Cosmological Magnetic Fields from the Electroweak Epoch

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(based on review in arXiv:2010.10525)

MHD & B

If the one-fluid MHD approximation holds, magnetic fields cannot be generated.

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\sigma} \nabla^2 \mathbf{B}$$

If initial B vanishes, then B=0 for all times.

MHD approximation may be insufficient in certain situations. (E.g. due to electron-proton mass difference as in Harrison mechanism at recombination.)

MHD is certainly insufficient during early universe phase transitions, e.g. electroweak epoch.

Electroweak physics

At the electroweak epoch, particles get masses and electroweak symmetry is broken to the Maxwellian symmetry.

<u>Maxwell</u>

$$\partial_\nu A^{\mu\nu} = j^\mu$$

Currents due to fermions in physical system.

<u>Electroweak</u>

$$D_\nu W^{\mu\nu a} = j^{\mu a}, \quad (a = 1, 2, 3)$$

$$\partial_\nu Y^{\mu\nu} = j^{\mu Y}$$

Currents due to fermions in physical system and the Higgs.

$$\Box \Phi + V'(\Phi) = 0$$

Predictions of electroweak equations are as reliable as predictions of Maxwell equations.

Philosophy: resist introducing ad hoc physics.

Electroweak to Maxwell

Electroweak: W_i^1, W_i^2, W_i^3, Y_i all massless

<Higgs> ↓ at T~100 GeV (10^{15} K), t~1 ns — electroweak plasma

Weak; E&M: $\{W_i^+, W_i^-, Z_i^0\}, A_i$ photon massless

Electroweak symmetry breaking

$$\hat{\Phi} = v \begin{pmatrix} \hat{\phi}_1 + i\hat{\phi}_2 \\ \hat{\phi}_3 + i\hat{\phi}_4 \end{pmatrix}$$

(hats denote quantum fields)

$$\Phi \equiv \langle \hat{\Phi} \rangle = v \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

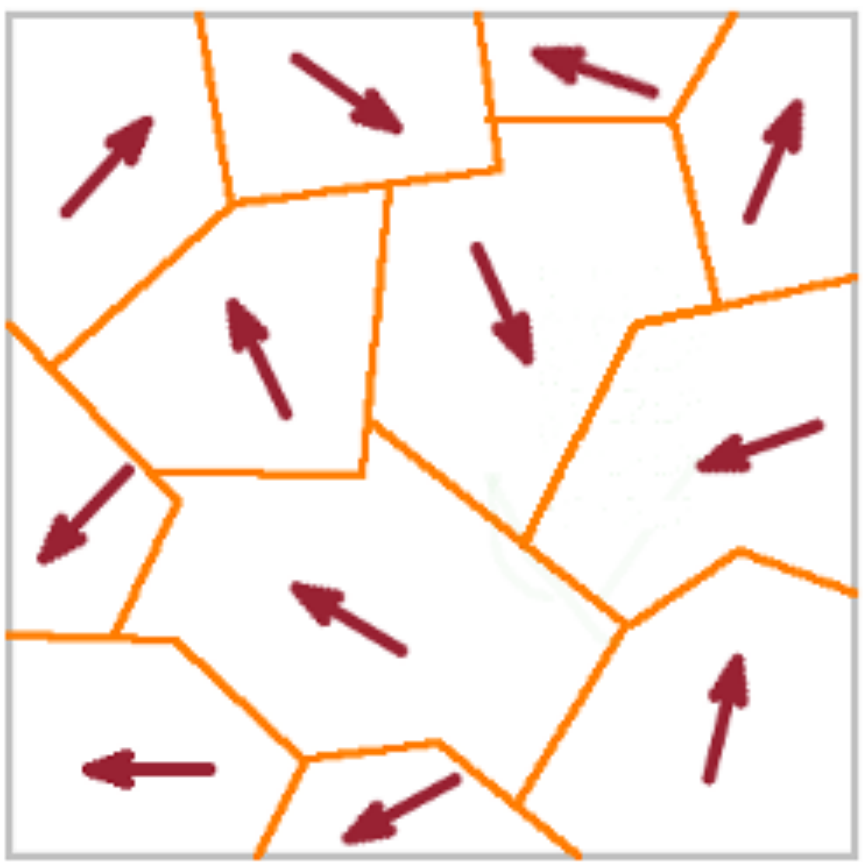
Electroweak phase: $\Phi = 0$

Maxwell phase: $\Phi = v \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$ $\qquad \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = 1$

During electroweak symmetry breaking, **Φ** acquires random values consistent with the constraints. I.e. **Φ** lies at any point on a three-sphere.

Kibble mechanism

During electroweak symmetry breaking, $\mathbf{\Phi}$ acquires *independent* random values on a three-sphere in well-separated spatial regions.



Basis for topological defect formation in cosmology and condensed matter systems.

What are the consequences of the Kibble mechanism in electroweak theory?

Embed Maxwell in Electroweak

Maxwell gauge field is a linear combination of the W and Y gauge fields.

$$A_\mu = \sin\theta_w \hat{n}^a W^a_\mu + \cos\theta_w Y_\mu \qquad \hat{n}^a \equiv -\frac{\Phi^\dagger \sigma^a \Phi}{\Phi^\dagger \Phi}$$

