

HI IM: Power spectrum modelling and foreground cleaning

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Outline

1. Modelling of HI IM power spectrum
2. Instrumental and systematic effects
3. Cosmological parameter estimation

Simplest *isotropic* model

The **power spectrum** tells us how clustered the matter is a function of inverse separation (k)

$$P^{\text{HI}}(k) = \bar{T}_{\text{HI}}^2 b_{\text{HI}}^2 P_m(k)$$

The diagram illustrates the components of the HI power spectrum equation. The equation is $P^{\text{HI}}(k) = \bar{T}_{\text{HI}}^2 b_{\text{HI}}^2 P_m(k)$. Green arrows point from each term to its corresponding label: \bar{T}_{HI}^2 points to "HI background temperature", b_{HI}^2 points to "HI bias", and $P_m(k)$ points to "Underlying dark matter power spectrum". A separate arrow points from the entire equation to "HI power spectrum".

HI background temperature

HI power spectrum

$\propto \Omega_{\text{HI}}(z)$

HI bias

Underlying dark matter power spectrum

Anisotropic model - RSD

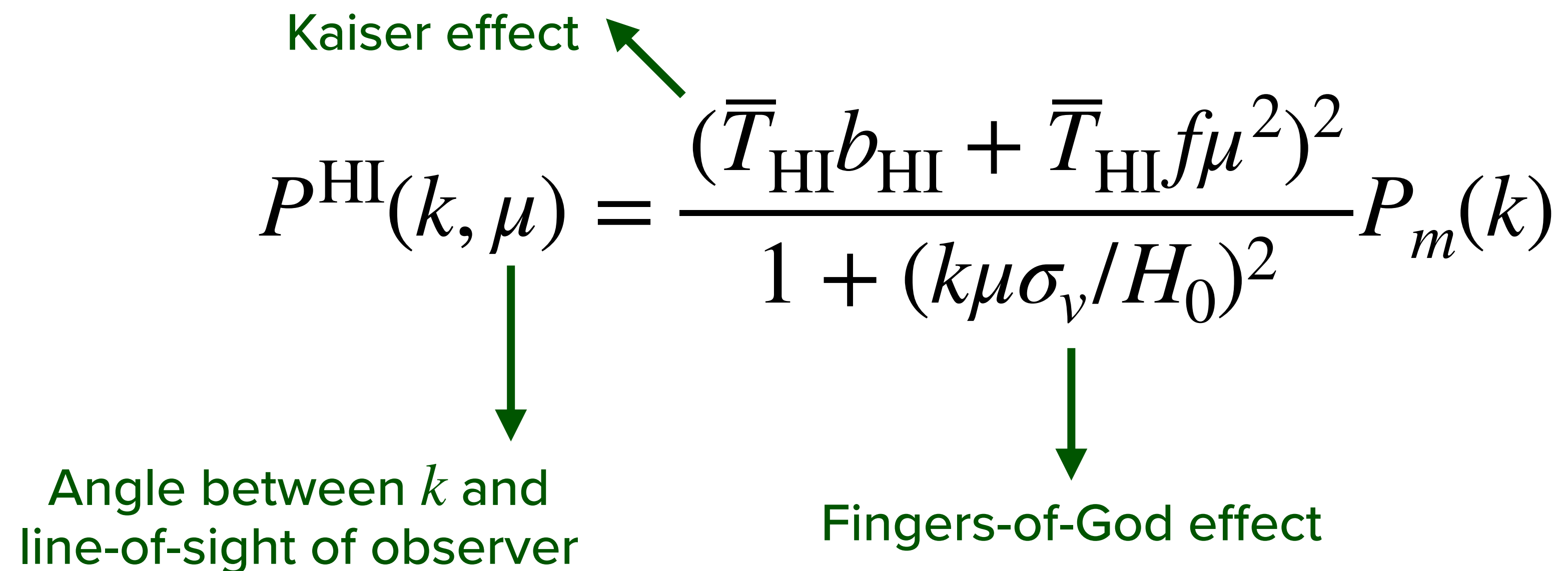
The **power spectrum** tells us how clustered the matter is as a function of inverse separation (k) **and angle between k and observer's line-of-sight (μ)**

$$P^{\text{HI}}(k, \mu) = \frac{(\bar{T}_{\text{HI}} b_{\text{HI}} + \bar{T}_{\text{HI}} f \mu^2)^2}{1 + (k \mu \sigma_v / H_0)^2} P_m(k)$$

Kaiser effect

Angle between k and line-of-sight of observer

Fingers-of-God effect




Anisotropic model - σ_8

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$$P^{\text{HI}}(k, \mu) = \frac{(\bar{T}_{\text{HI}} b_{\text{HI}} \sigma_8 + \bar{T}_{\text{HI}} f \sigma_8 \mu^2)^2 P_m(k)}{1 + (k\mu\sigma_v/H_0)^2} \frac{P_m(k)}{\sigma_8^2}$$

Multiply by: $\frac{\sigma_8^2}{\sigma_8^2}$


RMS of matter fluctuations within a sphere of radius 8 Mpc/h

Alcock-Paczynski effect

An effect introduced when we convert from redshift to distance

- We need to assume a *fiducial cosmology* to do this
- If this is different from the *true cosmology*, our power spectrum measurements will be distorted by the factors:

$$\alpha_{\perp} = \frac{D_A(z)}{D_A(z)^f}$$

$$\alpha_{\parallel} = \frac{H(z)^f}{H(z)}$$

Anisotropic model - AP

The **power spectrum** tells us how clustered the matter is as a function of inverse separation (k) **and angle between k and observer's line-of-sight (μ)**

Alcock-Paczynski effect

$$P^{\text{HI}}(k^f, \mu^f) = \alpha_{\parallel}^{-1} \alpha_{\perp}^{-2} \left[\frac{(\bar{T}_{\text{HI}} b_{\text{HI}} \sigma_8 + \bar{T}_{\text{HI}} f \sigma_8 \mu^2)^2 P_m(k)}{1 + (k \mu \sigma_v / H_0)^2} \frac{P_m(k)}{\sigma_8^2} + P_{\text{SN}} \right]$$

Shot noise

Anisotropic model

The **power spectrum** tells us how clustered the matter is a function of inverse separation (k) **and angle between k and observer's line-of-sight (μ)**

$$P^{\text{HI}}(k^f, \mu^f) = \alpha_{\parallel}^{-1} \alpha_{\perp}^{-2} \left[\frac{(\bar{T}_{\text{HI}} b_{\text{HI}} \sigma_8 + \bar{T}_{\text{HI}} f \sigma_8 \mu^2)^2}{1 + (k\mu\sigma_v/H_0)^2} \frac{P_m(k)}{\sigma_8^2} + P_{\text{SN}} \right]$$

Diagram illustrating the components of the HI power spectrum equation:

- α_{\parallel}^{-1} : Hubble constant
- α_{\perp}^{-2} : Angular diameter distance
- $\bar{T}_{\text{HI}} b_{\text{HI}} \sigma_8$: HI bias
- $\bar{T}_{\text{HI}} f \sigma_8 \mu^2$: Growth rate of structure

Anisotropic model

The **power spectrum** tells us how clustered the matter is a function of inverse separation (k) **and angle between k and observer's line-of-sight (μ)**

$$P^{\text{HI}}(k^f, \mu^f) = \alpha_{\parallel}^{-1} \alpha_{\perp}^{-2} \left[\frac{(\bar{T}_{\text{HI}} b_{\text{HI}} \sigma_8 + \bar{T}_{\text{HI}} f \sigma_8 \mu^2)^2}{1 + (k \mu \sigma_v / H_0)^2} \frac{P_m(k)}{\sigma_8^2} + P_{\text{SN}} \right]$$

$$\vec{\theta} = \{ \alpha_{\parallel}, \alpha_{\perp}, \bar{T}_{\text{HI}} f \sigma_8, \bar{T}_{\text{HI}} b_{\text{HI}} \sigma_8, \sigma_v, P_{\text{SN}} \}$$

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Telescope beam

- Because the beam damps small scales, we can model it as a Gaussian (very idealised!):

$$\tilde{B}_{\perp}(k, \mu) = \exp\left(\frac{-k^2 R^2 (1 - \mu^2)}{2}\right)$$

Beam size (determines resolution)

Anisotropic model - beam

The **power spectrum** tells us how clustered the matter is a function of inverse separation (k) **and angle between k and observer's line-of-sight (μ)**

