UNIVERSITÀ DEGLI STUDI DI TRIESTE

Osservatorio Astronomico di Trieste
Astronomical Observatory of Trieste

ifpU

euclid

Precision cosmology with galaxy clusters: preparing for Euclid

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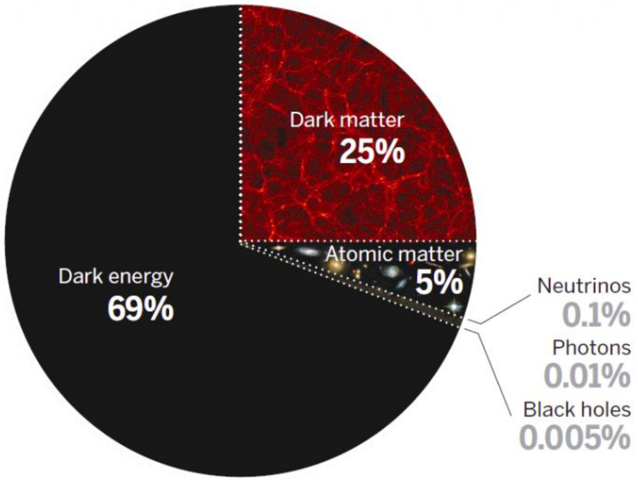
Dissecting Cluster Cosmology,
IFPU, 6 July 2023

Standard cosmological model

Flat ΛCDM cosmological model still suffers from some **problems:**

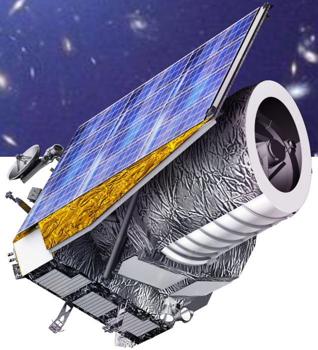
- nature of “dark components”
- behavior of gravity on cosmological scales
- gaussianity of initial conditions
- neutrino masses
- parameter tensions



Euclid:
space-based survey mission dedicated to investigate the origin of the accelerating expansion of the Universe and the nature of dark energy, dark matter and gravity

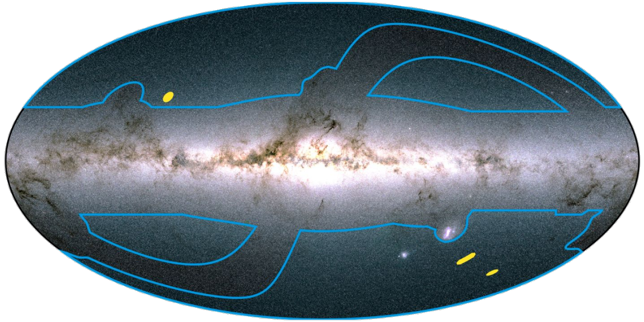
➡ improved statistics to try to address unresolved questions

Euclid mission



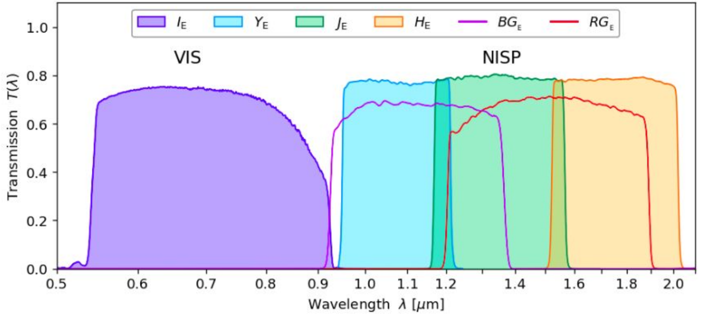
- **Visible/near-infrared** space telescope
  by the European Space Agency **(ESA)**

- Two major surveys:
  - **Euclid Wide Survey**: 15 000 deg^2 of the extra-galactic sky
  - **Euclid Deep Survey**: ~53 deg^2 split over three fields



- 1.2-meter-diameter telescope with two instruments:
  - Visible instrument (**VIS**)
  - Near Infrared Spectrometer and Photometer (**NISP**)

- Primary probes: weak lensing and galaxy clustering
  Secondary probes: **galaxy clusters**, strong lensing, …

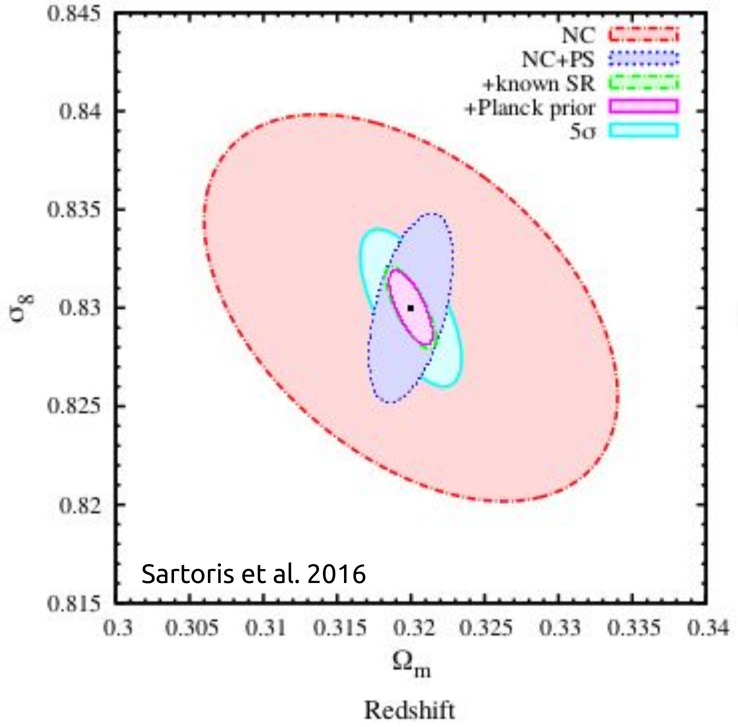


Euclid cluster survey

Cluster survey:

- area = 15000 deg^2
- redshift range z = 0.2 - 2
- mass range M > 0.9 - 1.0 x 10^{14} $M_{\odot}$
- Nobj ~ 2 x 10^5 if $N_{500}/\sigma > 5$
  ~ 2 x 10^6 if $N_{500}/\sigma > 3$

- Cluster detection:
  - photometric data
  - spectroscopic data
  - weak gravitational lensing



Systematics

$$\langle N(\Delta\lambda_i^{\mathrm{ob}}, \Delta z_j^{\mathrm{ob}})\rangle = \int_0^\infty \mathrm{d}z^{\mathrm{tr}}\, \Omega_{\mathrm{mask}} \frac{\mathrm{d}V}{\mathrm{d}z\,\mathrm{d}\Omega}(z^{\mathrm{tr}}) \int_0^\infty \mathrm{d}M \frac{\mathrm{d}n}{\mathrm{d}M}(M, z^{\mathrm{tr}})$$
$$\times \int_{\Delta\lambda_i^{\mathrm{ob}}} \mathrm{d}\lambda^{\mathrm{ob}} \int_0^\infty \mathrm{d}\lambda^{\mathrm{tr}}\, P(\lambda^{\mathrm{ob}} \mid \lambda^{\mathrm{tr}}, z^{\mathrm{tr}})\, P(\lambda^{\mathrm{tr}} \mid M, z^{\mathrm{tr}}) \int_{\Delta z_j^{\mathrm{ob}}} \mathrm{d}z^{\mathrm{ob}}\, P(z^{\mathrm{ob}} \mid z^{\mathrm{tr}}, \Delta\lambda_i^{\mathrm{ob}})$$

- cosmology-dependent quantities ➡ to constrain cosmological parameters
- mass-observable relation ➡ cluster mass not observable to infer through richness λ
- selection functions ➡ observational inaccuracy (photo-z error, projection effects, …)
- semi-analytical models ➡ calibrate on numerical simulations

+ cluster detection ➡ to ensure the best final catalog's completeness and purity

+ covariance matrix ➡ to describe statistical errors (shot-noise, sample variance, …)
Computed/validated on simulations

Covariance matrix

Inclusion of uncertainties of statistical quantities fundamental to constrain cosmological parameters

To compute the covariance:

- **Numerical matrix** from a large set of simulations

$$\hat{C}_{ij} = \frac{1}{N-1}\sum_{a=1}^{N}\left(\hat{d}_i^{\,a} - \langle \hat{d}_i \rangle\right)\left(\hat{d}_j^{\,a} - \langle \hat{d}_j \rangle\right)$$

  + all the contributes are included
  - noisy matrix due to finite number of simulations / high computational resources
  - cosmology-independent matrix

- **Analytical models**

$$C_{ij} = C_{ij}(\Theta), \quad \Theta = \{\Omega_{\mathrm{m}}, \sigma_8, \dots\}$$

  + noise free
  + cosmology-dependent
  - difficult to include all the terms (non-linearities, non-Gaussianity, window functions...)