but the field strength has an extra term,

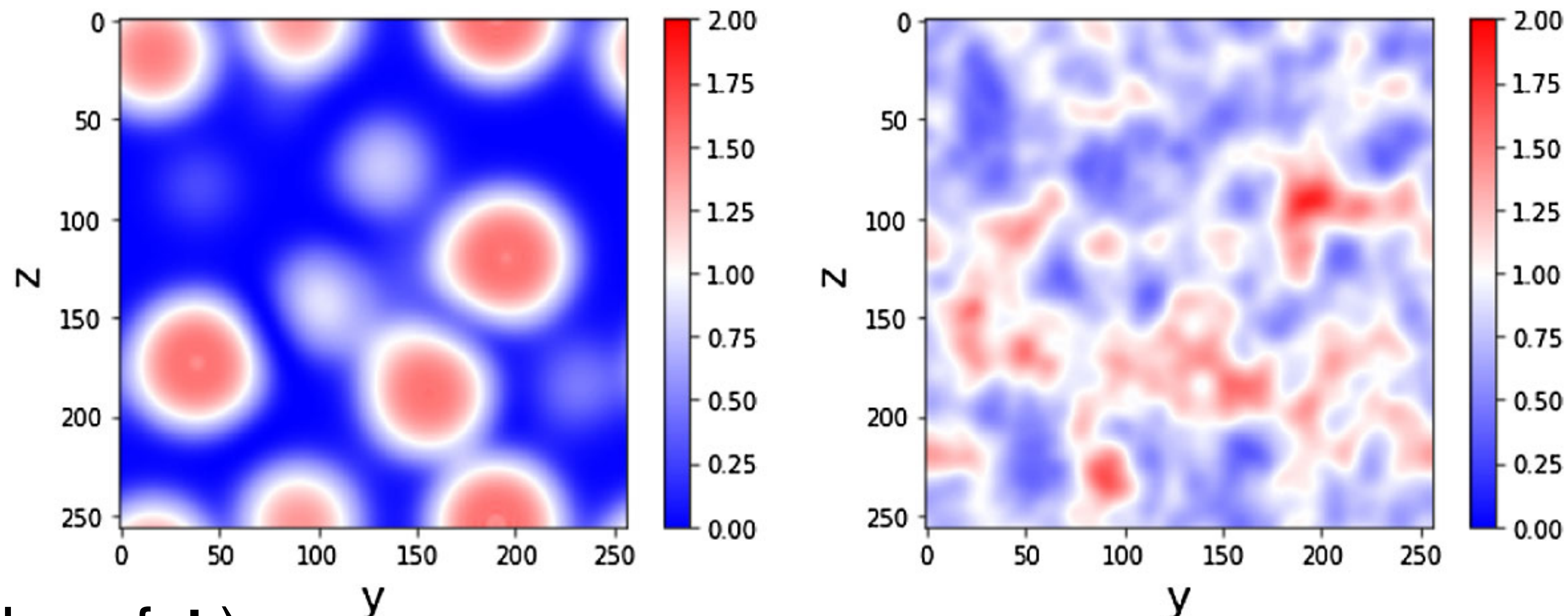
$$\mathbf{B} = \nabla \times \mathbf{A} - i\frac{2\sin\theta_w}{gv^2}\nabla\Phi^\dagger \times \nabla\Phi$$

Even if A=0, the magnetic field need not vanish.

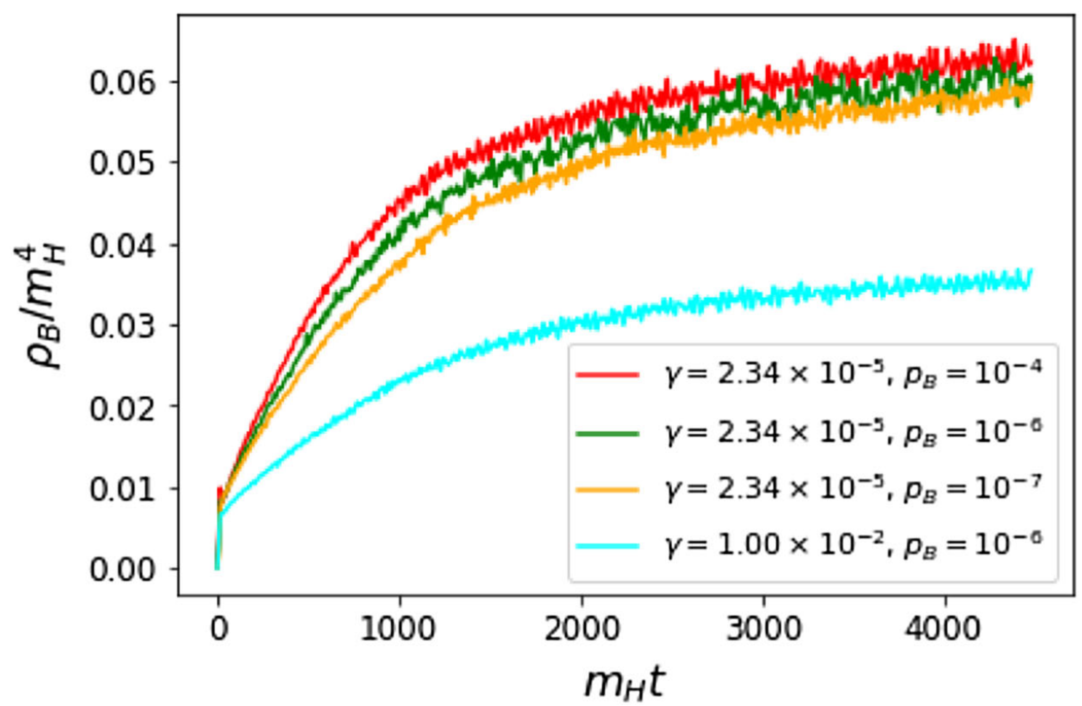
Random distributions of **Φ** will lead to a stochastic magnetic field.

Direct simulations of EWSB

Diaz-Gil, Garcia-Bellido, Perez & Gonzalez-Arroyo
Mou, Saffin & Tranberg
Zhang, Ferrer & TV



(bubbles of **Φ**)

