Implications of deviations from slow roll for inflationary magnetogenesis

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arXiv:2111.01478 and arXiv:2211.05834

Outline

- Observational evidence for magnetic fields
- Generation of magnetic fields during inflation
- Challenges in magnetogenesis for single field inflationary models generating features in scalar power spectra (SPS)
- Circumventing the challenges with the aid of two field model
- Imprints of PMFs on the CMB
- Conclusions

Observational evidence for magnetic fields

- In galaxies, strength of the observed magnetic field is $\sim 10^{-6}$ G which is coherent over scales of $1 - 10$ Kpc.[1]
- In clusters of galaxies, the strength is $\sim 10^{-7} - 10^{-6}$ G with coherent length of 10 Kpc $-$ 1 Mpc.[2]
- In intergalactic medium(IGM) voids the strength is $\geq 10^{-16}$ G which is coherent on scales above 1 Mpc.[3]

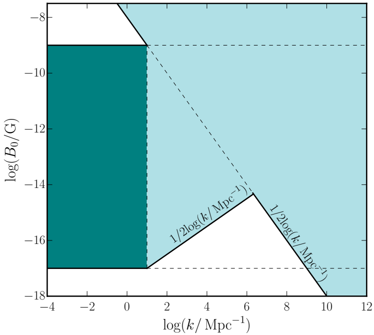
The origin of the seed magnetic field could be astrophysical or cosmological.

[1] R. Beck, Space Sci. Rev. **99**, 243 (2001); R. Beck and R. Wielebinski, "Magnetic Fields in Galaxies, in Planets, Stars and Stellar Systems," In: T. D. Oswalt and G. Gilmore, Eds., Springer, Vol. **5**, 641 (2013).

[2] T. E. Clarke, P. P. Kronberg and H. Böhringer, Astrophys. J. **547**, L111 (2001); F. Govoni and L. Feretti, Int. J. Mod. Phys. D **13**, 1549 (2004)

[3] A. Neronov and I. Vovk, Science **328**, 73 (2010)

Constraints on IGMF



Constraints on B_0, the strength of the magnetic fields observed today, as a function of the comoving wave number k.[4]

The origin of large scale magnetic fields can be explained using the processes during inflation in the early universe.

[4] Figure adapted from, T. Markkanen, S. Nurmi, S. Rasanen, and V. Vennin, JCAP **06**, 035 (2017)

Generation of primordial magnetic field (PMF)

The parity violating term is introduced to the action as

$$S[A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\,\sqrt{-g}\left[J^2(\phi)\,F_{\mu\nu}\,F^{\mu\nu} - \frac{\gamma}{2}\,J^2(\phi)\,F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right],$$

where $\widetilde{F}^{\mu\nu} = (\epsilon^{\mu\nu\alpha\beta}/\sqrt{-g})\,F_{\alpha\beta}$.
The equation of motion has the form

$$\mathcal{A}_k^{\sigma\,\prime\prime} + \left(k^2 + \frac{2\,\sigma\,\gamma\,k\,J'}{J} - \frac{J''}{J}\right)\mathcal{A}_k^\sigma = 0,$$

where $\sigma = \pm 1$ represents positive and negative helicity.
The power spectra of the helical magnetic and electric fields are given by[5]

$$\mathcal{P}_{\rm B}(k) = \frac{k^5}{4\,\pi^2\,a^4}\left[\left|\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^-\right|^2\right],$$

$$\mathcal{P}_{\rm E}(k) = \frac{k^3}{4\,\pi^2\,a^4}\left[\left|\mathcal{A}_k^{+\prime} - \frac{J'}{J}\mathcal{A}_k^+\right|^2 + \left|\mathcal{A}_k^{-\prime} - \frac{J'}{J}\mathcal{A}_k^-\right|^2\right].$$

[5] K Subramanian,Rept.Prog.Phys. **79**, 076901 (2016)

Electromagnetic (EM) power spectra

For the choice of coupling function $J(\eta) \propto a(\eta)^n$ (in de-Sitter $a = -1/H_{\rm I}\eta$), we obtain scale invariant $\mathcal{P}_{\rm B}(k)$ for $n = 2$.

