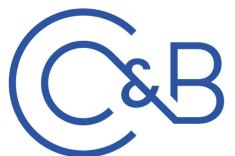


Statistical Physics of Avalanche Phenomena

CRACKLING NOISE

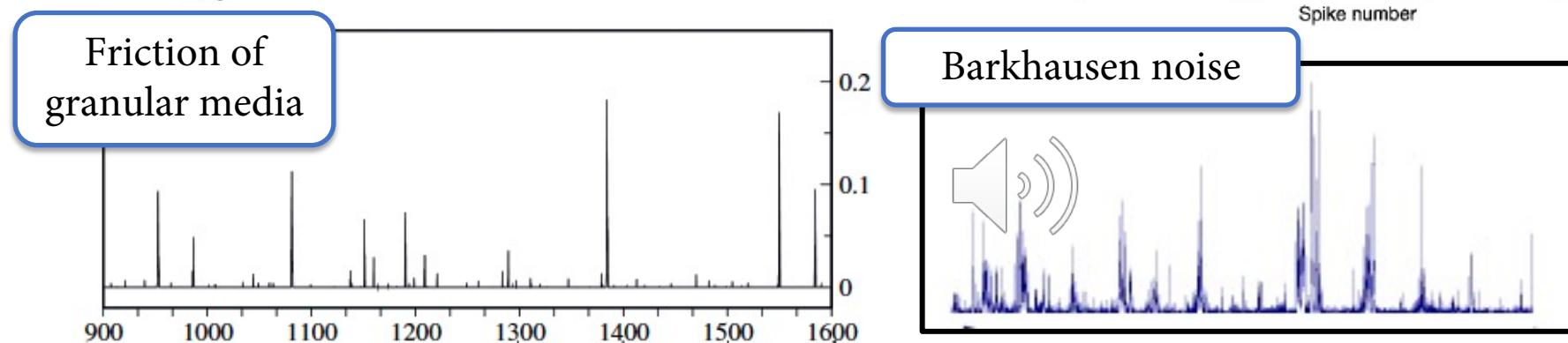
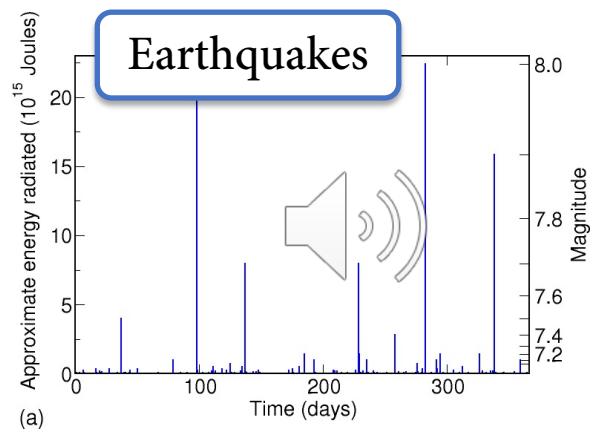
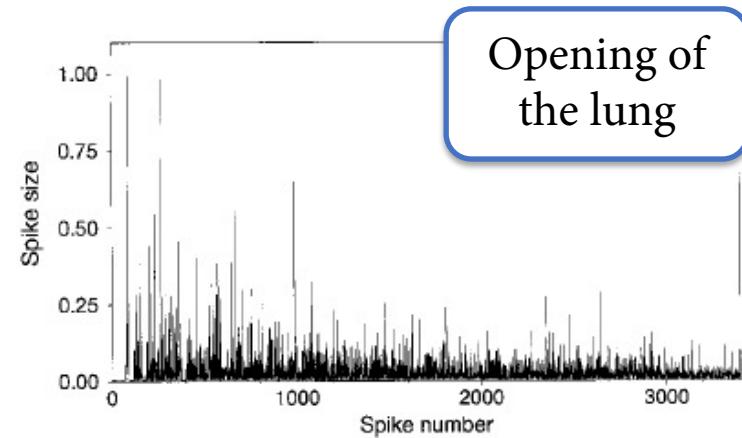
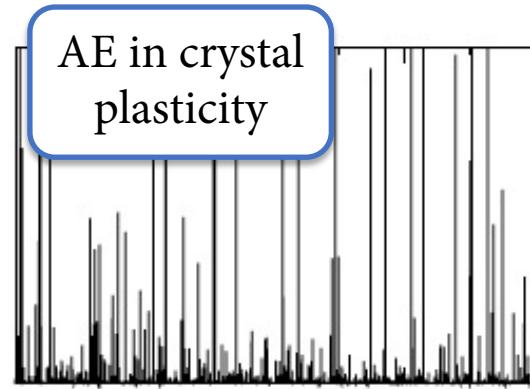
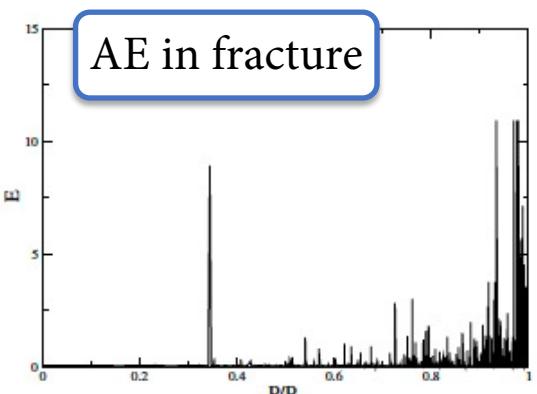


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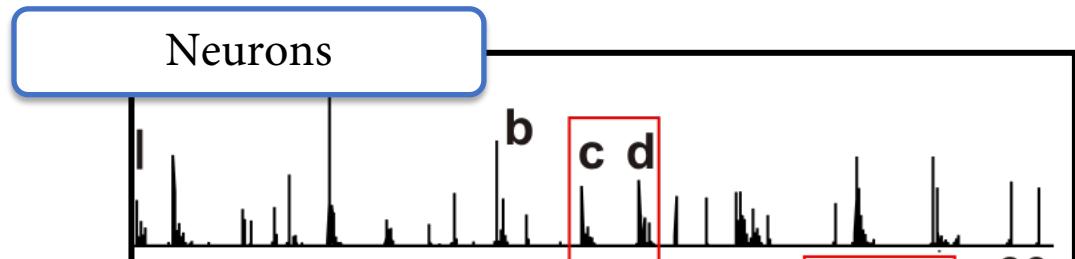
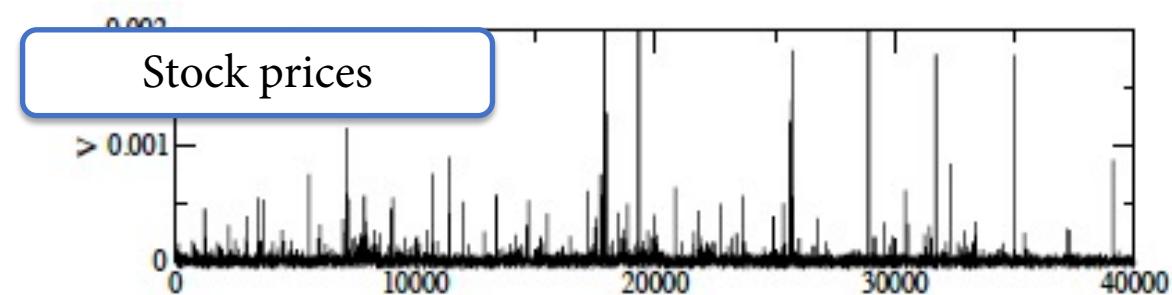
07 02 2023
IFPU focus week on
Dynamical Complexity in Astrophysics
Trieste

STEFANO ZAPPERI
University of Milan

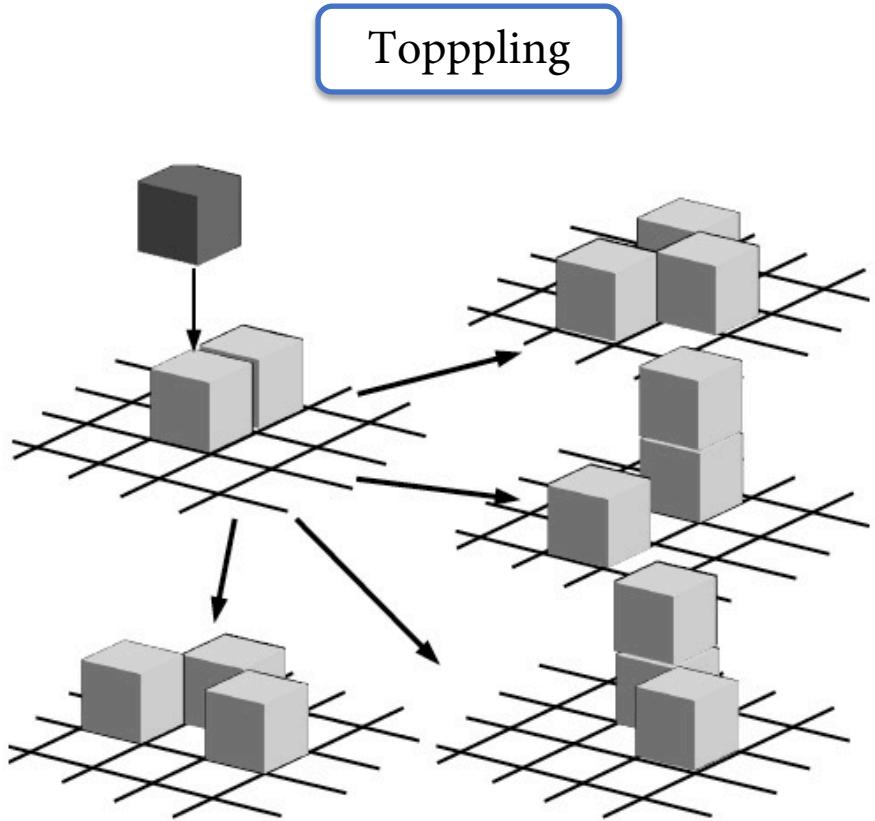
EXAMPLES OF CRACKLING NOISE



$$P(s) \sim s^{-\tau}$$



SANDPILE MODELS



Grains added when system is stable:

$$z_i \rightarrow z_i + 1$$

When: $z_i \geq z_c$



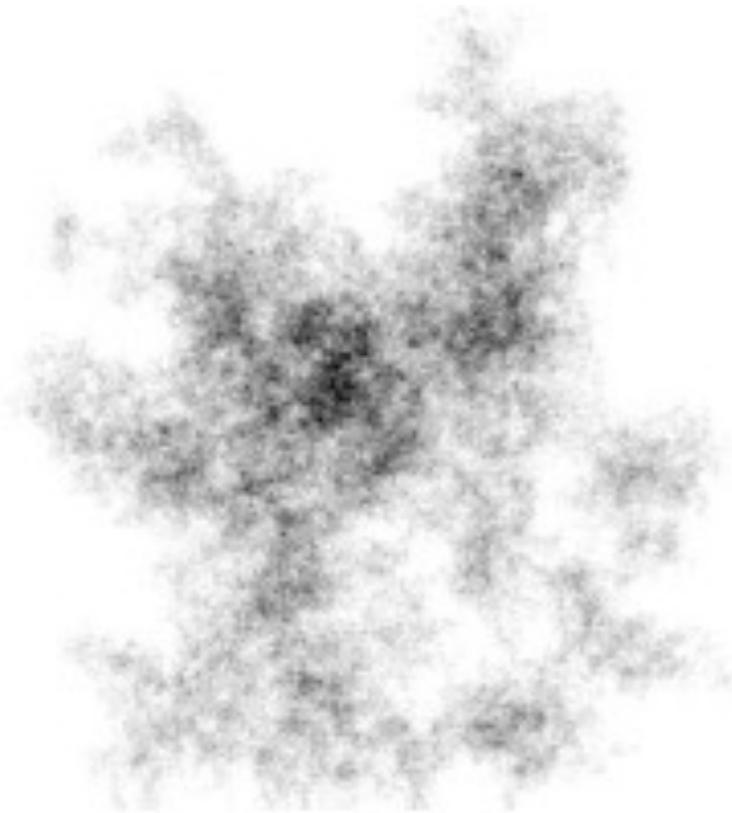
$$z_i \rightarrow z_i - z_c$$

$$z_{i+nn} \rightarrow z_{i+nn} + 1$$



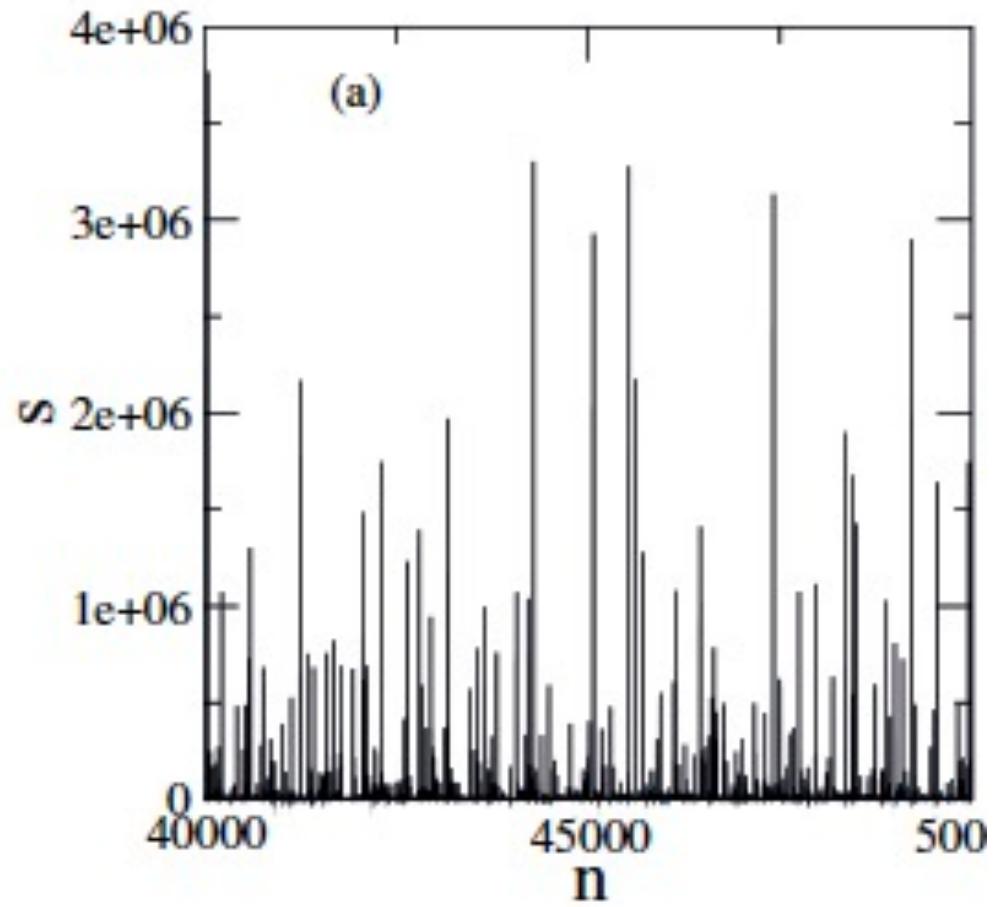
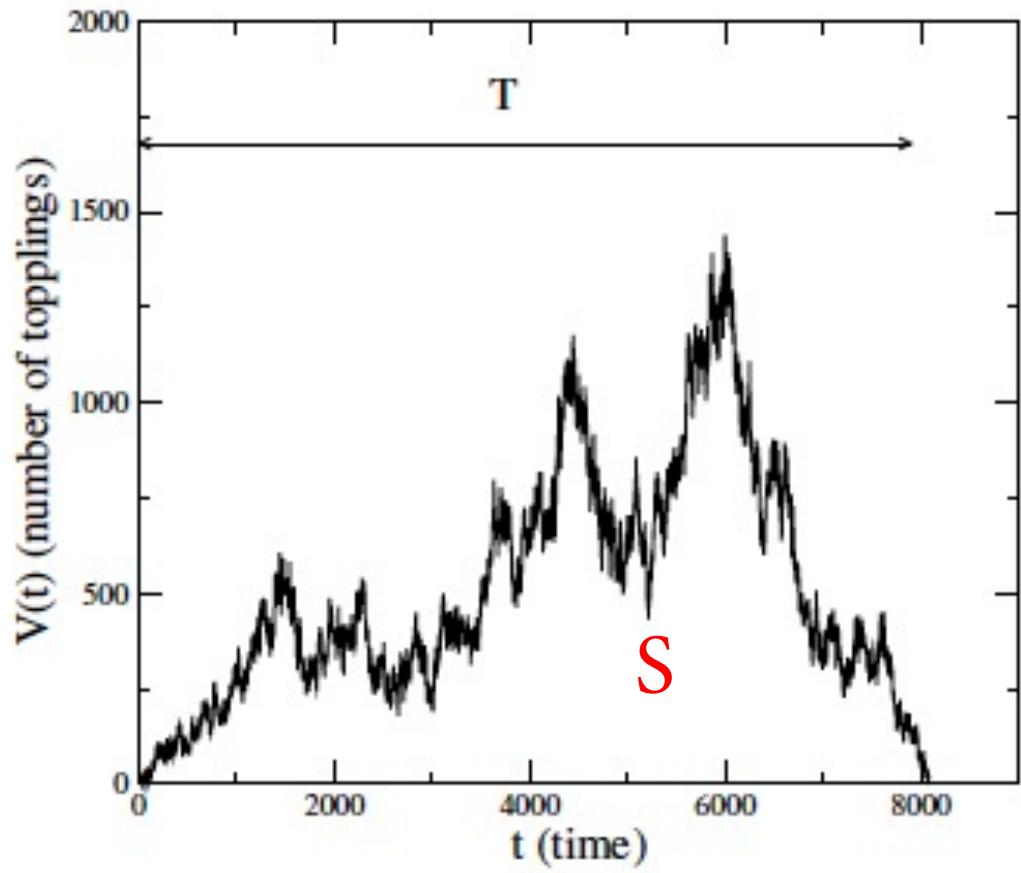
Avalanche

Grains exit from boundaries

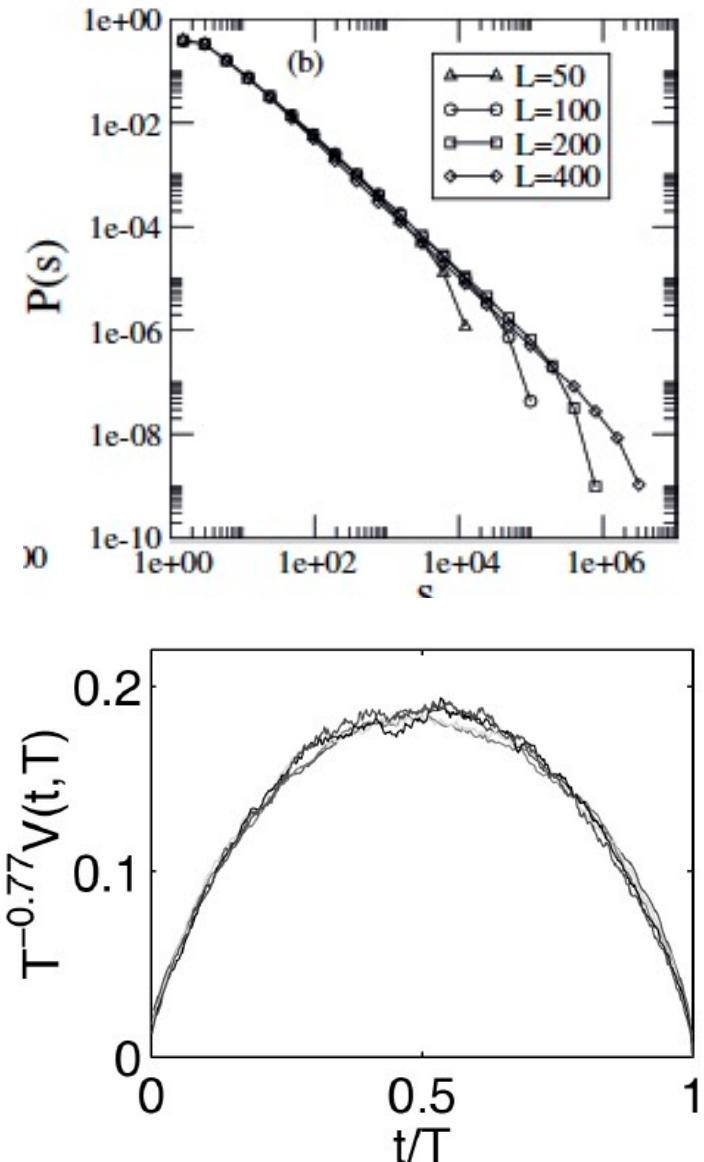


Stochastic vs Abelian
sandpiles

AVALANCHES



UNIVERSAL SCALING



$$P(s, L) = s^{-\tau} f(s/L^D)$$

$$P(T, L) = T^{-\alpha} g(T/L^\Delta)$$

$$\langle s(T) \rangle \propto T^{\gamma_{sT}} \quad \alpha = 1 + \gamma_{sT}(\tau - 1)$$

$$\langle F(\omega) \rangle \sim \omega^{-\gamma_{sT}} \quad \text{Sethna et al. (2001)}$$

$$\langle V(t, T) \rangle = T^{\gamma_{sT}-1} \mathcal{V}(t/T)$$

Exponents and scaling functions should be
UNIVERSAL

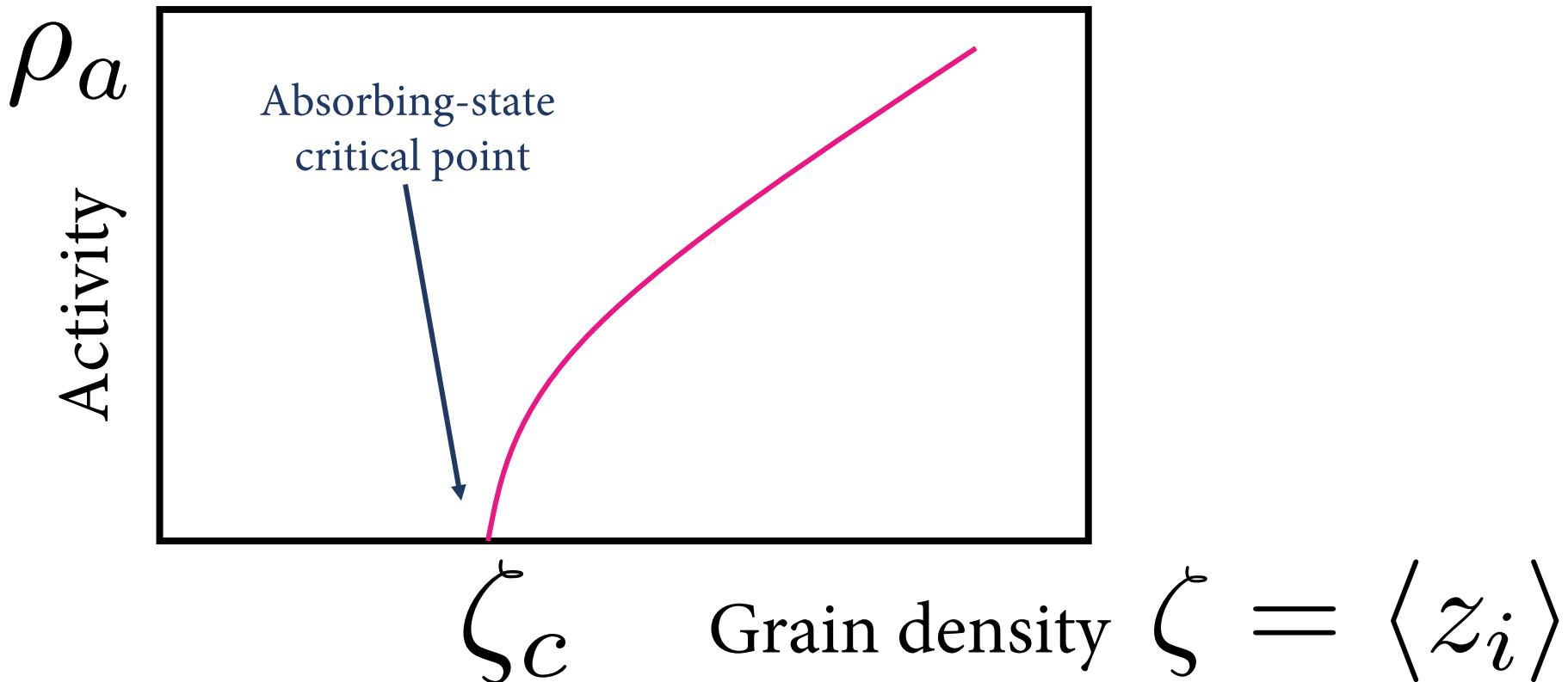
QUESTIONS

- WHAT KIND OF CRITICAL POINT?
- WHY/WHEN IT IS CRITICAL?
- IS IT RELEVANT FOR REAL CRACKLING NOISE?

CONSTANT-DENSITY SANDPILES

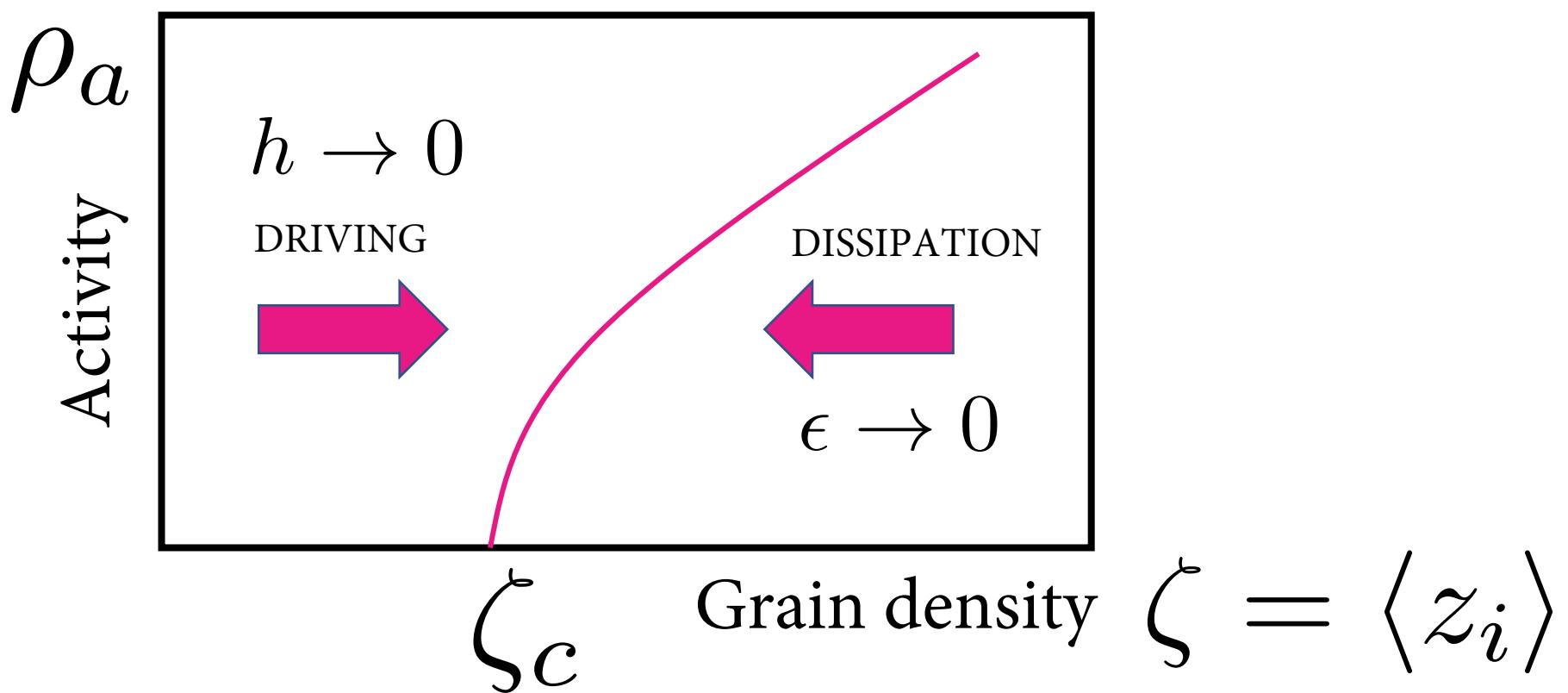
Driving rate (added of grains per unit time): $h = 0$