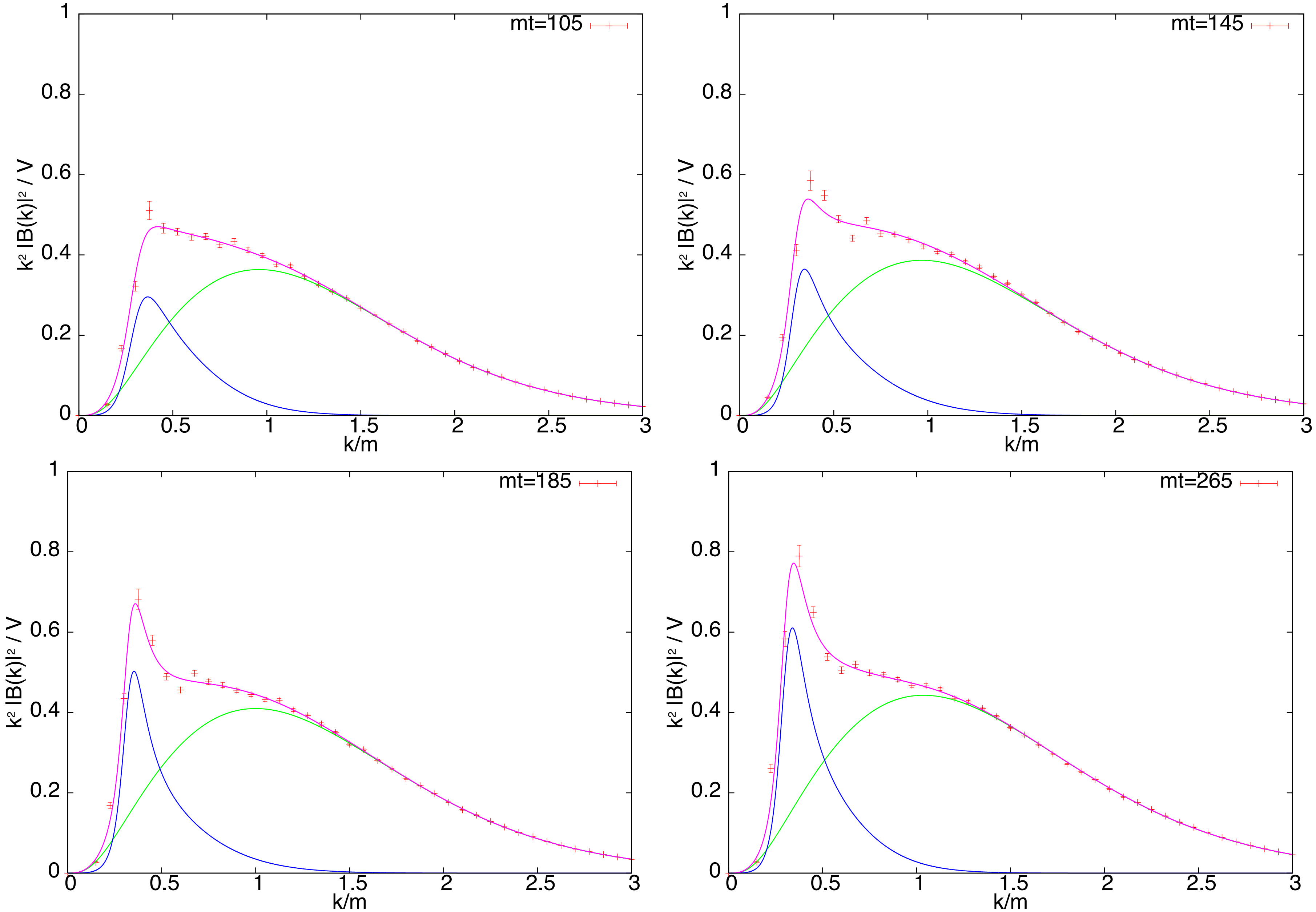


$$\left.\frac{\rho_B}{\bar{\rho}}\right|_{t_f} \approx 6\%$$

(bosonic degrees of freedom only)

B Spectrum

Diaz-Gil, Garcia-Bellido, Perez & Gonzalez-Arroyo



Pink=total

Green=thermal

Blue=total-thermal

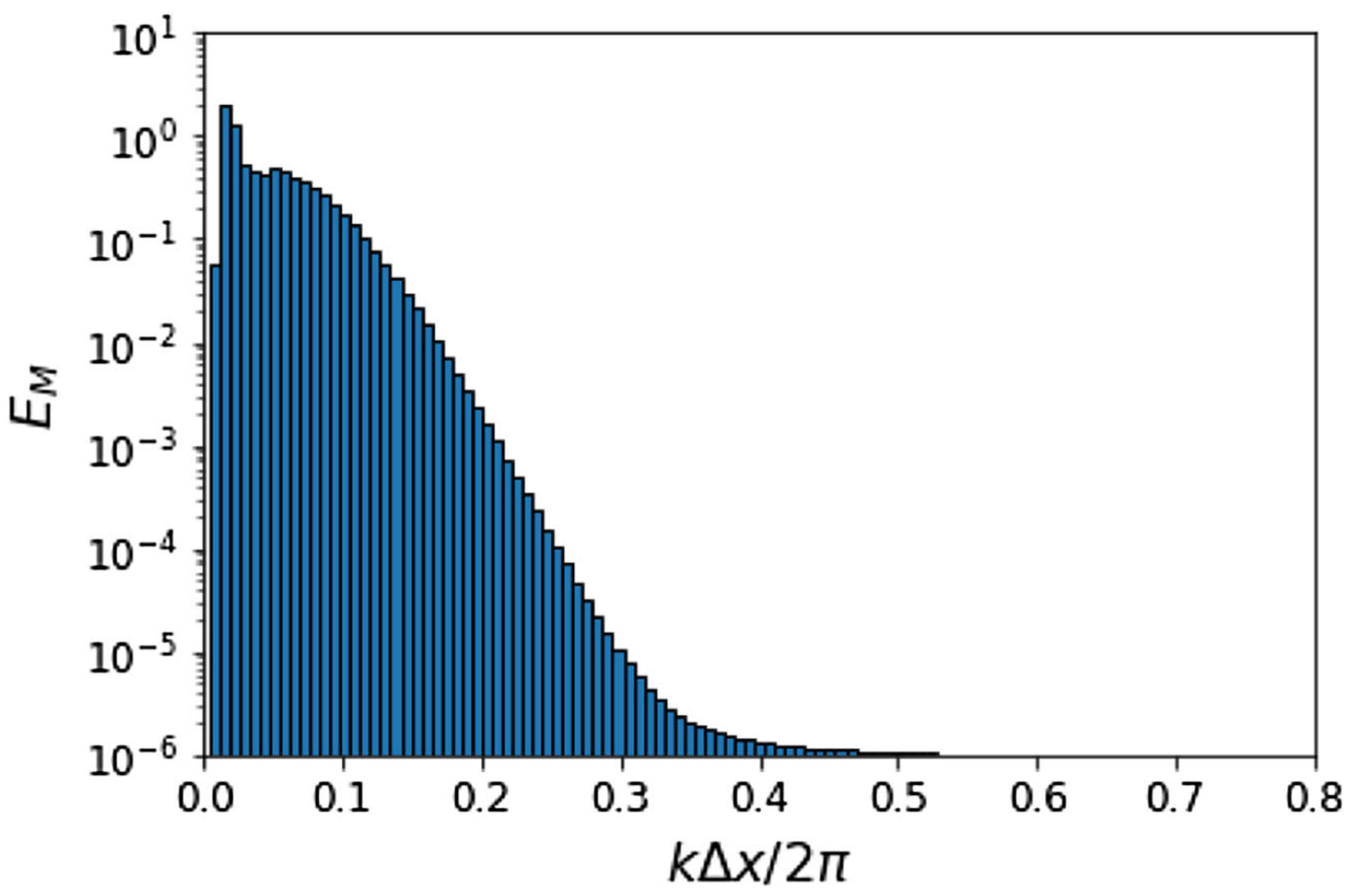
Peak at small k.

