Low-mass Stars Modelling
constraints from Gaia

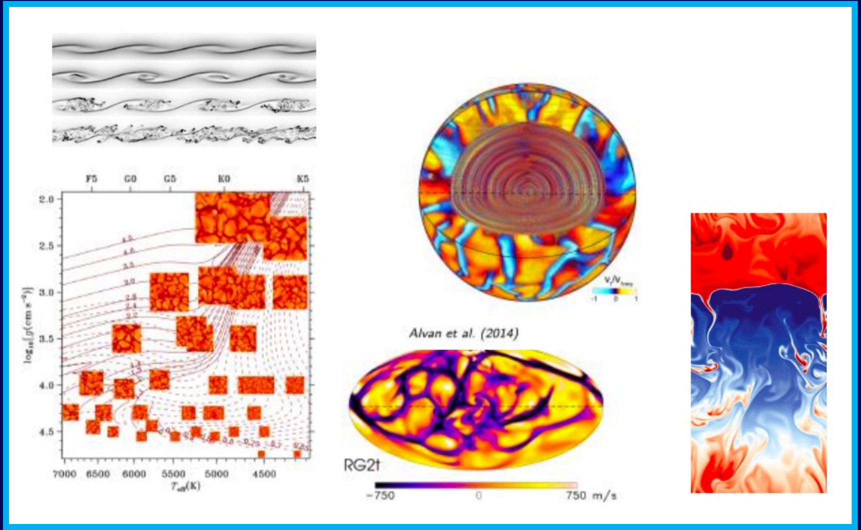
Santi Cassisi

INAF - Astronomical Observatory of Abruzzo, Italy

Golden age for stellar models

From a theoretical point of view…

- ✓ better knowledge of the input physics;
- ✓ 2-3D numerical simulation and laboratory experiments ⇒ prescriptions for multi-D MHD physical processes;



From an observational point of view…

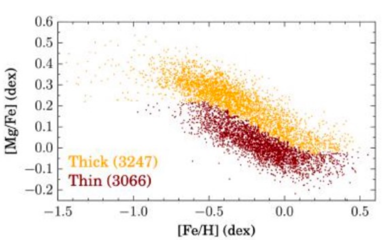
- ✓ Large scale surveys are revolutionazing Stellar Astrophysics;

Spectroscopy

Stellar parameters
Abundances
Rotation

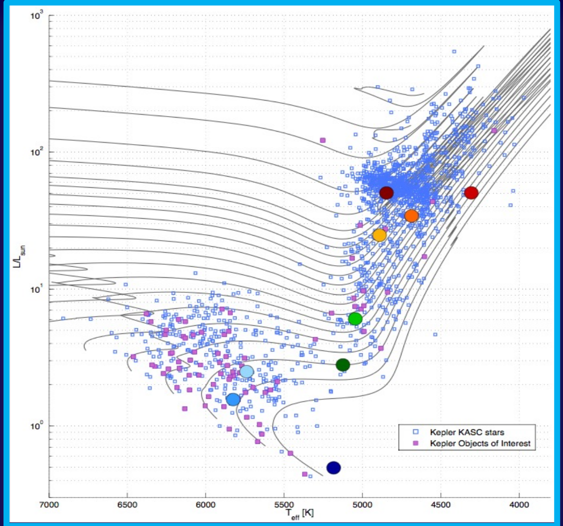
Spectropolarimetry

Magnetic field, rotation



Asteroseismology

- Internal properties
- Mass
- Evolutionary stage
- Age
- rotation



Chaplin & Miglio (2013)

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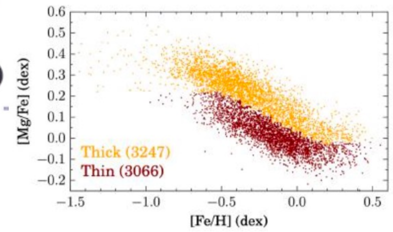

Spectropolarimetry

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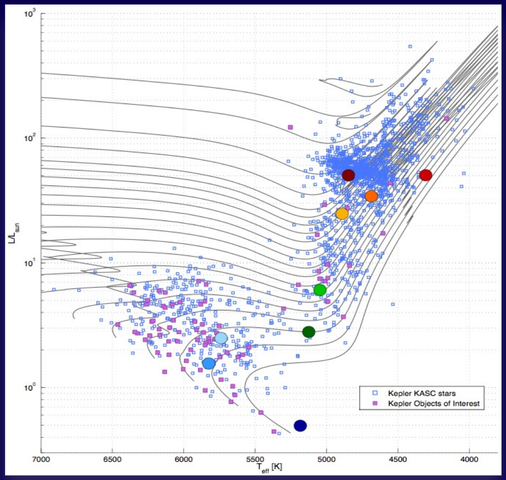
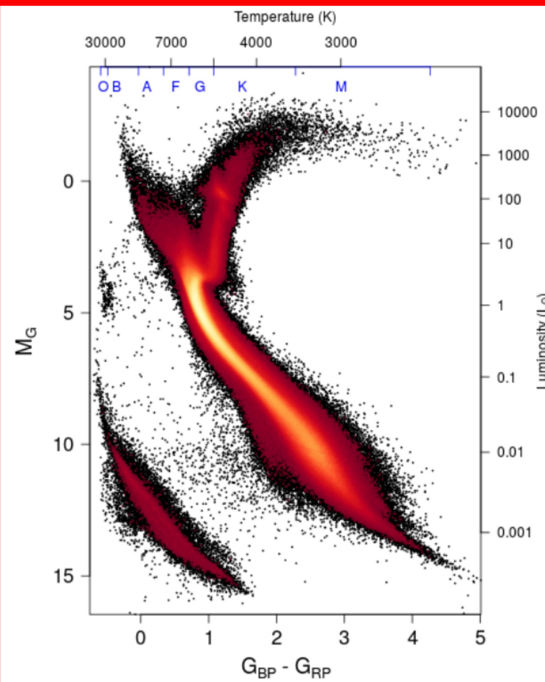


Chaplin & Miglio (2013)

Astrometry

- Distance
- Proper motions
- CMDs
- SED…



Babusiaux et al. (2018)

The “ingredients” for cooking a stellar model

$M<1M_\odot$ $M>1M_\odot$

Evolutionary code
- Numerics
- Boundary conditions

Physical inputs
- Equation of State
- Radiative opacity
- Conductive opacity
- Nuclear reaction rates

Macroscopic processes
- Overshooting
- Superadiabatic convection
- Semiconvection
- Breathing pulses
- Rotational mixing

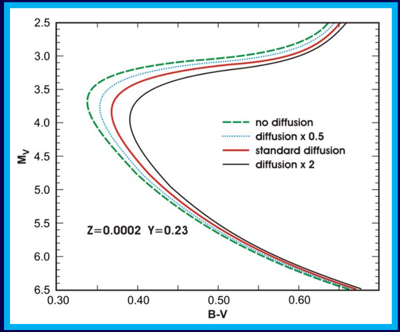
Microscopic mechanisms
- Diffusive processes

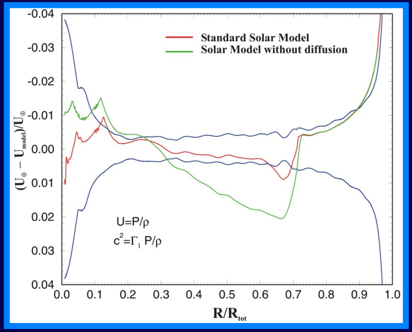
Additional mechanism
- Mass loss
- Magnetic field

Why do we need care of transport processes?

Any change in the chemical stratification affects:

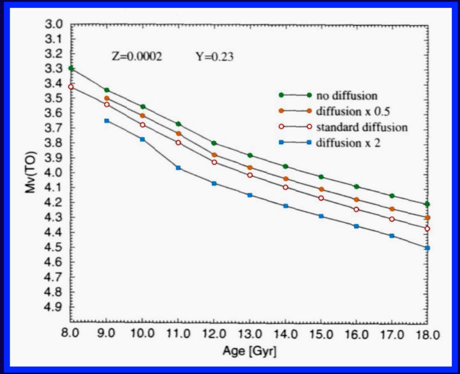
- **the thermal and opacitive properties of stellar matter**
  - ✓ Impact on structural and evolutionary properties;
  - ✓ Asteroseimic predictions;
  - ✓ Age dating of field stars;

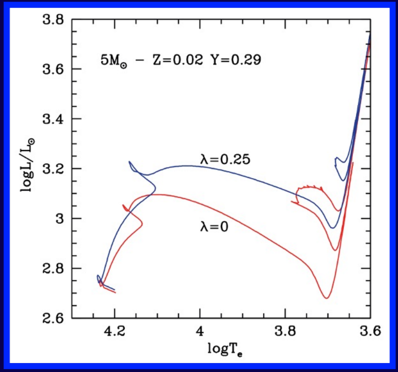




- **the amount of fuel available for burning**
  - ✓ Dramatic impact on evolutionary lifetimes;
  - ✓ Age dating of star clusters;

- **the properties of the stellar atmospheres**
  - ✓ Impact on the spectroscopic appearance;
  - ✓ Changes in Spectral Energy Distribution;





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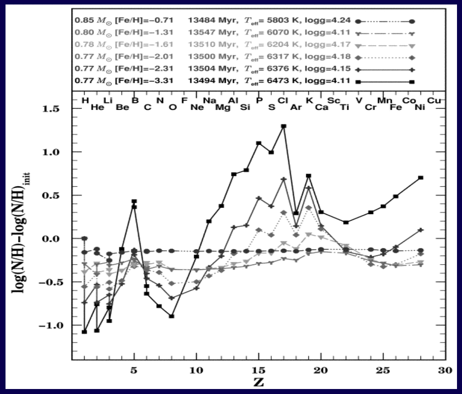
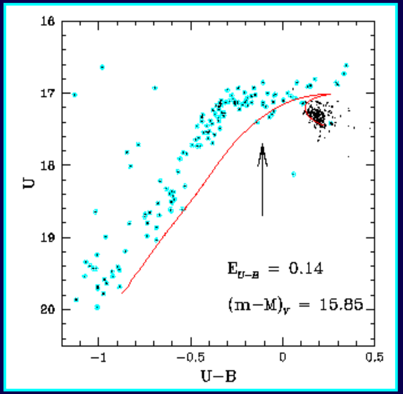
  - ✓ Age dating of star clusters;
- **the properties of the stellar atmospheres**
  - ✓ Impact on the spectroscopic appearance;
  - ✓ Changes in Spectral Energy Distribution;
- **in case of mixing, angular momentum transport**

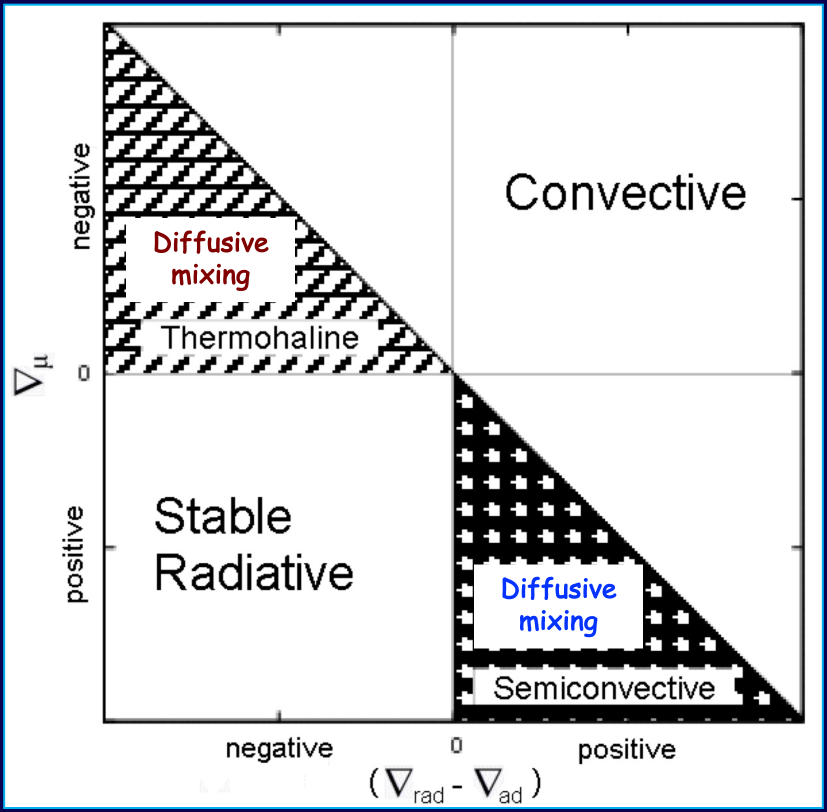




Why do we need to be worried about element transport?

