

# Quantum criticality and out-of-equilibrium scaling in long-range systems and beyond

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- Constructing effective theories of (universal) phenomena-

# Nicolò Defenu's Education and Experience

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A. Born in Rome on 19th May 1988.



B. 11/2010 B.Sc. with honors at La Sapienza University of Rome. Bachelor Thesis Advisor: M. Testa.



C. 10/2012 M.sc. with honors at La Sapienza University of Rome. Master Thesis Advisor: J. Lorenzana.



D. 01/2017 Ph.D. with honors at the International School for Advanced Studies (SISSA), Trieste.  
Ph.D. thesis advisors: S. Ruffo, A. Trombettoni, A. Codello.



E. 12/2016 Post Doc. at Heidelberg University, member of the collaborative research centre ISOQUANT  
on the study of out-of-equilibrium quantum matter.



F. 03/2020 Post Doc. at ETH Zurich, Member of the Institute of theoretical physics (ITP), working in  
collaboration between the condensed matter theory and the mathematical physics groups.



G. 01/2022 Italian Habilitation in theoretical condensed matter (02/B2) and high energy physics (02/A2).



# Constructing Effective (low-energy) Theories

• PA Murthy, ND, et al., *Science* 365, 268-272 (2019).

“Strong” long-range interacting systems

- ND, T Enss, M Kastner, G Morigi, *Phys. Rev. Lett.* 121, 240403 (2018).
- ND, *Proc. Nat. Acad. Sci.* 118, e2101785118 (2021).
- ND, *Comm. Phys.* 4, 150 (2021).

- ND, A Trombettoni, A Codello, *Phys. Rev. E* 92, 052113 (2015)
- ND, A Trombettoni, S Ruffo, *Phys. Rev. B* 94, 224411 (2016)
- ND, A Trombettoni, S Ruffo, *Phys. Rev. B* 96, 104432 (2017).
- ND, T Enss, JC Halimeh, *Phys. Rev. B* 100, 014434 (2019).
- ND, G Morigi, L Dell'Anna, T Enss, *Phys. Rev. B* 100, 184306 (2019).
- M Syed, T Enss, ND, *Phys. Rev. B* 103, 064306 (2021).
- AP Millán, G Gori, F Battiston, T Enss, ND, *Phys. Rev. Res.* 3, 023015 (2021).

Quantum systems with effective fractional dimension

Low dimensional quantum systems

- ND et al., *Phys. Rev. B* 96, 174505 (2017)
  - G Bighin, ND, et al., *Phys. Rev. Lett.* 123, 100601 (2019).
  - W Rządkowski, et al., *New J. Phys.* 22, 093026 (2020).
  - ND, A Trombettoni, D Zappalà, *Nucl. Phys. B* 964, 115295 (2021)
- 
- G Giachetti, ND, S. Ruffo, A. Trombettoni, *Phys. Rev. Lett.* 127, 156801 (2021).

1) “Long-range interacting quantum systems”, ND et al. arXiv:2109.01063, *invited review by Rev. Mod. Phys.*

2) “Out-of-equilibrium dynamics and criticality of quantum systems with long-range interactions”, ND, A. Lerose, S. Pappalardi, *invited review by Phys. Rep.*

# Funded projects and research supervision

## PI of 2 Exploratory Projects (300'000€):

1. Critical behavior of epidemic models on distinct network topologies and applications to the study of brain disease
2. Universality on Network Structures from Quantum Dynamics to Big Data