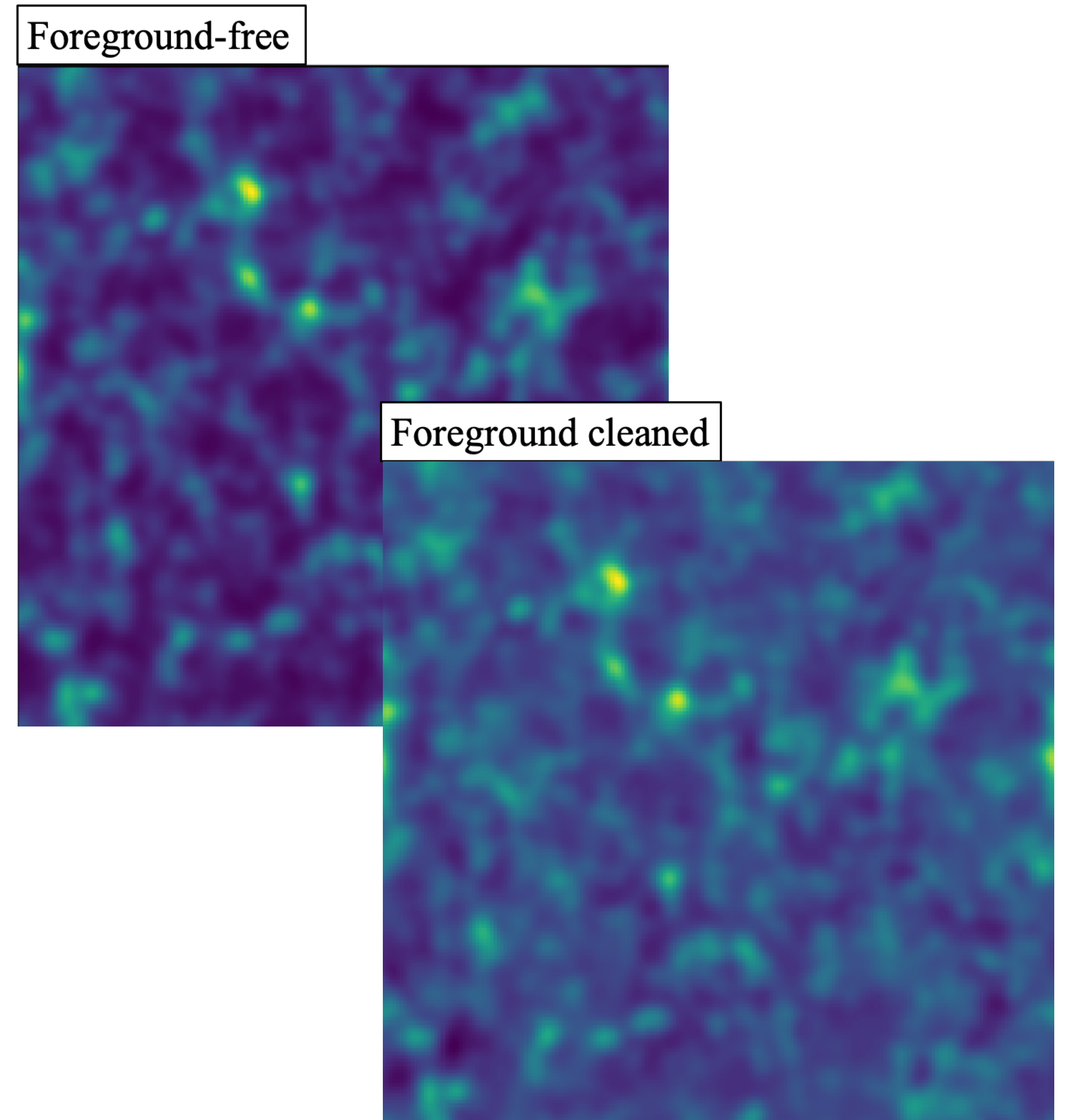
Beam effect



$$P^{\text{HI}}(k^f, \mu^f) = \tilde{B}_{\perp}^2(k, \mu) \alpha_{\parallel}^{-1} \alpha_{\perp}^{-2} \left[\frac{(\bar{T}_{\text{HI}} b_{\text{HI}} \sigma_8 + \bar{T}_{\text{HI}} f \sigma_8 \mu^2)^2}{1 + (k \mu \sigma_v / H_0)^2} \frac{P_m(k)}{\sigma_8^2} + P_{\text{SN}} \right]$$

Foreground removal

- We remove foregrounds using the fact that foregrounds are **smooth** in frequency and very **bright**, and HI is not
- However, the **largest** HI modes are smooth so get confused with foregrounds and **removed**
- **Power is damped** on large scale modes



Foreground model

Similar to the *beam* which damps power on **small** scales, we model foreground removal by damping power on **large** scales:

$$\tilde{B}_{\text{FG}}(k, \mu) = \left(1 - \exp \left[- \left(\frac{k}{N_{\perp} k_{\perp}^{\text{min}}} \right)^2 (1 - \mu^2) \right] \right) \times \left(1 - \exp \left[- \left(\frac{k}{N_{\parallel} k_{\parallel}^{\text{min}}} \right)^2 \mu^2 \right] \right)$$

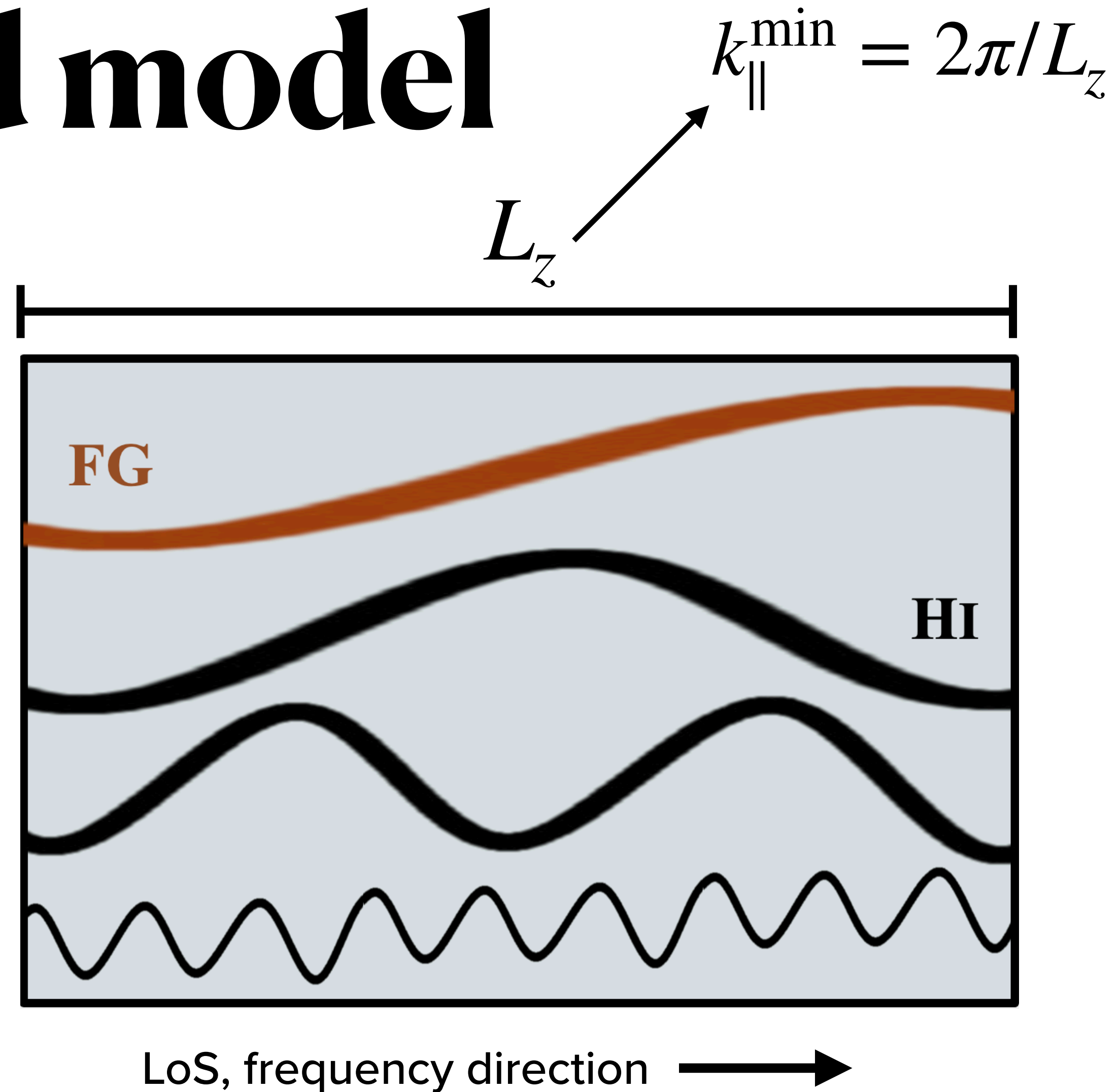
Free parameter

Largest physical
scale you can fit in
this direction

Foreground model

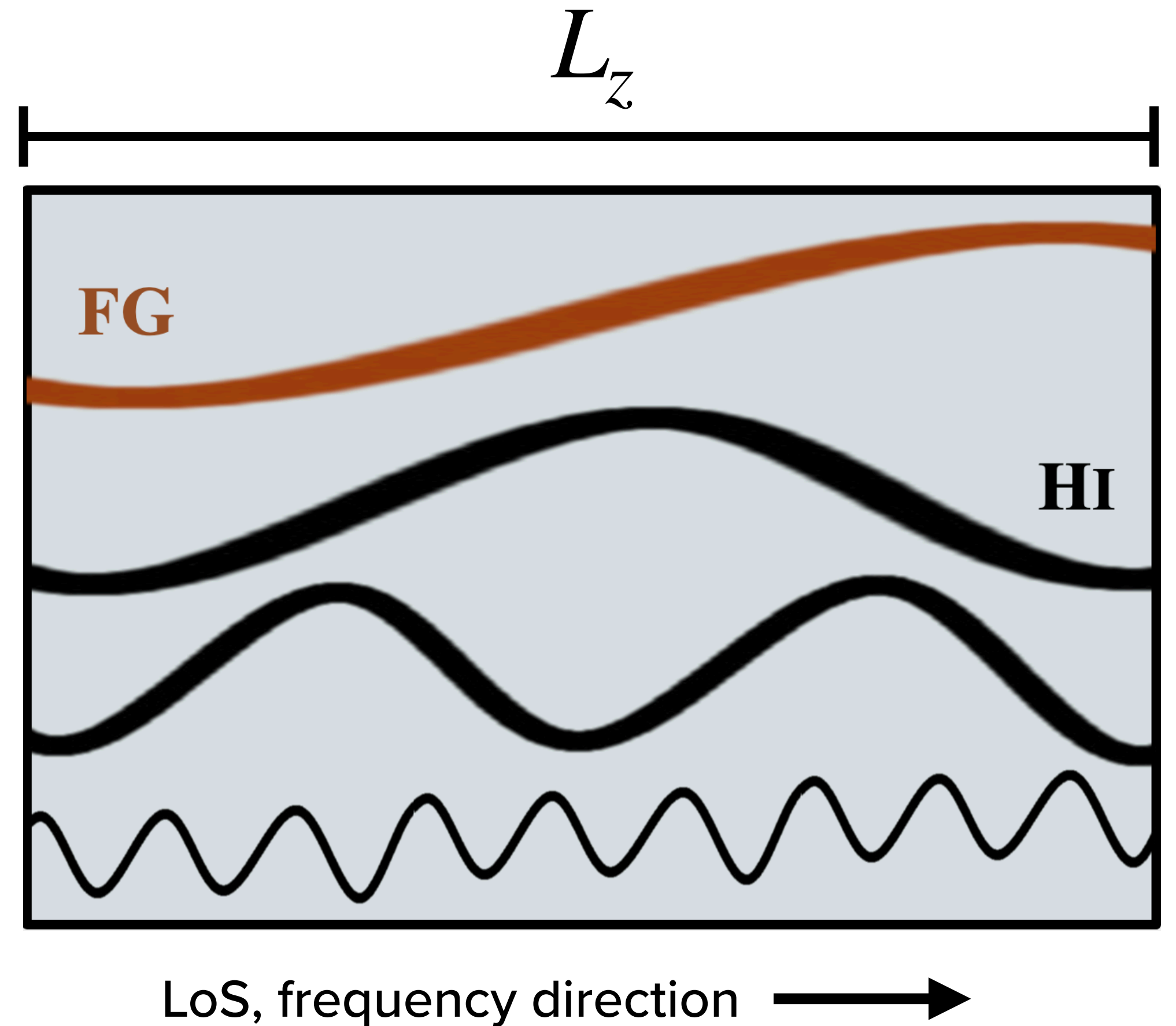
Along the line of sight direction:

- **FGs are spectrally smooth**
- **HI signal is not smooth**
 - However, the largest HI signal fluctuations that fit inside the box may appear smooth
 - So the threshold for differentiating it from FGs would be *half of the largest fluctuations we can fit in the box*
- Hence we expect $N_{\perp}, N_{\parallel} = 2$



Foreground model

- Keeping these as **free parameters** means it can model different types and levels of foreground removal
- However, we only tested this on **one** simulation and foreground removal method



Anisotropic model - foregrounds

The **power spectrum** tells us how clustered the matter is a function of inverse separation (k) **and angle between k and observer's line-of-sight (μ)**

Foreground removal
effect



$$P^{\text{HI}}(k^f, \mu^f) = \tilde{B}_{\perp}^2(k, \mu) \tilde{B}_{\text{FG}}(k, \mu) \alpha_{\parallel}^{-1} \alpha_{\perp}^{-2} \left[\frac{(\bar{T}_{\text{HI}} b_{\text{HI}} \sigma_8 + \bar{T}_{\text{HI}} f \sigma_8 \mu^2)^2 P_m(k)}{1 + (k \mu \sigma_v / H_0)^2} \frac{P_m(k)}{\sigma_8^2} + P_{\text{SN}} \right]$$

Multipole expansion

Useful way to **decompose** the power spectrum using Legendre polynomials

$$P_{\ell}^{\text{HI}}(k^f) = \frac{2\ell + 1}{2} \int_{-1}^1 d\mu^f \mathcal{L}_{\ell}(\mu^f) P^{\text{HI}}(k, \mu)$$

We look at $\ell = 0, 2, 4$:

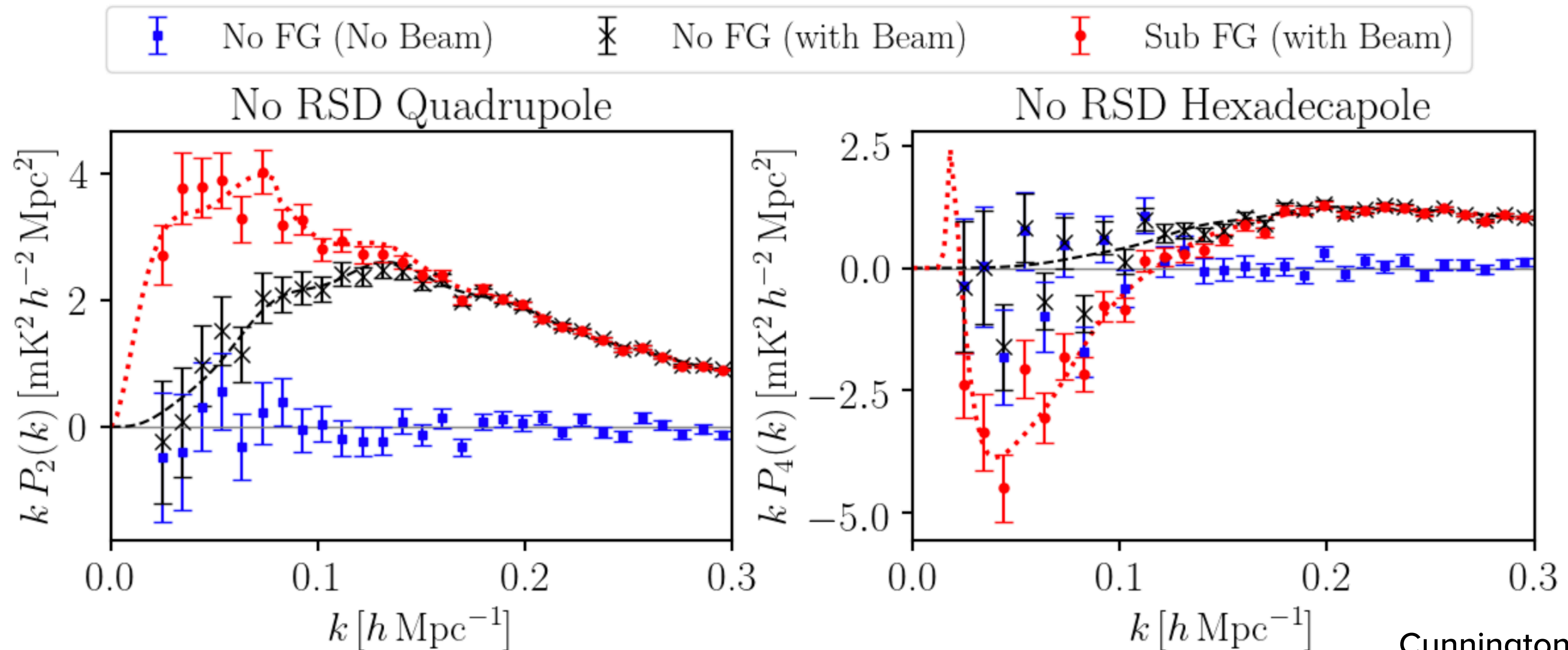
- As ℓ increases, we get **increasingly anisotropic information** (which can be harder to model), and should be fainter



Legendre polynomial

Multipole expansion

Higher multipoles should disappear without any RSD, but instrumental and systematic effects mean this is not true:



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Simulations

- **MultiDark-Galaxies** have used **MultiDark-Planck** N-body simulations and applied **SAGE** semi-analytical model
- Each galaxy has an associated cold gas mass, which we convert to HI mass, and then to HI brightness temperature

$$z = 0.82$$

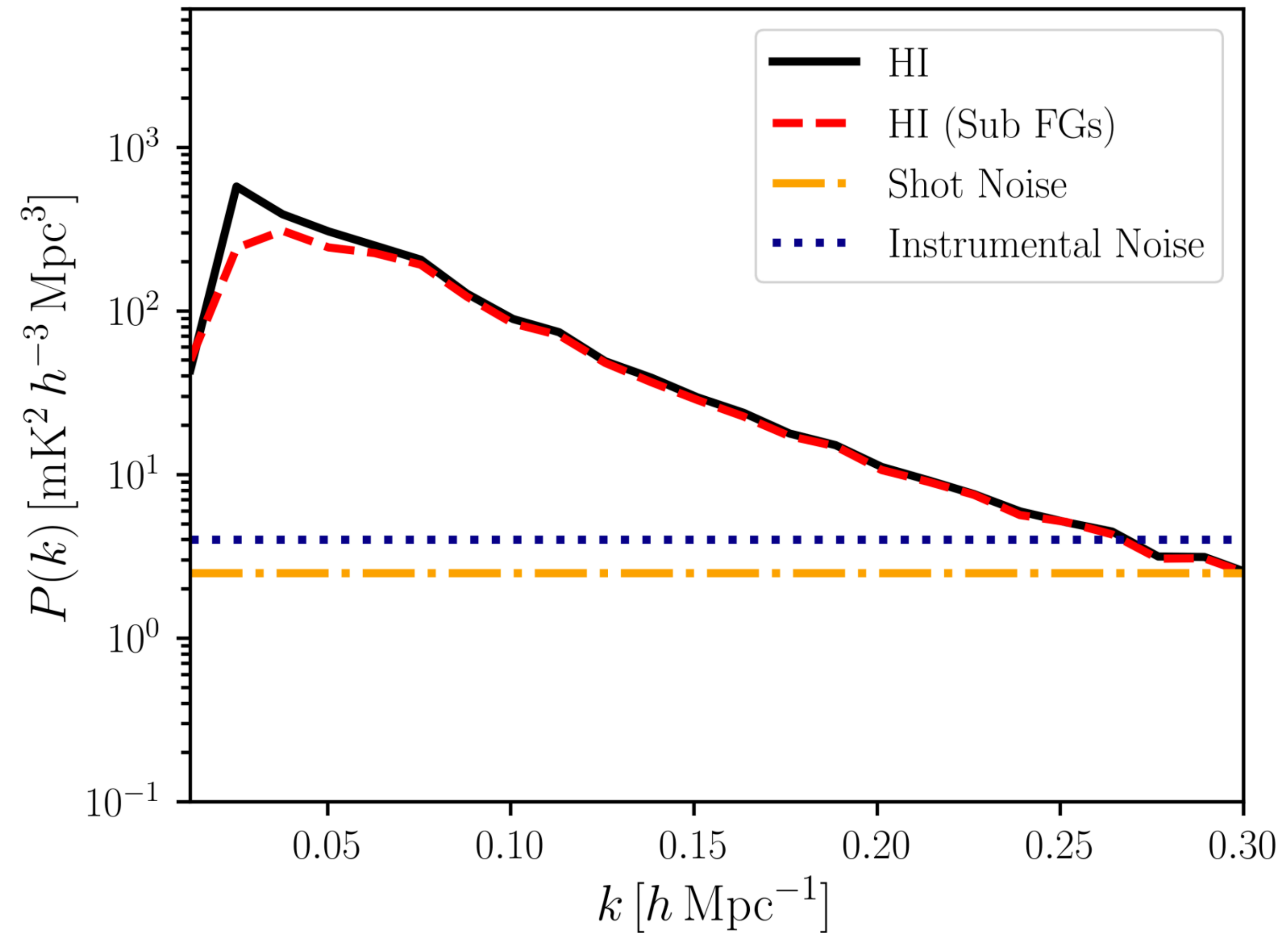
$$L_x = L_y = L_z = 1000 \text{ Mpc } h^{-1}$$

