Gravitational waves from (magneto)hydrodynamic turbulence at LISA and PTA

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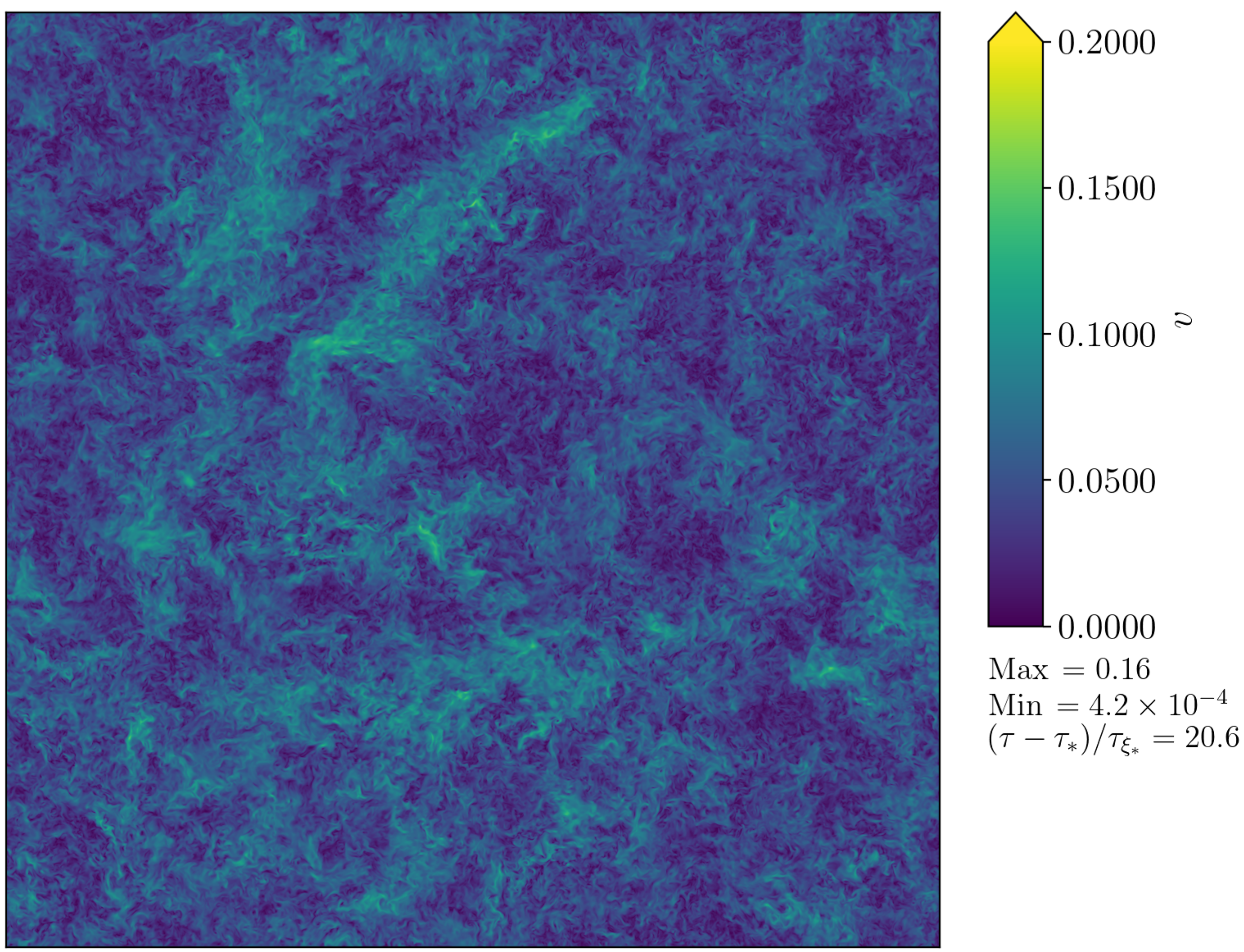


P. Auclair
D. Cutting
M. Hindmarsh
A. Neronov
A. Roper Pol
K. Rummukainen
D. Semikoz
D. Steer
D. Weir

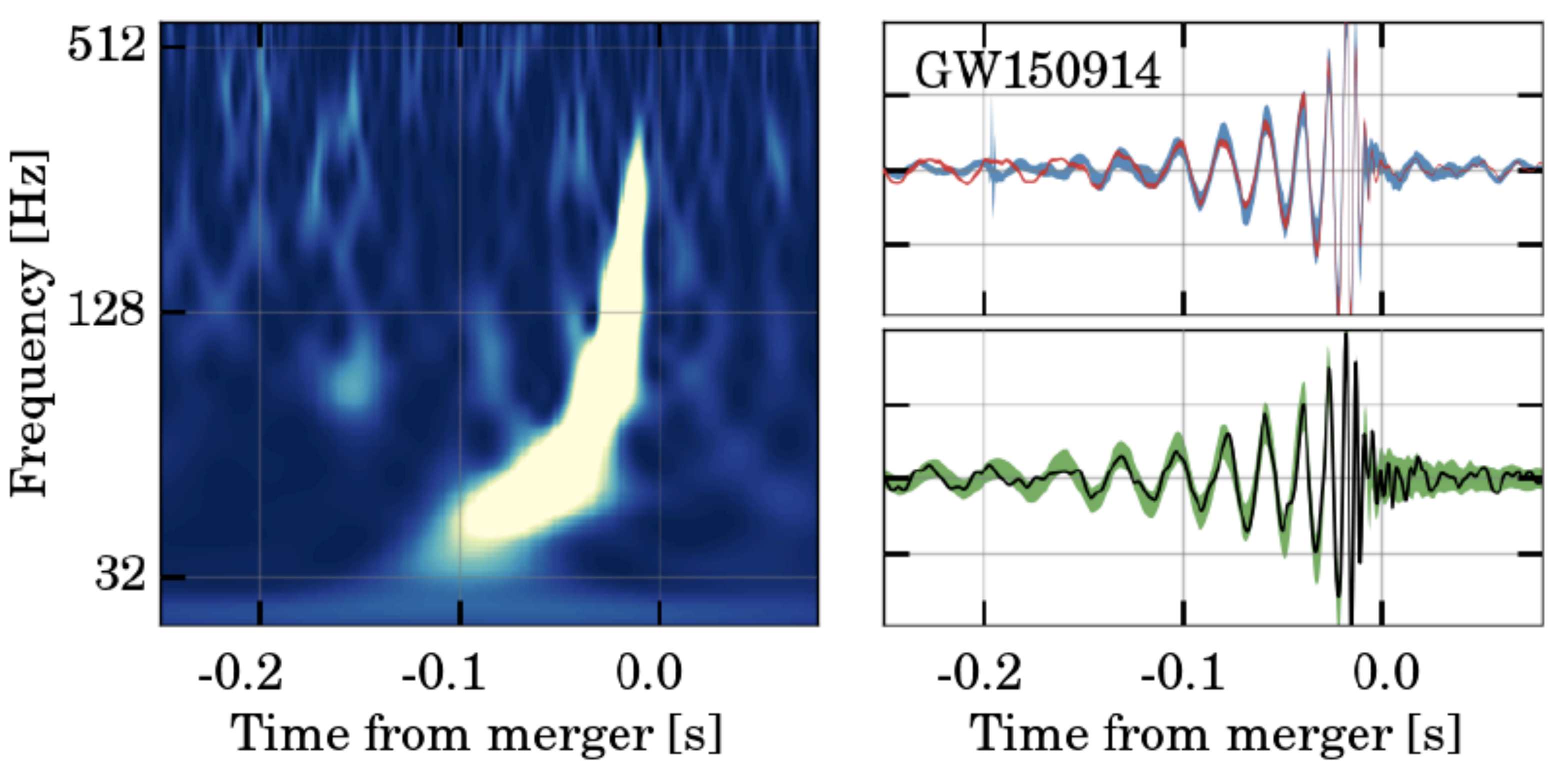
Outline

- Introduction to gravitational wave signals from the early universe
  - What they are
  - How we could detect them (LISA, PTA)
  - Discovery potential
  - How they are connected to primordial magnetic fields
- Results
  - Possible signals at LISA and PTA
  - (More technical) how is the signal predicted? What is new?

The stochastic gravitational wave background

the superposition of sources that cannot be resolved individually

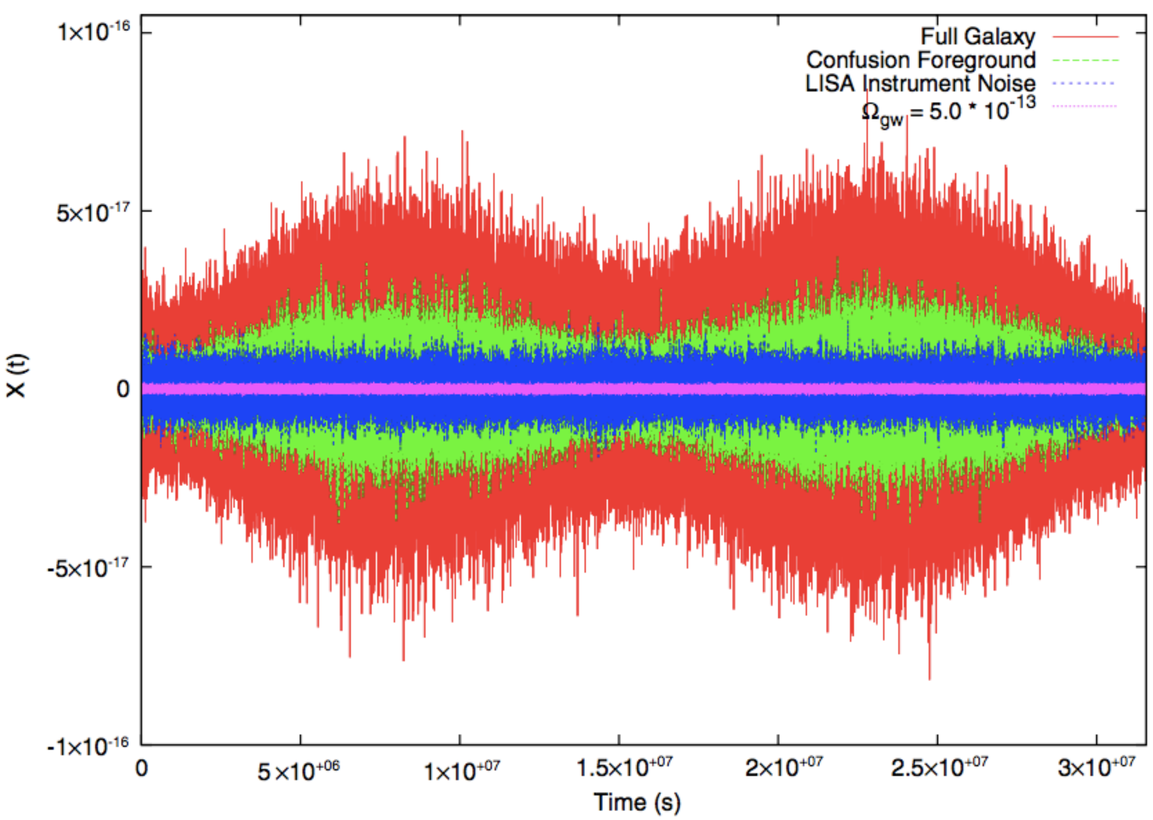
- Compact binaries too numerous and with too low SNR to be identified



The stochastic gravitational wave background

the superposition of sources that cannot be resolved individually

- Compact binaries too numerous and with too low SNR to be identified
- signals from the primordial universe with too small correlation scale (typically horizon at the time of production) with respect to the detector resolution



Adams and Cornish, 1307.4116

GWs can bring direct information from very early stages of the universe evolution, to which we have no direct access through em radiation

amazing discovery potential, linked to high energy physics ->
primordial magnetism

Space-based GW detection

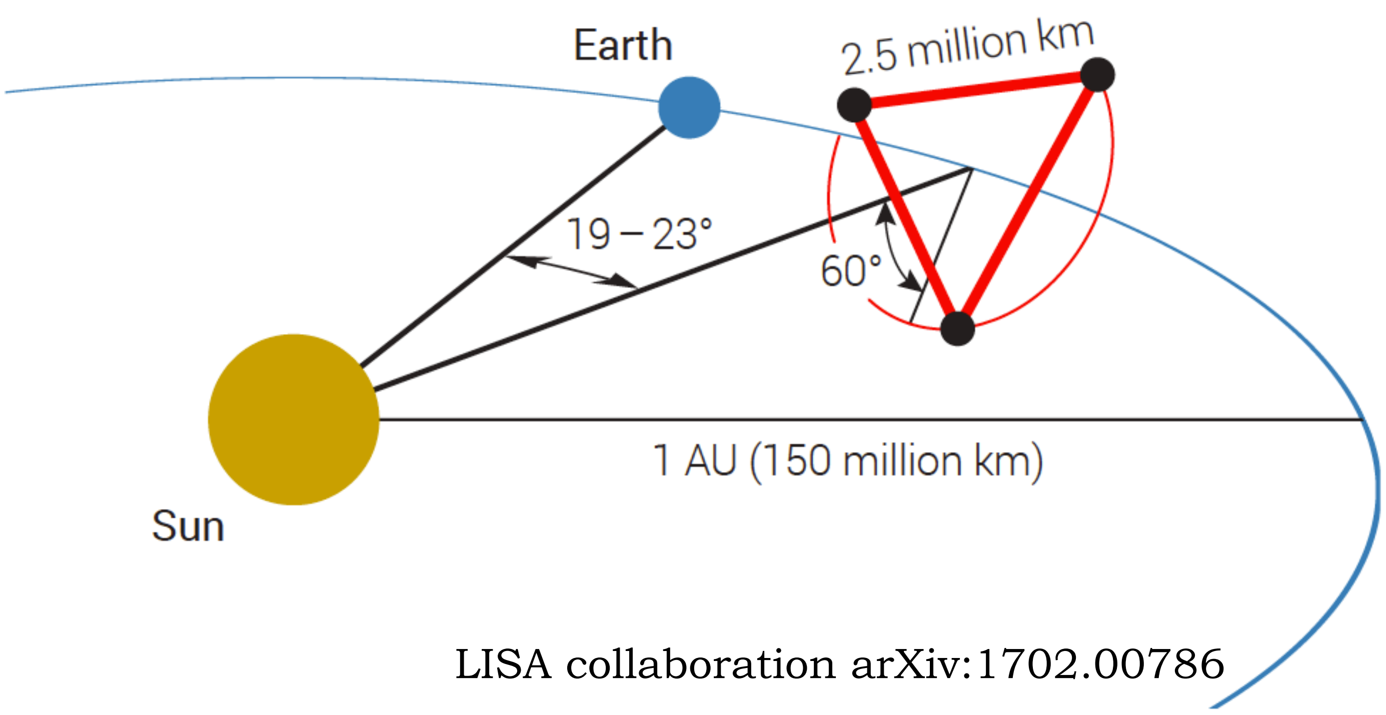
LISA: Laser Interferometer Space Antenna

