UNIVERSITY OF TRENTO

TIFPA Trento Institute for Fundamental Physics and Applications

"Cosmic Magnetism in Voids and Filaments"
Bologna, 23rd-27th Jan. 2023

INFLATIONARY HELICAL MAGNETIC FIELDS WITH A SAWTOOTH COUPLING

arXiv: 2301.07699

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January, 23rd 2023

OUTLINE

1. HELICAL MAGNETIC FIELDS DURING INFLATION
2. APPLICATION TO SCALE-INVARIANT GRAVITY
3. RESULTS

MAGNETOGENESIS DURING INFLATION: RATRA MODEL

B. Ratra ApJ 391 (1992)

- ➤ Amplification of magnetic field spatial irregularities
- ➤ Conformal invariance breaking
- ➤ Add a time-dependent coupling $I(t) = I[\phi, \ldots]$

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\, I^2[\phi(t)]\, F_{\mu\nu}F^{\mu\nu}\right] + \int d^4x\sqrt{-g}\,\mathscr{L}_{\phi}$$

↓ *Test* EM field

↓ Inflationary background evolution

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↓ *Test* EM field

↓ Inflationary background evolution

REHEATING: conductivity jumps to large values

- Electric field is shorted out
- Magnetic field decays $B \sim a^{-2}$

MODELLING THE COUPLING FUNCTION

$$I(\phi) \rightarrow I(\eta) = a(\eta)^n$$

$$\mathscr{A}''(\eta, k) + \left(k^2 - \frac{n(n+1)}{\eta^2}\right)\mathscr{A}(\eta, k) = 0$$

$$\mathscr{A}(\eta, k) \equiv \frac{A(\eta, k)}{I}$$

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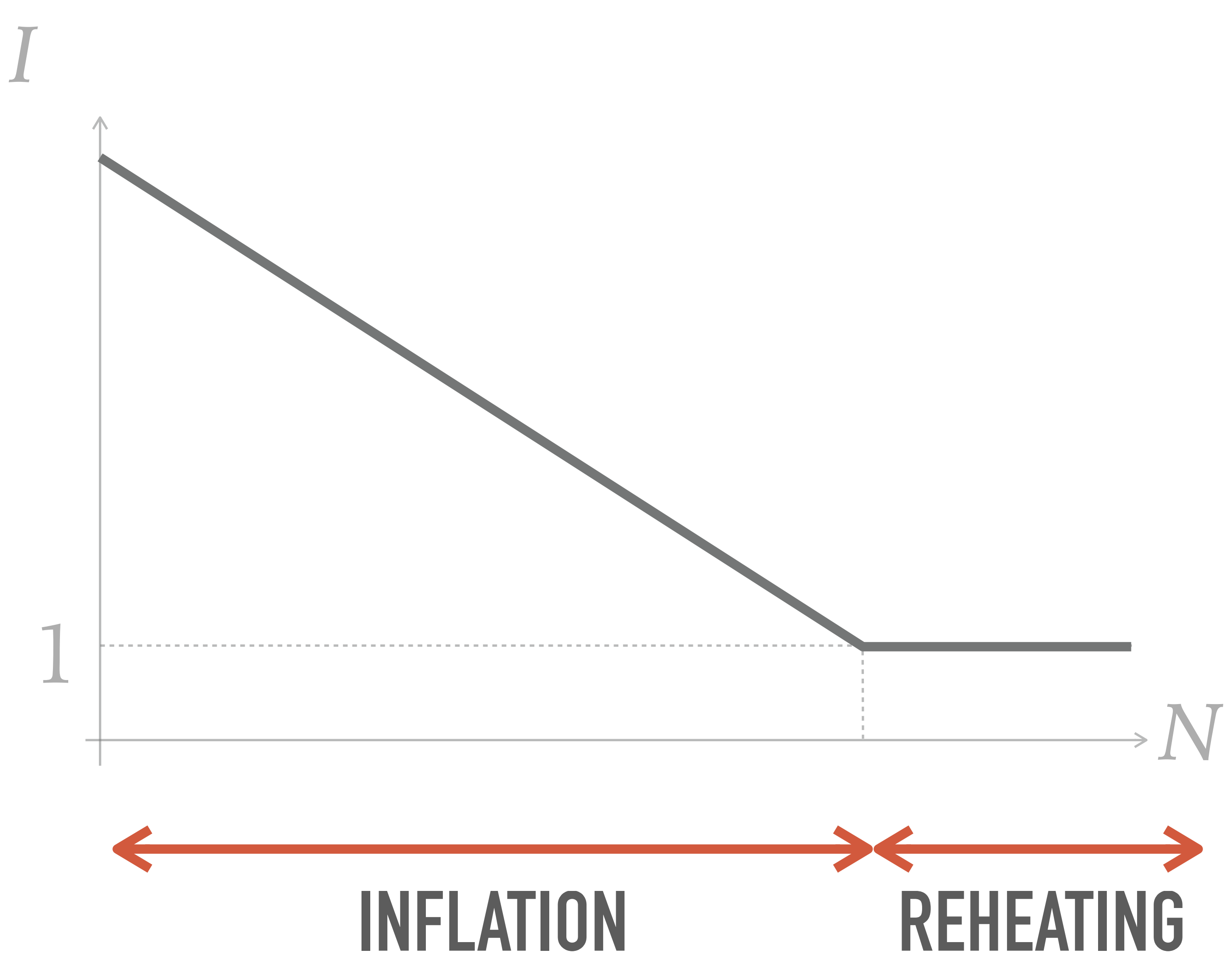
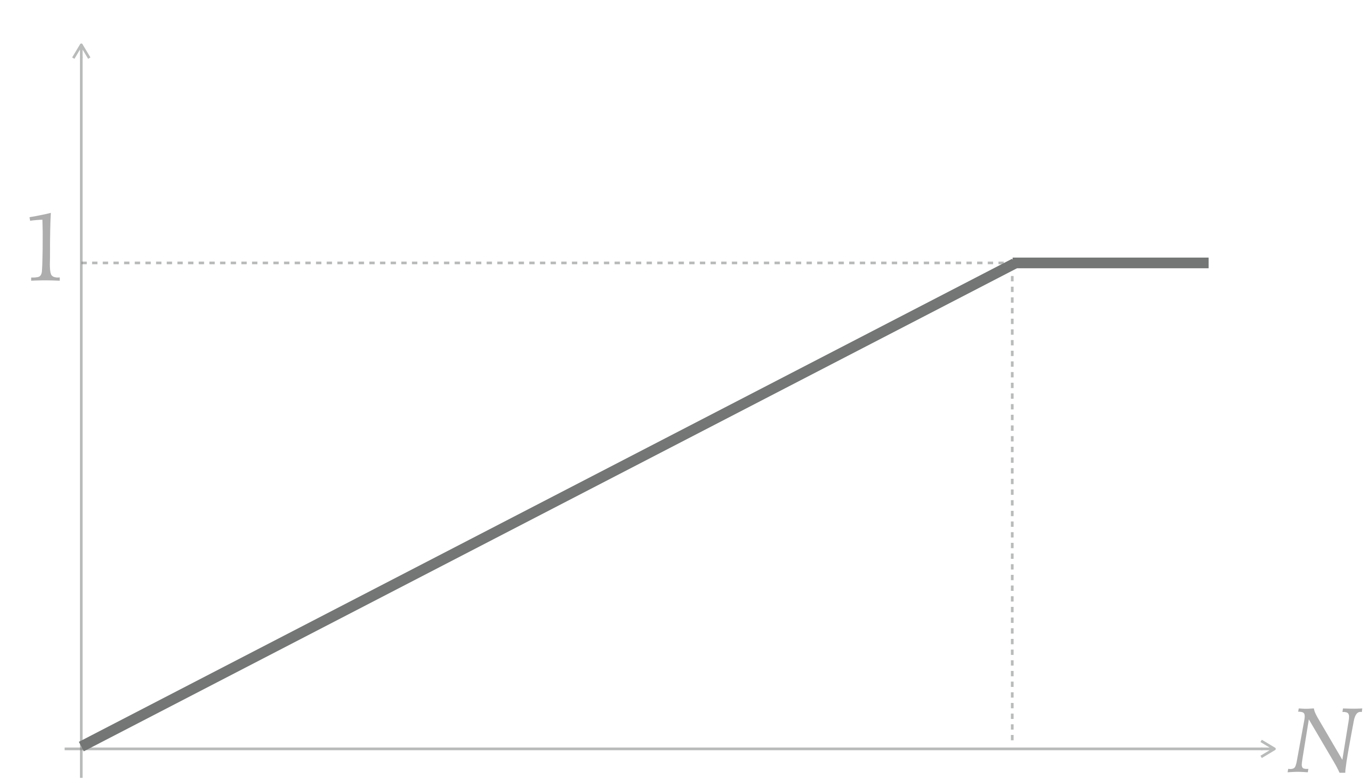
$$I(\phi) \to I(\eta) = a(\eta)^n$$





$$\mathscr{A}''(\eta,k) + \left(k^2 - \frac{n(n+1)}{\eta^2}\right)\mathscr{A}(\eta,k) = 0$$

