

Basic statistics, and applications to X-ray spectral fitting

- ✓ Normal error (Gaussian) distribution
 - most important in statistical analysis of data, describes the distribution of random observations for many experiments
- ✓ Poisson distribution
 - generally appropriate for counting experiments related to random processes (e.g., radioactive decay of elementary particles)
- ✓ Statistical tests: χ^2 and F-test (more on these in the XSPEC tutorial)
- ✓ Additional specific applications within XSPEC in the X-ray spectral analysis tutorial

All measurements should be provided with errors

- Measurement $X \pm \delta X$ (units of measure)



Error associated with the measurement X

- Significant digits:

g (gravitational acceleration of an object in a vacuum near the Earth surface)=
 $=9.82 \pm 0.02385 \text{ m/s}^2 \rightarrow 9.82 \pm 0.02 \text{ m/s}^2$

Another example: $v = 100.2 \pm 30 \text{ m/s} \rightarrow 100 \pm 30 \text{ m/s}$

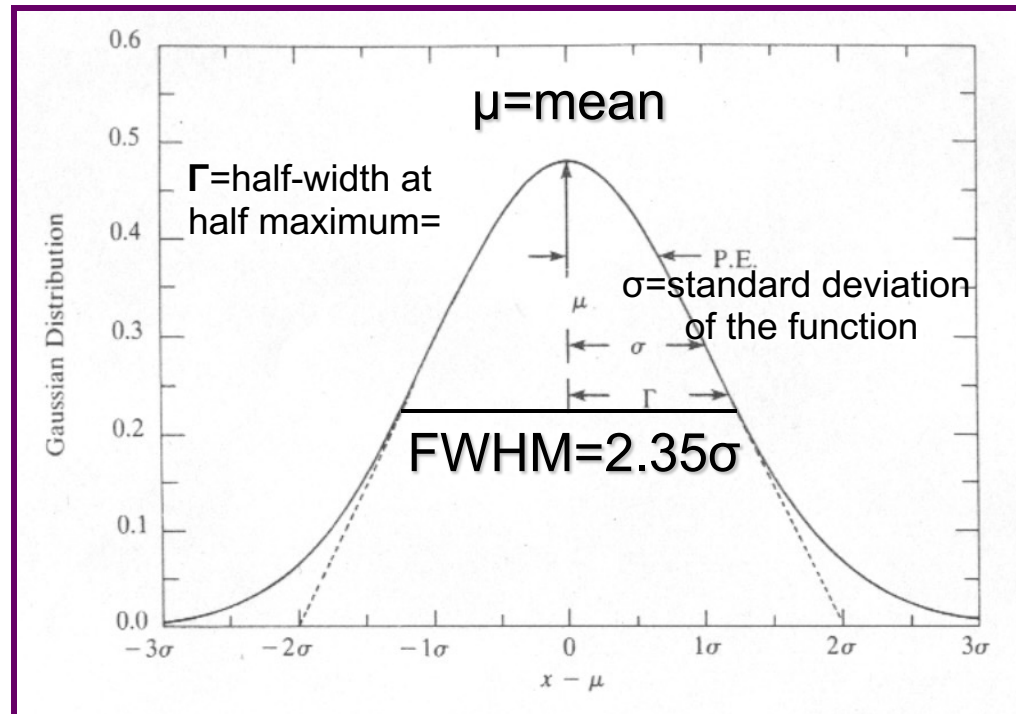
- Relative (fractionary) uncertainty: $\delta X/X$

The Gaussian (normal error) distribution. I

Averages of random variables (sufficiently large in number) independently drawn from independent distributions converge in distribution to the normal

Casual errors are above and below the “true” (most “common”) value

→ bell-shape distribution if systematic errors are negligible



The Gaussian probability function. II

$$P(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x - \mu)^2}{2\sigma^2}}$$

Probability Density Function
(centered on μ)