Dissipation rate (grains lost per unit time): $\epsilon = 0$



SELF-ORGANIZATION TO CRITICALITY

$$\frac{d\zeta}{dt} = h - \epsilon \rho_a + \nabla \rho_a \quad \xrightarrow{\hspace{1cm}} \quad \langle \rho_a \rangle = \frac{h}{\epsilon}$$



FIELD THEORY

Vespignani A, Dickman R, Munoz MA, Zapperi S. Driving, conservation, and absorbing states in sandpiles. Physical review letters. 1998 Dec 21;81(25):5676.

$$\begin{aligned}\frac{\partial \rho_a(\mathbf{x}, t)}{\partial t} = & D_a \nabla^2 \rho_a(\mathbf{x}, t) - r(\mathbf{x}) \rho_a(\mathbf{x}, t) - b \rho_a^2(\mathbf{x}, t) \\ & + w \rho_a(\mathbf{x}, t) \int_0^t dt' \nabla^2 \rho_a(\mathbf{x}, t') + \sqrt{\rho_a} \eta(\mathbf{x}, t).\end{aligned}$$

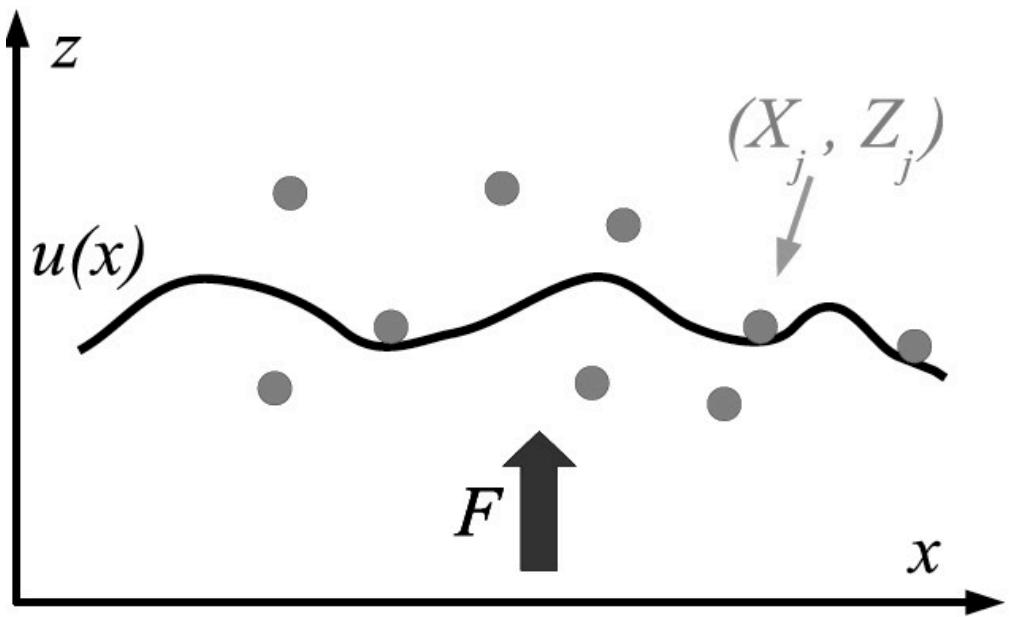
FUNCTIONAL RENORMALIZATION GROUP

Le Doussal P, Wiese KJ. Exact mapping of the stochastic field theory for Manna sandpiles to interfaces in random media. Physical review letters. 2015 Mar 18;114(11):110601.

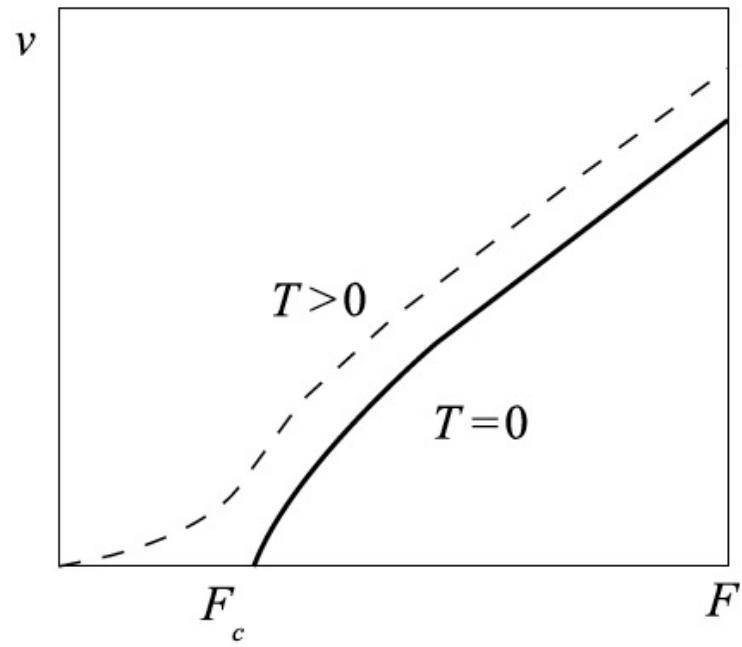
$d_c = 4$ ε -expansion available for critical exponents

DEPINNING TRANSITION

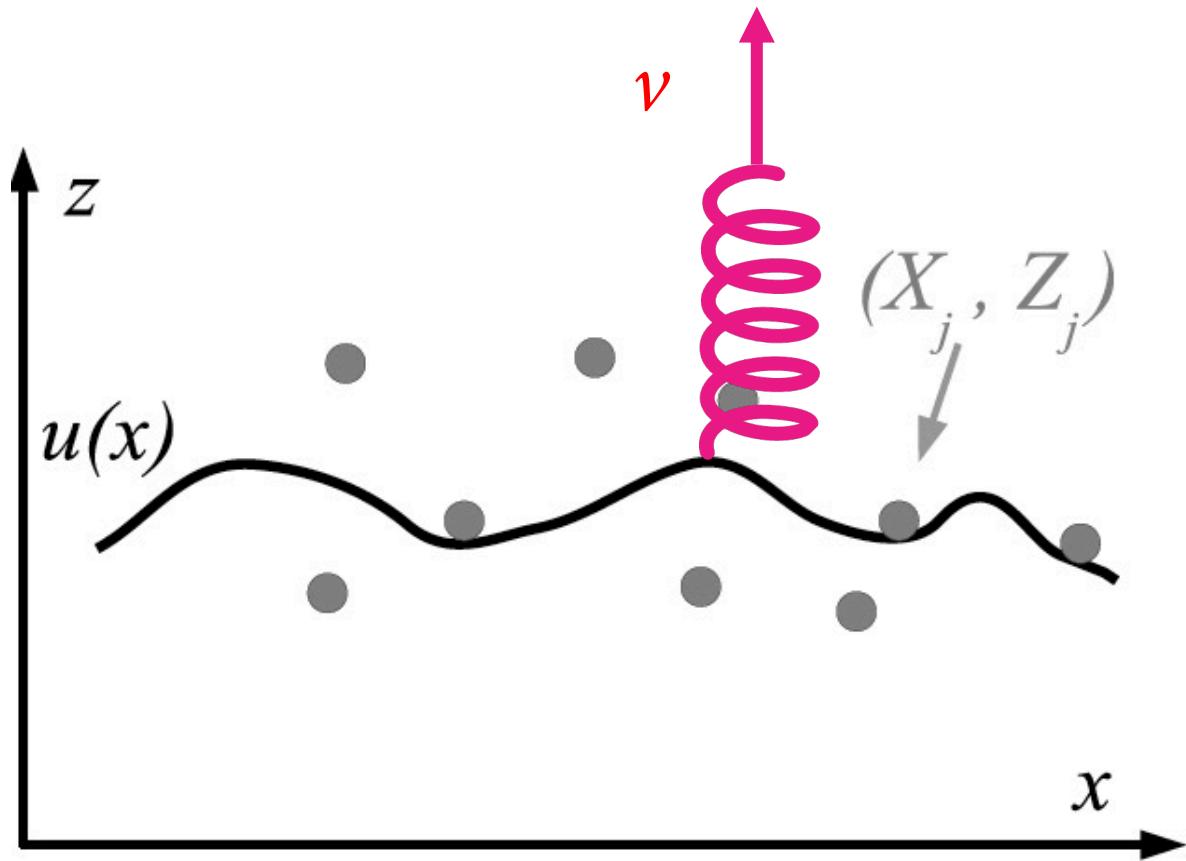
$$\frac{du(x, t)}{dt} = F + \int dx' K(x - x')(u(x', t) - u(x, t)) + \eta(x, u) + f(x, t)$$



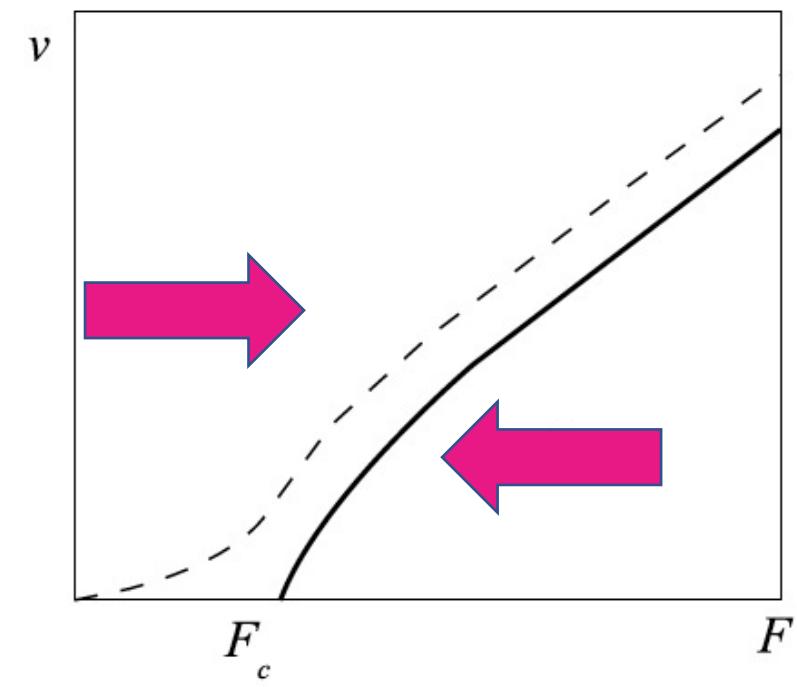
Quenched disorder Thermal noise



"SELF-ORGANIZED" DEPINNING



$$F = k(vt - u(x, t))$$



CLASSES OF CRACKLING NOISE

A. Tuned criticality:

$$F = \text{const}$$

$$P(s, F) = s^{-\tau} f(s(F_c - F)^{1/\sigma})$$

B. Self-organized:

$$F = k(vt - u(x, t))$$

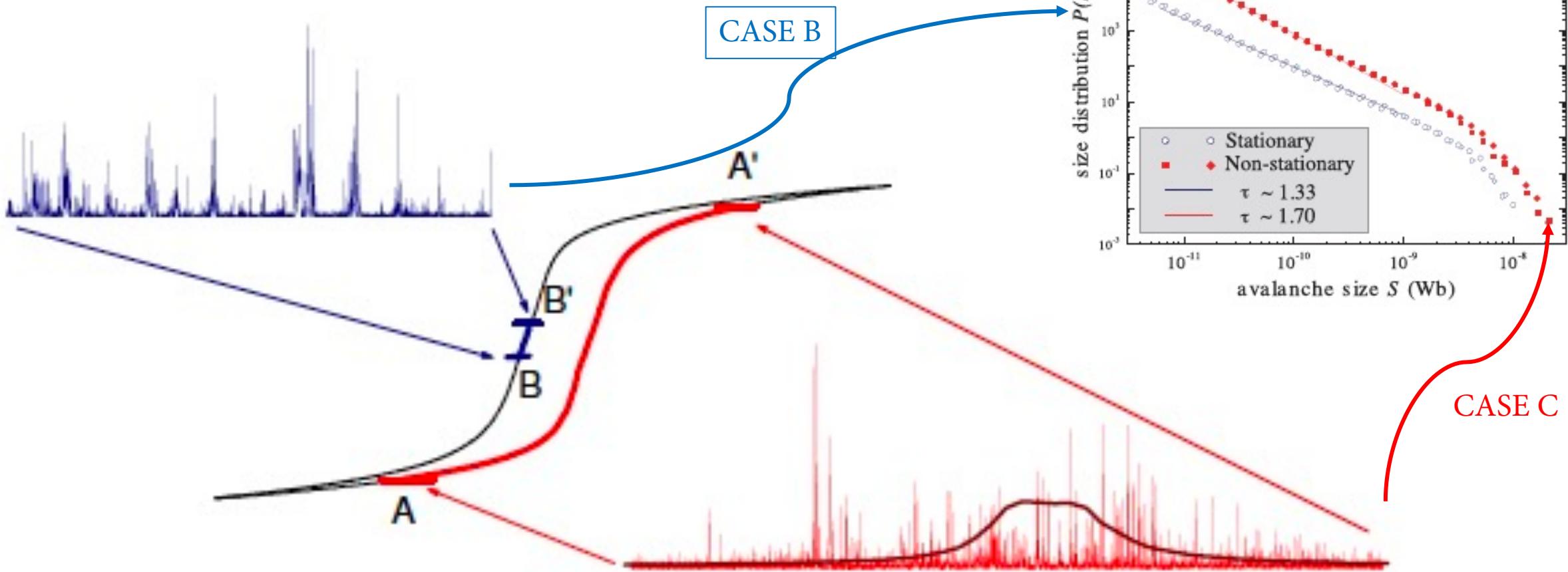
$$P(s, k) = s^{-\tau} f(sk^{1/\sigma_k})$$