## PI of SNSF project funding (500'000€):

1. Out-of-equilibrium criticality of long-range interacting quantum systems

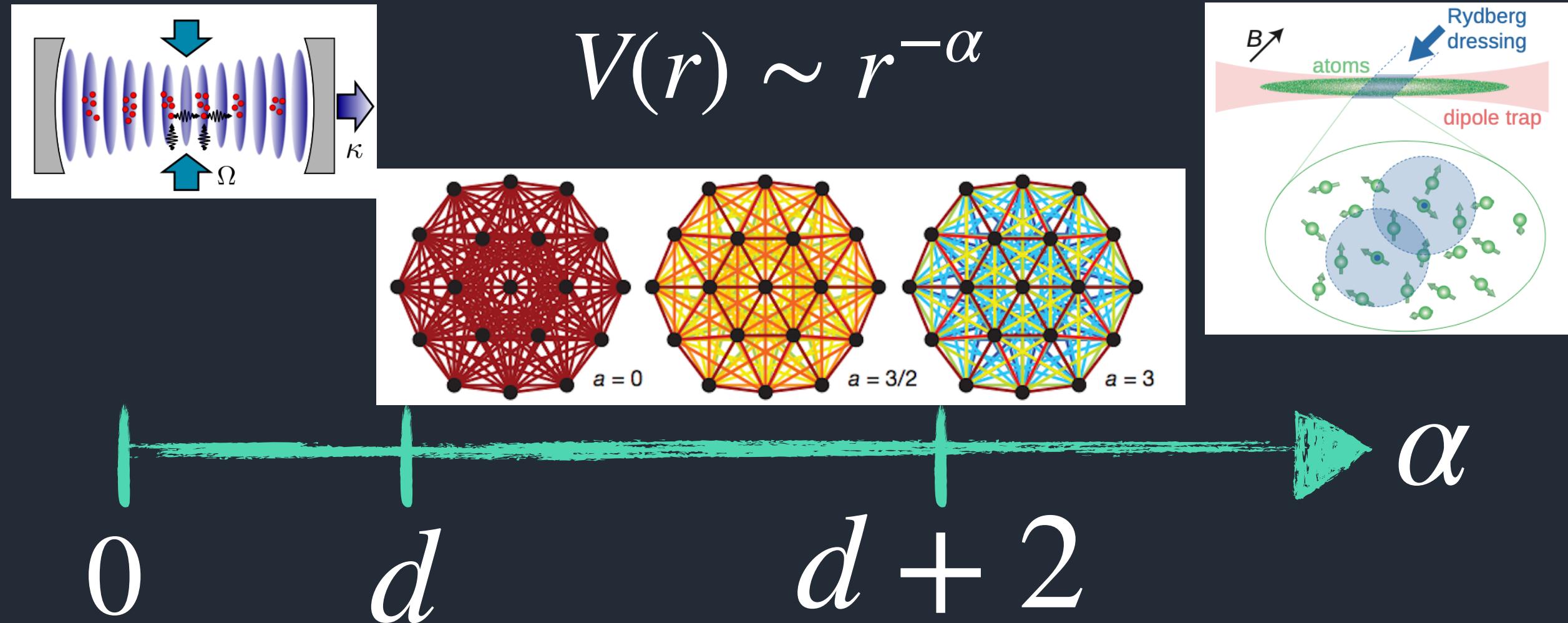
## M. Sc. & PhD Students Supervision

1. Marvin Syed
2. Guido Giachetti
3. Andrea Solfanelli
4. Benjamin Liegeois
5. Ka Rin Sim



1. M. Syed, T. Enss, ND, Phys. Rev. B 103, 064306 (2021)
2. M. Syed, T. Enss, ND, arXiv:2112.14180.
3. G. Giachetti, ND, arXiv:2112.11488.
4. G. Giachetti, A. Solfanelli, ND, *In preparation* (2022).
5. A. Solfanelli et al., *In preparation* (2022).
6. B. Liegeois, R. Chitra, ND, *In preparation* (2022)
7. K. R. Sim, P. Molignini, R. Chitra, ND, *In preparation* (2022)

# Long-range interactions across experiments



1) "Long-range interacting quantum systems", ND et al. arXiv:2109.01063, *invited review by Rev. Mod. Phys.*

2) "Out-of-equilibrium dynamics and criticality of quantum systems with long-range interactions", ND, A. Lerose, S. Pappalardi, *invited review by Phys. Rep.*

# My scientific path

< 6 >



# Long-range quantum Ising model $\alpha < d$

$$H = - \sum_{l < j} J_{ij} \sigma_l^x \sigma_j^x - h \sum_j \sigma_j^z,$$

Long-range interactions:  $J_{ij} = \frac{1}{N_\alpha} \frac{1}{|i-j|^\alpha}$

$$N_\alpha^{-1} \approx \begin{cases} (1-\alpha)2^{(1-\alpha)}N^{\alpha-1} & \text{if } \alpha < 1 \\ 1/\log(N) & \text{if } \alpha = 1 \\ 1/\zeta(\alpha) & \text{if } \alpha > 1. \end{cases}$$

$$J \gg h \Rightarrow |0\rangle \equiv \Pi_i |\uparrow\rangle_i \text{ or } \Pi_i |\downarrow\rangle_i \text{ and } \lim_{|i-j| \rightarrow \infty} \langle \sigma_i^z \sigma_j^z \rangle \propto N_0^2$$

$$J \ll h \Rightarrow |0\rangle \equiv \Pi_i |\rightarrow\rangle_i \text{ with } |\rightarrow\rangle \equiv |\uparrow\rangle + |\downarrow\rangle \text{ and } \lim_{|i-j| \rightarrow \infty} \langle \sigma_i^z \sigma_j^z \rangle \propto e^{-\frac{|i-j|}{\xi}}$$

