Parity violation and magnetic field amplification in relativistic and nonrelativistic systems

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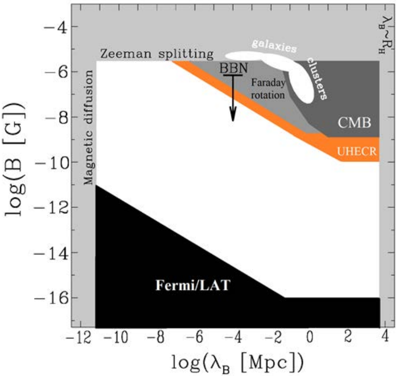
Overview

Magnetic fields in Universe

Magnetic fields exist in all astrophysical objects on all observable scales of the visible Universe:

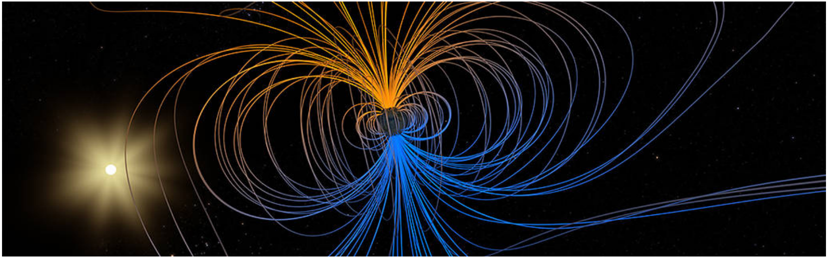
- **Neutron stars**: $10^{12} - 10^{15}$ G
- **Stars**: $1 - 10^3$ G
- **Planets**: ~ 1 G
- **Galaxies**: $\sim 10^{-5} - 10^{-6}$ G
- **Galaxy clusters**: $\sim 10^{-6} - 10^{-7}$ G



[Neronov&Vovk, Science **328**, 73 (2010); Neronov et al., arXiv:2112.08202]

Since 2010, there is evidence of MF detection also in the intergalactic medium — **in cosmic voids**: $10^{-16}\,\mathrm{G} \lesssim B_0 \lesssim 10^{-10}$ G [Tavecchio *et al.*, MNRAS **406**; Ando & Kusenko, Astrophys. J. Lett. **722**; Neronov & Vovk, Science **328**]

The origin of these magnetic fields?

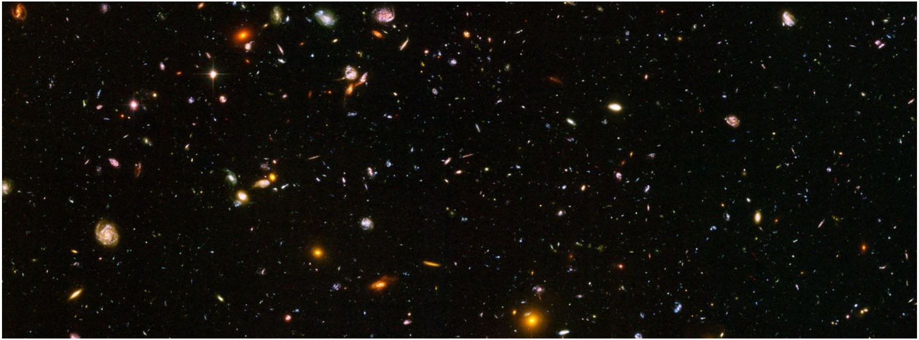


- some **seed field** existed in the plasma before structure formation
- it was amplified by **adiabatic contraction** when galaxies formed
- **AC** is not enough to explain the observed values of e.g. galactic MFs
- MF amplified by **MHD dynamos**, especially when plasma becomes turbulent
- This initial seed field could be of a primordial origin and, in this case, its properties could tell us a lot about the Early Universe!

- However, **dynamos** amplify MF by until it reaches saturation (e.g. at μG in galaxies, according to observations)
- As a consequence – the value of the seed field is not important!

Magnetic field in collapsed structures "looses memory" of its initial configuration and cannot help us to derive properties of the primordial magnetic field

Where to look for the genuine seed field unaffected by structure formation?



- Collapsed structure occupy only the small fraction of the volume of the Universe, while the primordial magnetic field is volume-filling
- Therefore, if we measure magnetic field outside the collapsed structure, in the intergalactic medium (IGM), we could be able to probe the properties of the primordial magnetic field

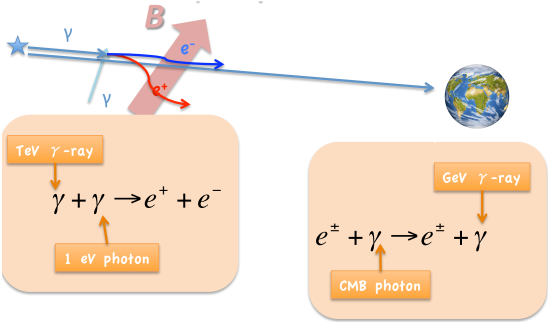
Magnetic field measurement in the IGM

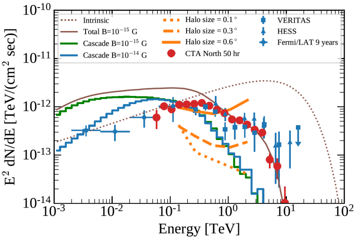
- Faraday rotation measure (RM) depends on the density of the medium,

$$\mathrm{RM} = \frac{e^3}{2\pi m_e^2}\int n_e B_{\|}\, d\ell, \tag{1}$$

  The effect is too small for the current experiments

- γ-ray astronomy has a potential to measure **long-range magnetic fields** in the **Intergalactic Medium** (IGM)





[2010.01349]

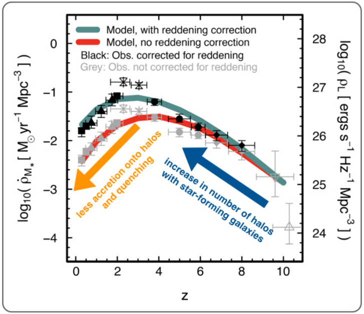
Galactic feedback

- The "genuine" properties of Intergalactic Magnetic Fields (IGMF) can be **affected by processes inside galaxies**
- Indeed, feedback from supernova and active galactic nuclei (AGNs) could **spread out** galactic matter and magnetic field at some distance around galaxies

- **To what extend IGMF are affected by galactic feedback?**

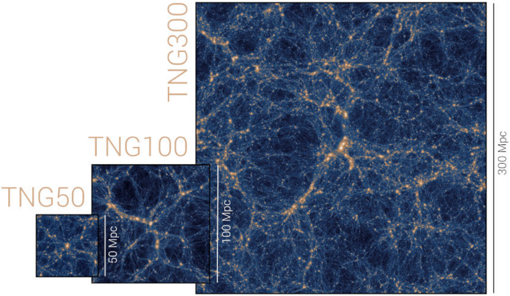
AGN feedback and observed properties of galaxies

- The strongest feedback comes from AGNs – supermassive black holes (SMBH) in the central parts of galaxies
- The source of energy for the SMBH is the accreted matter. This process is so effective, that $\mathcal{O}(10\%)$ of the mass of accreted matter transforms in radiation. This makes AGNs the most bright permanent sources of light in the Universe
- Feedback of AGNs heats up matter around and injects a lot of matter in the IGM. This affects star formation rate and creates Fermi bubbles seen in X-rays (see e.g. **[1204.4114]**)



