HITS - HI Intensity Mapping in Trieste
24 May, 2022

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Power Spectrum Multipoles

Introduction

"Multipole expansion for 21cm Intensity Mapping power spectrum: forecasts for the SKA Observatory" - **M. Berti**, M. Spinelli, M. Viel In preparation..

Berti et al. (2022)

- MeerKAT forecasts
- Dark Energy models
- (Limited) Tomographic data set

✓ Use the same tools
✳ SKAO forecasts
✳ Tomographic data set → more accurate
✳ Focus on ΛCDM
✳ Study of non linear scales

Soares et al. (2021)

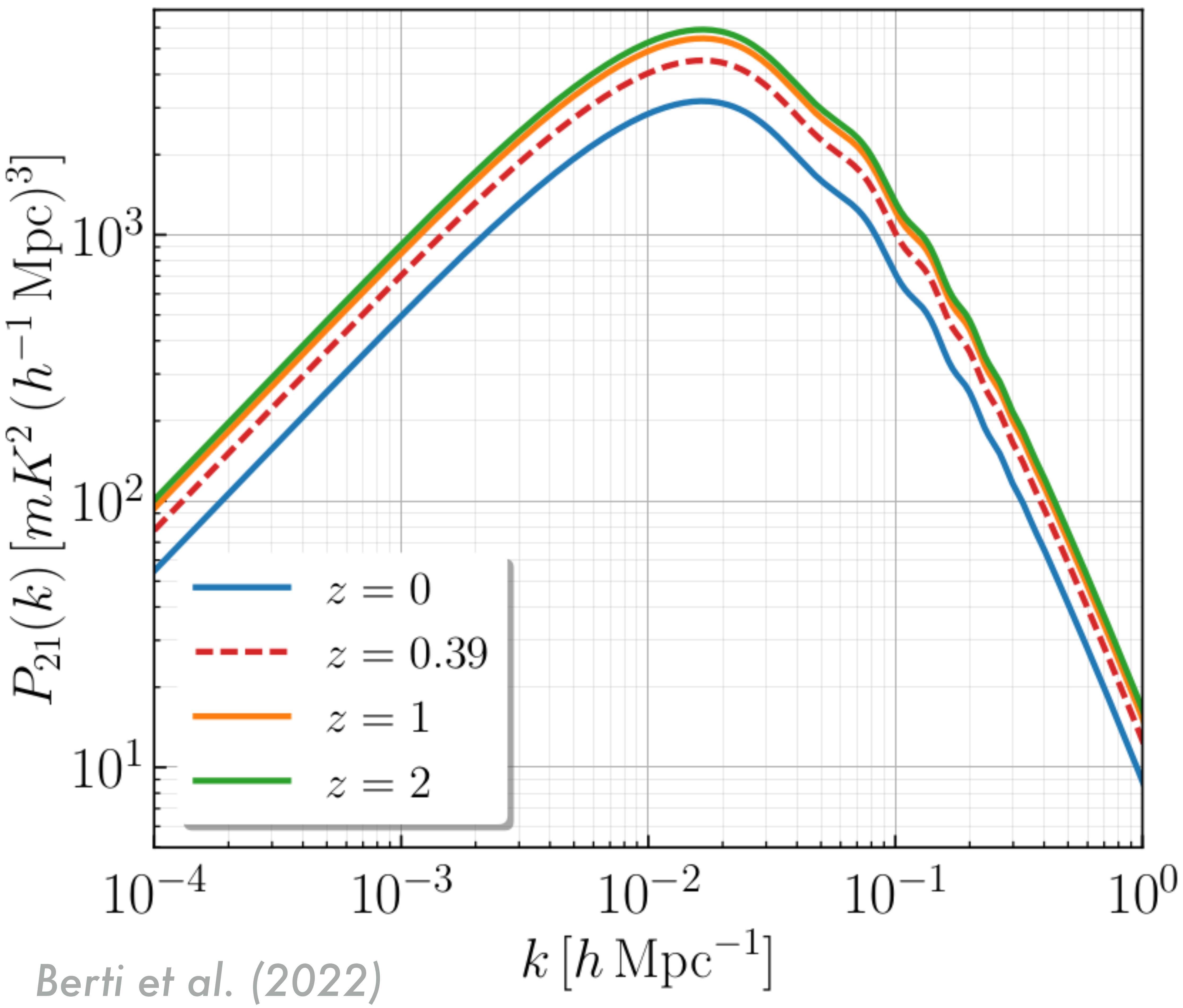
- Multipole expansion
- SKAO forecasts (MCMC)
- One redshift bin
- Foreground analysis

See Paula's talk!