Quantum Critical Point at  $h_c > 0$

# Quasi-Stationary-States

Initial State

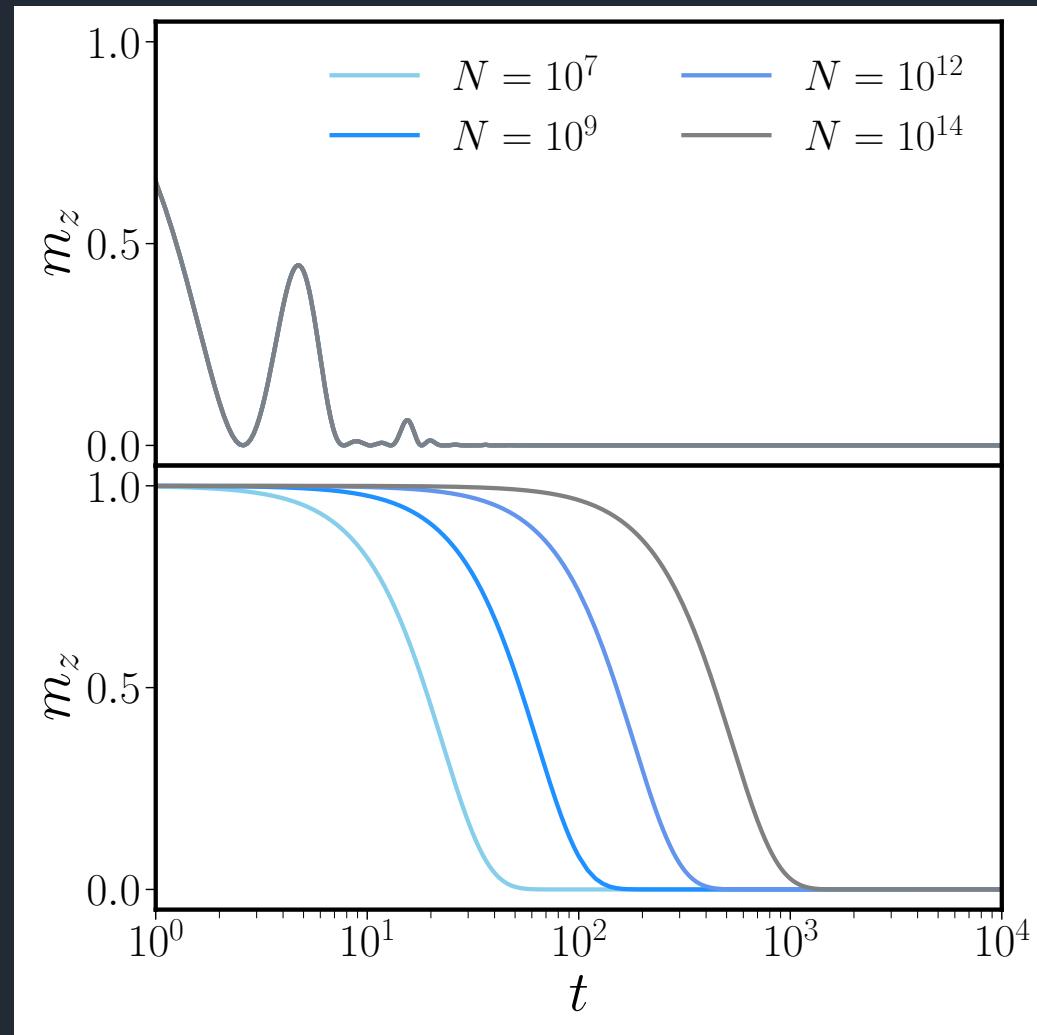
$$m_z = \langle \sum_i \sigma_i^z \rangle / N$$

$$\lim_{t \rightarrow 0} m_z = 1$$

Final Hamiltonian

$$H_f = - \sum_{l < j} J_{lj} \sigma_l^x \sigma_j^x$$

$$\lim_{t \rightarrow \infty} m_z = 0$$

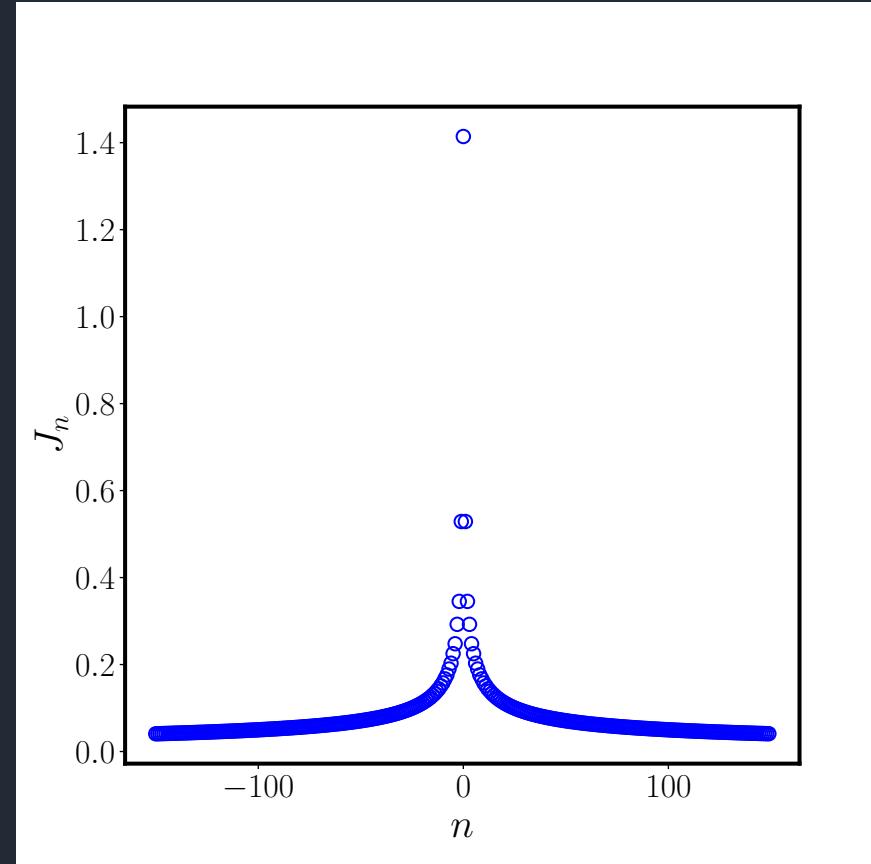


1) “Long-range interacting quantum systems”, ND et al. arXiv:2109.01063, invited review by Rev. Mod. Phys.