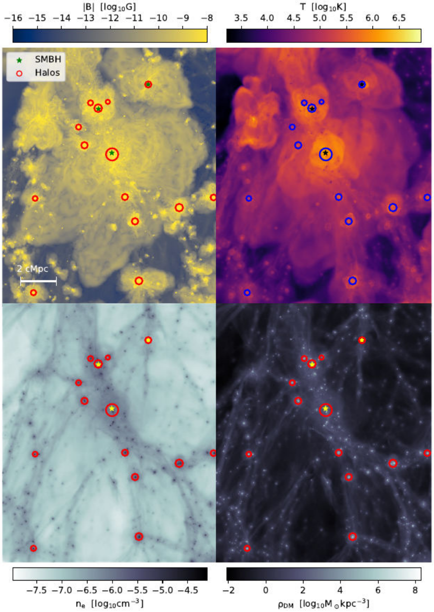
IllustrisTNG simulations

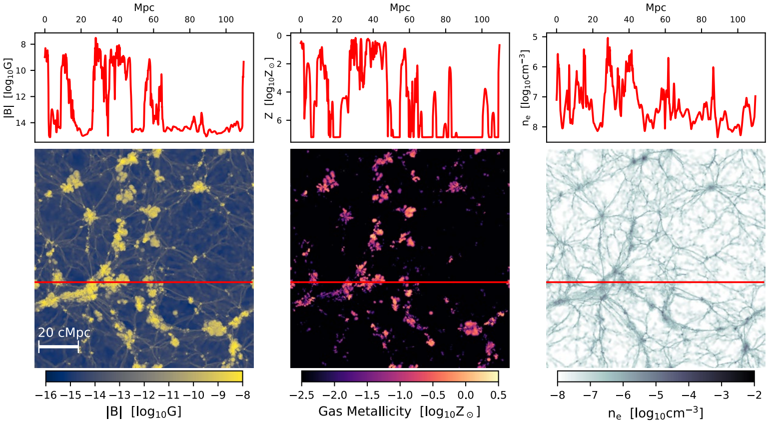
- **IllustrisTNG** (TNG) is a suite of large-volume cosmological gravo-magnetohydrodynamic simulations **[1707.03396]**
- It uses the moving-mesh AREPO code describe self-gravity and ideal MHD **[1108.1792]**



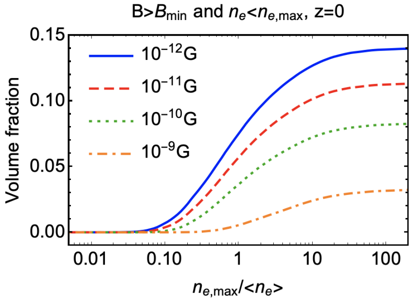
- TNG100 has a $L \sim 100$ cMpc box, 1820^3 of both DM and gas particles
- TNG includes a comprehensive galaxy formation model incorporating e.g. gas metal-line cooling and heating, star formation, stellar evolution, and heavy element enrichment, supermassive black hole growth

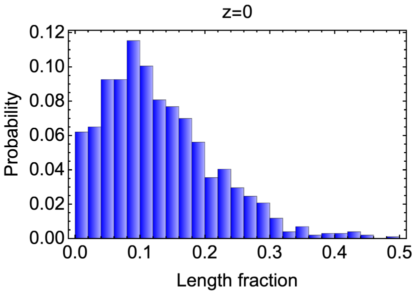
- We see that simulation volume is occupied by magnetic bubbles $\sim (10\ \mathrm{Mpc})^3$, we see presence of massive halos with AGNs inside them
- Magnetic field forms the butterfly-like configuration around the massive halo, suggesting that it was produced by outflows





- In TNG we have observed macroscopic (**tens Mpc**) regions around clusters of galaxies with **electron density** is as **low** as in the IGM and **magnetic field** is as strong as in clusters – **over-magnetized bubbles**
- Typical **sizes** of the bubbles are **order of magnitude larger** than virial radii of parent clusters





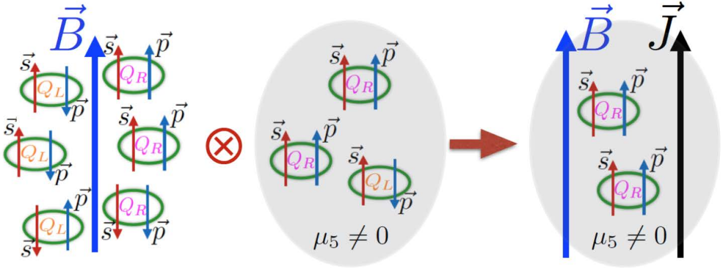
- At $z = 0$, the magnetic field is stronger than 10^{-12} G in 15% of the volume, while it is stronger than 10^{-9} G in 3% of the volume
- Alternatively, we can measure the fractional length, for a given line of sight, which intersects a strong magnetic field $B > 10^{-12}$ G
- We show the result for a sample of 1000 random sight-lines of length 100 Mpc

- **Physics of magnetic bubbles strongly depends on simulations of galaxy formation with dynamos**
- **It is important to deeply understand physical picture to be sure that simulations give physical magnetic field (rather than exponentially amplified numerical noize)!**

Numerical simulations of dynamos

- Understanding of the dynamo effect in galaxies is extremely important for the analysis of observations
- The most straightforward way — to include them into the simulations of galaxy formation, e.g., **IllustrisTNG**
- There are several problems with this:
  1. The Gauß constraint div $\boldsymbol{B} = 0$ can be violated on the lattice by numerical errors
  2. Numerical errors may cause fake instability
  3. Since dynamo leads to exponential growth, even a small numerical error at the beginning can go out of control later
  4. In dynamo effect, the instability originates from small-scale turbulence which cannot be resolved well on the lattice
- Thus, we need an analytical model of dynamo in order to have the control over simulations.

Chiral magnetic effect



- **Magnetic field** make the spins aligned $\langle \boldsymbol{s} \rangle \propto \boldsymbol{B}$
- **Chirality** determines the momentum direction: $\boldsymbol{p} \uparrow\uparrow \boldsymbol{s}$ (R), $\boldsymbol{p} \uparrow\downarrow \boldsymbol{s}$ (L)
- **Chiral imbalance $\mu_L - \mu_R \equiv \mu_5 \neq 0$:** one of the chiralities is dominating

$$\Downarrow$$

Electric current appears: $\boldsymbol{j}_{CME} = \frac{e^2}{2\pi^2}\mu_5 \boldsymbol{B}.$

Maxwell equations

- As a result Maxwell equations contain current, **proportional to** μ_5: [Vilenkin (1978); Fröhlich & Pedrini (2000–2001); Joyce & Shaposhnikov (1997)]

$$
\begin{aligned}
\operatorname{curl} \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\
\operatorname{curl} \boldsymbol{B} &= \sigma \boldsymbol{E} + \frac{e^2}{4\pi^2}\mu_5 \boldsymbol{B} \quad \leftarrow \text{Chiral magnetic effect}
\end{aligned}
$$

- **time-reversal symmetry** $\mathcal{T}$:

$$
\left.
\begin{aligned}
\boldsymbol{j} &\xrightarrow{\mathcal{T}} -\boldsymbol{j} \\
\boldsymbol{B} &\xrightarrow{\mathcal{T}} -\boldsymbol{B} \\
\boldsymbol{E} &\xrightarrow{\mathcal{T}} +\boldsymbol{E}
\end{aligned}
\right\} \Longrightarrow
\begin{cases}
\sigma \xrightarrow{\mathcal{T}} -\sigma & \text{– dissipative current} \\
\mu_5 \xrightarrow{\mathcal{T}} \mu_5 & \text{– non-dissipative current}
\end{cases}
$$

Magnetic field amplification

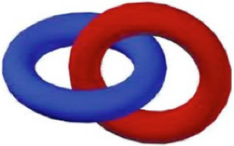
In the presence of CME current, the induction equation reads as

$$\partial_t \boldsymbol{B} = \underbrace{\frac{1}{\sigma}\nabla^2 \boldsymbol{B}}_{\text{magnetic diffusion}} + \underbrace{\frac{e^2\mu_5}{4\pi^2}\,\mathrm{curl}\,\boldsymbol{B}}_{\text{instability}},$$

A simple solution to this equation is

$$\boldsymbol{B} = B_0(\cos(kz), \pm\sin(kz), 0)e^{\lambda_\pm t}, \qquad \lambda_\pm = -\frac{k^2}{\sigma} \pm \frac{e^2\mu_5}{4\pi^2}k$$

- One circular polarization always decays ($\lambda_- < 0$)
- Another polarization **grows** ($\lambda_+ > 0$) if $k < \dfrac{e^2\sigma\mu_5}{4\pi^2}$.