Similar results from other simulations.

B Spectrum contd.

Zhang, Ferrer & TV

Another simulation.

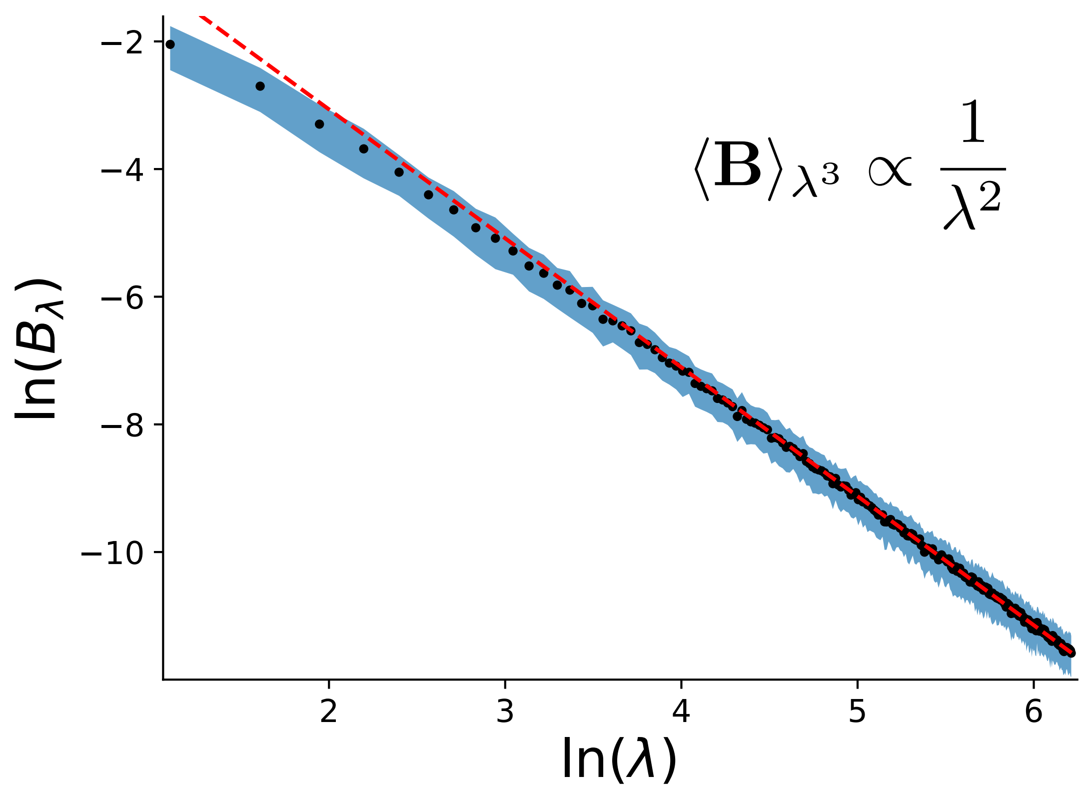


Analytical estimate

$$\mathbf{B} = \nabla \times \mathbf{A} - i\frac{2\sin\theta_w}{g}\nabla\hat{\Phi}^\dagger \times \nabla\hat{\Phi}$$

Contribution to volume-averaged magnetic field:

$$\langle \mathbf{B} \rangle_V = \frac{1}{V}\int_V d^3x\,\mathbf{B} = -i\frac{2\sin\theta_w}{gV}\int_{\partial V} d\mathbf{S} \times (\hat{\Phi}^\dagger \nabla\hat{\Phi})$$



$$\langle \mathbf{B} \rangle_{\lambda^3} \propto \frac{1}{\lambda^2}$$ or, $$\mathbf{B}(k) \propto k^2$$

Summary: Magnetized Universe

Fractional cosmic energy density in magnetic fields:

$$\Omega_B(t_{EW}) \sim 1\%$$

(by counting degrees of freedom)

with spectrum:

$$B_\lambda(t_{\rm EW}) \sim 2\sqrt{\rho_{EW,B}}\left(\frac{k}{k_*}\right)^2, \quad k \leq k_*$$

$$E_M(k) \sim \frac{4\,\rho_{EW,B}}{k_*}\left(\frac{k}{k_*}\right)^3, \quad k \leq k_*$$

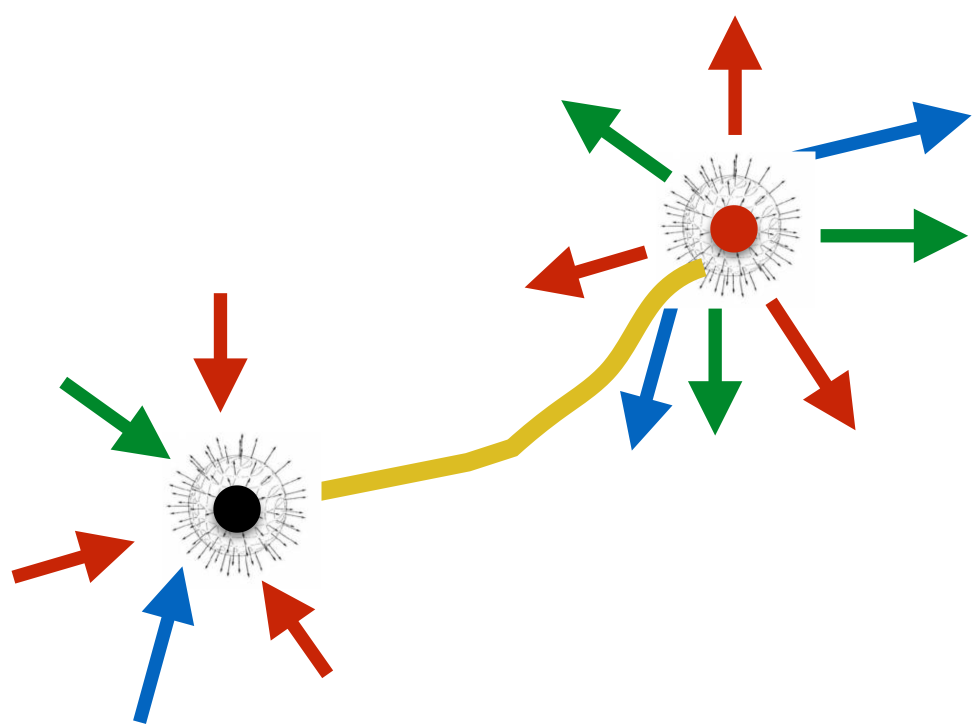
What is k_*?