$$N_x = N_y = N_z = 225$$

Simulations

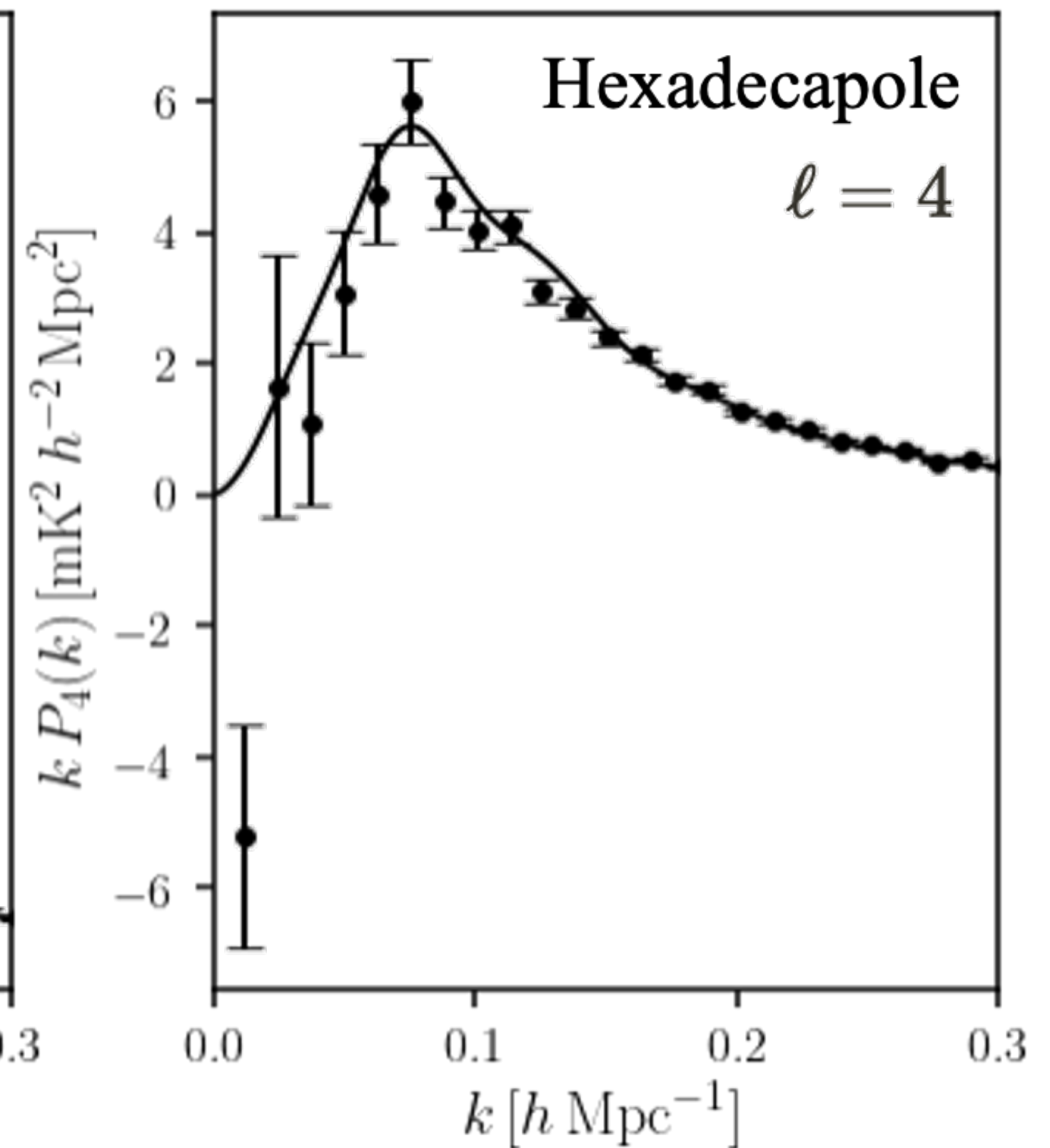
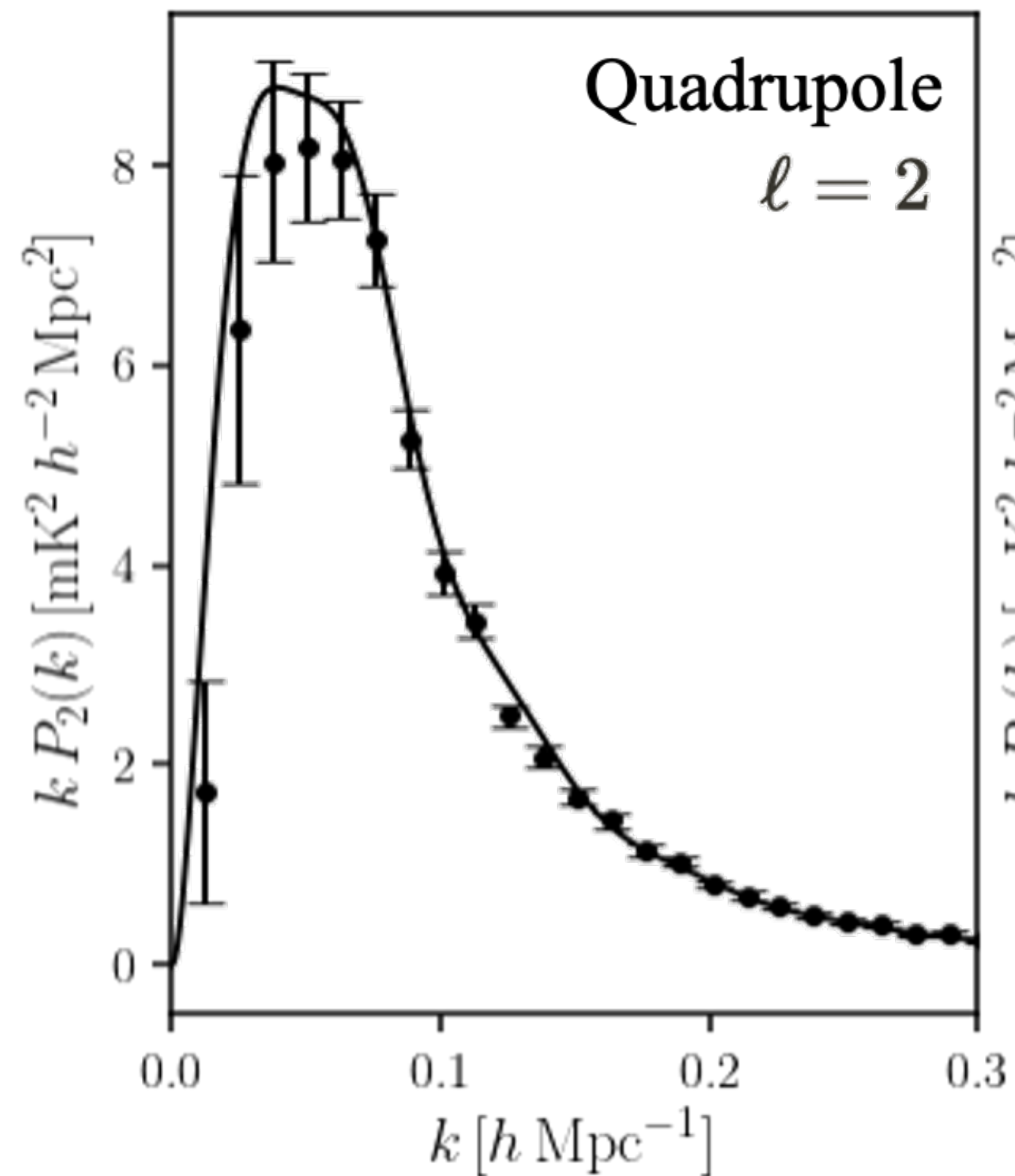
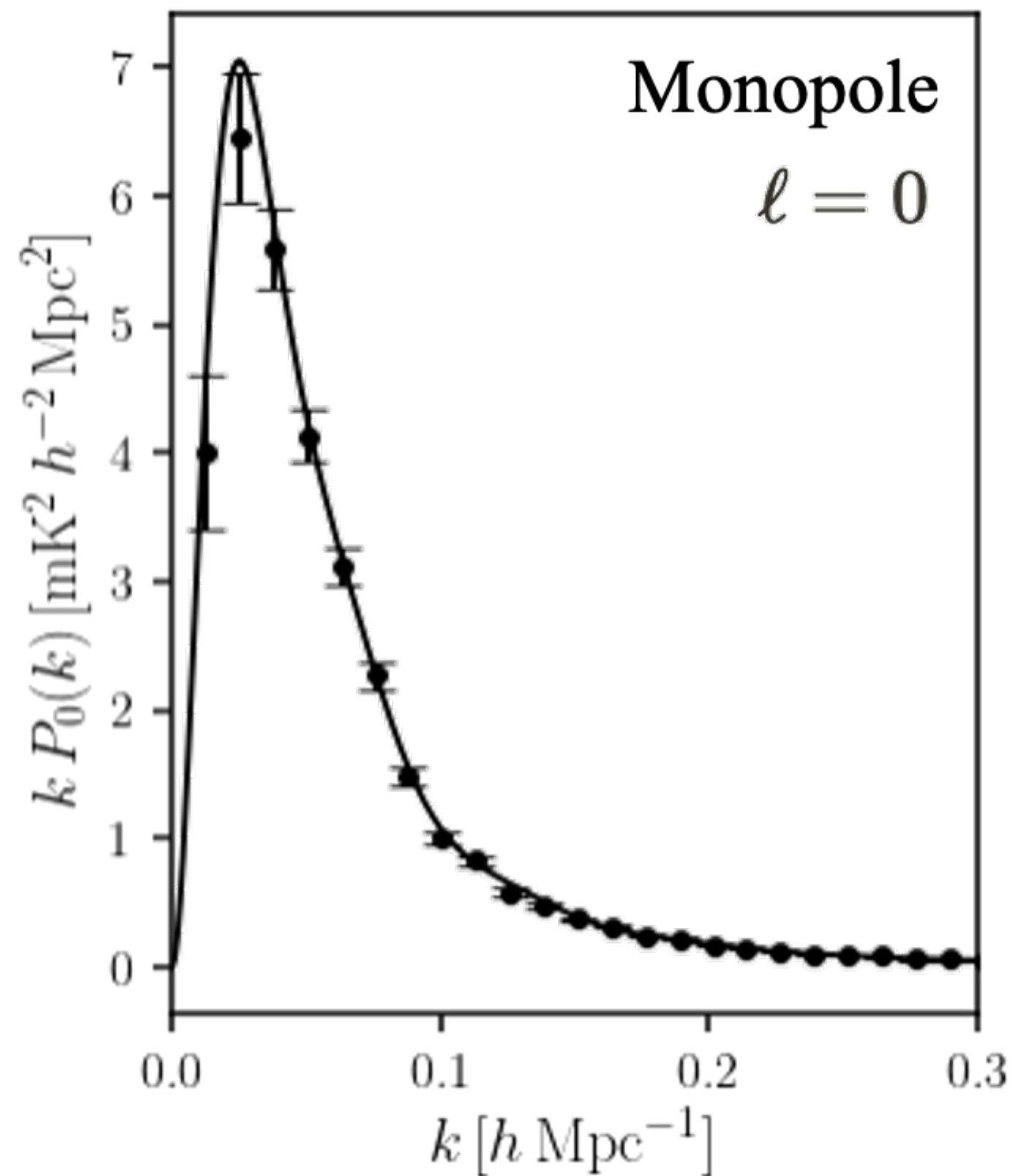
- We simulate foregrounds using the **Global Sky Model** and also realisations of a diffuse emission model power spectrum for finer detail
- No **noise** is added but we instead consider this in our modelling
- Add HI + foregrounds, then **smooth** each frequency slice with a telescope beam of SKA-like size
- Remove foregrounds using **FastICA**