$$\mathscr{A}(\eta,k) \equiv \frac{A(\eta,k)}{I}$$

BACK-REACTION PROBLEM

- The EM field spoils inflation

$$\rho_{EM} > \rho_\phi$$

STRONG COUPLING PROBLEM

- Out of perturbative regime

$$I \sim g^{-1} > 1$$

MODELLING THE COUPLING FUNCTION

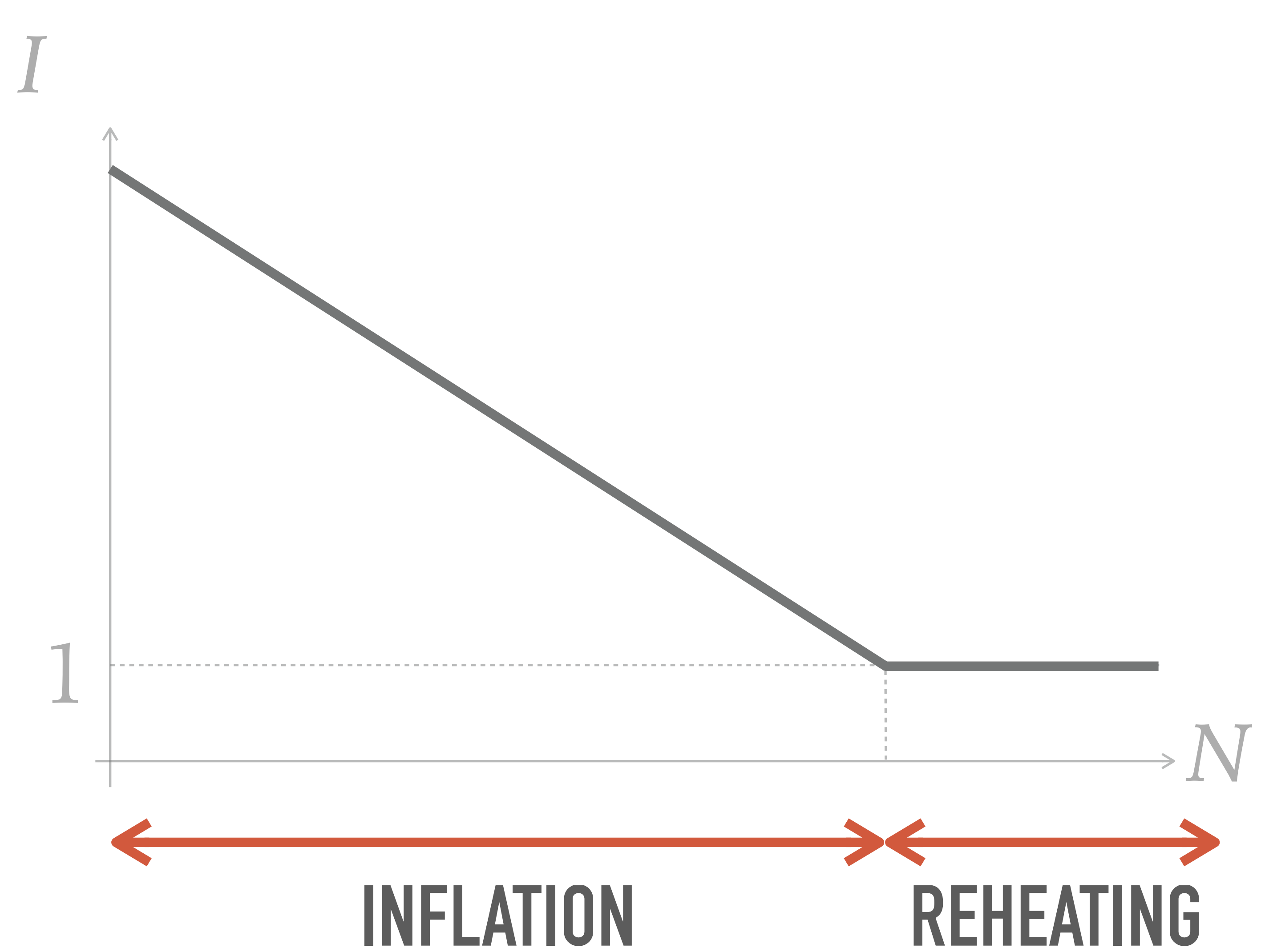
$$I(\phi) \to I(\eta) = a(\eta)^n$$

$$\mathcal{A}''(\eta, k) + \left(k^2 - \frac{n(n+1)}{\eta^2}\right)\mathcal{A}(\eta, k) = 0$$

$$\mathcal{A}(\eta, k) \equiv \frac{A(\eta, k)}{I}$$

POSSIBLE SOLUTION

$$-2 < n < 0$$



- Lower H
- Reduce ΔN

- Less back-reaction but smaller B_f !

$$B_f \sim \mathcal{O}(1)\frac{H^2}{I_f}$$

BEYOND RATRA: ADDING HELICITY

C. Caprini and L. Sorbo JCAP 1410 (2014)

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\, I^2[\phi(t)]\left[F_{\mu\nu}F^{\mu\nu} - \gamma F_{\mu\nu}\tilde{F}^{\mu\nu}\right] + \int d^4x\sqrt{-g}\,\mathscr{L}_\phi$$

$$\mathscr{A}''_\pm(\eta,k) + \left(k^2 \pm \frac{2k\gamma n}{\eta} - \frac{n(n+1)}{\eta^2}\right)\mathscr{A}_\pm(\eta,k) = 0$$

Exponential amplification of the positive helicity mode

WHY HELICITY?