- no seismic noise
- much longer arms than on Earth: 2.5 million km

frequency range of detection: $10^{-4}\,\mathrm{Hz} < f < 1\,\mathrm{Hz}$

TARGET SOURCES:

- Coalescing massive BH binaries: 10^4 to 10^7 solar masses
- Inspiralling black hole binaries of few to hundred solar masses
- Inspiralling galactic binaries (white dwarfs, neutron stars...)
- Extreme Mass Ratio Inspirals
- Stochastic GW background from astrophysical and cosmological sources



LISA collaboration arXiv:1702.00786

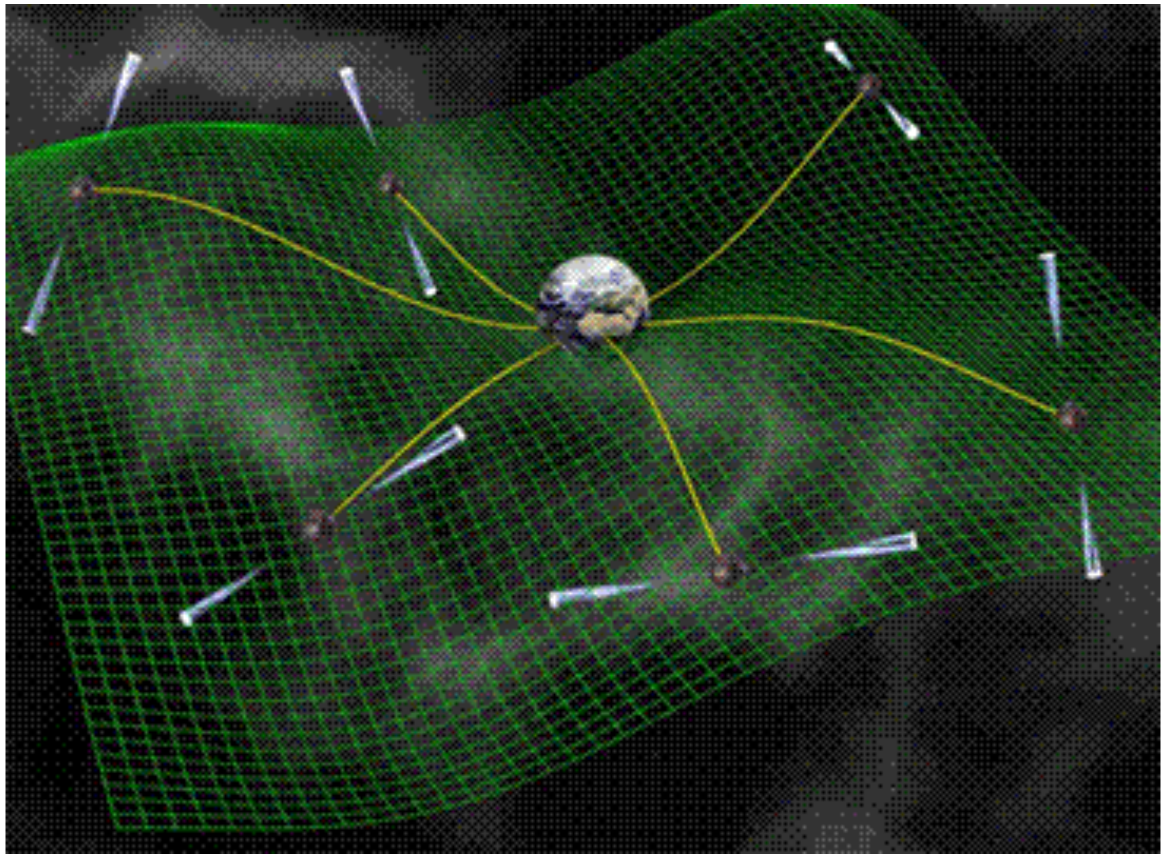
Space-based GW detection

Pulsar timing array

frequency range of detection: $10^{-9}\,\mathrm{Hz} < f < 10^{-7}\,\mathrm{Hz}$

OBSERVABLE:
correlated shifts in time of arrivals of radio pulses due to GW propagation between Pulsar and Earth

TARGET SOURCE:
Super Massive BH binaries (masses of order 10^9 solar masses):
stochastic background from inspirals and/or resolved signals



Recent discovery of correlated noise in all Pulsar networks!
(NanoGrav, Parkes, European, International)

Z. Arzoumanian et al, arXvi: 2009.04496, B. Goncharov et al, arXiv:2107.12112, S. Chen et al, arXiv:2110.13184

Space-based GW detection

Pulsar timing array

- There is a strong statistical support for the presence of a common red noise
- There is no evidence yet for a quadrupolar signal
- Possible explanation: background from SMBHBs (but *compatible with MHD turbulence from the early universe*)

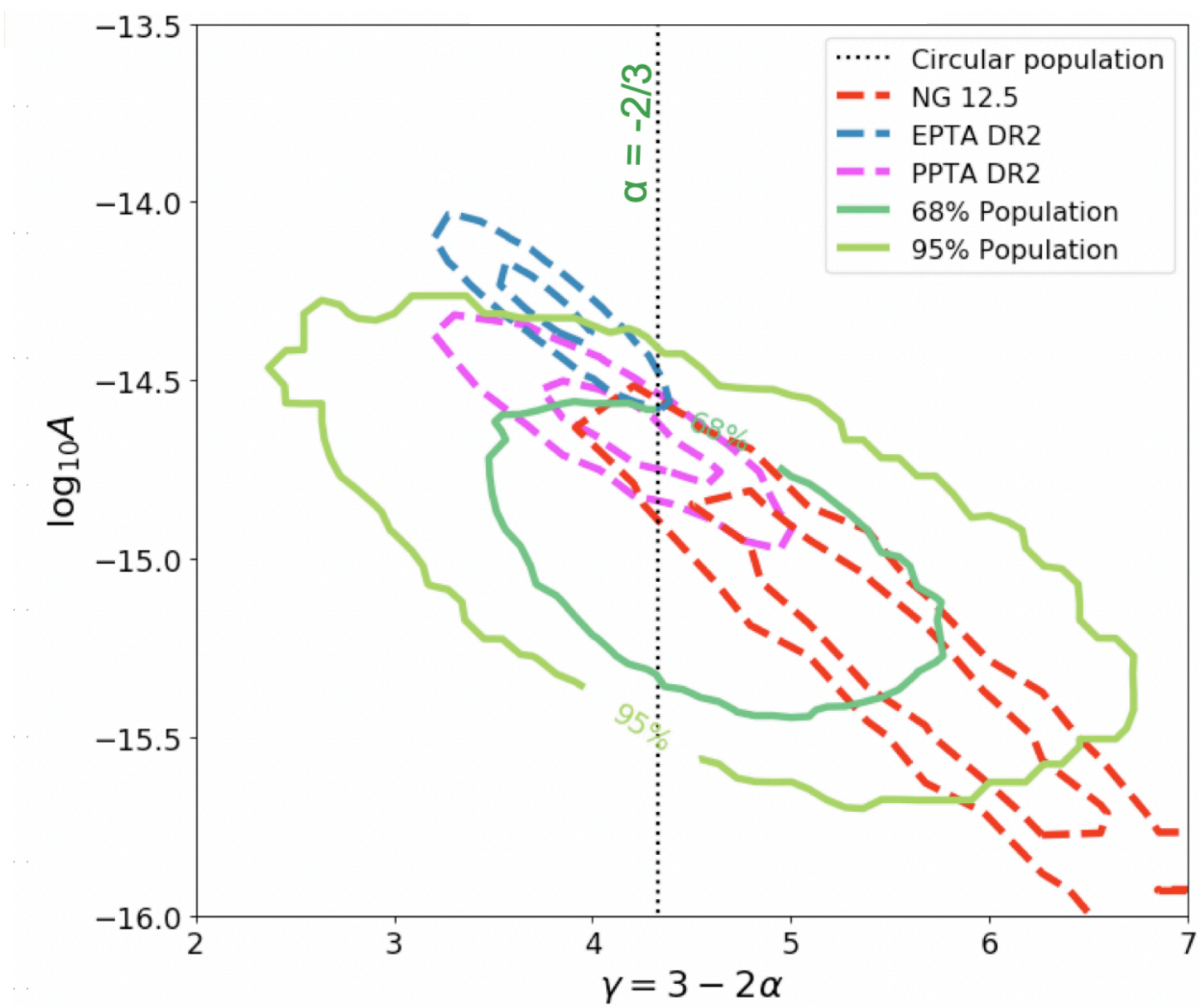


Figure from:
S. Babak & S. Chen, 2021

Z. Arzoumanian et al, arXvi: 2009.04496, B. Goncharov et al, arXiv:2107.12112, S. Chen et al, arXiv:2110.13184

GWs in the early universe

GWs are *tensor perturbations* of the FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$|h_{ij}| \ll 1$ $h^i_i = \partial_j h^j_i = 0$ superimposed on the homogeneous and isotropic background

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G\,(\bar{T}_{\mu\nu} + \delta T_{\mu\nu})$$

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = \boxed{16\pi G\,\Pi^{TT}_{ij}}$$

ACTIVE GW SOURCE

tensor anisotropic stress, for example from EM field:

$$\Pi^{TT}_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$$

A GW source acting at time t_* in the early universe cannot produce a signal correlated on length/time scales larger than the causal horizon at that time

$$\ell_* \leq H_*^{-1}$$

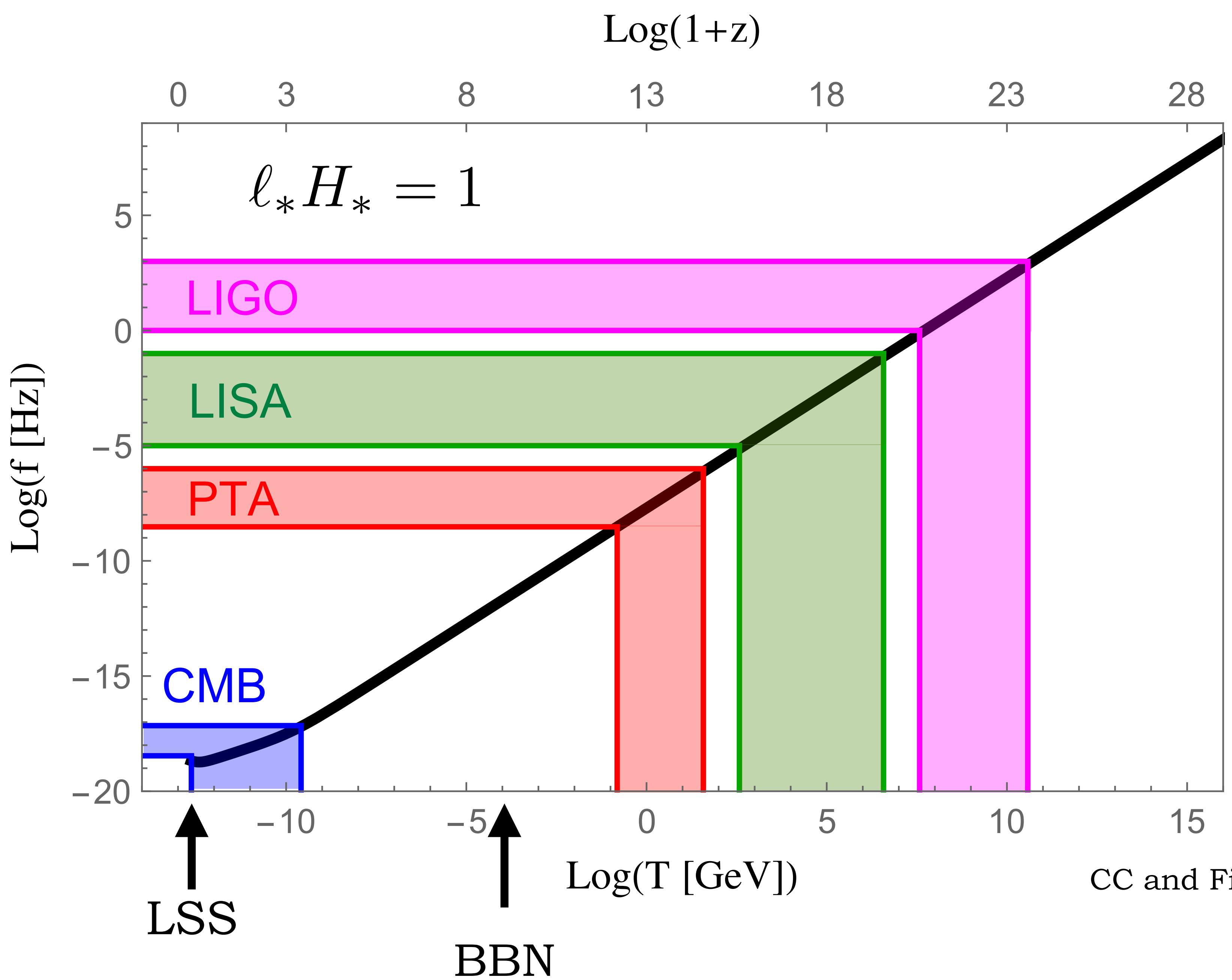
ℓ_* characteristic length/time-scale of the source
typical size/time of the tensor anisotropic stresses

characteristic frequency of the GW signal $$f_* = \frac{1}{\ell_*} \geq H_*$$

$\ell_* H_*$ Ratio of the typical length/time-scale of the GW sourcing process to the Hubble scale at the generation time