C. Swept towards critical point:

$$F = Ct$$

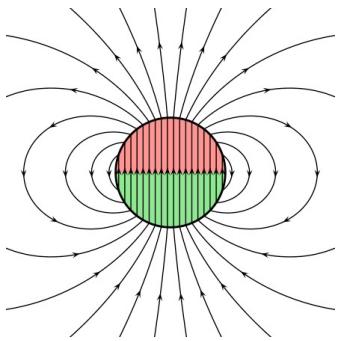
$$P(s) = \int dF s^{-\tau} f(s(F_c - F)^{1/\sigma}) \sim s^{-(\tau+\sigma)}$$

BARKHAUSEN NOISE



with Gianfranco Durin

BARKHAUSEN NOISE



Demagnetizing field

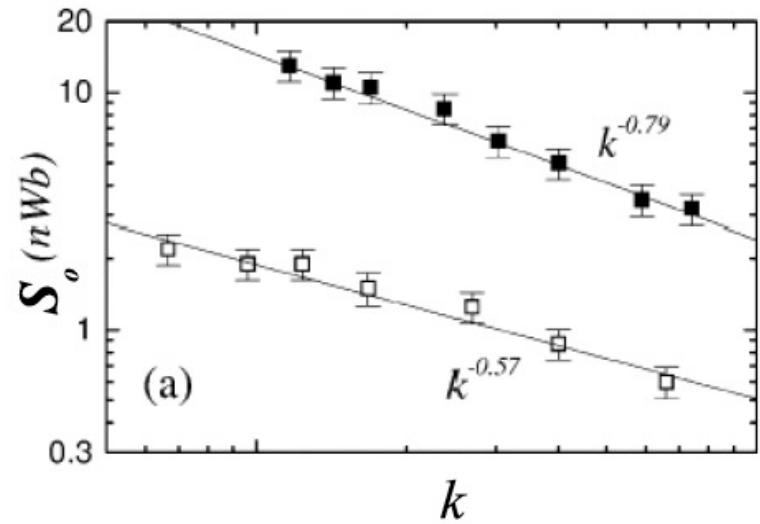
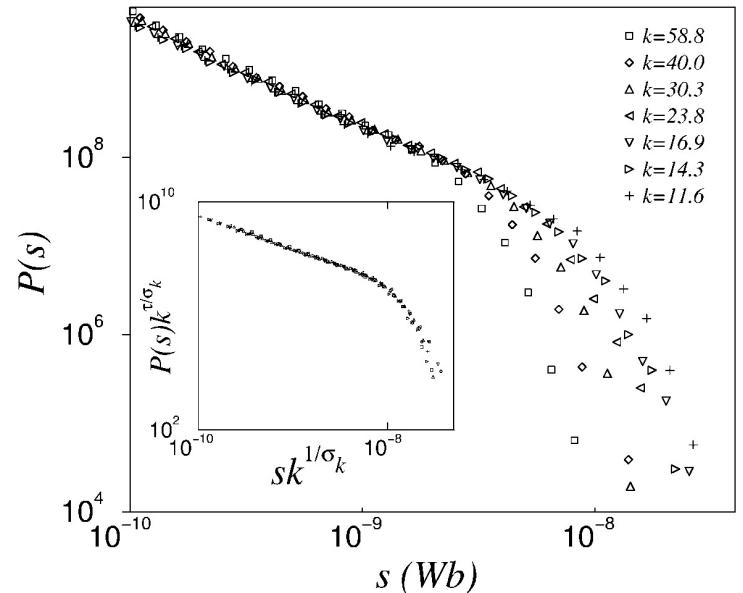
$$H_{eff} = H(t) - k_d M$$

$$\frac{\partial u(\mathbf{r}, t)}{\partial t} = H - \bar{k}\tilde{u} + \gamma_W \nabla^2 u(\mathbf{r}, t) +$$

$$\int d^2 r' K(\mathbf{r} - \mathbf{r}') (u(\mathbf{r}') - u(\mathbf{r})) + \eta(\mathbf{r}, h)$$

$$\bar{k} \equiv 4\mu_0 k M_s^2 / V \quad K(r) = \mu_0 M_s^2 \frac{1 - 3(\cos \theta)^2}{r^3}$$

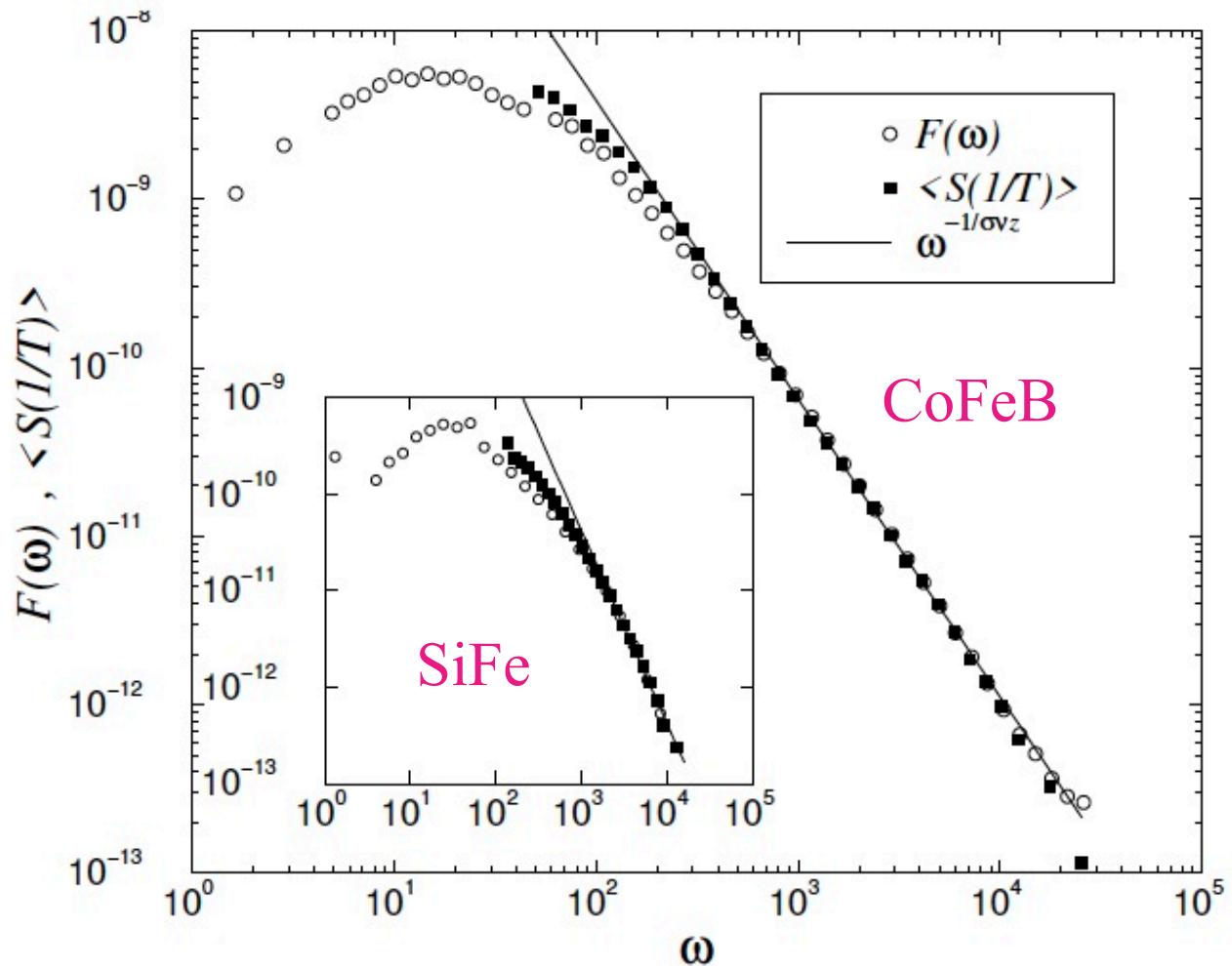
with Gianfranco Durin



BARKHAUSEN NOISE: POWER SPECTRUM

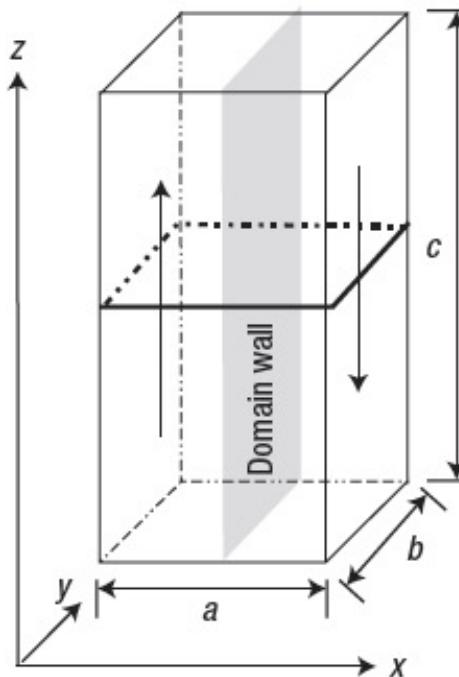
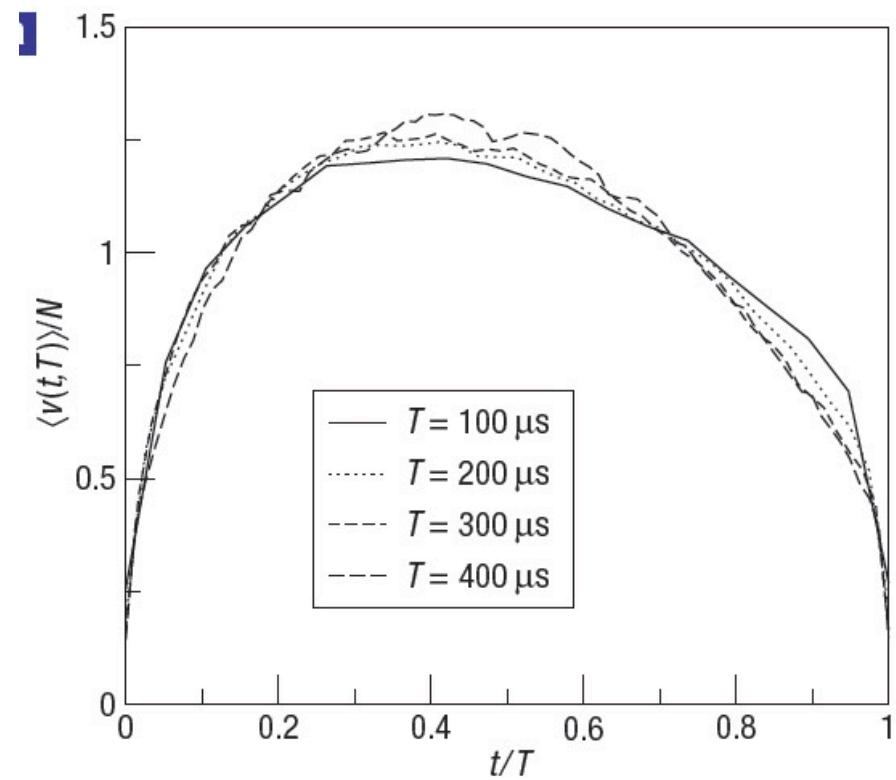
$$\langle s(T) \rangle \propto T^{\gamma_{sT}}$$

$$\langle F(\omega) \rangle \sim \omega^{-\gamma_{sT}}$$



with Gianfranco Durin

BARKHAUSEN NOISE: «NEGATIVE» MASS



$$\sigma\mu \frac{\partial H_e}{\partial t} - \nabla^2 H_e = 0. \quad \text{Eddy currents}$$

$$\frac{\partial H_e(0^+, y, t)}{\partial x} - \frac{\partial H_e(0^-, y, t)}{\partial x} = 2\sigma I_s v_x(t).$$

$$P_e = \frac{2I_s}{b} \int_{-b/2}^{b/2} H_e(0, y, t) dy = \int dt' f(t-t') v_x(t').$$

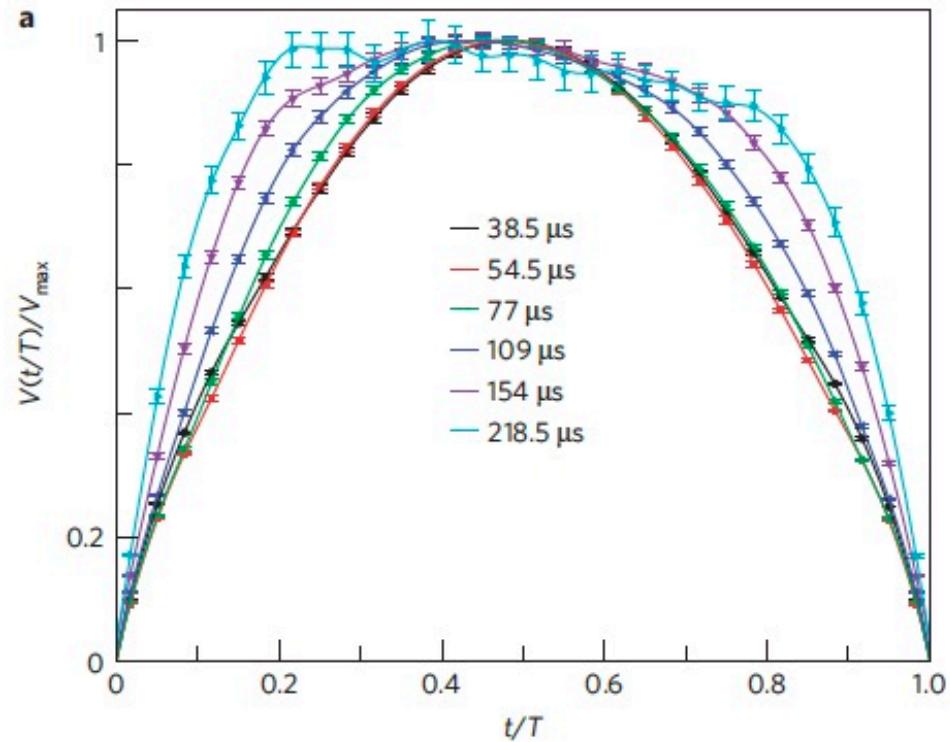
$$\tilde{P}_e = (\beta + i\omega M^*) \tilde{v}_x$$