Aim : validate covariance models for number counts and clustering of galaxy clusters, to properly describe the sources of uncertainty that can affect the cluster observables at the Euclid level of accuracy

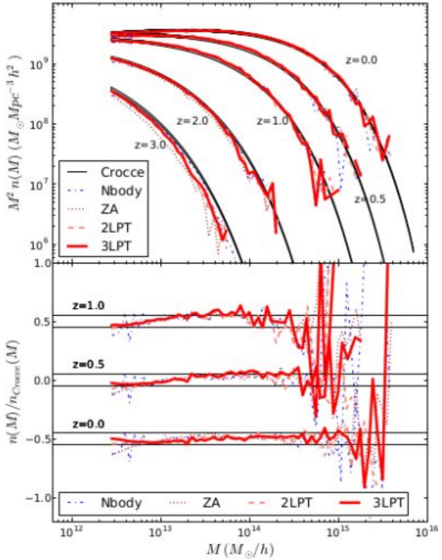
Simulations: PINOCCHIO algorithm

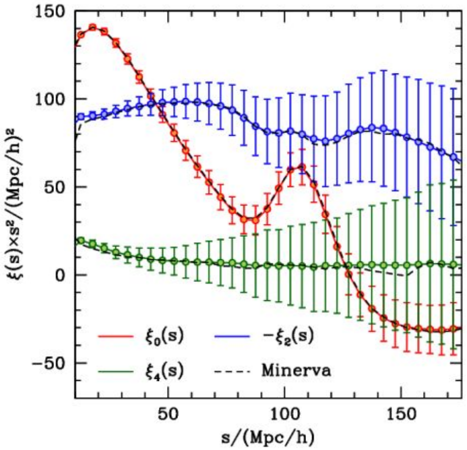
Covariance matrices require **large sets of simulations (~10^3):**

➔ not feasible with N-body simulations due to high computational costs
➔ **approximate methods**: less accurate but faster

PINOCCHIO algorithm (Monaco et al. 2002):

- dark matter halo catalogs through LPT and ellipsoidal collapse
- ~10^3 times faster than N-body
- 5 – 10% accuracy in reproducing 2-point statistics, mass function and bias

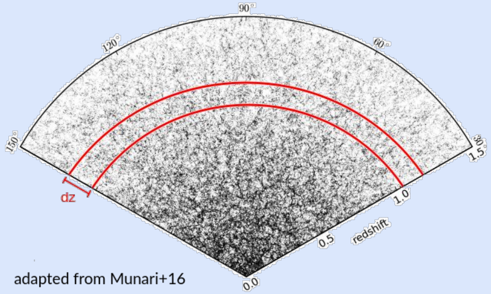




Simulations

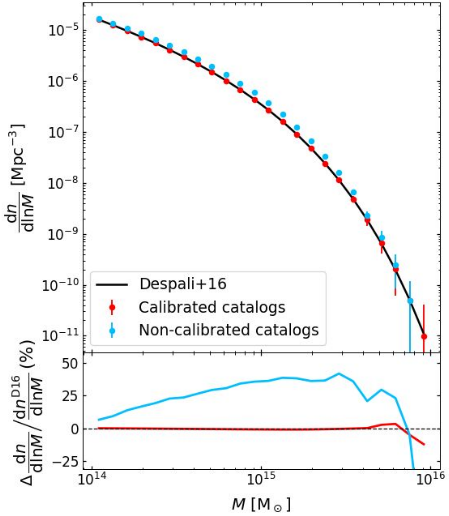
1000 Euclid-like lightcones:

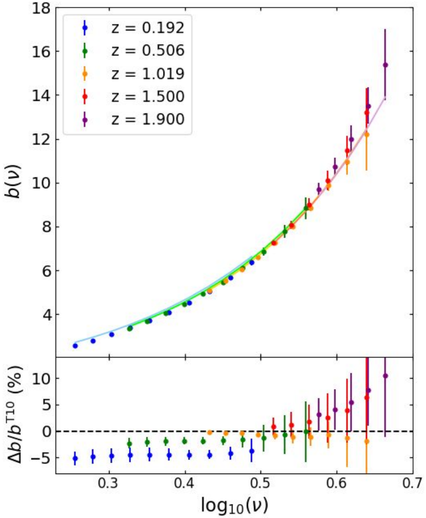
- area ≃10300 deg² (a quarter of the sky)
- redshift range z = 0 - 2
- mass range M = 10^{14}-10^{16} M☉
- number of objects ~3x10^5



adapted from Munari+16

- Masses rescaled to **Despali+16/Castro+22 halo mass function**
- **Tinker+10 halo bias** model
  5-10% agreement with simulations





Number counts: covariance

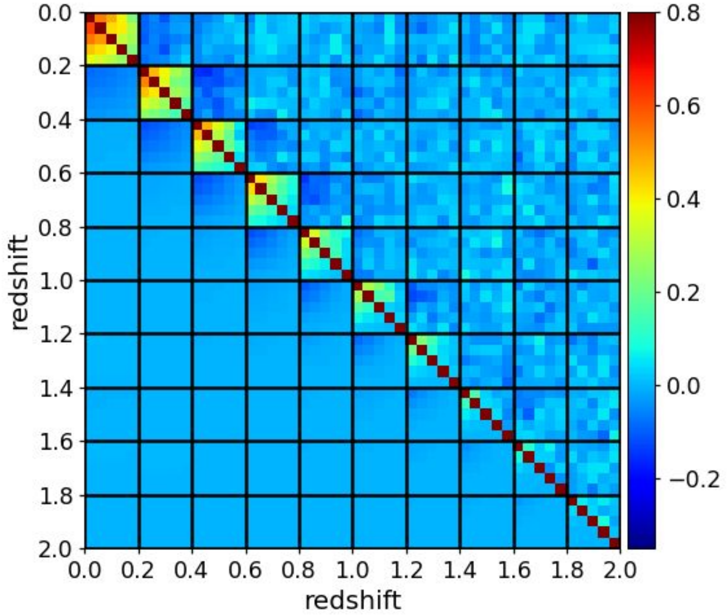
Euclid Collaboration: Fumagalli et al. 2021

Covariance matrix from **Hu&Kravtsov (2003)** model:

$$C_{\alpha\beta ij} = \langle N \rangle_{\alpha i} \delta_{\alpha\beta} \delta_{ij} + \langle Nb \rangle_{\alpha i} \langle Nb \rangle_{\beta j} S_{\alpha\beta}$$

$$S_{\alpha\beta} = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \sqrt{P(k; z_\alpha) P(k; z_\beta)} W_\alpha(\mathbf{k}) W_\beta(\mathbf{k})$$

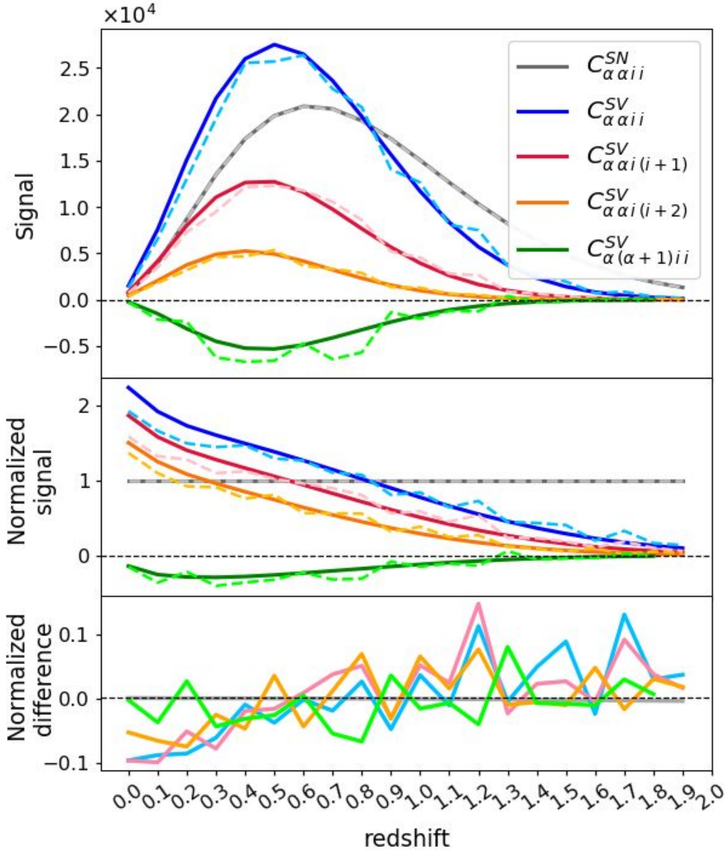
$$W_\alpha(\mathbf{k}) = \frac{4\pi}{V_\alpha} \int_{\Delta z_\alpha} \mathrm{d}z \, \frac{\mathrm{d}V}{\mathrm{d}z} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} (\mathrm{i})^\ell \, j_\ell[k\, r(z)] \, Y_{\ell m}(\hat{\mathbf{k}}) \, K_\ell$$



Number counts: covariance

Euclid Collaboration: Fumagalli et al. 2021

- non-negligible sample variance (higher than shot-noise at low mass/low redshift)
- good agreement between numerical and analytical matrix (deviations $\lesssim$10%)



Number counts: likelihood

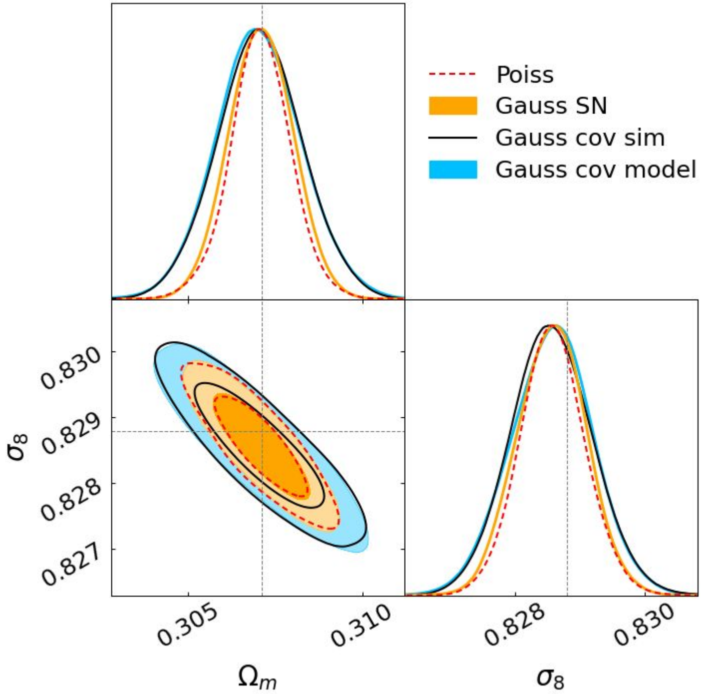
Euclid Collaboration: Fumagalli et al. 2021

Model validation and likelihood comparison:

- **No differences** between numerical and analytical covariance
  ➡ **ΔFoM = +0%**

