Inferring primordial magnetic properties from the resulting gravitational waves

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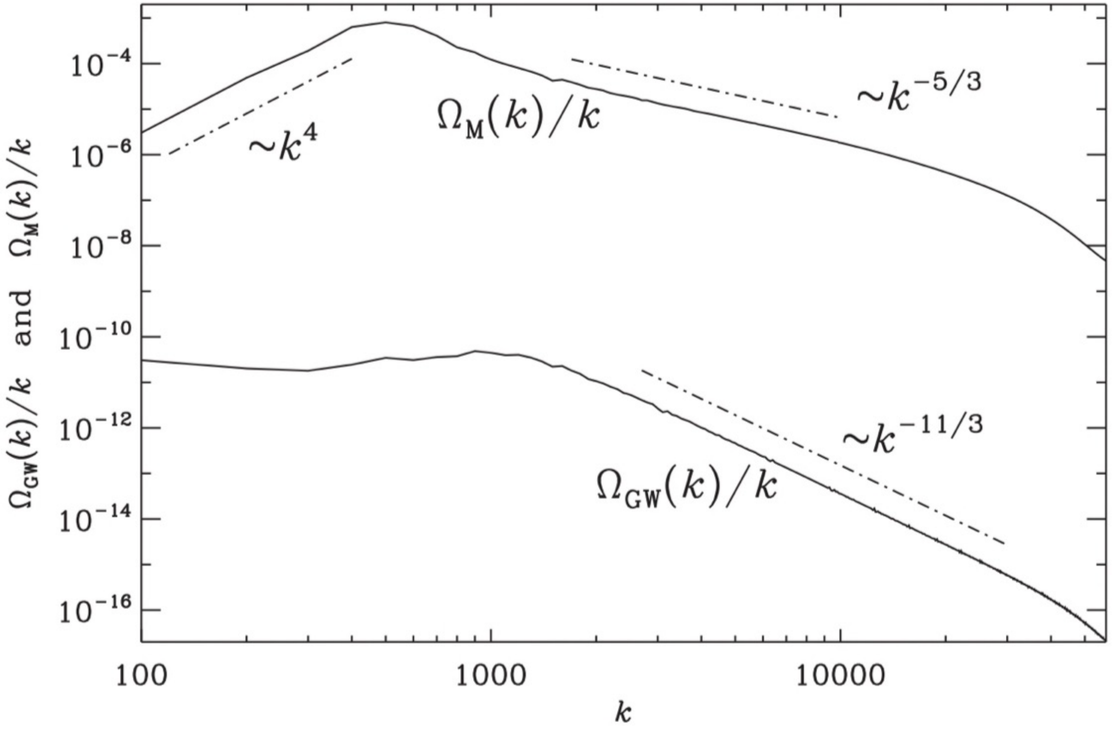
$$\left(\partial_t^2 + 3H\partial_t - c^2\nabla^2\right) h_{ij}(\boldsymbol{x},t) = \frac{16\pi G}{c^2}\, T_{ij}^{\rm TT}(\boldsymbol{x},t)$$

Relation between spectra:

$$\mathrm{Sp}(\dot{\mathbf{h}}) \approx k^2 \mathrm{Sp}(\mathbf{h}) \approx k^{-2}\mathrm{Sp}(\mathbf{T}),$$