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\ell_* H_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{\mathrm{GeV}} \, \mathrm{Hz}$$

Characteristic frequency of the GW signal

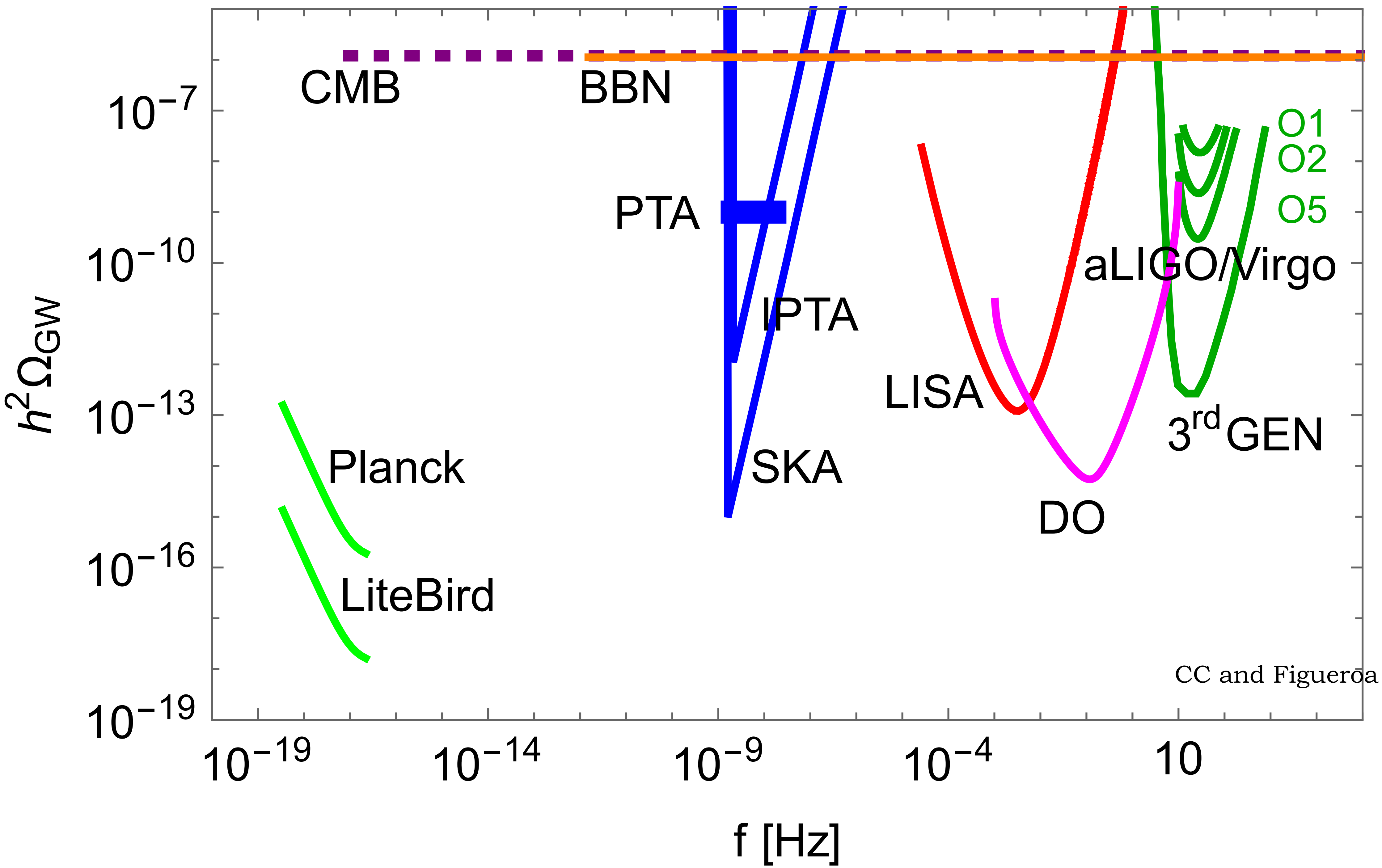


CC and Figueroa arXiv:1801.04268

$T_{\rm QCD} \sim 100\,{\rm MeV}$ $\quad \ell_* H_* \sim 0.1 \longrightarrow f \sim 10\,{\rm nHz}$ PTA

$T_{\rm EW} \sim 100\,{\rm GeV}$ $\quad \ell_* H_* \sim 0.01 \longrightarrow f \sim {\rm mHz}$ LISA

What is/will be known about a stochastic GW background:



CC and Figueroa arXiv:1801.04268

Are there signals to populate this diagram?

Examples of SGWB sources in the early universe

- Background from the very early universe

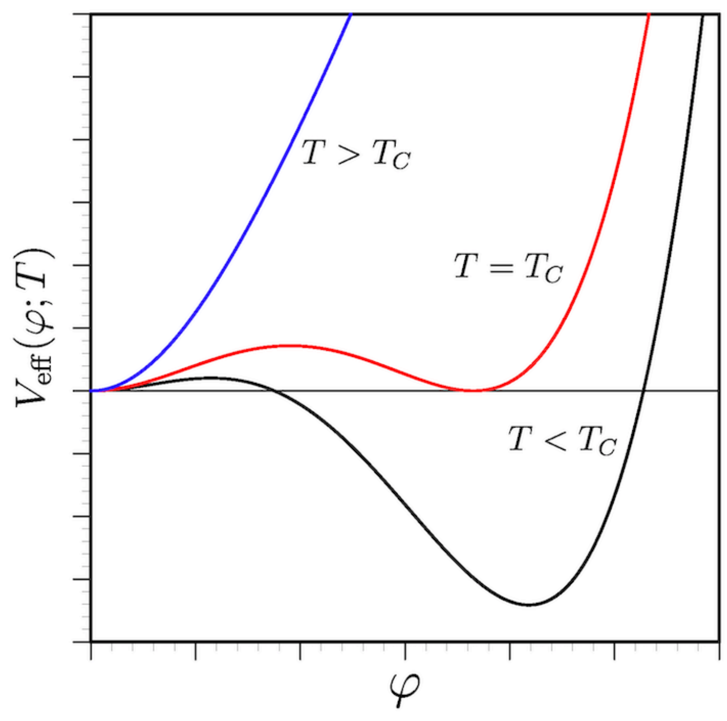
  - Inflation:
    - quantum tensor fluctuations (at first and second order)
    - tensor modes from additional fields (scalar, gauge…)
    - GWs linked to primordial BHs
    - preheating
    - modifications of gravity
    - …

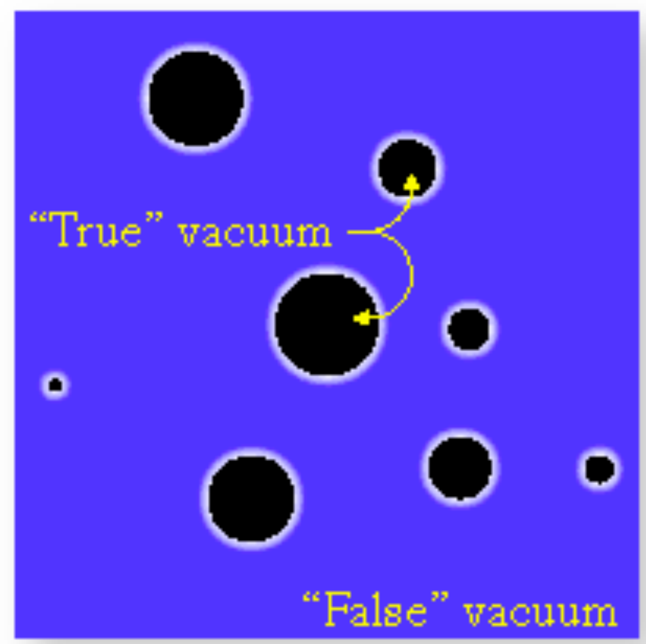
  - Other phase transitions:
    - stable topological defects (in particular strings)
    - *first order* phase transitions
      - bubble wall collisions
      - bulk fluid motion (compressional and vortical)
      - magnetic fields

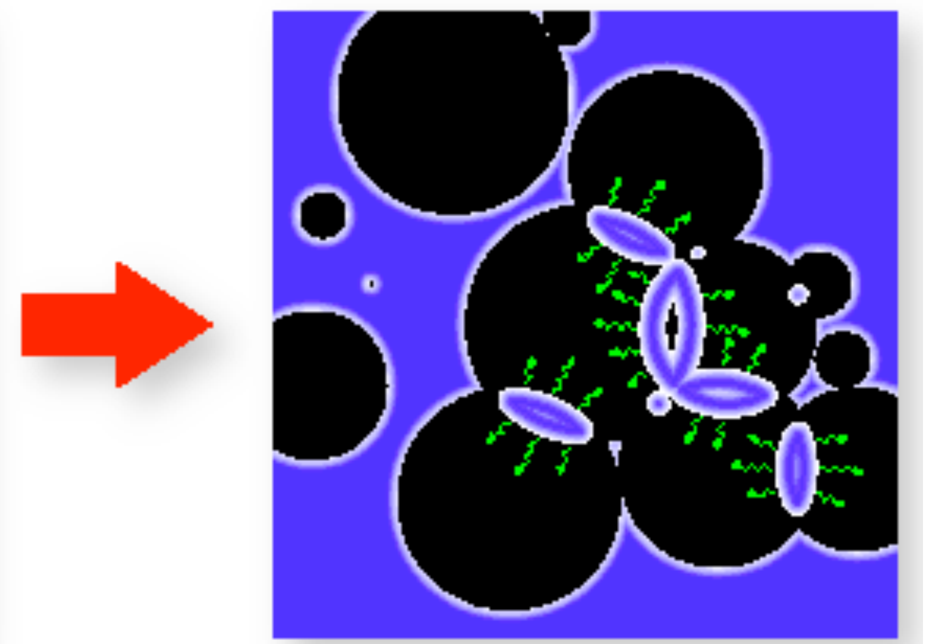
- Foreground from astrophysical sources (galactic binaries, stellar origin BHB…)
  - *to be accounted for or subtracted if the spectral shape is known*

Sources of tensor anisotropic stress at a first order phase transition:

GW sourcing process $$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$







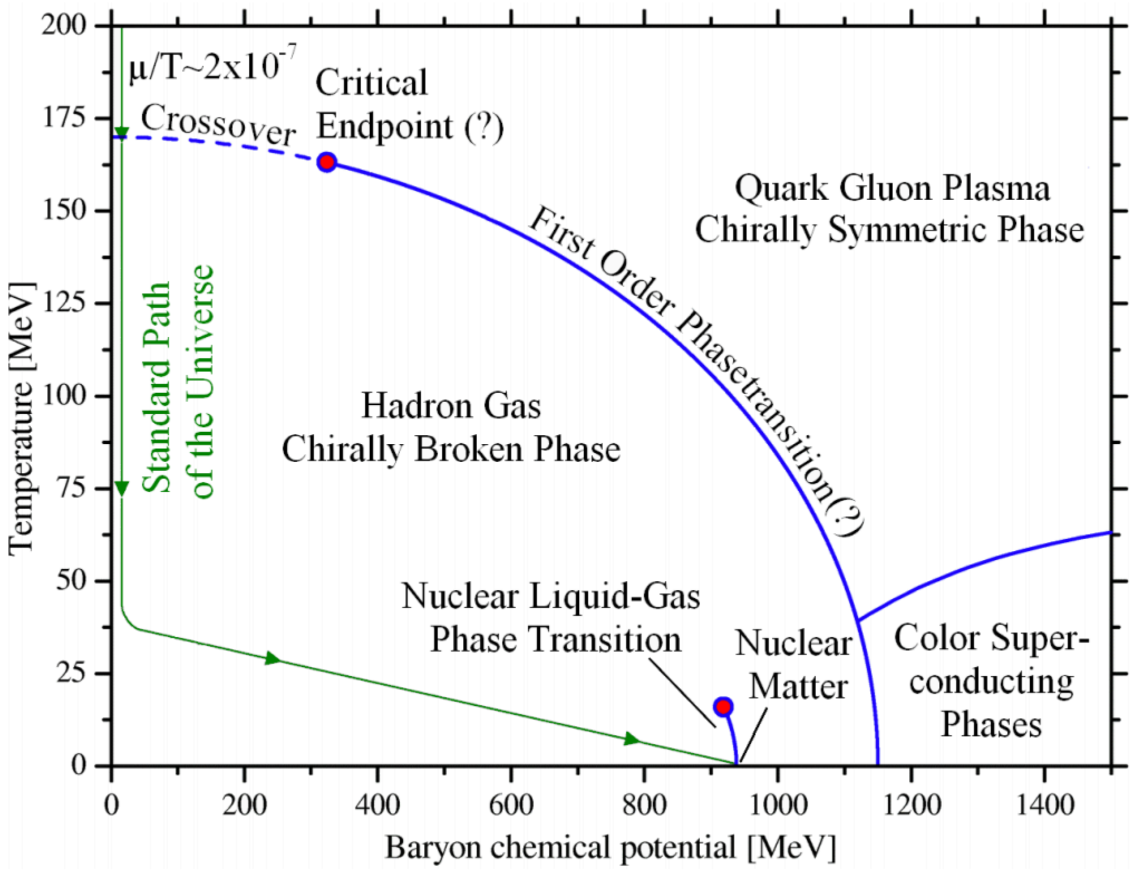
- Bubble collision (scalar field gradients) $$\Pi_{ij}^{TT} \sim [\partial_i\phi\partial_j\phi]^{TT}$$
- Bulk fluid motion $$\Pi_{ij}^{TT} \sim [\gamma^2(\rho+p)v_iv_j]^{TT}$$
- Electromagnetic fields $$\Pi_{ij}^{TT} \sim [-E_iE_j - B_iB_j]^{TT}$$