- **Only shot-noise underestimate the error** on parameters
  ➡ **ΔFoM = -60%**

Full covariance needed not to underestimate the error on parameters



Number counts: likelihood

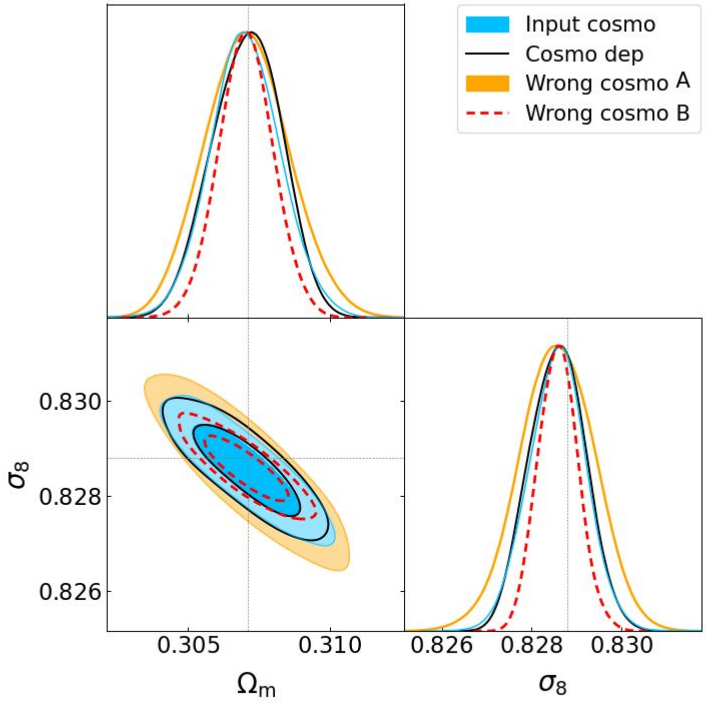
Euclid Collaboration: Fumagalli et al. 2021

Cosmology dependence of the covariance:

- Wrong cosmology in the covariance (2σ from Planck 2018)
  ➡ ΔFoM = ±40/80%

- Cosmo-dependent covariance
  ➡ ΔFoM = +0%

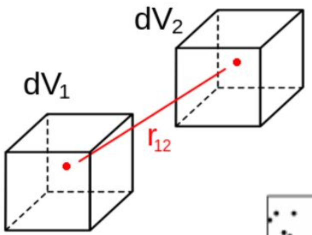
Cosmology-dependent covariance needed not to under/overestimate the posterior error

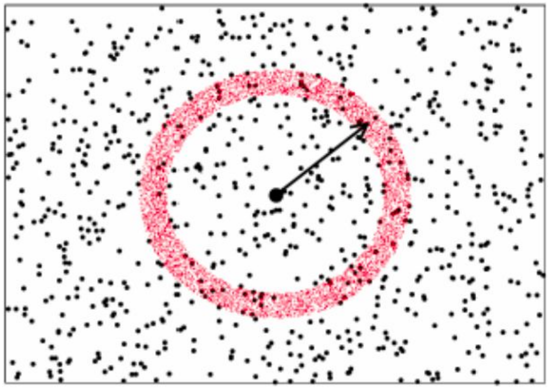


Clustering: 2-point correlation function

2-point correlation function:

excess number of pairs of a given radial separation, relative to that expected for a random distribution





Measurement: Landy-Szalay estimator:

$$\hat{\xi}_{ai} = \frac{1}{RR}\left[\left(\frac{n_R}{n_D}\right)^2 DD - 2\left(\frac{n_R}{n_D}\right) DR + RR\right]$$

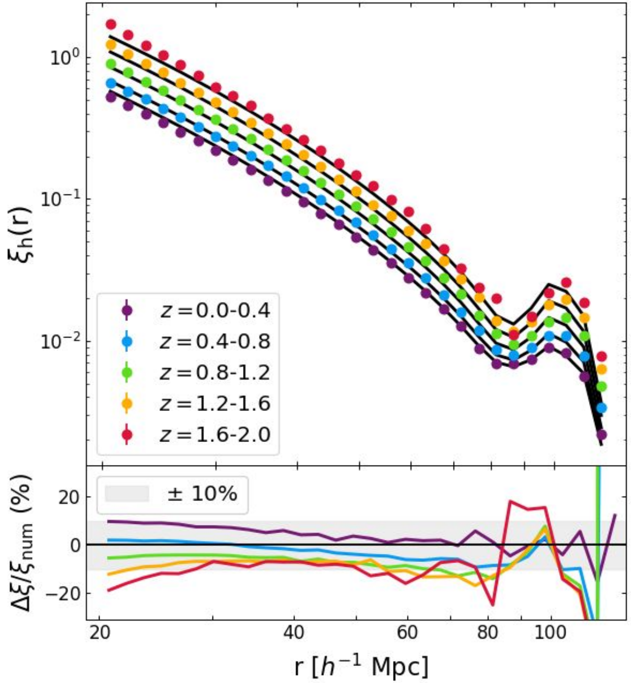
Clustering: 2-point correlation function

Theoretical prediction:

Fourier transform of the (linear) power spectrum
+ radial and redshift binning

$$\xi_{\rm h}^{ai} = \int \frac{{\rm d}k\, k^2}{2\pi^2} \left\langle \bar{b}^2\, P_{\rm m}(k) \right\rangle_a W_i(k)$$

r = 20 - 130 Mpc/h $\Rightarrow$ linear scales + BAO peak



Clustering: covariance

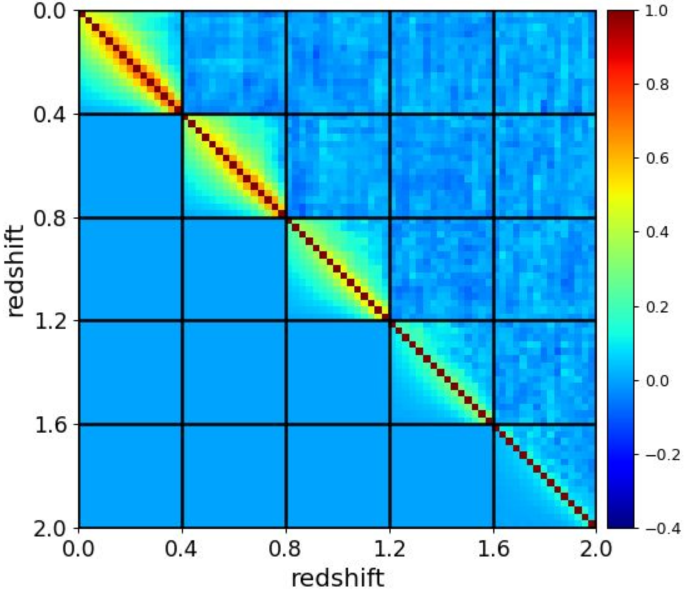
Euclid Collaboration: Fumagalli et al. 2022

Covariance matrix from **Meiksin & White (1999)** model
(Fourier transform of power spectrum covariance)

$$C_{aij} = \frac{2}{V_a}\int \frac{\mathrm{d}k\,k^2}{2\pi^2}\left[\left\langle \bar{b}^2 P_{\mathrm{m}}(k)\right\rangle_a + \left\langle \frac{1}{\bar{n}}\right\rangle_a\right]^2 W_i(k)\,W_j(k)$$

$$+ \frac{2}{V_a V_i}\int \frac{\mathrm{d}k\,k^2}{2\pi^2}\left\langle \bar{b}^2 P_{\mathrm{m}}(k)\right\rangle_a \left\langle \frac{1}{\bar{n}}\right\rangle_a^2 W_j(k)\,\delta^{\mathrm{D}}_{ij}$$

$$\cancel{+ \text{ high order terms}(\propto B, T)}$$



Gaussian term
Low-order non-Gaussian term
High-order non-Gaussian terms

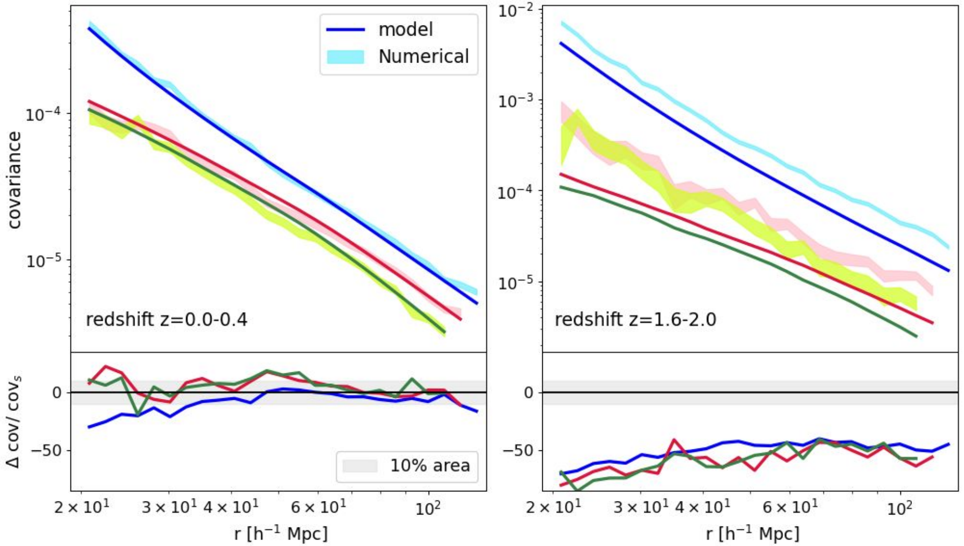
Clustering: covariance

Euclid Collaboration: Fumagalli et al. 2022

- too approximate model:
  <10% difference at low redshift
  ~30-60% difference at high redshift