GW slope by k^2 steeper
Peak at twice magnetic peak

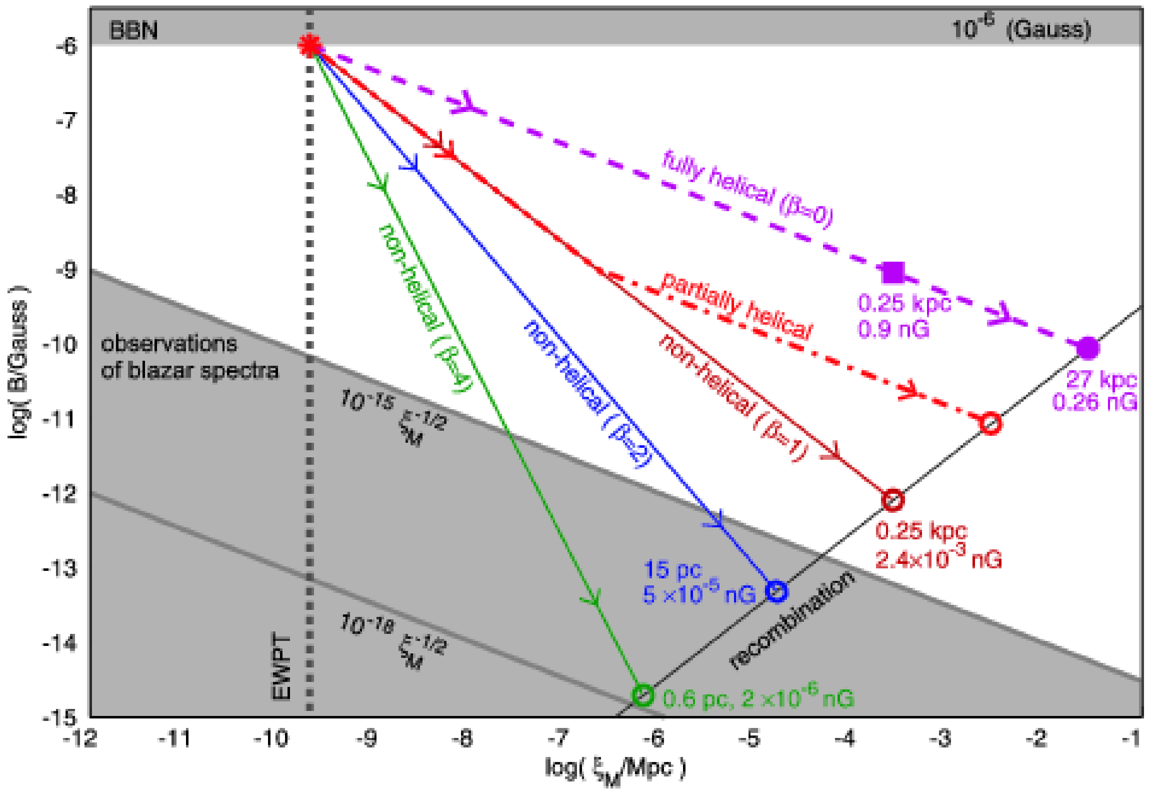


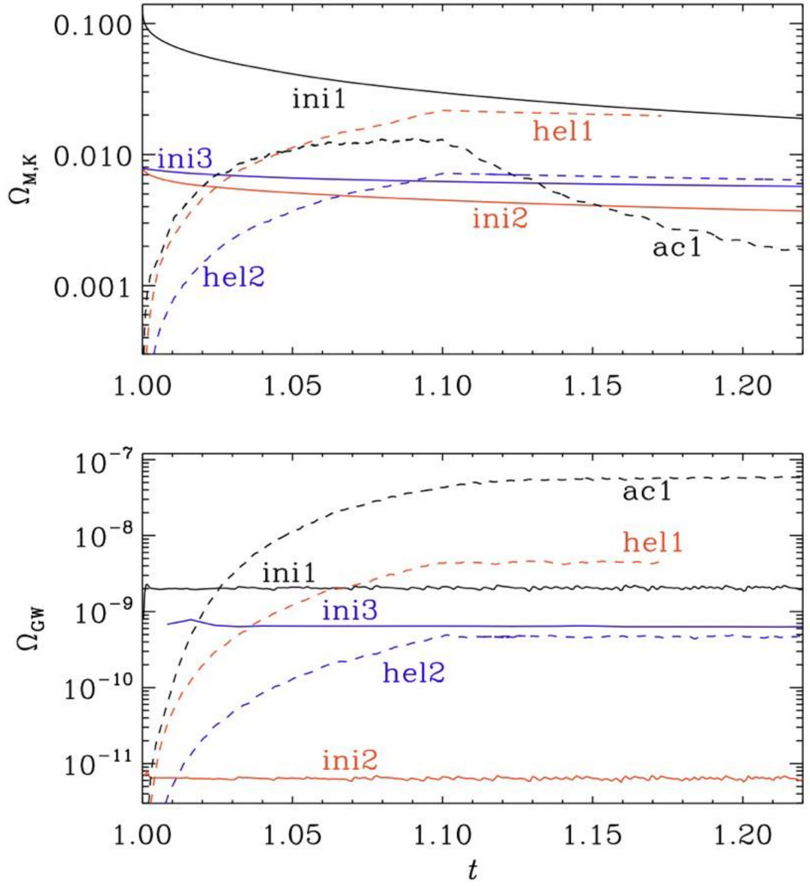
Roper Pol+20

Evolution since electroweak era

See Tanmay's talk of Monday

Brandenburg+17



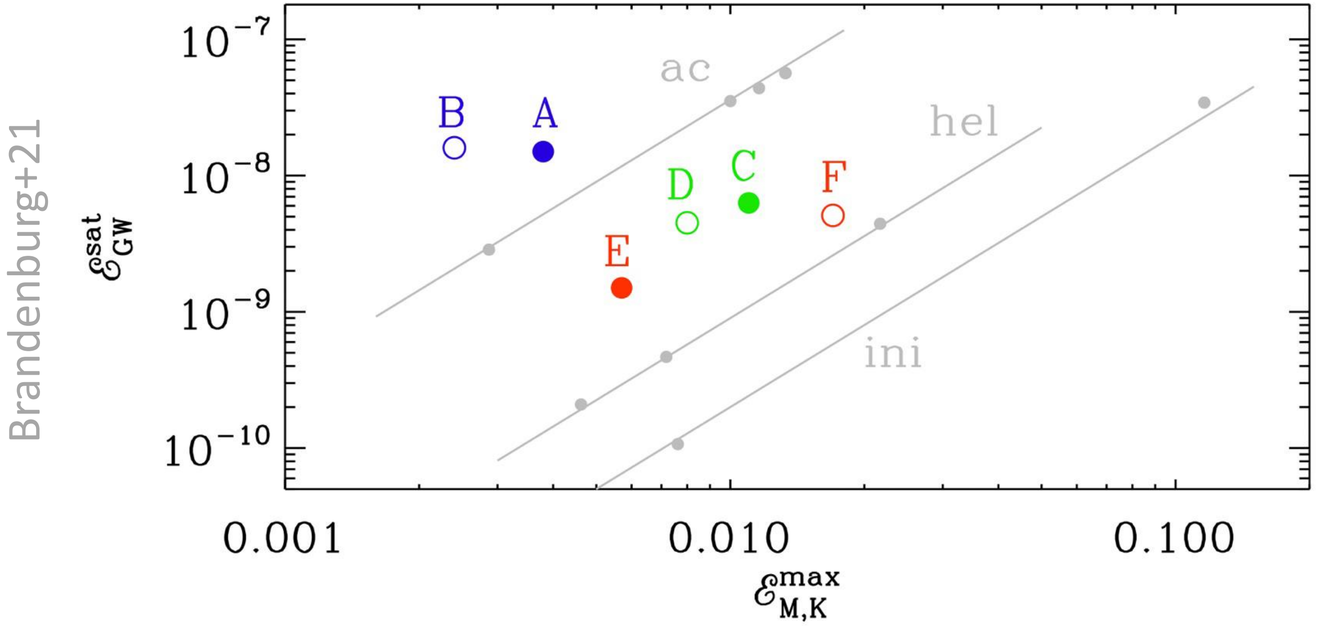


Roper Pol+20

- Power law decay: slope -(β+1)/2
- Hosking+Schekochihin: β=3/2
- Earlier termination (reconnection?)

- Magnetic energy decays
- GW energy does not!

GW energy depends quadratically on energy input & scale



$$\mathcal{E}_{\rm GW}^{\rm sat} \approx (q\mathcal{E}_{\rm M}^{\rm max}/k_{\rm peak})^2$$

Acoustic turbulence more efficient (q~30)

Vortical turbulence less efficient (q<5)