# Discrete Spectrum

$$\lim_{N \rightarrow \infty} J_k = \lim_{N \rightarrow \infty} \frac{1}{N_\alpha} \sum_{r=1}^{N/2-1} \frac{\cos(kr)}{r^\alpha} \approx \frac{c_\alpha}{N} \sum_{r=1}^{N/2} \frac{\cos\left(2\pi n \frac{r}{N}\right)}{(r/N)^\alpha} \equiv J_n$$

$$J_n \equiv c_\alpha \int_0^{\frac{1}{2}} \frac{\cos(2\pi n s)}{s^\alpha} ds .$$



# Effective models I: Kitaev Chain

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$$\hat{H}_{\text{KLR}} = - \sum_{i=1}^N \sum_{r=1}^{N/2-1} J_r (\hat{c}_i^\dagger \hat{c}_{i+r} + \hat{c}_i^\dagger \hat{c}_{i+r}^\dagger - \hat{c}_i \hat{c}_{i+r} - \hat{c}_i \hat{c}_{i+r}^\dagger) - h \sum_i (1 - 2\hat{c}_i^\dagger \hat{c}_i).$$

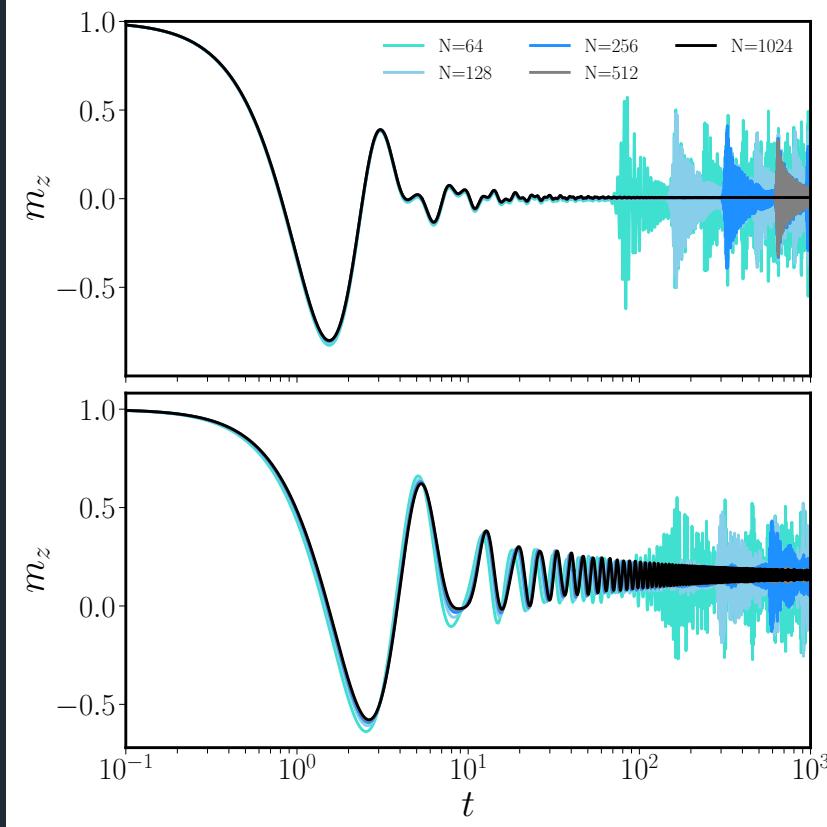
$$\hat{c}_k = u_k \hat{\gamma}_k + v_{-k}^* \hat{\gamma}_{-k}^\dagger \quad \text{with} \quad (u_k, v_k) = \left( \cos \frac{\theta_k}{2}, \sin \frac{\theta_k}{2} \right)$$