As a result, **magnetic helicity** of the amplified field is nontrivial:

$$\mathcal{H} = \int d^3\boldsymbol{x}\,\boldsymbol{A}\cdot\boldsymbol{B} \neq 0.$$

Dynamics of μ_5

Boyarsky, Fröhlich, **O.R.** PRL (2012)

$$\mathrm{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\mathrm{curl}\,\vec{B} = \sigma \vec{E} + \frac{2\alpha}{\pi}\mu_5 \vec{B}$$

Chiral magnetic effect

$$\frac{\partial \mu_5}{\partial t} \propto \frac{2\alpha}{\pi}\int d^3x\, \vec{E}\cdot\vec{B} - \Gamma_{\mathrm{flip}}\mu_5$$

Chiral anomaly

Finite mass breaks chiral symmetry

- Without B chirality flipping reactions drive $\mu_5 \to 0$ ($\mu_5 = \mu_0 e^{-\Gamma_{\mathrm{flip}} t}$)
- Without μ_5 finite conductivity drives $B \to 0$ ($B_k = B_0 e^{-\frac{k^2 t}{\sigma}}$)

Analysis of quasi-homogeneous system

PRL'12 [1109.3350]

- Introduce $\mathcal{H}_k = \frac{k}{2\pi^2}(|B_k^+|^2 - |B_k^-|^2)$
- The system of equations

$$\begin{cases} \dfrac{\partial \mathcal{H}_k}{\partial t} = -\dfrac{2k^2}{\sigma}\mathcal{H}_k + \dfrac{\alpha}{\pi}\dfrac{k\mu_5}{\sigma}\mathcal{H}_k \,, \\ \dfrac{\partial \mu_5}{\partial t} = -(c_\Delta \alpha)\displaystyle\int dk \, \dfrac{\partial \mathcal{H}_k}{\partial t} - \Gamma_f \mu_5 \end{cases}$$

- ...can re-write this as

$$\frac{\partial \mu_5}{\partial t} = -(\Gamma_B + \Gamma_f)\mu_5 + S_B$$

- where

$$\Gamma_B(t) \equiv \frac{c_\Delta \alpha^2}{\pi\sigma}\int dk \, k\mathcal{H}_k; \qquad S_B(t) \equiv 2\frac{c_\Delta \alpha}{\sigma}\int dk \, k^2 \mathcal{H}_k \,.$$

Tracking solution

$$\frac{\partial \mu_5}{\partial t} = -(\Gamma_B + \Gamma_f)\mu_5 + S_B$$

- Can we have a "tracking solution":

$$\bar{\mu}_5 \approx \frac{S_B}{\Gamma_B + \Gamma_f}$$

- If $\Gamma_B \gg \Gamma_f$ (i.e. magnetic field is sufficiently large)

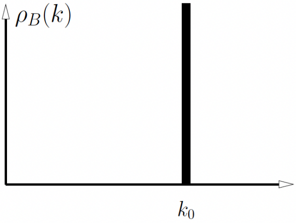
$$\bar{\mu}_5 = \frac{S_B}{\Gamma_B} \approx \frac{\pi k_{\text{peak}}}{\alpha}$$

where magnetic energy/helicity is maximal at k_{peak}
recall $\Gamma_B \propto \int dk k \mathcal{H}_k$, $S_B \propto \int dk k^2 \mathcal{H}_k$

Attractor solution

- For maximally helical **monochromatic** magnetic field it reduces to
  Energy density: ρ_B. Magnetic diffusion time $t_\sigma = \frac{2\sigma}{k_0^2}$

$$\frac{d\mu_5}{dt} = -\rho_B\left(\mu_5 - \bar{\mu}_5\right)$$
$$\frac{d\rho_B}{dt} = \frac{\rho_B}{t_\sigma}\left(\frac{\mu_5}{(\bar{\mu}_5)} - 1\right)$$

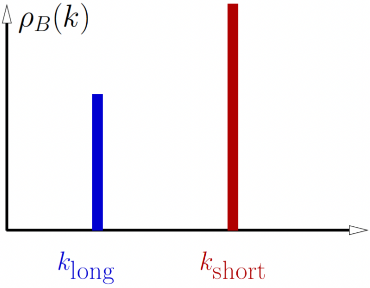


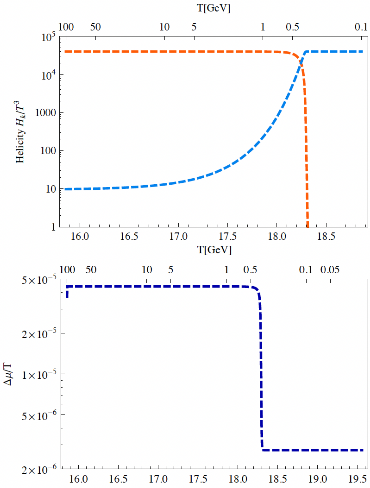
- Large ρ_B drives μ_5 to an **attractor solution** $\boldsymbol{\bar{\mu}_5 = \frac{2\pi k_0}{\alpha}}$

Boyarsky, PRL 2012 **[1109.3350]**

- Electric conductivity of the plasma is **finite** but **magnetic diffusion** is compensated by the presence of μ_5

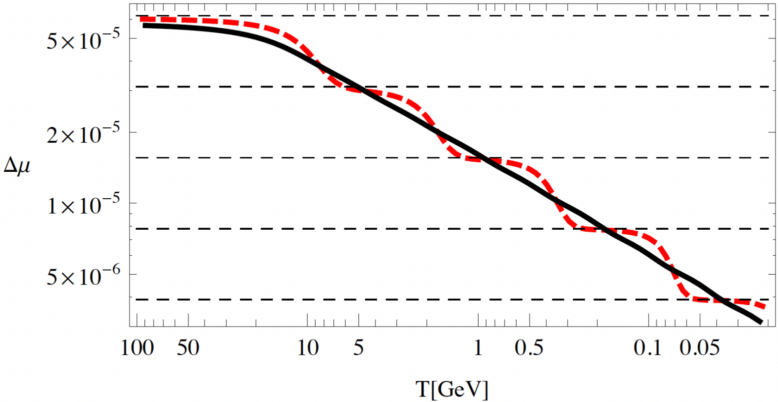
Two modes





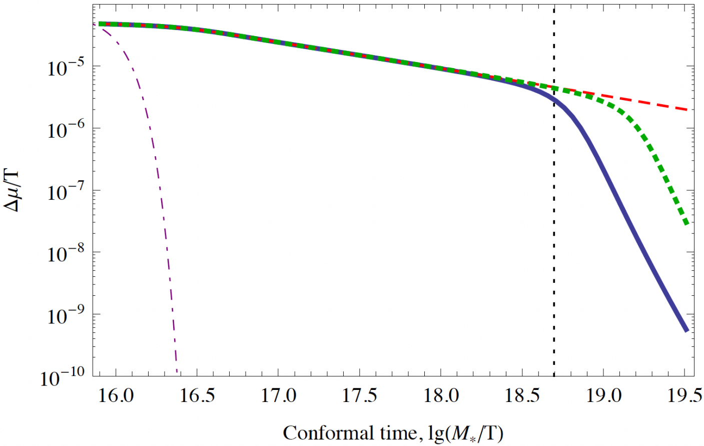
- In case of two modes the helicity gets transferred from the shorter one to the longer one
- Chemical potential follows the wave-number of the mode with higher helicity $\mu = \frac{\pi k}{\alpha}$