QCD phase transition and Pulsar Timing Array noise excess

- In the Standard Model at zero baryon chemical potential it is a cross-over, negligible GW production
- It depends on the (uncertain) conditions of the early universe

D. Schwarz and Stuke, arXiv:0906.3434
M. Middeldorf-Wygas et al, arXiv:2009.00036

T. Boekel and J. Schaffner-Bielich, arXiv:1105.0832



PTA (nHz) are sensitive to energy scales around the **QCD scale**, so they can probe physical processes connected to the QCDPT IF it is first order

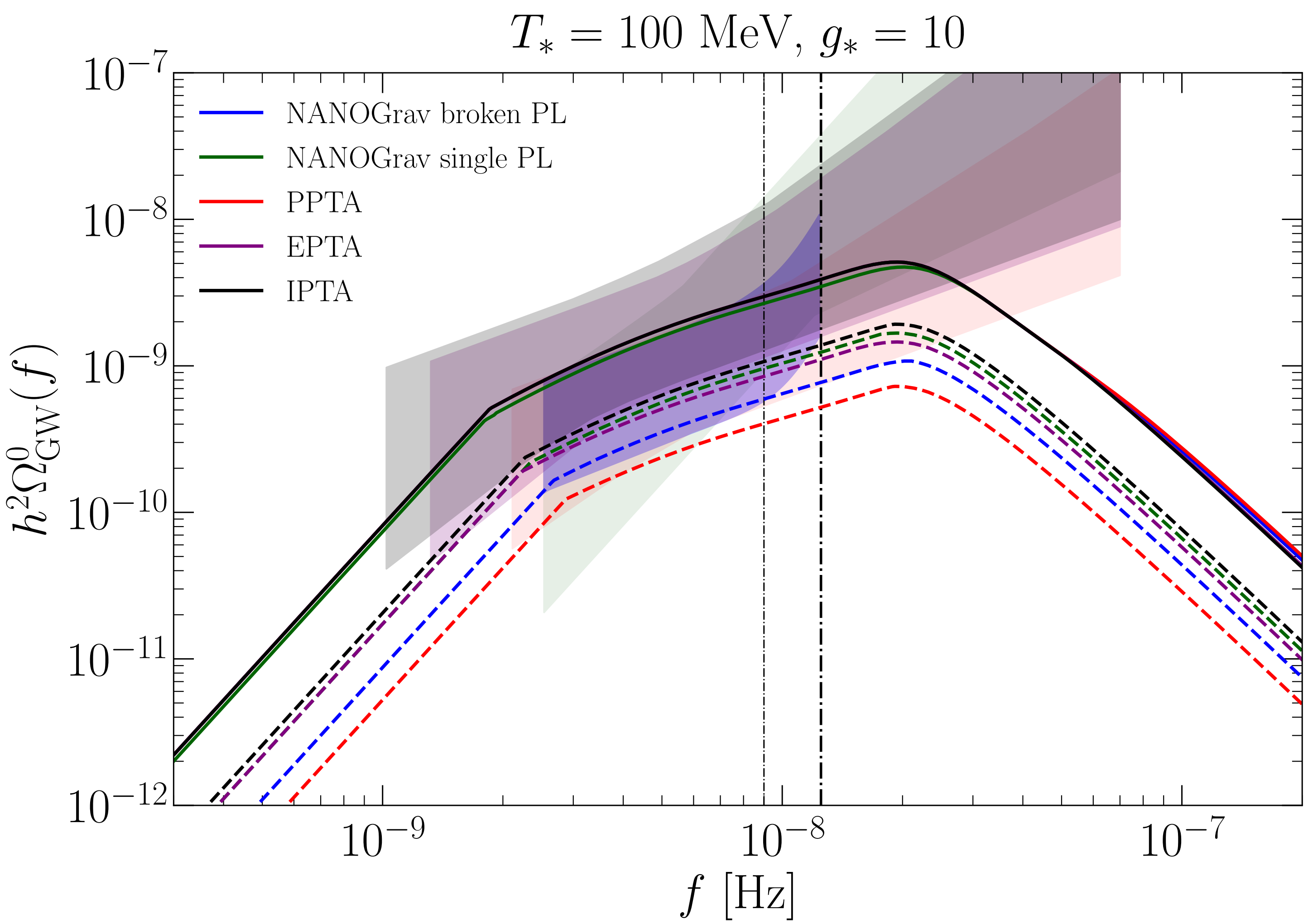
- PTA observatories (NANOGrav, Parkes, European) have recently measured common noise excess

Z. Arzoumanian et al, arXvi: 2009.04496, B. Goncharov et al, arXiv:2107.12112, S. Chen et al, arXiv:2110.13184

- It is compatible with the GW generated by fully developed MHD turbulence at the QCD scale

A. Neronov et al, arXiv:2009.14174

QCD phase transition and PTA noise excess: MHD turbulence from first order PT?



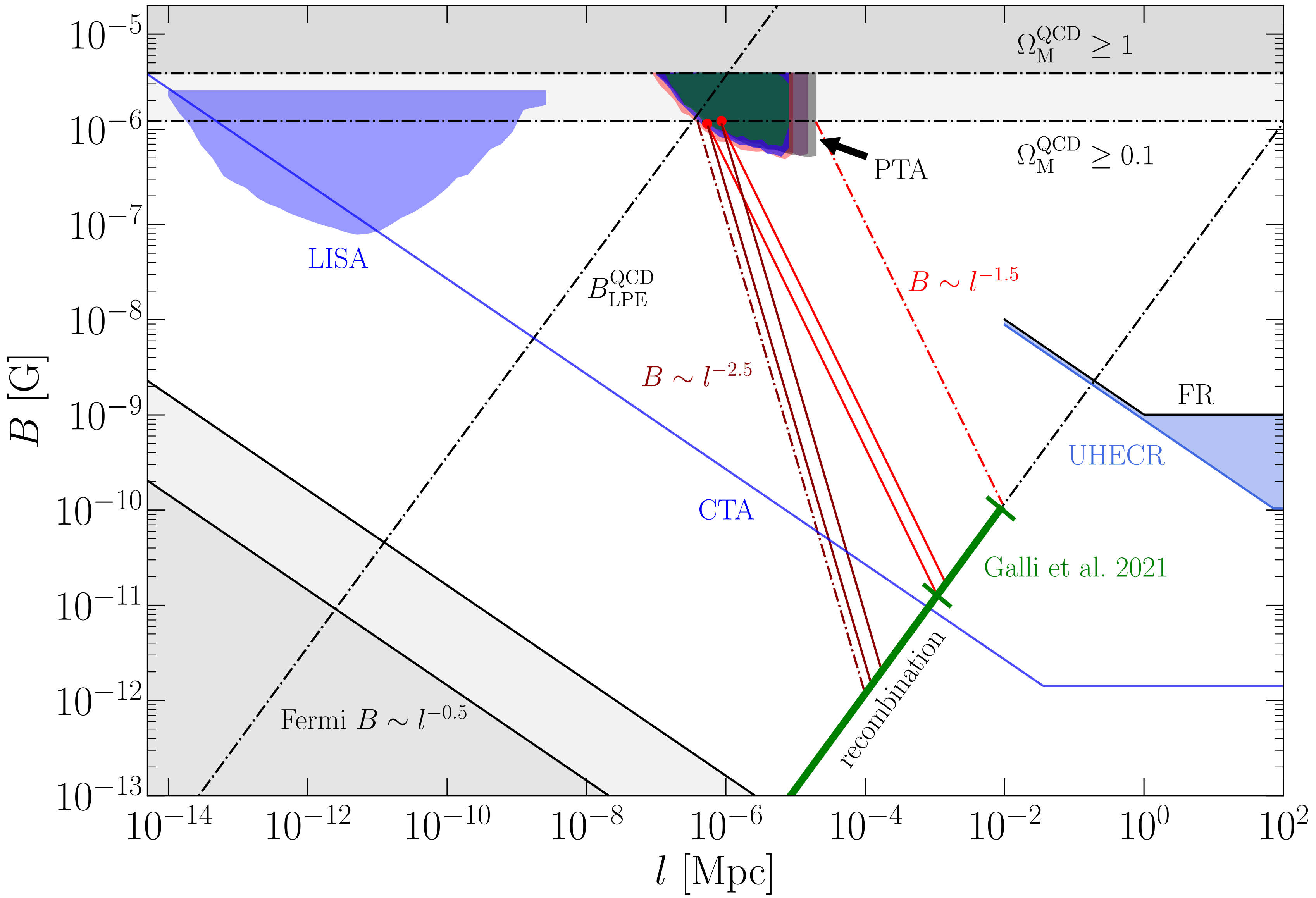
Regions compatible with the PTA observations, a GW spectrum must lie within them

The parameters are

$$(T_*,\ \Omega_B,\ \ell_* \mathcal{H}_*)$$

A. Roper Pol et al, arXiv:2201.05630

- For QCD temperature scales, the part of the GW spectrum falling in the region of best quality PTA data is the sub-peak one
  - Slopes (k^3 or k^1) fully compatible with PTA constraints
  - Visible break in the spectrum occurring at $k \sim \mathcal{H}_{QCD}$
- The temperature scale is constrained to 2 MeV < T_* < 200 MeV, the magnetic field energy density must be close to 10% of the radiation energy density and the magnetic correlation scale must be close to the horizon



- The magnetic field giving rise to the GW signal evolves in the radiation era

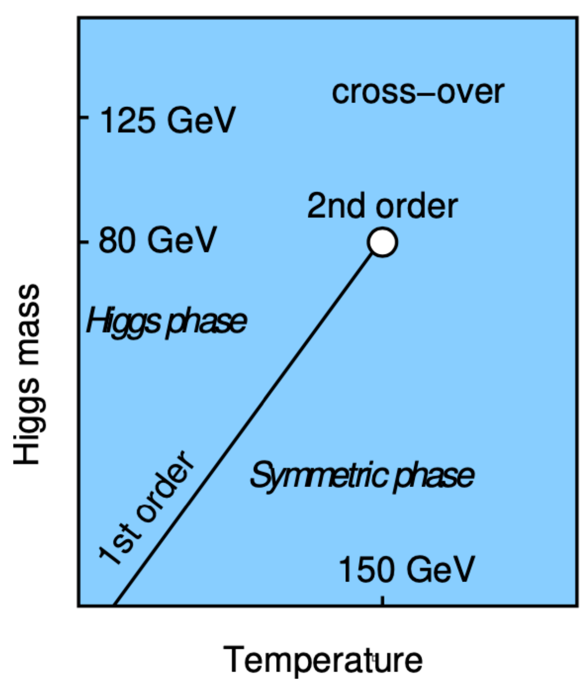
Banerjee and Jedamzik arXiv:0410032,
Durrer and Neronov, arXiv:1303.7121

- It might modify the CMB spectrum and ease the Hubble tension at recombination, seed the magnetic fields observed today in matter structures, and be constrained by future gamma-ray telescopes

S. Galli et al, arXiv:2109.03816
Jedamzik and Pogosian, arXiv:2004.09487
Korochin et al, arXiv:2007.14331

Electroweak phase transition: phase transition of the Higgs field, driven by the temperature decrease as the universe expands