k_*

Simulations suggest some large coherence scale but are limited by dynamic range.

Instead we can argue in the following way —

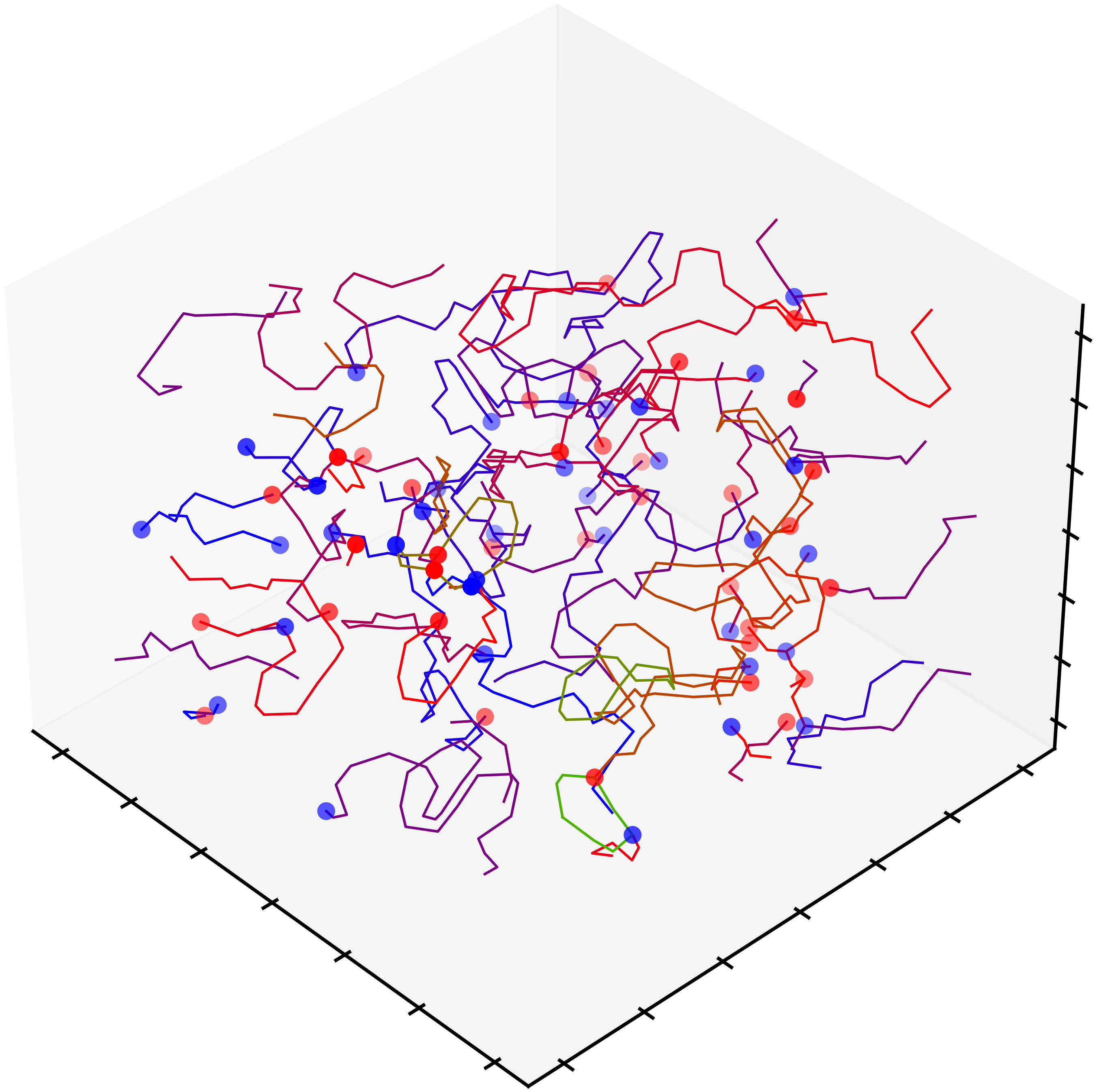
the Higgs term in the magnetic field strength implies “Nambu dumbbells” in the electroweak model.



“Magnetic monopoles connected by strings”