- Observations
- Inverse-cascade evolution

- γ sets the amplitude
- n sets the spectral index

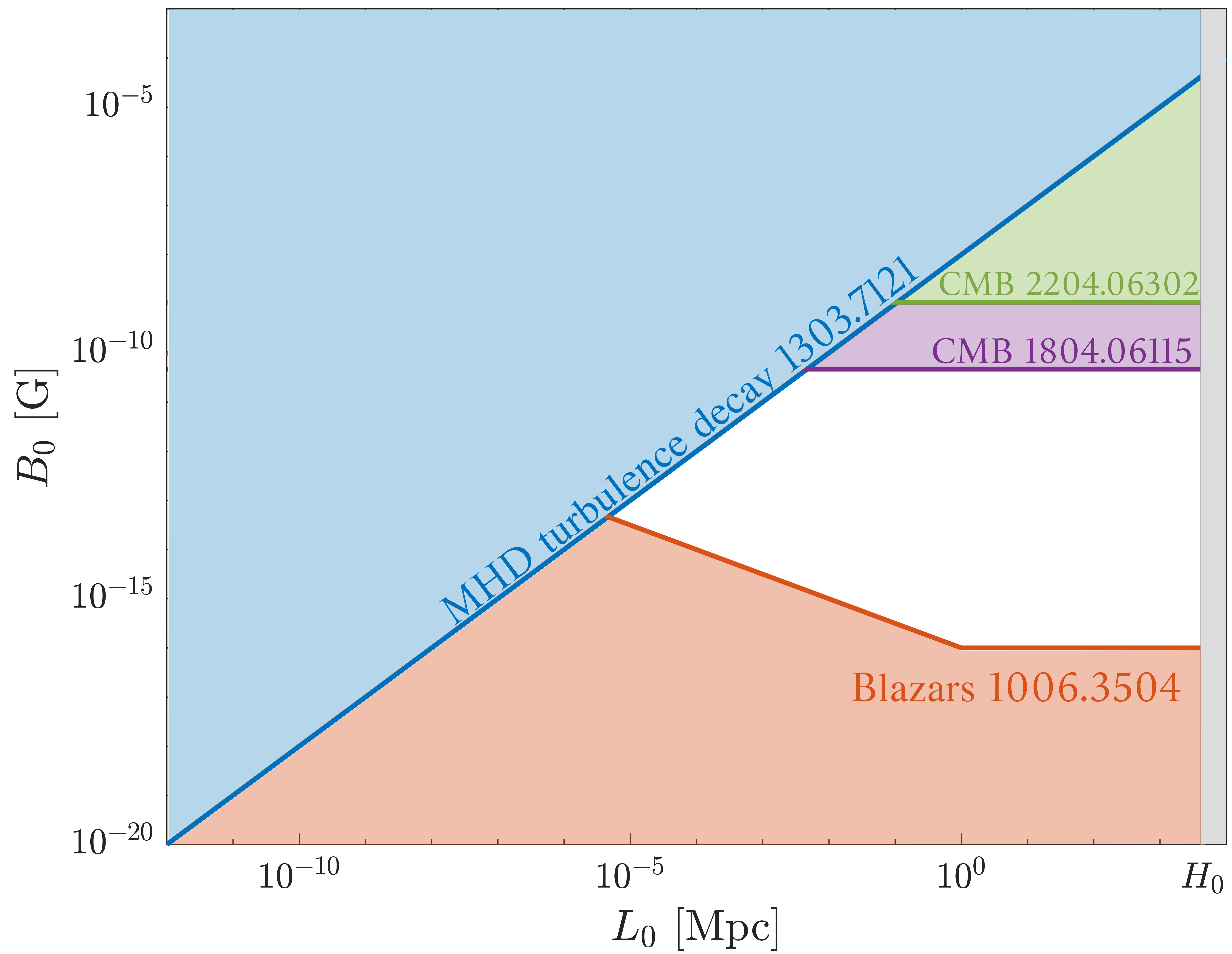
BEYOND RATRA: ADDING HELICITY

RESULTS

- ➤ (B_0, L_0) in agreement with bounds

ASSUMPTIONS

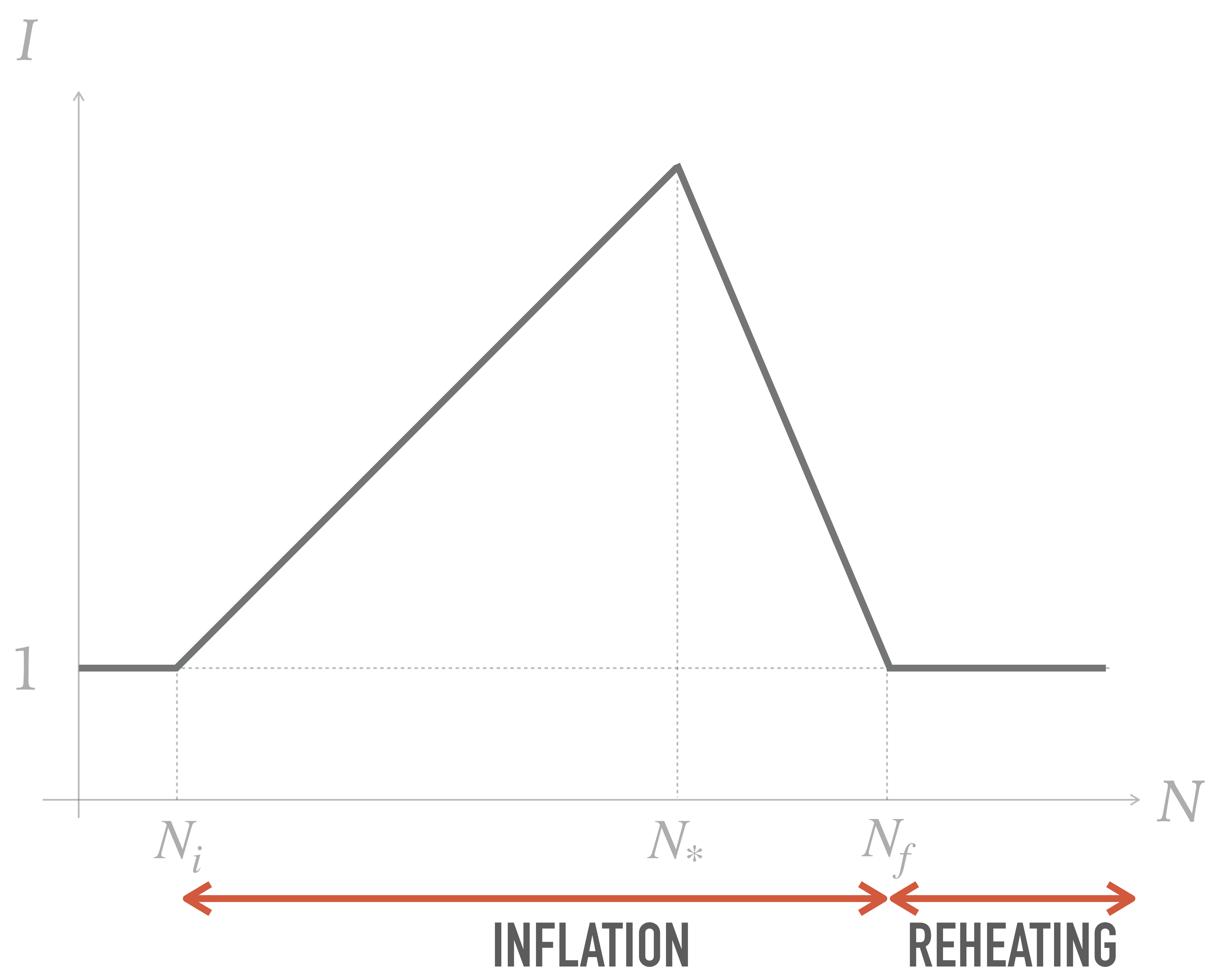
- ➤ $\rho_{inf}^{1/4} < 10^8\,\text{GeV}$
- ➤ ϕ is an auxiliary field (not the inflaton) slowly rolling during inflation



SAWTOOTH COUPLING

R. Ferreira, R.K. Jain, & M. Sloth, JCAP10 (2013) 004
R. Sharma, S. Jagannathan, T. R. Seshadri & K. Subramanian, Phys Rev D 96 (2017)

- A possibility is to take advantage of both the increasing and the decreasing mode
- Get larger (L_0, B_0) with larger $\rho_{inf}^{1/4}$



$$I = \begin{cases} \mathscr{C}\left(\frac{a}{a_*}\right)^{\nu_1} & a_i > a > a_* \\ \mathscr{C}\left(\frac{a}{a_*}\right)^{-\nu_2} & a_* > a > a_f \end{cases}$$

$$\nu_1, \nu_2 \geq 0$$

- *Fix $\mathscr{C}$ and a_* to have $I_i = I_f = 1$*

SAWTOOTH COUPLING: POWER SPECTRA

- Match the super horizon solutions at each transition: continuity of $A(k,\eta)$ and $A'(k,\eta)$

FIRST STAGE

$$\frac{d\rho_{B,I}}{d\ln k}^{\pm} \propto e^{\pm\pi\gamma\nu_1} H^4 (-k\eta)^{4-2\nu_1}$$

$$\frac{d\rho_{E,I}}{d\ln k}^{\pm} \propto e^{\pm\pi\gamma\nu_1} H^4 (-k\eta)^{4-2\nu_1}$$

SECOND STAGE

$$\frac{d\rho_{B,II}}{d\ln k}^{\pm} \propto e^{\pm\pi\gamma\nu_1} H^4 (-k\eta)^{8-2\nu_1} \left(\frac{\eta_*}{\eta}\right)^{2-2\nu_1+2\nu_2}$$

$$\frac{d\rho_{E,II}}{d\ln k}^{\pm} \propto e^{\pm\pi\gamma\nu_1} H^4 (-k\eta)^{6-2\nu_1} \left(\frac{\eta_*}{\eta}\right)^{2-2\nu_1+2\nu_2}$$

- Due to matching, we cannot have a scale-invariant magnetic spectrum in both stages!
- $\nu_1 = 2 \rightarrow$ **both spectra** are scale-invariant in the first stage.
  Result of helicity+sawtooth.

SCALE-INVARIANT MODIFIED GRAVITY MODEL

M. Rinaldi and L. Vanzo PR D 94 (2016)

- $\mathscr{L}_{EH} \longrightarrow f(R, \phi)$

$$\mathscr{L} = \sqrt{-g}\left[\frac{\alpha}{36}R^2 + \frac{\xi}{6}\phi^2 R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4\right], \qquad \alpha, \lambda, \xi > 0$$

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C. Wetterich , Nuclear Physics B, 115326 (2021)
A. Strumia & A. Salvio, J High Energ Phys, 6 (2017)

SCALE-INVARIANCE AS A FUNDAMENTAL SYMMETRY

- Perturbations on the CMB
- Flat inflationary potentials
- Standard Model
- Dynamical mass generation
- Dark matter candidates