Standard Model
of particle physics:
Cross-over
Negligible GW production



M. Hindmarsh et al,
arXiv:2008.09136

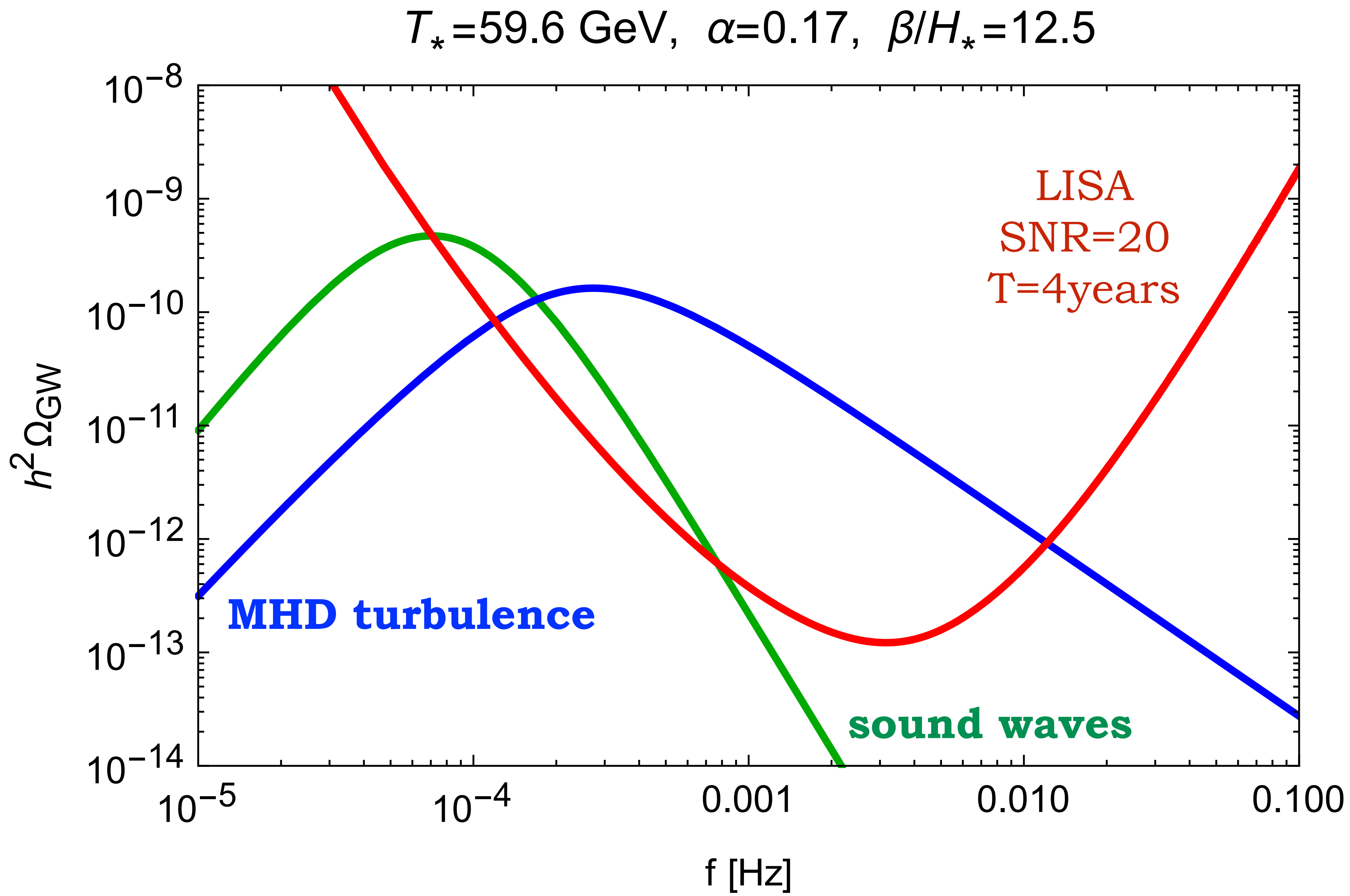
Beyond the Standard Model:
First order phase transition
Possibly observable GW production

Examples of scenarios leading to observable signals:

- singlet/multiplet extensions of SM or MSSM (SUSY motivated or not)
- SM plus dimension six operator (EFT approach)
- Dark Matter sector uncoupled to the SM
- Warped extra dimensions
- …

One example of GW signal from the EW phase transition
“Higgs portal” scenario

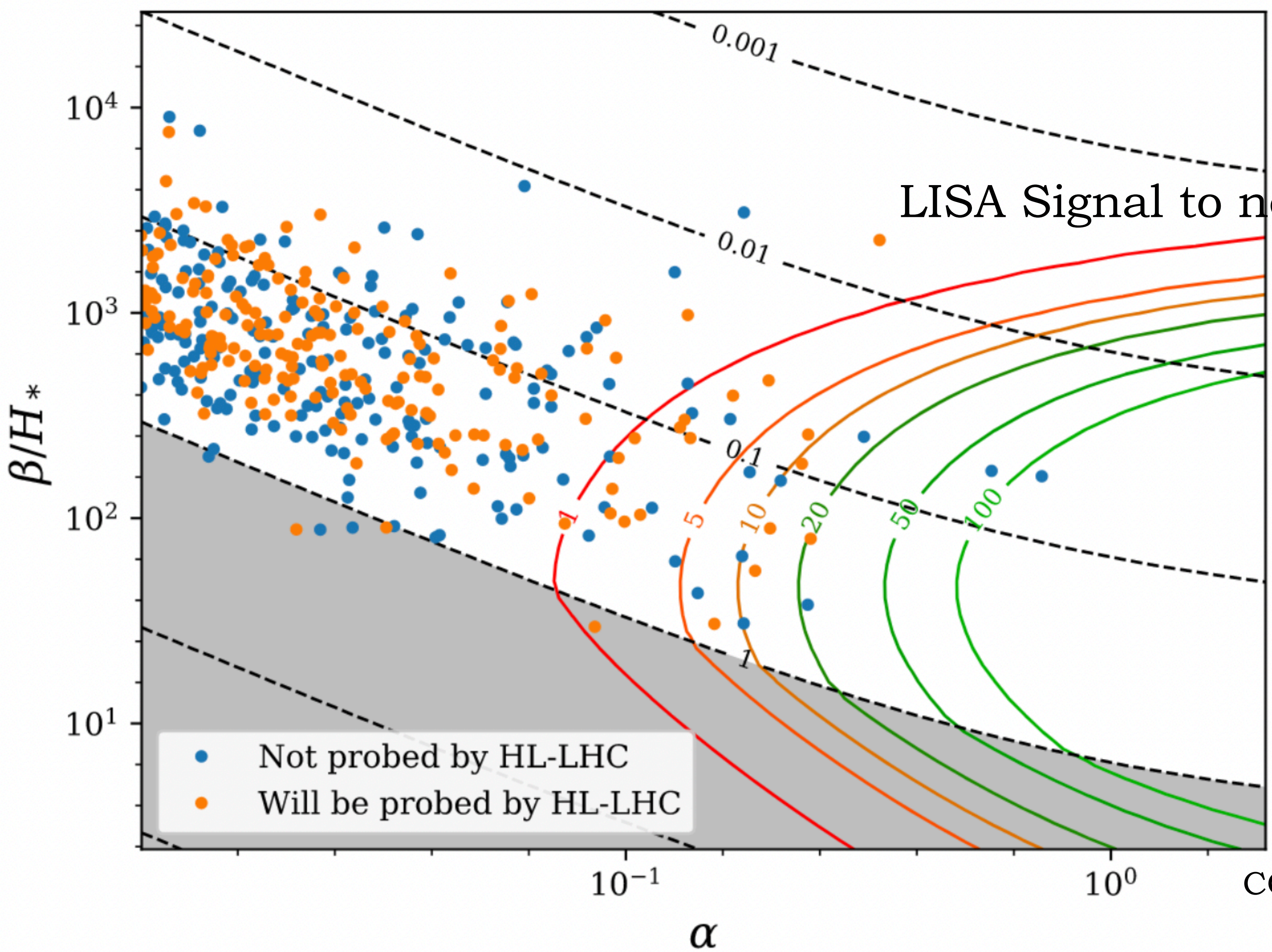
$$\tau_{\rm nl} \sim \frac{\ell_*}{v_{\rm rms}} = \frac{0.54}{\mathcal{H}_*}$$



$$\langle v^2 \rangle_{\rm turb} = 0.25 \, \langle v^2 \rangle_{\rm sound} \qquad \qquad \delta t_{\rm fin} = 5 \, \delta t_e$$

Can be probed both at LISA and at the High Luminosity LHC

Inverse duration of the first order EW phase transition



LISA Signal to noise ratio curves

CC et al, arXiv:1910.13125

Strength of the first order EW phase transition

Prediction of the stochastic GW signal from magneto-hydrodynamic turbulence in the aftermath of a first order PT

Magneto-hydrodynamic turbulence from a first order PT

- Turbulence is a phase of chaotic fluid motion, in general it presents both dilatational and vortical components

- It arises when the advection term in Navier-Stokes equation is larger than the diffusion one: large Reynolds number

$$\mathrm{Re} = \frac{v_{\rm rms}\,\ell}{\nu} > 1$$

$$T_* \sim 100 \text{ GeV},\ v_{\rm rms} \sim 0.01,\ \ell_* \mathcal{H}_* \sim 0.01 \quad \longrightarrow \quad \mathrm{Re}(\ell_*) \simeq 3 \cdot 10^{13}$$

Kinetic viscosity from the particle content (neutrinos)

- Turbulence is expected to occur in the early universe

- If an initial electromagnetic field is also present, the magnetic field will be amplified to equipartition with the kinetic energy, while the electric one will be dissipated (the conductivity is very high)

$$P_{\rm m} = \nu\sigma > 1$$

Magneto-hydrodynamic turbulence from a first order PT

- The full system is composed by: scalar field driving the transition, surrounding fluid to which the field is coupled, magnetic field "frozen into the fluid"
- This system can be highly non-linear for strongly first order PTs, when the energies in the game (vacuum, kinetic) are high: it needs to be tackled through simulations

- SCOTTS code (Helsinki group): coupled dynamics of the field-fluid system, no magnetic field, relativistic

Helsinki/Sussex group,
M. Hindmarsh et al,
arXiv:1304.2433 and following

D. Cutting et al, arXiv:1906.00480

- Pencil code (Nordita group): simulates MHD turbulence (present in the initial conditions or induced by adapted forcing), relativistic up to order v^2

A. Roper Pol et al,
arXiv:1903.08585 and other works by the Nordita group

Both codes output the GW signal

Magneto-hydrodynamic turbulence from a first order PT

It is challenging to observe the onset of MHD turbulence in these simulations
It can be done only with the SCOTTS code which simulates the phase transition

strength of the PT (bag EoS) $\alpha = \frac{\rho_{\rm vac}}{\rho^*_{\rm rad}} < 0.1 \longrightarrow \tau_{\rm nl} \sim \frac{\ell_*}{v_{\rm rms}} \gg$ time of the simulation

- In this conditions, the main source of GW are sound waves
- Bubble walls collision is subdominant, no turbulence seen (linear regime)
  Helsinki/Sussex group, M. Hindmarsh et al, arXiv:1304.2433 and following
- But in simulations reaching the mildly non-linear regime, vorticity is generated
  D. Cutting et al, arXiv:1906.00480

Our results are produced with a
fully developed turbulent spectrum
as initial conditions for the random velocity/magnetic field

Analysis of the GW signal from (M)HD turbulence

- We construct a model of non-relativistic (M)HD turbulence and its anisotropic stresses improving on previous analytical analyses

e.g. CC et al, arXiv:0909.0622 and Niksa et al, arXiv:1803.02271

- We validate it with (M)HD simulations using *both codes*, but with (M)HD turbulent spectra (velocity and magnetic) inserted in the initial conditions

- We calculate the SGWB spectrum
  1. From the simulations
  2. From numerical integration of the source model (purely kinetic for now but work in progress for MF)
  3. With analytical assumptions

- We provide an analytical template for the SGWB spectrum as a function of the (M)HD turbulence parameters, which can be easily used to estimate the signal

P. Auclair et al, arXiv:2205.02588
A. Roper Pol et al, arXiv:2201.05630

Model of the source

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

$$\Pi_{ij}^{TT} \sim [\gamma^2(\rho+p)v_i v_j]^{TT}$$

$$\Pi_{ij}^{TT} \sim [-B_i B_j]^{TT}$$

The SGWB spectrum:

$$\left.\frac{\mathrm{d}\Omega_{\rm gw}}{\mathrm{d}\ln k}\right|_{\eta_0} = \Omega^0_{\rm rad}\frac{g_{\rm fin}}{g_0}\left(\frac{g_{s,0}}{g_{s,\rm fin}}\right)^{4/3} k^3 \iint_{\eta_{\rm ini}}^{\eta_{\rm fin}} \cos[k(\eta_0-\tau)]\cos[k(\eta_0-\zeta)] P_\Pi(k,\zeta,\tau)\frac{\mathrm{d}\tau}{\tau}\frac{\mathrm{d}\zeta}{\zeta}$$

The anisotropic stress unequal time correlation function

$$\langle\tilde\Pi_{ij}(\mathbf{k},\zeta)\tilde\Pi_{ij}(\mathbf{q},\tau)\rangle = \delta(\mathbf{k}-\mathbf{q})P_\Pi(k,\zeta,\tau)$$

Assuming a gaussian, divergence-free source:

$$P_\Pi(k,\zeta,\tau) = \int \mathrm{d}^3p\, P_s(p,\tau,\zeta)P_s(q,\tau,\zeta)[1+(\hat{k}\cdot\hat{p})^2][1+(\hat{k}\cdot\hat{q})^2]$$

$$\langle s_i(\mathbf{k},\tau)s_j^*(\mathbf{q},\zeta)\rangle = \delta(\mathbf{k}-\mathbf{q})(\delta_{ij}-\hat{k}_i\hat{k}_j)P_s(k,\tau,\zeta)$$