✓ Modelling multipoles and errors
✓ MCMC analysis, with different tools
✳ Constraints on cosmological parameters
✳ 6 redshift bins
✳ 21cm combined with CMB data
– We do not model foregrounds

Theoretical Model

Modelling the 21cm Linear Power Spectrum



Berti et al. (2022)

We model it as[1]

$$P_{21}(z,k,\mu) = \bar{T}_b^2(z)\left[\, b_{HI}(z) + f(z)\,\mu^2 \,\right]^2 P_m(z,k)$$

where

- $\bar{T}_b^2(z)$ is the mean brightness temperature
- $b_{HI}(z)$ is the HI bias
- $f(z)$ is the growth rate
- $\mu = \hat{k} \cdot \hat{z}$
- $P_m(z,k)$ is the matter power spectrum

✓ in good agreement with hydrodynamical simulations results (Villaescusa-Navarro et al. 2018)

[1] Kaiser (1987), Bacon et al. (2019)

Modelling the 21cm Linear Power Spectrum

$$P_{21}(z,k,\mu) = \bar{T}_b^2(z)\left[b_{HI}(z) + f(z)\mu^2\right]^2 P_m(z,k)$$

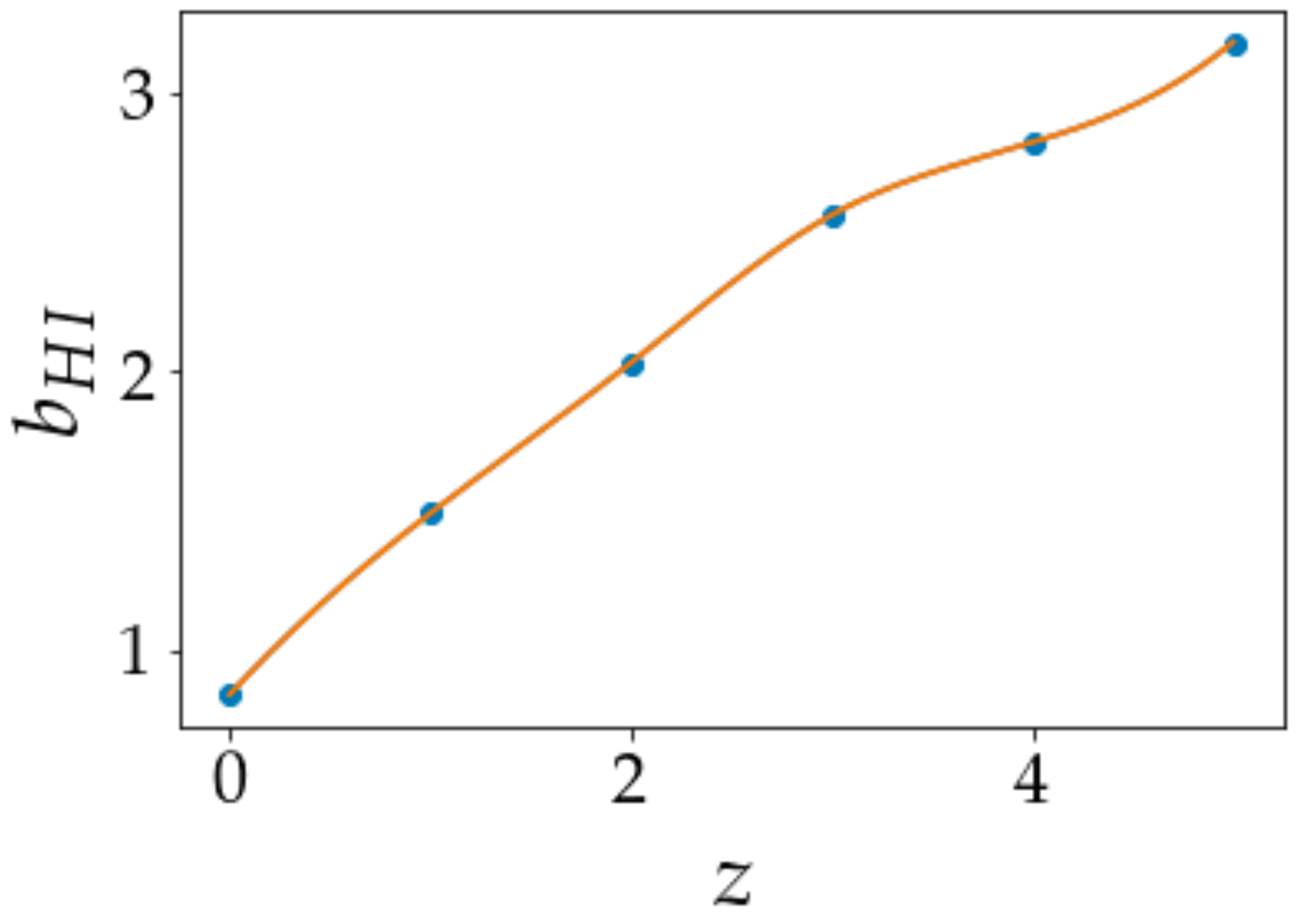
Computed numerically with CAMB *Lewis (2000)*