$$\hat{m}_z = 1 - \frac{2}{N} \sum_i \hat{c}_i^\dagger \hat{c}_i$$

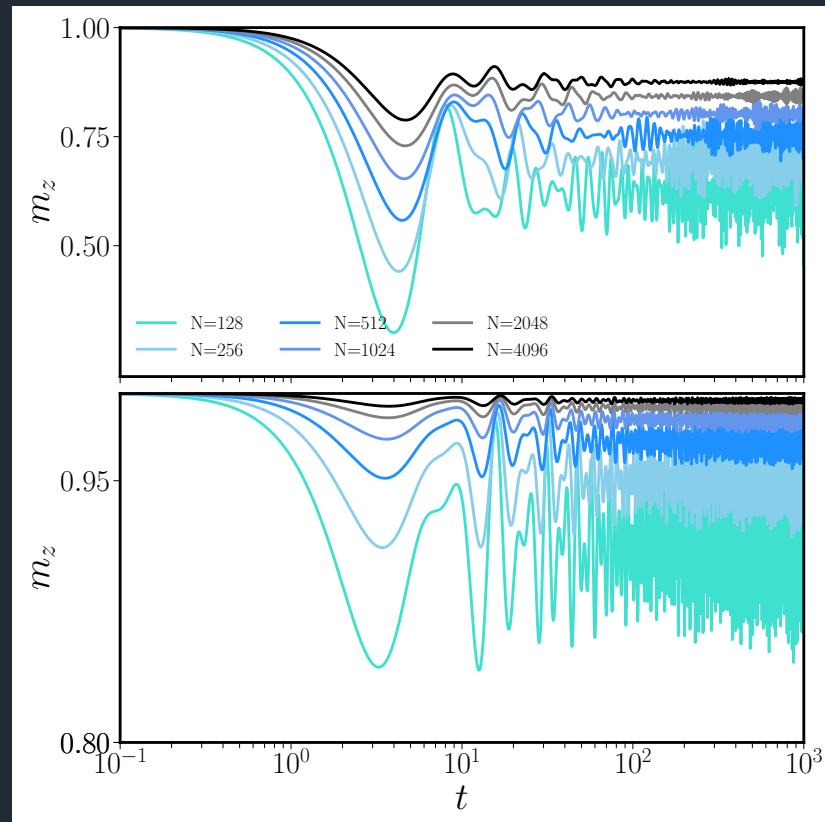
# Quench Dynamics

Quench from the normal phase  $h_i \gg 1$  to the topological phase  
 $h_f < 1$

$$\alpha > 1$$



$$\alpha < 1$$





## RESEARCH ARTICLE

# Metastability and discrete spectrum of long-range systems

Nicolò Defenu

[+ See all authors and affiliations](#)PNAS July 27, 2021 118 (30) e2101785118; <https://doi.org/10.1073/pnas.2101785118>

Edited by Giorgio Parisi, Università degli Studi di Roma La Sapienza, Rome, Italy, and approved May 24, 2021 (received for review January 29, 2021)

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## Significance

The linear dynamics of closed quantum systems produce well-known difficulties in the definition of quantum chaos. This leads to several issues in the theoretical justification of the equilibration and thermalization dynamics observed in closed experimental systems. In the case of large harmonic baths these issues are partially resolved due to the continuous nature of the spectrum, which produces divergent Poincaré recurrence times. As is shown in the following, such a scenario does not apply to long-range interacting models, whose spectrum remains discrete up to the thermodynamic limit, in contradiction with the textbook description of infinitely extended systems.

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## ARTICLE CLASSIFICATIONS

Physical Sciences » Physics

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Long-range interactions:

$$J_{ij} = \frac{1}{N_\alpha} \frac{1}{|i-j|^\alpha}$$
$$N_\alpha^{-1} \approx \begin{cases} (1-\alpha)2^{(1-\alpha)}N^{\alpha-1} & \text{if } \alpha < 1 \\ 1/\log(N) & \text{if } \alpha = 1 \\ 1/\zeta(\alpha) & \text{if } \alpha > 1. \end{cases}$$

$$J \gg h \Rightarrow |0\rangle \equiv \Pi_i |\uparrow\rangle_i \text{ or } \Pi_i |\downarrow\rangle_i \text{ and } \lim_{|i-j| \rightarrow \infty} \langle \sigma_i^z \sigma_j^z \rangle \propto N_0^2$$

$$J \ll h \Rightarrow |0\rangle \equiv \Pi_i |\rightarrow\rangle_i \text{ with } |\rightarrow\rangle \equiv |\uparrow\rangle + |\downarrow\rangle \text{ and } \lim_{|i-j| \rightarrow \infty} \langle \sigma_i^z \sigma_j^z \rangle \propto e^{-\frac{|i-j|}{\xi}}$$

Quantum Critical Point at  $h_c > 0$

# A new world, full of treasures

## Quantum Finite Range

Tunable  
defect  
formation

## Classical Finite Range

Entanglement Scaling  
in Quantum Circuits



Non-ergodicity

## Quantum Flat ( $\alpha = 0$ )

Open Dynamics of  
Quasi-Stationary States

## Classical Long-Range

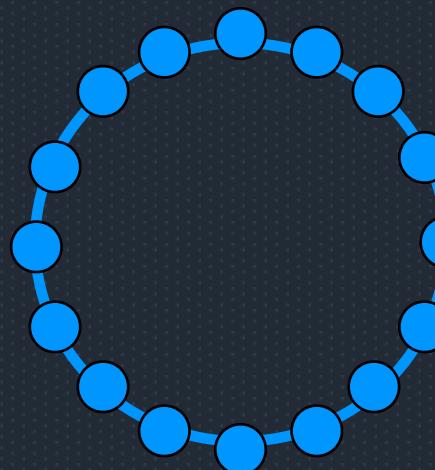
# New tools are needed

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Theoretical understanding is limited as the scaling of couplings becomes super-extensive