We need to model the velocity and magnetic field unequal time correlators

Model of the source

Source spectral density

fully developed turbulent/magnetic field spectrum, dissipating with time

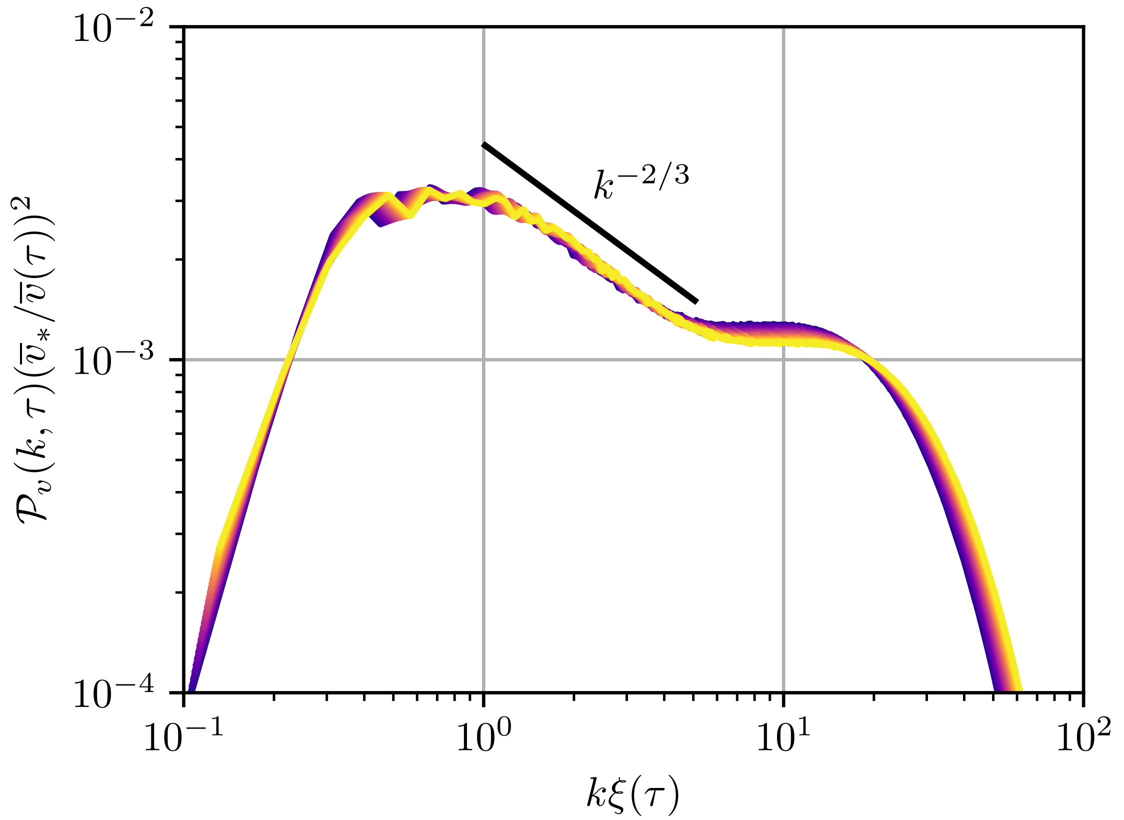
$$P_s(k,\tau) \sim \Omega_s(\tau)\ell^5(\tau)\frac{k^2}{\{1+[k\ell(\tau)]^2\}^{17/6}}$$

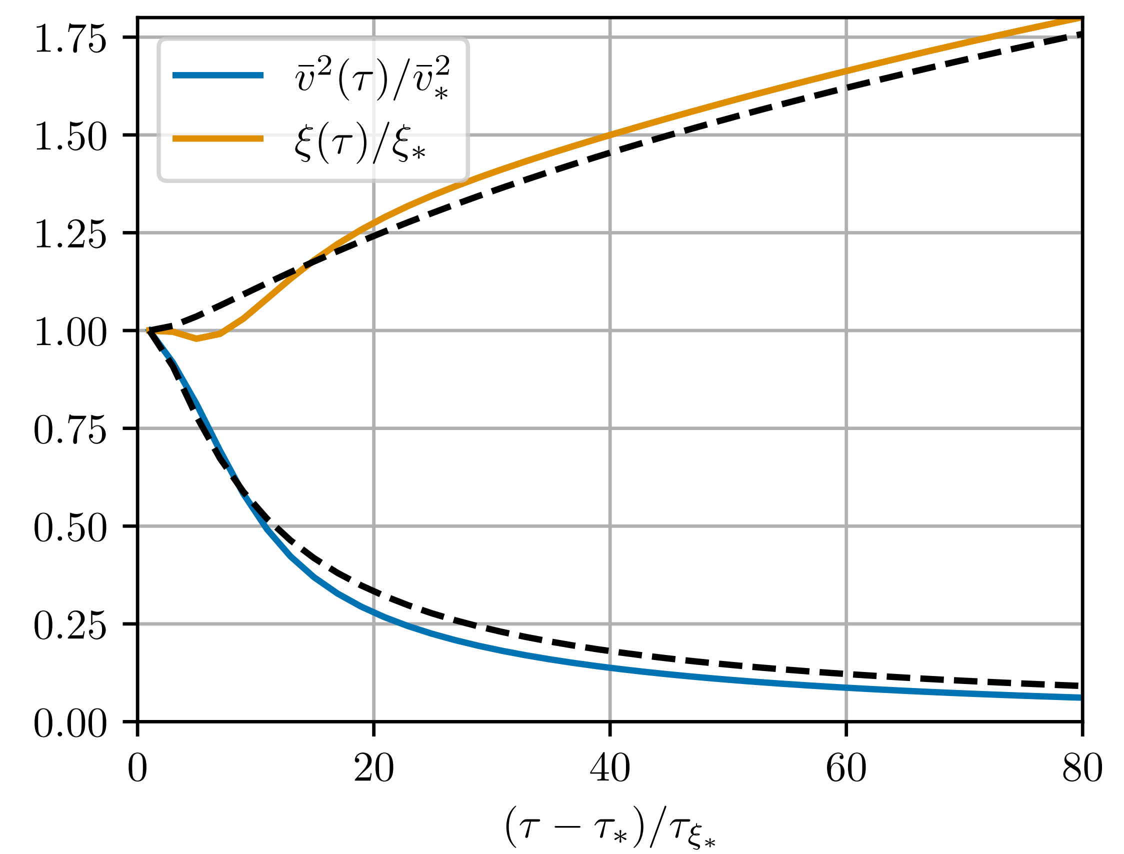
k^2 → causality

$\{1+[k\ell(\tau)]^2\}^{17/6}$ → Kolmogorov

$\ell(\tau)$ is the characteristic scale, at which the kinetic/magnetic energy is concentrated, connected to bubble size towards the end of the phase transition

$\Omega_s(\tau) = \frac{\rho_{v,B}}{\rho}$ is the source energy density parameter





Model of the source

Unequal time correlator

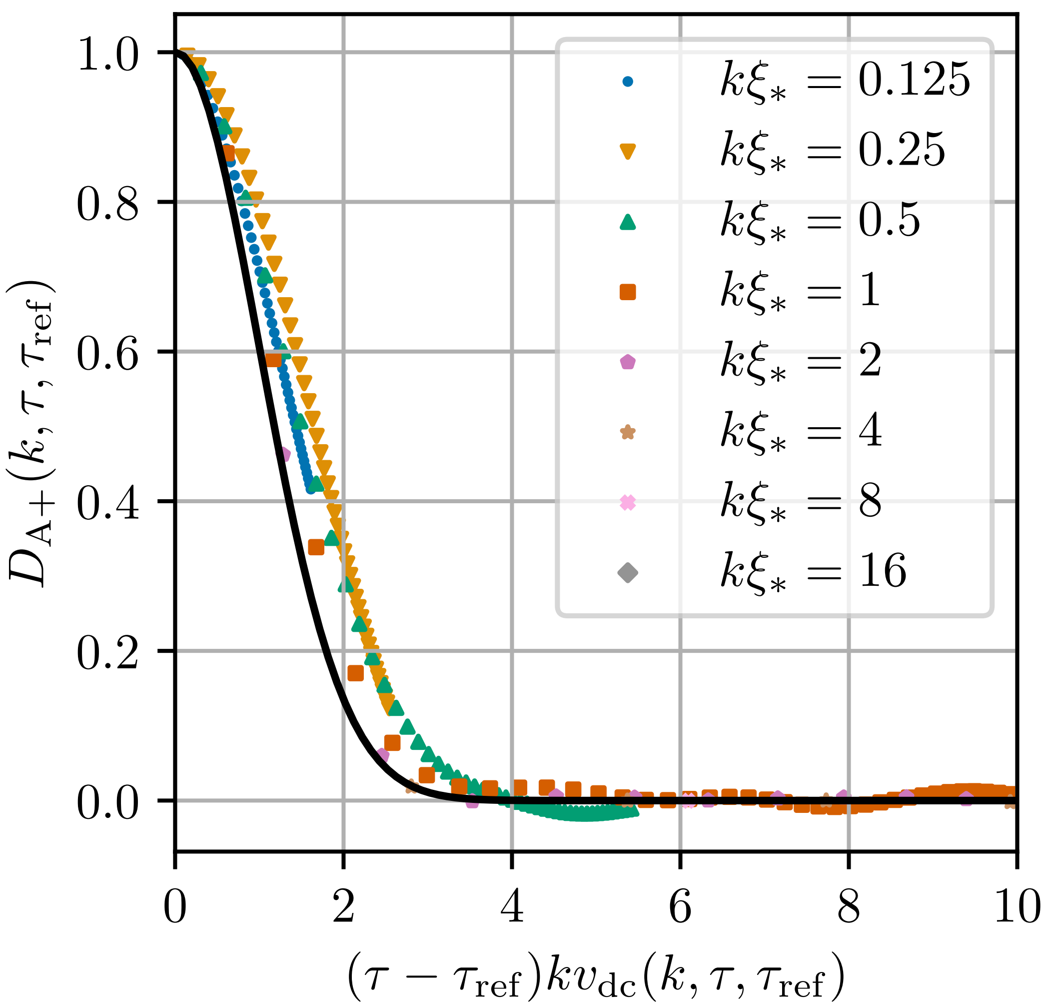
1. *Velocity field*: we build an extended version of Kraichnan decorrelation, based on Mercer's condition for positive kernels

$$P_v(k,\tau,\zeta) = \sqrt{P_v(k,\tau)P_v(k,\zeta)}\sqrt{\frac{2v_{\rm dc}(k,\tau)v_{\rm dc}(k,\zeta)}{v_{\rm dc}^2(k,\tau)+v_{\rm dc}^2(k,\zeta)}}\exp[-k^2\frac{(\tau-\zeta)^2}{2}v_{\rm dc}^2(k,\tau,\zeta)]$$

Normalisation Symmetrisation Gaussian decorrelation

Valid beyond the inertial range

$$v_{\rm dc}^2(k,\tau) \simeq \frac{\langle v^2\rangle}{3}\left(\frac{1+0.2K}{\sqrt{5/2}+0.2K}\right)^2$$

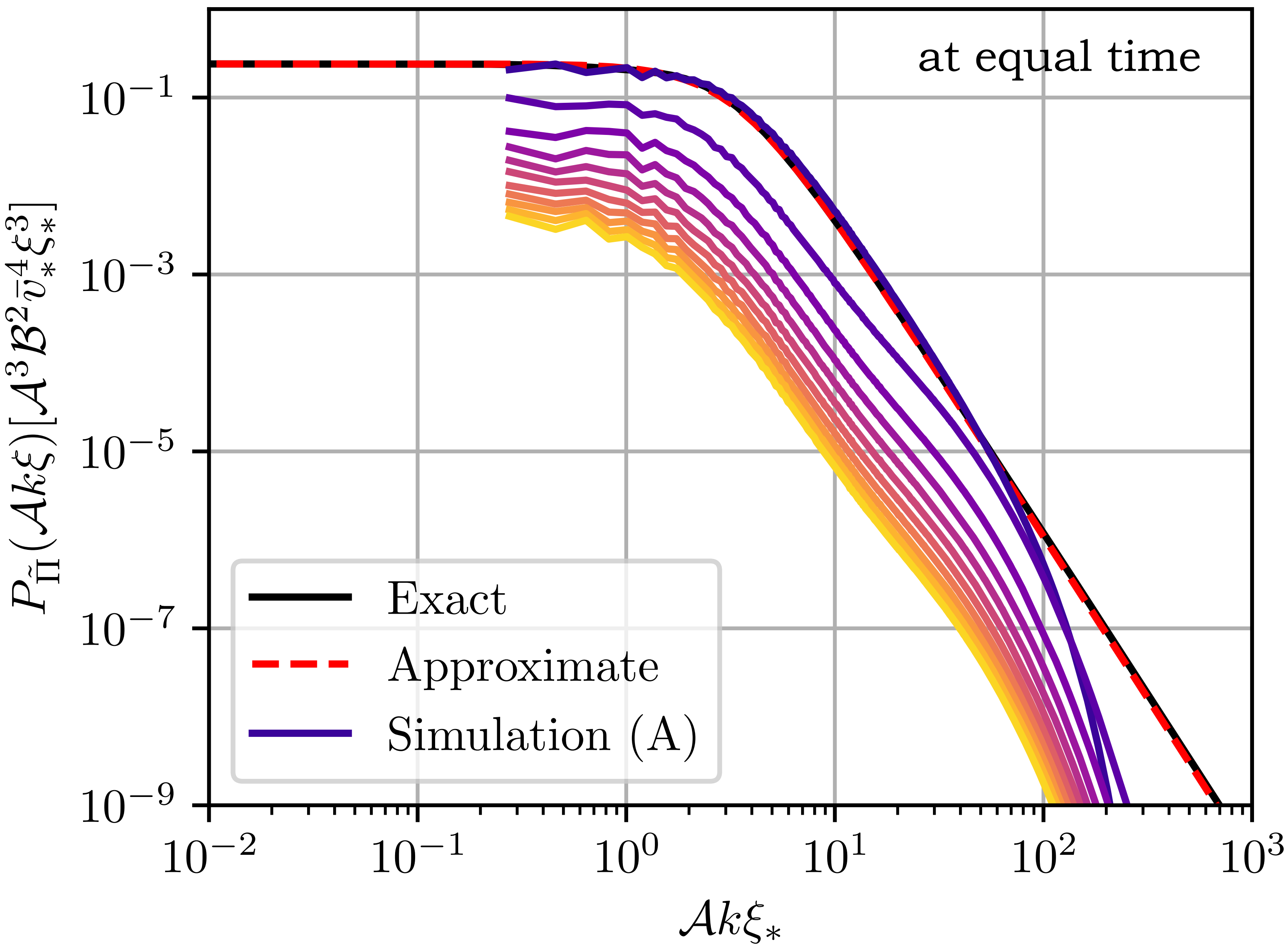


2. *Magnetic field*: similar decorrelation?
Work in progress...

