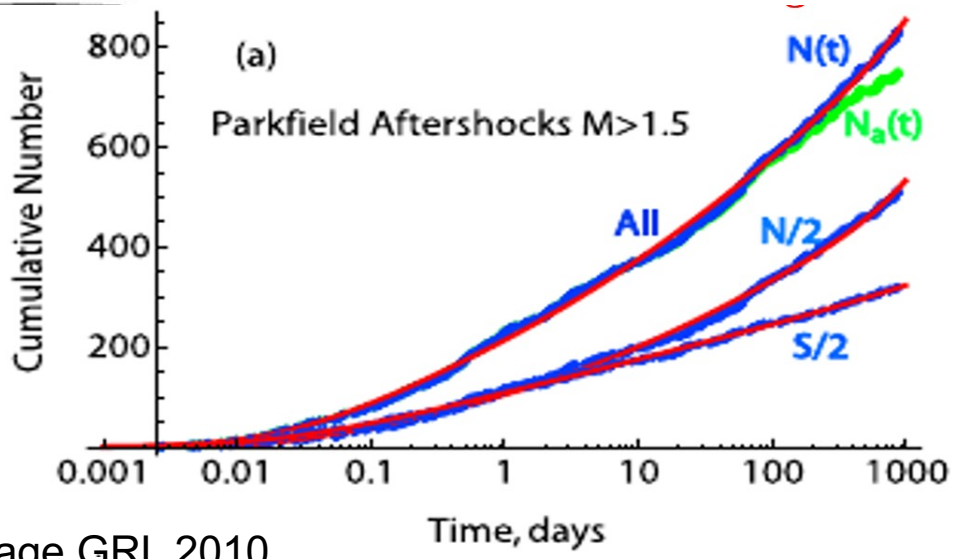


Perfettini & Avouac 2004

$$u(t) = u_0 + \rho_0 \log\left[1 + e^{\frac{\Delta\tau}{k\rho_0}}\left(\frac{t}{t_R}\right)\right]$$