μ =mean (expectation) value
 σ =standard deviation
 σ^2 =variance

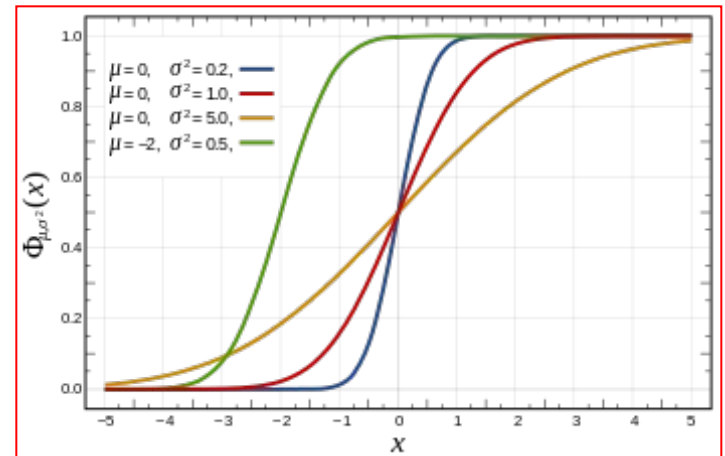


normalization factor, so that $\int f(x) dx = 1$

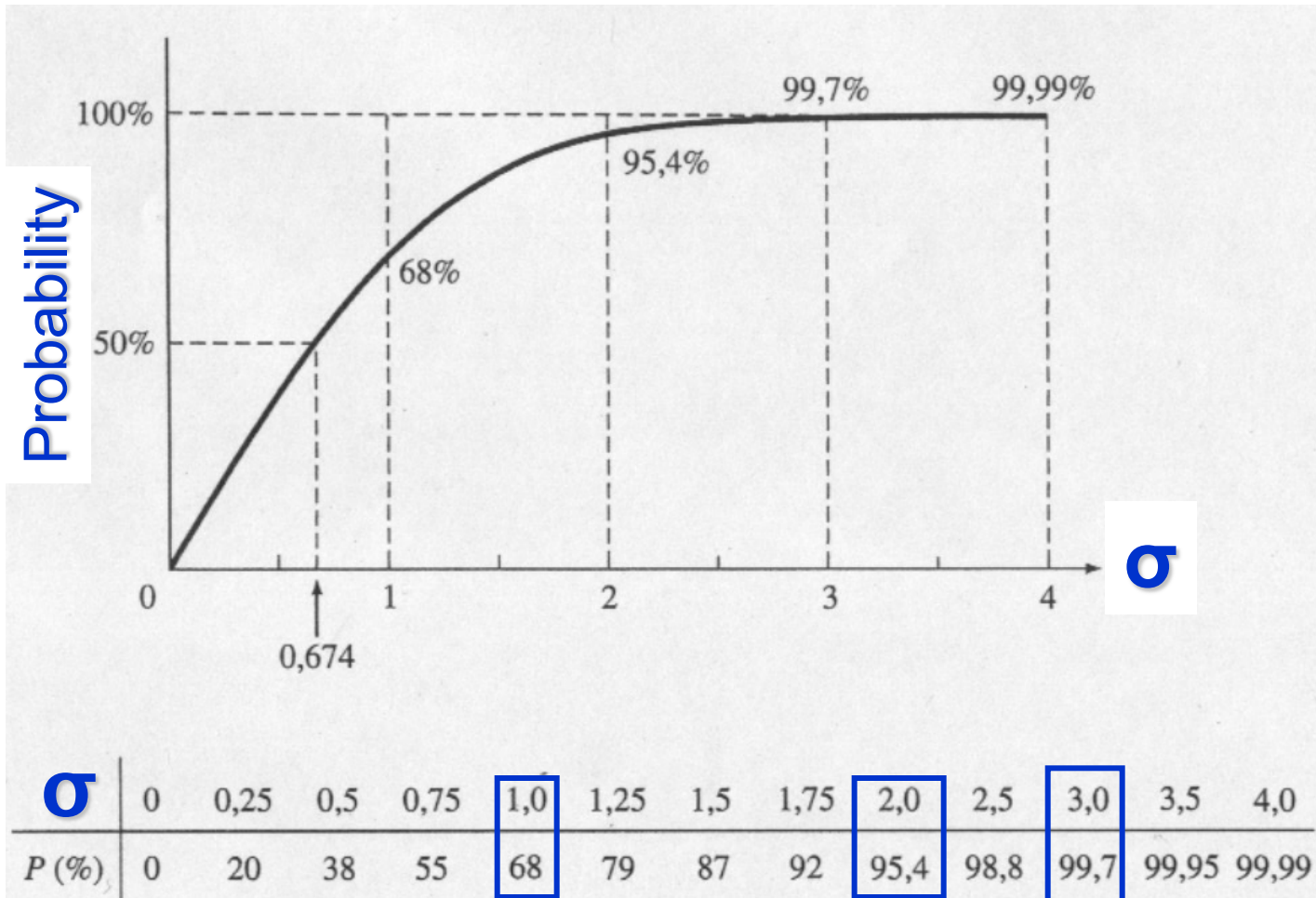


$$e^{-x^2 / 2\sigma^2}$$

function centered on 0



The Gaussian probability function. III



$$F(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^x e^{-(x-\mu)^2 / 2\sigma^2} dx$$