Model of the source

Anisotropic stress unequal time correlator

$$\langle \tilde{\Pi}_{ij}(\mathbf{k},\zeta)\tilde{\Pi}_{ij}(\mathbf{q},\tau)\rangle = \delta(\mathbf{k}-\mathbf{q})P_\Pi(k,\zeta,\tau)$$



Model of the GW production

- Characteristic time of the source evolution (both decay and decorrelation): eddy turnover time

$$\delta t_e = \frac{\ell_*}{v_{\rm rms}}$$

For the magnetic field: $v_A = \frac{\langle B^2 \rangle}{\langle \rho + p \rangle}$

- Characteristic time of the GW production from the Green's function: $\delta t_{\rm gw} \sim 1/k$

- GW production goes faster than source evolution for all relevant wave-numbers including t spectrum peak

$$k > \frac{v_{{\rm rms},A}}{\ell_*}$$

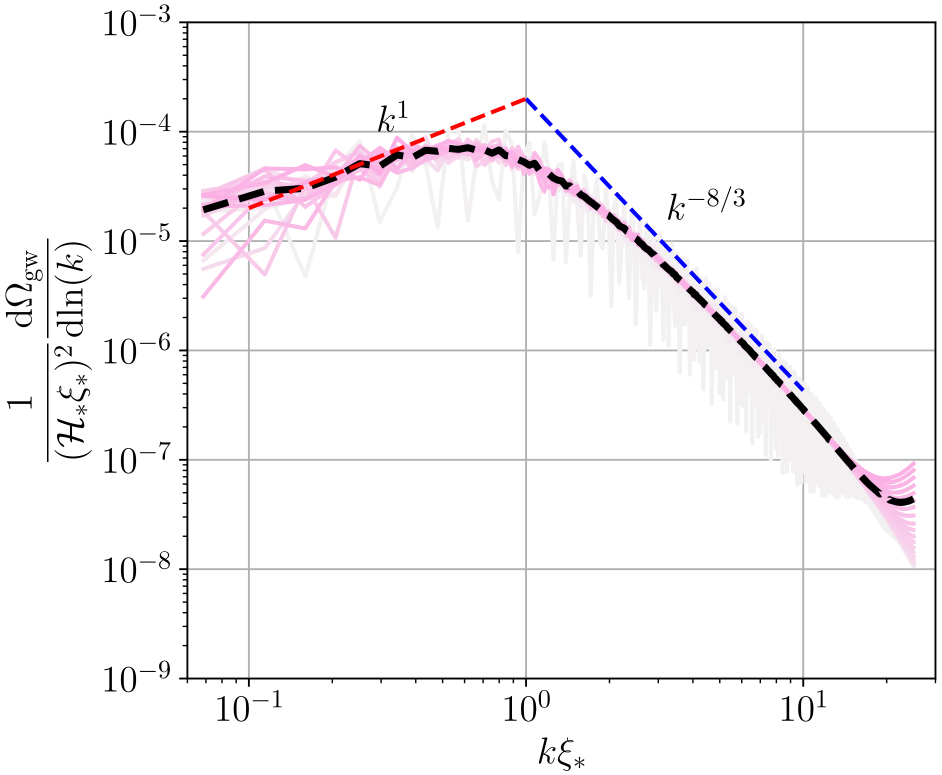
- We assume that the source is constant in time for a finite time interval

$$\delta t_{\rm fin} \sim \mathcal{N} \delta t_e$$

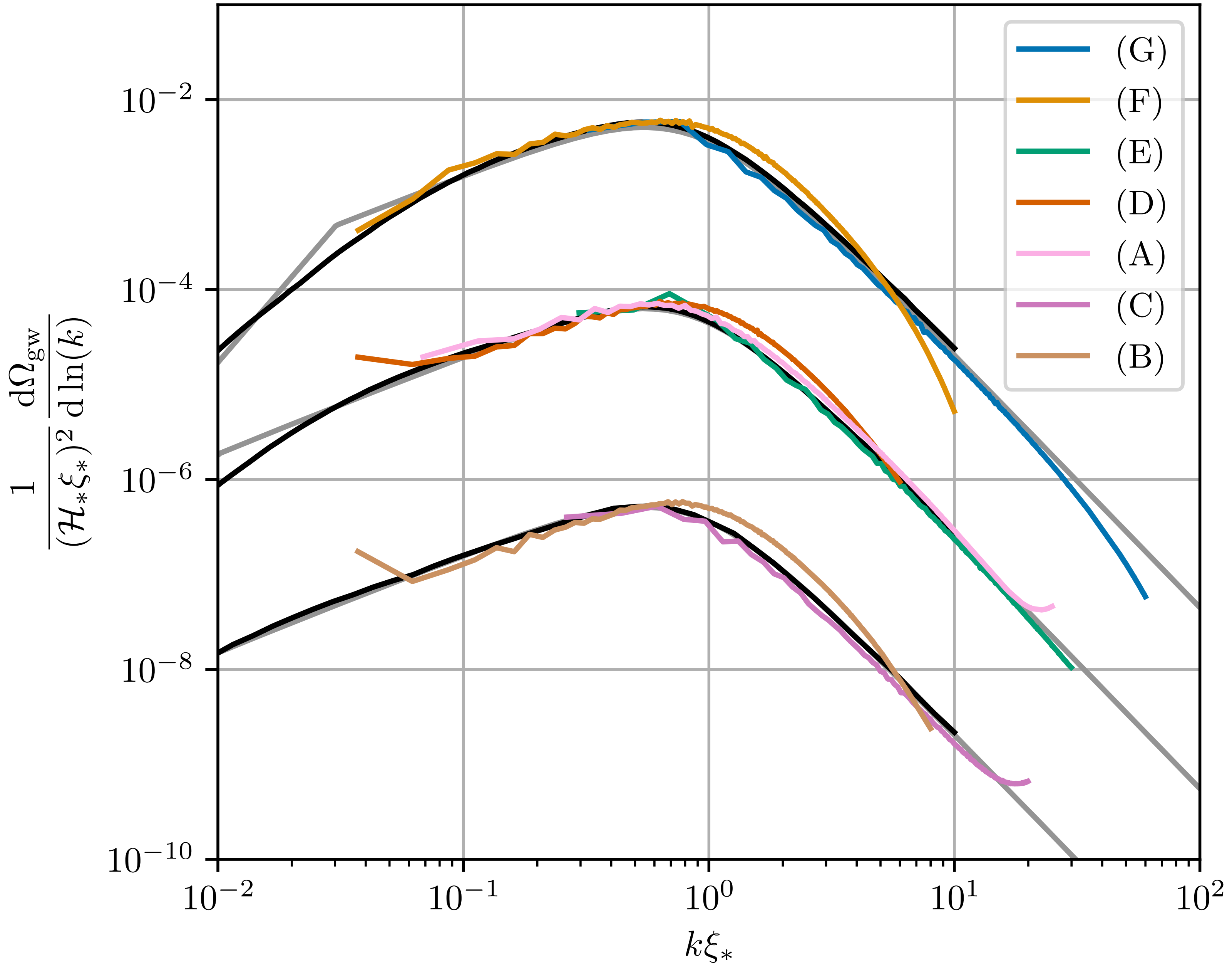
- We can then easily integrate to find the GW spectrum $\Omega_s = \frac{\rho^*_{v,B}}{\rho_*}$

$$\Omega_{\rm gw}(k, t_{\rm fin}) \approx (k\ell_*)^3 \, \Omega_s^2 \; \bar{P}_\Pi(k\ell_*) \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\rm fin}] & \text{if } k\,\delta t_{\rm fin} < 1 \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k\,\delta t_{\rm fin} \geq 1 \end{cases}$$

Validation of the source semi-analytical model with simulations: kinetic, SCOTTS code



Gravitational wave power spectrum

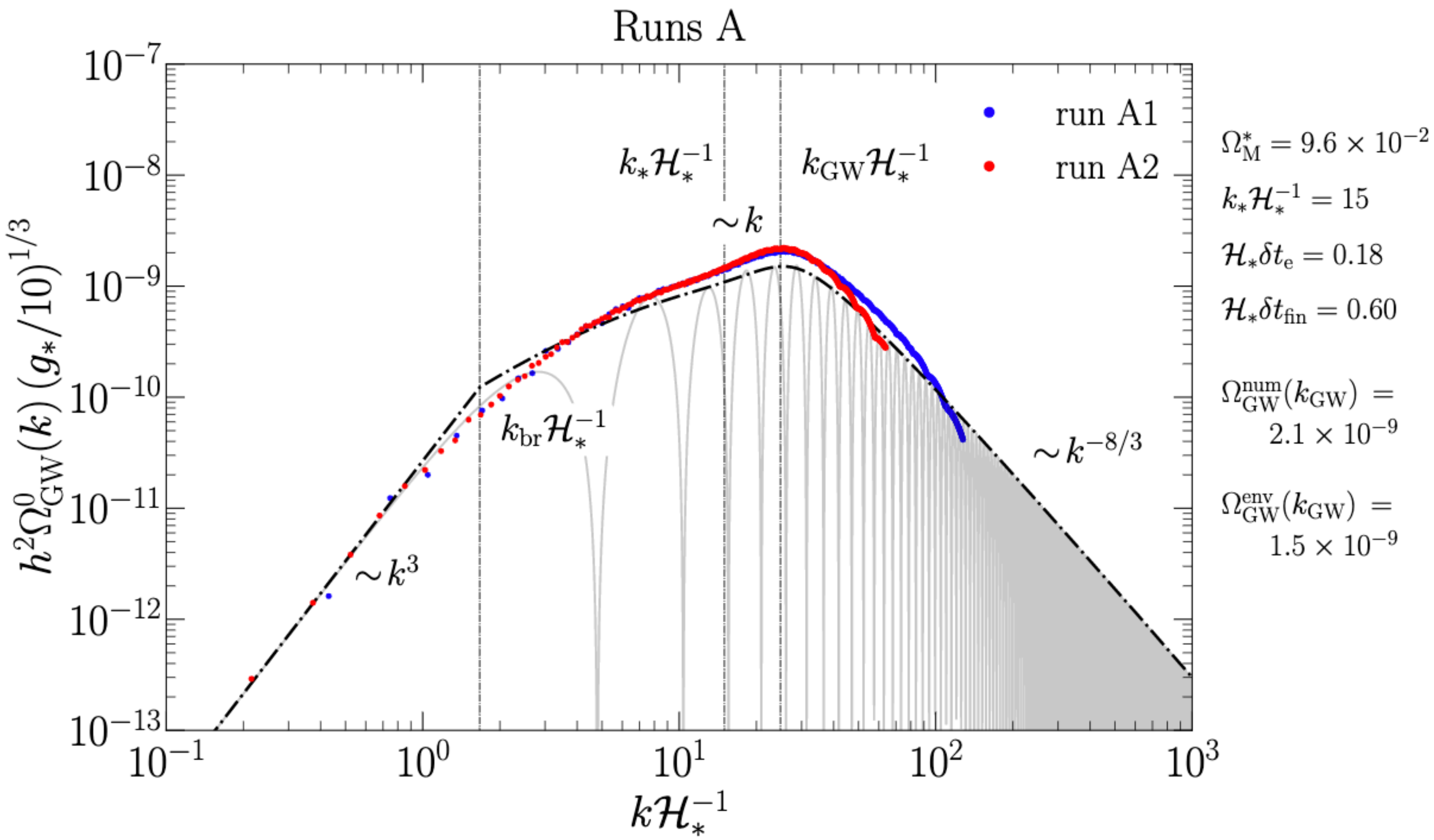


$$\sqrt{\langle v^2 \rangle} = 0.3, 0.1, 0.03$$

$$\mathcal{H}_* \ell_* = 0.001$$

(simulations are in flat-space time)

Validation of the source semi-analytical model with simulations: MHD, Pencil code