- ✓ *Mixing and element transport do not arise from the solution of the stellar structure equations*
- ✓ *These processes have to be included in evolutionary computations following recipes that involve a number of free parameter/ad hoc assumptions and/or are affected by relevant uncertainties;*

The stability plane: local criteria…



$$\nabla_{\rm rad} > \nabla_{\rm ad} - \frac{\chi_\mu}{\chi_T}\nabla_\mu \equiv \nabla_{\rm L}$$

Theoretical issues:

- ✓ Convective boundaries
- ✓ timescales ⟶ { Instantaneous / diffusive

$$\left.\frac{\partial X_i}{\partial t}\right|_{M_r} = \frac{1}{\rho r^2}\frac{\partial}{\partial r}\left(D_{ov}\,\rho r^2 \frac{\partial X_i}{\partial r}\right)$$

These diffusion coefficients have to be estimated in someway…

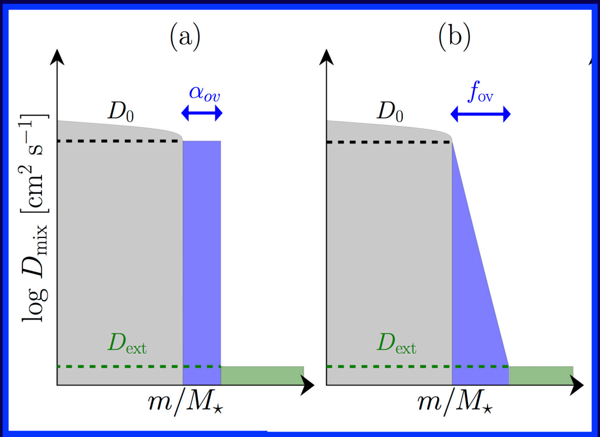
This is not the whole story…

- ✓ Microscopic processes: atomic diffusion & radiative levitation
- ✓ Rotationally induced mixings

Convective mixing

✓ **How extended is the mixed region beyond the formal convective boundary?**

Various implementations…



Pedersen et al. (2018)

a) Step overshooting: mixing assumed constant and instantaneous

convective boundary@distance

$$\alpha_{ov} = \lambda \times H_P$$

from the formal border

What gradient in the mixed region?

Mixing does not affect thermal structure: $\nabla=\nabla_{rad}$ (overshooting)

Mixing does affect thermal structure: $\nabla=\nabla_{ad}$ (penetrative conv.)

b) Exponential diffusive overshooting (Freytag+96): mixing efficiency assumed to decrease further away the convective core

$$\left.\frac{\partial X_i}{\partial t}\right|_{M_r} = \frac{1}{\rho r^2}\frac{\partial}{\partial r}\left(D_{ov}\rho r^2 \frac{\partial X_i}{\partial r}\right)$$

with

$$D_{ov} = D_c \exp\left(-\frac{2z}{f H_p}\right)$$

$$D_c = (1/3)\alpha_{MLT} v_c H_P$$

✓ **How do we reduce to zero the extension of the mixing region when convective core masses approach zero?**

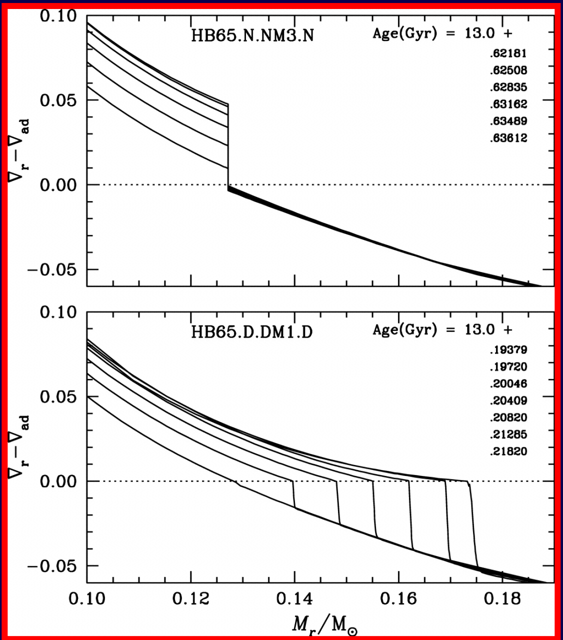
SEMICONVECTIVE MIXING: the central He-burning stage

At the beginning of the core He-burning stage: $\log T \approx 8.1$ $\log\rho \approx 4.1$

The opacity is dominated by electron scattering – not dependent on the chemical composition -, with some contribution from *free-free* transitions: $\kappa = \kappa_{es} + \kappa_{ff}$ and $\kappa_{ff}/\kappa \sim 0.25$

$$\kappa_{ff} \propto \sum \frac{X_i Z_i^2}{A_i} \rho T^{-7/2}$$

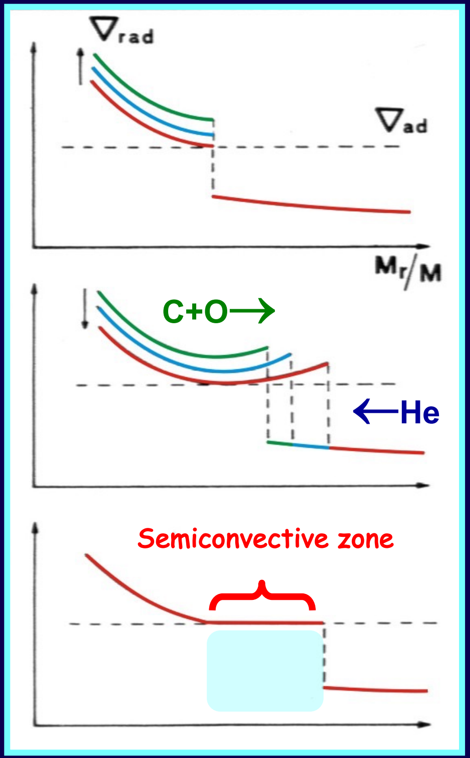
He↓ C↑ O↑ κ_{ff} increases in the convective core



Michaud et al. (2007)

What mechanism to extend the convective boundary?

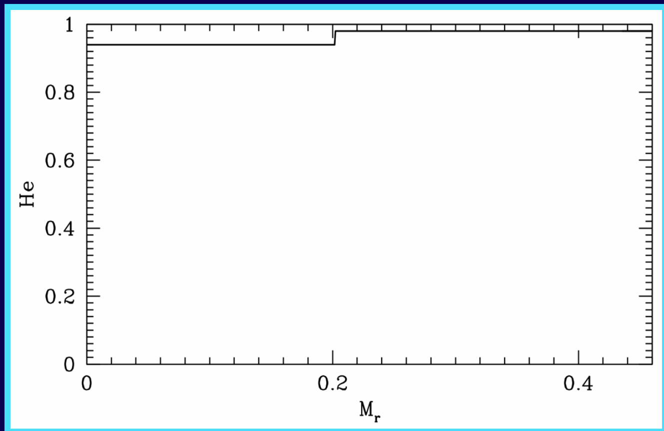
- Overshooting – self-driving process;
- Atomic diffusion;
- Shear instability;



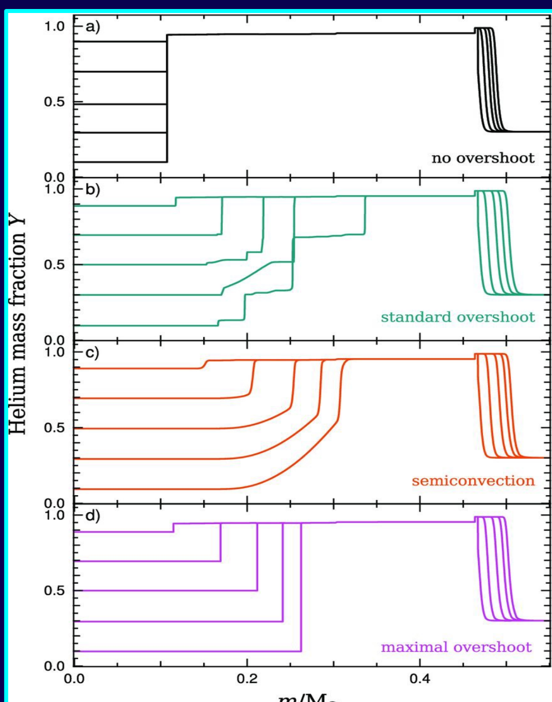
How to manage this mixing process in core He-burning stellar models?