Simulations



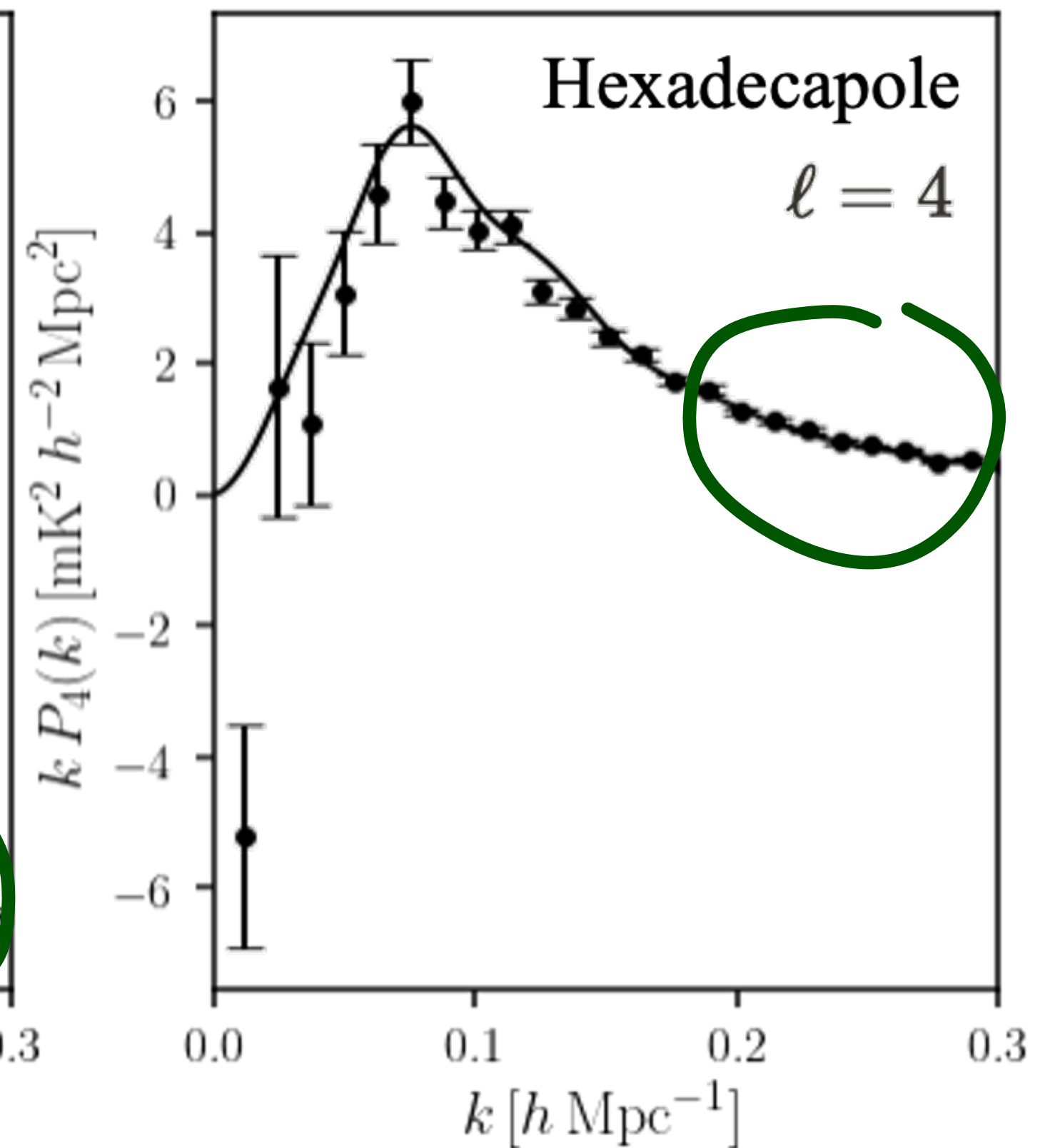
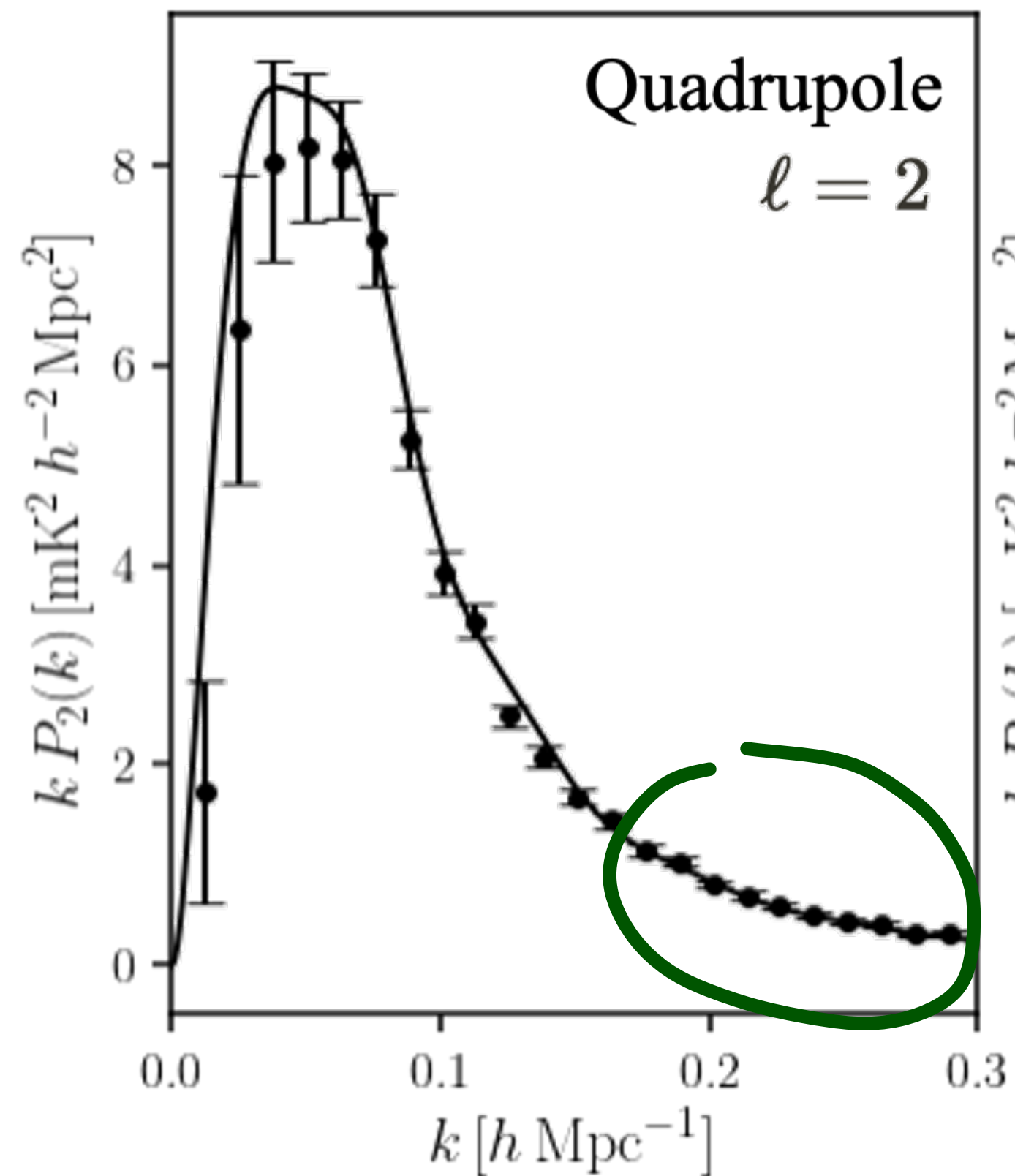
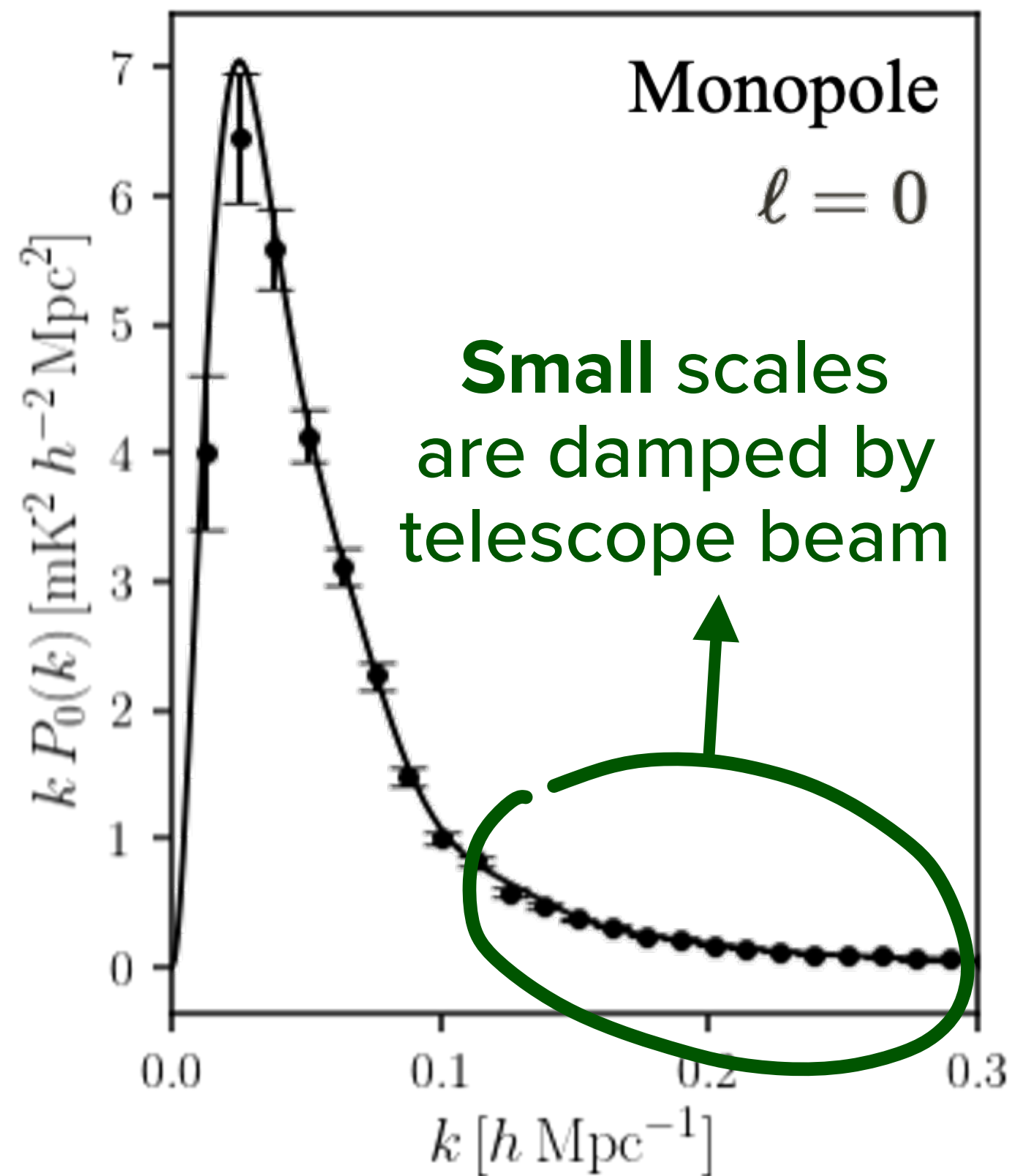
Model vs. simulation