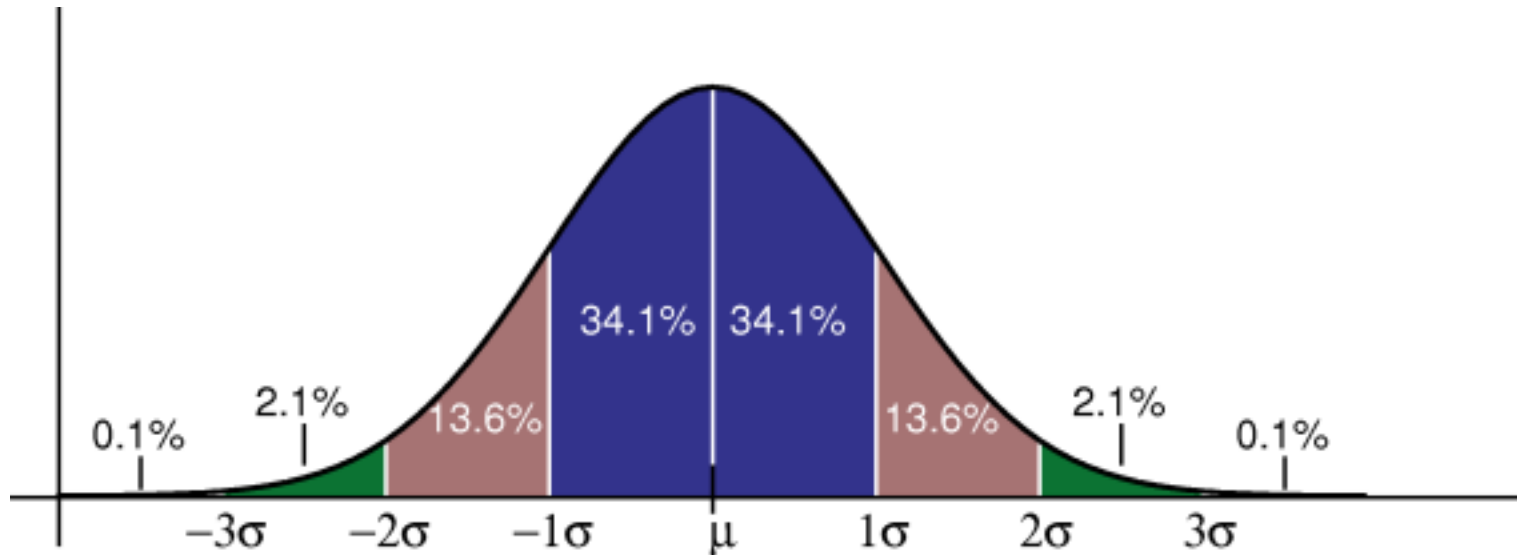
Cumulative Distribution Function

Chance to be
outside the range

Probability within $\pm n\sigma$

1σ	1/3
2σ	1/22
3σ	1/370
4σ	1/15787
5σ	1/1744277

1σ	68.3%
2σ	95.45%
3σ	99.730%
4σ	99.99367%
5σ	99.999943%



$$F(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^x e^{-(x-\mu)^2 / 2\sigma^2} dx$$