Monopole-string distribution

Teerthal Patel & TV, 2021

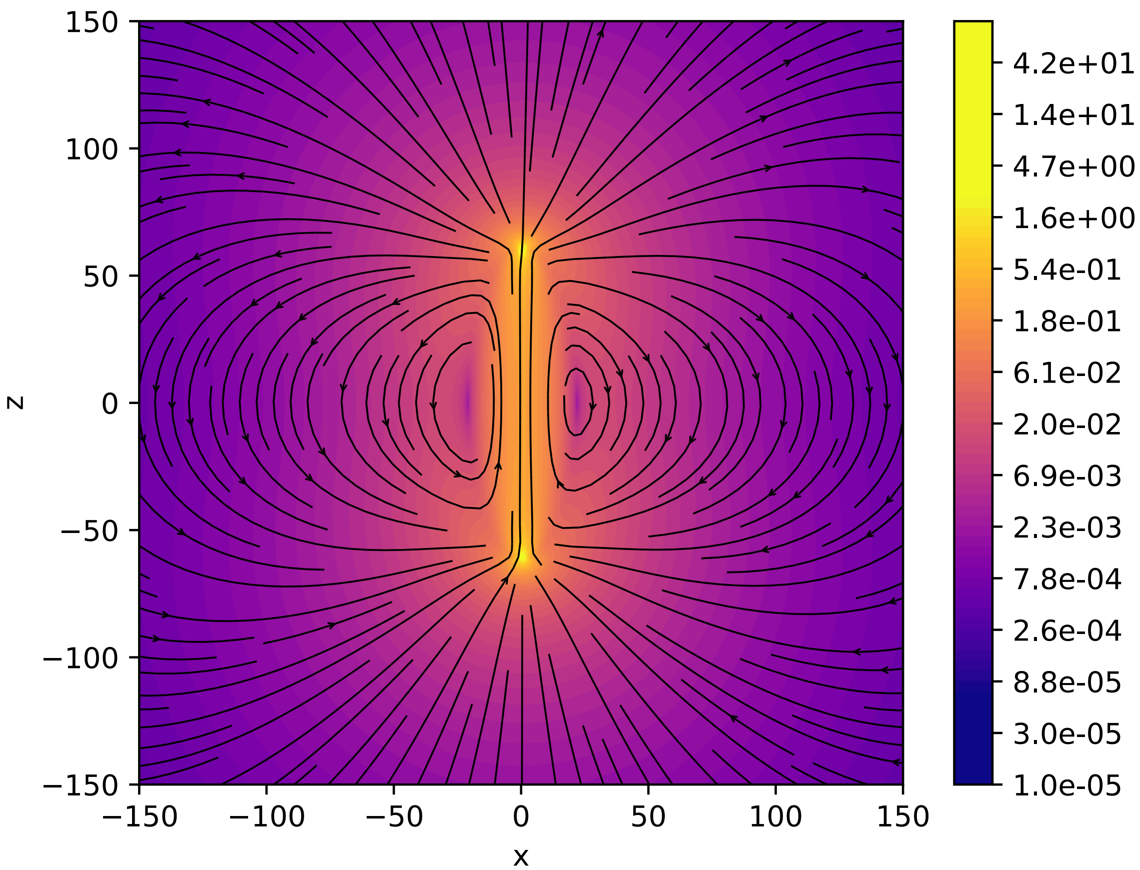


Magnetic field of dumbbells

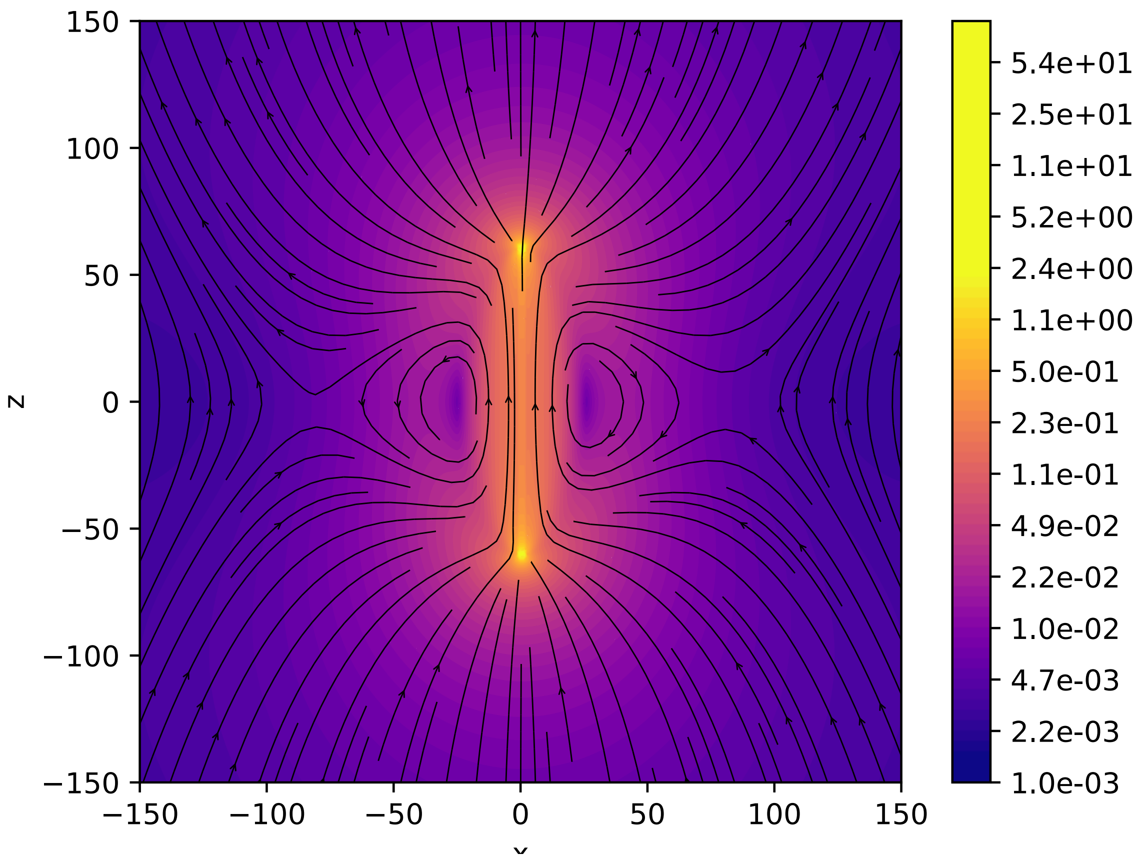
Teerthal Patel & TV

Impose topological constraint: fix monopole-antimonopole positions.

Relax field configuration for different Higgs field configurations.