Good agreement



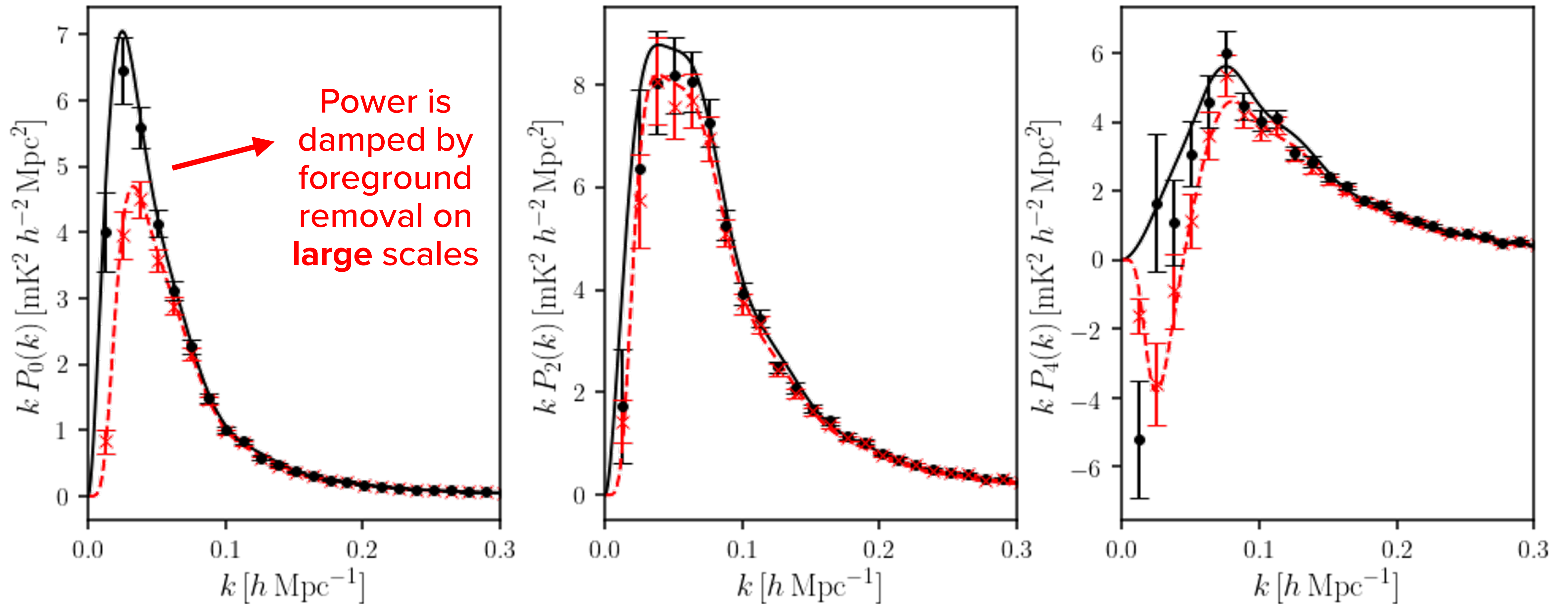
Model vs. simulation

Good agreement

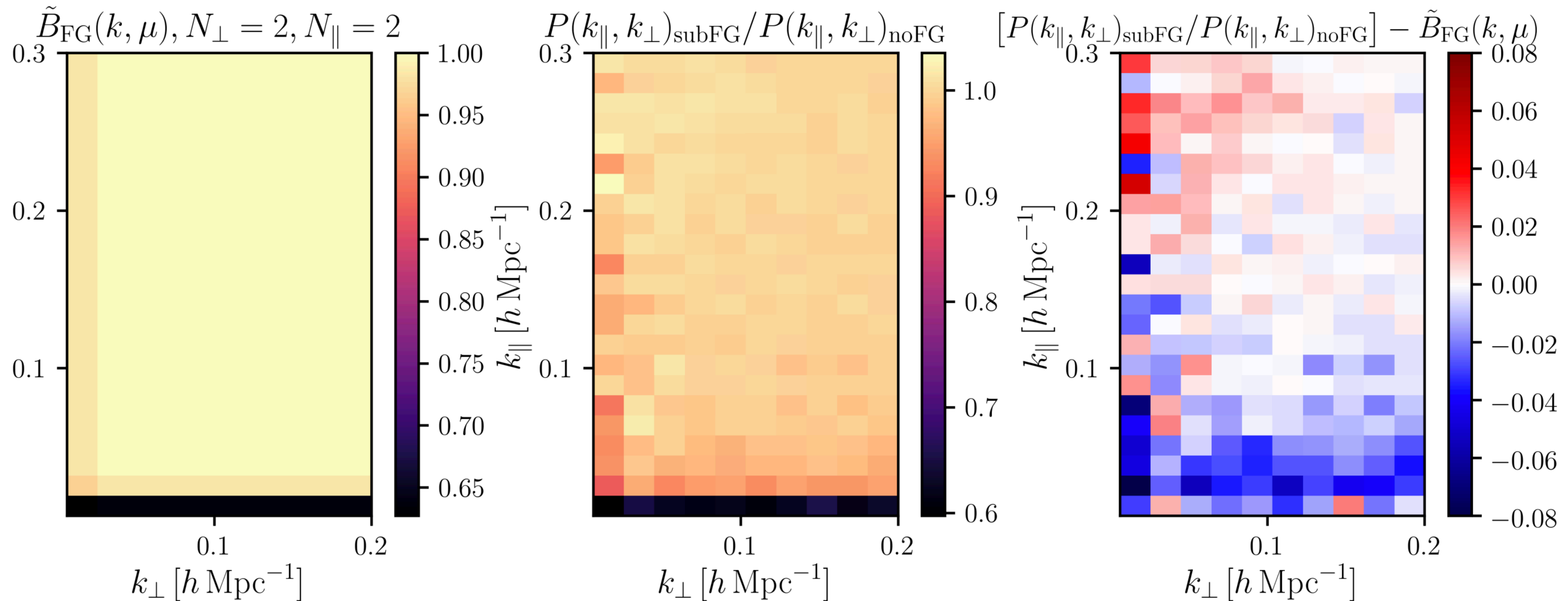


Foreground model vs. simulation

Also good agreement between model and data (using $N_{\perp}, N_{\parallel} = 2$).