$$n(t) \sim \frac{1}{t} \Rightarrow N(t) \sim \log(t) \sim u(t)$$

2010



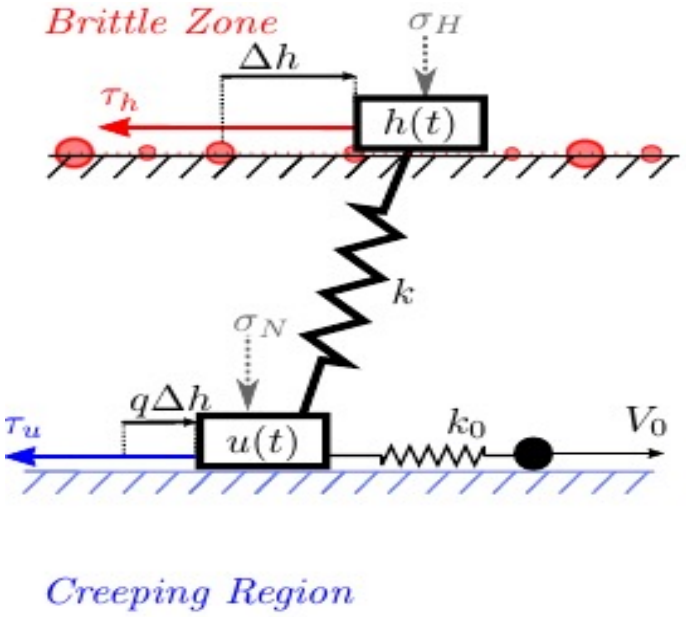
From Savage GRL 2010

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

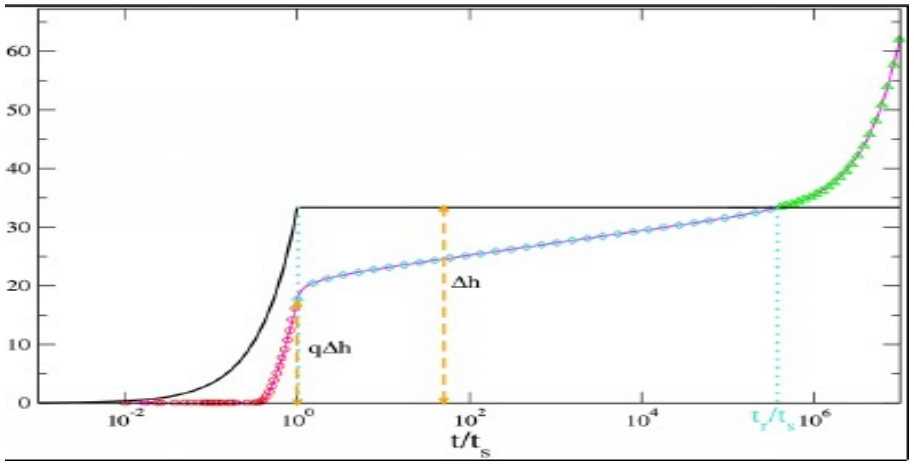
The 2 Layer qEW model

Generalization of the two block model

Two block model (Lippiello et al. 2018)



The slip of h(t) induces the coseismic Slip of the u(t) which subsequent relaxes logarithmically because of velocity strengthening friction