Helical MHD turbulence least efficient

- Large-scale fields → more GW energy
- Generation at electroweak era: need strong fields
- Generation during inflation & reheating

GW energy depends quadratically on energy input & scale

Rogachevskii, Boyarsky+17

Chiral magnetic effect (CME), use $[\mu_5]=[k]$

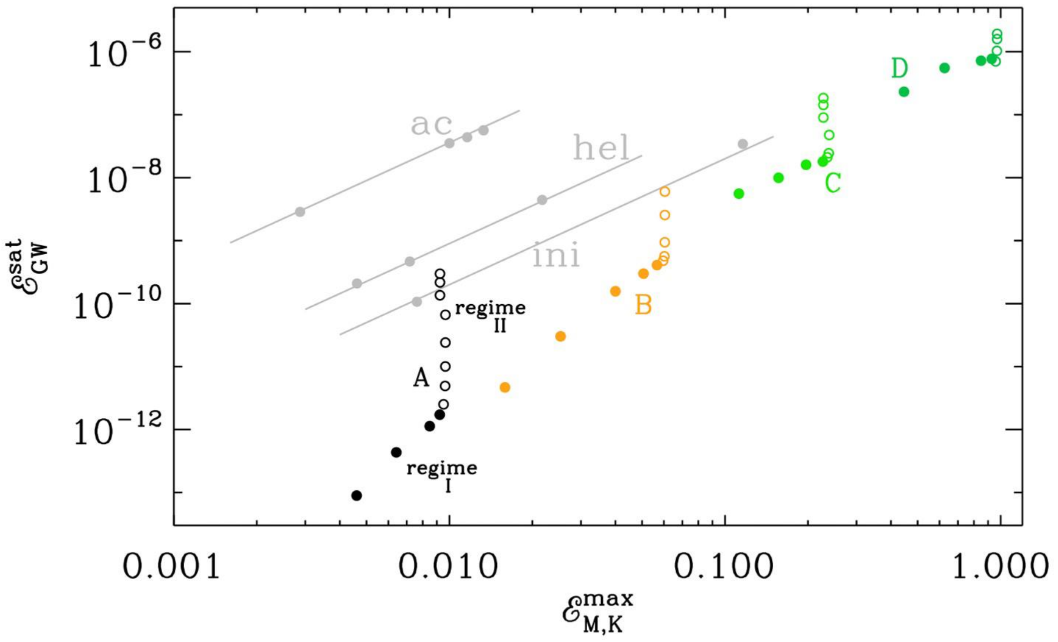
$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{u} \times \boldsymbol{B} + \eta(\mu_5 \boldsymbol{B} - \boldsymbol{J})], \quad \boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B}$$

$$\frac{D\mu_5}{Dt} = -\lambda\, \eta(\mu_5 \boldsymbol{B} - \boldsymbol{J}) \cdot \boldsymbol{B} + D_5 \nabla^2 \mu_5 - \Gamma_{\rm f} \mu_5$$