For $\gamma = 0$ (non-helical fields)

$$\begin{aligned}\mathcal{P}_{\rm B}(k) &= \frac{9\, H_{\rm I}^4}{4\,\pi^2}, \\ \mathcal{P}_{\rm E}(k) &= \frac{H_{\rm I}^4}{4\,\pi^2}\,(-k\,\eta_{\rm e})^2.\end{aligned}$$

For $\gamma \neq 0$ (helical fields)

$$\begin{aligned}\mathcal{P}_{\rm B}(k) &= \frac{9\, H_{\rm I}^4}{4\,\pi^2}\, f(\gamma), \\ \mathcal{P}_{\rm E}(k) &= \frac{9\, H_{\rm I}^4}{4\,\pi^2}\, f(\gamma) \left[\gamma^2 - \frac{\sinh^2(2\,\pi\,\gamma)}{3\,\pi\,\left(1+\gamma^2\right)\,f(\gamma)}\,(-k\,\eta_{\rm e}) \right. \\ &\quad \left. + \frac{1}{9}\,\left(1 + 23\,\gamma^2 + 40\,\gamma^4\right)\,(-k\,\eta_{\rm e})^2\right],\end{aligned}$$

where $f(\gamma) = \frac{\sinh\,(4\,\pi\,\gamma)}{4\,\pi\,\gamma\,(1+5\,\gamma^2+4\,\gamma^4)}$. For $\gamma = 1, \quad f(\gamma) \simeq 10^3$

Construction of $J(\phi)$ for slow roll (SR) models

In terms of e-folds, the coupling function is expected to be

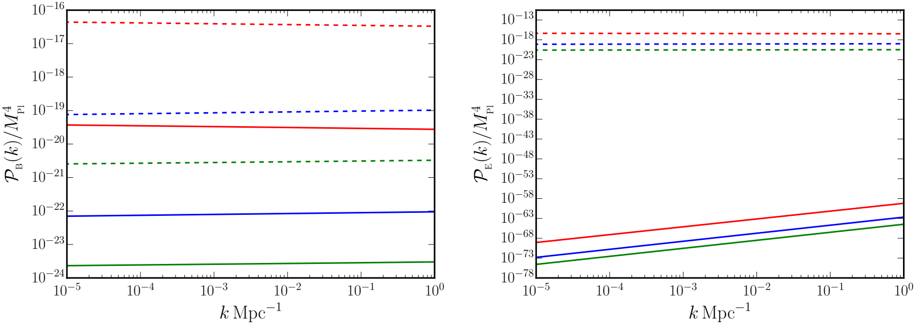
$$J(N) = \exp\left[n\left(N - N_{\mathrm{e}}\right)\right].$$

The Klein-Gordon equation for inflaton field is

$$\ddot{\phi} + 3H\dot{\phi} + V_{\phi} = 0.$$

SR Model	Potential	Coupling fuction $[J(\phi)]$
Quadratic potential (QP)	$\frac{m^2}{2}\phi^2$	$\exp\left[-\frac{n}{4M_{\mathrm{Pl}}^2}\left(\phi^2 - \phi_{\mathrm{e}}^2\right)\right]$
Small field model (SFM)	$V_0\left[1 - \left(\frac{\phi}{\mu}\right)^q\right]$	$\left(\frac{\phi}{\phi_{\mathrm{e}}}\right)^{n\mu^2/2M_{\mathrm{Pl}}^2}\exp\left[-\frac{n}{4M_{\mathrm{Pl}}^2}\left(\phi^2 - \phi_{\mathrm{e}}^2\right)\right]$
First Starobinsky model (FSM)	$V_0\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]^2$	$\exp\left\{-\frac{3n}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_{\mathrm{Pl}}}\right) - \exp\left(\sqrt{\frac{2}{3}}\frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right) - \sqrt{\frac{2}{3}}\left(\frac{\phi}{M_{\mathrm{Pl}}} - \frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right)\right]\right\}$

EM power spectra



The spectra of the magnetic (on the left) and electric (on the right) fields for the QP (in red), the SFM (in blue) and the FSM (in green) in both the non-helical (as solid lines) and helical (as dashed lines) cases.[6]