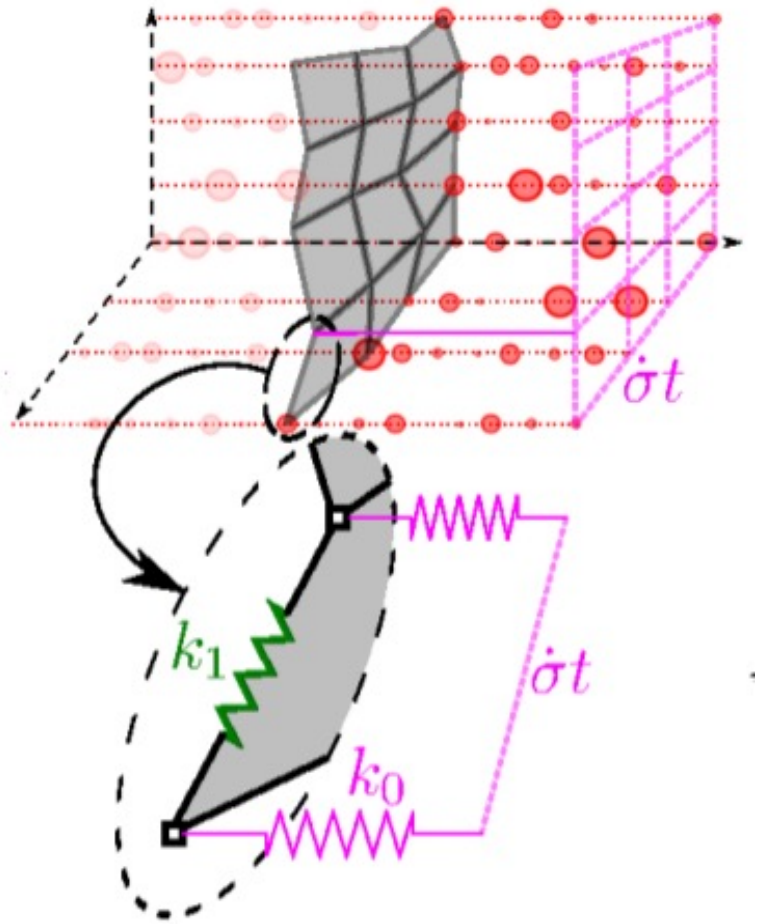


Separation of time scales

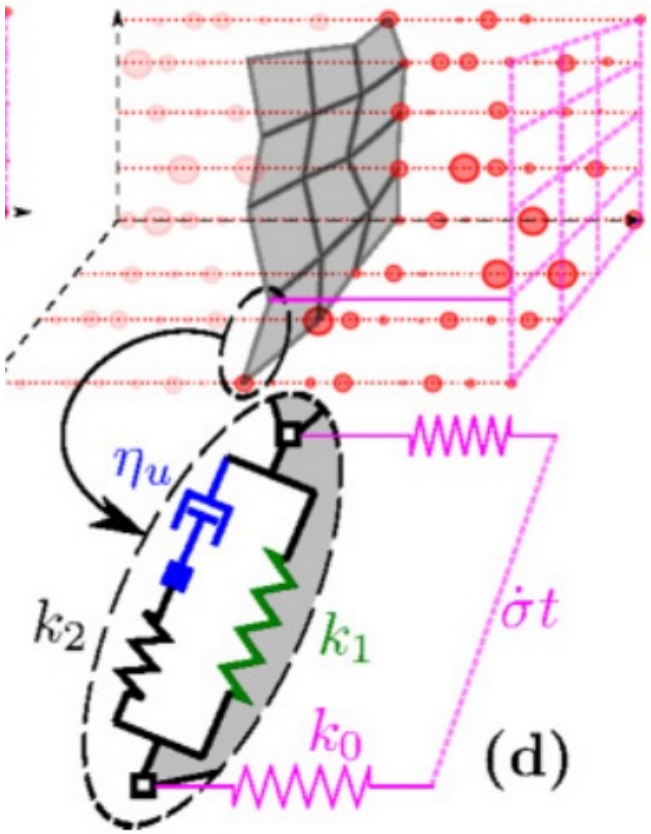
- Strain accumulation rate cm/year
- Post seismic defomation velocity cm/day
- Strain propagation velocity km/sec

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

qEW model



2L qEW model



The Viscoelastic or 2L qEW model

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

2L qEW model