Evolution of chemical potential



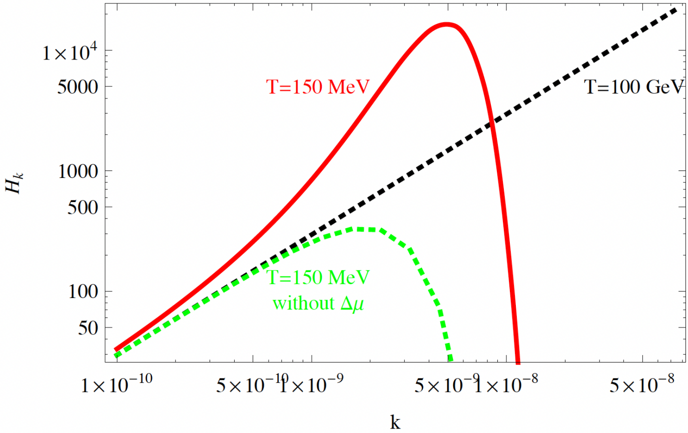
μ_5 tracks wavenumber where helicity peaks, $\mu_5 = \frac{\pi k_n}{\alpha}$

Evolution of chemical potential



Continuous initial spectrum with $\mathcal{H}_k \propto k$ and fraction of magnetic energy density 5×10^{-5} (blue) or 5×10^{-4} (green). Red – evolution without flip

Evolution of helicity spectrum



Inverse cascade without turbulence!

[Joyce & Shaposhnikov (1997)]

Mean-field dynamo: differences from the case of CME

CME instability

- One homogeneous pseudoscalar function $\mu_5 = \mu_R - \mu_L$
- Parity violation by $\mu_5 \neq 0$
- Backreaction is due to chiral anomaly
- Energy is released from the flipping of chirality of fermions

Mean-field dynamo

- A new vector field – velocity of the fluid $\boldsymbol{U}$
- Parity is violated by nontrivial correlators of the velocity and/or magnetic field – **which correlators are important?**
- The backreaction is due to interaction between the short-scale turbulent motion and large-scale fields – **what exactly is the mechanism?**
- Energy is pumped from the short-scale turbulent motion – **what is the mechanism?**

Equations of magnetohydrodynamics

- Induction equation

$$\partial_t \boldsymbol{B} = \underbrace{\frac{1}{\sigma}\nabla^2 \boldsymbol{B}}_{\text{magnetic diffusion}} + \underbrace{\nabla \times [\boldsymbol{U} \times \boldsymbol{B}]}_{\text{Frozen-in MF}}$$

- Navier-Stokes equation

$$\partial_t \boldsymbol{U} + \underbrace{(\boldsymbol{U}\nabla)\boldsymbol{U}}_{\text{Convective term}} = \underbrace{\nu\nabla^2 \boldsymbol{U}}_{\text{viscosity}} + \underbrace{\frac{1}{\rho}[\boldsymbol{J} \times \boldsymbol{B}]}_{\text{Ampère's force}} - \frac{1}{\rho}\nabla P + \boldsymbol{f}$$

- The terms in **red** are nonlinear and will lead to interaction between different scales (crucial for dynamo).
- The terms in **blue** lead to dissipation.

Separation of scales. EOM for large-scale field

- Analytical model of dynamo can be built only in the case of ideal scale separation:

$$\lambda_{\text{turbulence}} \ll \lambda_{\text{large scale fields}} \qquad \tau_{\text{turbulence}} \ll \tau_{\text{large scale fields}}$$

- Decompose

$$\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}, \qquad \boldsymbol{U} = \overline{\boldsymbol{U}} + \boldsymbol{u}$$

where averaging is performed on some intermediate scale L.

- For simplicity, assume $\overline{\boldsymbol{U}} = 0$
- The induction equation gives

$$\partial_t \overline{\boldsymbol{B}} = \underbrace{\frac{1}{\sigma}\boldsymbol{\nabla}^2 \overline{\boldsymbol{B}}}_{\text{magnetic diffusion}} + \underbrace{\boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}}}_{\text{source of instability}}$$

$$\boldsymbol{\mathcal{E}} = \overline{[\boldsymbol{u} \times \boldsymbol{b}]} \qquad \text{– the electromotive force.}$$

Infinite chain of equations on correlators

- Let us consider for a moment that velocity flow $\boldsymbol{u}$ is fixed and $\partial_t \boldsymbol{u} = 0$. The dynamics of $\boldsymbol{b}$ is given by

$$\partial_t \boldsymbol{b} = \frac{1}{\sigma}\boldsymbol{\nabla}^2 \boldsymbol{b} + \boldsymbol{\nabla} \times ([\boldsymbol{u} \times \boldsymbol{b}] - \overline{[\boldsymbol{u} \times \boldsymbol{b}]}) + \boldsymbol{\nabla} \times [\boldsymbol{u} \times \overline{\boldsymbol{B}}] \tag{2}$$

- This equation contains two-point correlator $\overline{[\boldsymbol{u} \times \boldsymbol{b}]}$ (electromotive force) that is not known. We can find equation on correlator,

$$\partial_t \overline{[\boldsymbol{u} \times \boldsymbol{b}]} = \overline{[\boldsymbol{u} \times (\partial_t \boldsymbol{b})]} = \frac{1}{\sigma}\overline{[\boldsymbol{u} \times \boldsymbol{\nabla}^2 \boldsymbol{b}]} + \overline{[\boldsymbol{u} \times \boldsymbol{\nabla} \times [\boldsymbol{u} \times \boldsymbol{b}]]} + \ldots \tag{3}$$

- We see that evolution of the electromotive force depends on other two-point correlators as well as on three-point correlators
- Equations of motion for three-point correlators contain four-point correlators, so we get infinite chain of equations
- How to break this chain?

Dynamics of small-scale fields

- Assumptions:
  1. fluctuations are small so we can neglect quadratic terms in their EOM
  2. gradients of pressure and external forces are negligible
- Equations of motion:

$$\begin{cases} \partial_t \boldsymbol{b} - \frac{1}{\sigma}\boldsymbol{\nabla}^2 \boldsymbol{b} = \boldsymbol{\nabla} \times [\boldsymbol{u} \times \overline{\boldsymbol{B}}], \\ \partial_t \boldsymbol{u} - \nu \boldsymbol{\nabla}^2 \boldsymbol{u} = \frac{1}{4\pi\mu\rho}\left\{[\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}] \times \boldsymbol{b} + [\boldsymbol{\nabla} \times \boldsymbol{b}] \times \overline{\boldsymbol{B}}\right\}, \end{cases}$$

- Expanding $\boldsymbol{b} = \boldsymbol{b}^{(0)} + \boldsymbol{b}^{(1)}[\overline{\boldsymbol{B}}] + \ldots$, $\boldsymbol{u} = \boldsymbol{u}^{(0)} + \boldsymbol{u}^{(1)}[\overline{\boldsymbol{B}}] + \ldots$, we get for the electromotive force

$$\boldsymbol{\mathcal{E}}_i = \underbrace{\overline{[\boldsymbol{u}^{(0)} \times \boldsymbol{b}^{(0)}]_i}}_{=0} + \underbrace{\overline{[\boldsymbol{u}^{(0)} \times \boldsymbol{b}^{(1)}]_i} + \overline{[\boldsymbol{u}^{(1)} \times \boldsymbol{b}^{(0)}]_i}}_{\text{linear in } \overline{\boldsymbol{B}}} + \ldots$$