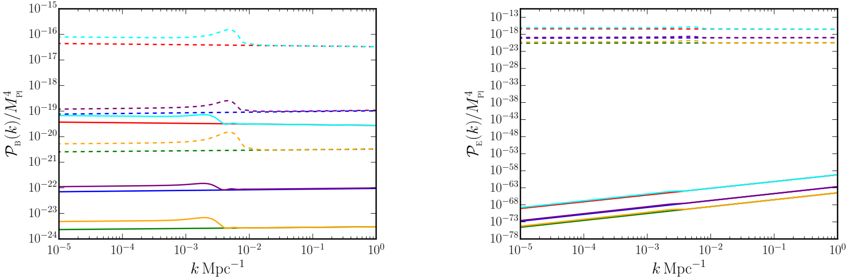
[6] **S. Tripathy**, D. Chowdhury, R. K. Jain, L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022)

Models generating features in SPS

Features over large scales

(a) **Introducing a step in the slow roll potential**[7]

$$V_{\rm step}(\phi) = V(\phi)\left[1 + \alpha \tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right)\right]$$



The spectra of the magnetic (on the left) and electric field (on the right) for potential with step for QP (in cyan), the SFM (in purple) and the FSM (in orange) in both the non-helical (as solid lines) and helical (as dashed lines) magnetic fields.[8]

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[7] J. A. Adams, B. Cresswell, and R. Easther, Phys. Rev. D **64**, 123514 (2001).

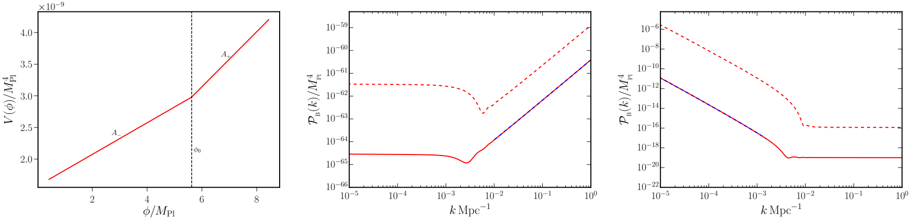
[8] **S. Tripathy**, D. Chowdhury, R. K. Jain, L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022)

Features over large scales continued

Model with change in slope in the potential

The second Starobinsky model is described by the potential[9]

$$V(\phi) = \begin{cases} V_0 + A_+ (\phi - \phi_0), & \text{for } \phi > \phi_0, \\ V_0 + A_- (\phi - \phi_0), & \text{for } \phi < \phi_0. \end{cases}$$



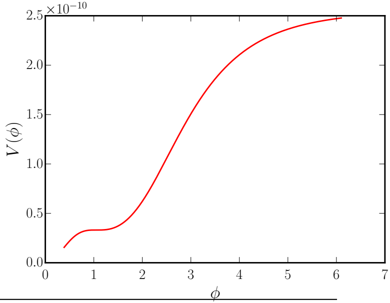
[9] A. A. Starobinsky, JETP Lett. **55**, 489 (1992)

Potentials with a point of inflection

Features over large scales

(c) **First punctuated inflation model** [10]

$$V(\phi) = \frac{m^2}{2}\phi^2 - \frac{2m^2}{3\phi_0}\phi^3 + \frac{m^2}{4\phi_0^2}\phi^4$$



Features over small scales

(a) **Ultra slow roll model** [11]

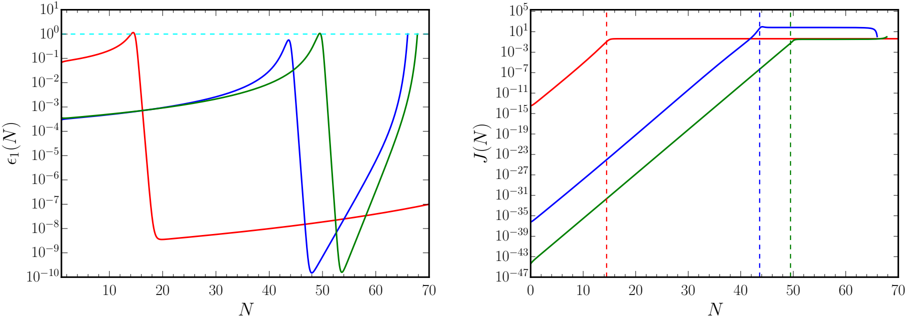
$$V(\phi) = V_0\left\{\tanh\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right) + A\,\sin\left[\frac{1}{f_\phi}\tanh\left(\frac{\phi}{\sqrt{6}M_{\rm Pl}}\right)\right]\right\}^2$$