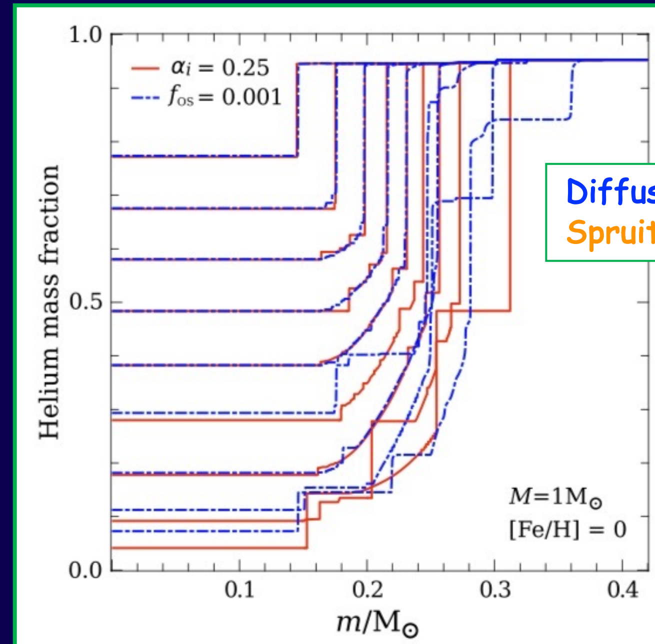
Various mixing schemes…

Semiconvection



Other schemes





Diffusive mixing
Spruit overshoot

Maximum rate of He ingestion

$$\dot{m}_i = \alpha_i \frac{12}{5} \frac{L}{RT} \left(1 - \frac{\nabla_{ad}}{\nabla_{rad}}\right)$$

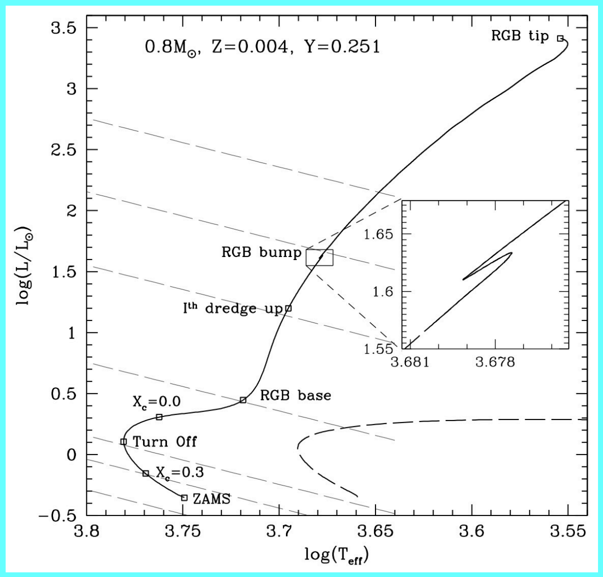
Spruit (2015)

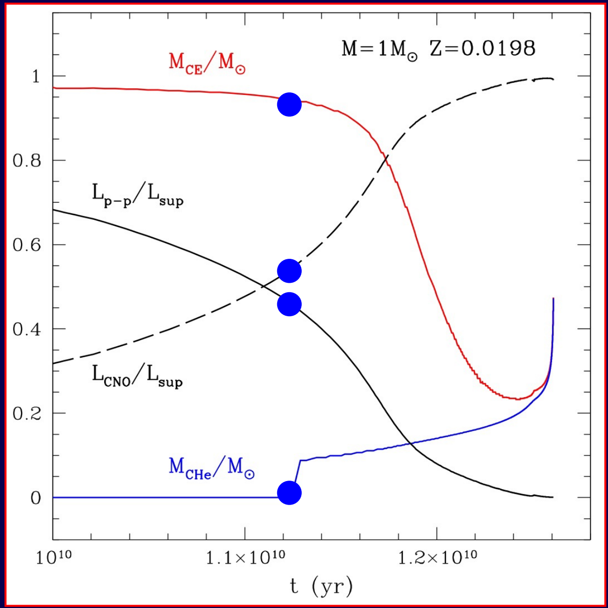
All these formalisms rely on *ad hoc* assumptions and/or free parameters

Sizeable effects on:

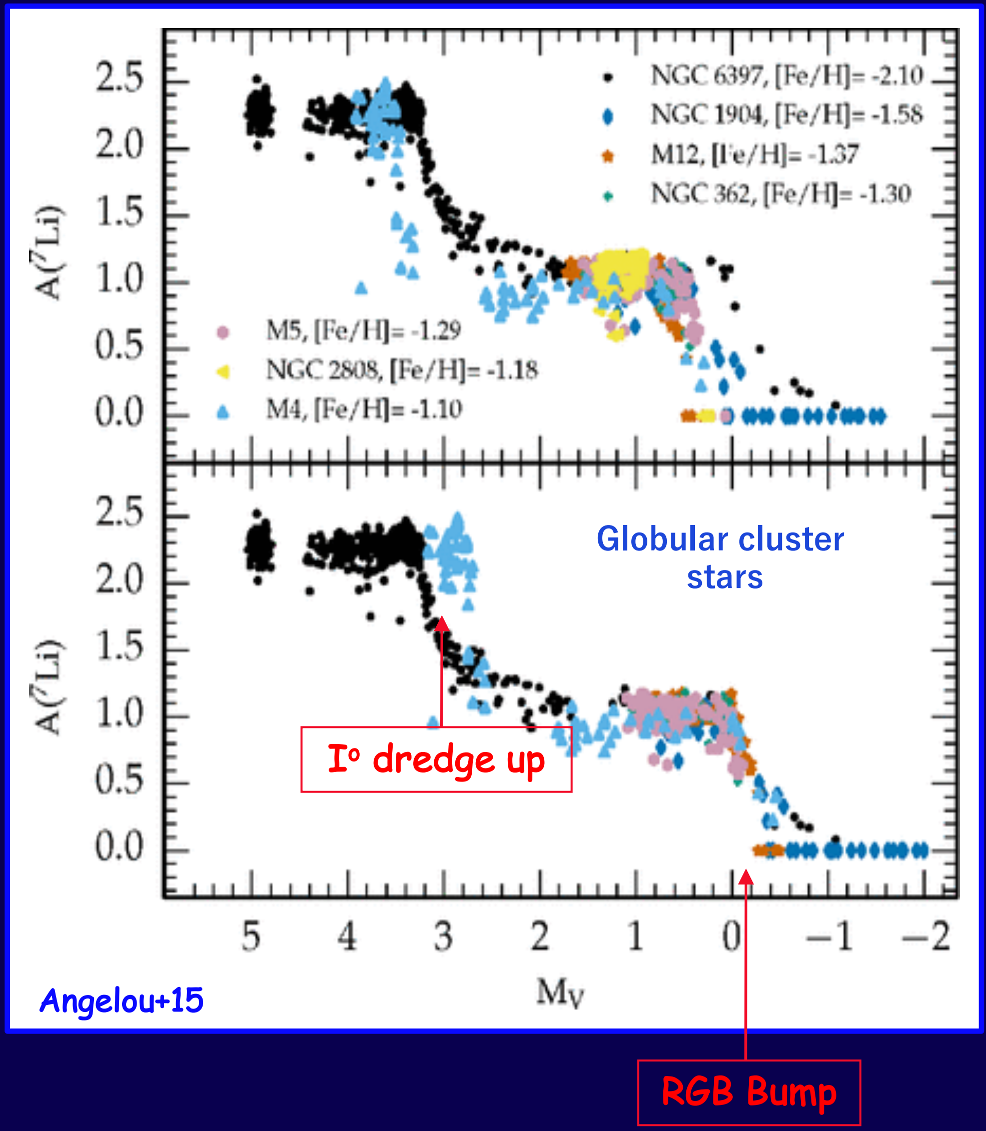
Lifetimes, Blue loop extension, AGB clump luminosity, C/O profile…

Thermohaline mixing





Why do we need it?

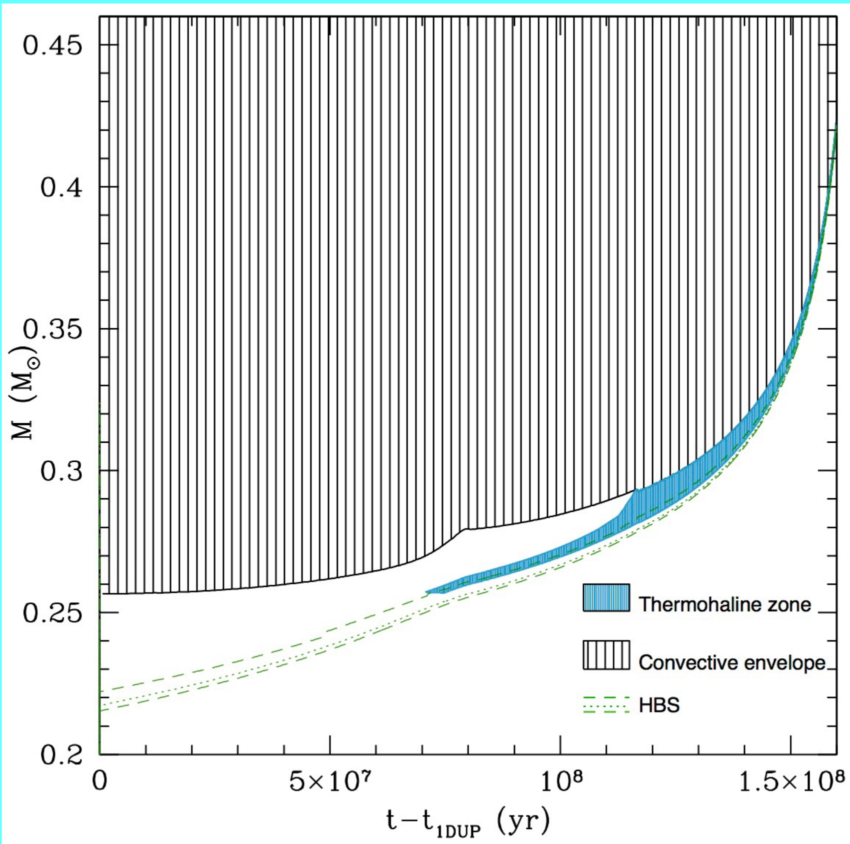


$$D_t = C_t K \left(\frac{\varphi}{\delta}\right) \frac{-\nabla_\mu}{(\nabla_{ad} - \nabla)} \quad \text{for } \nabla_\mu < 0,$$

with K the thermal diffusivity.

$$C_t = \frac{8}{3}\pi^2\alpha^2,$$

and with $\alpha = 5$ (Ulrich 1972) this coefficient is $C_t = 658.$

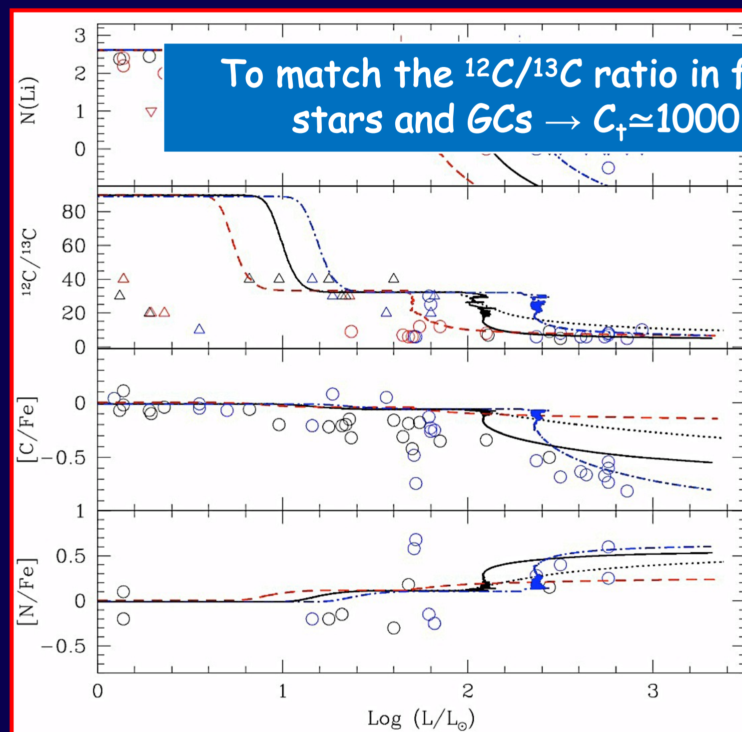


Charbonnel & Lagarde (2010)