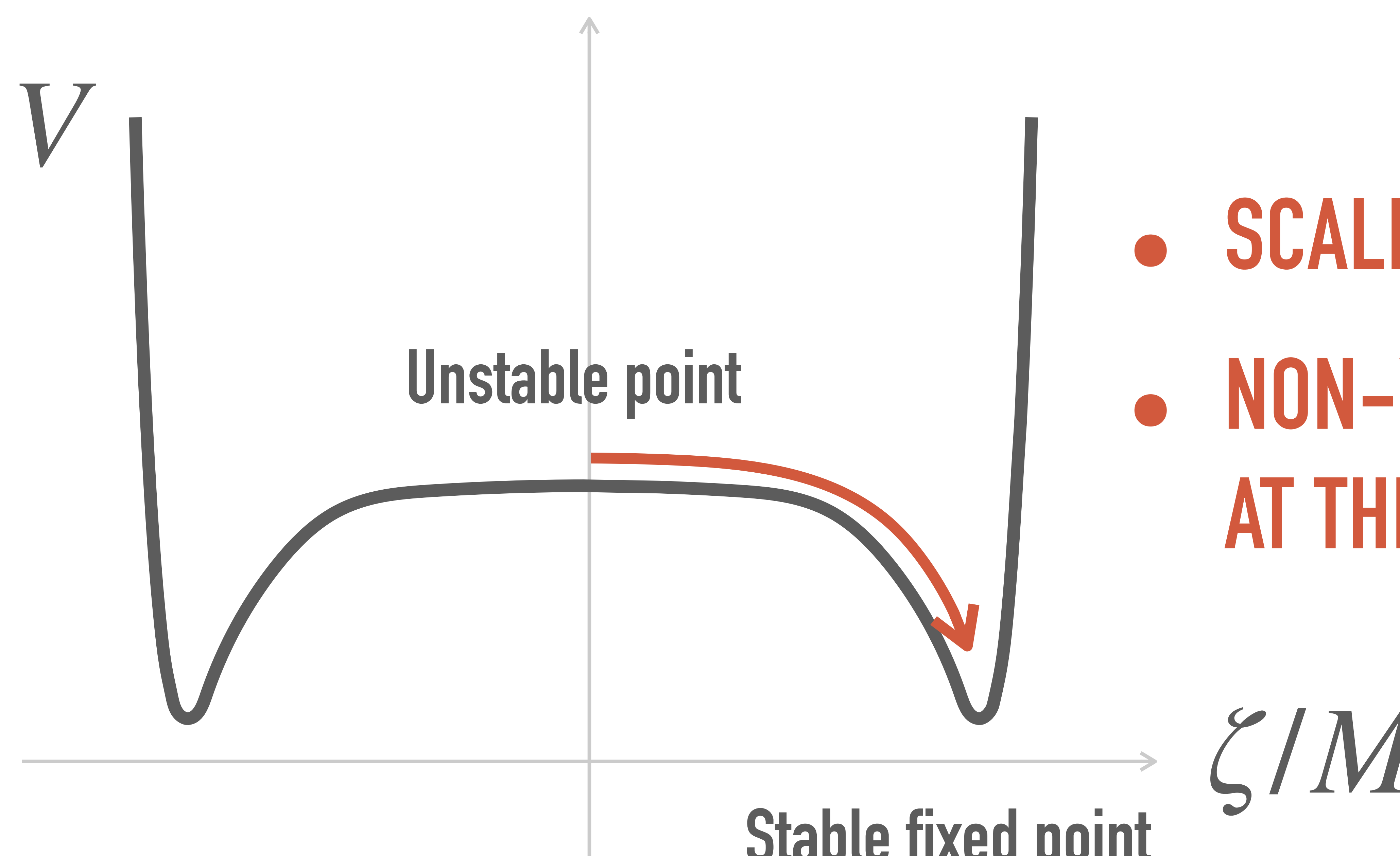
SCALE-INVARIANT MODIFIED GRAVITY MODEL

G. Tambalo and M. Rinaldi Gen Relativ Gravit 49 (2017)

1. JORDAN FRAME $\longrightarrow$ EINSTEIN FRAME
2. FIELDS REDEFINITION

ρ GOLDSTONE BOSON

ζ INFLATON

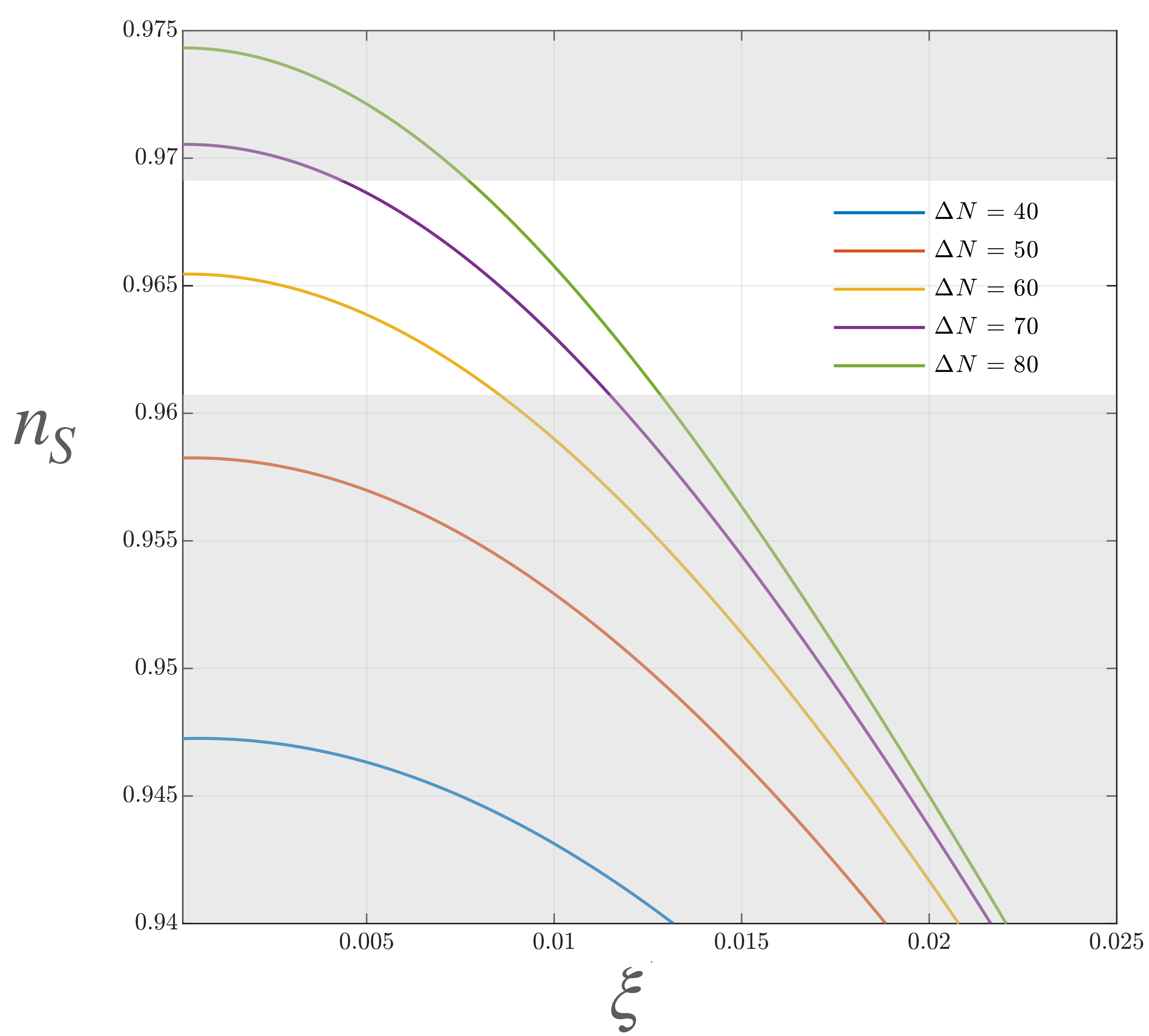


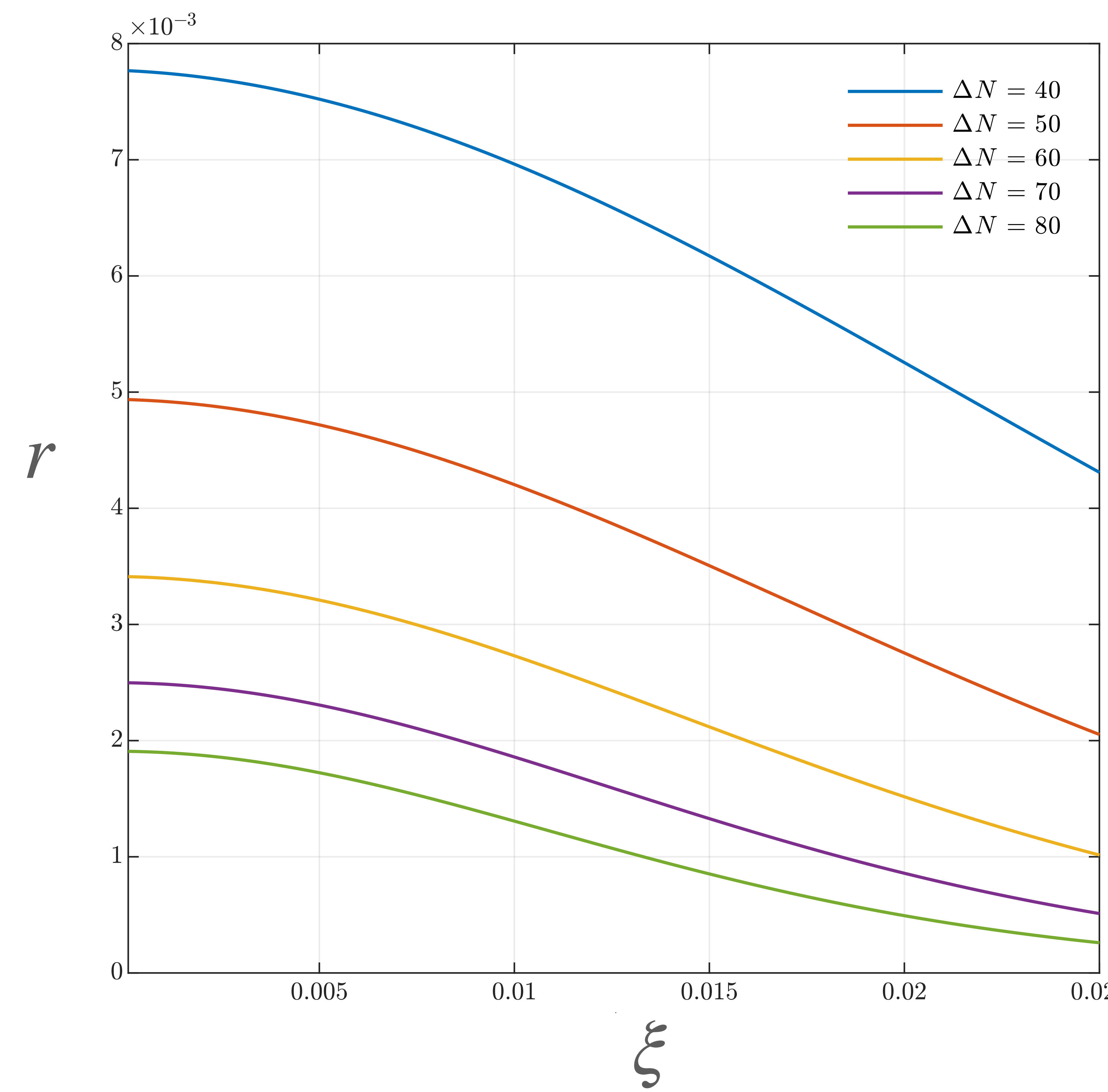
- SCALE SYMMETRY BREAKING
- NON-VANISHING POTENTIAL AT THE MINIMA

SCALE-INVARIANT MODIFIED GRAVITY MODEL

3. SPECTRAL INDICES

- $\Omega = \alpha\lambda + \xi^2 \lesssim 1.15\,\xi^2$
- $\alpha \gtrsim 2 \times 10^{10}$
- $\xi \lesssim 1.3 \times 10^{-2}$
- $\Delta N \gtrsim 55$