Foreground model vs. simulation



Covariance matrix

Covariance of our power spectrum:

$$\sigma^2(k, \mu) = \frac{(P^{\text{HI}}(k, \mu) + P_N)^2}{N_{\text{modes}}(k, \mu)}$$

Assumes no coupling between different k bins

HI power spectrum

Instrumental noise (Gaussian)

Number of modes (number of measurements that make up this bin)

Covariance matrix

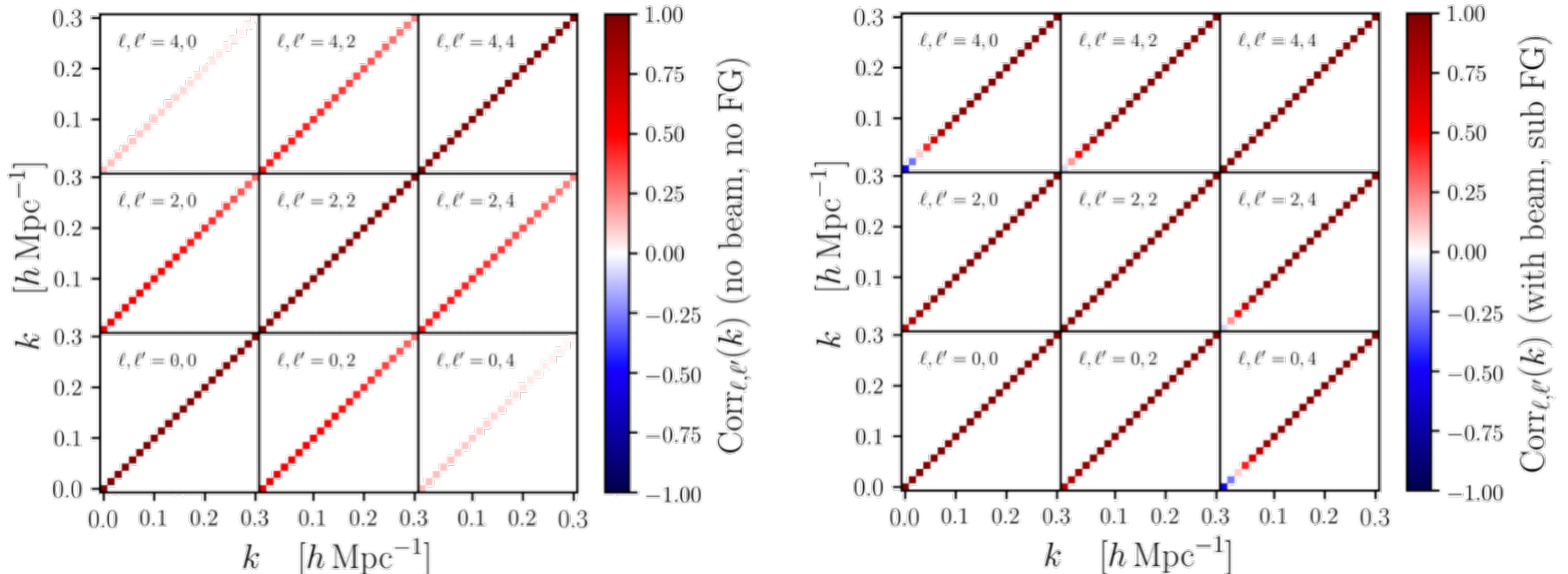
$$C_{\ell\ell'}(k) = \frac{(2\ell + 1)(2\ell' + 1)}{2} \int_{-1}^1 d\mu \sigma^2(k, \mu) \mathcal{L}_\ell(\mu) \mathcal{L}_{\ell'}(\mu)$$

- **Don't ignore covariance between different multipoles!** Systematic and instrumental effects enhance these.

Correlation matrix

$$\text{Corr}_{\ell\ell'}(k) = \frac{C_{\ell\ell'}(k)}{\sqrt{C_{\ell\ell}(k)C_{\ell'\ell'}(k)}}$$

- Demonstrates how the beam and FG enhance correlation (and hence covariance) between different multipoles.

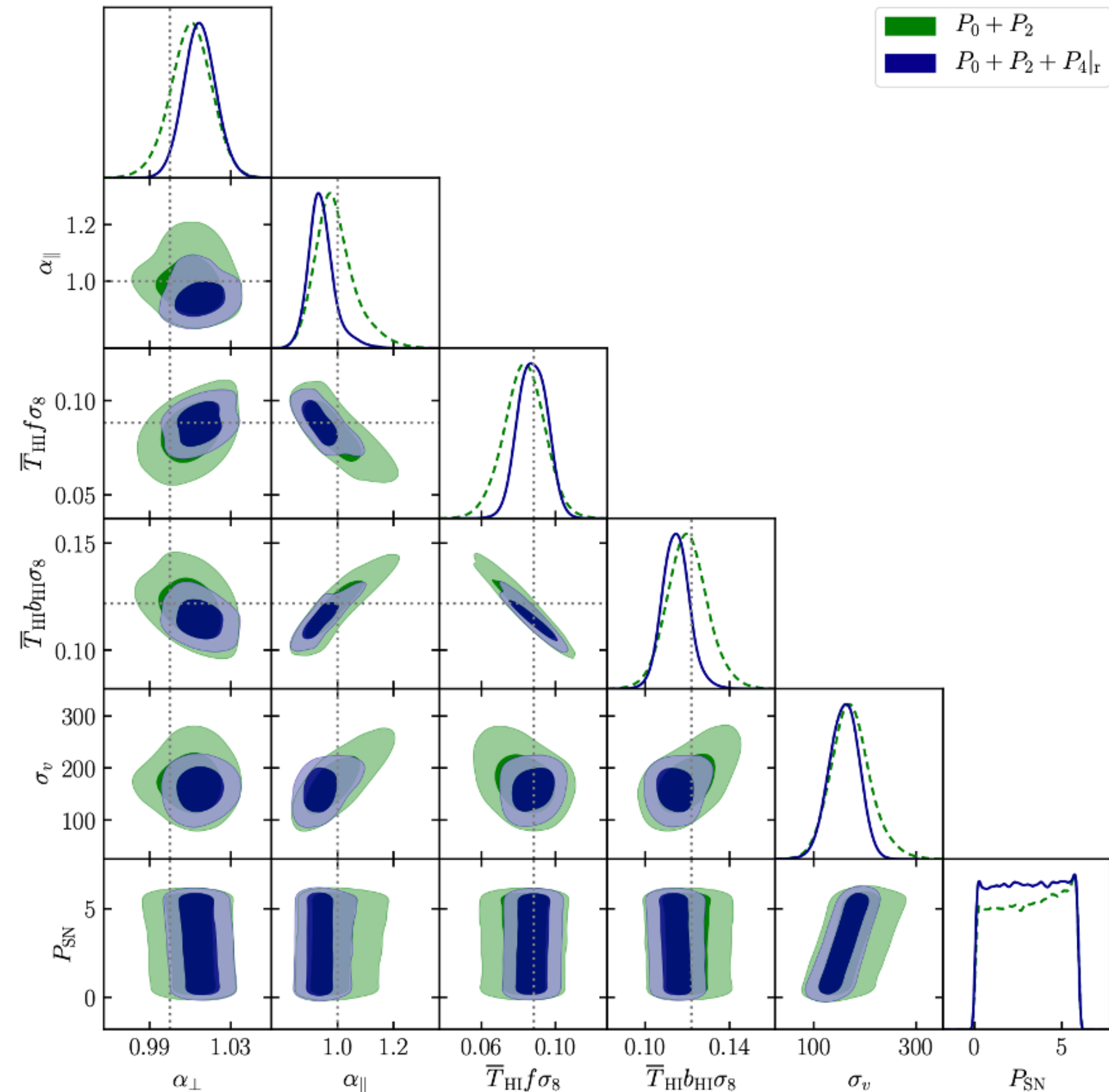


Parameter estimation - FG free

Performed an **MCMC analysis** using our model, covariance, and simulation.

- Unbiased results
- Sub-10% uncertainty on all parameters of interest
- Including the hexadecapole improves constraints, but needs smaller k_{\max}

Parameter	$P_0 + P_2$	$P_0 + P_2 + P_4 _r$
α_{\perp}	1.0%	0.8%
α_{\parallel}	7.6%	5.3%
$\bar{T}_{\text{HI}} f \sigma_8$	13.3%	8.8%
$\bar{T}_{\text{HI}} b_{\text{HI}} \sigma_8$	8.1%	5.7%

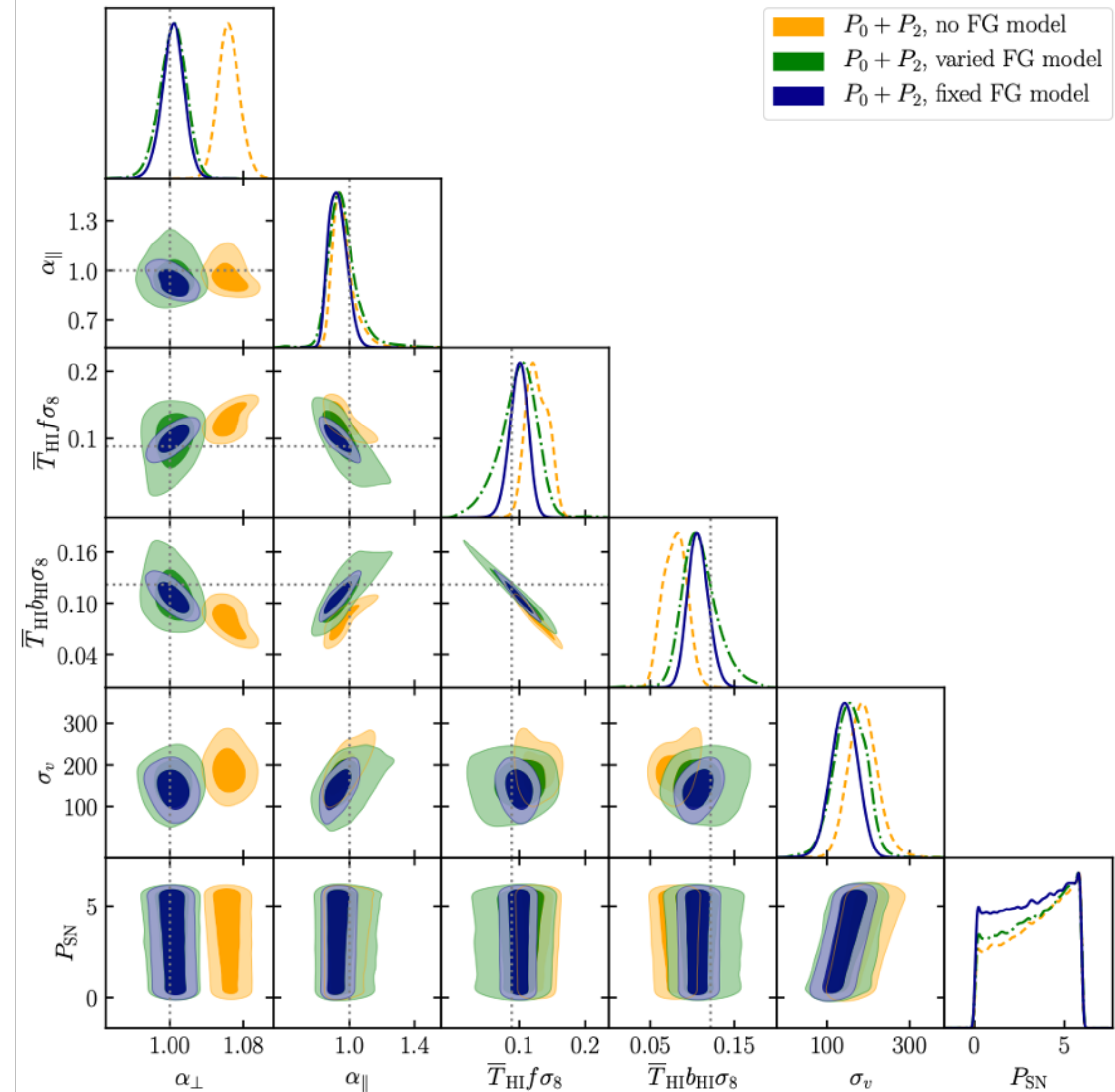


Parameter estimation - FG

If we don't use any foreground damping model, our results are **biased**.