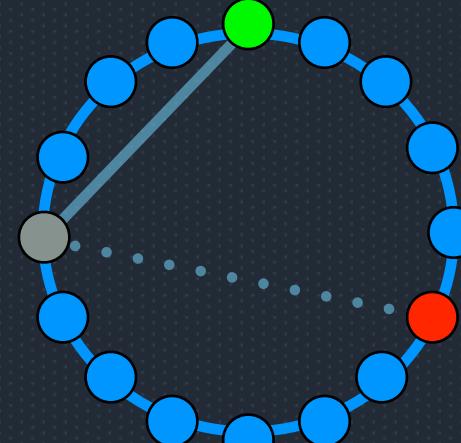
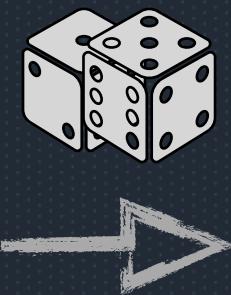
All to all couplings severely reduce numerical performance

# My solution



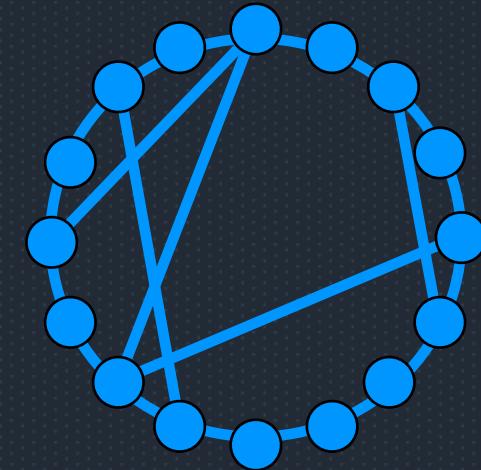
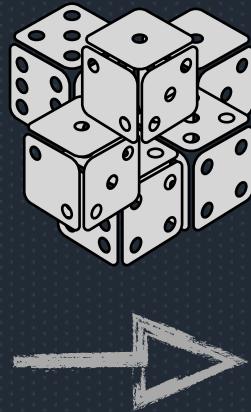
Regular lattice  
in  $d$ -dimension