EVOLUTION OF THE INFLATON

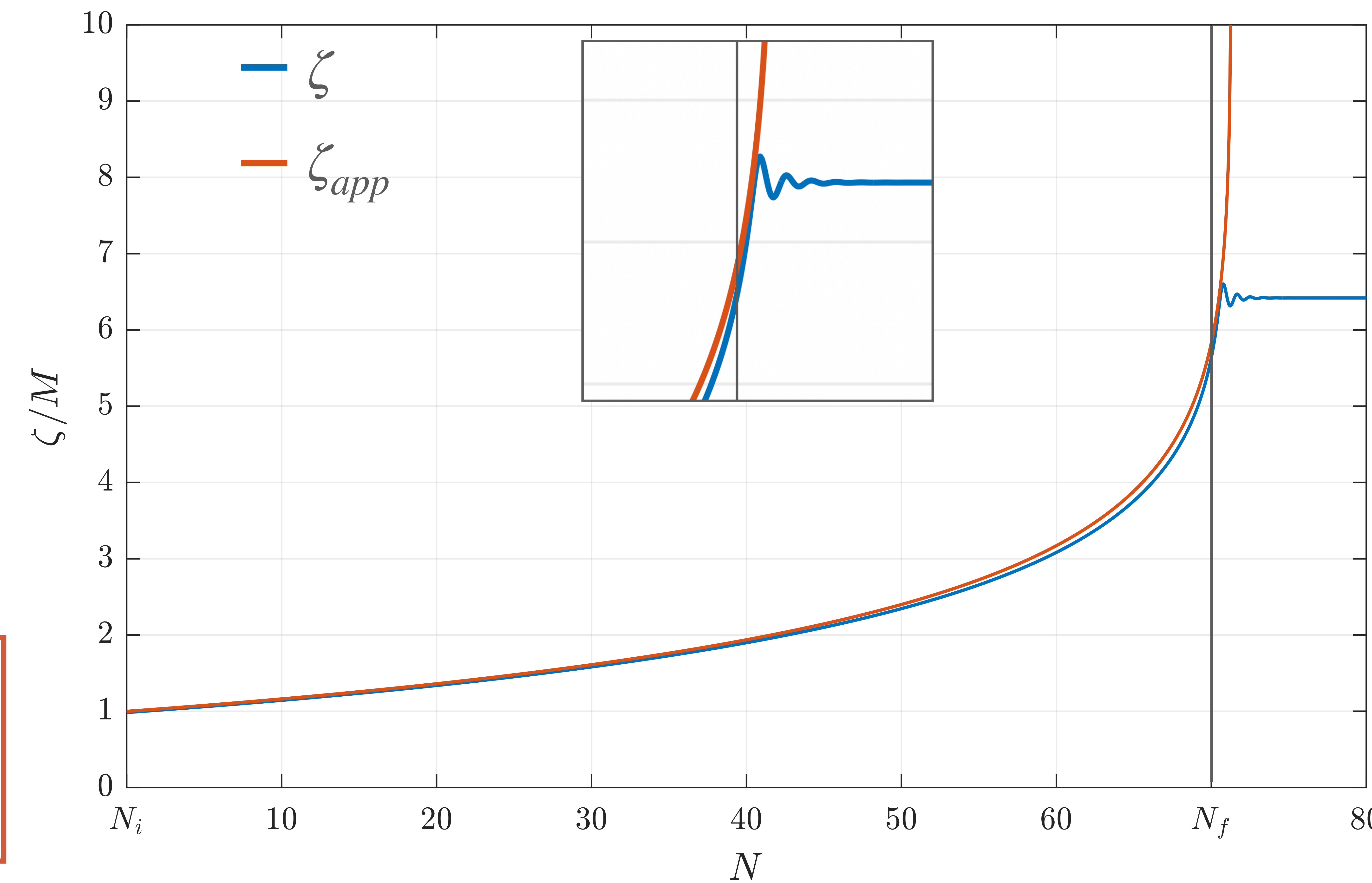
APPROXIMATIONS

- Slow-roll
- $\xi \ll 1$
- $\Omega \to \xi^2$

↓

$$\zeta(a) \simeq \sqrt{6}\, M \mathrm{ArcTanh}\left[\mathscr{C}\, a^{\frac{4}{3}\xi}\right]$$

$$I(\zeta) = \propto \tanh\left[\frac{\zeta}{\sqrt{6}M}\right]^{\pm\frac{3}{4\xi}\nu_1} \simeq a^{\pm\nu_i}$$



RESULTS

- ➤ Scale-invariant magnetic power spectrum is favoured by Planck + inflation
- ➤ CMB observational window: first e-folds of inflation

$\nu_1 = 2$

FIRST STAGE

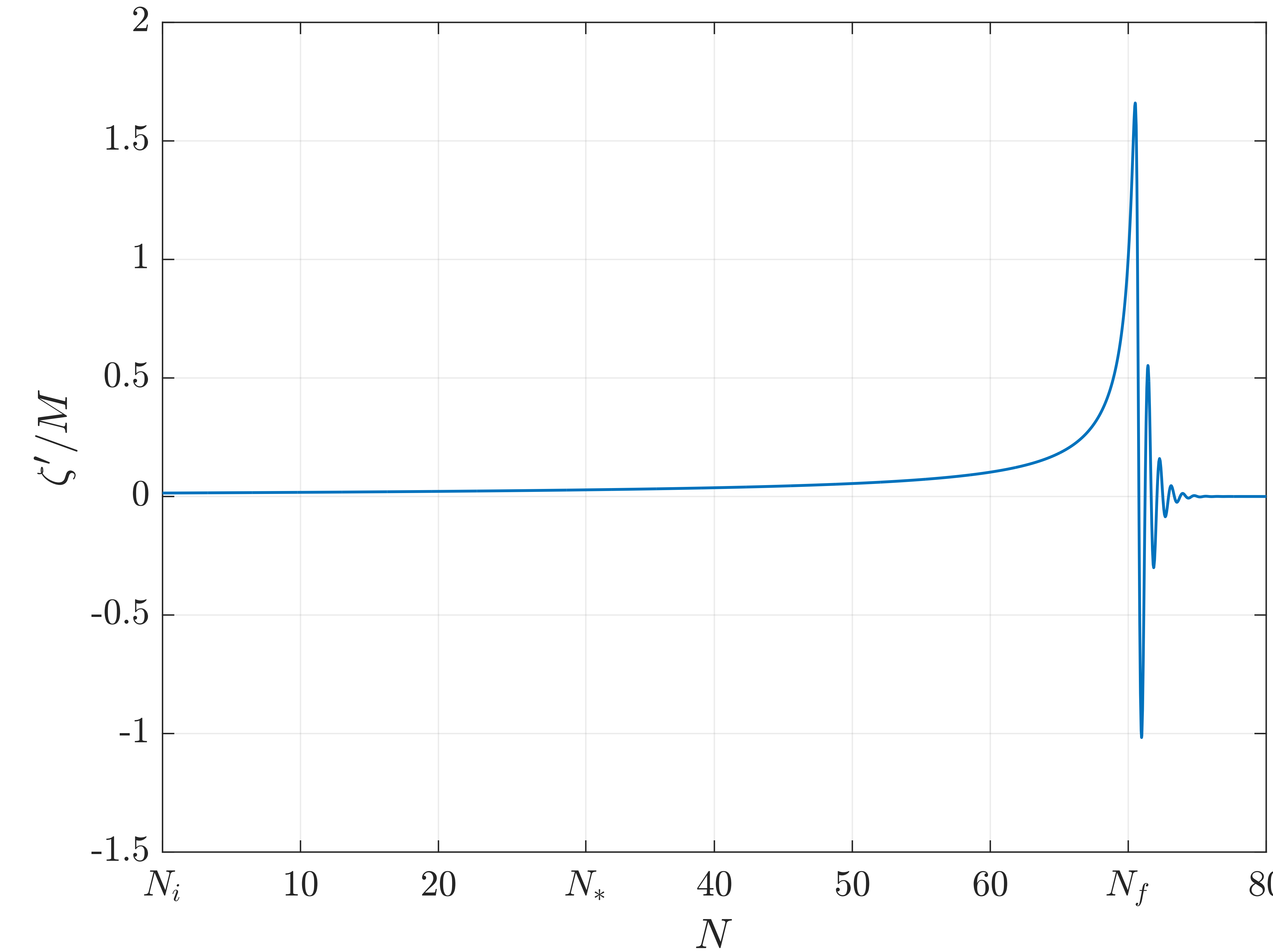
$\rightarrow \dfrac{d\rho_{B,I}}{d\ln k}^{\pm}$ is scale-invariant

$\rightarrow \dfrac{d\rho_{E,I}}{d\ln k}^{\pm}$ is scale-invariant

SECOND STAGE

$\rightarrow \dfrac{d\rho_{B,II}}{d\ln k}^{\pm} \propto k^4$

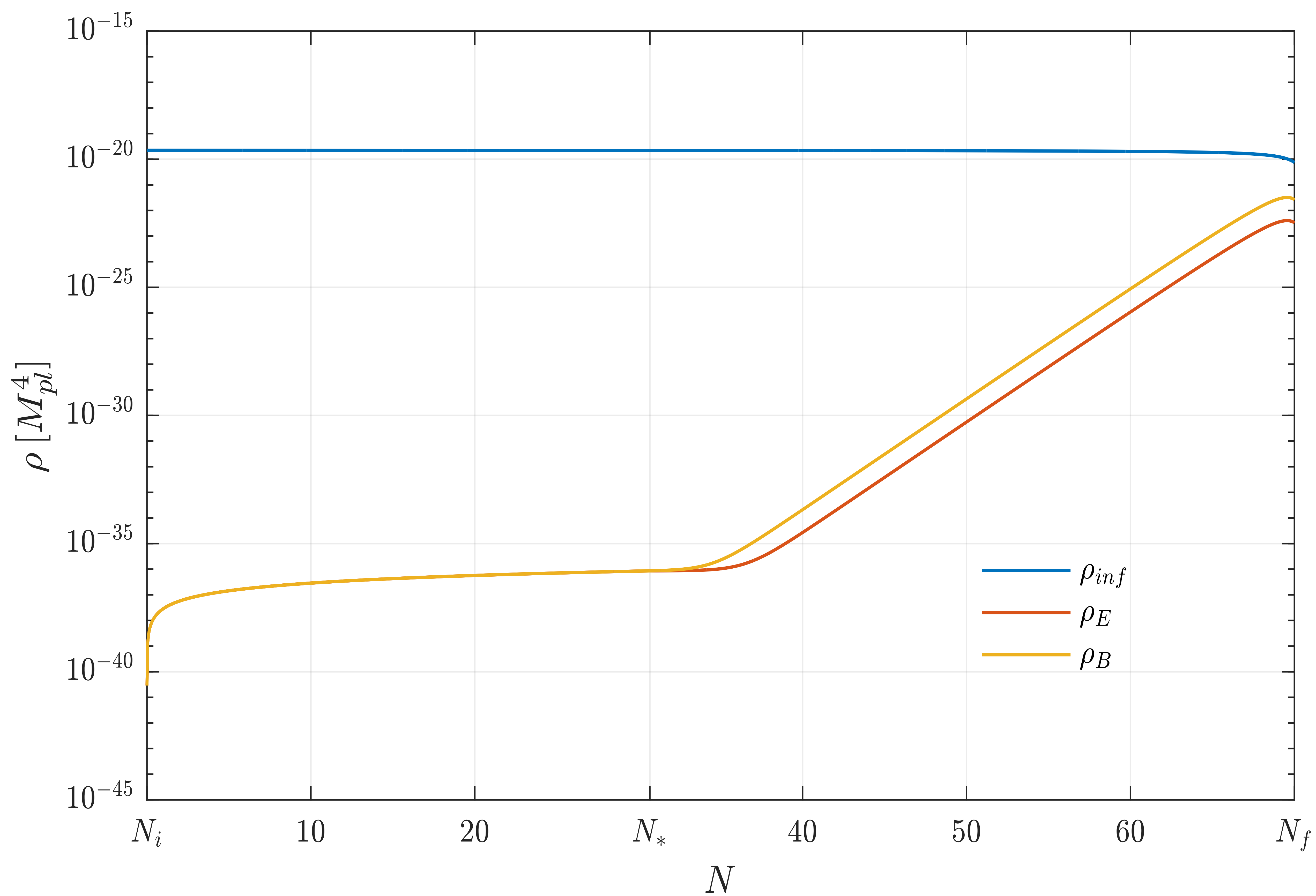
$\rightarrow \dfrac{d\rho_{E,II}}{d\ln k}^{\pm} \propto k^2$