$$\bar{T}_{HI}(z) = 180\ \Omega_{HI}(z)\frac{h\,H_0}{H(z)}(1+z)^2\mathrm{mK}$$

Battye et al. (2013)

$$\Omega_{HI}(z) = 4.\times 10^{-4}(1+z)^{0.6}$$

Crighton et al. (2015)

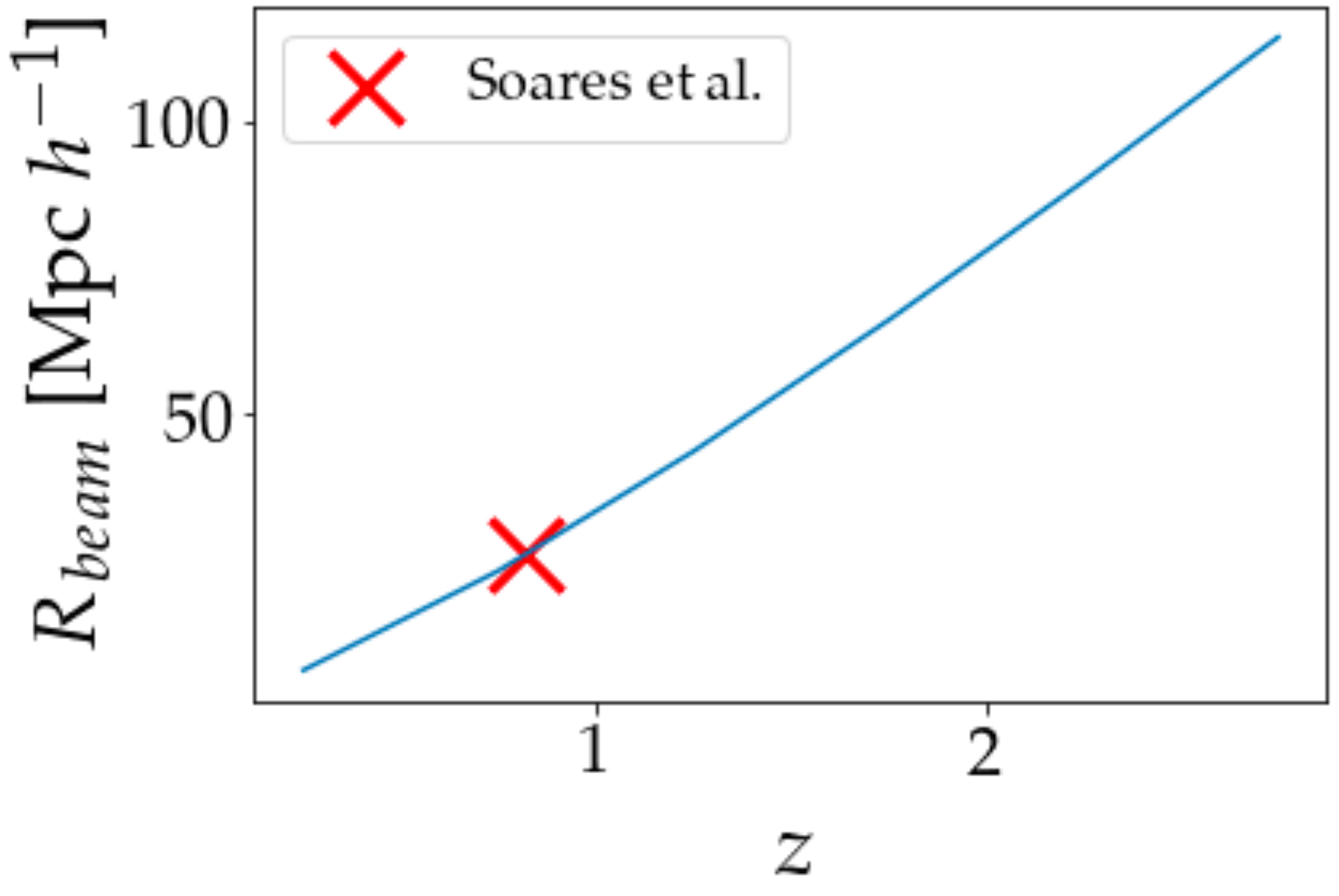


Villaescusa-Navarro et al. (2018)

The Effect of the Telescope Beam

$$P_{21}(z,k,\mu) = \bar{T}_b^2(z)\left[b_{HI}(z) + f(z)\,\mu^2\right]^2 P_m(z,k)$$

$$R_{beam}(z) = \sigma_\theta r(z) = \frac{\theta_{FWHM}}{2\sqrt{2\ln 2}} r(z)$$



Beam Smoothing Factor

Damping of the signal depending on the scale of the beam

$$\tilde{B}(z,k,\mu) = \exp\left[\frac{-k^2 R_{beam}^2(z)(1-\mu^2)}{2}\right]$$

$$P_{21}(z,k,\mu) = \bar{T}_b^2(z)\,\tilde{B}^2(z,k,\mu)\left[b_{HI}(z) + f(z)\,\mu^2\right]^2 P_m(z,k)$$

Power Spectrum Multipoles Expansion