Cumulative Distribution Function

Value \pm error at 1σ confidence level: if we make a measurement N times, in 68.3% of the times we obtain such value. Every measurement should be reported and considered along its own error

Percentage probability P within $\pm\sigma$: $P = \int_{X-t\sigma}^{X+t\sigma} G(x)dx$



Bevington textbook

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80
2.3	97.86	97.91	97.97	98.02	98.07	98.12	98.17	98.22	98.27	98.32
2.4	98.36	98.40	98.45	98.49	98.53	98.57	98.61	98.65	98.69	98.72
2.5	98.76	98.79	98.83	98.86	98.89	98.92	98.95	98.98	99.01	99.04
2.6	99.07	99.09	99.12	99.15	99.17	99.20	99.22	99.24	99.26	99.29
2.7	99.31	99.33	99.35	99.37	99.39	99.40	99.42	99.44	99.46	99.47
2.8	99.49	99.50	99.52	99.53	99.55	99.56	99.58	99.59	99.60	99.61
2.9	99.63	99.64	99.65	99.66	99.67	99.68	99.69	99.70	99.71	99.72

3.0 99.73
3.5 99.95

4.0 99.994
4.5 99.9993

5.0 99.99994

→ $3\sigma=99.73\%$: in 1000 experiments you can get results outside this $\pm 3\sigma$ range three times

$5\sigma=99.99994\%$:

6 cases out of 10^6

The Poisson distribution

Describes experimental results where events are counted and the uncertainty is not related to the measurement but reflects the intrinsically casual behavior of the process (e.g., radioactive decay of particles (Geiger counter), X-ray photons, etc.)

$$P(x) = e^{-\mu} \mu^x / x! \quad (x=0,1,2, \dots)$$

Probability of obtaining x events when μ events are expected
 x =observed number of events in a time interval (frequency of events)

average
number
of events

$$\bar{x} = \sum_{x=0}^{\infty} xP(x) = \sum_{x=0}^{\infty} x e^{-\mu} \mu^x / x! = \mu$$