➤ Departure from scale invariance: inflaton ↔ EM field

RESULTS

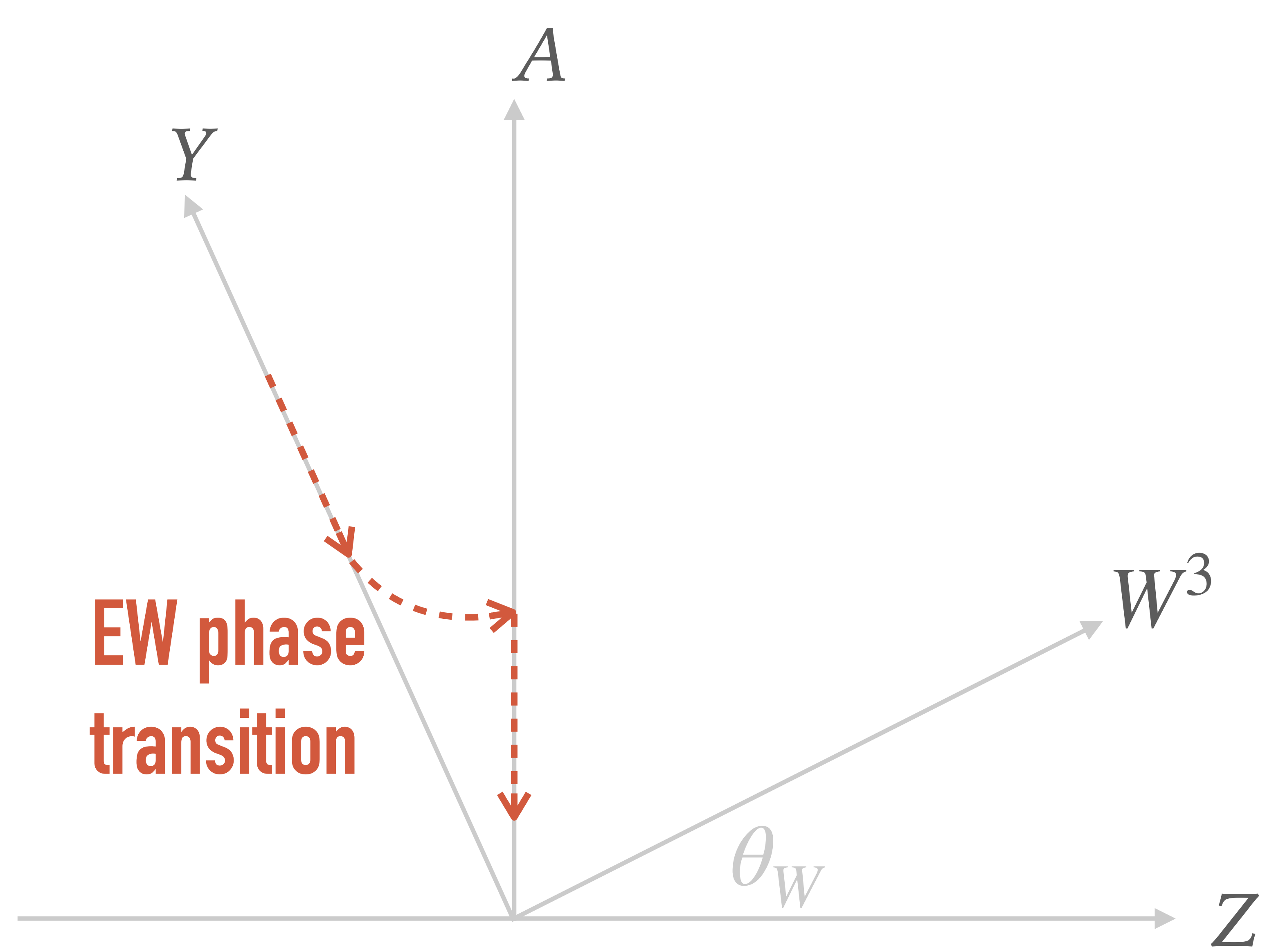
$\rho_{inf}^{1/4}$ [GeV]	ΔN	ν_1	ν_2	γ
5×10^{14}	60	2	1.4	1
$\rightarrow L_0 = 0.13$ Mpc $\rightarrow B_0(L_0) = 1.3$ nG $\rightarrow B_0(\ell = 1\,\text{Mpc}) = 0.02$ nG				
3×10^{13}	60	2	1.5	1
$\rightarrow L_0 = 6.7 \times 10^{-3}$ Mpc $\rightarrow B_0(L_0) = 6.7 \times 10^{-2}$ nG $\rightarrow B_0(\ell = 1\,\text{Mpc}) = 3 \times 10^{-6}$ nG				



BARYOGENESIS

K. Kamada and A. J. Long PRD 94 (2016)

- If helical PMF existed before the EW transitions, baryon asymmetry is generated
- Decaying hyper magnetic helicity→ (B+L) asymmetry



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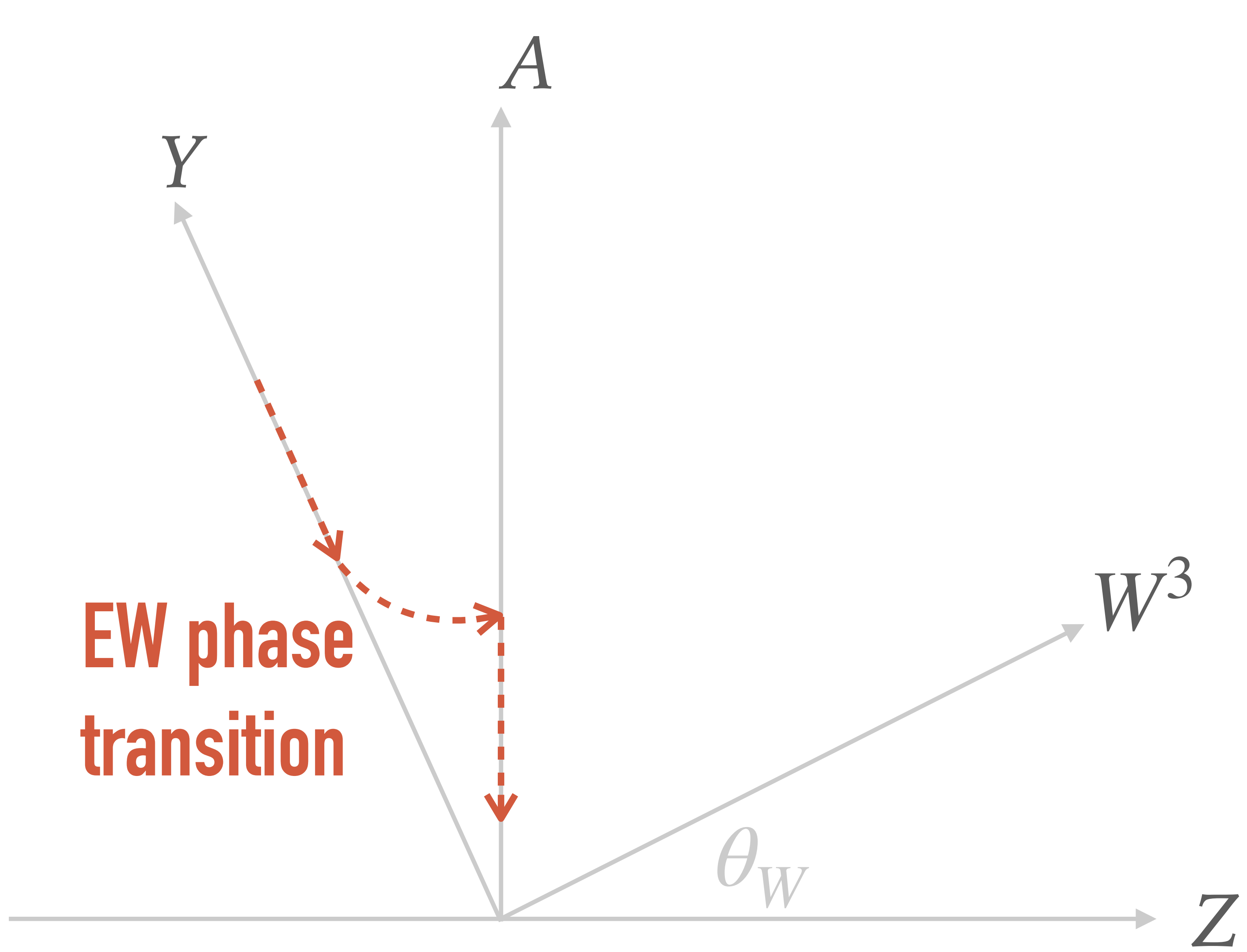