(b) **Second punctuated inflation model** [12]

$$V(\phi) = V_0\left[c_0 + c_1\,\tanh\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right) + c_2\,\tanh^2\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right) + c_3\,\tanh^3\left(\frac{\phi}{\sqrt{6}\,M_{\rm Pl}}\right)\right]^2$$

[11] R. K. Jain, P. Chingangbam, L. Sriramkumar, and T. Souradeep, Phys. Rev. D **82**, 023509 (2010)
[12] I. Dalianis, A. Kehagias, and G. Tringas, JCAP **01**, 037 (2019)
[13] I. Dalianis and K. Kritos, Phys. Rev. D **103**, 023505 (2021)

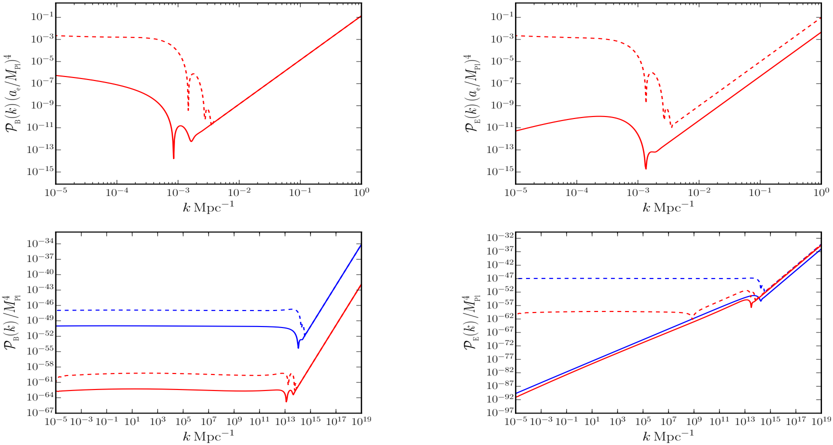
ϵ_1 and $J(\phi)$ of potentials with a point of inflection



The evolution of ϵ_1 and $J(N)$ for the first and second punctuated inflation model and ultra slow model (in solid red, green and blue, respectively) as a function of the e-fold N.[13]

[13] **S. Tripathy**, D. Chowdhury, R. K. Jain, L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022)

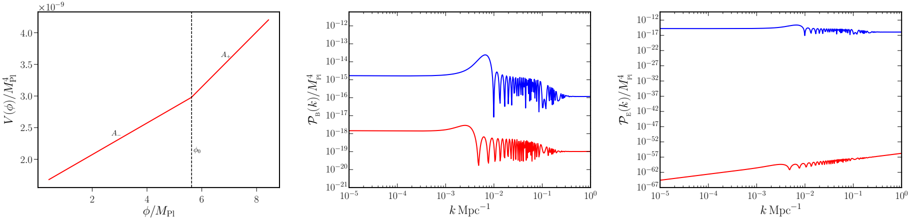
EM spectra for potentials with a point of inflection



The spectra of the magnetic (on the left) and electric (on the right) fields for both the non-helical (in solid red) and helical (in dashed red) cases.[14]

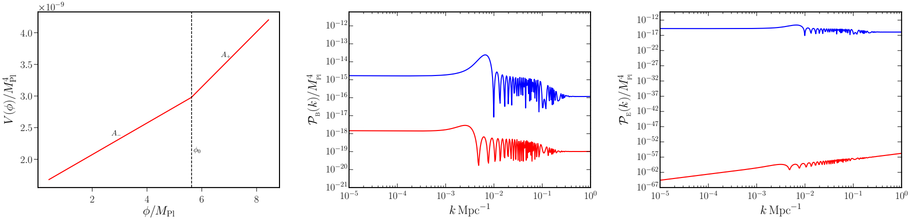
[14] **S. Tripathy**, D. Chowdhury, R. K. Jain, L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022)

An attempt to iron out the features



The spectra of the magnetic (on the left) and electric (on the right) fields for both the non-helical (in red) and helical (in blue) cases for the second Starobinsky model.