$$M^* \approx -\frac{8I_s^2 b^3 \mu \sigma^2}{\pi^5}$$

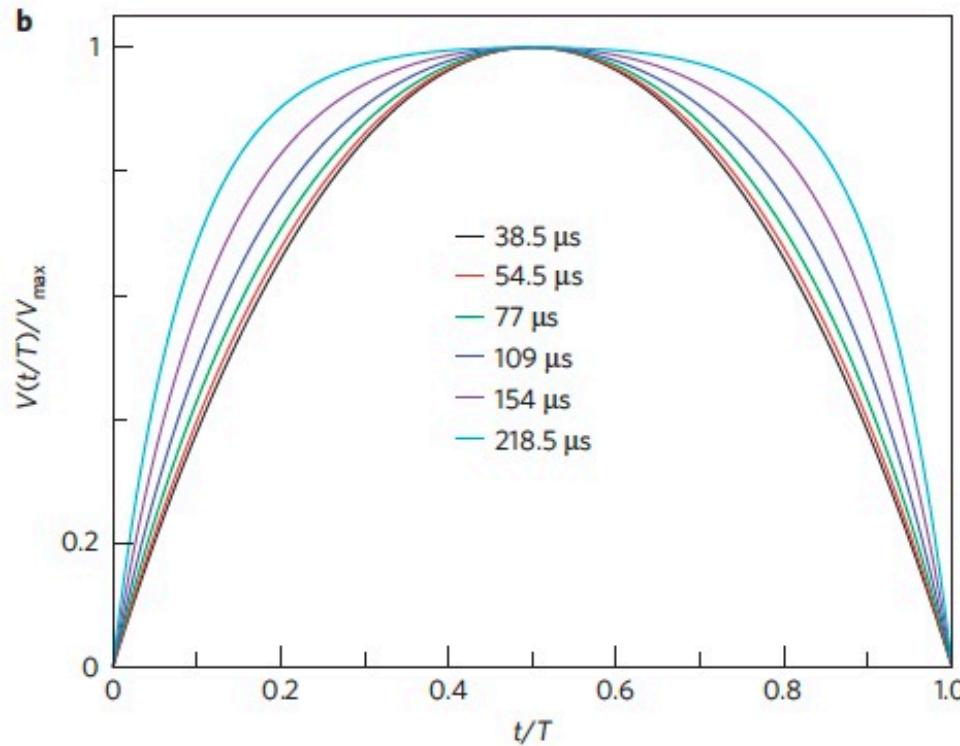
Zapperi, Castellano, Colaiori, Durin
Nature Phys 2005

BARKHAUSEN NOISE: THIN FILMS

EXPERIMENTS



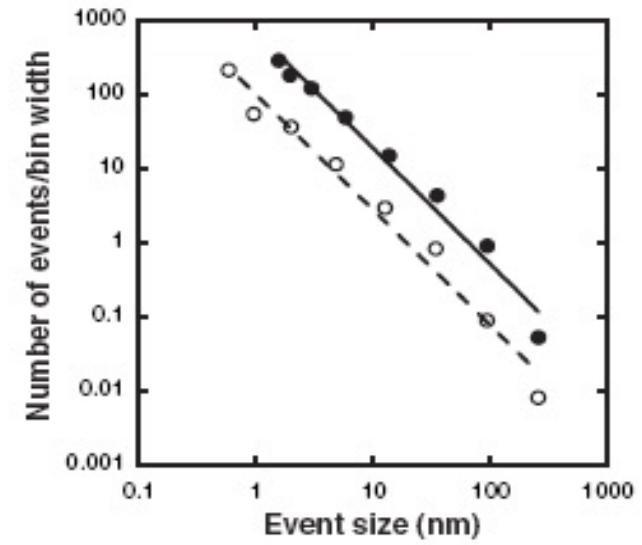
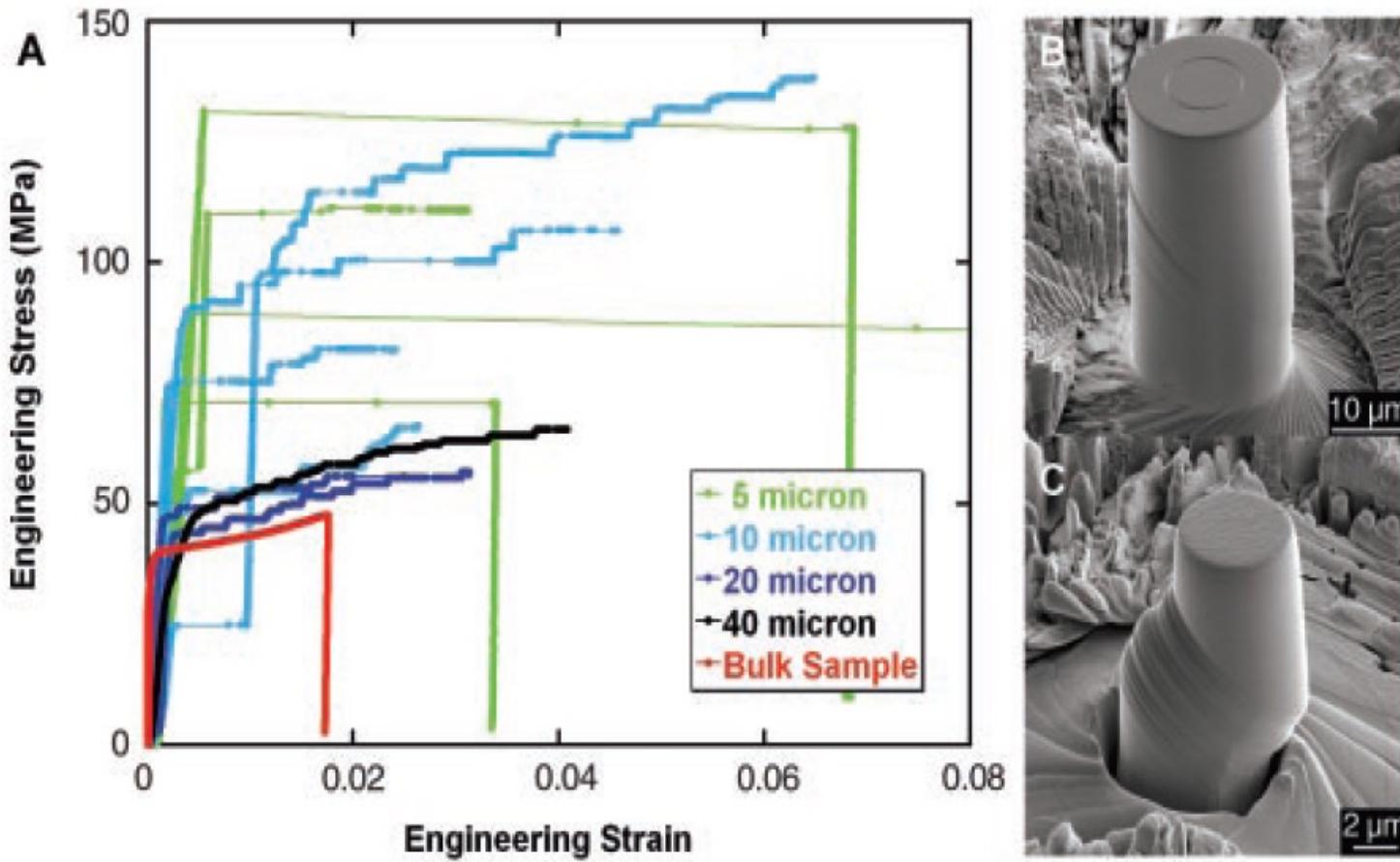
THEORY



$$M^* \approx -\frac{8I_s^2 b^3 \mu \sigma^2}{\pi^5} \quad b \rightarrow 0$$

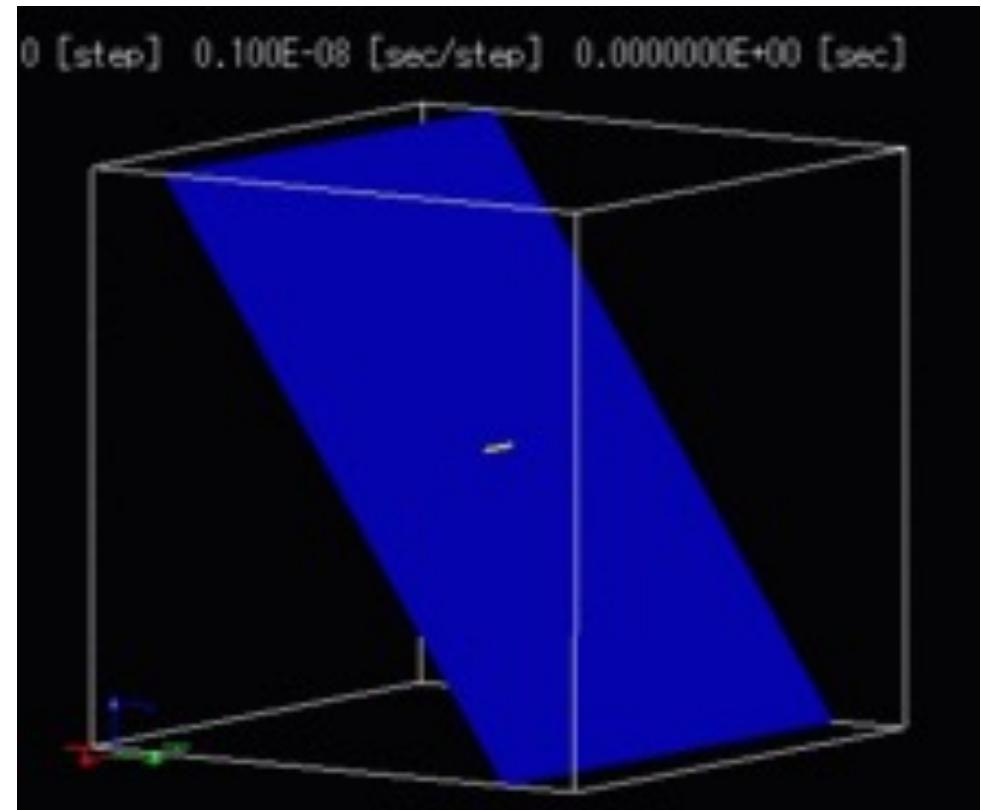
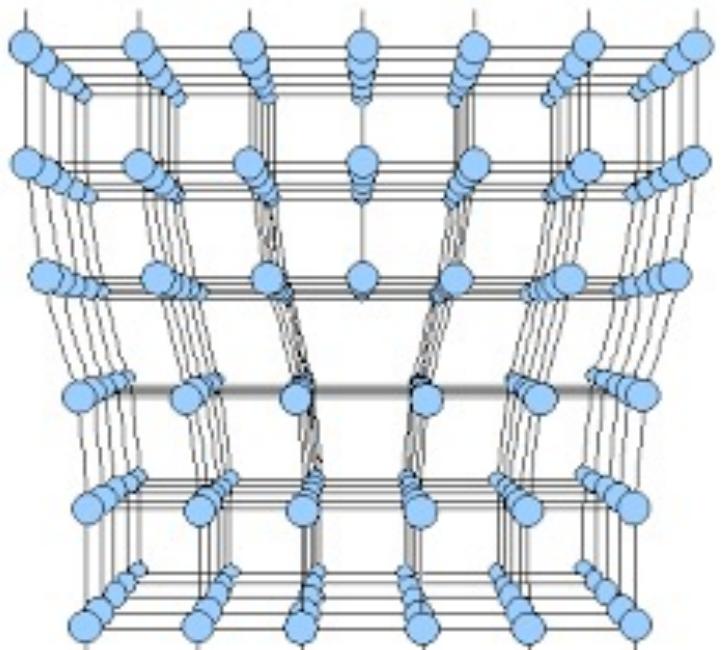
Papanikolaou, ... Durin, Zapperi, Sethna
Nature Phys 2011