Ideal case $\nu = 1/\sigma = 0$

- Equations of motion:

$$\begin{cases} \partial_t \boldsymbol{b} = \boldsymbol{\nabla} \times [\boldsymbol{u} \times \overline{\boldsymbol{B}}], \\ \partial_t \boldsymbol{u} = \frac{1}{4\pi\mu\rho}\Big\{[\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}] \times \boldsymbol{b} + [\boldsymbol{\nabla} \times \boldsymbol{b}] \times \overline{\boldsymbol{B}}\Big\}, \end{cases}$$

$$\begin{cases} \boldsymbol{b}^{(1)} = \int d\tau \boldsymbol{\nabla} \times [\boldsymbol{u}^{(0)} \times \overline{\boldsymbol{B}}](t-\tau), \\ \boldsymbol{u}^{(1)} = \frac{1}{4\pi\mu\rho}\int d\tau \Big\{[\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}] \times \boldsymbol{b}^{(0)} + [\boldsymbol{\nabla} \times \boldsymbol{b}^{(0)}] \times \overline{\boldsymbol{B}}\Big\}\Big|_{t-\tau}, \end{cases}$$

- Electromotive force:

$$\begin{aligned} \boldsymbol{\mathcal{E}}_i^{(1)} &= \overline{[\boldsymbol{u}^{(0)} \times \boldsymbol{b}^{(1)}]_i} + \overline{[\boldsymbol{u}^{(1)} \times \boldsymbol{b}^{(0)}]_i} = \\ &= \int d\tau \overline{[\boldsymbol{u}^{(0)}(t) \times [\boldsymbol{\nabla} \times [\boldsymbol{u}^{(0)}(t-\tau) \times \overline{\boldsymbol{B}}]]]} + \text{similar with } \boldsymbol{b}^{(0)} = \\ \big|\text{isotropic turb.}\big| &\approx -\frac{1}{3}\int d\tau \overline{\boldsymbol{u}^{(0)}(t) \times [\boldsymbol{\nabla} \times \boldsymbol{u}^{(0)}(t-\tau)]}\overline{\boldsymbol{B}} - \\ &- \frac{1}{3}\int d\tau \overline{\boldsymbol{u}^{(0)}(t)\boldsymbol{u}^{(0)}(t-\tau)}[\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}] + \text{similar with } \boldsymbol{b}^{(0)} = \\ &= \alpha\overline{\boldsymbol{B}} - \beta[\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}]. \end{aligned}$$

General case

- Assumptions for the field $\overline{\boldsymbol{B}}$:
  1. slowly varying is space and time
  2. weak enough, so that $|u^{(1)}| \ll |u^{(0)}|$, $|b^{(1)}| \ll |b^{(0)}|$
- Expand the integrand

$$\boldsymbol{\mathcal{E}}_i = \int_0^\infty d\tau \int_V d^3\boldsymbol{\xi} \, \Big[\mathcal{A}_{ij}(t,\,\boldsymbol{x};\,\tau,\,\boldsymbol{\xi})\overline{B}_j(t-\tau,\,\boldsymbol{x}+\boldsymbol{\xi}) + \\ + \mathcal{B}_{ijk}(t,\,\boldsymbol{x};\,\tau,\,\boldsymbol{\xi})\frac{\partial \overline{B}_j(t-\tau,\,\boldsymbol{x}+\boldsymbol{\xi})}{\partial x_k} + \dots\Big].$$

- $\mathcal{A}_{ij}$, $\mathcal{B}_{ijk}$ are nontrivial for $|\boldsymbol{\xi}| \sim \lambda_{\text{turbulence}}$ and $\tau \sim \tau_{\text{turbulence}}$. $\overline{\boldsymbol{B}}$ is constant on these scales:

$$\boldsymbol{\mathcal{E}}_i = \alpha_{ij}\overline{B}_j(t,\,\boldsymbol{x}) + \beta_{ijk}\frac{\partial \overline{B}_j(t,\,\boldsymbol{x})}{\partial x_k} + \dots$$

Homogeneous and isotropic turbulence

- For homogeneous and isotropic fluctuations, $\alpha_{ij} \propto \delta_{ij}$, $\beta_{ijk} \propto \epsilon_{ijk}$, so that

$$\boxed{\boldsymbol{\mathcal{E}} = \alpha \overline{\boldsymbol{B}} - \beta [\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}]}$$

- Coefficients α and β are expressed in terms of the background turbulence correlators:

$$\alpha = \frac{\tau^{(\alpha B)}}{3} \frac{1}{4\pi\mu\rho} \overline{\boldsymbol{b}^{(0)} \cdot [\boldsymbol{\nabla} \times \boldsymbol{b}^{(0)}]} - \frac{\tau^{(\alpha U)}}{3} \overline{\boldsymbol{u}^{(0)} \cdot [\boldsymbol{\nabla} \times \boldsymbol{u}^{(0)}]}$$

$$\beta = \frac{\tau^{(\beta)}}{3} \overline{(\boldsymbol{u}^{(0)})^2}$$

- Characteristic times $\tau^{(\alpha B)}$, $\tau^{(\alpha U)}$, $\tau^{(\beta)}$ are of the order $\tau_{\text{turbulence}}$. They arise when we replace time integrals with local expressions (neglect memory effects)

$$\int d\tau \overline{u_i(t) u_j(t-\tau)} \approx \tau^{(\alpha,\beta)} \overline{u_i(t) u_j(t)}$$

α and β effects

- Under several assumptions (weak, slowly varying $\overline{\boldsymbol{B}}$, homogeneous and isotropic turbulence, neglecting memory effects) we finally have the expression for the electromotive force:

$$\boxed{\boldsymbol{\mathcal{E}} = \alpha\overline{\boldsymbol{B}} - \beta[\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}]}$$

- Induction equation for the large-scale field $\overline{\boldsymbol{B}}$ reads as

$$\partial_t\overline{\boldsymbol{B}} = (\frac{1}{\sigma} + \beta)\boldsymbol{\nabla}^2\overline{\boldsymbol{B}} + \alpha[\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}].$$

- Coefficient β leads to stronger dissipation of the magnetic field
- Coefficient α is analogous to the CME current and leads to **instability**.
- Thus, correlators of the type $\overline{\boldsymbol{b} \cdot [\boldsymbol{\nabla} \times \boldsymbol{b}]}$, $\overline{\boldsymbol{u} \cdot [\boldsymbol{\nabla} \times \boldsymbol{u}]}$ (which violate parity) are responsible for the instability.