An attempt to iron out the features



The spectra of the magnetic (on the left) and electric (on the right) fields for both the non-helical (in red) and helical (in blue) cases for the second Starobinsky model.

Is there a better way to overcome these challenges and obtain the desired shape and amplitude of $\mathcal{P}_{\rm B}(k)$?

Circumventing the challenges with two field models

- The action governing two field model is given as,

$$S[\phi,\chi] \;=\; \int \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi - \frac{f(\phi)}{2}\,\partial_\nu\chi\,\partial^\nu\chi - V(\phi,\chi)\right].$$

- Here the form of the non-canonical coupling $f(\phi) = \mathrm{e}^{b(\phi)}$.
- Deviations from slow roll can be naturally achieved in two field models due to a sharp turn in the trajectory in the field space for non-zero values of $b(\phi)$.
- The equations of motion describing the evolution of the scalar fields are,

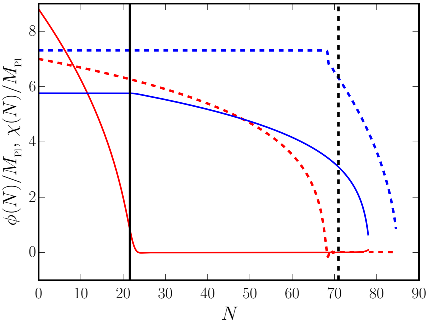
$$\ddot{\phi} + 3\,H\,\dot{\phi} + V_\phi \;=\; b_\phi\,\mathrm{e}^{2\,b}\,\dot{\chi}^2,$$

$$\ddot{\chi} + (3\,H + 2\,b_\phi\,\dot{\phi})\,\dot{\chi} + \mathrm{e}^{-2\,b}\,V_\chi \;=\; 0.$$

Two field models generating features in SPS

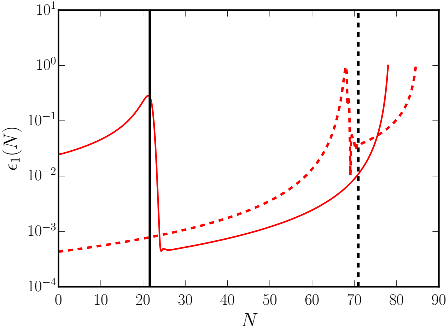
The potential that leads to suppression in scalar power over large scales has the form[15]

$$V(\phi,\chi) = \frac{m_\phi^2}{2}\,\phi^2 + V_0\,\frac{\chi^2}{\chi_0^2+\chi^2}.$$



The potential that leads to enhancement in scalar power over small scales has the form[16]

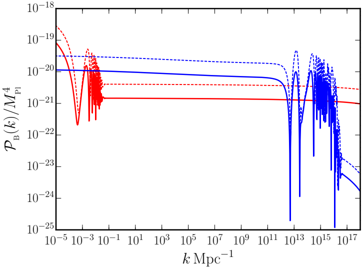
$$V(\phi,\chi) = V_0\,\frac{\phi^2}{\phi_0^2+\phi^2} + \frac{m_\chi^2\,\chi^2}{2}.$$

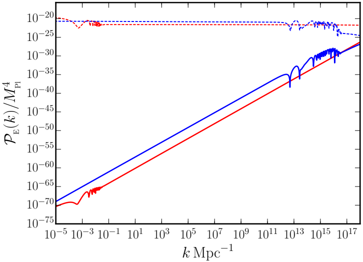


[17] R. Kallosh, A. Linde, and Y. Yamada, JHEP 01, 008 (2019); M. Braglia, D. K. Hazra, L. Sriramkumar, and F. Finelli, JCAP **08**, 025 (2020)

[16] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar, and A. A. Starobinsky, JCAP **08**, 001 (2020)

EM power spectra for two field models





The power spectra for magnetic (on the left) and electric (on the right) fields are presented.[17]

[17] **S. Tripathy**, D. Chowdhury, H.V. Ragavendra, R.K. Jain, L. Sriramkumar, arXiv:2211.05834

[18] A. Zucca, Y. Li, L. Pogosian, Phys. Rev. D **95**, 063506 (2017)