$\rho_{inf}^{1/4}$ [GeV]	ΔN	ν_1	ν_2	γ
2×10^7	55	3.4	0.3	0.2

$$\rightarrow L_0 = 4.6\times10^{-8}\ \mathrm{Mpc}$$

$$\rightarrow B_0(L_0) = 4.6\times10^{-7}\ \mathrm{nG}$$

$$\rightarrow B_0(\ell = 1\,\mathrm{Mpc}) = 1.8\times10^{-11}\ \mathrm{nG}$$

- ➤ Observed BAU
- ➤ Seed for dynamo

BACKUP SLIDES

INFLATIONARY BACKGROUND: MODIFIED GRAVITY

f(R) THEORIES OF MODIFIED GRAVITY

- A scalar condensate can be imitated within gravity
- Einstein gravity $\mathscr{L}_{EH} = R - 2\Lambda$ as a low-curvature limit of a more complicated theory

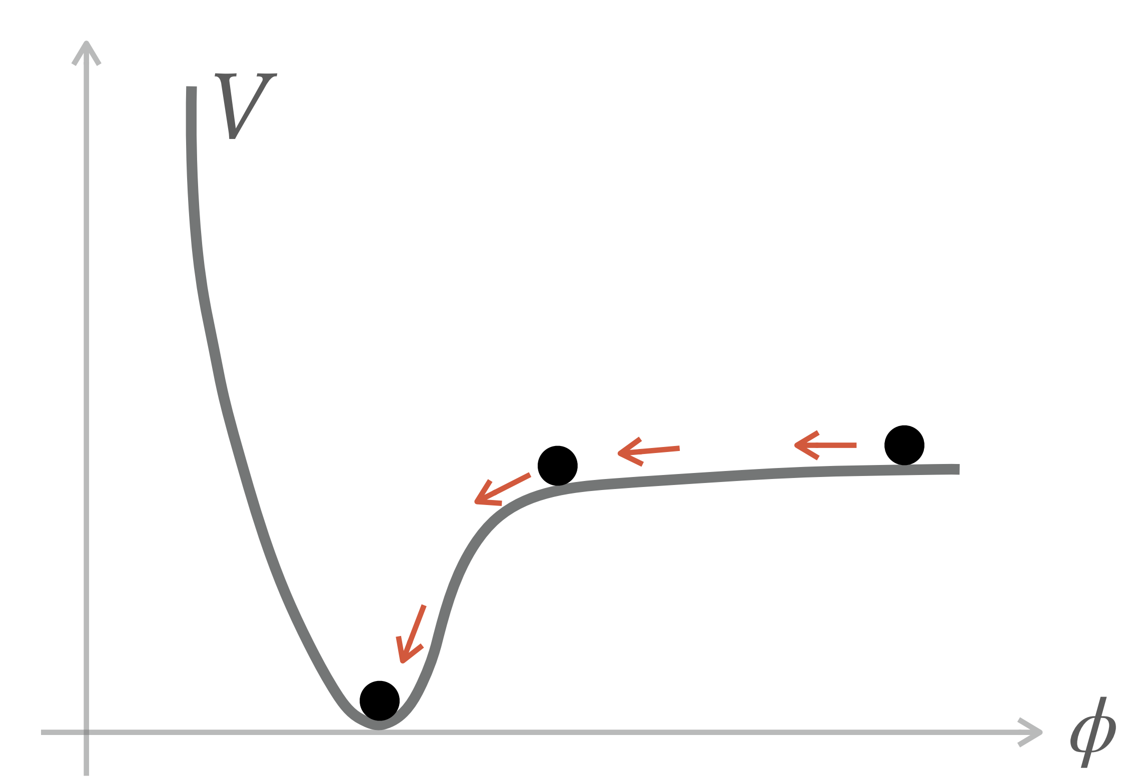
JORDAN FRAME —— Conformal transformation ——→ **EINSTEIN FRAME**

$$S = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g}\, f(R)$$

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{pl}^2}{2} \tilde{R} + \mathscr{L}_\phi \right]$$

STAROBINSKY'S MODEL

$$f(R) = R + \frac{R^2}{6M^2}$$



SCALE-INVARIANT MODIFIED GRAVITY MODEL: SYMMETRIES

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$$\mathscr{L} = \sqrt{-g}\left[\frac{\alpha}{36}R^2 + \frac{\xi}{6}\phi^2 R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4\right], \qquad \alpha, \lambda, \xi > 0$$

SYMMETRIES

- Scale symmetry (dilations)
  - $\bar{g}_{\mu\nu}(x) = g_{\mu\nu}(\ell x)$
  - $\bar{\phi}(x) = \ell\phi(\ell x)$
- Rigid internal Weyl symmetry
  - $\bar{g}_{\mu\nu}(x) = L^2 g_{\mu\nu}(x)$
  - $\bar{\phi}(x) = L^{-1}\phi(x)$

SCALE-INVARIANT MODIFIED GRAVITY MODEL

EINSTEIN FRAME $g^*_{\mu\nu} = \Omega^2 g_{\mu\nu}$

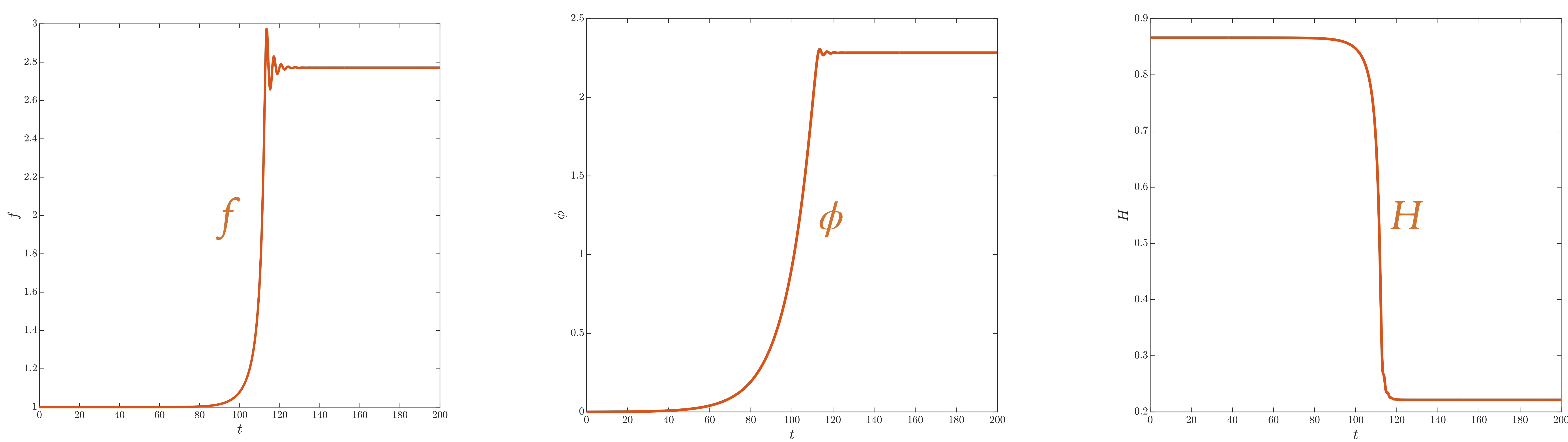
REDUNDANT MASS SCALE

$$\mathscr{L}_E = \sqrt{-g}\left[\frac{M^2}{2}R - \frac{3M^2}{f^2}(\partial f)^2 - \frac{f^2}{2M^2}(\partial\phi)^2 - V(f,\phi)\right]$$

$f = -M\Omega$ SCALARON

$$V(f,\phi) = \frac{9M^4}{4\alpha} + f^2\phi^2\left(-\frac{3\xi}{2\alpha} + \frac{K}{M^4}f^2\phi^2\right)$$

$$K = \frac{1}{4}\left(\lambda + \frac{\xi^2}{\alpha}\right)$$



SCALE-INVARIANT MODIFIED GRAVITY MODEL

FIELDS REDEFINITION

Single-field inflation can be attained via field redefinition.

GOLDSTONE BOSON

$$\rho = \frac{M}{2}\log\left[\frac{\phi^2}{2M^2} + 3\frac{M^2}{f^2}\right]$$

$$\zeta = \sqrt{6}M\mathrm{ArcSinh}\left[\frac{f\phi}{\sqrt{6}M^2}\right]$$

INFLATON

$$\mathscr{L}_E = \sqrt{-g}\left(\frac{M^2}{2}R - \frac{1}{2}\partial_\mu\zeta\,\partial^\mu\zeta - 3\mathrm{Cosh}\left[\frac{\zeta}{\sqrt{6}M}\right]^2\partial_\mu\rho\partial^\mu\rho - U(\zeta)\right);$$

$$U(\zeta) = \frac{9M^4}{4\alpha}\left(1 - 4\,\xi\,\mathrm{Sinh}\left[\frac{\zeta}{\sqrt{6}M}\right]^2 + 4\,\Omega\,\mathrm{Sinh}\left[\frac{\zeta}{\sqrt{6}M}\right]^4\right);$$

$$\Omega = \alpha\lambda + \xi^2$$

SAWTOOTH COUPLING WITHOUT HELICITY

POWER SPECTRA

- Match the super horizon solutions at each transition: continuity of $A(k,\eta)$ and $A'(k,\eta)$

FIRST STAGE

$$\frac{d\rho_B}{d\ln k}^{(1)} = \mathscr{F}(\nu_1)\, H^4 \,(-k\eta)^{4-2\nu_1}$$

$$\frac{d\rho_E}{d\ln k}^{(1)} = \mathscr{G}(\nu_1)\, H^4 \,(-k\eta)^{6-2\nu_1}$$

SECOND STAGE

$$\frac{d\rho_B}{d\ln k}^{(2)} = \mathscr{F}_2(\nu_1,\nu_2)\, H^4 \,\left(-k\eta\right)^{8-2\nu_1} \left(\frac{\eta_*}{\eta}\right)^{2-2\nu_1+2\nu_2}$$

$$\frac{d\rho_E}{d\ln k}^{(2)} = \mathscr{G}_2(\nu_1,\nu_2)\, H^4 \,\left(-k\eta\right)^{6-2\nu_1} \left(\frac{\eta_*}{\eta}\right)^{2-2\nu_1+2\nu_2}$$