Attempt to take into account backreaction

- If $\alpha = \text{const}$, magnetic field grows exponentially. Naturally that some mechanism of backreaction should exist to stop this growth. One can try to derive the equation of motion for α.
- Additional assumptions:
  1. α depends on full fluctuations, not on the background ones, i.e., $\boldsymbol{u}^{(0)} \to \boldsymbol{u}$, $\boldsymbol{b}^{(0)} \to \boldsymbol{b}$
  2. Fluctuations are dominated by one (energy caring) mode with momentum $\boldsymbol{q}$
  3. Energy of fluctuation is constant (equivalent to weak $\overline{\boldsymbol{B}}$ and small dissipation)
  4. Neglecting higher than first derivatives of $\overline{\boldsymbol{B}}$
- EOM for α reads

$$\partial_t \alpha = \underbrace{-\frac{4}{3} q^2 \tau \overline{\boldsymbol{B}} \cdot \boldsymbol{\Gamma}}_{\text{backreaction}} - \underbrace{(\nu + \tfrac{1}{\sigma}) q^2 \alpha}_{\text{dissipation}}$$

where a new quantity appeared

$$\Gamma_i = -\frac{1}{q^2} \overline{[(\partial_i \partial_j)\boldsymbol{u} \times \boldsymbol{b}]_j}$$

Backreaction

- Using the single-mode approximation for fluctuations and symmetry properties, we conclude that

$$\Gamma_i = -\frac{1}{q^2}\overline{[(\partial_i\partial_j)\boldsymbol{u}\times\boldsymbol{b}]_j} \approx \frac{1}{q^2}\overline{[(q_i q_j)\boldsymbol{u}\times\boldsymbol{b}]_j} \approx c\mathcal{E}_i, \qquad c = \mathcal{O}(1).$$

- Using that $\mathcal{E} = \overline{[\boldsymbol{u}\times\boldsymbol{b}]} = \alpha\overline{\boldsymbol{B}} - \beta[\boldsymbol{\nabla}\times\overline{\boldsymbol{B}}]$, we rewrite EOM for α:

$$\partial_t\alpha = -\Big[\underbrace{\frac{4c}{3}q^2\tau\overline{\boldsymbol{B}}^2}_{\text{backreaction}} + \underbrace{(\nu+\tfrac{1}{\sigma})q^2}_{\text{dissipation}}\Big]\alpha + \frac{4c}{3}q^2\tau\beta\overline{\boldsymbol{B}}\cdot[\boldsymbol{\nabla}\times\overline{\boldsymbol{B}}]$$

- Indeed, nonzero $\overline{\boldsymbol{B}}^2$ leads to the decrease of α. This will finally stop the growth of magnetic field.
- Note, that we got completely analogous equation as in the case with chiral anomaly

$$\partial_t\mu_5 = -(\underbrace{\Gamma_B}_{\propto\overline{\boldsymbol{B}}^2} + \Gamma_f)\mu_5 + \underbrace{S_B}_{\propto\overline{\boldsymbol{B}}\cdot[\boldsymbol{\nabla}\times\overline{\boldsymbol{B}}]}$$

Conclusion

Physics of the dynamo effect is qualitatively similar to that of the chiral magnetic effect:

- The parity violation leads to the electric current $\boldsymbol{j} \propto \overline{\boldsymbol{B}}$ which leads to an instability in Maxwell's equation
- Only one helicity is amplified – resulting magnetic field is helical
- The energy is pumped from the subsystem which initially violates parity (fermions in CME, small-scale turbulence in dynamo)
- The backreaction leads to amelioration of instability (in CME this is due to conservation of chirality+helicity; in dynamo this is also connected to conservation of total large- and small-scale helicity)
- As μ_5 or α decreases, the modes with longer and longer wavelength are amplified: inverse cascade process

Problems

There are, however, difficulties in the derivation of effective mean-field dynamo theory:

- We have not just one pseudoscalar function μ_5, but two small-scale vector fields $\boldsymbol{u}$ and $\boldsymbol{b}$.
- EOM for these quantities are, generally speaking, nonlinear
- Expression for the electromotive force $\boldsymbol{\mathcal{E}} = \alpha\overline{\boldsymbol{B}} - \beta[\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}]$ is approximate and the underlying assumptions seem to be violated once the magnetic field $\overline{\boldsymbol{B}}$ gets amplified – therefore, it is more convenient to consider $\boldsymbol{\mathcal{E}}$ as unknown function and write EOM for it
- EOM for the small-scale correlators are, in general, not closed as they involve new and new correlators of different structure

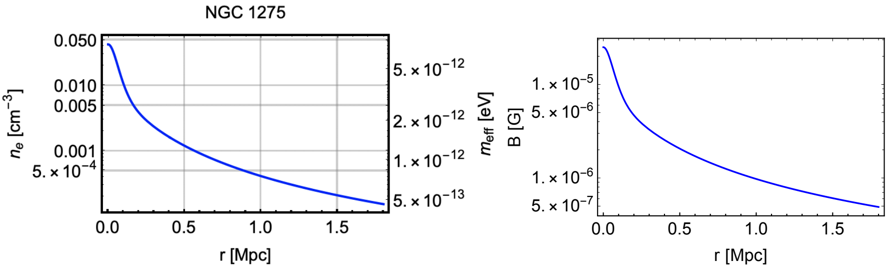
The only possible strategy: to introduce some minimal set of small-scale correlators whose EOM can be closed under a number of physically reasonable assumptions.

Backup slides

- **How do we measure magnetic fields in the Universe?**
- In **dense structures** we can to measure MFs using the **Faraday effect**
- Reminder: the Faraday effect causes a polarization rotation $\Delta\theta$,

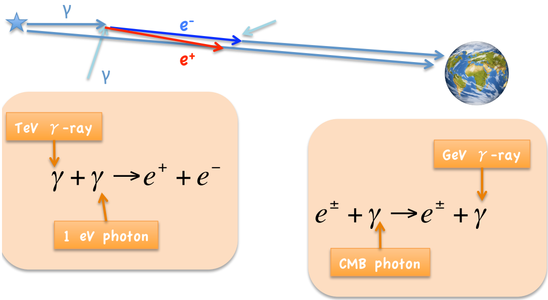
$$\Delta\theta = \mathrm{RM}\lambda^2, \qquad \mathrm{RM} = \frac{e^3}{2\pi m_e^2}\int n_e B_{||}\, d\ell, \tag{4}$$

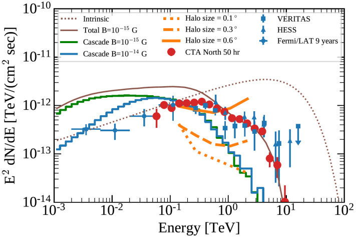
- Typical values of the observed MFs: $\sim 10^{-6}$ G in galaxies and central parts of clusters, $\sim 10^{-8}$ G in filaments between two close clusters [2101.09331]



[1907.05475]

Magnetic fields in voids

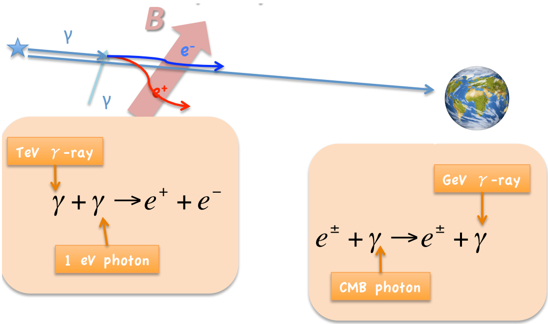


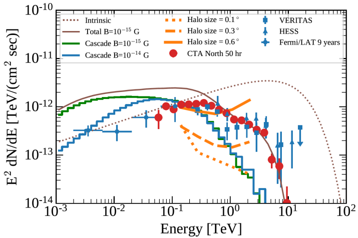


[2010.01349]

- γ-ray astronomy has a potential to measure **long-range magnetic fields** in the **Intergalactic Medium** (IGM)