$$P_{21}(z,k,\mu) = \bar{T}_b^2(z)\,\tilde{B}(z,k,\mu)\left[b_{HI}(z) + f(z)\,\mu^2\right]^2 P_m(z,k)$$

Expand in μ

$$P_{21}(k,\mu) = \Sigma_\ell\, P_\ell(k)\mathscr{L}_\ell(\mu)$$

where the Legendre polynomials are

$$\mathscr{L}_0(\mu) = 1$$

$$\mathscr{L}_2(\mu) = \frac{3\mu^2}{2} - \frac{1}{2}$$

$$P_\ell(z,k) = \frac{(2\ell+1)}{2}\int_{-1}^{1} d\mu\, \mathscr{L}_\ell(\mu) P_{21}(z,k,\mu)$$

$$P_\ell(z,k) = \frac{(2\ell+1)}{2}\bar{T}_b^2(z)\,P_m(z,k)\int_{-1}^{1} d\mu\, \mathscr{L}_\ell(\mu)\,\tilde{B}(z,k,\mu)\left[b_{HI}(z) + f(z)\,\mu^2\right]^2$$

⟶ we forecast observations of the monopole $P_0(z,k)$ and the quadrupole $P_2(z,k)$

Data Set and Likelihood

Modelling the Sources of Error

I. Instrumental Noise

$$P_{\rm N}(z) = \frac{T^2_{\rm sys} 4\pi f_{\rm sky}}{N_{\rm dish} t_{\rm obs} \delta\nu} \frac{V_{\rm bin}(z)}{\Omega_{\rm sur}}$$