$$v_\lambda = \mu_{50}/\lambda^{1/2}, \qquad v_\mu = \mu_{50}\eta.$$

$$\eta k_1 < v_\mu < v_\lambda \quad \text{(regime I)},$$

$$\eta k_1 < v_\lambda < v_\mu \quad \text{(regime II)},$$

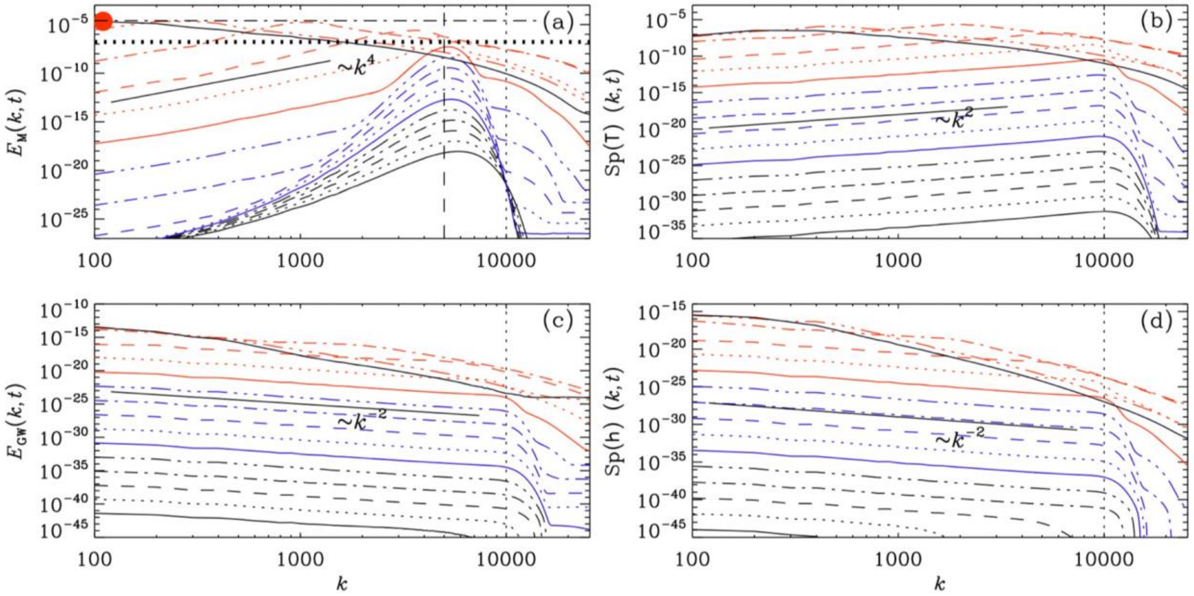


Brandenburg+21

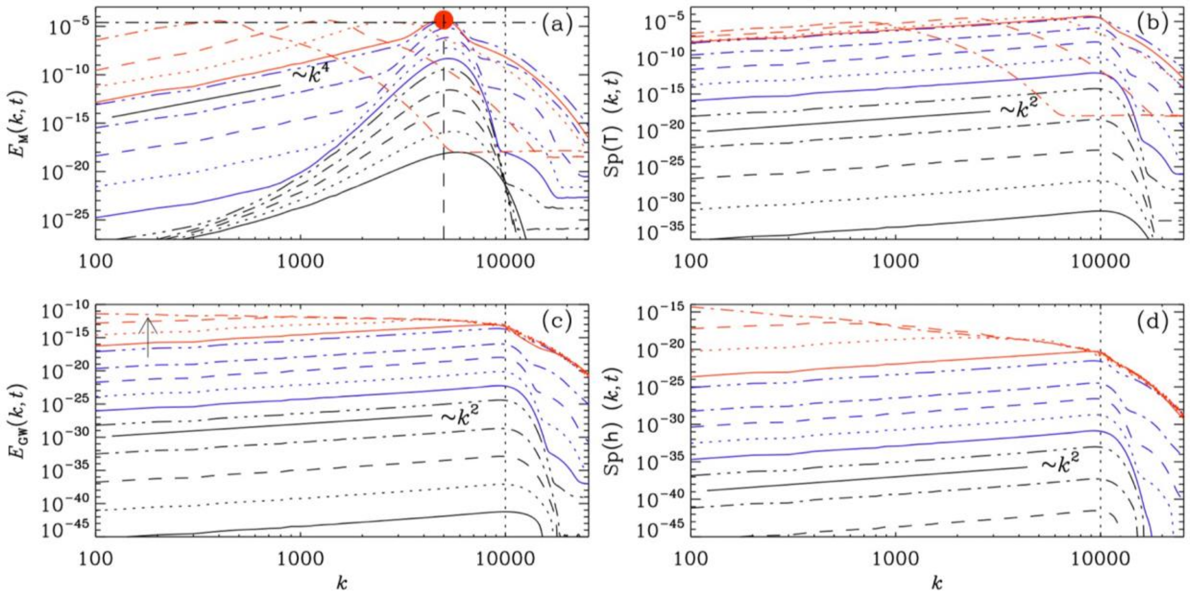
- Regime II: is more resistive → unrealistic, but large GW energies
- Regime I: → realistic, but small scales & less GW energy