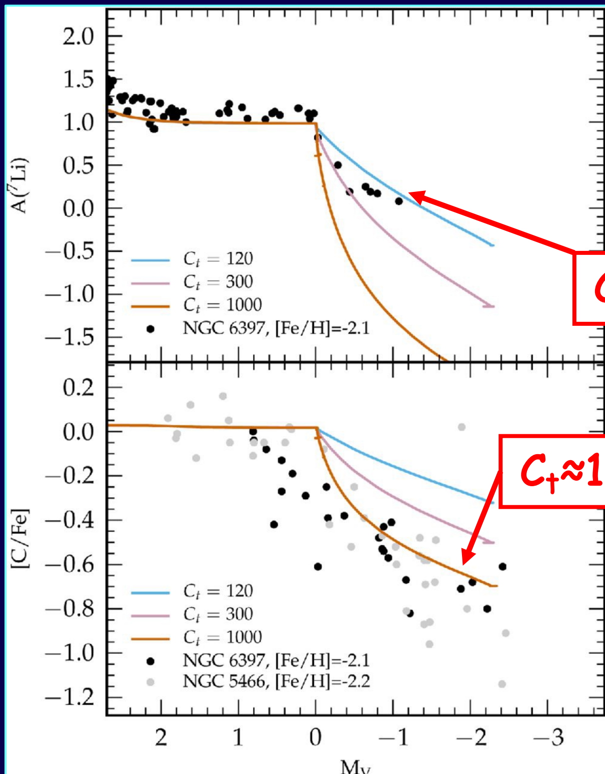
This convective instability is:

- a turbulent process;
- it occurs very fast

To match the $^{12}C/^{13}C$ ratio in field stars and GCs → $C_t \simeq 1000$



Theoretical issues: What value for C_t?



Angelou+15

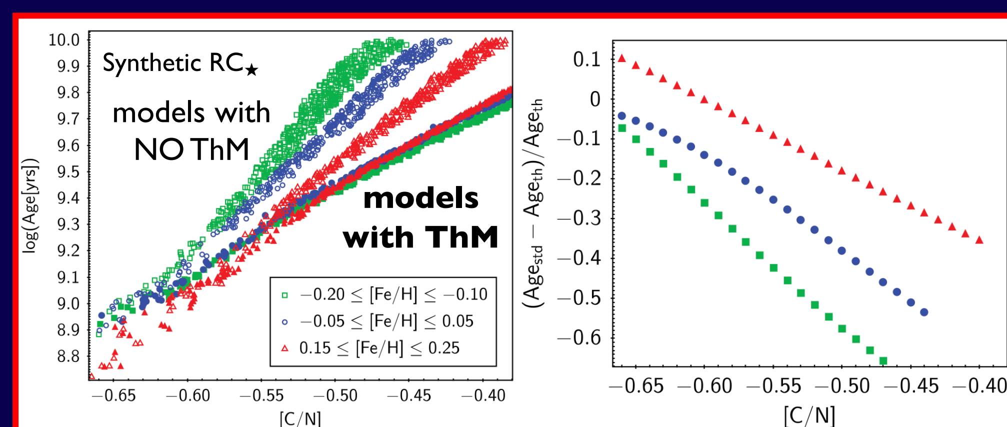
Problem: not consistent values of C_t from distinct spectroscopic constraints…

Additional problem…: C_t~200-1000 needed to reproduce observations, but C_t~10 (or lower…) is obtained from hydro-simulations (Denissenkov+10,11; Traxler+11, Wachlin+11, Brown+13)

Additional hydro-simulations are mandatory…

Crucial constraints provided by synergy between spectroscopic, photometric and seismic surveys!
(see the talk by Lagarde)

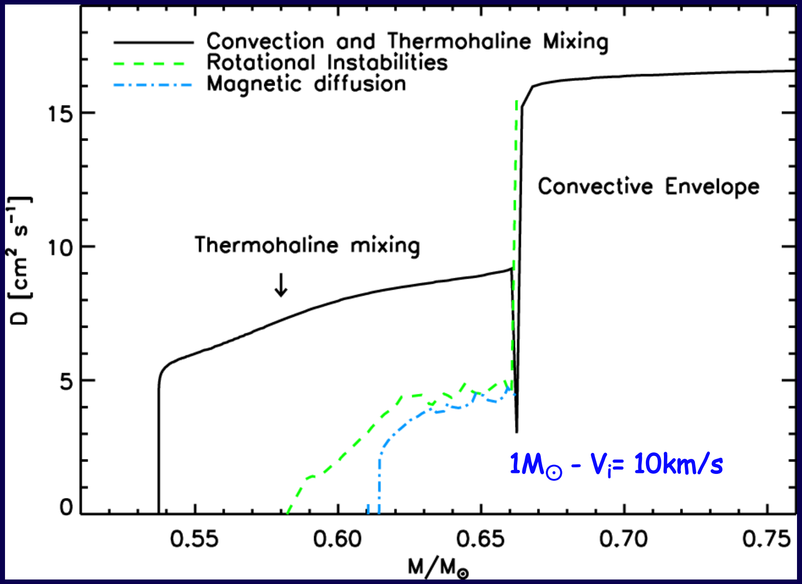
Pay attention in using age indicators based on the chemical tagging of RGB stars



Lagarde+18

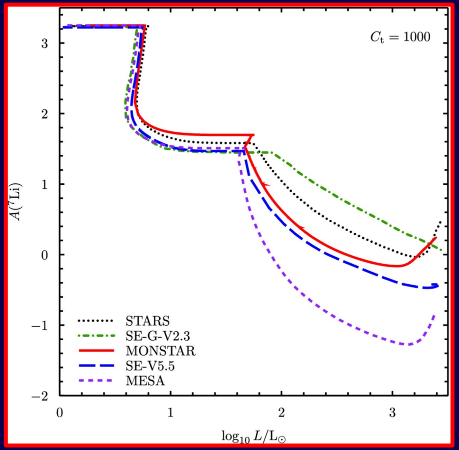
Some issues about thermohaline mixing

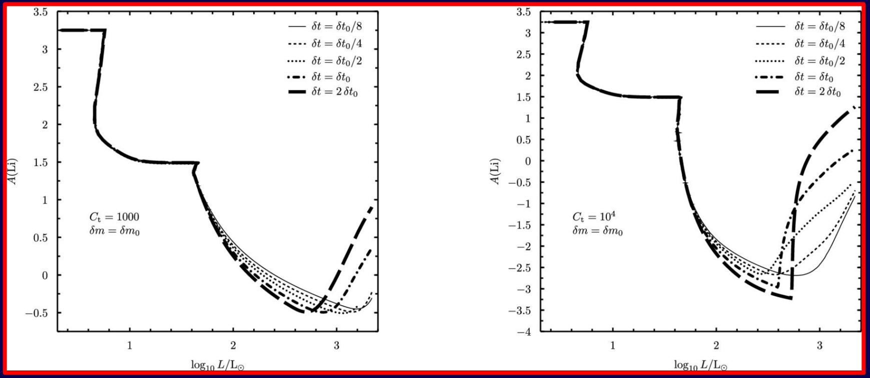
Rotational instabilities and magnetic mixing have lower diffusion coefficients than thermohaline mixing



- ✓ What is the interaction among rotation, magnetic fields and thermohaline?
- ✓ Any impact on the thermohaline mixing efficiency…?

Evolutionary predictions strongly model-dependent and affected by numerical accuracy:





Diffusive processes

Microscopic effects due to collisions among gas particles induce slow element transport within radiative regions

Burgers formalism

$$\frac{dX_i}{dt} = \frac{d}{dm}(4\pi r^2 \rho X_i w_i)$$

$$w_i(r) = D_P(i)\frac{\partial \ln(P)}{\partial r} + D_T(i)\frac{\partial \ln(T)}{\partial r} + D_c(i)\frac{\partial \ln(c_i)}{\partial r}$$

✓ Atomic diffusion

- Pressure gradient (gravitational settling)
- Temperature gradient
- Chemical gradients

Theoretical issues:

- Non classical interactions between 2-point-charge particles
- The limitations of the Burgers formalism…

Atomic diffusion efficiency is probably predicted with an accuracy of about 10-20%

2. Classical treatment of element diffusion

The diffusion of elements in a multicomponent fluid can be treated either according to the Chapman & Cowling (1970) or the B69 formalism. We are following the latter description, which is equivalent to the so-called "second approximation" in Chapman & Cowling (1970). It would be desirable to use a higher order approximation, because an uncertainty of the order of 10% is introduced by using only the second one (Roussel-Dupré 1982), independent of the accuracy in the resistance coefficients. But this can only be done in the scheme of Chapman & Cowling (1970), which becomes very cumbersome for gases with more than two components.

✓ Radiative levitation

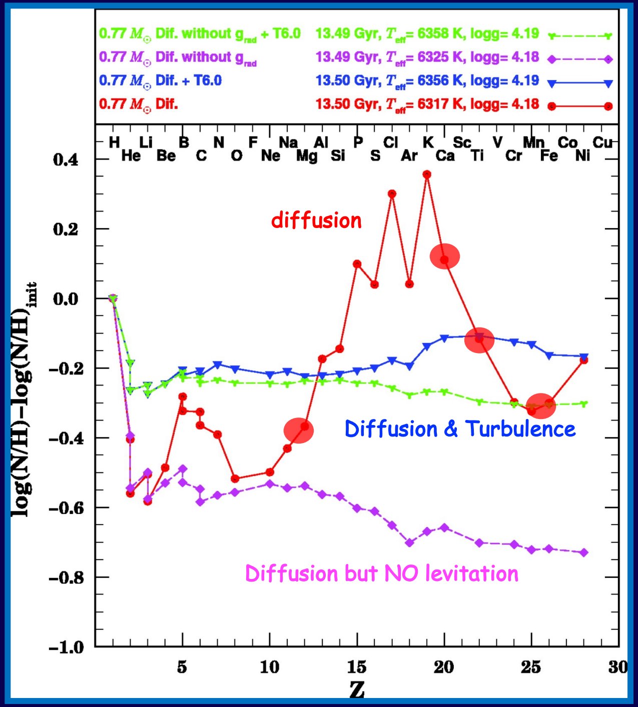
- (strong) radiation field
- Not fully ionized elements (so effective only in the outer layers)

$$w_i = D_c\left(-\frac{dc_i}{dr} + \frac{m_i(g^i_{rad} - g)}{K_B T}\right)$$

Theoretical issues:

- The evaluation of accurate g_{rad} implies a very detailed prediction of the ionization levels for all chemical species…
- An extremely computing-demanding task in stellar model computations…