MICROPLASTICITY

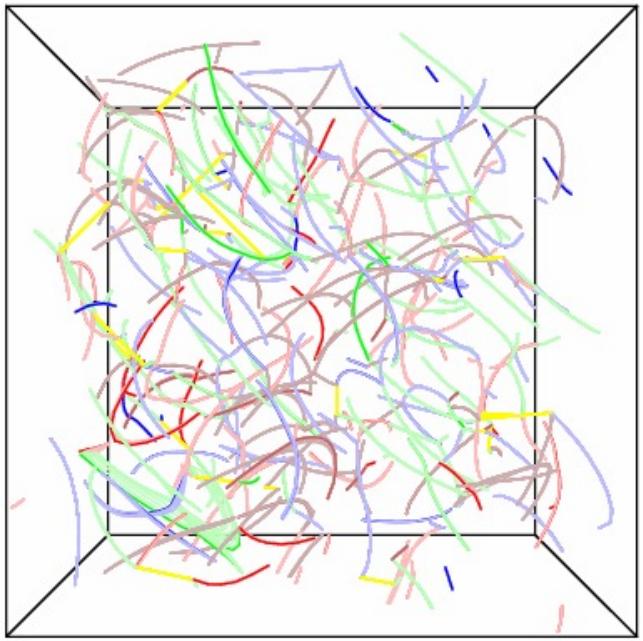


D. M. Dimiduk et al.,
Science 312, 1188 (2006).

DISLOCATIONS

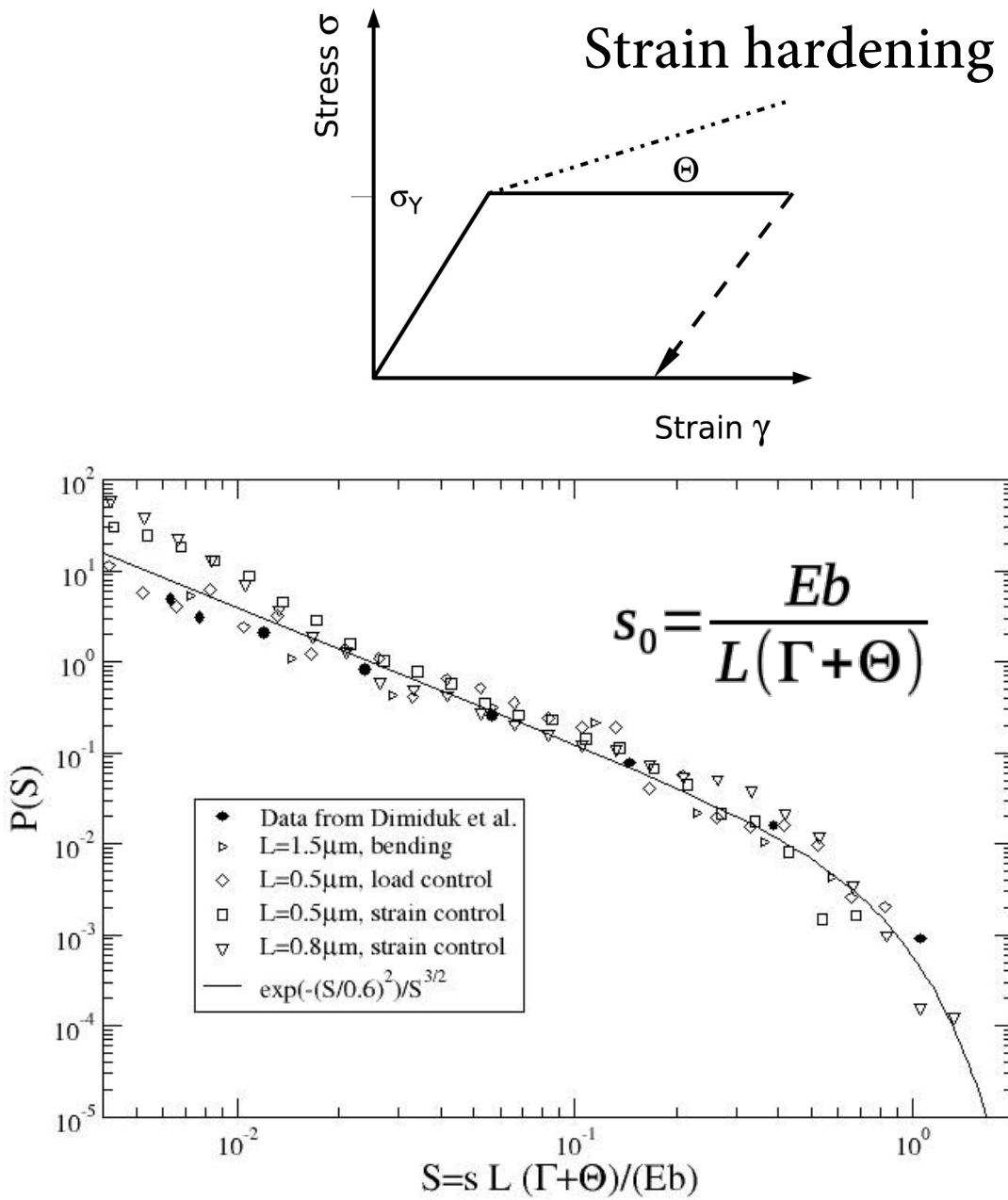


DISLOCATION DYNAMICS



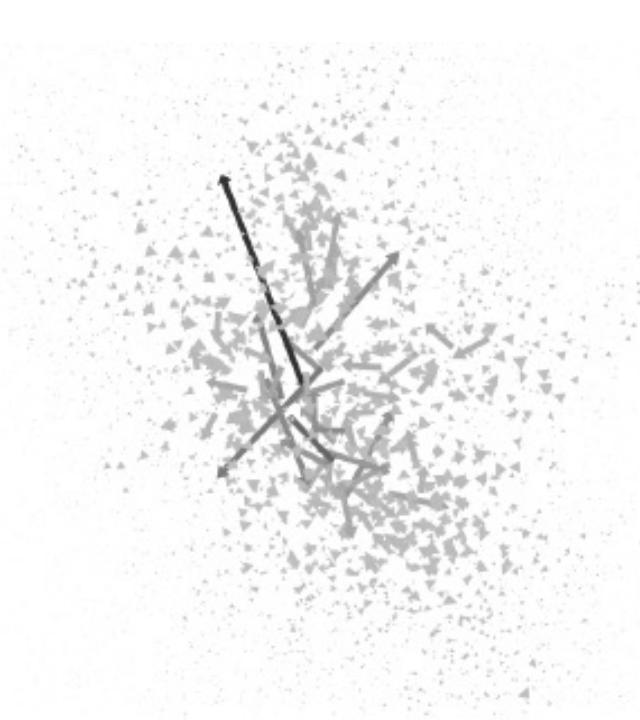
Dislocation Avalanches, Strain Bursts, and the Problem of Plastic Forming at the Micrometer Scale

Ferenc F. Csikor,^{1,2} Christian Motz,³ Daniel Weygand,³ Michael Zaiser,² Stefano Zapperi^{4,5*}



GLASSES

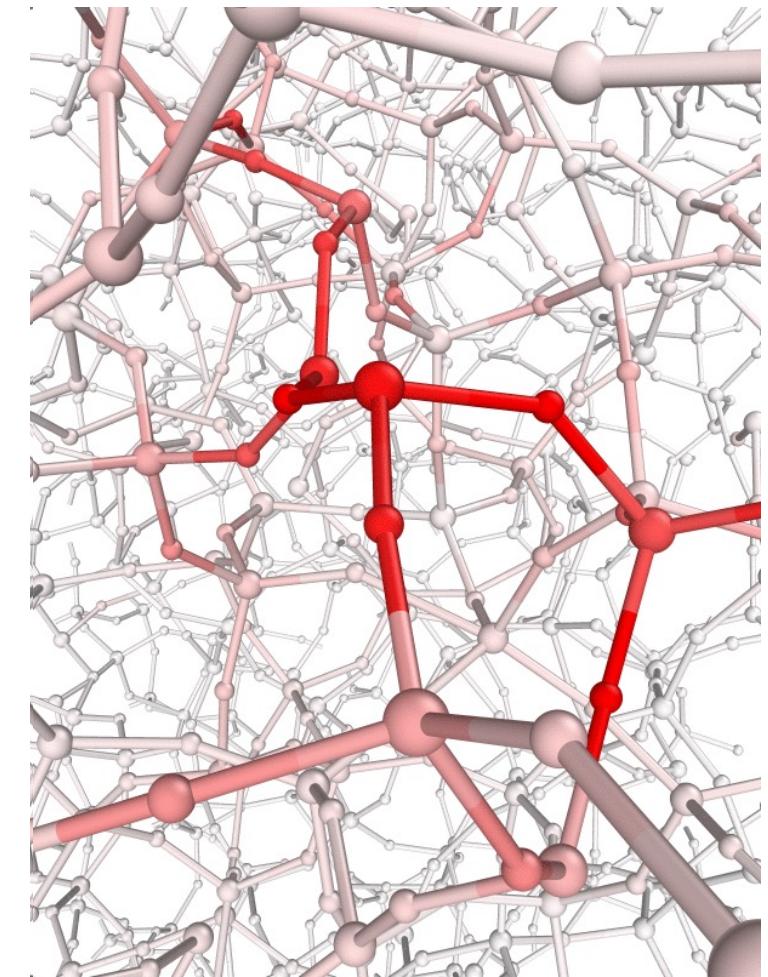
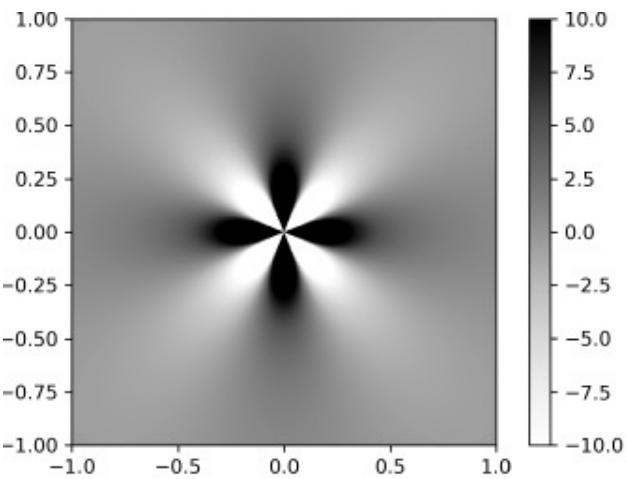
Bonfanti, Guerra, Procaccia, Zapperi PRE 2019