GW energy depends quadratically on energy input & scale

Regime I



Regime II



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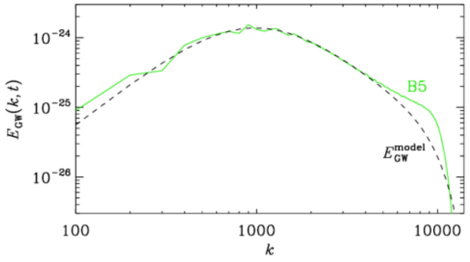
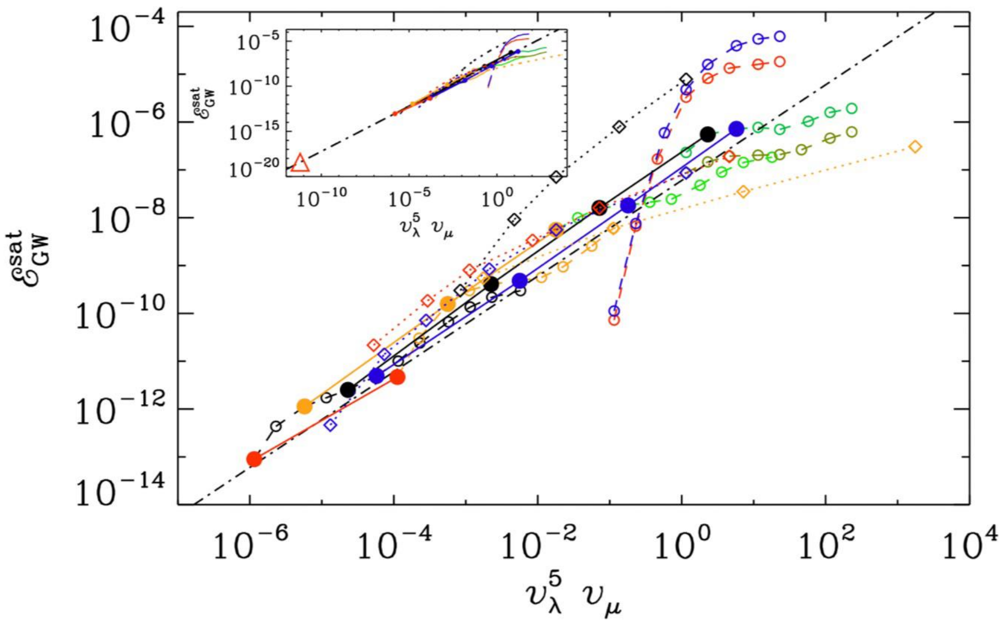
$$\tilde{h}(k,\, t) = \frac{6\tilde{T}_0(k)}{4\gamma_0^2 + k^2}\left[e^{2\gamma_0\tau} - \cos k\tau - \frac{2\gamma_0}{k}\sin k\tau\right]_{\tau=t-1}$$

Kinematic phase:
Peak determined
By growth rate

$$\gamma_0 = \eta\mu_{50}^2/4$$



GW energy depends quintically on limiting CME speed v_λ



$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times [\boldsymbol{u} \times \boldsymbol{B} + \eta(\mu_5 \boldsymbol{B} - \boldsymbol{J})], \quad \boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B}$$

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$$v_\lambda = \mu_{50}/\lambda^{1/2}, \qquad v_\mu = \mu_{50}\eta.$$

$$\eta k_1 < v_\mu < v_\lambda \quad \text{(regime I)},$$
$$\eta k_1 < v_\lambda < v_\mu \quad \text{(regime II)},$$

Brandenburg+21

- For realistic parameters → very weak GW energy
- Need larger length scales

Magnetic energy also weak

$$\langle \boldsymbol{B}^2 \rangle\, \xi_{\rm M} = \epsilon\, (k_{\rm B} T_0)^3 (\hbar c)^{-2},$$

Brandenburg+17

Inflationary magnetogenesis

- Early Universe Turbulence
  - Source of gravitational waves
  - Information from young universe
- Magnetogenesis
  - Inflation/reheating
  - No particles yet, no conductivity
  - Coupling with electromagn field

$$f^2 F_{\mu\nu} F^{\mu\nu},$$

  - Breaking of conformal invariance
  - Quantum fluct → field stretched

$$\tilde{\mathbf{A}}'' + \left(\mathbf{k}^2 - \frac{f''}{f}\right)\tilde{\mathbf{A}} = 0,$$

$$\tilde{h}''_{+/\times} + \left(\mathbf{k}^2 - \frac{a''}{a}\right)\tilde{h}_{+/\times} = \frac{6}{a}\tilde{T}_{+/\times},$$

Code (Pencil Code Collaboration et al. 2021), where

$$\mathbf{A}' = \mathbf{u} \times \mathbf{B} + \sigma^{-1}\nabla^2 \mathbf{A} \quad (8)$$

is solved in real space, which includes the induction effect from the velocity and the finite conductivity. It is solved together with (Brandenburg et al. 1996)

$$\mathbf{u}' = -\mathbf{u} \cdot \nabla \mathbf{u} - \frac{1}{4}\nabla \ln \rho + \frac{3}{4\rho}\mathbf{J} \times \mathbf{B} + \boldsymbol{\mathcal{F}}_\nu + \boldsymbol{\mathcal{F}}, \quad (9)$$