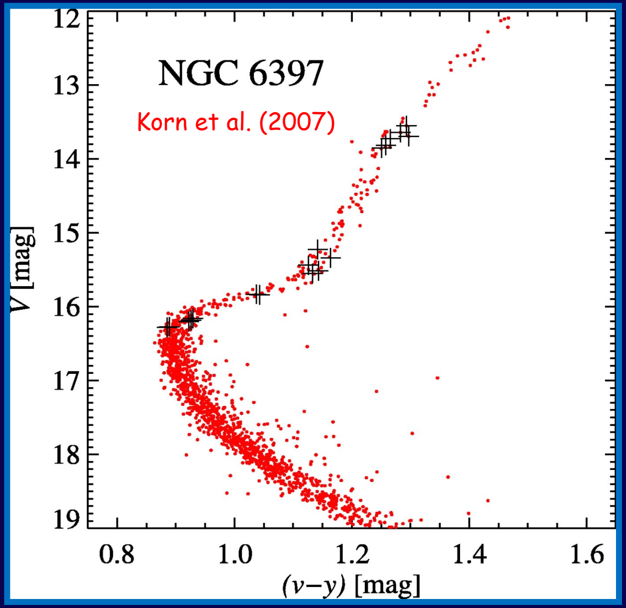
Diffusive processes: a thorny problem…

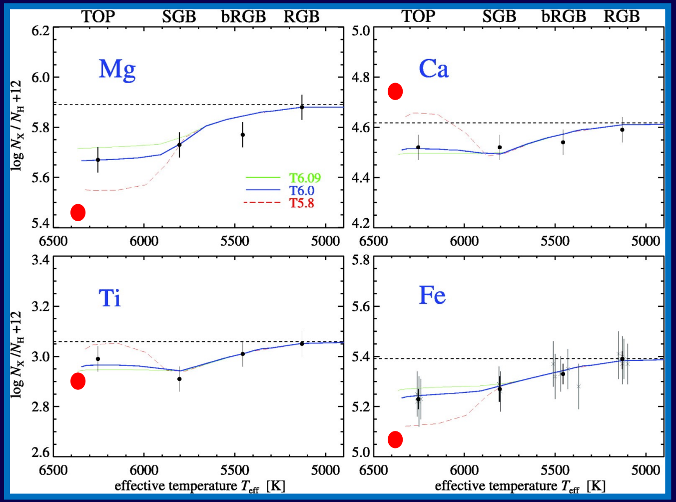


Effect of diffusion on surface abundances increases with age and decreasing thickness of convective envelopes

Along an isochrone the effect of diffusion on the surface abundances is **maximized around the Main Sequence Turn-off**, while the surface initial abundances **are restored during the RGB phase**

But…



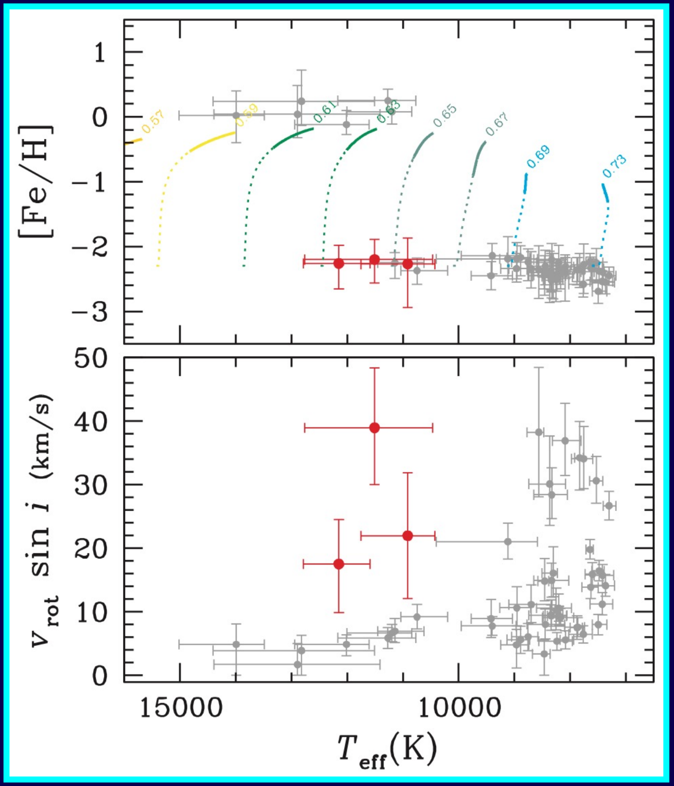


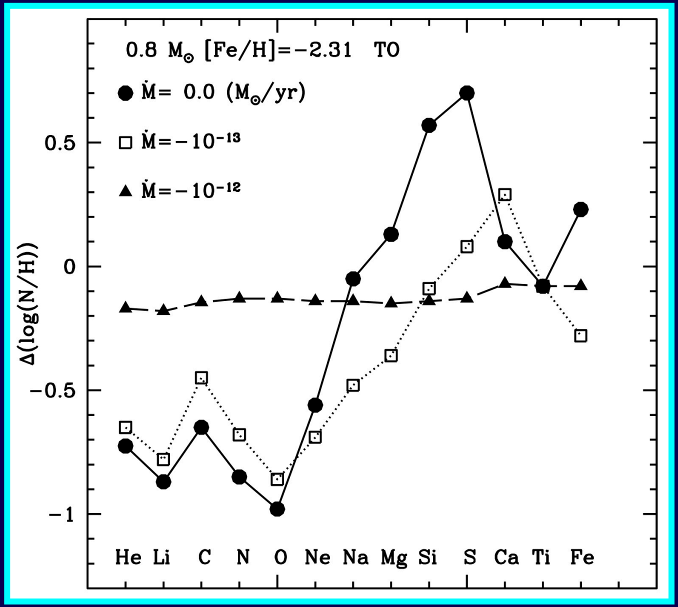
Canonical stellar models predict a too huge impact of diffusive processes on surface chemical abundances

Inhibition of atomic diffusion efficiency

- Rotational mixings
  - ✓ Why do HB stars rotate?
  - ✓ Why do they rotate with this Teff trend?
  - ✓ Also limiting levitation efficiency too large overabundances for cool HB stars…
- Mass loss
  - ✓ A poorly understood physical process
  - ✓ The requested mass loss rates are about 2 orders of magnitude larger than current solar mass loss rate…

HB stars in Galactic GCs





Inhibition of atomic diffusion efficiency

- ## Rotational mixings
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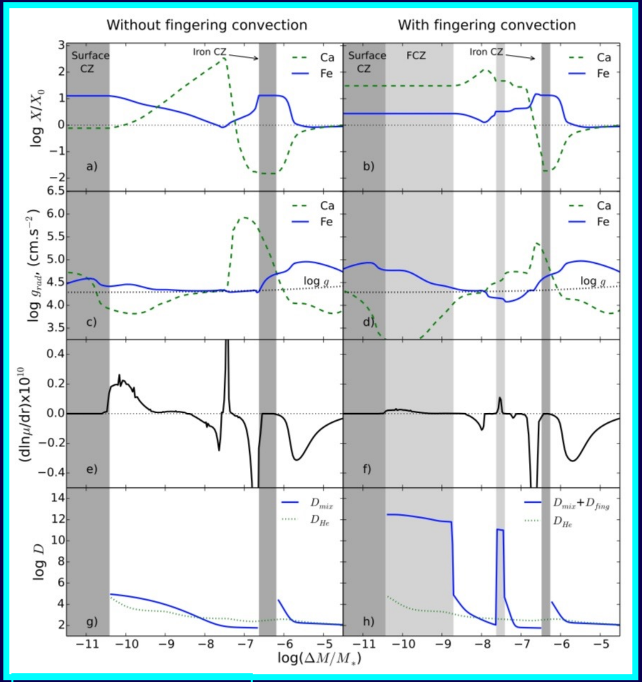
  - ✓ Also limiting levitation efficiency too large overabundances for cool HB stars…

- ## Mass loss
  - ✓ A poorly understood physical process
  - ✓ The requested mass loss rates are about 2 orders of magnitude larger than current solar mass loss rate…

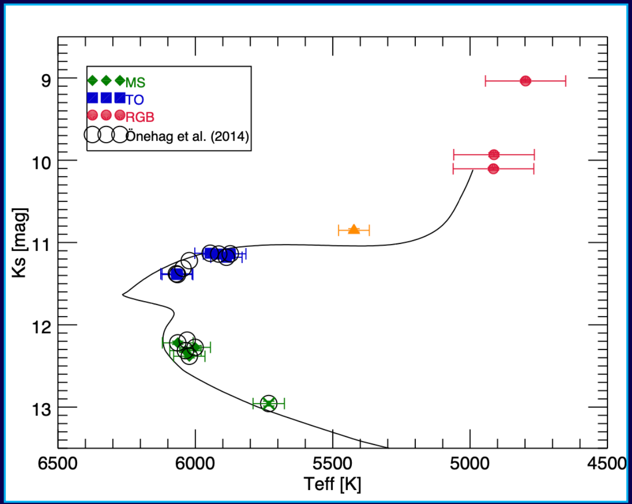
- ## Thermohaline mixing (due to the impact of radiative levitation)
  - ✓ Only tested for A-F type stars
  - ✓ The spectroscopic measurements are not reproduced…
  - ✓ The fit improves only (arbitrarily) connecting the two FC zones…
  - ✓ The impact of rotational mixing and mass loss can not be ruled out…

- ## Generic turbulence
  - ✓ an additional *ad-hoc* term to the chemical evolution equations, added to reduce/suppress the development of chemical abundance gradients ⇨ $D_{turbulence}$
  - ✓ No physics behind it
  - ✓ Different choices of $D_{turbulence}$ calibrated on empirical constraints

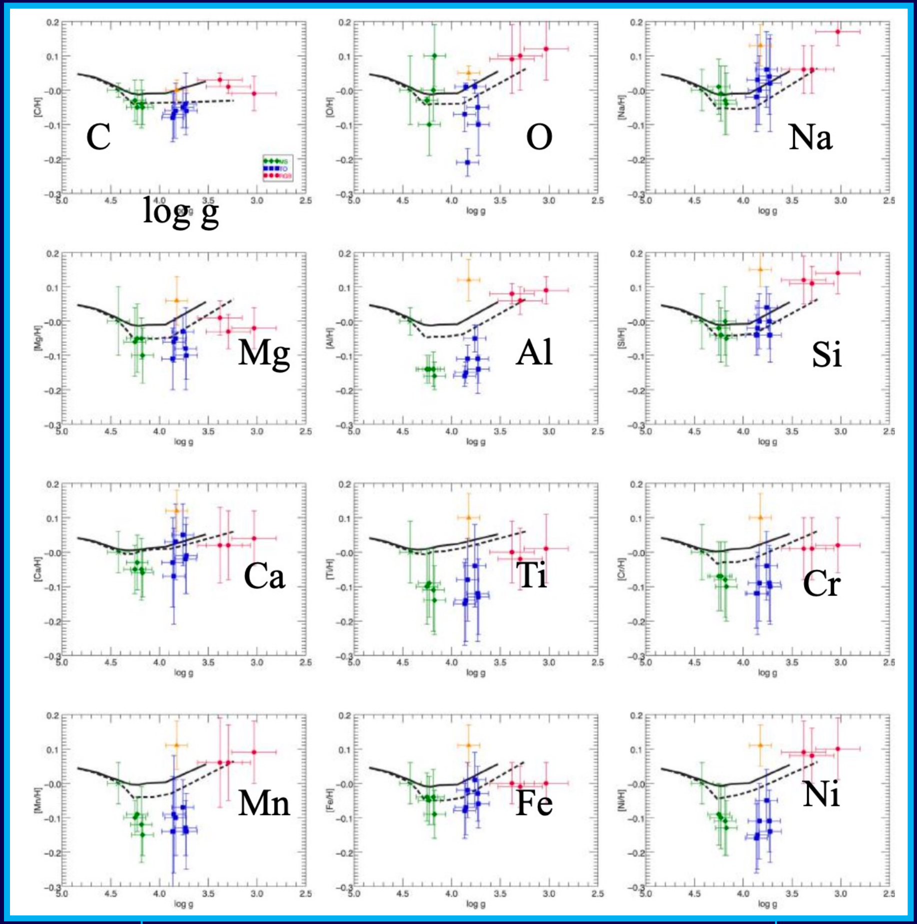


Dean et al. (2016)