  - inaccurate bias model
  - non-Poissonian shot noise
  - high-order terms

- add three nuisance parameters fitted from simulations following Fumagalli et al. 2022



$$C_{aij} = \frac{2}{V_a} \int \frac{\mathrm{d}k\, k^2}{2\pi^2} \left[\left\langle (\beta\, \bar{b}\,)^2 P_{\rm m}(k) \right\rangle_a + \left\langle \frac{1+\alpha}{\bar{n}} \right\rangle_a \right]^2 W_i(k)\, W_j(k)$$

$$+ \frac{2}{V_a V_i} \int \frac{\mathrm{d}k\, k^2}{2\pi^2} \left\langle (\beta\, \bar{b}\,)^2 P_{\rm m}(k) \right\rangle_a \left\langle \frac{1+\gamma}{\bar{n}} \right\rangle_a^2 W_j(k)\, \delta^{\rm D}_{ij} \,,$$

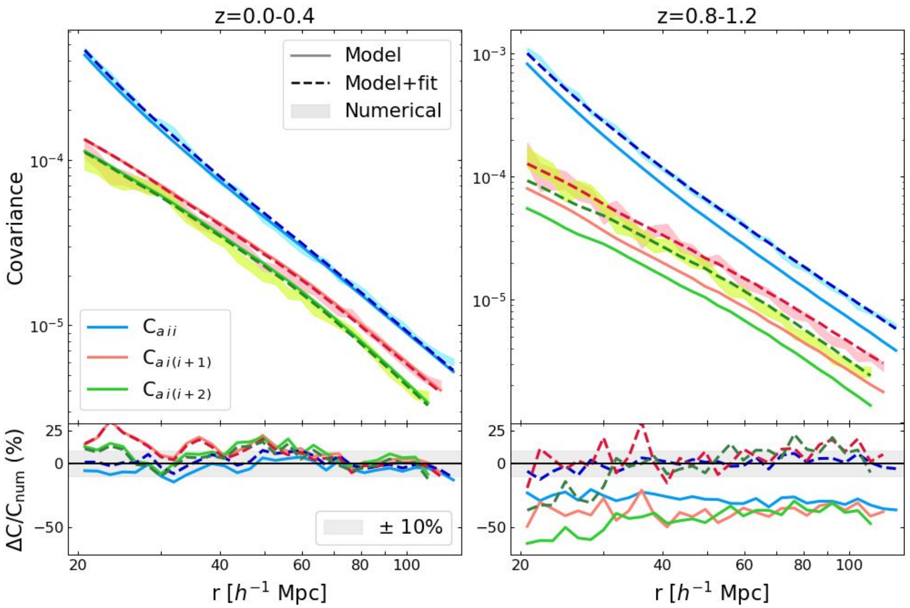
Clustering: covariance

Euclid Collaboration: Fumagalli et al. 2022

$$C_{aij} = \frac{2}{V_a} \int \frac{\mathrm{d}k\, k^2}{2\pi^2} \left[\left\langle (\beta\, \bar{b})^2 P_\mathrm{m}(k) \right\rangle_a + \left\langle \frac{1+\alpha}{\bar{n}} \right\rangle_a \right]^2 W_i(k)\, W_j(k) + \frac{2}{V_a V_i} \int \frac{\mathrm{d}k\, k^2}{2\pi^2} \left\langle (\beta\, \bar{b})^2 P_\mathrm{m}(k) \right\rangle_a \left\langle \frac{1+\gamma}{\bar{n}} \right\rangle_a^2 W_j(k)\, \delta^\mathrm{D}_{ij}\,,$$

Table 1. Best-fit values for the covariance model parameters introduced in Eq (27).

Redshift	α	β	γ
0.0 – 0.4	0.111 ± 0.008	0.979 ± 0.008	−0.027 ± 0.047
0.4 – 0.8	0.109 ± 0.008	1.055 ± 0.009	−0.083 ± 0.037
0.8 – 1.2	0.134 ± 0.008	1.181 ± 0.013	−0.129 ± 0.027
1.2 – 1.6	0.157 ± 0.008	1.270 ± 0.022	−0.199 ± 0.024
1.6 – 2.0	0.188 ± 0.008	1.460 ± 0.045	−0.263 ± 0.026
Reference	0	1	0



With **few simulations (~10^2)**, accurate **noise-free, cosmology-dependent** covariance matrix

Clustering: covariance

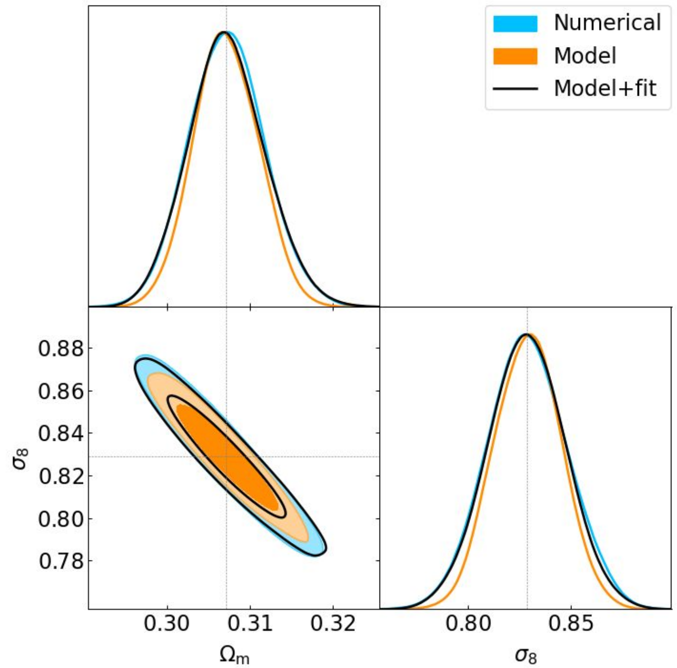
Euclid Collaboration: Fumagalli et al. 2022

Covariance comparison through likelihood analysis :

- **Model:** underestimates the numerical result (reference)
  ➡ **ΔFoM = +40%**

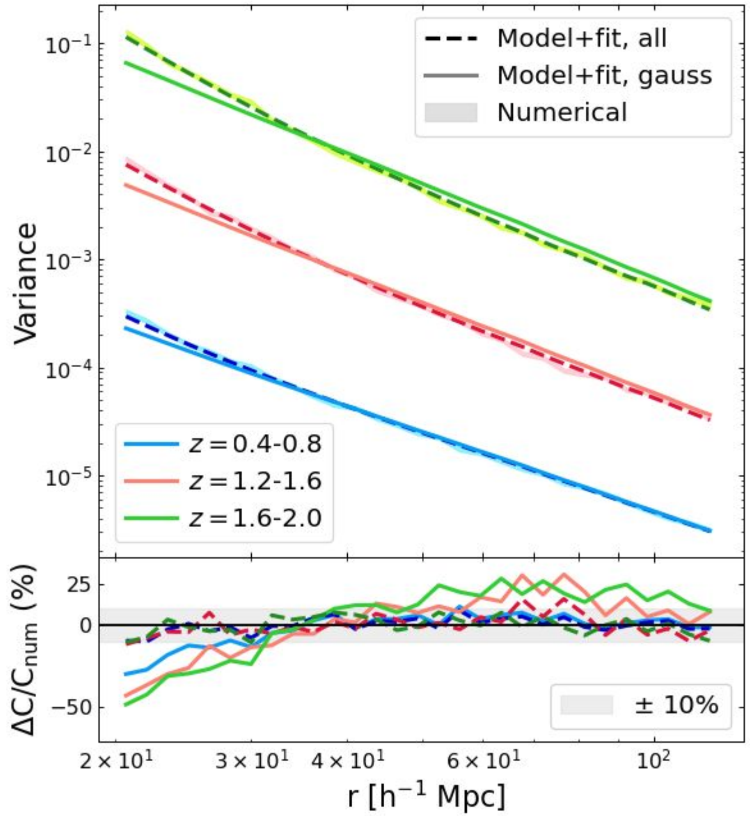
- **Model+parameters:** correctly reproduces numerical result
  ➡ **ΔFoM = +5%**

When adding fitted parameters,
accurate description of the covariance



Clustering: Gaussian covariance

Euclid Collaboration: Fumagalli et al. 2022



$$C_{aij} = \frac{2}{V_a}\int \frac{\mathrm{d}k\,k^2}{2\pi^2}\left[\left\langle(\beta\,\bar{b})^2 P_{\mathrm{m}}(k)\right\rangle_a + \left\langle\frac{1+\alpha}{\bar{n}}\right\rangle_a\right]^2 W_i(k)\,W_j(k)$$

$$\cancel{+\frac{2}{V_a V_i}\int \frac{\mathrm{d}k\,k^2}{2\pi^2}\left\langle(\beta\,\bar{b})^2 P_{\mathrm{m}}(k)\right\rangle_a \left\langle\frac{1+\gamma}{\bar{n}}\right\rangle_a^2 W_j(k)\,\delta^{\mathrm{D}}_{ij}}\,,$$

- **Gaussian covariance:** incomplete model
  ➡ **ΔFoM = +20%**

Gaussian covariance **not enough** for cluster clustering covariance

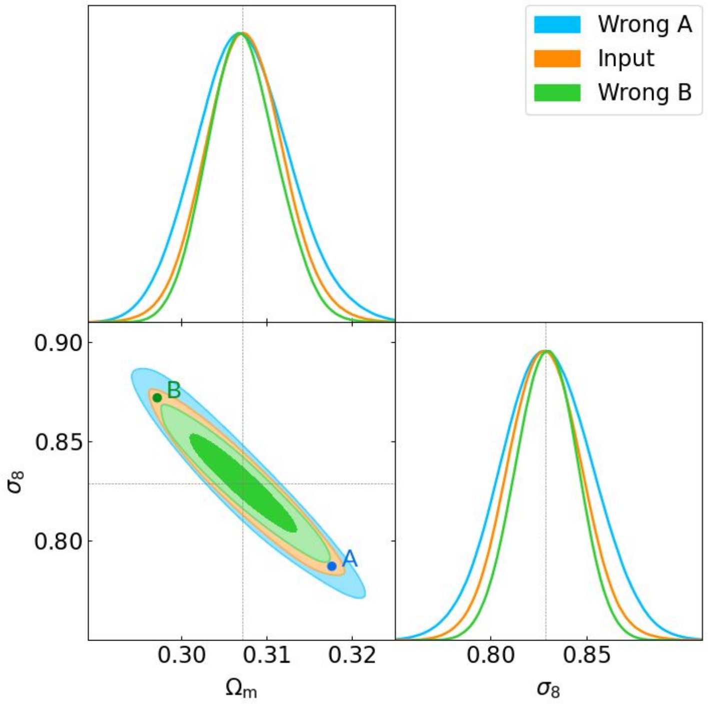
Clustering: cosmo-dependent covariance

Euclid Collaboration: Fumagalli et al. 2022

Cosmology dependence of the covariance:

- **Wrong-cosmology** covariance (2σ from Planck 2018)
  ➡ **ΔFoM = ±35%**
- **Cosmo-dependent** ξ+covariance
  ➡ **ΔFoM ~ 150%**

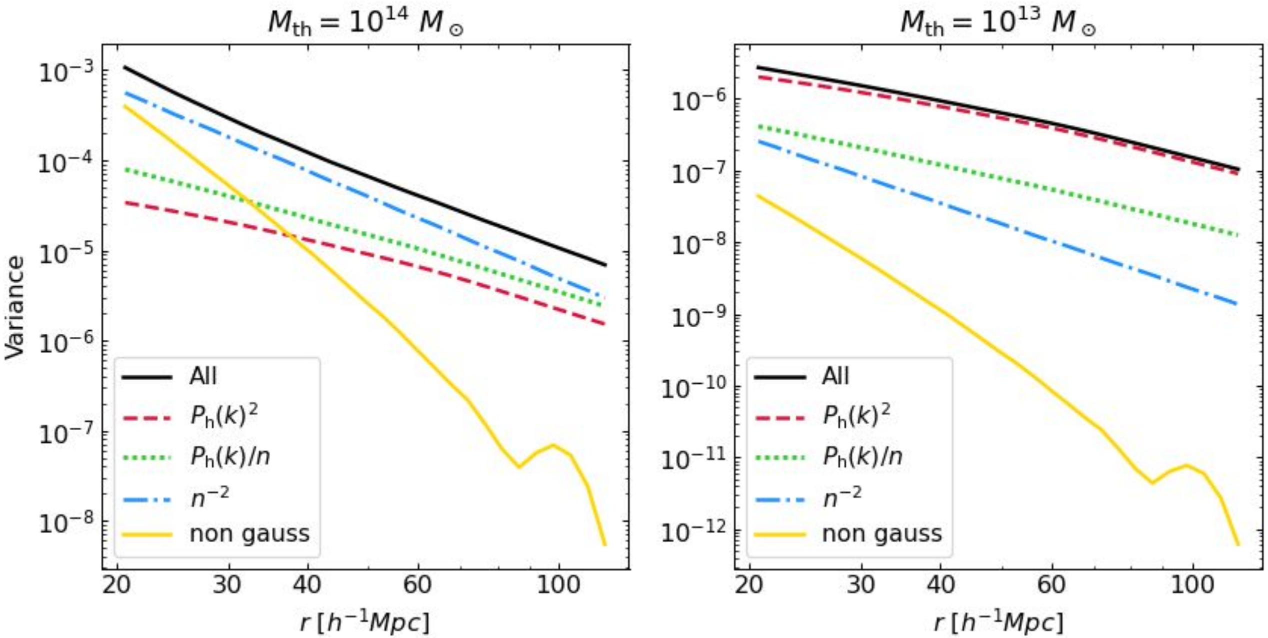
Covariance with **different degeneracy** on parameters w.r.t. ξ due to shot-noise ∝ mass function



Clustering: covariance

Euclid Collaboration: Fumagalli et al. 2022

$$C_{aij} = \frac{2}{V_a}\int \frac{\mathrm{d}k\,k^2}{2\pi^2}\left[\boxed{\left\langle(\beta\,\bar{b})^2 P_\mathrm{m}(k)\right\rangle_a^2} + \boxed{2\left\langle(\beta\,\bar{b})^2 P_\mathrm{m}(k)\right\rangle_a \left\langle\frac{1+\alpha}{\bar{n}}\right\rangle_a} + \boxed{\left\langle\frac{1+\alpha}{\bar{n}}\right\rangle_a^2}\right] W_i(k)\,W_j(k)$$
$$+ \frac{2}{V_a V_i}\int \frac{\mathrm{d}k\,k^2}{2\pi^2}\boxed{\left\langle(\beta\,\bar{b})^2 P_\mathrm{m}(k)\right\rangle_a \left\langle\frac{1+\gamma}{\bar{n}}\right\rangle_a^2} W_j(k)\,\delta^\mathrm{D}_{ij}\,,$$



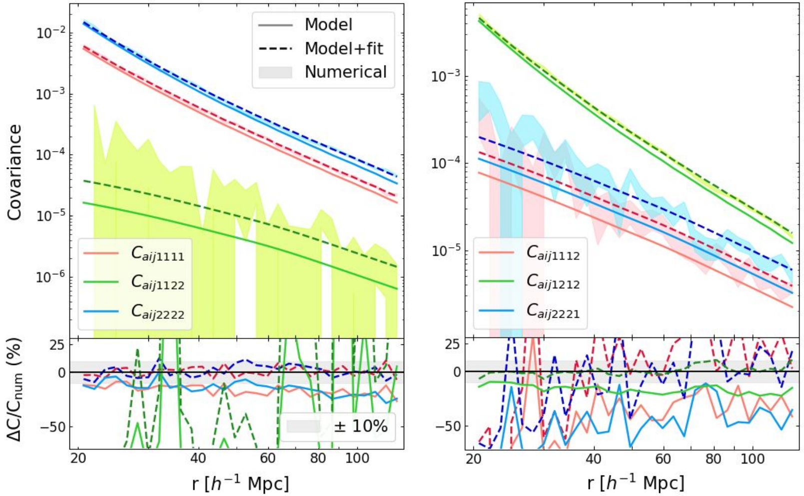
Clustering: mass binning

Euclid Collaboration: Fumagalli et al. 2022

Add mass binning to quantify the information in the mass-dependence of the halo bias

$$
\begin{aligned}
C'_{aijmnpq} = {} & \frac{2}{V_a} \int \frac{\mathrm{d}k\, k^2}{2\pi^2} \left[\left\langle \bar{b}_m\, \bar{b}_n\, P_\mathrm{m}(k) \right\rangle_a + \left\langle \frac{\delta^\mathrm{D}_{mn}}{\bar{n}_m} \right\rangle_a \right] \\
& \times \left[\left\langle \bar{b}_p\, \bar{b}_q\, P_\mathrm{m}(k) \right\rangle_a + \left\langle \frac{\delta^\mathrm{D}_{pq}}{\bar{n}_p} \right\rangle_a \right] W_i\, W_j \\
& + \frac{2}{V_a} \int \frac{\mathrm{d}k\, k^2}{2\pi^2} \left\langle \bar{b}_m\, \bar{b}_p\, P_\mathrm{m}(k) \right\rangle_a \left\langle \frac{\delta^\mathrm{D}_{mn}}{\bar{n}_m} \right\rangle_a \left\langle \frac{\delta^\mathrm{D}_{pq}}{\bar{n}_p} \right\rangle_a \frac{W_j}{V_i} \delta^\mathrm{D}_{ij} .
\end{aligned}
$$

$$
\longrightarrow \qquad C_{aijmnpq} = \frac{C'_{aijmpnq} + C'_{aijmqnp}}{2} ,
$$



Likelihood forecasts

Table 3. Figure of merit for the different mass binning cases. In the third column, percent difference with respect to the "Mass threshold" case in the upper part, and "2 mass bins" numerical case in the lower one.

Case	FoM	ΔFoM / FoM$_{\mathrm{num}}$
Mass threshold	29759 ± 554	–
2 mass bins	36555 ± 349	+ 23 %
3 mass bins	35243 ± 308	+ 18 %
4 mass bins	37160 ± 497	+ 25 %
Model	48500 ± 738	+ 33 %
Model + fit	37980 ± 543	+ 4 %
Cosmo-dependent	121921 ± 615	+ 230 %

Observable space

Euclid Collaboration: Fumagalli et al. *in preparation*

Richness-mass relation:

$$P(\lambda|M,z) = \frac{1}{\lambda\sqrt{2\pi\sigma^2_{\ln\lambda}}}\exp\left[-\frac{(\ln\lambda - \langle\ln\lambda(M,z)\rangle)^2}{2\sigma^2_{\ln\lambda}}\right]$$

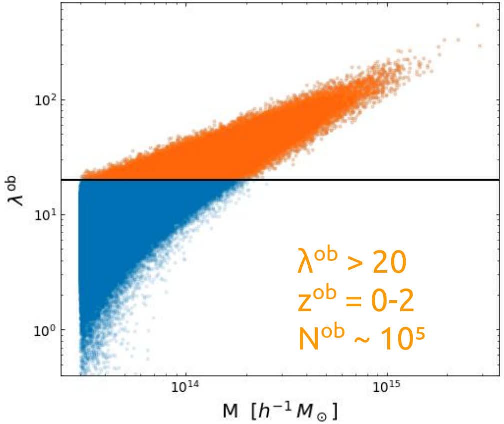
with

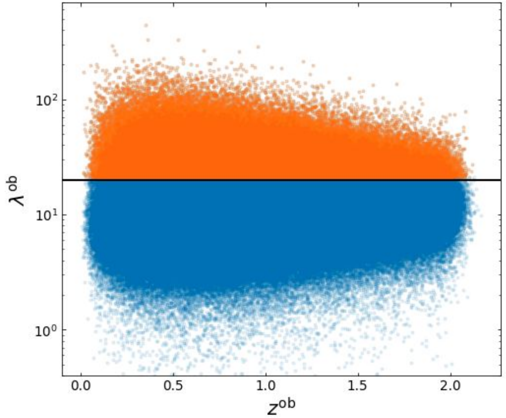
$$\langle\ln\lambda(M,z)\rangle = \ln(A_\lambda) + B_\lambda \ln\left(\frac{M}{3\times10^{14}\,h^{-1}\,M_\odot}\right) + C_\lambda \ln\left(\frac{1+z}{1+0.45}\right)$$

$$\sigma_{\ln\lambda}(M,z) = D_\lambda\,,$$

+ selection functions $P(\lambda^{ob}|\lambda,z)$ and $P(z^{ob}|z)$ to add observational inaccuracy