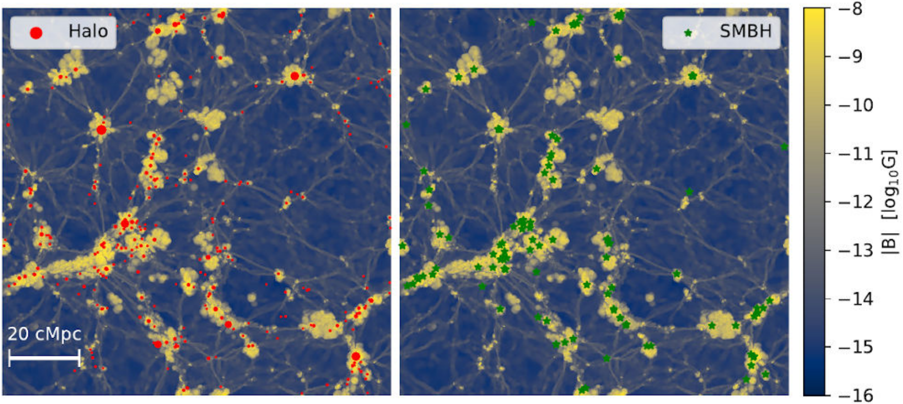
Magnetic fields in voids





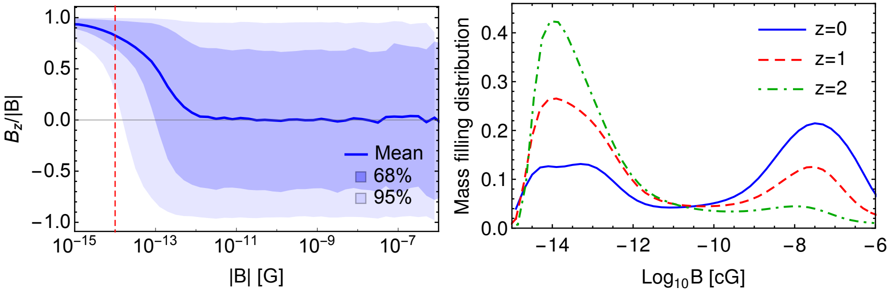
[2010.01349]

- γ-ray astronomy has a potential to measure **long-range magnetic fields** in the **Intergalactic Medium** (IGM)



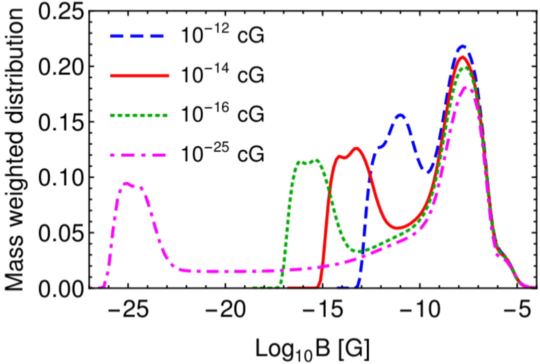
- Magnetic bubbles are produced by the **outflows caused by AGNs and supernovae**

MFs in bubbles forget initial conditions

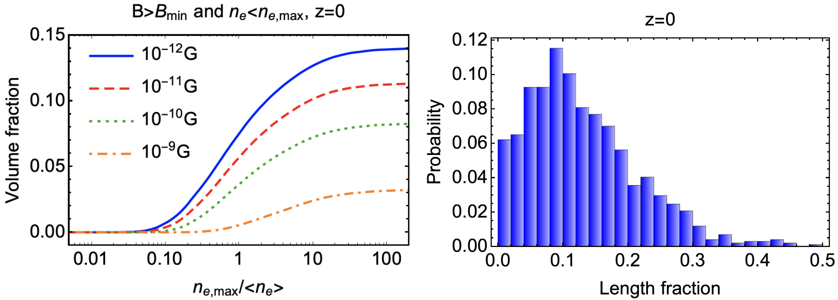


- At $z = 0$ for MFs with $B > 10^{-12}$ cG there is **no** longer a **preferred direction** of the field (for seed field was along z axis with $B = 10^{-14}$ cG)
- Over-magnetized bubbles **formed quite recently**, at redshifts $z \lesssim 2$
- Simulated magnetic fields in bubbles "**forget**" the initial orientation of the seed magnetic field!

MFs in bubbles forget initial conditions



- Moreover, different seed magnetic field values results in similar distribution of magnetic field in bubbles

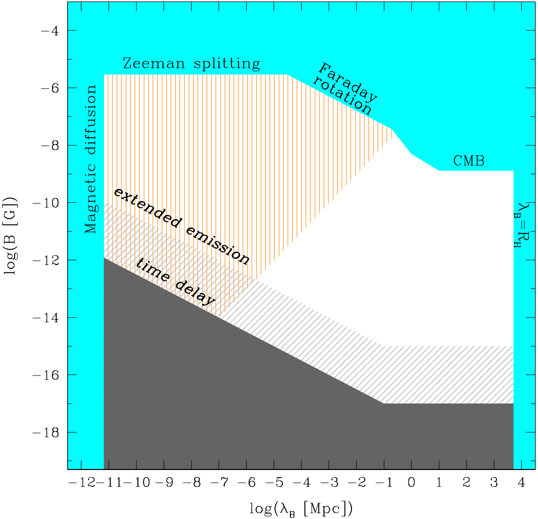


- At $z = 0$, the magnetic field is stronger than 10^{-12} G in 15% of the volume, while it is stronger than 10^{-9} G in 3% of the volume
- Alternatively, we can measure the fractional length, for a given line of sight, which intersects a strong magnetic field $B > 10^{-12}$ G
- We show the result for a sample of 1000 random sight-lines of length 100 Mpc

Cosmological magnetic fields observed?

Neronov & Vovk, Science (2010); Dolag et al. (2010); Tavecchio et al. (2011)

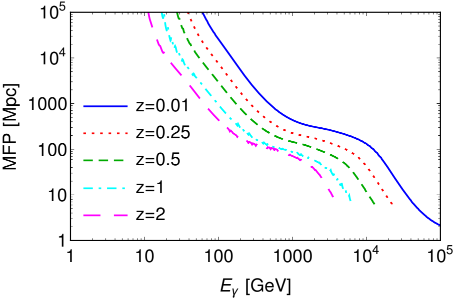
- Summary of different bounds on cosmological magnetic fields **[Taylor et al. (2011)]**
- **Can the lower bound be affected by bubbles?**



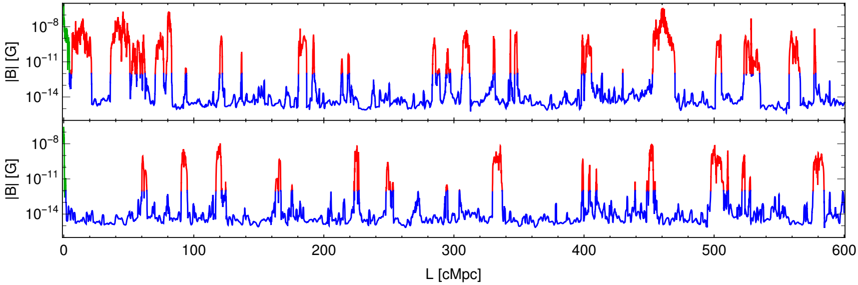
Influence of over-magnetized bubbles

- The electron-positron pair could be created anywhere along the line of sight. Over-magnetized bubbles have volume filling fraction of $10-15\%$, so **it seems that their presence should not significantly influence the secondary emission**

- This is correct only for the gamma-rays with long enough **mean free path (MFP)**
- Highest-energy gamma-rays have short means free path and secondary emission from them is **sensitive to the local environment** around the source



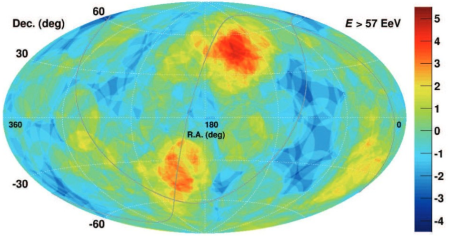
Influence of over-magnetized bubbles



- Each individual source can be unlucky enough the have an **extended over-magnetized bubble** along the line of sight to the Earth
- In our recent paper **[2106.02690]** we make a preliminary study of this effect. We found that for individual objects 70% of energy of secondary emission was lost, but for most of the systems the missing energy fraction is below 50%
- One way to deal with this problem is to increase statistic. With CTA we will be significantly increase amount and quality of observed sources

Example 2: propagation of UHECR

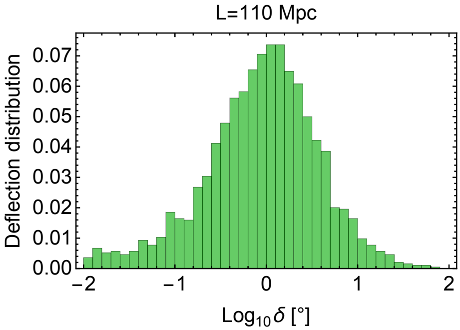
- The identification of the sources of **ultra-high energy cosmic rays (UHECRs)** is one of the central problems of astroparticle physics
- No strong signatures of sources have been seen in the data so far – the observed UHECRs show a surprisingly **high level of isotropy**, with no significant small scale clustering



[J. Matthews, 2017]

- This absence of small scale clustering is believed to arise from the **deflection of UHECRs in magnetic fields** during their propagation between the sources and Earth
- For protons outside of the galactic plane with energy 5×10^{19} eV the deflection angles is $\sim 1^\circ$ **[1904.08160]**
- What is a contribution of outflow-driven bubbles to the total deflection angle?