Constraints from the Gaia-ESO survey: the case of M67



Bertelli Motta+18



Theoretical predictions by Michaud+04
- dashed: pure diffusive models
- solid: diffusive + turbulence models

- The abundance effects do depend on the evolutionary stage;
- Smaller empirical errors are necessary to better constrain the theoretical framework…

Rotation & rotational mixings

Rotation has not generally been an ingredient of standard stellar models due to:

- ✓ **Increase in complexity and uncertainty;**
- ✓ **(in the past) empirical constraints on internal rotation have been scarce;**

Evolution of angular momentum (in a radiative zone):

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\Omega\right) = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4 \Omega u_r\right] + \frac{1}{r^4}\frac{\partial}{\partial r}\left[\rho \nu_v r^4 \frac{\partial \Omega}{\partial r}\right]$$

circulation

turbulence

Evolution of chemical species:

$$\rho \frac{\mathrm{d}c}{\mathrm{d}t} = \rho c_{\mathrm{nuc}} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \rho V_{ip} c\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho r^2 \left(D_{\mathrm{eff}} + D_v\right)\frac{\partial c}{\partial r}\right]$$

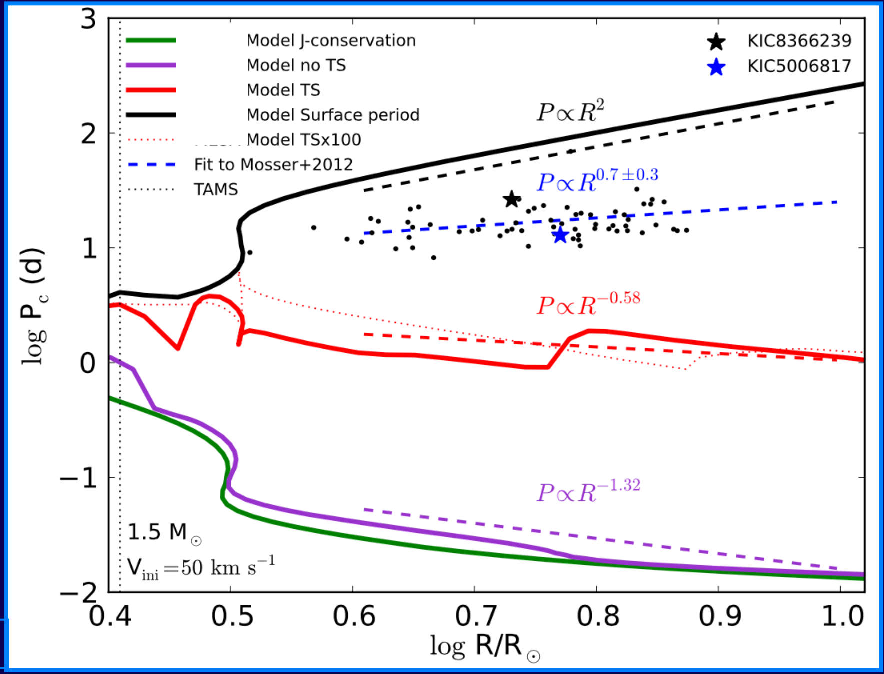
nuclear burning

atomic diffusion

In a convective region...:

- **the interaction between convection and meridional currents is not well-understood;**
- **what is the efficiency of angular momentum transport? Solid body rotation or specific angular momentum conservation?**

The state-of-the-art of theoretical modelling



Cantiello+14

Asteroseismology plays a crucial role

Some evidences:

- the envelope slows down as expected due to expansion and AM conservation;
- the core rotates about 10 – 10^3 times faster than empirical values;
- the trend of the core rotational rate is reversed with respect the observed ones;

Theoretical issue:

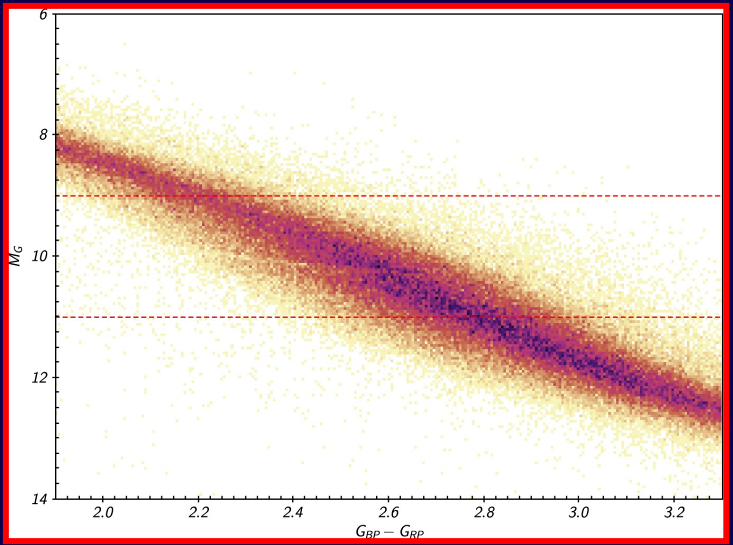
the amount of torque between the core and envelope is underestimated by current RGB stellar models (see, e.g. Fellay+2021)

A query for additional mechanisms:

- ✓ Internal Gravity Waves
- ✓ Large scale magnetic field
- ✓ Mass loss

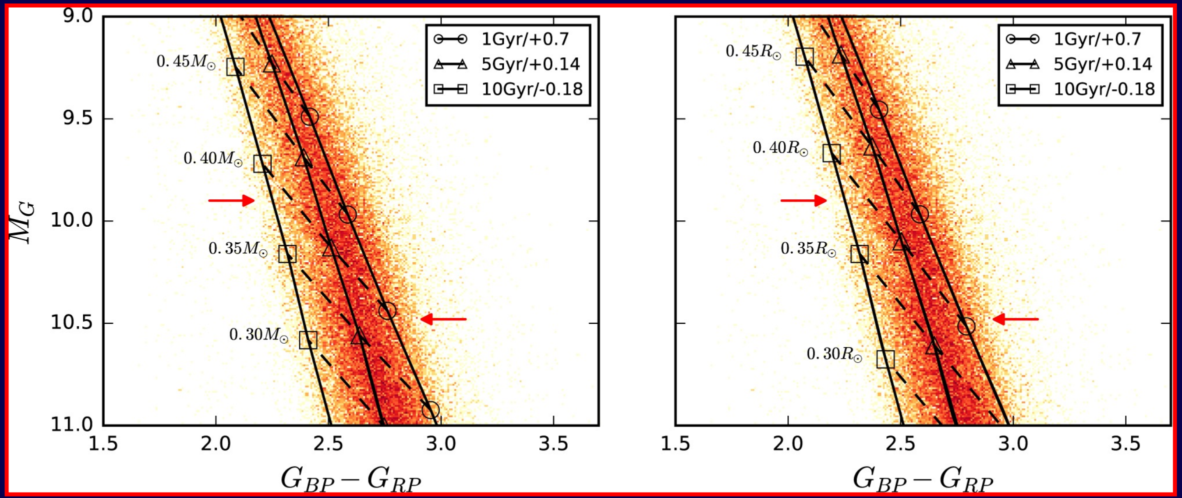
Gaia data as a direct proof/test of Stellar Evolution Theory

The Gaia gap



Jao+2018

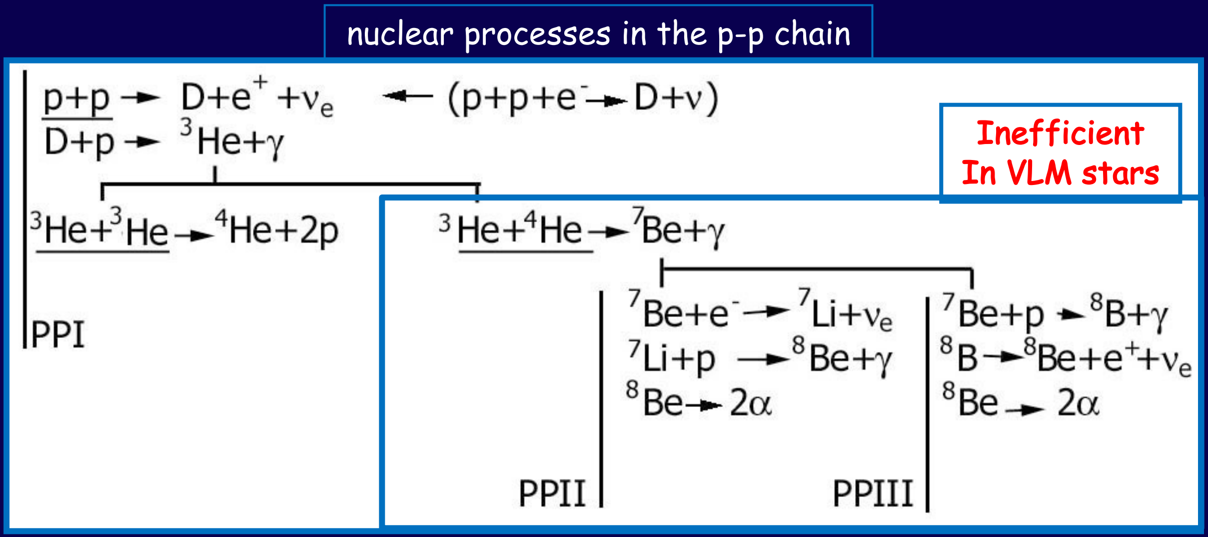
- Gaia data revealed the existence of a gap near M_G~10 mag and G_{BP}-G_{RP} ~ 2.3-2.5 (Spectr. Type M3.0);
- It is present both in optical and near-IR photometry, so it is not due to an atmospheric feature that could depend on the wavelength;
- The gap is very narrow: ~ 0.05 mag