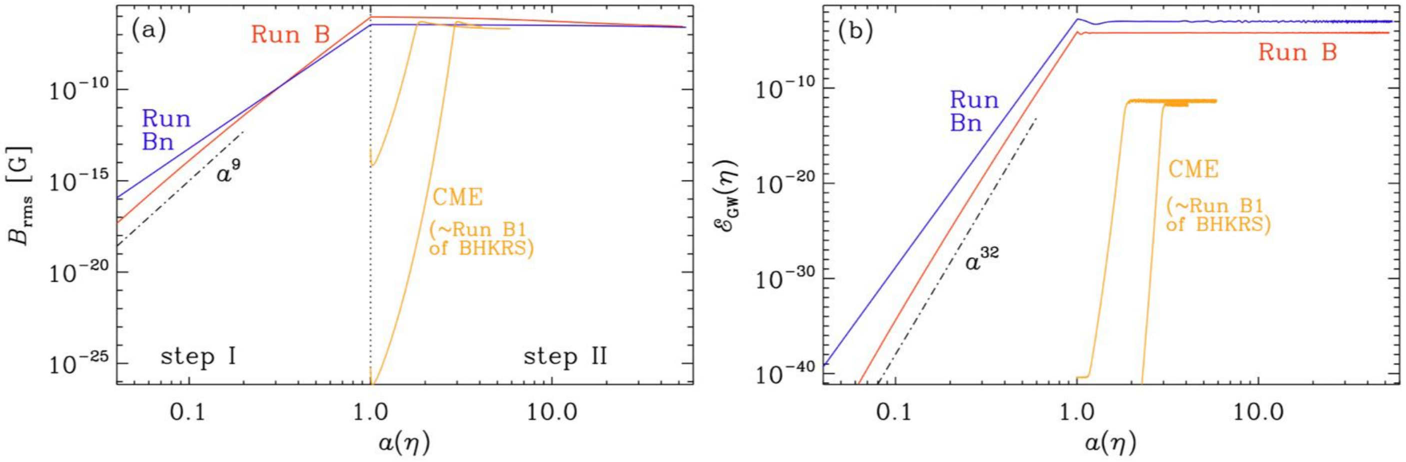
$$(\ln \rho)' = -\frac{4}{3}(\nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \ln \rho) + \mathcal{H}, \quad (10)$$

where $\boldsymbol{\mathcal{F}} = (\nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \ln \rho)\mathbf{u}/3 - [\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) + \mathbf{J}^2/\sigma]\mathbf{u}/\rho$, and $\mathcal{H} = [\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) + \mathbf{J}^2/\sigma]\rho$ are higher

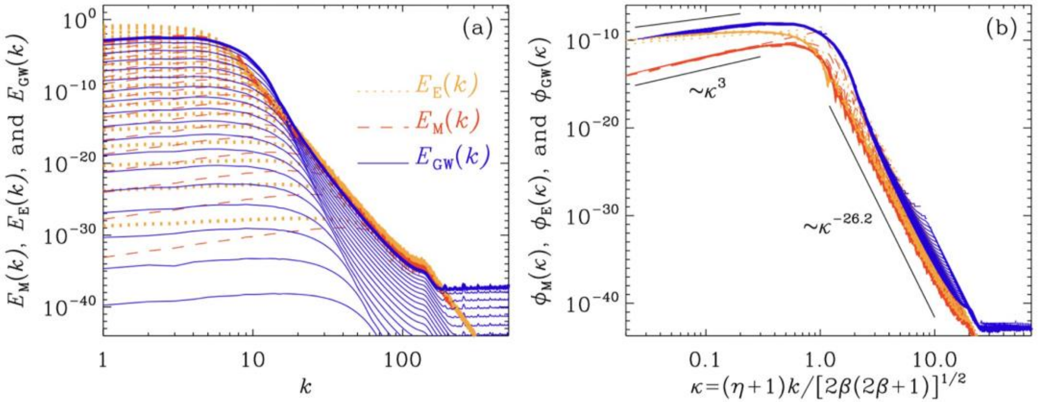
Software and Data Availability. The source code used for the simulations of this study, the PENCIL CODE (Pencil Code Collaboration et al. 2021), is freely available on https://github.com/pencil-code/. The doi of the code is 10.5281/zenodo.2315093 (Brandenburg 2018). The simulation setup and the corresponding data are freely available from doi:10.5281/zenodo.4448211, see also http://www.nordita.org/~brandenb/projects/GWfromCME/ for easier access.