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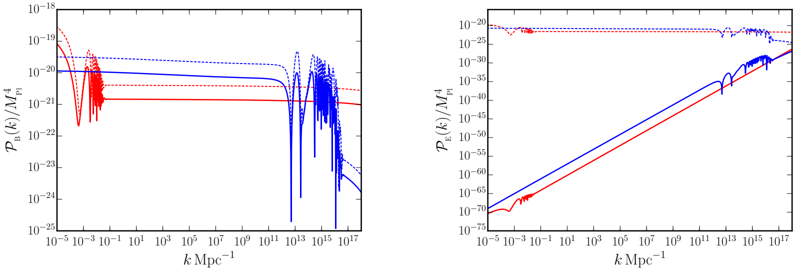


The power spectra for magnetic (on the left) and electric (on the right) fields are presented.[17]

The magnitude of the magnetic field is constrained by the quantity B_λ which is related to $\mathcal{P}_{\rm B}(k)$ as

$$B_\lambda^2 = \frac{1}{4\pi}\int {\rm d}^3\boldsymbol{k}\, {\rm e}^{-k^2\lambda^2}\, \frac{\mathcal{P}_{\rm B}(k)}{k^3}.$$

where $\lambda = 1$ Mpc is the coherence length.

For the two field models of interest the estimates of B_λ^0 turn out to be $\mathcal{O}(10^{-1})$ nG, well within the constraint of $B_\lambda^0 < 1.2$ nG.[18]

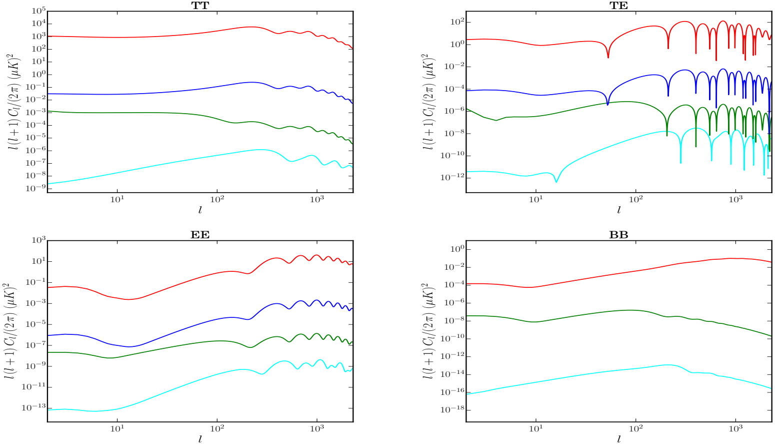
[17] **S. Tripathy**, D. Chowdhury, H.V. Ragavendra, R.K. Jain, L. Sriramkumar, arXiv:2211.05834
[18] A. Zucca, Y. Li, L. Pogosian, Phys. Rev. D **95**, 063506 (2017)

Imprints of PMF on CMB

- MagCAMB: A code to compute the contributions of PMF to the CMB angular spectra[19].
- It assumes as power law form for the magnetic power spectrum and arrives at the corresponding C_ℓs due to both compensated and passive modes.
- We estimated such an angular spectrum for the two field model generating features over small scales in SPS.

[19] J. R. Shaw and A. Lewis, Phys. Rev. D **81**, 043517 (2010); A. Lewis, A. Challinor, and A. Lasenby, Astrophys. J. **538**, 473 (2000); A. Zucca, Y. Li, L. Pogosian, Phys. Rev. D **95**, 063506 (2017)

The CMB spectra



We present the standard CMB spectra (in red) and the respective contributions from PMFs arising from the model that generates features over small scales due to compensated (in cyan), passive (in green) and inflationary magnetic modes (in blue).[20]

[20] **S. Tripathy**, D. Chowdhury, H. V. Ragavendra, R. K. Jain, L. Sriramkumar, arXiv:2211.05834

Conclusions

- Deviation from slow roll inflation
  - Features in the scalar power spectrum
  - Features in the electromagnetic spectra
- Challenges in single field models of inflation
  - Fine tuning of $J(\phi)$
  - Scale dependence of EM spectra over small scales
  - Supression of the amplitude of EM spectra over large scales
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  - Consistent with CMB observations

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Thank You