Covariance model validation for richness-selected clusters
➡ confirm the results in mass-space



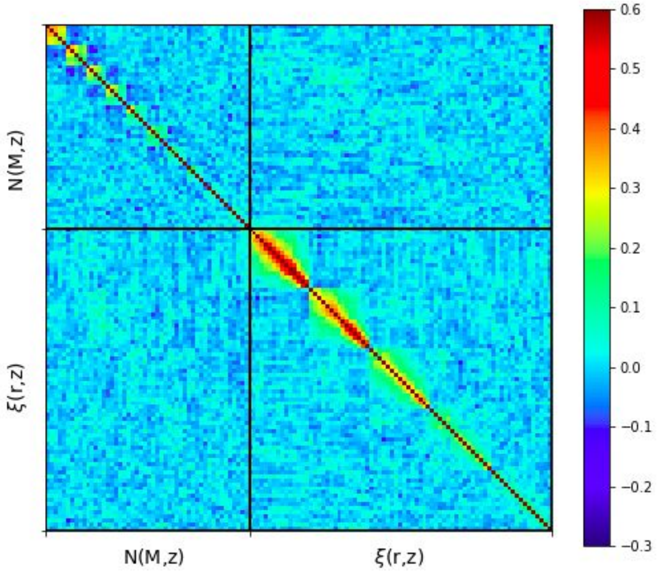


Joint analysis

Euclid Collaboration: Fumagalli et al. *in preparation*

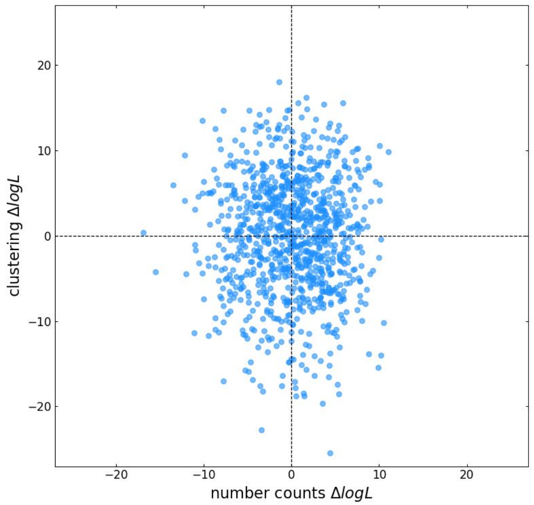
Negligible cross-correlation:

- number counts and clustering independent observables
- Pearson correlation coefficient: **ρ = -0.015 ± 0.032**
- Result in agreement with Mana et al. 2013 results



Independent likelihood functions:

$$\Longrightarrow \mathscr{L}_{TOT} = \mathscr{L}_{NC} \times \mathscr{L}_{CL}$$



$$\Delta\log\mathscr{L}^n = \log\mathscr{L}^n - \langle\log\mathscr{L}\rangle$$

$n = 1, \ldots, N_{mocks}$

Joint analysis

Euclid Collaboration: Fumagalli et al. *in preparation*

Likelihood forecasts for Euclid-like survey: constraints on cosmological and mass-observable relation (MoR) parameters

$A_\lambda, B_\lambda, C_\lambda, D_\lambda$ with Gaussian priors with 0,1,3,5% amplitude

Combined analysis:
cosmological constraints improved by 20 - 90 %, depending on MoR uncertainty

MoR prior	Probe	FoM	ΔFoM / FoM_{NC}
0 %	NC	1971760 ± 24660	–
	CL	81018 ± 231	−95 %
	NC + CL	2409898 ± 62519	+ 22 %
1 %	NC	186780 ± 4254	–
	CL	50875 ± 954	−72 %
	NC + CL	225765 ± 3593	+ 20 %
3 %	NC	39301 ± 708	–
	CL	22826 ± 839	−41 %
	NC + CL	64562 ± 1208	+ 64 %
5 %	NC	20572 ± 315	–
	CL	14766 ± 253	−28 %
	NC + CL	39052 ± 720	+90 %

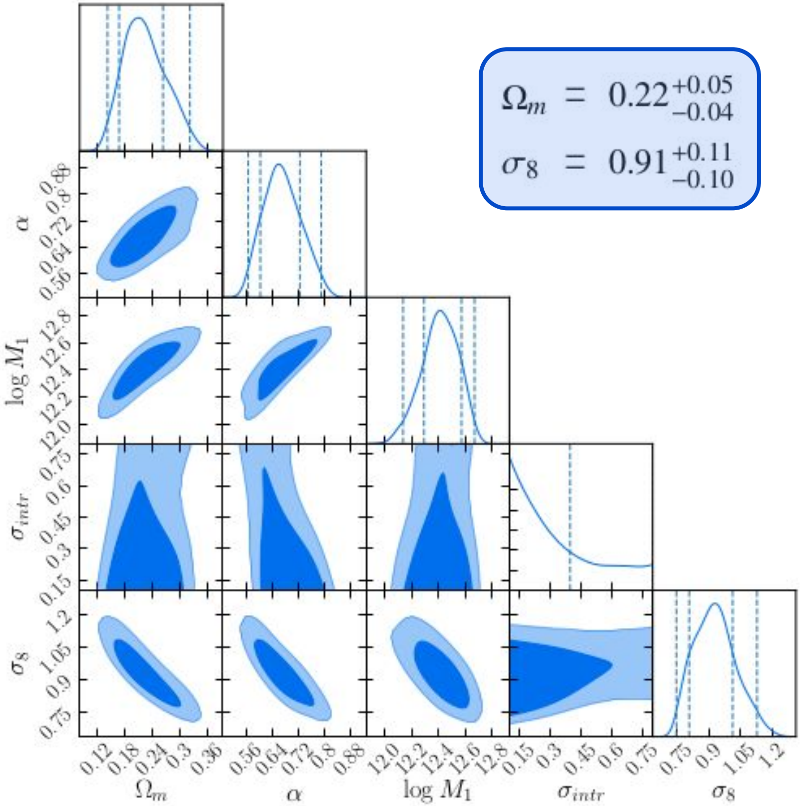
Application to SDSS data

Fumagalli et al. *in preparation*

Dataset:
redMaPPer cluster catalog from
Sloan Digital Sky Survey data release 8 (SDSS DR8)

Catalog:
Sky area: $\Omega \sim 10\,000\ \mathrm{deg}^2$
Redshift: $z = 0.1 - 0.3$
Richness: $\lambda \geq 20$
#cluster: $N \sim 7 \times 10^4$

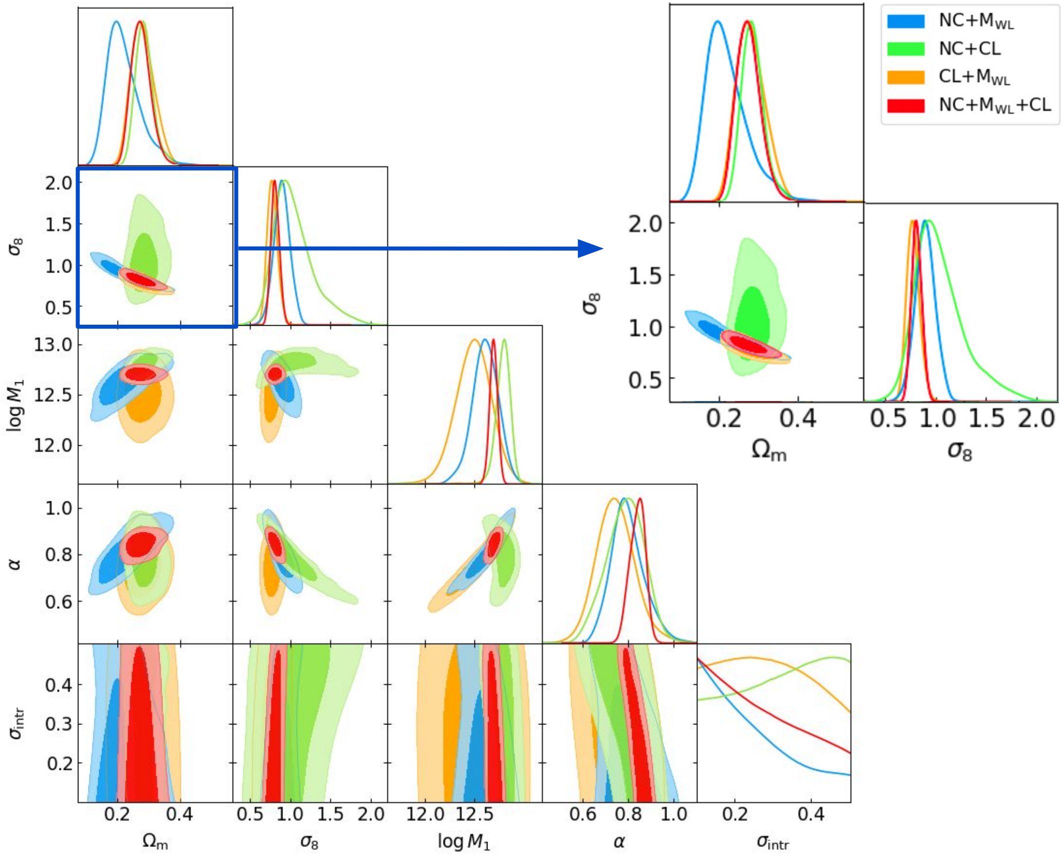
Analysis:
repeat the analysis by Costanzi et al. 2018
(number counts + weak lensing masses)
with addition of cluster clustering



$$\Omega_m = 0.22^{+0.05}_{-0.04}$$
$$\sigma_8 = 0.91^{+0.11}_{-0.10}$$

Application to SDSS data

Fumagalli et al. *in preparation*



Results:

- CL helps to constrain Ω_{m}
- NC+CL don't constrain MoR ⇒ σ_8
- M_{WL} +CL better than NC+M_{WL}
- high improvement from NC+CL+M_{WL}
- different systematics on CL and NC

Conclusions

Number counts:

- accurate analytical covariance model
- Gaussian likelihood with cosmology-dependent full covariance

Clustering:

- accurate semi-analytical covariance model
- 2PCF covariance contains cosmological information ($\propto 1/n$) that is not present in the mean value
- useful information also when considering mass binning

Joint:

- improved cosmological constraints (~ 20-90% improvement)
- ~160% improvement from combined analysis($\Omega_{\square}$ = 0.27 ± 0.03 , σ_8 = 0.81 ± 0.05)
- different systematics between cluster counts and clustering
  ⇒ can help to improve mass calibration?