→ μ =average number of expected events if the experiment is repeated many times

$$\sigma^2 = \langle (x - \mu)^2 \rangle =$$

expectation value of the
square of the deviations

$$= \sum_{x=0}^{\infty} (x - \mu)^2 \frac{\mu^x}{x!} e^{-\mu} = \mu$$



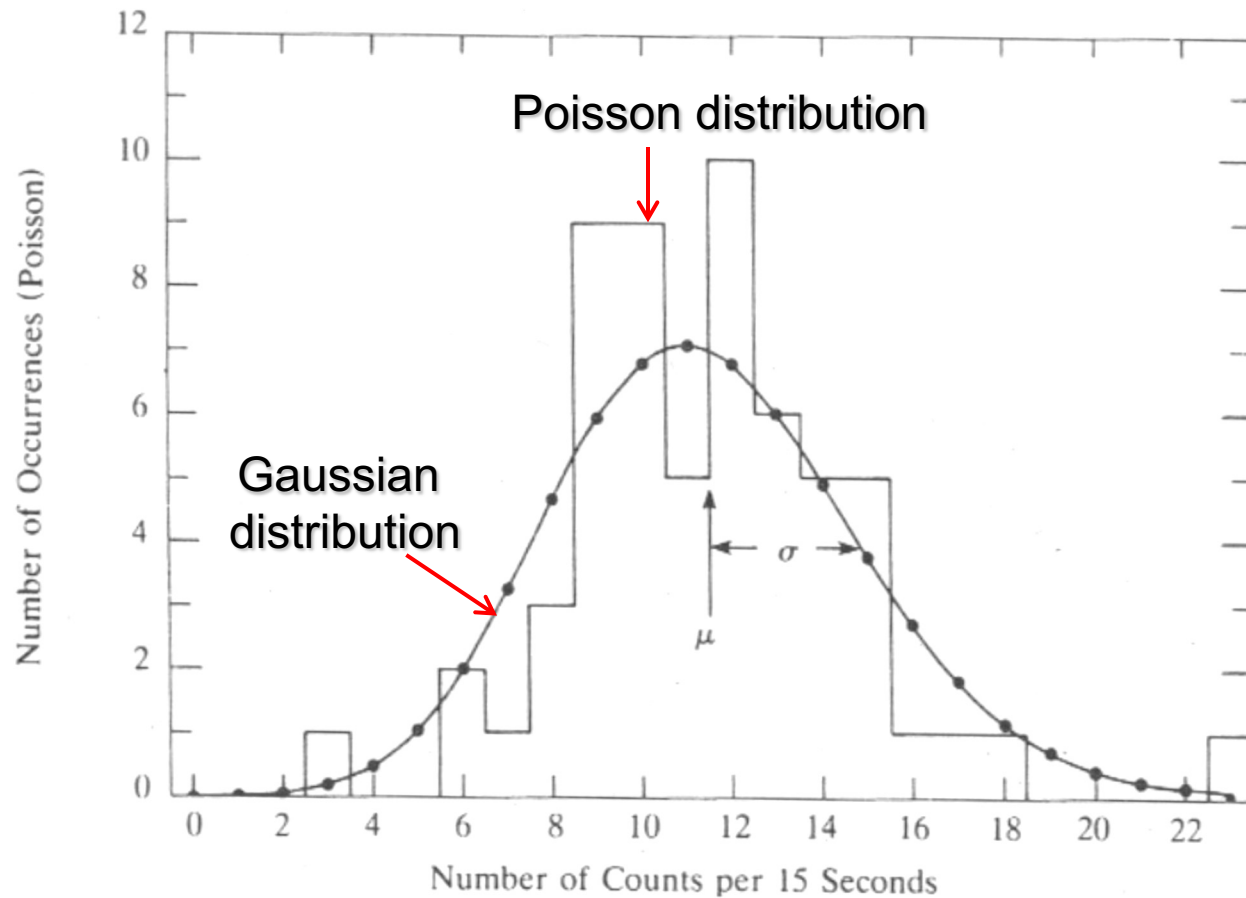
the Poisson distribution with average counts= μ has standard deviation $\sqrt{\mu}$



Example: $N_{\text{counts}} \pm \sqrt{N}$

High μ : the Poisson distribution is approximated by the Gaussian distribution

defined by only one parameter μ



Statistical test: χ^2 – more on XSPEC tutorial

Test to compare the observed distribution of the results with that expected

$$\chi^2 = \sum_{k=1}^n \frac{(O_k - E_k)^2}{\sigma_k^2}$$

It provides a measure on how much the data differ from the expectations (model), taking into account the errors associated with the measurement (e.g., datapoints)

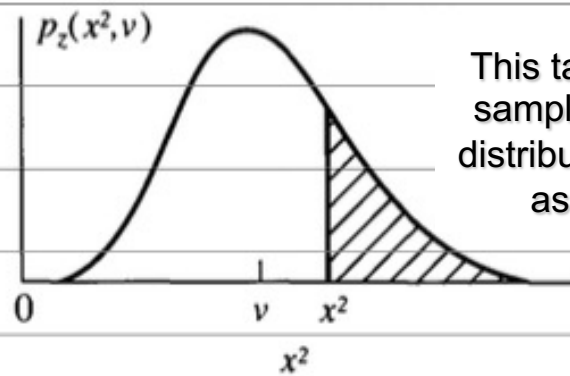
- O_k =observed values (e.g., spectral datapoints)
- E_k =expected values (model, i.e. predicted distribution)
- σ_k =error on the measured values (e.g., error on each spectral bin)
- k =number of datapoints (bins after rebinning)

$$\chi^2 / dof \approx 1$$



the observed and expected distributions are similar

dof=degrees of freedom = #datapoints – #free parameters



This table gives the probability that a random sample of data, when compared to its parent distribution, would yield values of X^2/v as large as (or larger than) the observed value