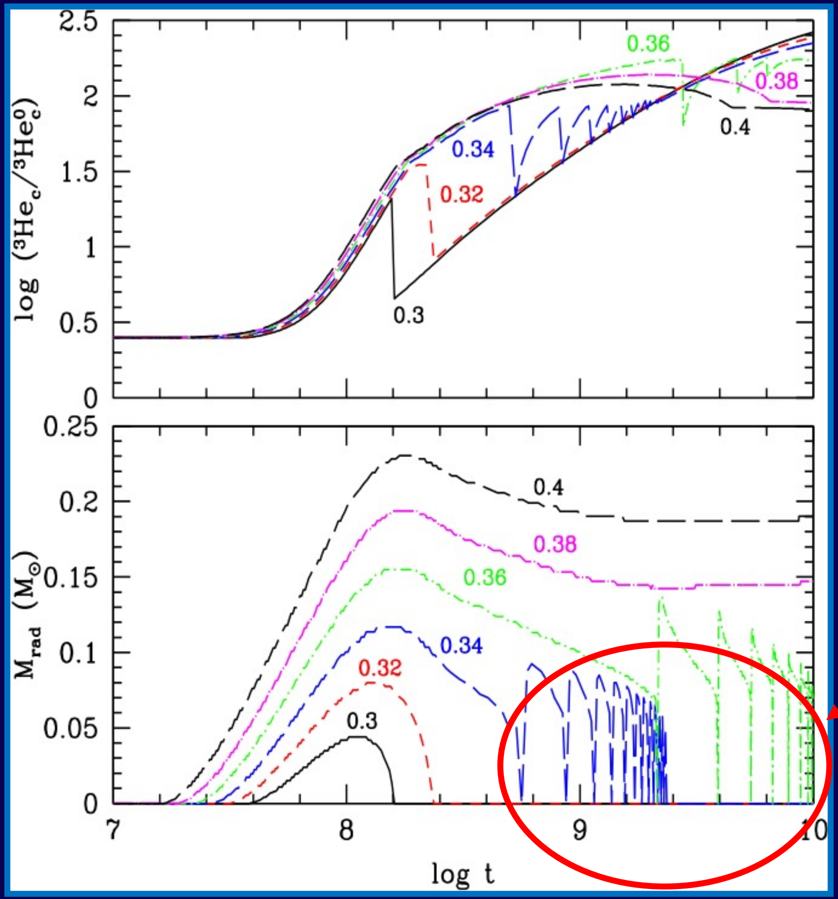
The location of Gap is near to the region where M dwarf stars transition from partially to fully convective;

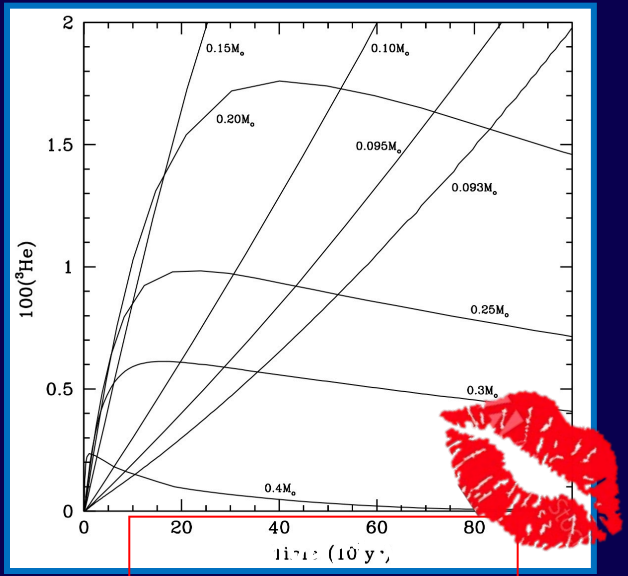
The feature seems to have a shallower slope than the either the equal-mass line or equal-radius one;

The H-burning phase in M dwarf stars



The H-burning phase in M dwarf stars



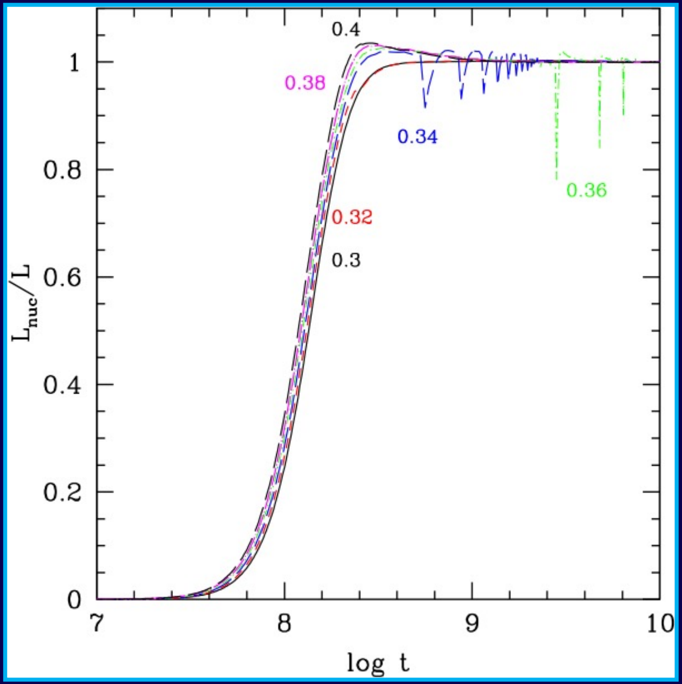


kissing instability

Saders & Pinsonneault (2012)
Baraffe & Chabrier (2018)
MacDonald & Gizis (2018)

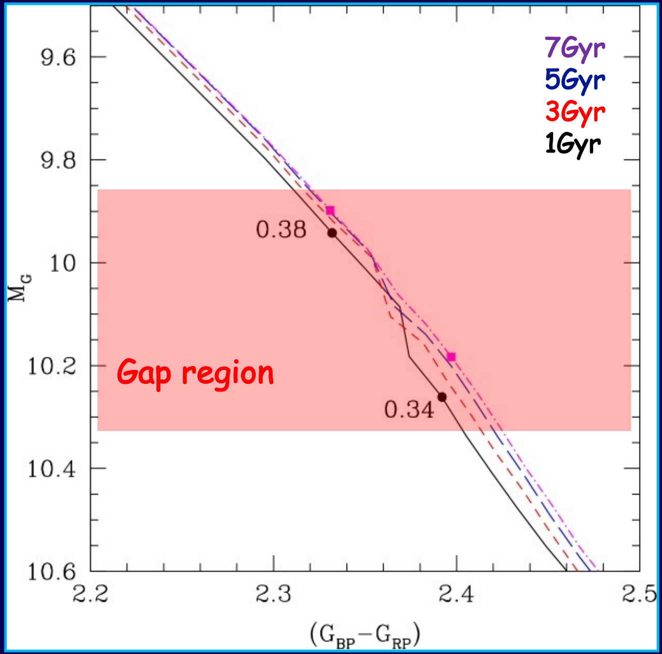
- $M<0.34M_\odot$ after a short episode during which they show a radiative core become fully convective
- $M>0.36M_\odot$ once they develop a radiative core, they stop to be fully convective
- $0.34<M/M_\odot<0.36$ show a complex behaviour with burst events due to episodic merging of the convective core and envelope

The impact on the nuclear burning efficiency



The merging between the convective core and the envelope causes:

- a decrease of the central abundance of 3He;
- hence a decrease of the nuclear burning efficiency (note the dips in the fig.)
- hence an overall contraction that affects the evolution of the stellar radius;



This transition region could have interesting implications:

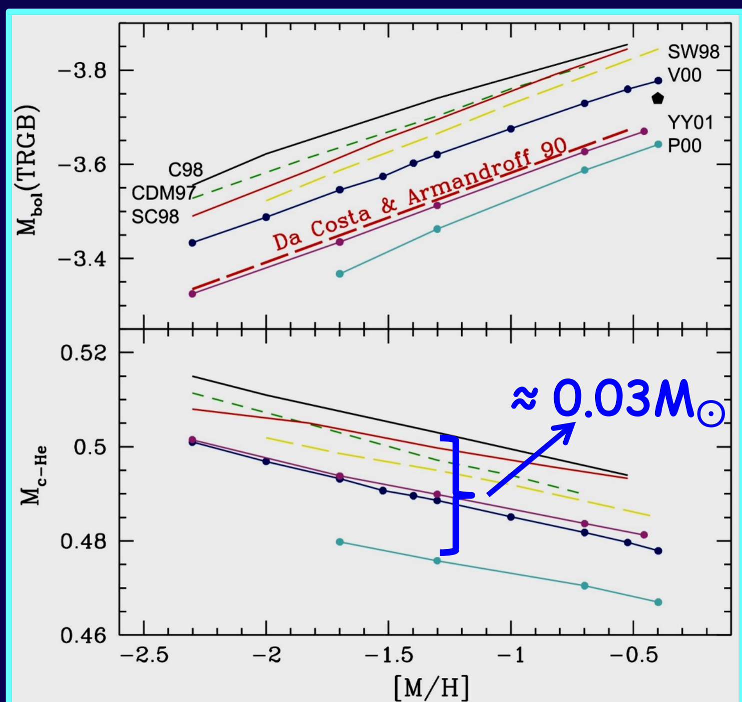
- ✓ The spectral type (M3.0) of stars at the Gaia gap, is very close to the spectral type (M2.0) where a signature of a change in the chromospheric activity has been detected (Mullan+20);
- ✓ Is this coincidence suggesting something about the Physics of Magnetic Fields in M dwarf stars?
- ✓ Could more than one dynamo ($\alpha\Omega$, α^2, $\alpha^2\Omega$) be at work in the same time?

Gaia as a direct proof/test of Stellar Physics

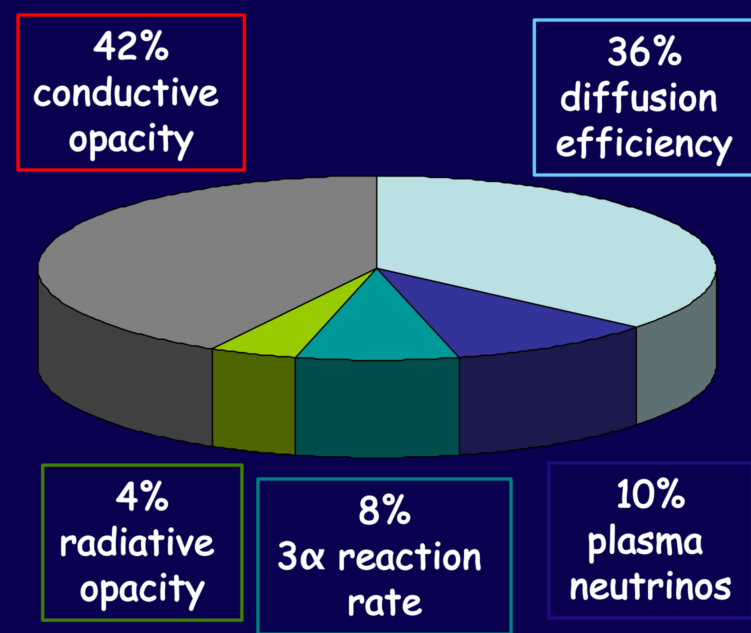
The luminosity level of RR Lyrae

$$\left.\frac{\partial \log L_{3.85}}{\partial M_{cHe}}\right|_{Y,Z} \approx 3.04$$

$\Delta^{MAX} M_{cHe} \approx 0.01M \Rightarrow \Delta M_{V,ZAHB} \sim 0.1$ mag



Who is really governing the uncertainty in the M_{cHe} predictions?