Backup slides

Observable space

$$z_{\rm obs} = z_{\rm c} + \frac{v_\parallel}{c}(1+z_{\rm c}) \pm \sigma_z$$

$$P(k,\mu) = P_{\rm DM}(k)(b + f\mu^2)^2 \exp(-k^2\mu^2\sigma^2)$$

$$\xi(s) = b^2\xi'(s) + b\xi''(s) + \xi'''(s)$$

$$P'(k) = P_{\rm DM}(k)\frac{\sqrt{\pi}}{2k\sigma}\mathrm{erf}(k\sigma)$$

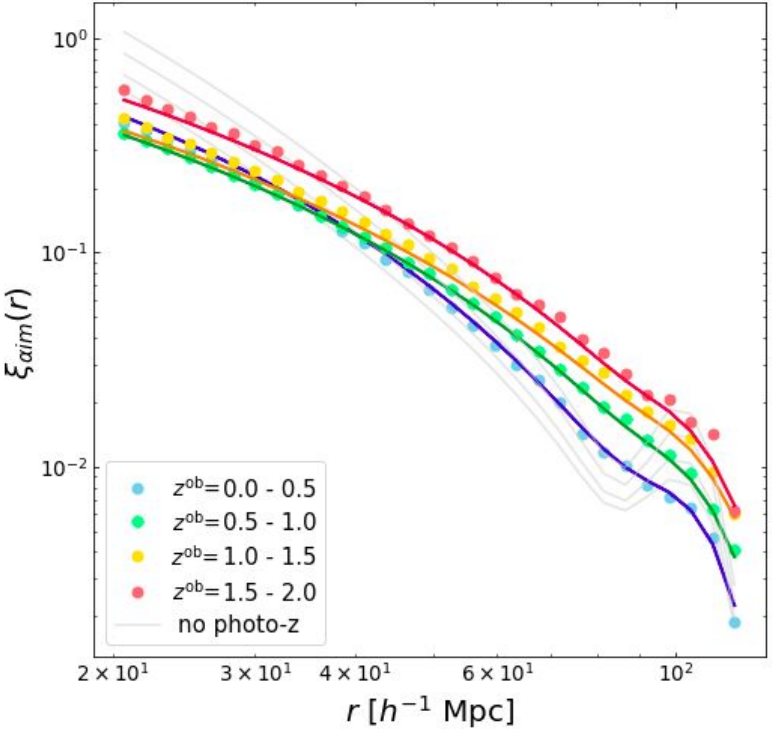
$$P''(k) = \frac{f}{(k\sigma)^3}P_{\rm DM}(k)\left[\frac{\sqrt{\pi}}{2}\mathrm{erf}(k\sigma) - k\sigma\exp(-k^2\sigma^2)\right]$$

$$P'''(k) = \frac{f^2}{(k\sigma)^5}P_{\rm DM}(k)\left\{\frac{3\sqrt{\pi}}{8}\mathrm{erf}(k\sigma) - \frac{k\sigma}{4}\left[2(k\sigma)^2+3\right]\exp(-k^2\sigma^2)\right\}$$