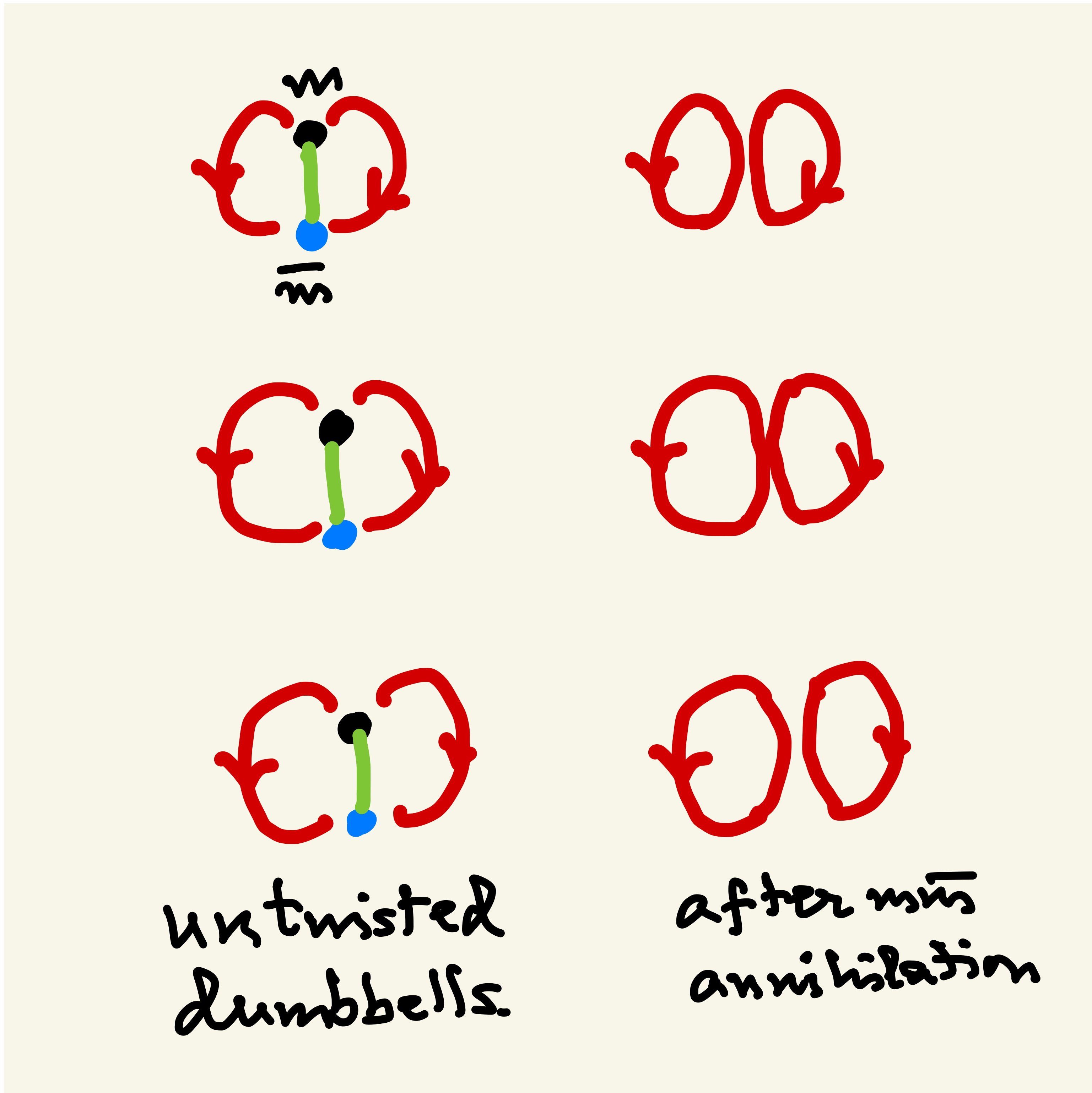


twist=0

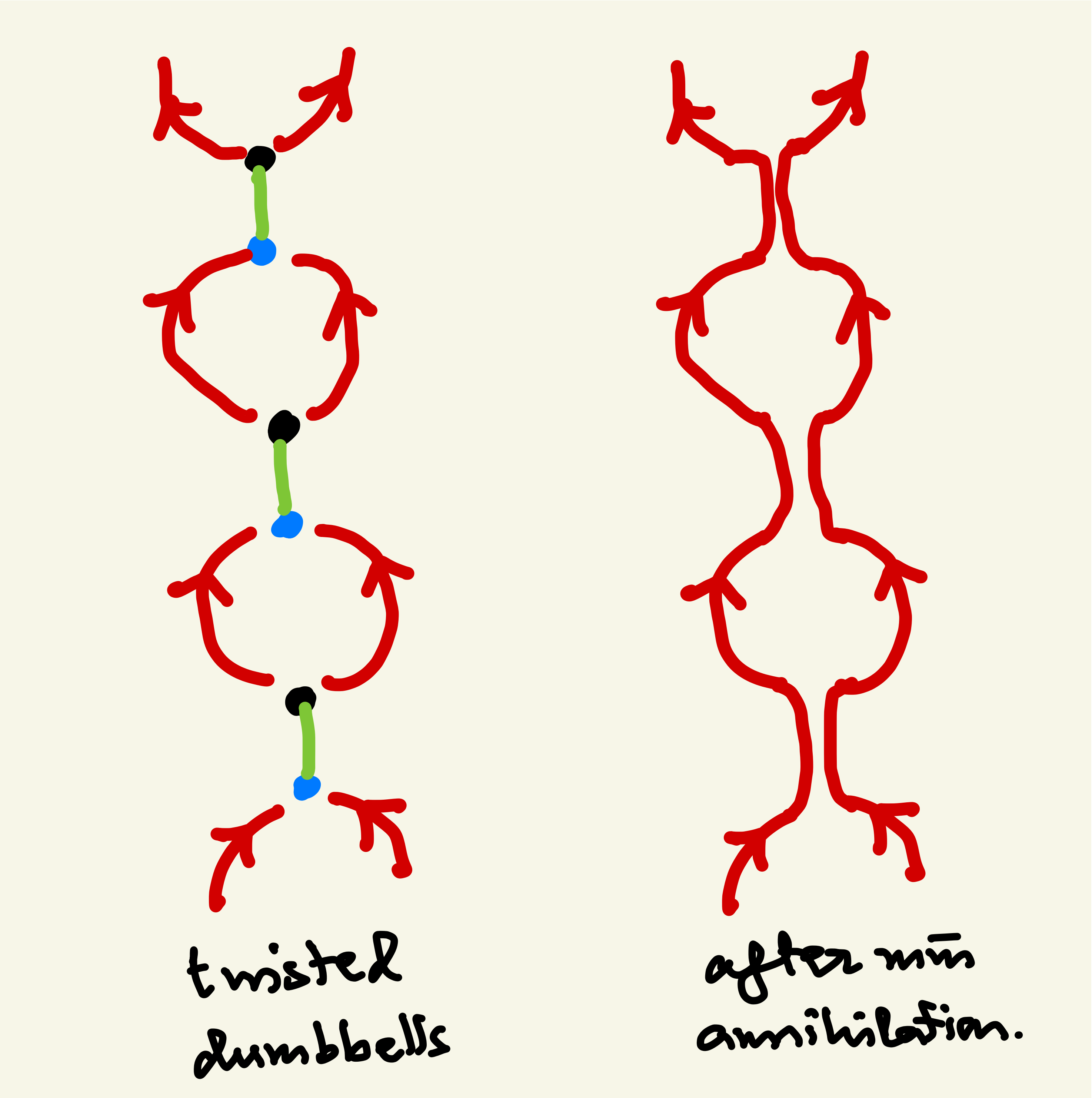


twist=pi

Gas of untwisted dumbbells

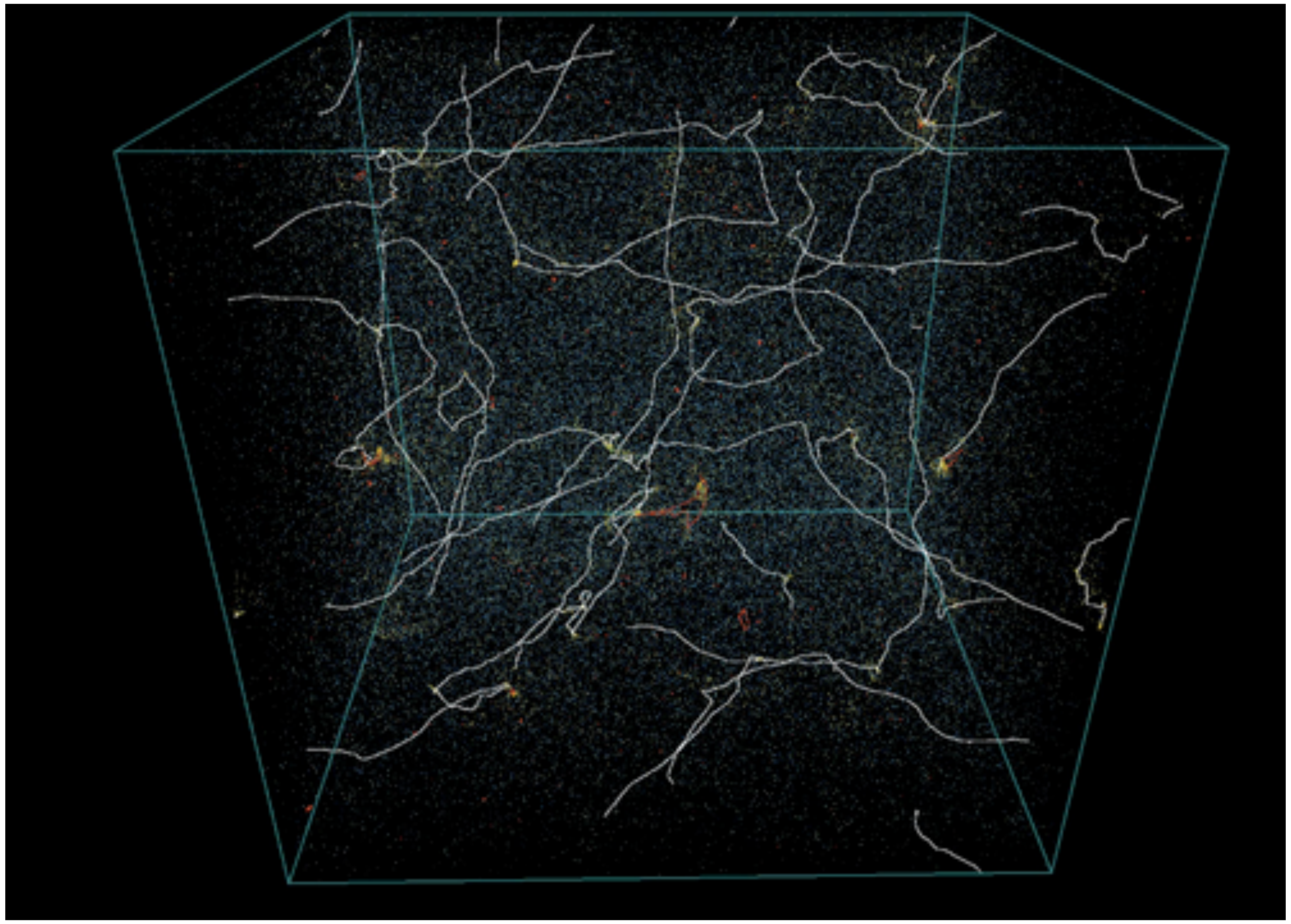


Gas of twisted dumbbells



After dumbbell annihilation

If cosmic strings are a guide, a large fraction of the magnetic field energy is on the largest length scales.



~80% of the energy is in infinite strings.

Magnetic field lines random walk in 3D and never close on themselves.

$k_\star$ is presumably set by the horizon scale.

Magnetic Fields at the present epoch

Many experts: Banerjee, Brandenburg, Jedamzik, Kahniashvili, Sigl, Subramanian,…

Non-helical:

Only the long wavelength tail of the distribution can survive dissipation.

$$B_{1\,\mathrm{kpc}} \sim 10^{-18}\,\mathrm{G}$$

(1 kpc dissipation scale)

Helical: $$B_{10\,\mathrm{kpc}} \sim 10^{-11}\,\mathrm{G}$$

(would be consistent with blazar observations)

(Evolution by **Hosking & Schekochihin** arXiv: 2203.03573 claims stronger magnetic fields in non-helical case.)

Other electroweak effects

Fermionic sector: plasma, chirality.

CP violation and helicity: how much? in what interactions?

Helicity fluctuations: …

Conclusions

★ Electroweak physics is well-established since the discovery of the Higgs.

★ Electroweak symmetry breaking leads to magnetic fields with significant energy density. Coherence scale may be reasonably large.

★ Predictions for present day magnetic fields depends on the helicity and evolutionary details.

★ If the magnetic field is helical, or if the evolution is significantly different from the “standard” picture, magnetic fields generated at the electroweak epoch are consistent with blazar bounds.

★ Magnetic field observations may be a window into electroweak physics and provide hints for accelerator physics (CP violation).