Additional random  
bonds



$$p \sim \frac{1}{r_{ij}^\alpha}$$

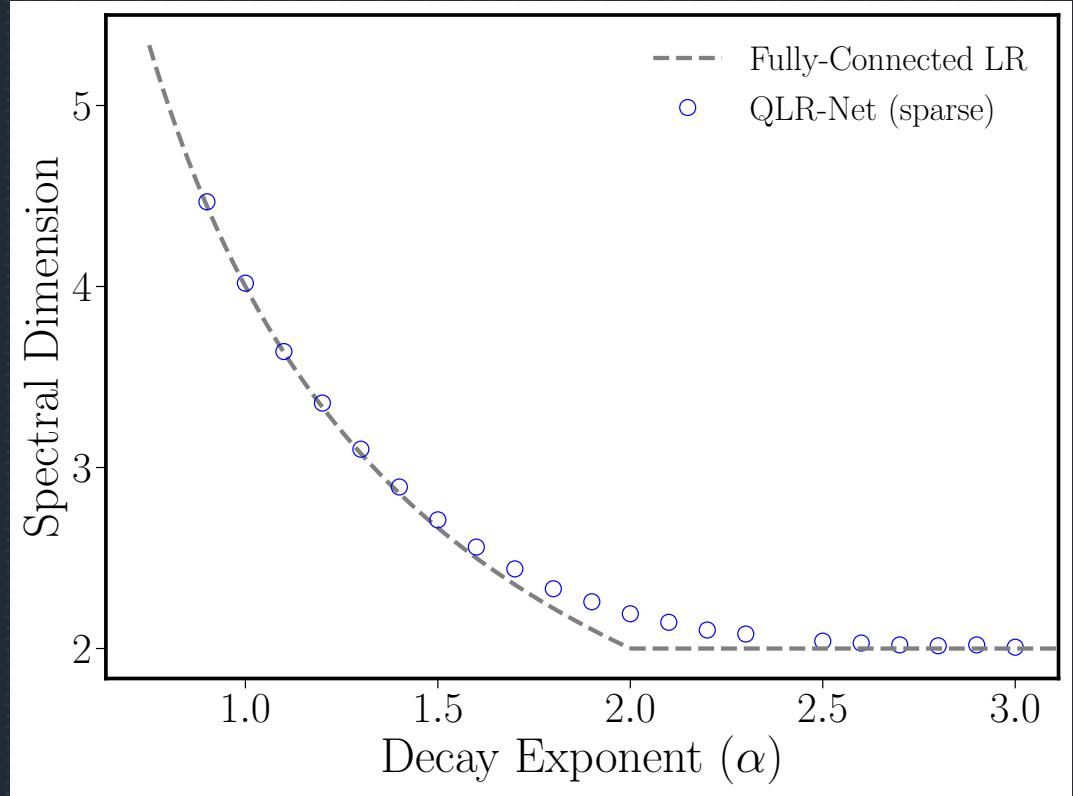
$\sim N$  new bonds



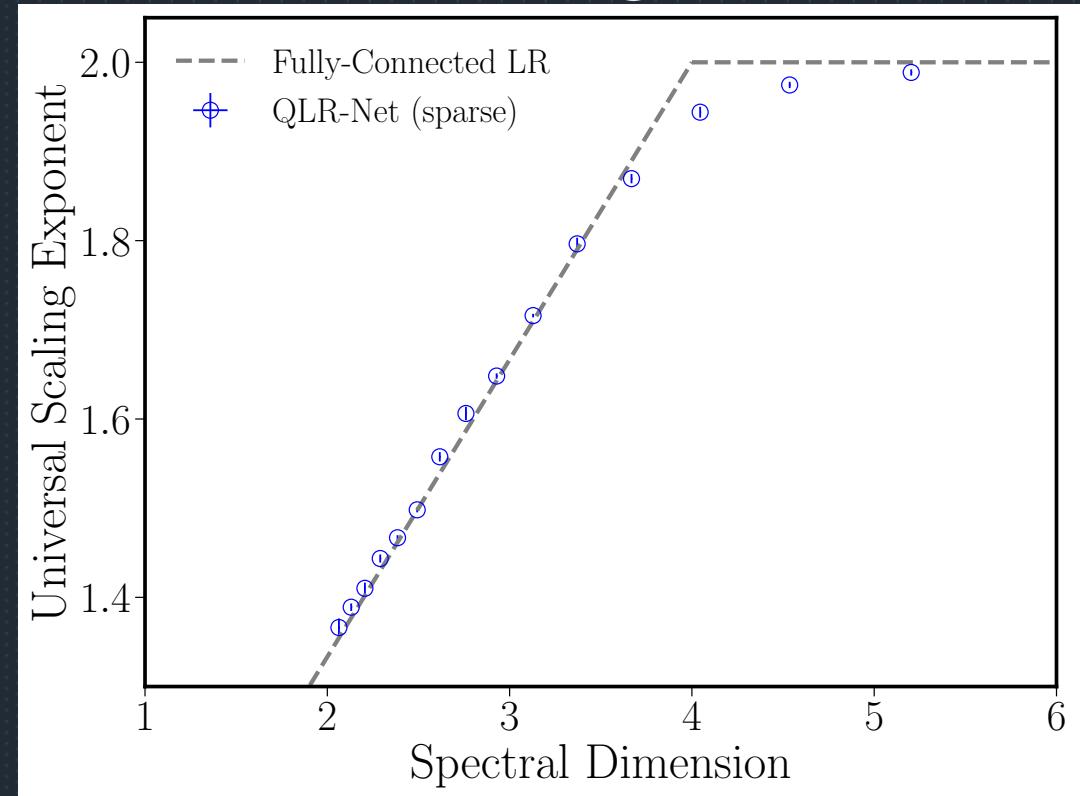
Self-averaging graph  
structure

# The universality conjecture

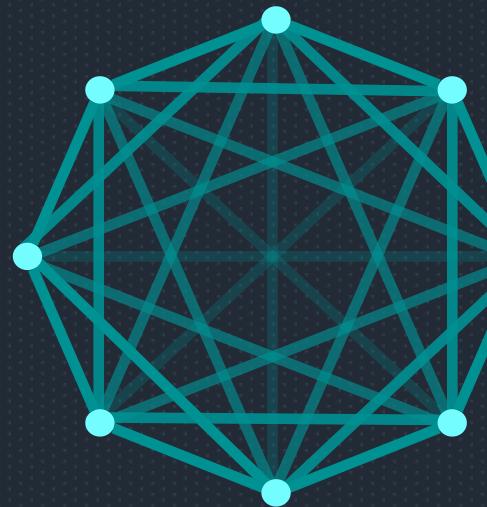
Match the low-energy spectra



Reproduce the universal scaling

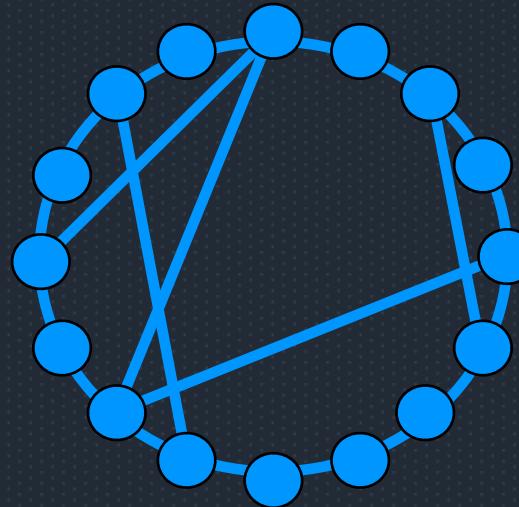


# My goal



Fully connected

VS



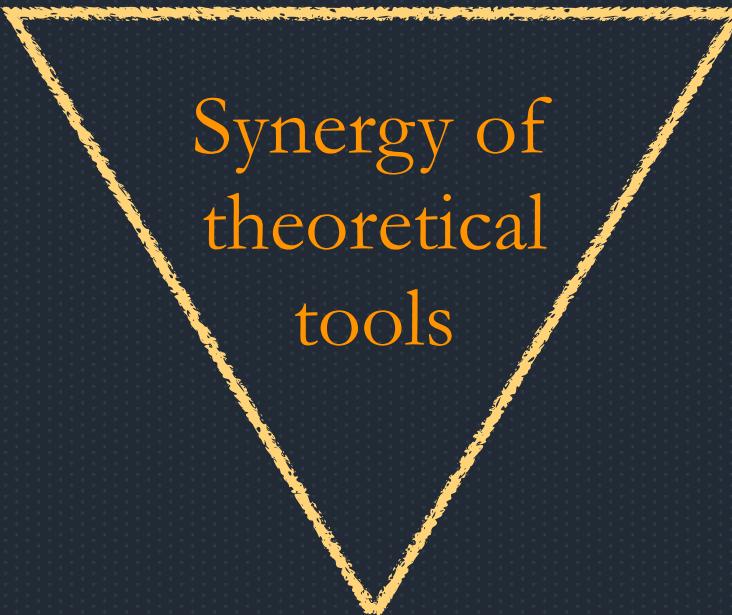
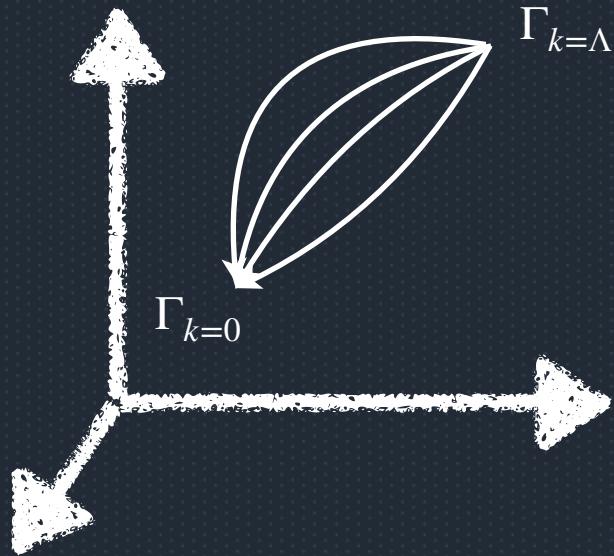
Sparse

Advantages:

- Fast entanglement spreading inherited by long-range couplings
- Amenable numerics thanks to sparse coupling matrices