TABLE C.4

χ^2 distribution. Values of the reduced chi-square $\chi_v^2 = \chi^2/v$ corresponding to the probability $P_\chi(\chi^2; v)$ of exceeding χ^2 versus the number of degrees of freedom v

$v = \text{dof} = \# \text{datapoints} - \# \text{free parameters}$

v	P							
	0.99	0.98	0.95	0.90	0.80	0.70	0.60	0.50
1	0.00016	0.00063	0.00393	0.0158	0.0642	0.148	0.275	0.455
2	0.0100	0.0202	0.0515	0.105	0.223	0.357	0.511	0.693
3	0.0383	0.0617	0.117	0.195	0.335	0.475	0.623	0.789
4	0.0742	0.107	0.178	0.266	0.412	0.549	0.688	0.839
5	0.111	0.150	0.229	0.322	0.469	0.600	0.731	0.870
6	0.145	0.189	0.273	0.367	0.512	0.638	0.762	0.891
7	0.177	0.223	0.310	0.405	0.546	0.667	0.785	0.907
8	0.206	0.254	0.342	0.436	0.574	0.691	0.803	0.918
9	0.232	0.281	0.369	0.463	0.598	0.710	0.817	0.927
10	0.256	0.306	0.394	0.487	0.618	0.727	0.830	0.934
11	0.278	0.328	0.416	0.507	0.635	0.741	0.840	0.940
12	0.298	0.348	0.436	0.525	0.651	0.753	0.848	0.945
13	0.316	0.367	0.453	0.542	0.664	0.764	0.856	0.949
14	0.333	0.383	0.469	0.556	0.676	0.773	0.863	0.953
15	0.349	0.399	0.484	0.570	0.687	0.781	0.869	0.956

Statistical test: F-test – more on XSPEC tutorial

If two statistics following the χ^2 distribution have been determined, the ratio of the reduced chi-squares is distributed according to the F distribution

$$P_f(f; \nu_1, \nu_2) = \frac{\chi_1^2 / \nu_1}{\chi_2^2 / \nu_2}$$



$$\propto \Delta\chi^2 / k$$

with k=number of additional terms (parameters)



Example: Use the F-test to evaluate the improvement to a spectral fit due to the assumption of a different model, with additional terms

Conditions: (a) the simpler model is nested within the more complex model;
(b) the extra parameters have Gaussian distribution (not truncated by the parameter space boundaries)

→ see the F-test tables for the corresponding probabilities (specific command in XSPEC)

An application of the F-test within XSPEC

```

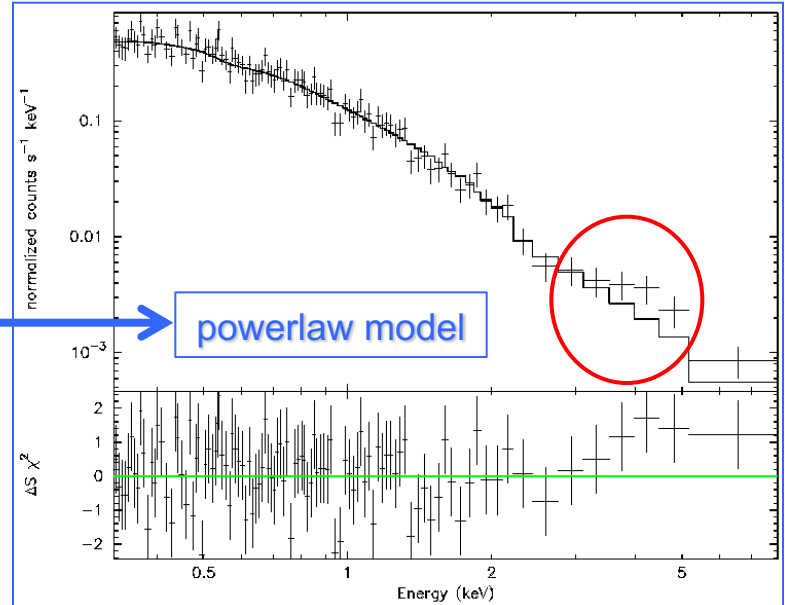
=====
Model phabs<1>*powerlaw<2> Source No.: 1   Active/On
Model Model Component Parameter Unit      Value
par  comp
  1    1   phabs      nH          10^22    1.59000E-02  frozen
  2    2   powerlaw   PhoIndex      2.72811    +/-  0.0
  3    2   powerlaw   norm          1.51490E-04 +/-  0.0
    
```

Using energies from responses.