All the dynamics can be described in terms of the local stress value

$$F_i=(1-\theta)F_i^{(fast)}+\theta\, F_i^{(slow)}(t)$$

$F_i^{(fast)}(t)$ is the elastic force caused by usual elastic interaction

$F_i^{(slow)}(t)$ is the «visco-elastic» force (intraplate force) it decays with a given law g(t)

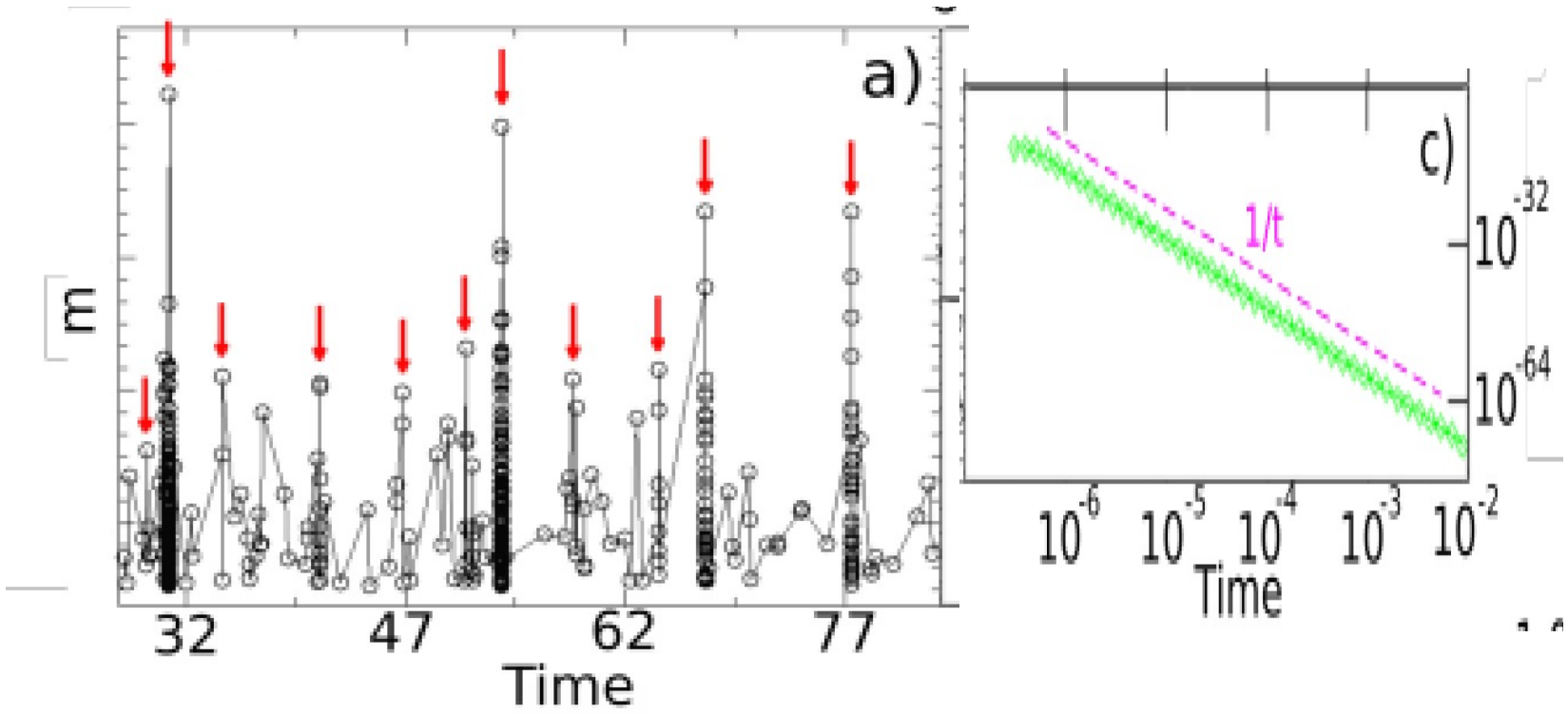
Infinite time separation results are independent of g(t)