- Universal tool to describe long-range interacting systems

# Methodology

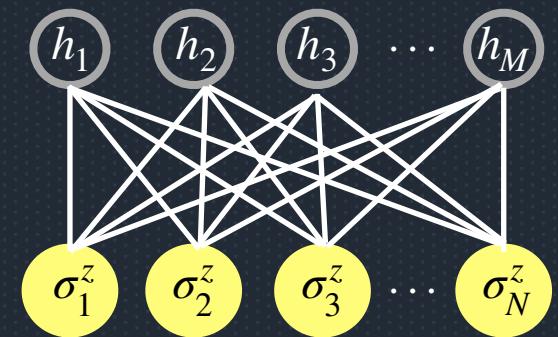
(Out-of-equilibrium)  
RG study at  $\alpha > d$



(Variational) Quantum MC  
simulations at all  $\alpha$

$1/N$ -expansion at  $\alpha < d$

$$= + \text{[Diagram]} + \text{[Diagram]} =$$



# Entering a new era of quantum information

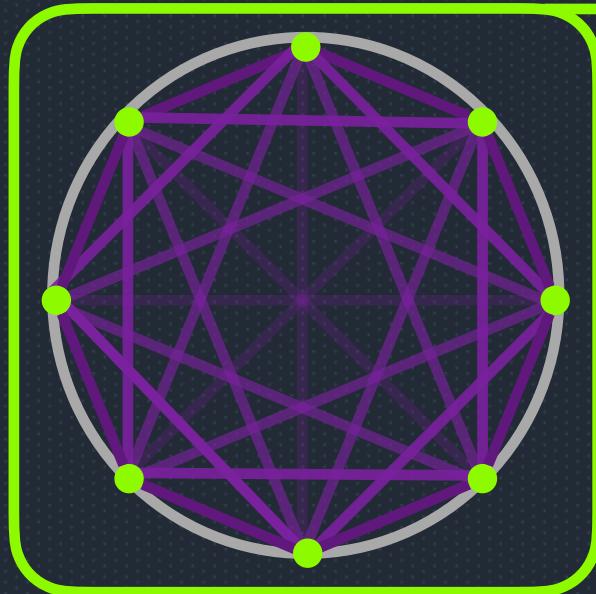
Manipulate fundamental physics laws via complex geometric structures

Tunable Spectrum



Strong Interactions

Novel fundamental  
physics phenomena



Novel proposals for  
Quantum Technologies

ERC-Stg rating: A (Excellent) (2022)