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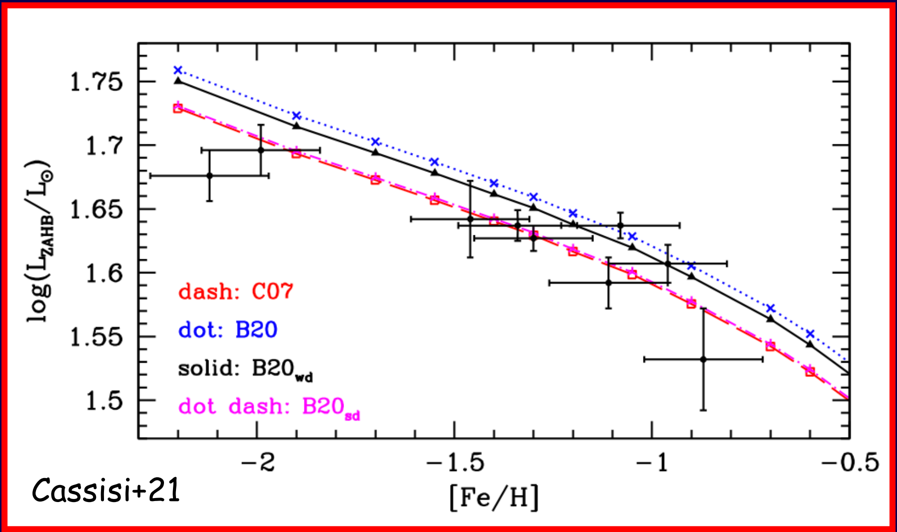
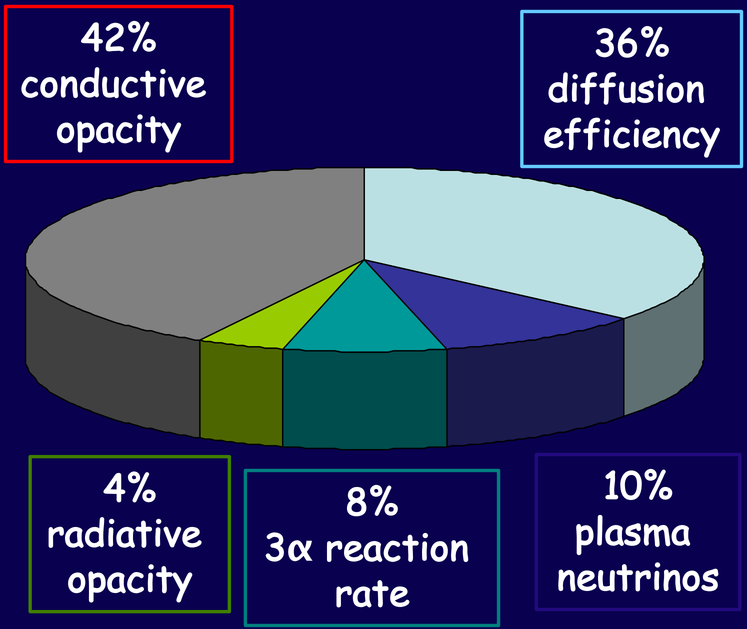
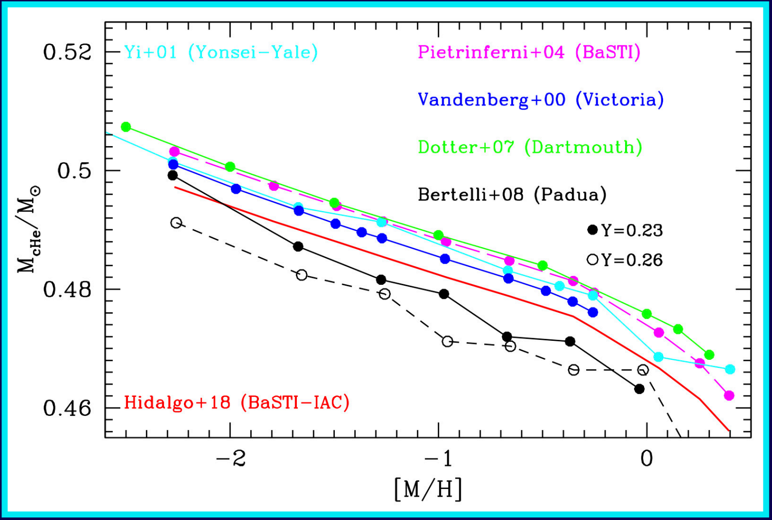
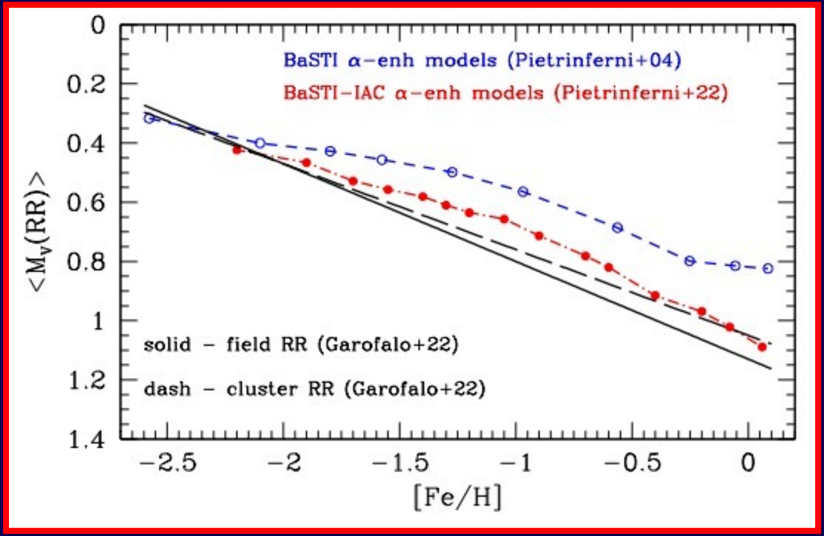


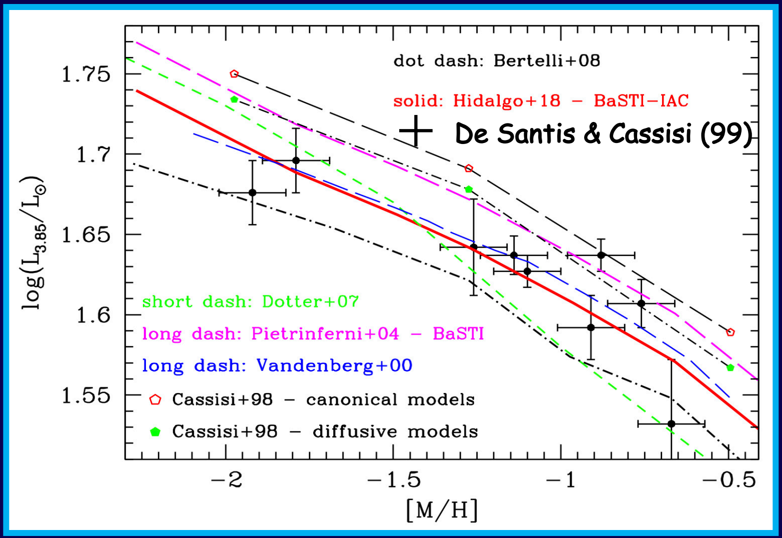


In this field the contribution of Gaia (Muraveva+18, Garofalo+22) can be crucial…

see Kovacs' talk







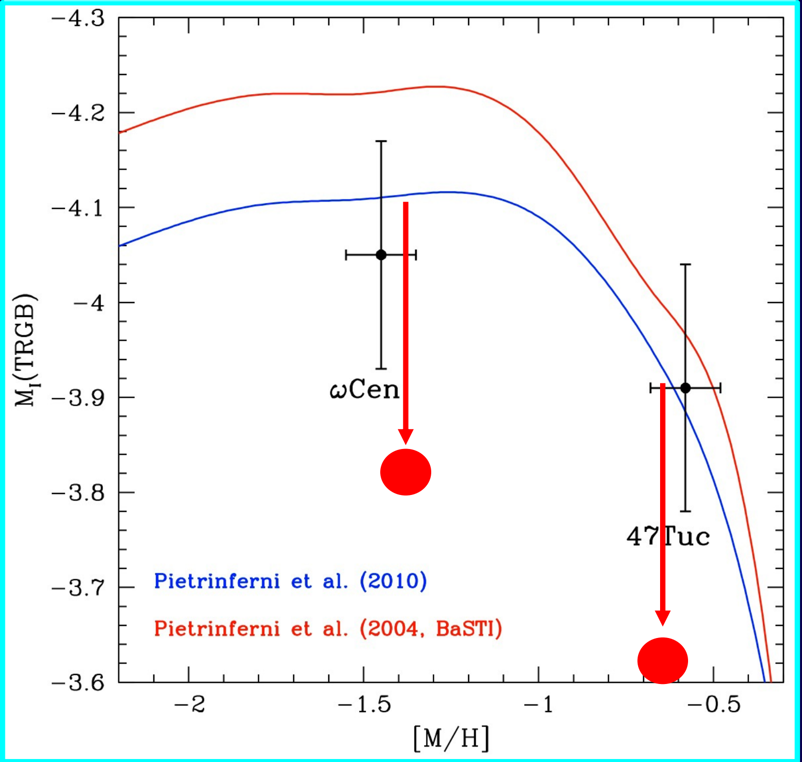
Some issues:

- the metallicity scale…
- evolutionary effects…
- linear or not linear?

What constraints on stellar modelling/physics we get from this comparison?

The brightness of the RGB Tip

GC data by Bellazzini+04
Gaia eDR3 distances by Baumgardt & Vasiliev (2021)



Updated RGB models are now in agreement with empirical data at the level of better than 0.5σ

$\Delta M_{V,RR}$~0.2 mag

$$\left.\frac{\partial \log L_{3.85}}{\partial M_{cHe}}\right|_{Y,Z} \approx 3.04$$

$\Delta^{MAX} M_{cHe} \approx -0.026 M_{\odot}$

$$\left.\frac{\partial \log L_{TRGB}}{\partial M_{cHe}}\right|_{Y,Z} \approx 4.7$$

$\Rightarrow \Delta M_{I,TRGB}$~0.3 mag

Take home messages

- ✓ 3D hydrodynamics simulations (rescaled to match stellar conditions) predict an efficiency of thermohaline mixing orders of magnitude (Denissenkov+11, Traxler+11) lower than required by stellar evolution models to match observations (Lattanzio+15);
- ✓ 2D hydro-simulations of chemical mixing induced by Internal Gravity Wave (Rogers & McElwaine 2017) are providing a diffusion coefficient for this mixing;
- ✓ Lots of recent activity from the point of view of 2D and 3D hydro-simulations to determine the mixing in overshooting regions (e.g. Prat+17)…
- ✓ Asteroseismology is providing strong constraints on element transports in stars. .. (Montalban's talk!)

It is (maybe) not too optimistic to think that information from hydro-simulations, spectroscopic and photometric surveys and asteroseismology could – in the (very) near future - lead to reliable and parameter-free descriptions of element transport in stellar evolution calculations