Brandenburg & Sharma 2106:03857

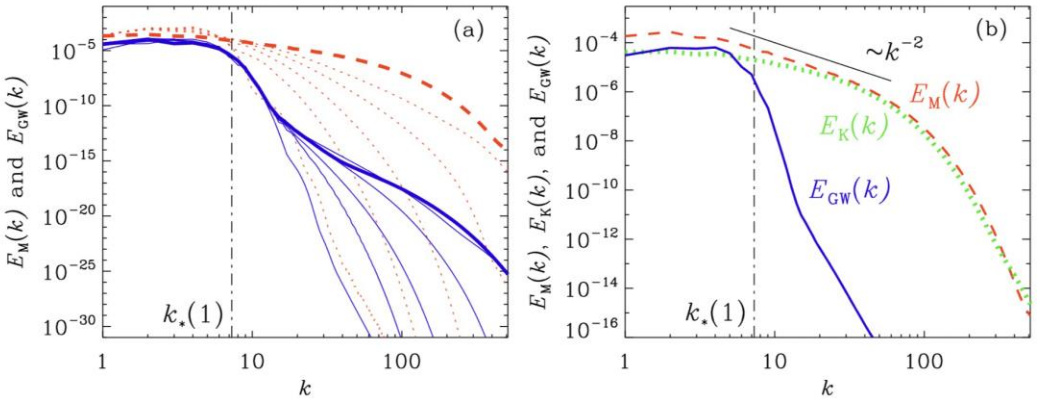
Inflationary growth & magnetic decay



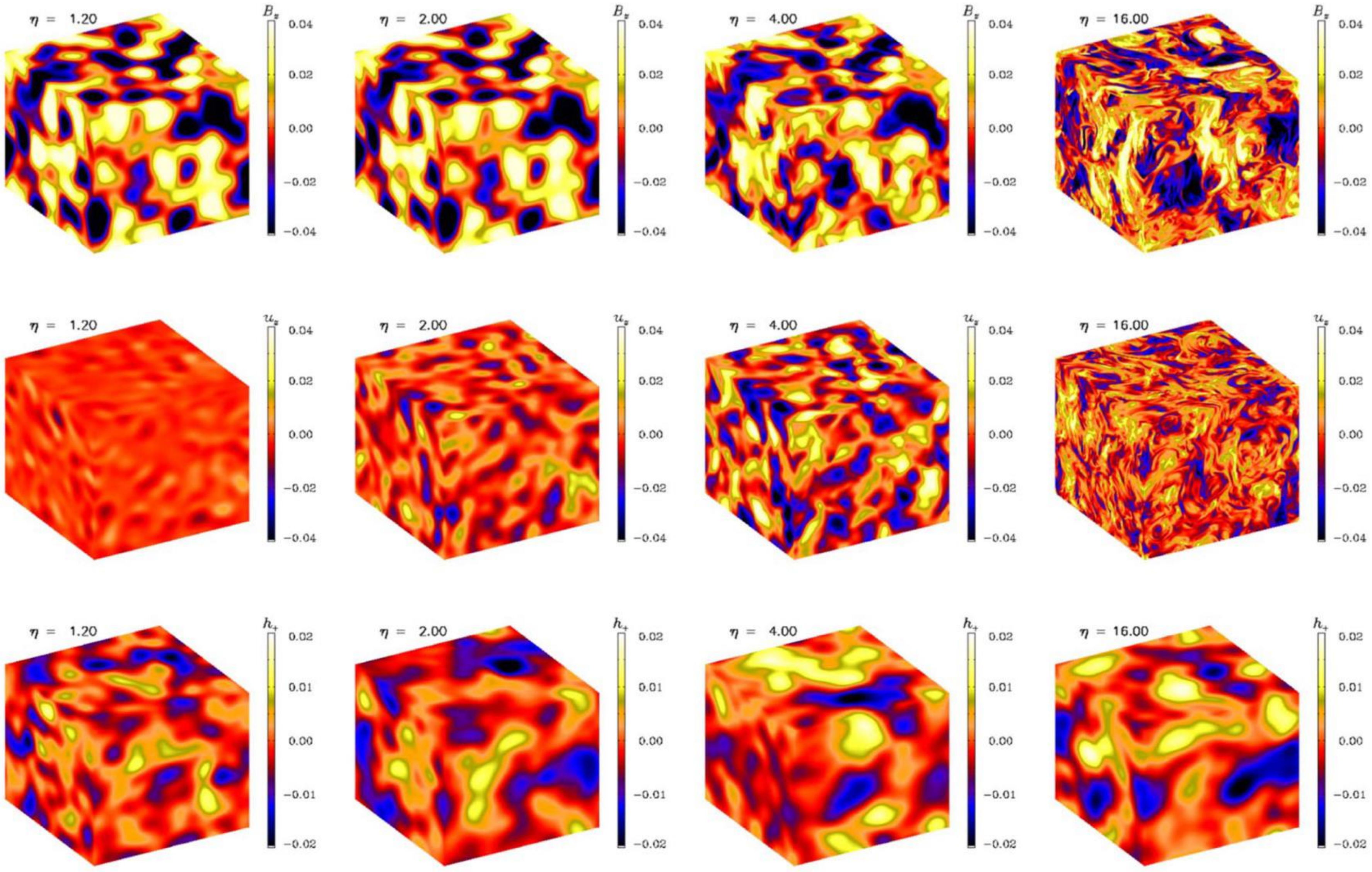
Inflationary growth: electric, magnetic, and GW grow