→ depends on SKAO specifics

Parameter		Value
$D_{\rm dish}$ [m]	SKAO dish diameter	15
$N_{\rm dish}$	SKAO dishes	133
$t_{\rm obs}$ [h]	observing time	10000
$T_{\rm sys}$ [K]	system temperature	25
$\delta\nu$ [MHz]	frequency range	1
$\Omega_{\rm sur,1}$ [sr]	survey area (Band 2)	1.5
$\Omega_{\rm sur,2}$ [sr]	survey area (Band 2)	6.1
$f_{\rm sky,2}$	covered sky area (Band 2)	0.12
$f_{\rm sky,1}$	covered sky area (Band 1)	0.48
Δz	width of the redshift bins	0.5

Bacon et al. (2018)

II. Variance per k and μ Bin

$$\sigma^2(z,k,\mu) = \frac{\left(P_{21}(z,k,\mu) + P_{\rm N}(z)\right)^2}{N_{\rm modes}(z,k,\mu)}$$

with the number of modes per bin being

$$N_{\rm modes}(z,k,\mu) = \frac{k^2 \Delta k(z) \Delta\mu(z)}{4\pi^2} V_{\rm bin}(z)$$

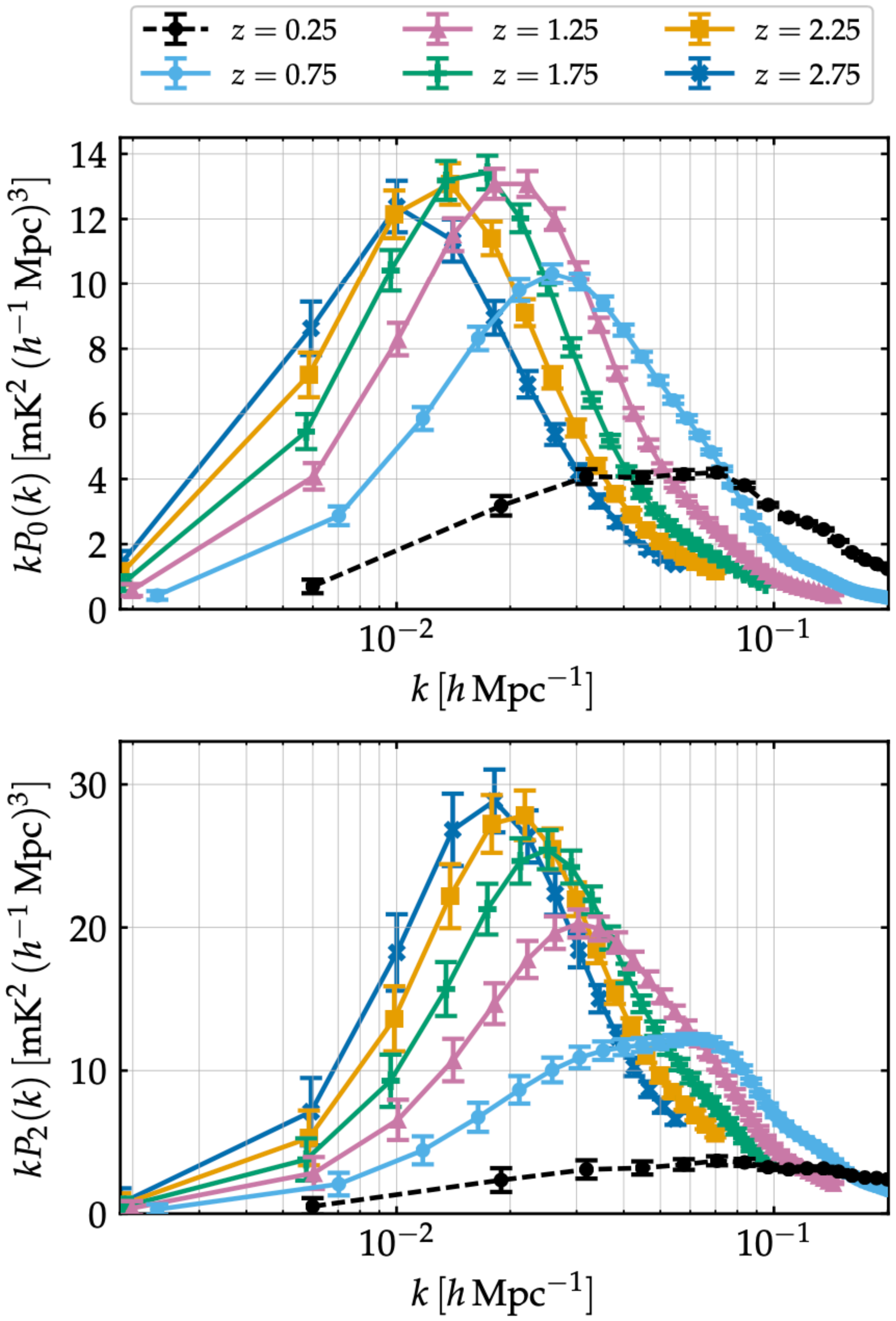
III. Multipole Covariance

$$C_{\ell\ell'}(z,k) = \frac{(2\ell+1)(2\ell'+1)}{2} \int_{-1}^{1} {\rm d}\mu \, \mathscr{L}_\ell(\mu) \, \mathscr{L}_{\ell'}(\mu) \, \sigma^2(z,k,\mu)$$

The error on the single data point is ($\ell' = \ell$)

$$\sigma_{P_\ell}(z,k_i) = \sqrt{C_{\ell\ell}(z,k_i)}$$

The Forecasted Tomographic Data Set



- Forecasted observations for the SKAO telescope at the effective central redshifts