Only one parameter θ (coupling between the two layers)

For θ=0 we have the qEW model

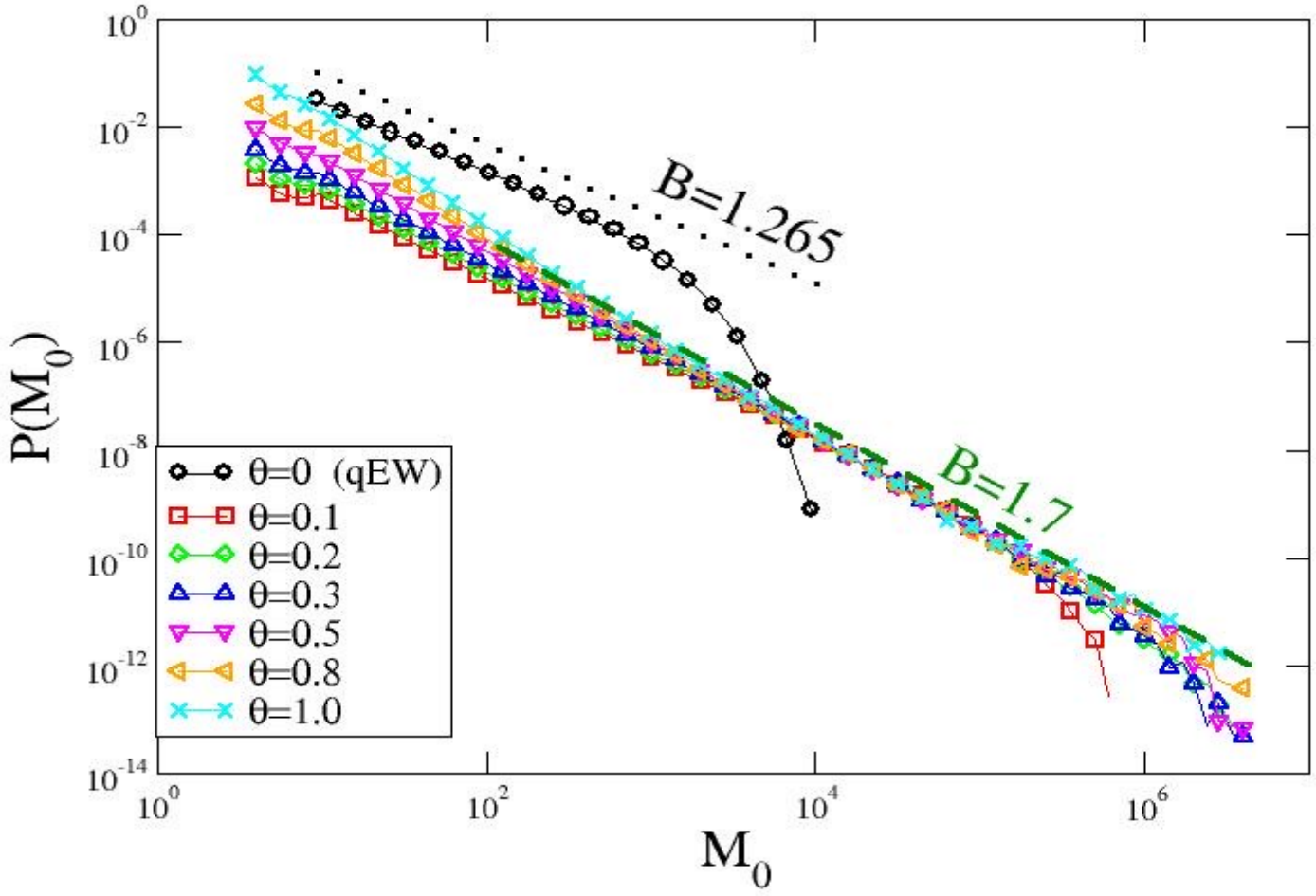
Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