Example 2: propagation of UHECR



- The distribution of deflection angles is quite wide with an average value around 1° **[2101.07207]**
- The influence of intergalactic magnetic fields on the propagation of the UHECRs could be important and must to be taken into account when searching for the sources of these particles

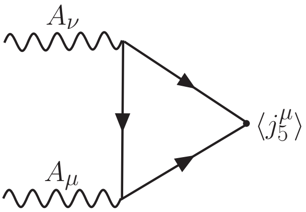
Chiral anomaly

- Gauge interactions respects chirality ($D_\mu = \partial_\mu + eA_\mu$)...

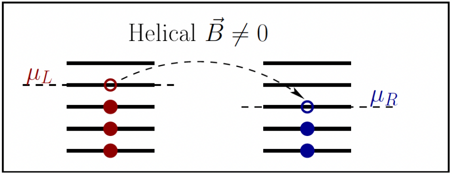
$$\begin{pmatrix} \cancelto{0}{-m} & i(D_t + \boldsymbol{\sigma}\cdot\boldsymbol{D}) \\ i(D_t - \boldsymbol{\sigma}\cdot\boldsymbol{D}) & \cancelto{0}{-m} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

- ...but the difference of left and right-movers is **not conserved** once the quantum corrections are taken into account — **chiral anomaly**

$$\frac{dQ_5}{dt} = \frac{d(N_L - N_R)}{dt} = \int d^3\boldsymbol{x}\left(\partial_\mu j_5^\mu\right) = \frac{e^2}{4\pi^2}\int d^3x\ \boldsymbol{x}\boldsymbol{E}\cdot\boldsymbol{B}$$



Axial anomaly at finite densities



- μ_L – Fermi level for left fermions
- μ_R – Fermi level for right fermions

- Change helicity of $\vec{B}$-field $\Rightarrow$ due to axial anomaly of $N_{L,R}$ changes:

$$\delta N_{L,R} = \int_{t_i}^{t_f} dt\, \dot{N}_{L,R}(t) = \mp \int dt \frac{\alpha}{\pi} \int dV\, E \cdot B$$

- The energy: $\delta \mathcal{E} = \delta N_L \mu_L + \delta N_R \mu_R = \frac{\alpha(\mu_L - \mu_R)}{2\pi} \int dV\, A \cdot B$
- Free energy of magnetic fields : $\delta \mathcal{F} = \int dV\, \frac{\alpha(\mu_L - \mu_R)}{2\pi} A \cdot B$

Nielsen & Ninomiya (1983);

Rubakov (1986)

Free energy of magnetic field

- The free energy for static magnetic fields $\mathcal{F}[\vec{A}]$ has the form

$$\mathcal{F}[A] = \frac{1}{2}\int d^3k\, A_i(\vec{k})\Pi_{ij}(k)A_j(-\vec{k}) + \mathcal{O}(A^3)$$

(magnetic field $\vec{B} = \nabla \times \vec{A}$)

- Polarization operator **in plasma** Π_{ij} should be rotation invariant and gauge invariant (i.e. transversal: $k_i\Pi_{ij} = 0$). The most general form:

$$\Pi_{ij}(\vec{p}) = (k^2\delta_{ij} - k_ik_j)\Pi_1(k^2) \quad + \quad i\epsilon_{ijn}k^n\Pi_2(k^2)$$

parity-even part — parity-odd part

here and below we will speak only about $\Pi_2(0)$ that we denote simply by Π_2

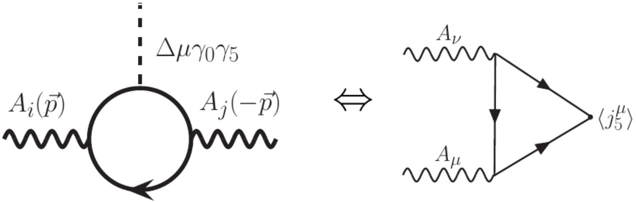
- In vacuum $\Pi_{\mu\nu}(k) = (\eta_{\mu\nu}k^2 - k_\mu k_\nu)\Pi(k^2)$

Thermal equilibrium and chiral anomaly

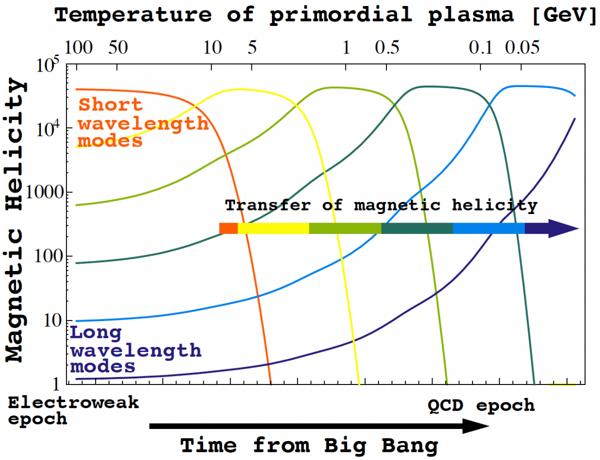
- Can have chem. potentials only for conserved charges: no separate μ_L, μ_R in real system – charged fermions are massive!
- If fermions interact with the chiral gauge field (only left components are charged), then then for the **charged** particles $\mu_L^{\mathbf{c}} - \mu_R^{\mathbf{c}} = \mu$

Redlich & Wijewardhana (1985)

$$\Pi_2 = \frac{\alpha}{\pi}(\mu_R - \mu_L)$$



Evolution of helicity spectrum



Process continues while $\Gamma_B \gg \Gamma_{\text{flip}}$ (recall that $\Gamma_B \propto \rho_B$)

Analysis of non-linear equations

- The non-linear system of equations:

$$\begin{cases} \operatorname{curl}\vec{B} = \sigma\vec{E} + \frac{\alpha}{\pi}\Big(\dot{\theta}_5\vec{B} + \nabla\theta_5 \times \vec{E}\Big) \\ \Lambda^2\ddot{\theta}_5 + \text{gradient terms} = \frac{2\alpha}{\pi}\vec{E}\cdot\vec{B} + \text{chirality flip}, \\ \vec{B}\cdot\nabla\dot{\theta}_5 = 0, \\ \mu_5 = \dot{\theta}_5 \end{cases}$$

- Tracking solution?
- Take $\vec{B}(x) = B(t)\big(\sin(kz), \cos(kz), 0\big)$. In this case $\vec{E}\cdot\vec{B}$ is **constant in space**
- $\Rightarrow \mu_5 = \dot{\theta}_5$ depends **on time only**. The gradient terms are not important. But the equation is still non-linear!