$$z_c = \{0.25, 0.75, 1.25, 1.75, 2.25, 2.75\}$$

SKA-MID Band 2

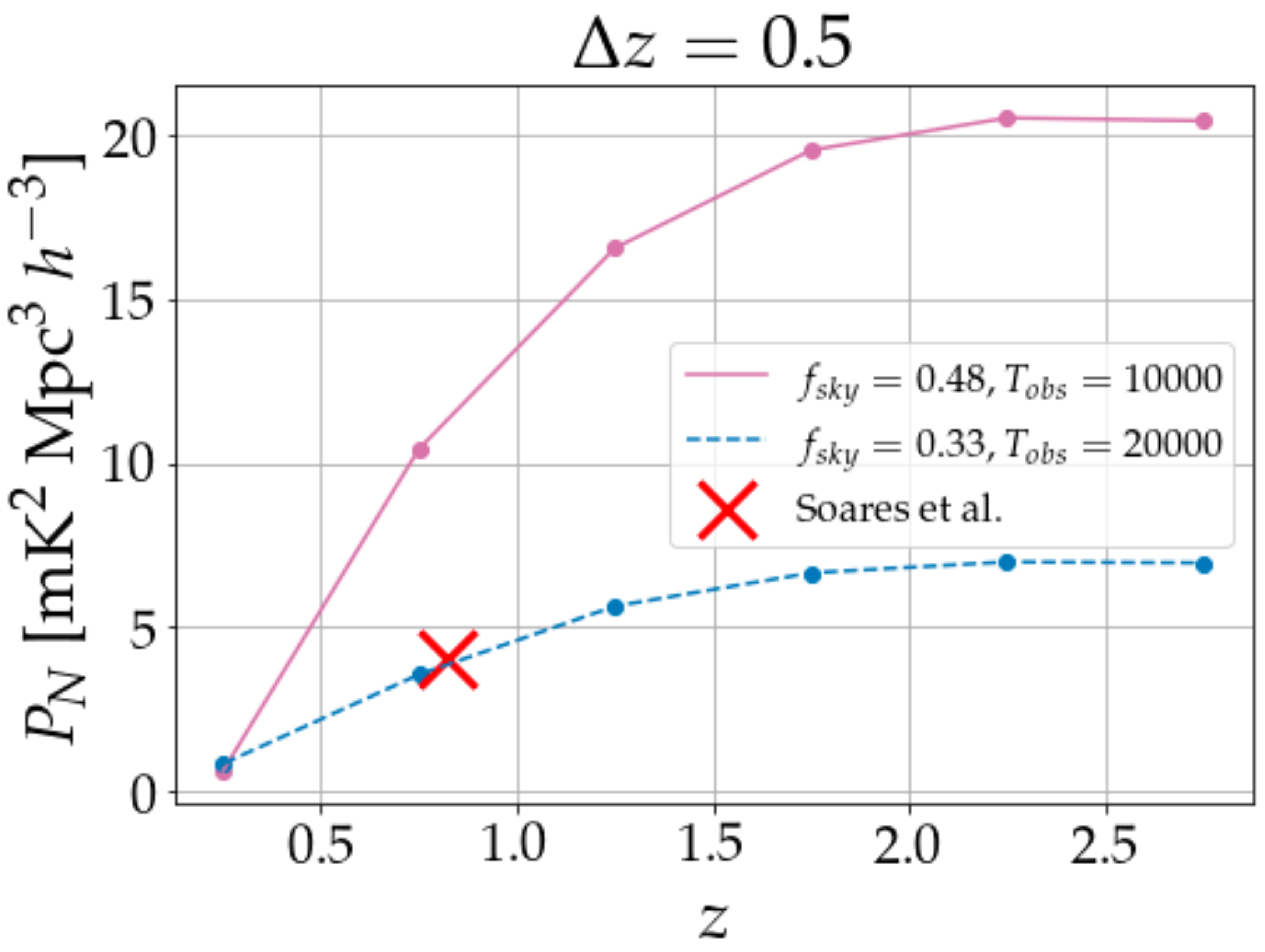
5 000 deg^2

z range 0-0.5

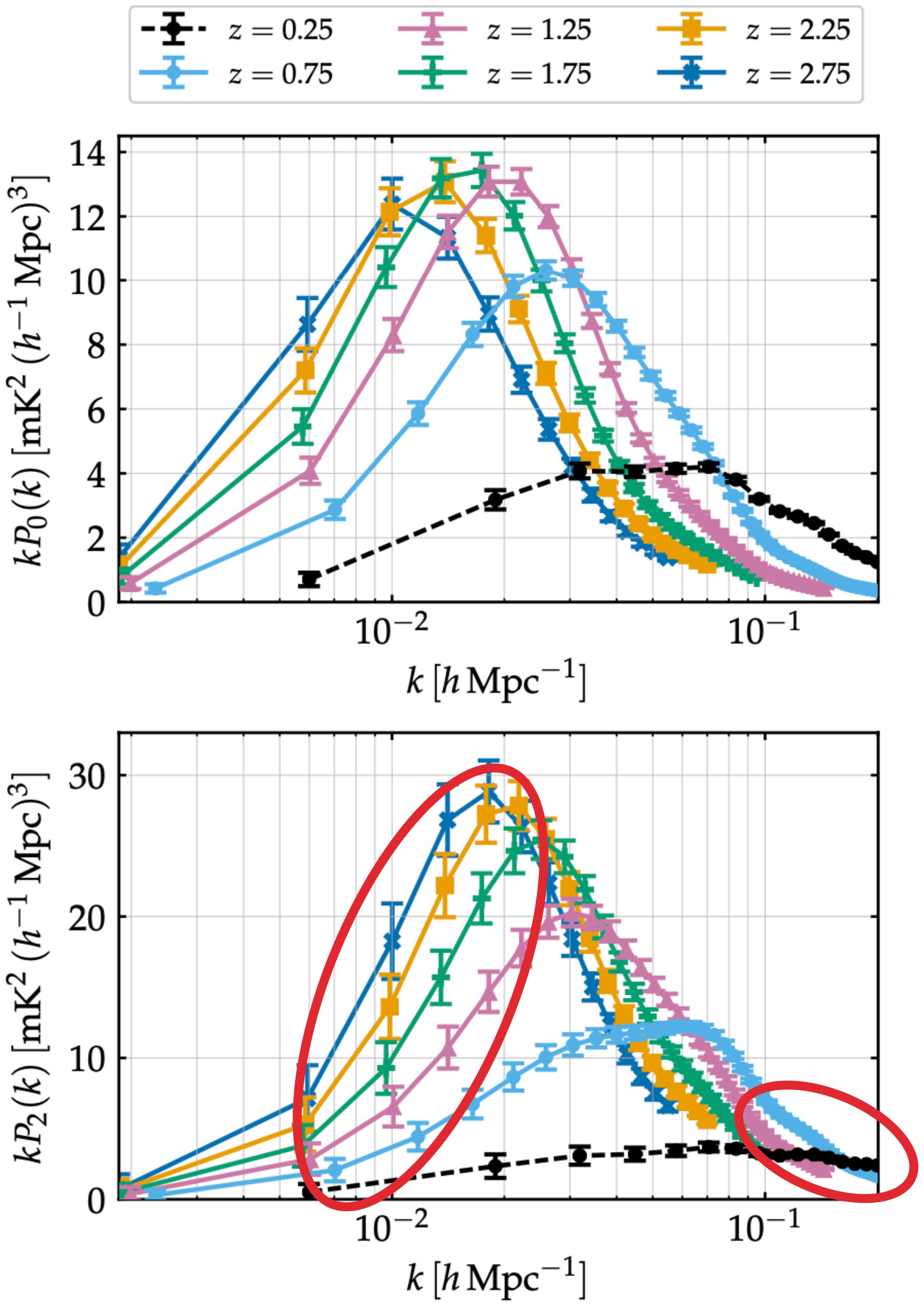
SKA-MID Band 1

20 000 deg^2

z range 0.35-3



The Forecasted Tomographic Data Set



- Forecasted observations for the SKAO telescope at the effective central redshifts

$$z_c = \{0.25, 0.75, 1.25, 1.75, 2.25, 2.75\}$$

SKA-MID Band 2
5 000 deg^2
z range 0-0.5

SKA-MID Band 1
20 000 deg^2
z range 0.35-3

- Planck 2018 as fiducial cosmology