Timing of events in the 2LqEW model



Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

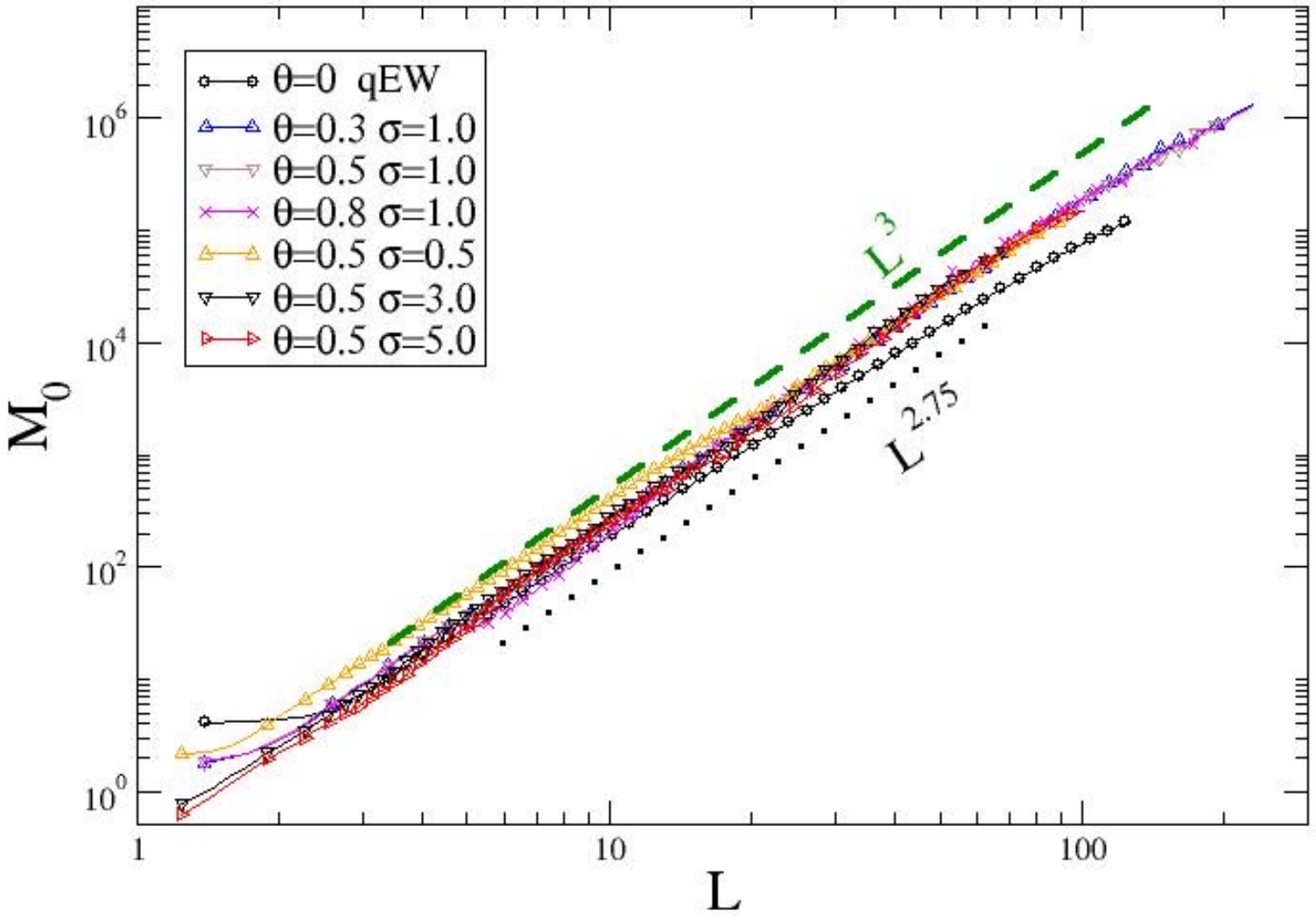
Size distribution in the 2LqEW model



B=1.7 indepedent of θ

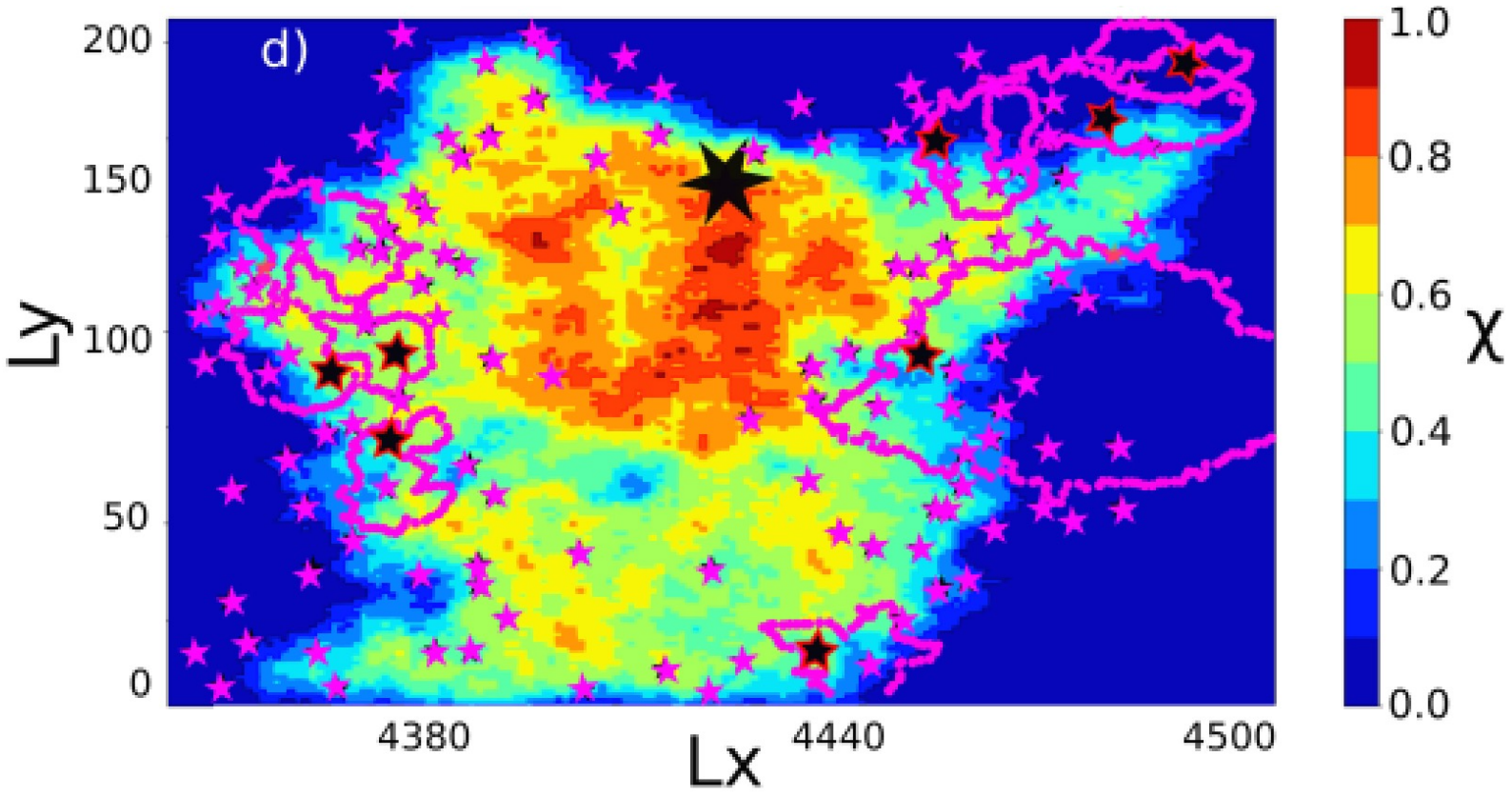
Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