localized shear
transformations

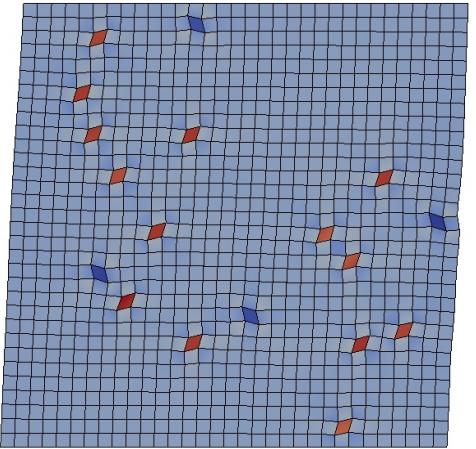


$$G(r, \theta) = \frac{\cos 4\theta}{r^2}$$



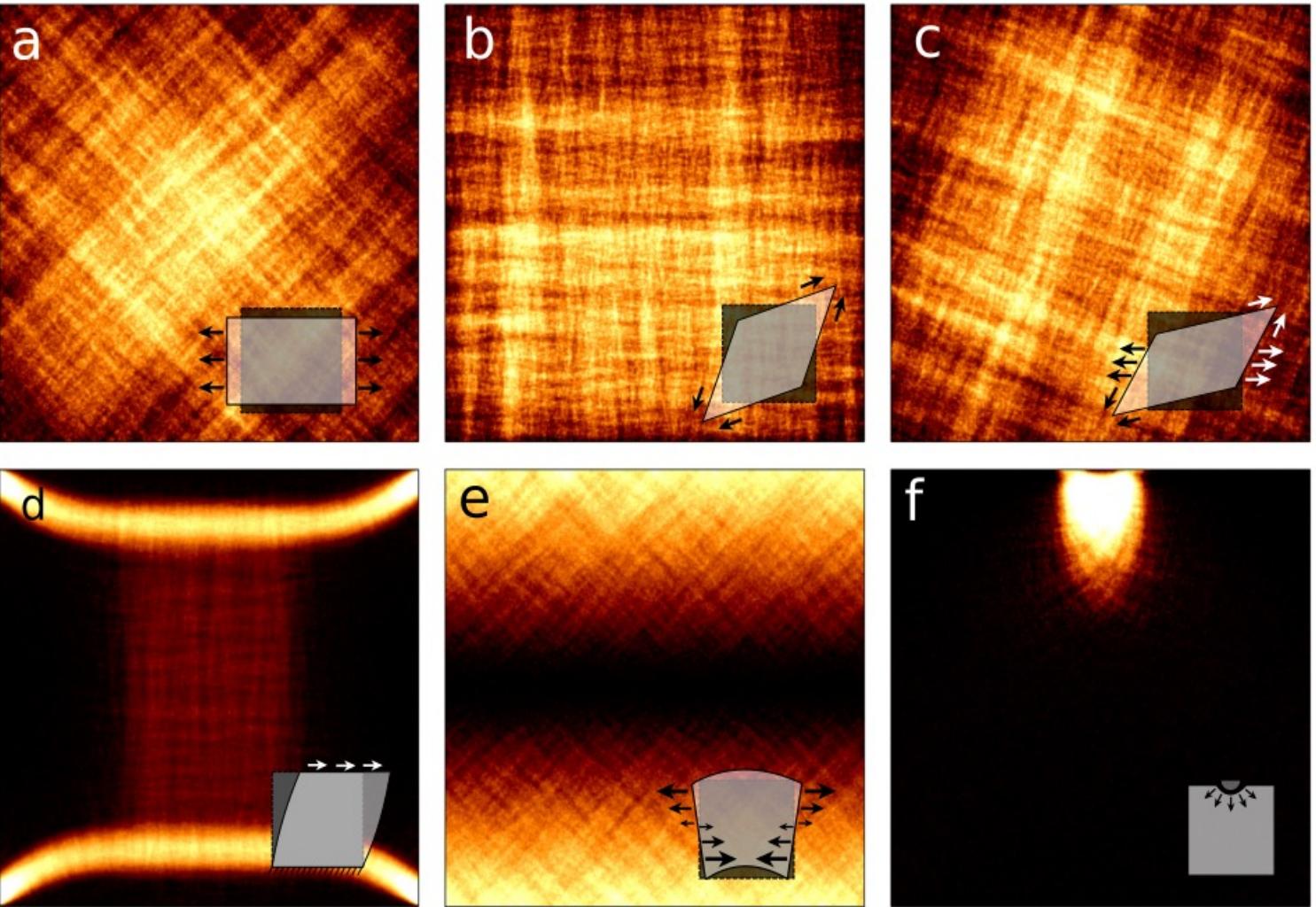
SiO_2

ELASTO-PLASTIC MODEL

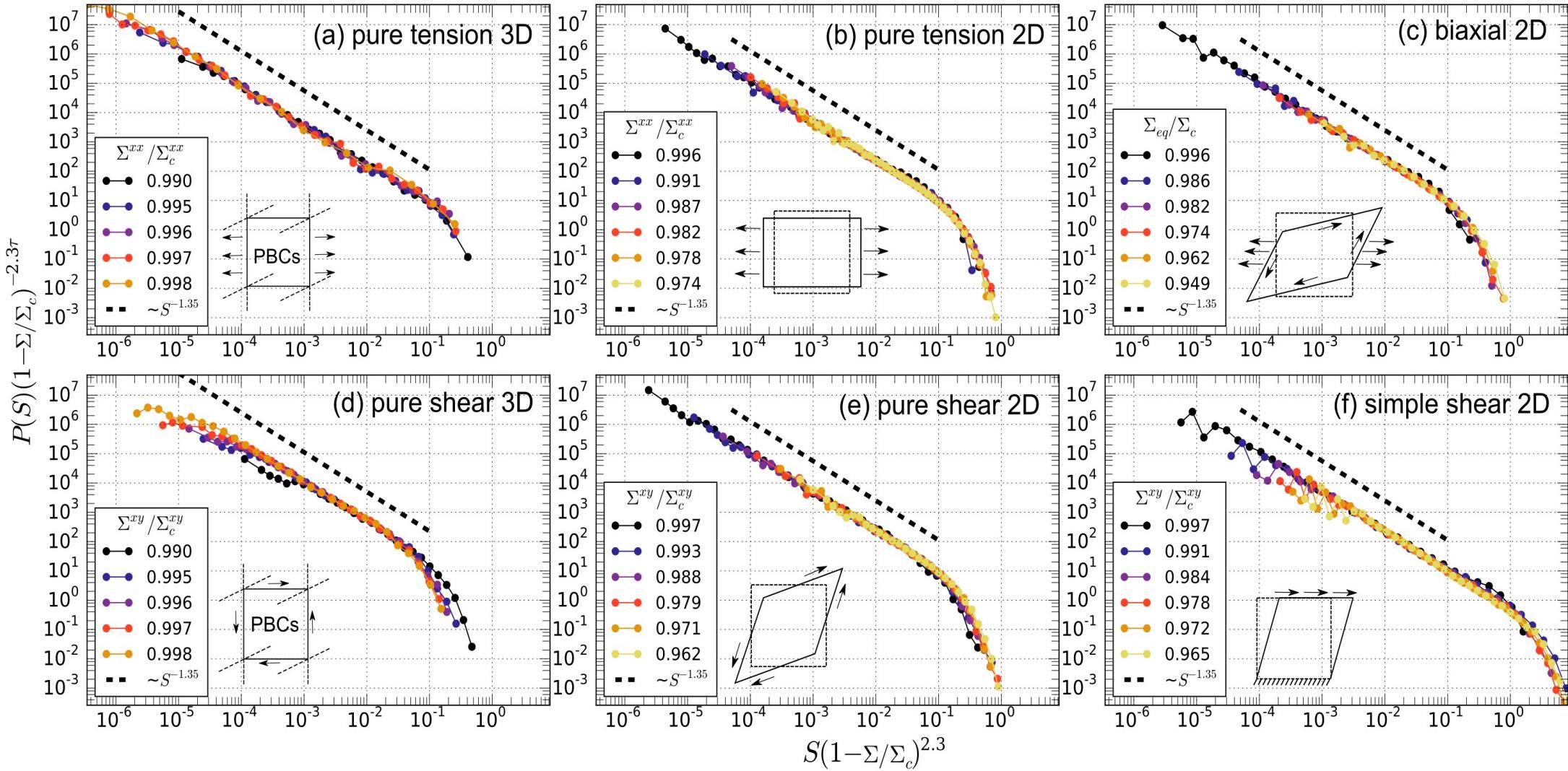


$$\Sigma_{ij}(\vec{r}) = \int \sigma_{ij}(\vec{r}') G_\xi(\vec{r} - \vec{r}') d^3 r'$$

Budrikis, Castellanos, Sanfeld,
Zaiser, Zapperi
Nature Comm 2017

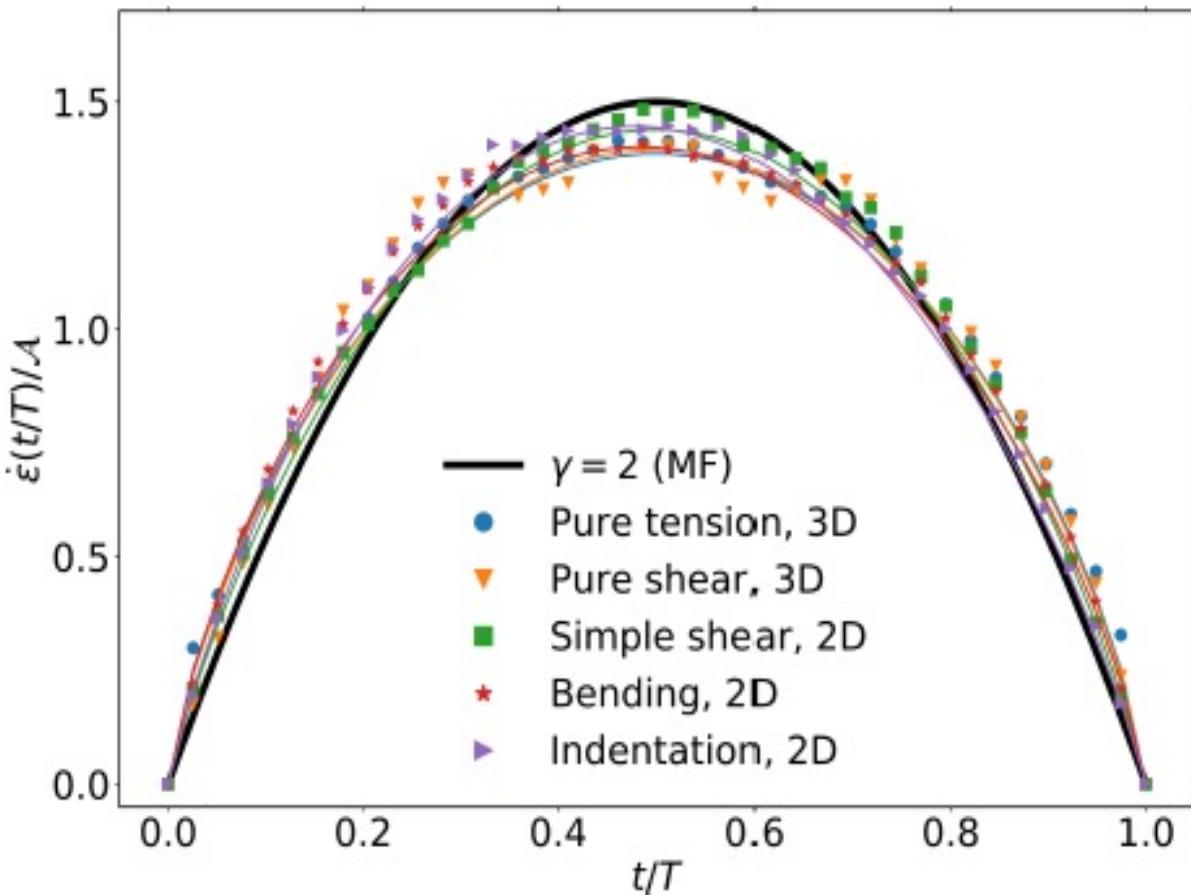


UNIVERSAL SCALING



Budrikis, ... Zaiser, Zapperi
Nature Comm 2017

UNIVERSAL SCALING



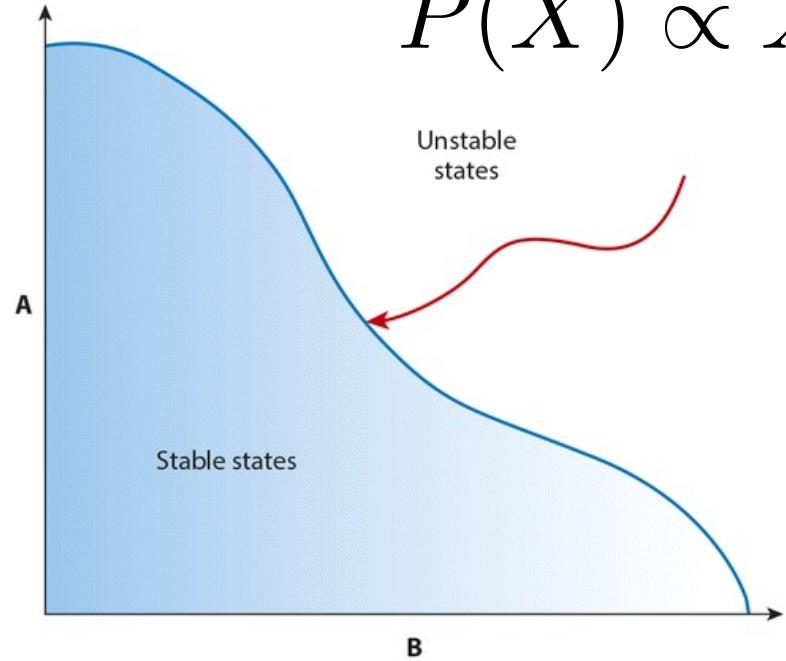
Budrikis, Castellanos, Sanfeld,
Zaiser, Zapperi
Nature Comm 2017

$$\langle \dot{\epsilon}(t') \rangle = (t'(1-t'))^{1-\gamma} (1 - a(t' - 1/2)) \frac{\Gamma(2\gamma)}{(\Gamma(\gamma))^2},$$

MARGINAL STABILITY

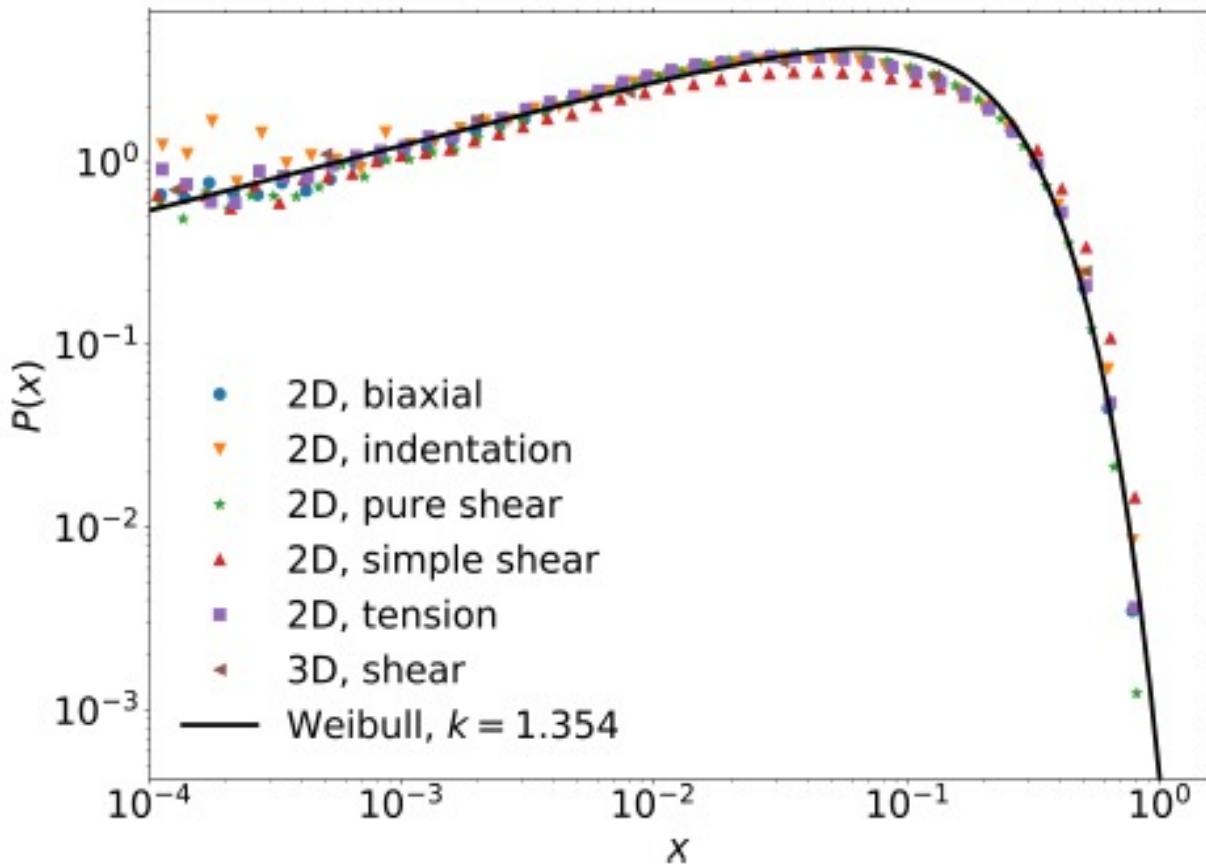
Budrikis, Castellanos, Sanfeld,
Zaiser, Zapperi
Nature Comm 2017

$$P(X) \propto X^\theta$$

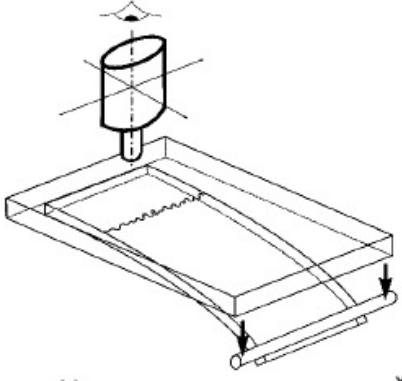


A Müller M, Wyart M. 2015.
Annu. Rev. Condens. Matter Phys. 6:177–200

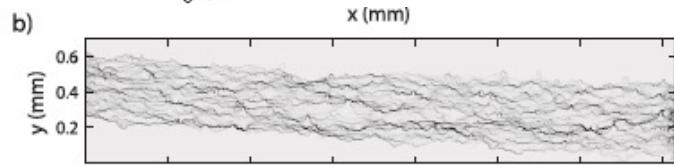
H. G. E. Hentschel, Smarajit Karmakar,
Edan Lerner, and Itamar Procaccia
Phys. Rev. E 83, 061101 – (2011)