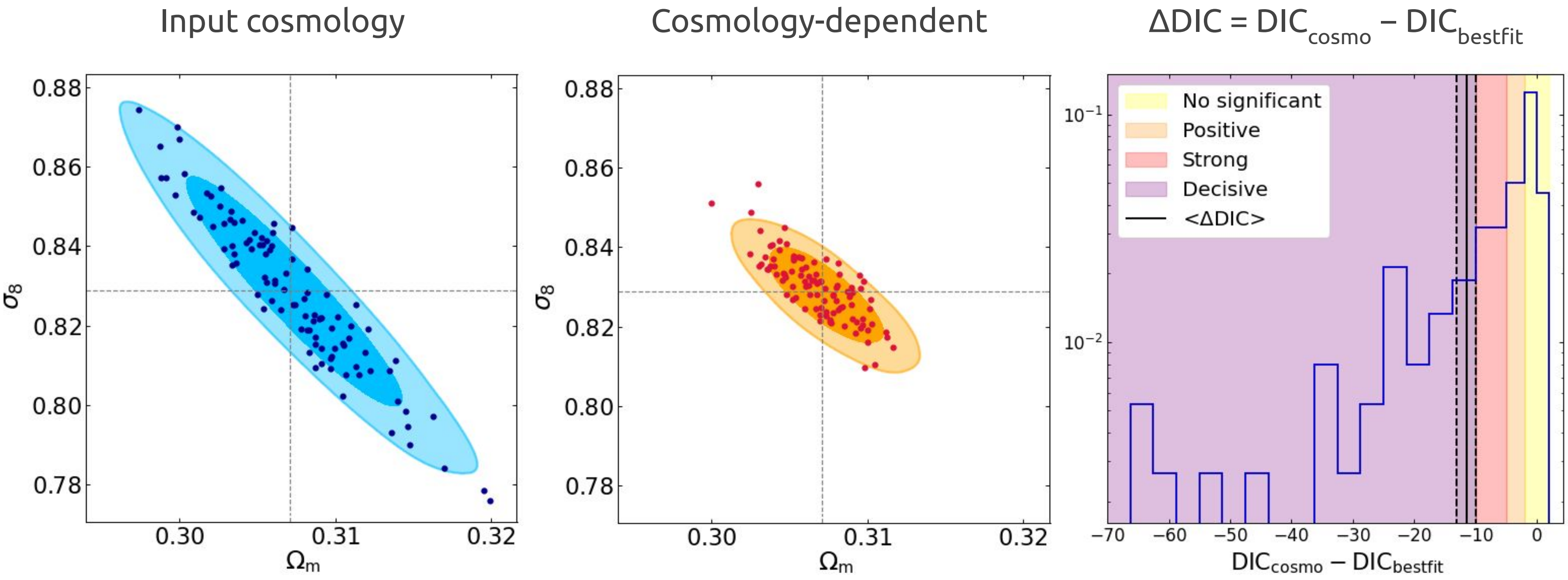
$$\sigma = \frac{c\sigma_z}{H(z)}$$

from Sereno et a. 2015

Cluster clustering

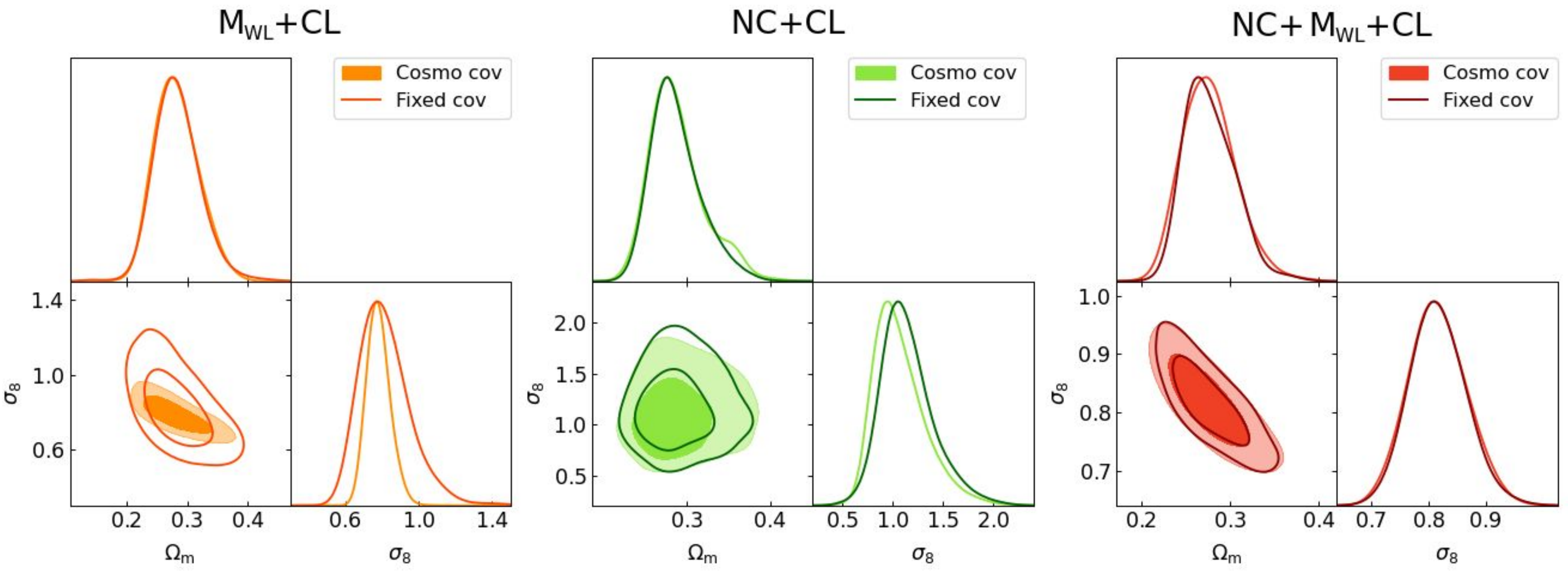


Clustering: cosmology-dependence

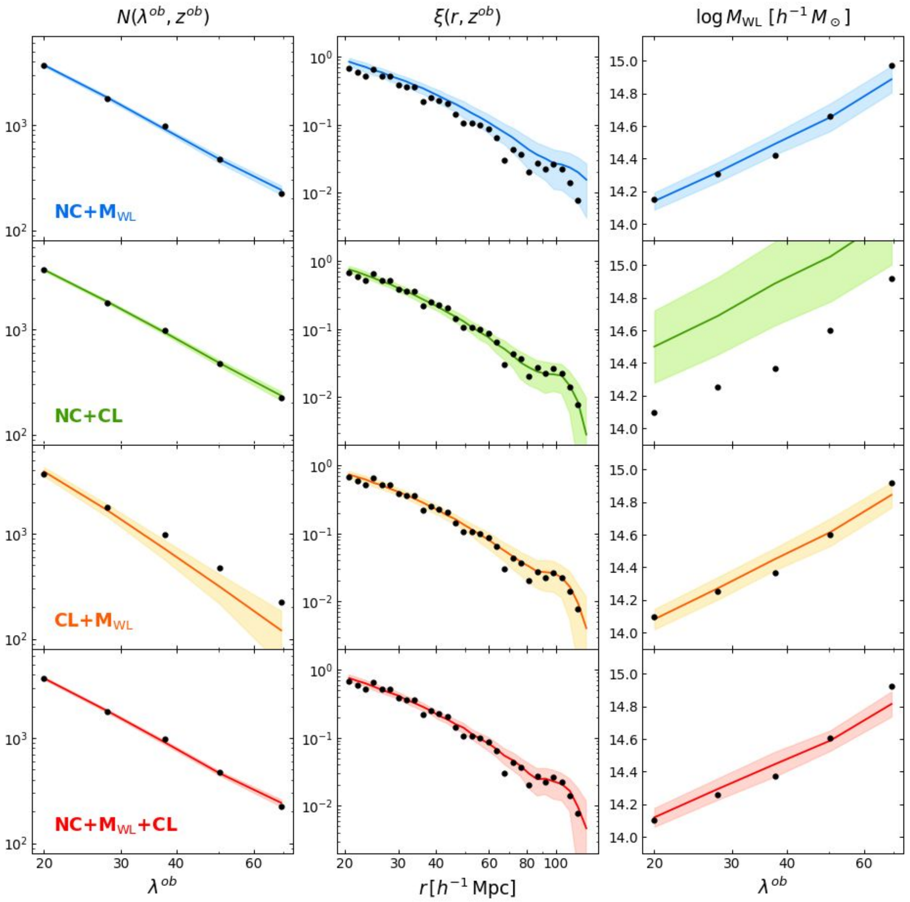


Cosmology-dependent covariance statistically preferred (<ΔDIC> = −11.5 ± 1.6)

Application to SDSS data



Application to SDSS data



Covariance fit

Set of M simulations, producing measurements **mi** with dimension N
Model covariance C(θ), with θ=model parameters

Gaussian likelihood

$$\mathcal{L}(\mathbf{d}|\theta) = \prod_{i=1\ldots M} |C(\theta)|^{-1/2} \; \exp\left(-\tfrac{1}{2}\, \mathbf{d}_i^T C^{-1}(\theta)\, \mathbf{d}_i \right)$$

$$\mathbf{d}_i = \mathbf{m}_i - \langle \mathbf{m} \rangle$$

$$\log \mathcal{L}(\mathbf{d}|\theta) = -\frac{M}{2}\left(\log |C(\theta)| + \mathrm{Tr}(C^{-1}(\theta) C_n)\right)$$

$$C_n = \frac{1}{M} \sum_{i=1\ldots M} \mathbf{d}_i \mathbf{d}_i^T$$

MCMC process to maximise log$\mathcal{L}$ and fit θ

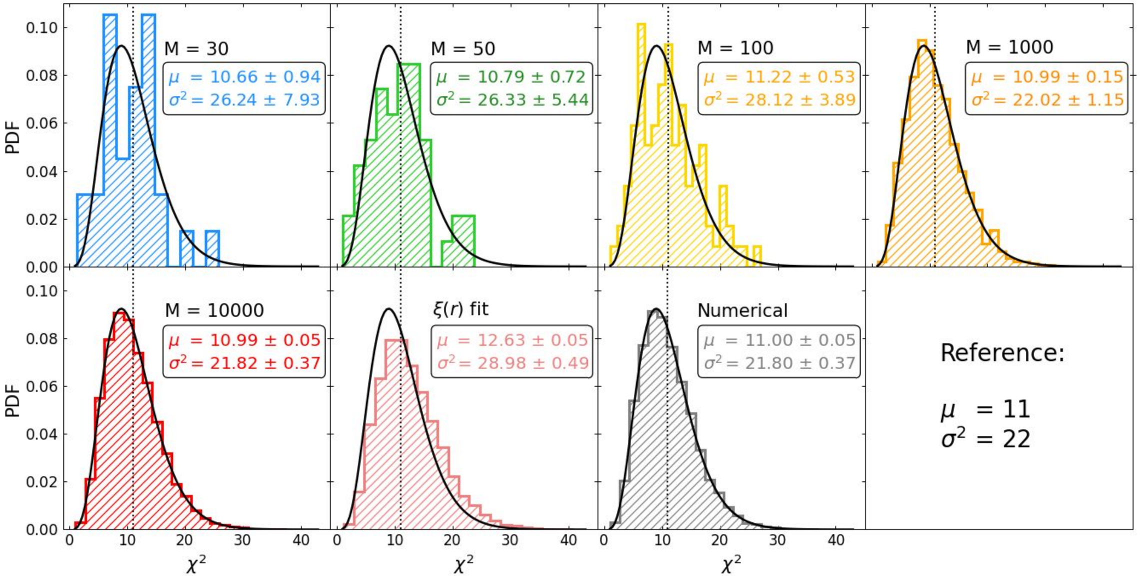
Covariance parameters fit

Fumagalli et al. 2022

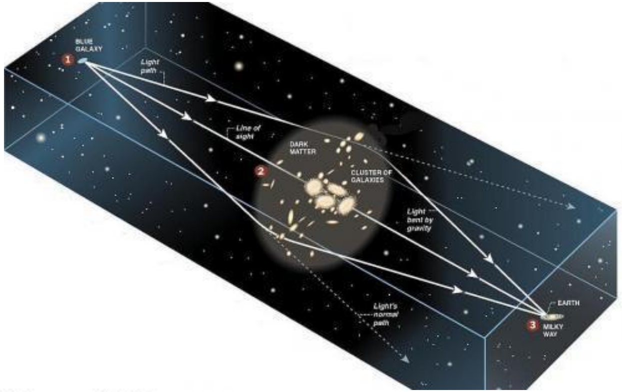
Bayesian method for fitting the covariance model parameters by examining the χ^2 **distributions** from simulations:

A good covariance produces χ_i^2 values distributed following a χ^2 distribution

$$\chi_i^2 = (\mathbf{m}_i - \langle \mathbf{m} \rangle) C^{-1} (\mathbf{m}_i - \langle \mathbf{m} \rangle)^T \qquad i = 1, \ldots, M$$



Weak-lensing mass calibration



Weak-lensing signal: tangential alignment of background galaxies around the foreground cluster due to gravitational lensing

$\Longrightarrow$ probe the projected mass distribution of clusters

Observed WL mass:

$$\log_{10}\hat{M}_{\rm WL}(\Omega_{\rm m}) = \log_{10}\hat{M}_{\rm WL}\big|_{\Omega_{\rm m}=0.3} + \frac{{\rm d}\log_{10}M_{\rm WL}}{{\rm d}\Omega_{\rm m}}(\Omega_{\rm m}-0.3)$$

Expected WL mass:

$$M_{ai}^{\rm WL} = \frac{1}{N_{ai}}\Omega_{\rm sky}\int_0^\infty {\rm d}z\,\frac{{\rm d}V}{{\rm d}z\,{\rm d}\Omega}(z)\langle M_i n_i\rangle(z)\int_{\Delta z_a^{\rm ob}}{\rm d}z^{\rm ob}P(z^{\rm ob}|z,\Delta\lambda_i^{\rm ob})$$

$$\langle M_i n_i\rangle(z) = \int_0^\infty {\rm d}M\,M\,\frac{{\rm d}n}{{\rm d}M}(M,z)\int_{\Delta\lambda_i^{\rm ob}}{\rm d}\lambda^{\rm ob}\int_0^\infty {\rm d}\lambda\,P(\lambda^{\rm ob}|\lambda,z)\,P(\lambda|M,z)$$

Methods

- **Validate analytical models**, by comparison with numerical matrix:
  evaluate which terms are important or negligible, add nuisance parameters to improve accuracy, . . .

- **Test covariance models in cosmological analysis,** by constraining Ω_m and σ_8 through Bayesian inference:
  comparison of different likelihood and covariance configurations

$$p(\theta|d) \propto \mathcal{L}(d|\theta)\, p(\theta)$$

d = data
θ = parameters in the model

p(θ|d) = posterior distribution
$\mathcal{L}$(d|θ) = likelihood function
p(θ) = prior distribution

$$\mathrm{FoM}(\Omega_\mathrm{m}, \sigma_8) = \frac{1}{\sqrt{\det C(\Omega_\mathrm{m}, \sigma_8)}}$$

$$\mathcal{L}(\mathbf{d}|\boldsymbol{\theta}) = \frac{\exp\left\{-\frac{1}{2}[\mathbf{d}-\mathbf{m}(\boldsymbol{\theta})]^T C^{-1} [\mathbf{d}-\mathbf{m}(\boldsymbol{\theta})]\right\}}{\sqrt{(2\pi)^N |C|}}$$