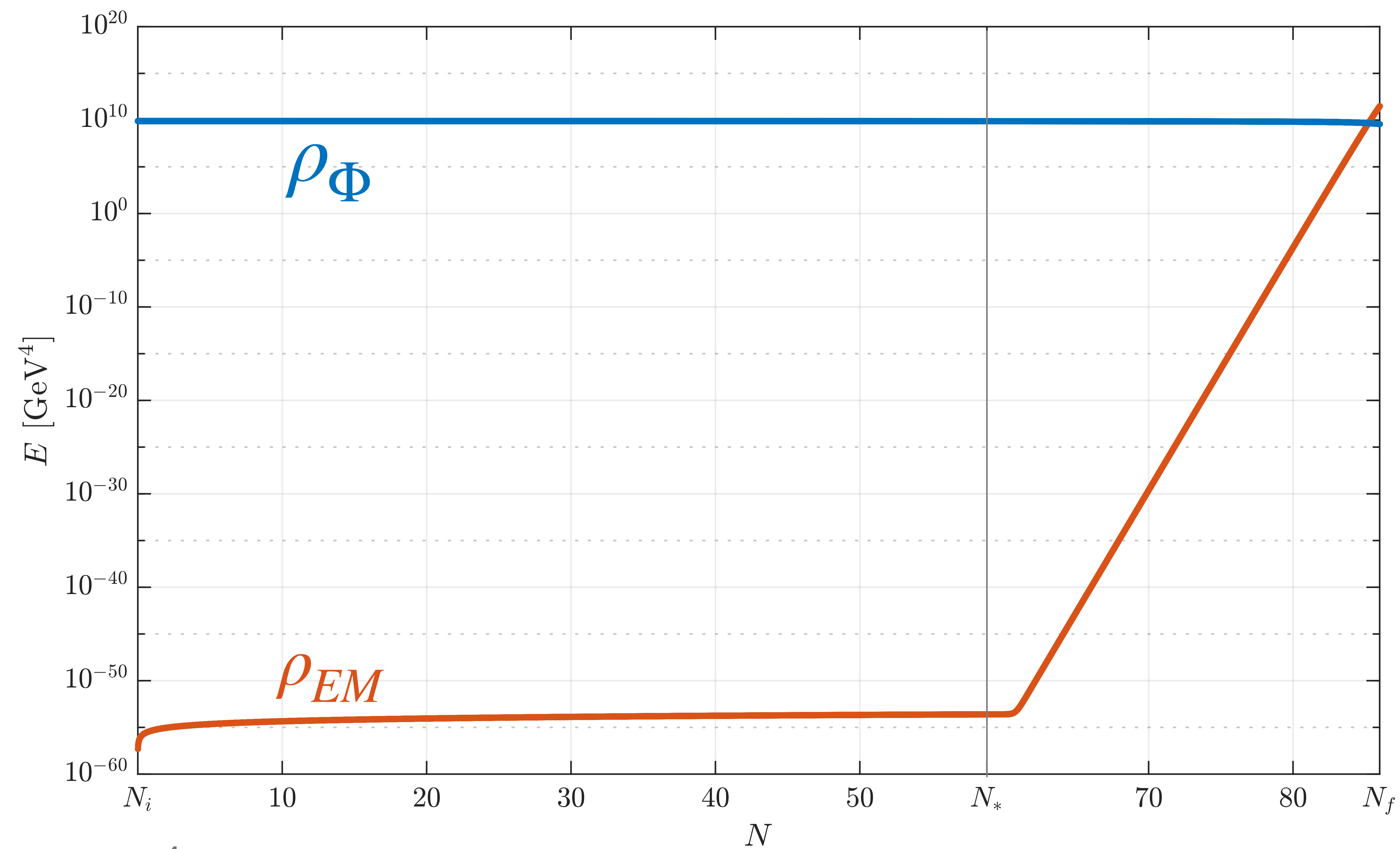
- Due to matching, we cannot have a scale-invariant magnetic spectrum in both stages!

SAWTOOTH COUPLING WITHOUT HELICITY

$\nu_1 = 2$: SCALE-INVARIANCE IN THE FIRST STAGE

$$\rho_B^{(2)} \sim \rho_E^{(2)} \sim H_I^4 \left(\frac{a}{a_*}\right)^{-2+2\nu_2}$$

$$B(\lambda_{ph}, a) \sim \frac{1}{\lambda_{ph}^2} \left(\frac{a}{a_*}\right)^{-1+\nu_2}$$



$\nu_2 = 4$

$\Delta N = 85$

$\alpha = 10^{64}$

$\longrightarrow$

$B_0 = 1.7 \times 10^{-20}\,\mathrm{G}$

$E_I = \left(3H_I^2 M_{Pl}^2\right)^{1/4} = 298\,\mathrm{GeV}$ (!)

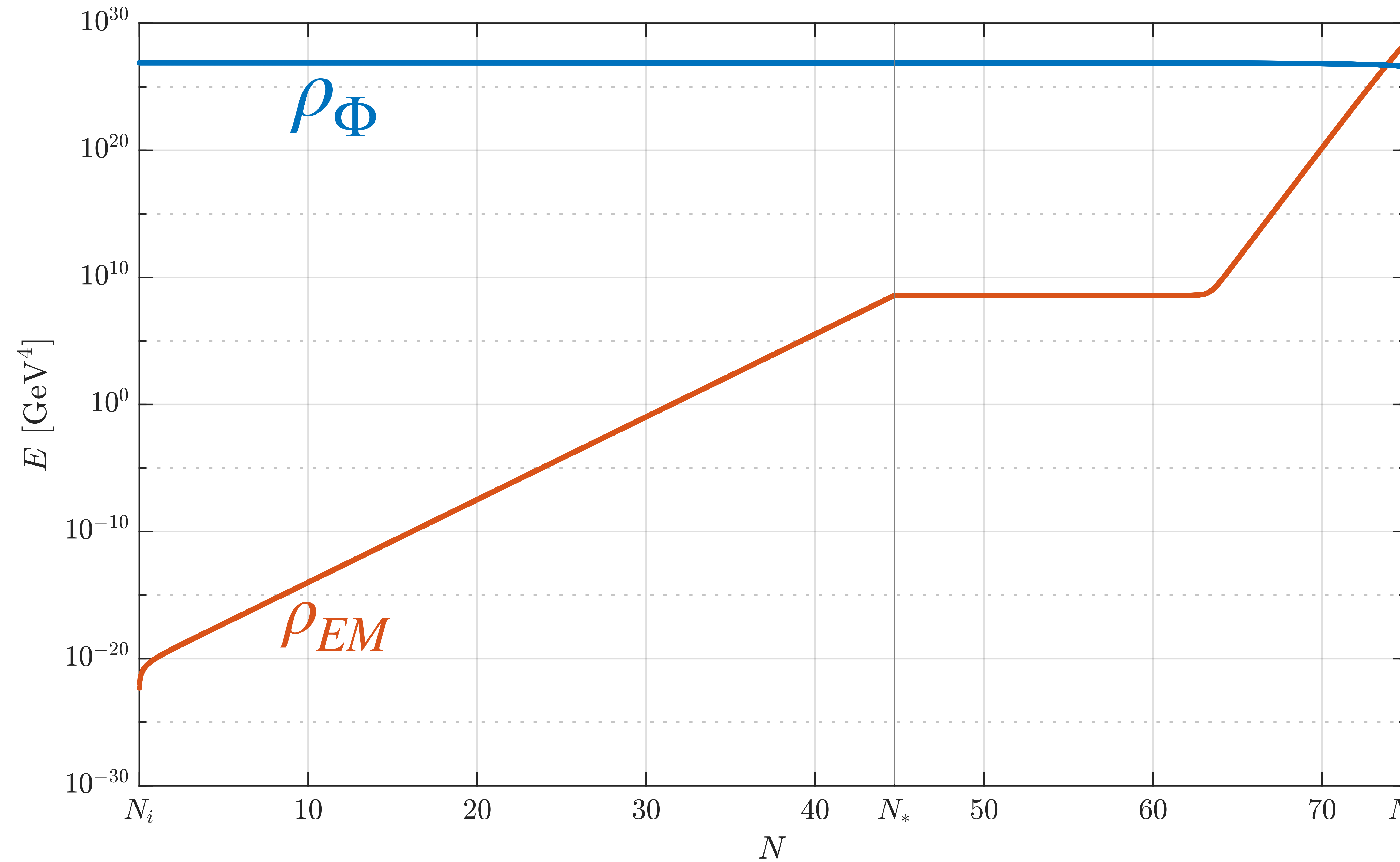
SAWTOOTH COUPLING

$2 < \nu_1 < 3$: DEVIATIONS FROM SCALE-INVARIANCE

$$\rho_B^{(1)} \sim H_I^4 \left(\frac{a}{a_i}\right)^{2\nu_1-4}$$

$$\rho_B^{(2)} \sim \rho_E^{(2)} \sim H_I^4 \left(\frac{a}{a_*}\right)^{2-2\nu_1+2\nu_2}$$

$$B(\lambda_{ph}, a) \sim \frac{H_I^{\nu_1-2}}{\lambda_{ph}^{4-\nu_1}} \left(\frac{a}{a_*}\right)^{1-\nu_1+\nu_2}$$



$\nu_1 = 2.75 \qquad \Delta N = 74$

$\nu_2 = 3.78 \qquad \alpha = 10^{47}$

$\longrightarrow$

$B_0 = 3.2 \times 10^{-20}\,\mathrm{G}$

$\lambda_B = 3.9\,\mathrm{Mpc}$

$E_I = 5.3 \times 10^{6}\,\mathrm{GeV}$