Model1

```

Chi-Squared =          97.23 using 105 PHA bins.
Reduced chi-squared =          0.9440 for    103 degrees of freedom
Null hypothesis probability =    6.417127e-01
    
```



```

=====
Model phabs<1>(laor<2> + powerlaw<3>) Source No.: 1   Active/On
Model Model Component Parameter Unit      Value
par  comp
  1    1   phabs      nH          10^22    1.59000E-02  frozen
  2    2   laor       lineE       keV      5.23582    +/-  0.0
  3    2   laor       Index        3.00000    frozen
  4    2   laor       Rin(G)       1.23500    frozen
  5    2   laor       Rout(G)      400.000    frozen
  6    2   laor       Incl         deg      30.0000    frozen
  7    2   laor       norm          6.83065E-06 +/-  0.0
  8    3   powerlaw   PhoIndex      2.77137    +/-  0.0
  9    3   powerlaw   norm          1.48123E-04 +/-  0.0
    
```

Using energies from response

Model1+extra component

```

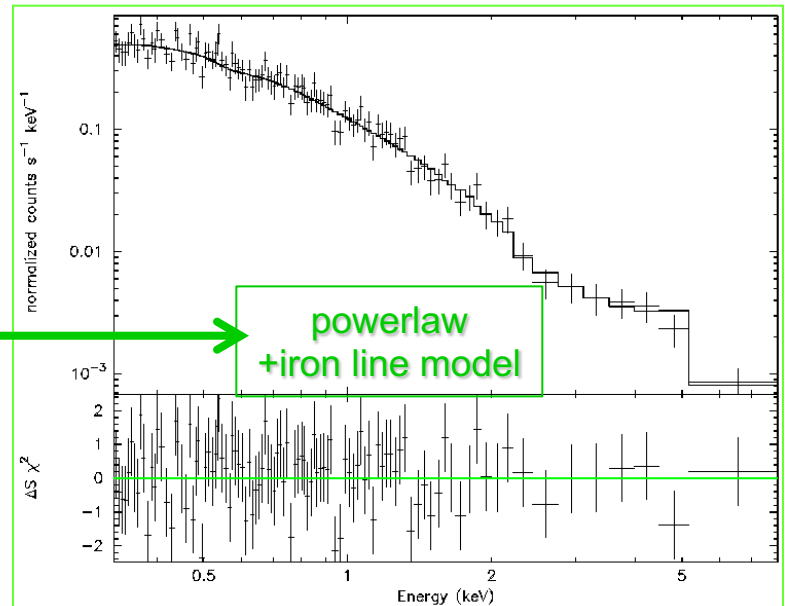
Chi-Squared =          90.84 using 105 PHA bins.
Reduced chi-squared =          0.8994 for    101 degrees of freedom
Null hypothesis probability =    7.557789e-01
    
```

Current data and model not fit
Weighting method: standard

XSPEC12>ftest 90.84 101 97.2 103

F statistic value = 3.53567 and probability 0.0327981

low F value ⇒ low significance of the added component



F-test probability in XSPEC: probability of exceeding F (see tabulated values)

Fit (2) = Fit (1) + one component

```
xspec> ftest  $\chi^2$  (best fit)  $\text{dof}$  (best fit)  $\chi^2$  (previous fit)  $\text{dof}$  (previous fit)
```

```
xspec> ftest 90.8 101 97.2 103 → ftest=3.54 → prob=0.0328
```

$$F_t = \left(\frac{\chi^2(\text{dof}) - \chi^2(\text{dof} - k)}{\text{dof} - (\text{dof} - k)} \right) / \left(\chi^2(\text{dof} - k) / (\text{dof} - k) \right) =$$
$$= (\Delta\chi^2 / k) / \chi^2_{\nu}$$