Lorentz force drives smaller scales: surprisingly weak



No small scales in GW field



Circular polarization in chiral inflationary magnetogenesis

$$\iota f^2 F_{\mu\nu} {}^*F^{\mu\nu}$$

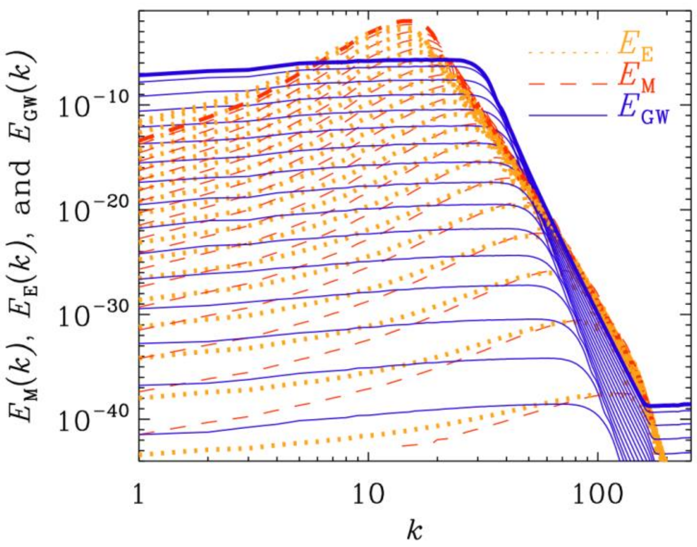
$$f(a) = a^{-\beta}, \quad \text{where } a = (\eta + 1)^2/4$$

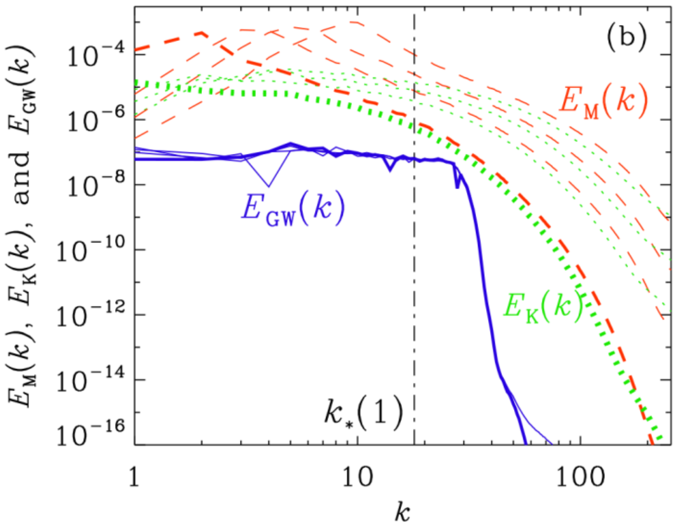
$$\tilde{A}''_\pm + \left(k^2 \pm 2\iota k \frac{f'}{f} - \frac{f''}{f}\right) \tilde{A}_\pm = 0,$$

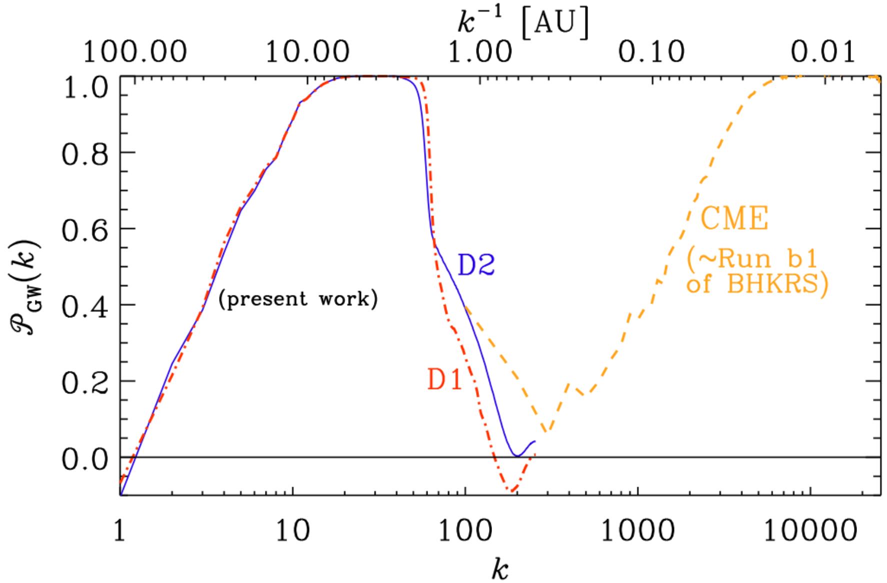
$$\frac{f'}{f} = -\frac{2\beta}{\eta + 1}, \qquad \frac{f''}{f} = \frac{2\beta(2\beta + 1)}{(\eta + 1)^2}.$$

- Step I: spectra peaked
- Step II: Inverse cascade
- GW: circularly polarized

$$\mathcal{P}(k) = \int 2\,\mathrm{Im}\,\tilde{h}_+ \tilde{h}_\times^*\, k^2 \mathrm{d}\Omega_k \Big/ \int \left(|\tilde{h}_+|^2 + \tilde{h}_\times|^2\right) k^2 \mathrm{d}\Omega_k$$







Helical field from CME or inflation: Always ~100% circular polarized

Conclusions

- Sp(dh/dt) ~ k^{-2} Sp(T) ~ k^{-2} Sp(B)
- B decays, GW do not → messenger
- CME: interesting, but weak B and GWs
- Inflation & reheating: large scale
  - Small scales during radiation era
  - But surprisingly inefficient
  - $f^{-8/3}$ spectrum characteristic of turbulence
- NANOGrav result ←→ low energy scale