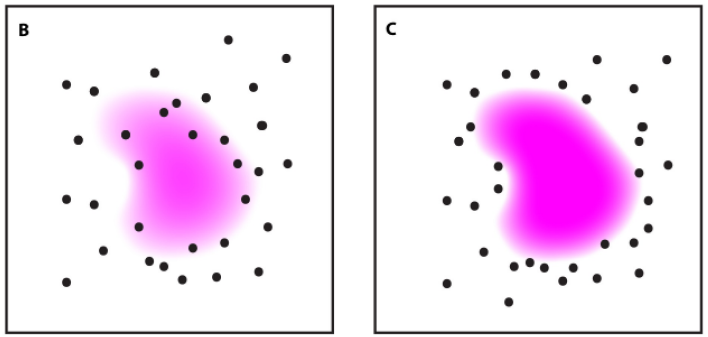
Scaling in the 2LqEW model



Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

Space organization of events in the 2LqEW model





Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

Final Summary

- The sand-pile model does not capture the temporal organization of earthquakes and also solar flares
- The introduction of an intermediate time scale within the model for depinning transitions leads to a new universality class sharing the same exponents of real seismic catalogs
- The new universality class indicates that large earthquakes are NOT intrinsically unpredictable

Dipartimento di Matematica e Fisica
Università degli Studi della Campania Luigi Vanvitelli

Final Summary

"A prince who, moved by a noble heart allows himself to be induced by these misfortunes that touch mankind to remove the misery of war from those who are already threatened on all sides by grave misfortunes, is a beneficent instrument working in the benevolent hands of God and a gift that He bestows on the peoples of the earth whose worth they will never know how to estimate in its greatness"

I. Kant, *Allgemein Naturgeschichte und Theorie des Himmels", 1755*

References Earthquakes

Statistical physics approach to earthquake occurrence and forecasting
L de Arcangelis, C Godano, JR Grasso, E Lippiello
Physics Reports 628, 1-91

The influence of the brittle-ductile transition zone on aftershock and foreshock occurrence
G Petrillo, E Lippiello, FP Landes, A Rosso
Nature communications 11 (1), 1-10 (2020)

References Solar Flares

Universality in solar flare and earthquake occurrenceL de Arcangelis, C Godano, E Lippiello, M Nicodemi
Physical Review Letters 96 (5), 051102

Different triggering mechanisms for solar flares and coronal mass ejections
E Lippiello, L De Arcangelis, C Godano
Astronomy & Astrophysics 488 (2), L29-L32

Time-energy correlations in solar flare occurrence
E Lippiello, L de Arcangelis, C Godano
Astronomy & Astrophysics 511, L2