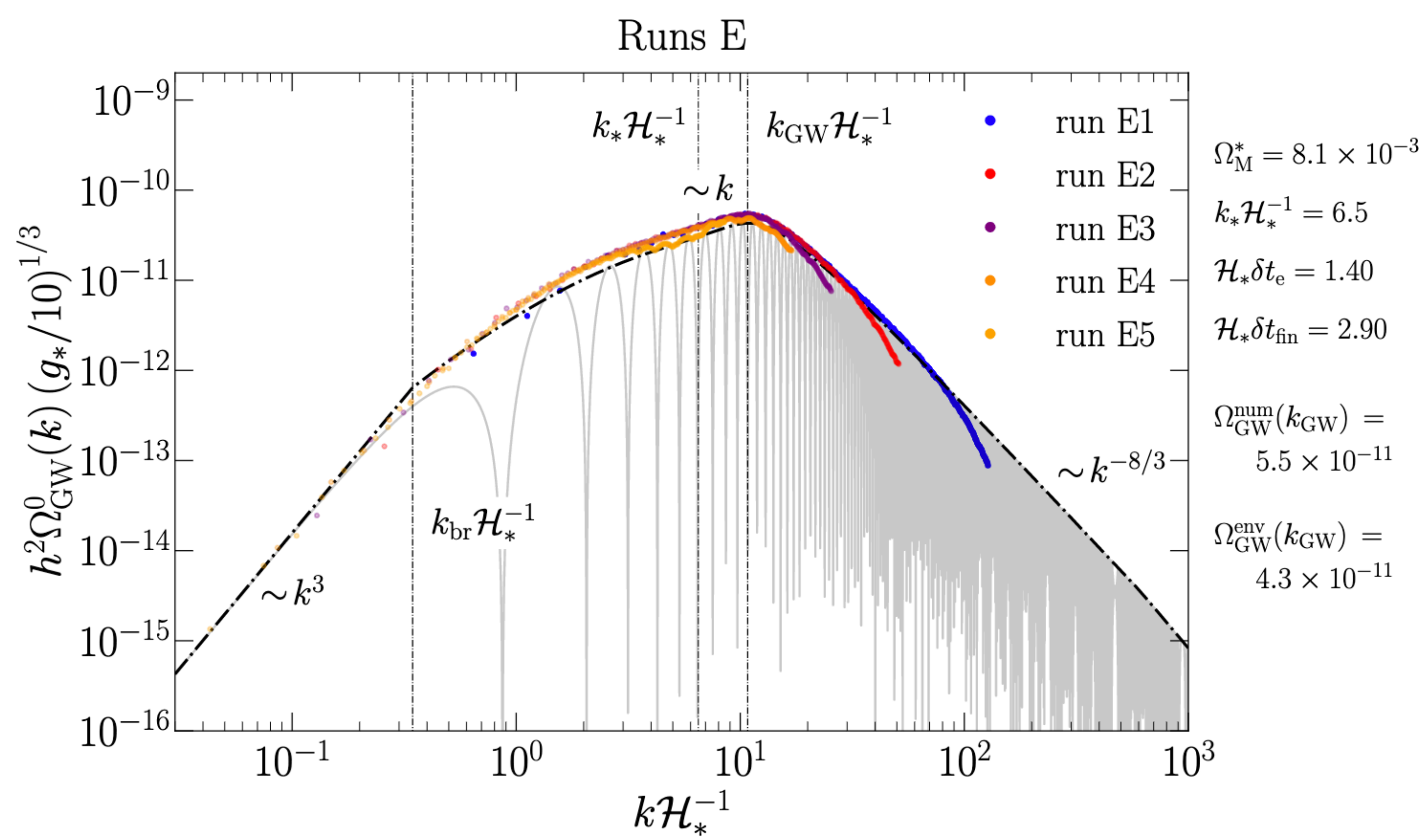


With expansion of the universe -> we could simulate large scales

$$k_{\rm peak} \simeq 4\pi/\ell_*$$

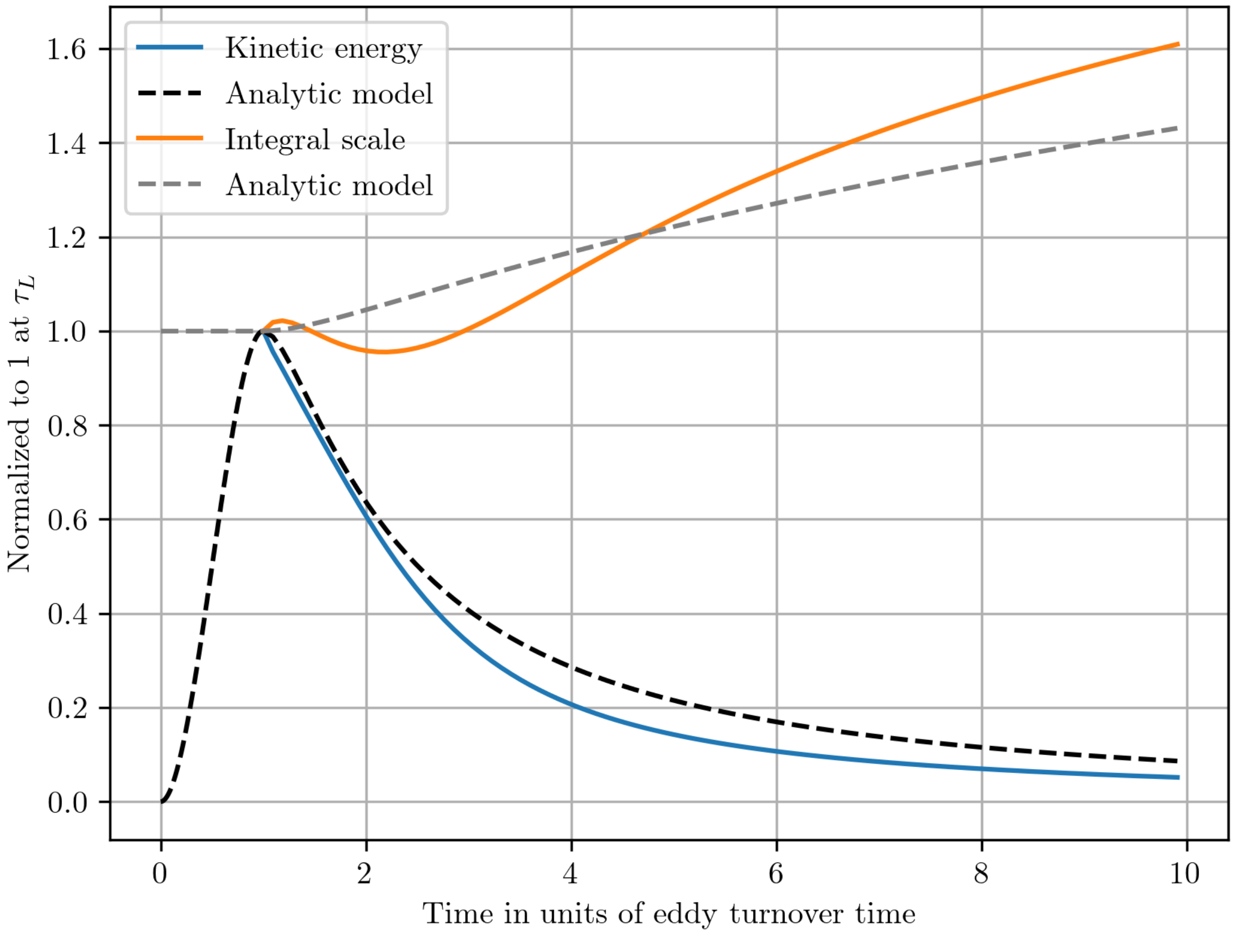
$$\Omega_{\rm gw,\,peak} \propto \Omega_s^2\,(\mathcal{H}_* \ell_*)^2$$

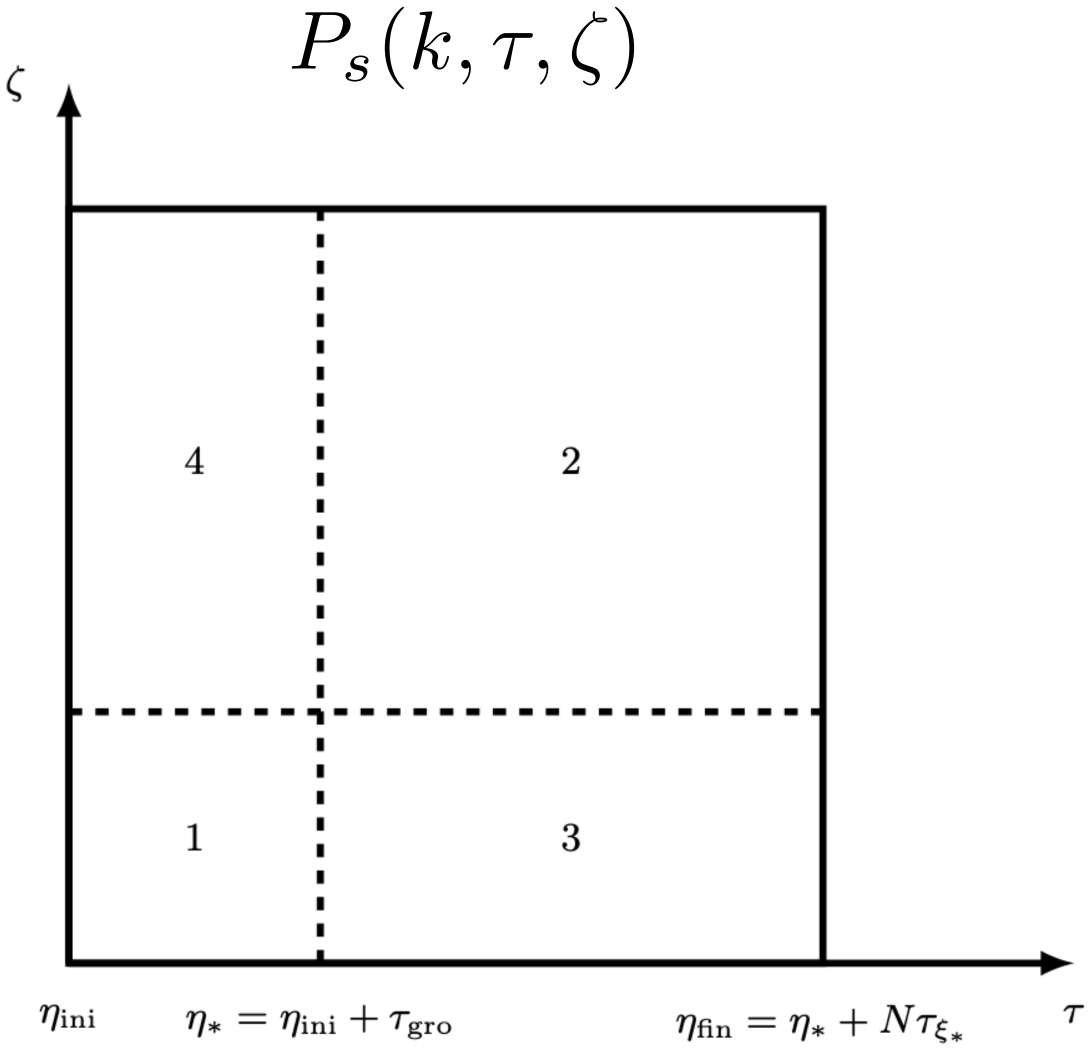
- Transition from k^3 to k^1 at $k \simeq 1/\delta t_{\rm fin}$
- Can be smoother (logarithmic) if $\delta t_{\rm fin} > 1/\mathcal{H}_*$



Model of the source: the main open problem

However, we have so far ignored the period of
turbulence/magnetic field formation

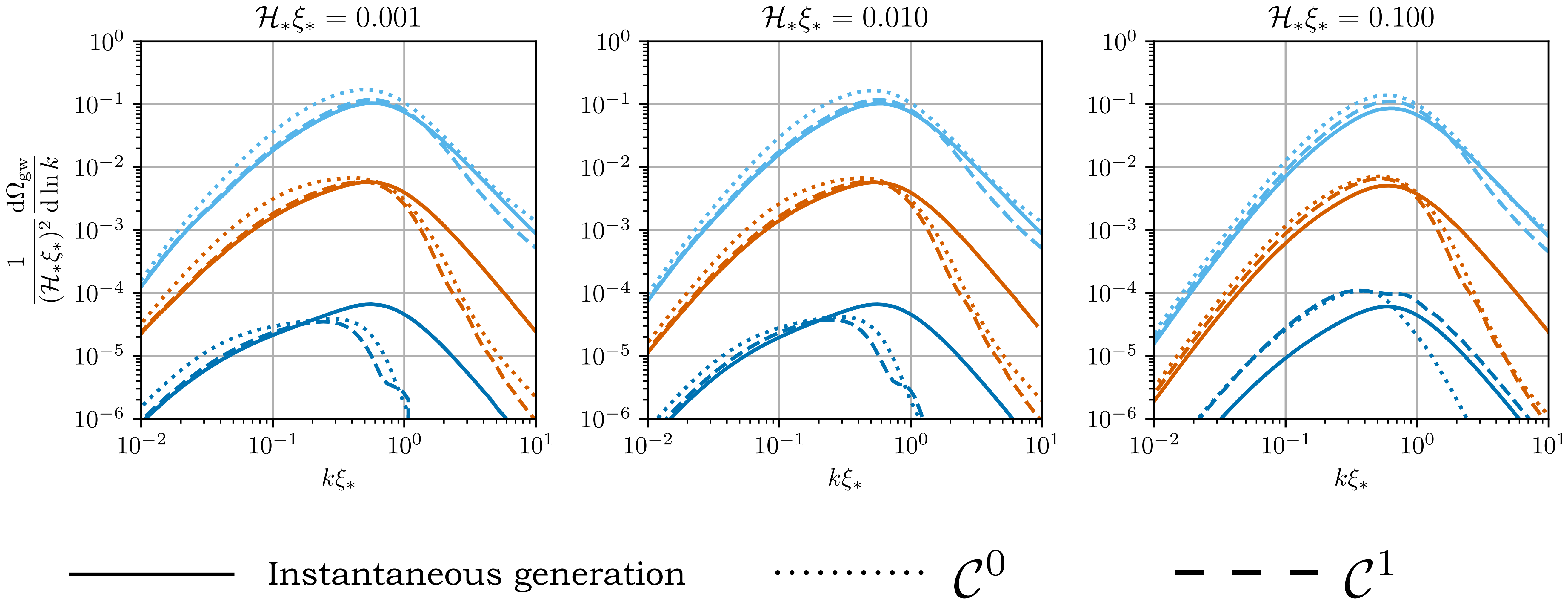




- The duration of the growth phase can be related to the eddy turnover time, or to the PT characteristic time… it depends on the scenario
- The UETC is difficult to model

Model of the source: the main open problem

The period of turbulence formation strongly influences the SGWB spectral shape, especially around the peak



$$\langle v^2 \rangle = 0.1,\ 0.3,\ 0.6$$

In the simple case of a power law growth, the continuity of the growth phase in time is what influences the most the spectral shape

To summarise:

- There are many potential SGWB sources, offering discovery space for energy scales otherwise untested: in particular, several phase transitions might have occurred in the early universe, leading to appreciable GW production if of first order

- Electroweak PT: GW signal can be accessed/constrained only for models beyond the standard model of particle physics —> tests of models, complementary to particle colliders: **Interesting for LISA**

- QCD PT: **Interesting for PTA**, the relic magnetic field has also other effects (CMB…)

- We attempted to accurately construct the SGWB from (M)HD turbulence

- **Main result**: analytical formula for the SGWB spectrum validated by simulations

- Still to be solved:
    - How to deal with realistic initial conditions?
    - How much turbulence is generated from sound waves?
    - How precisely is the magnetic field sourced?

- SGWBs from MHD turbulence in the primordial universe might offer a complementary test of the presence of primordial magnetic fields (provided observational challenges are solved)

- Magnetic fields from inflation also source GWs - neglected here