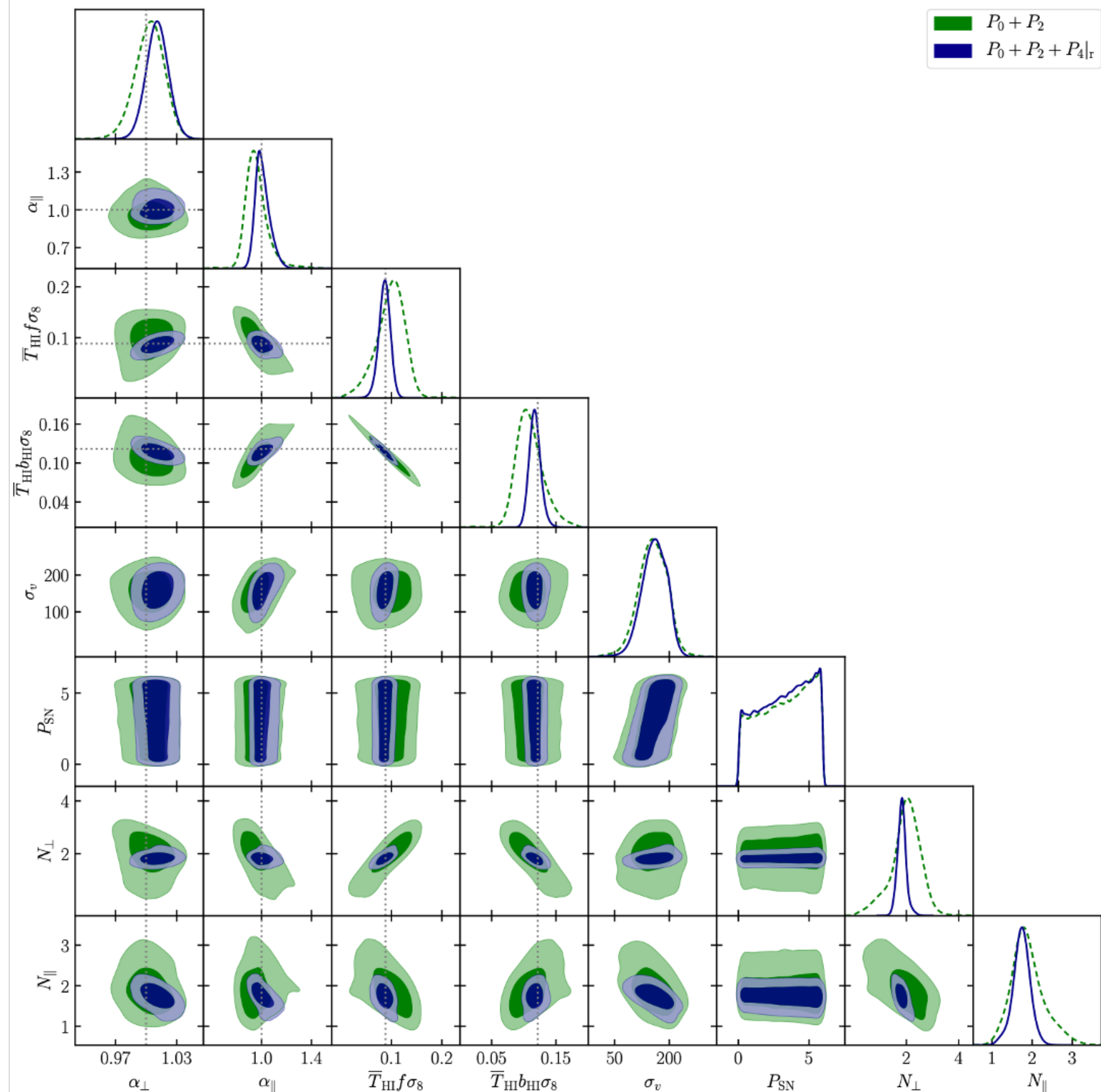
If we use our foreground damping model, results are **unbiased**.

Uncertainties are larger, since we are varying more parameters.



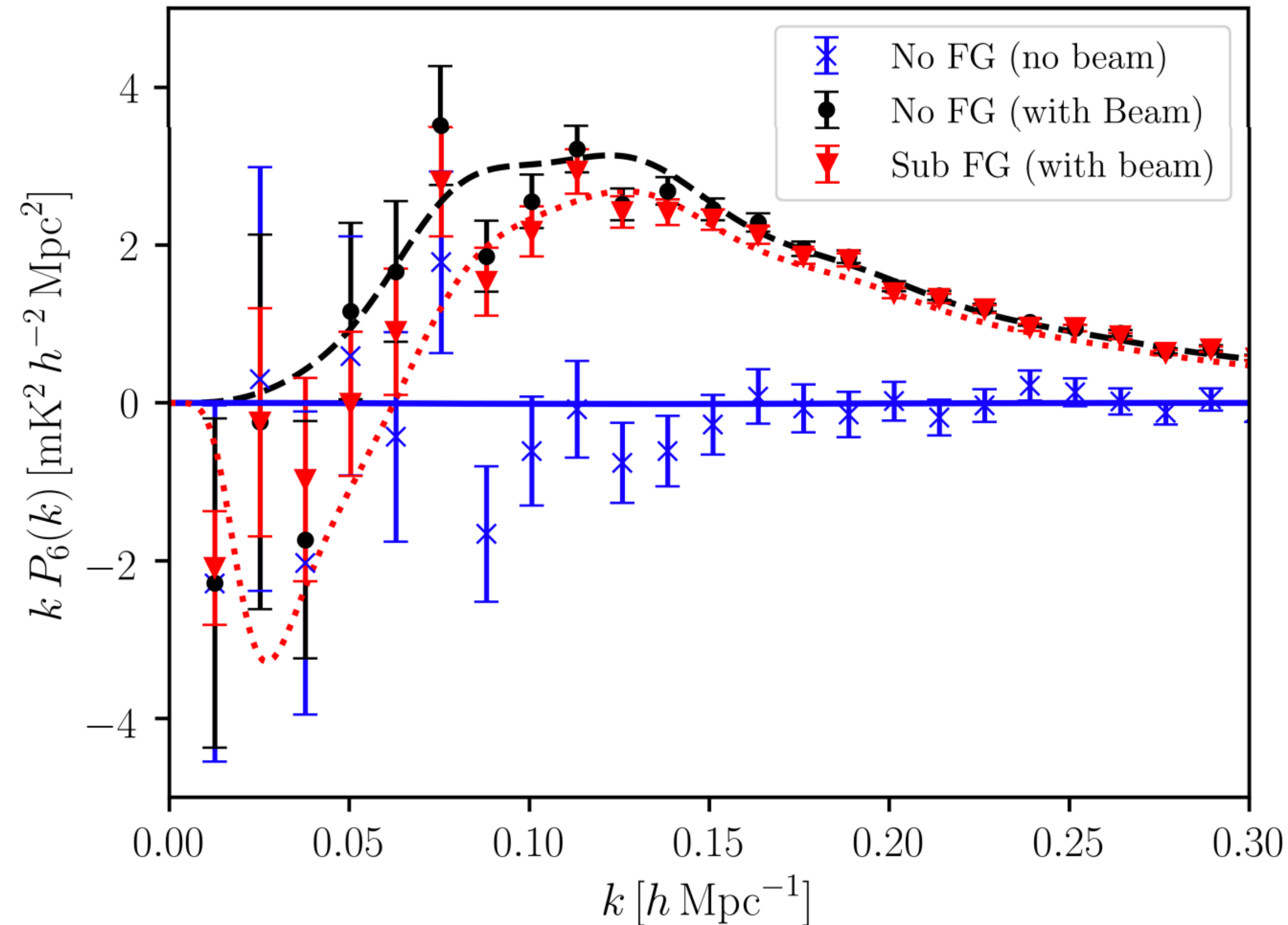
Including the hexadecapole and letting the FG model vary, we again get an improvement on parameter constraints:

Parameter	$P_0 + P_2 + P_4 _r$ Varied N_\perp, N_\parallel
α_\perp	1.1%
α_\parallel	5.9%
$\overline{T}_{\text{HI}} f \sigma_8$	13.3%
$\overline{T}_{\text{HI}} b_{\text{HI}} \sigma_8$	7.8%



Hexacontatetrapole ($\ell = 6$)

Should be zero for no RSD, but again systematic and instrumental effects enhance it:



Key takeaways

- ★ It is possible to conduct competitive cosmological parameter estimation with HI IM, but leads to **biased results if foreground removal is not properly accounted for**
- ★ We present a foreground removal model and show that it **unbiases** results
- ★ Systematic and instrumental effects significantly impact the covariance between different multipoles, making them more correlated