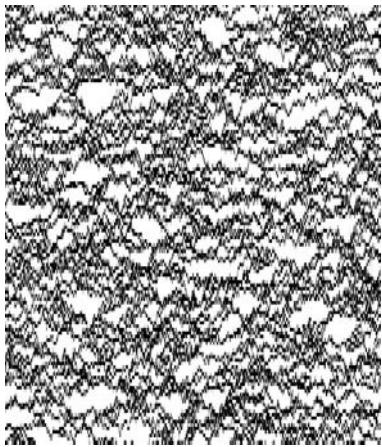
CRACK FRONTS



Maloy, Santucci et al. 2006



Simulations



Avalanches is composed
by multiple clusters

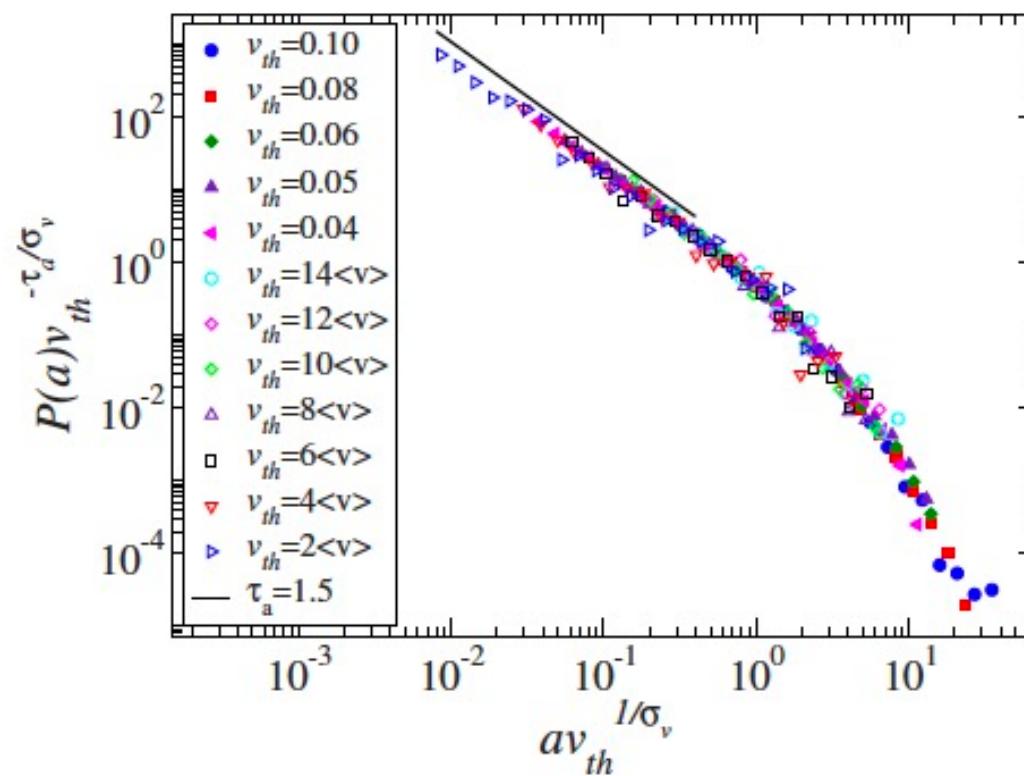
$$\tau_c = 2\tau - 1$$

Laurson, Santucci, Zapperi PRE 2010

$$\Gamma \frac{\partial h}{\partial t} = K_{ext} + K(\{u(x, t)\}) + K_c(x, u(x, t))$$

$$K(\{u(x, t)\}) = K_0 \int dx' \frac{u(x', t) - u(x, t)}{(x - x')^2}$$

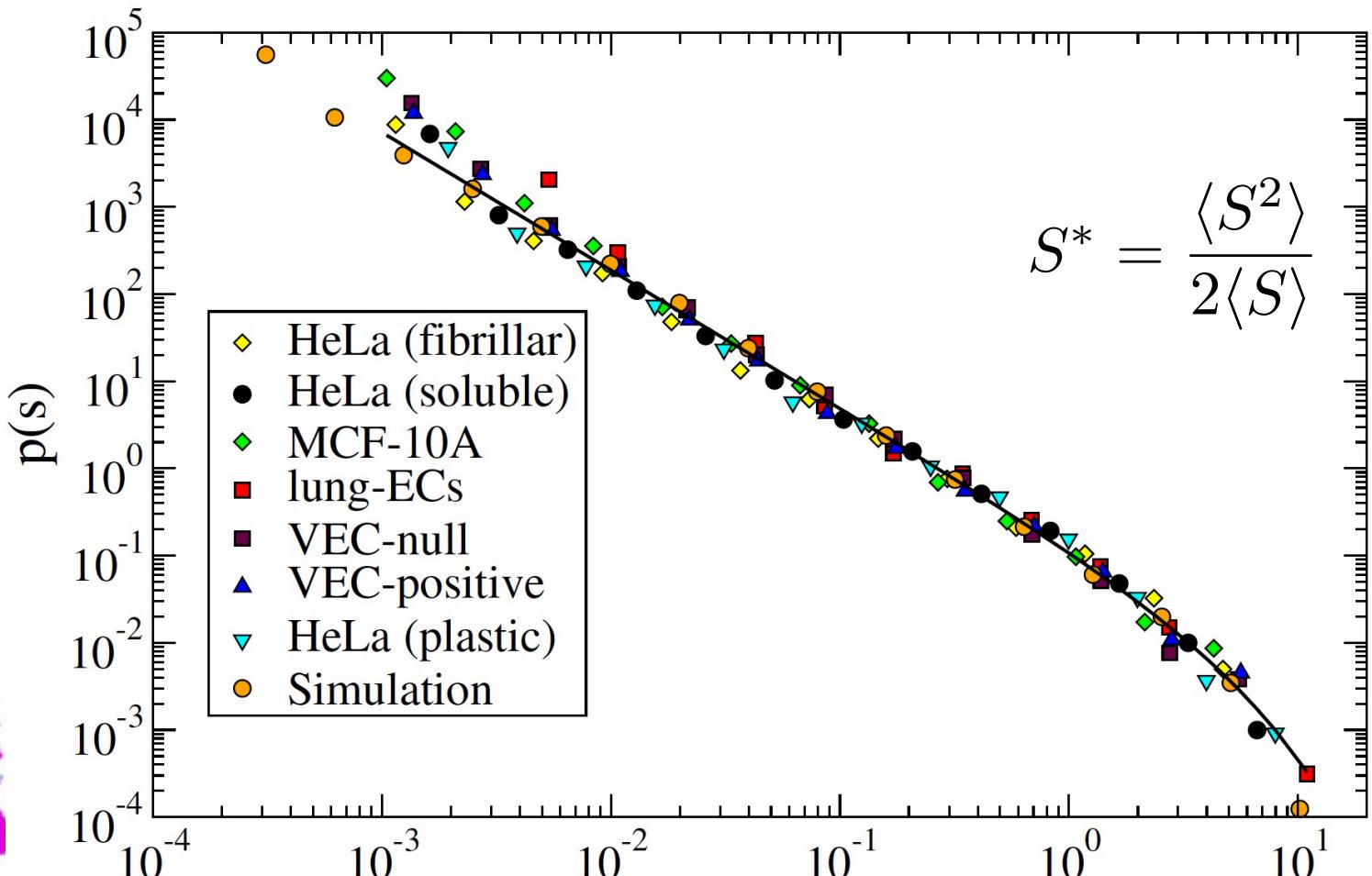
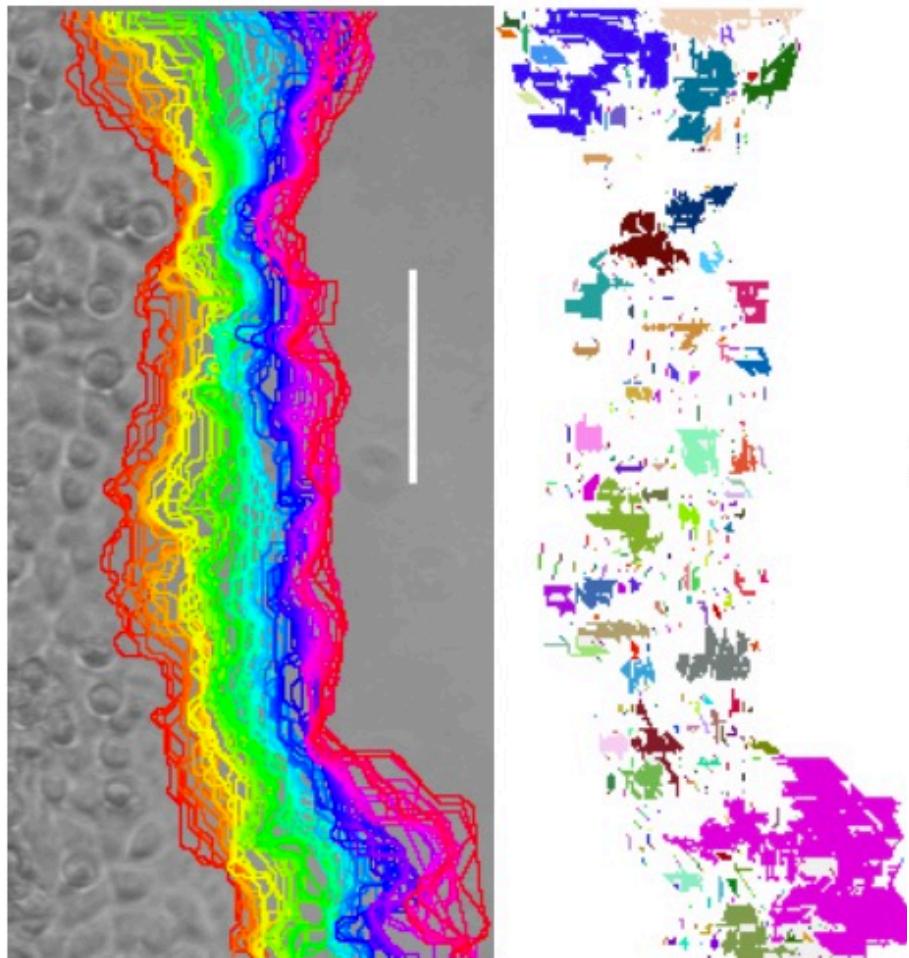
Stress-intensity factor



CASE A

CANCER CELL FRONTS

$$P(S) = \frac{\langle S \rangle}{S^*} p\left(\frac{S}{S^*}\right)$$



$$S^* = \frac{\langle S^2 \rangle}{2\langle S \rangle}$$

Chepizhko, ... Zapperi, La Porta PNAS 2016

$$s = \frac{S}{S^*}$$

ROADMAP

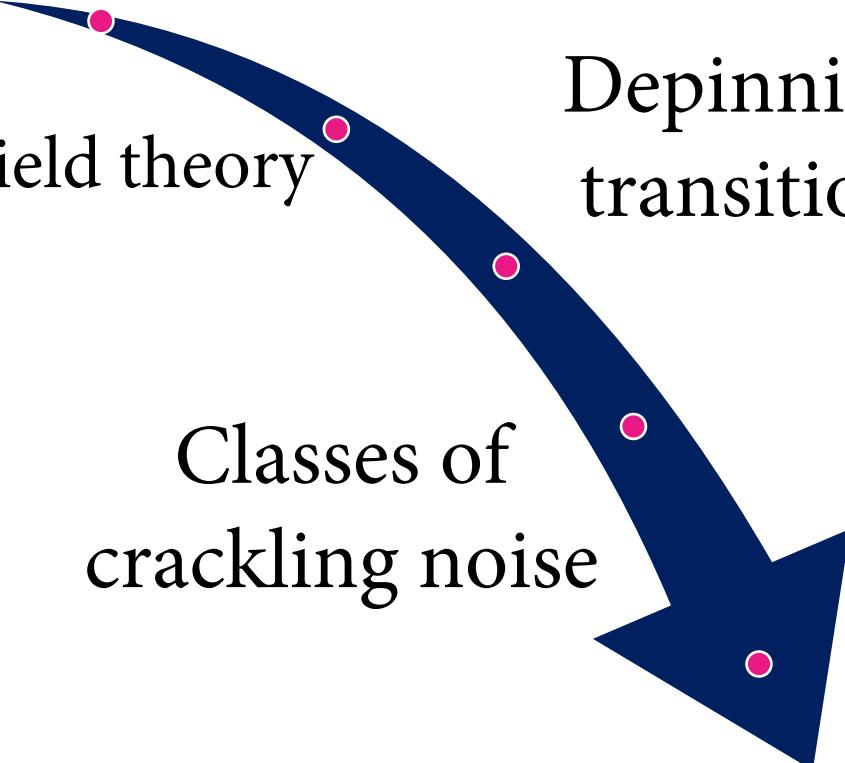
Idealized
sandpile models

Field theory

Classes of
crackling noise

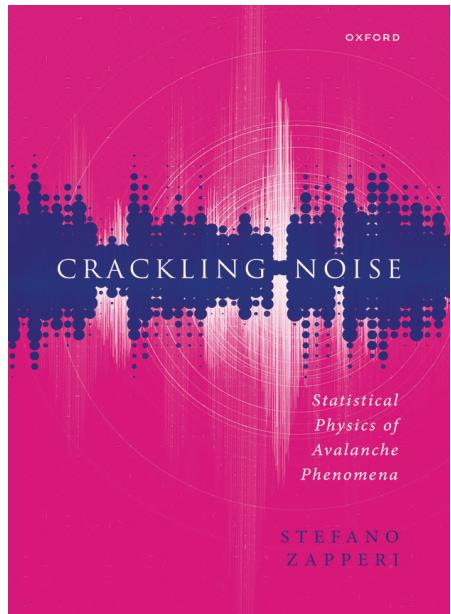
Depinning
transition

Experiments

- 
- {
- Origin of the observed scaling
 - Identification of control parameters
 - Prediction for scaling exponents
 - Predictions for scaling functions

Crackling Noise

Statistical Physics of Avalanche Phenomena



The response of materials and the functioning of devices is often associated with noise. In this book, Stefano Zapperi concentrates on a particular type of noise, known as crackling noise, which is characterized by an intermittent series of broadly distributed pulses. While representing a nuisance in many practical applications, crackling noise can also tell us something useful about the microscopic processes ruling a material's behavior.

Features

- Provides a comprehensive overview of key concepts and theoretical models
- Explores the many applications of the theory of crackling noise in materials science
- Includes expansive discussions considering implications for the life sciences

THE AUTHOR: STEFANO ZAPPERI

Stefano Zapperi is Professor of Theoretical Condensed Matter Physics and Coordinator of the Center for Complexity and Biosystems at the University of Milan.

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THANK YOU FOR YOUR ATTENTION!

...and many many thank to:

M. J. Alava, F. Bohn, S. Bonfanti, Z. Budrikis,
D. F. Castellanos, O. Chepizhko, P. Cizeau, F. Colaiori,
F. Csikor, R. Dickmann, G. Durin, R. Guerra, L.
Laurson, C. A. M. La Porta, C. Mondal, M. A. Munoz,
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J. P. Sethna, R. L. Sommer, H. E. Stanley, A. Vespignani,
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