Ex: $\chi^2(103) = 97.23$
 $\chi^2(101) = 90.84$
 $\rightarrow \Delta\chi^2 = 6.39, k = 2 \rightarrow F_t = (6.39/2) / (90.84/101) = 3.55$

F_t follows the F distribution with $v_1=k=\Delta(\text{dof})$ and $v_2=\text{dof}-k(-1)$

Search in the F-distribution tables for the probability of the null hypothesis (H_0)
for $v_1=2$ and $v_2 \sim 100$

The significance of the improvement is given by
 $P=1-\text{prob}=1-0.032=96.8\%$ (i.e., not particularly significant)

Note of caution: F-test is an approximation (BUT quick); optimal solution would be running simulations (see Protassov+2002)

You simulate N times (1000, 10000 trials) within XSPEC (command *fakeit*) data (source and background) of the same quality as that of your original data (including also response matrices ARF and RMF) and fit them with the same modeling without the line (e.g., a powerlaw); you then verify how many times your feature is found purely by chance

If you find it X times, the significance of the line
$$=(1-X)/(\text{number of trials})$$

Percentage probability P within $\pm\sigma$: $P = \int_{X-\sigma}^{X+\sigma} G(x) dx$



shaded region
X between $-\sigma$ and $+\sigma$

(t means # in units of σ)

Compute the
significance of the
improvement in terms
of σ given $P=0.0328$,
hence $(1-P)=0.968$

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80
2.3	97.86	97.91	97.97	98.02	98.07	98.12	98.17	98.22	98.27	98.32
2.4	98.36	98.40	98.45	98.49	98.53	98.57	98.61	98.65	98.69	98.72
2.5	98.76	98.79	98.83	98.86	98.89	98.92	98.95	98.98	99.01	99.04
2.6	99.07	99.09	99.12	99.15	99.17	99.20	99.22	99.24	99.26	99.29
2.7	99.31	99.33	99.35	99.37	99.39	99.40	99.42	99.44	99.46	99.47
2.8	99.49	99.50	99.52	99.53	99.55	99.56	99.58	99.59	99.60	99.61
2.9	99.63	99.64	99.65	99.66	99.67	99.68	99.69	99.70	99.71	99.72
3.0	99.73									
3.5	99.95									
4.0	99.994									
4.5	99.9993									
5.0	99.99994									

$P=96.8\% \rightarrow \approx 2.1\sigma$

Gaussian probability table

$$v_1=2$$

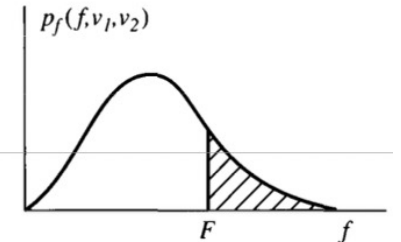
TABLE 5 (Contd.)

$P(F) = 0.05$

$f_2 \backslash f_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.09	251.14	252.20	253.25	254.32
2	18.51	19.00	19.25	19.39	19.50	19.58	19.65	19.71	19.76	19.80	19.84	19.87	19.90	19.92	19.94	19.96	19.98	19.99	20.00
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.84	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.66	2.60	2.53	2.45	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.54	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.44	2.37	2.30	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.17	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

$P(F)=0.05$

F-test probability in XSPEC: probability of exceeding F (see tabulated values)



$$F_{\text{xspect}}=3.54$$

$P(F) = 0.025$

$f_2 \backslash f_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63	976.71	984.87	993.10	997.25	1001.4	1005.6	1009.8	1014.0	1018.3
2	38.51	39.00	39.16	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.42	39.43	39.45	39.46	39.47	39.48	39.49	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.74	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.76	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.58	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.06	1.99	1.91	1.83
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.04	1.97	1.89	1.81
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.43	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.31	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.49
120	5.11	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.33
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00