$$\{\Omega_b h^2 = 0.022383, \Omega_c h^2 = 0.12011, n_s = 0.96605, \ln(10^{10} A_s) = 3.0448, \tau = 0.0543, h = 0.6732, \Sigma m_\nu = 0.06\mathrm{eV}\}$$

- Scale range limited by the bin volume and the beam size

$$k_{\rm min}(z_c) = 2\pi / \sqrt[3]{V_{\rm bin}(z_c)}$$

$$k_{\rm max}(z_c) = 2\pi / R_{\rm beam}(z_c)$$

NB hard limit at $k_{\rm max} = 0.2\, h\mathrm{Mpc}^{-1}$

Covariance Matrix

For N_k number of data points, $N_\ell = 2$ number of multipoles

$$C(z) = \begin{pmatrix} C_{00}(z) & C_{02}(z) \\ C_{02}(z) & C_{22}(z) \end{pmatrix} \quad N_\ell \times N_k$$

$N_\ell \times N_k$

Covariance between monopole and quadrupole

If we switch it off

$$C_{\rm diag}(z) = \begin{pmatrix} C_{00}(z) & 0 \\ 0 & C_{22}(z) \end{pmatrix}$$

See Paula's talk!

$$C_{\ell\ell'}(z) = \begin{pmatrix} C_{\ell\ell'}(z,k_1) & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & C_{\ell\ell'}(z,k_N) \end{pmatrix} \quad N_k$$

N_k

$$C_{\ell\ell'}(z,k) = \frac{(2\ell+1)(2\ell'+1)}{2}\int_{-1}^{1} \mathrm{d}\mu\, \mathscr{L}_\ell(\mu)\, \mathscr{L}_{\ell'}(\mu)\, \sigma^2(z,k,\mu)$$

Likelihood and Signal-To-Noise

Defining $\vec{\Theta}(z_c) = \left(\vec{P}_0(z_c), \vec{P}_2(z_c)\right)$

$\vec{P}_\ell(z_c) = (P_\ell(z_c, k_1), \ldots, P_\ell(z_c, k_N))$

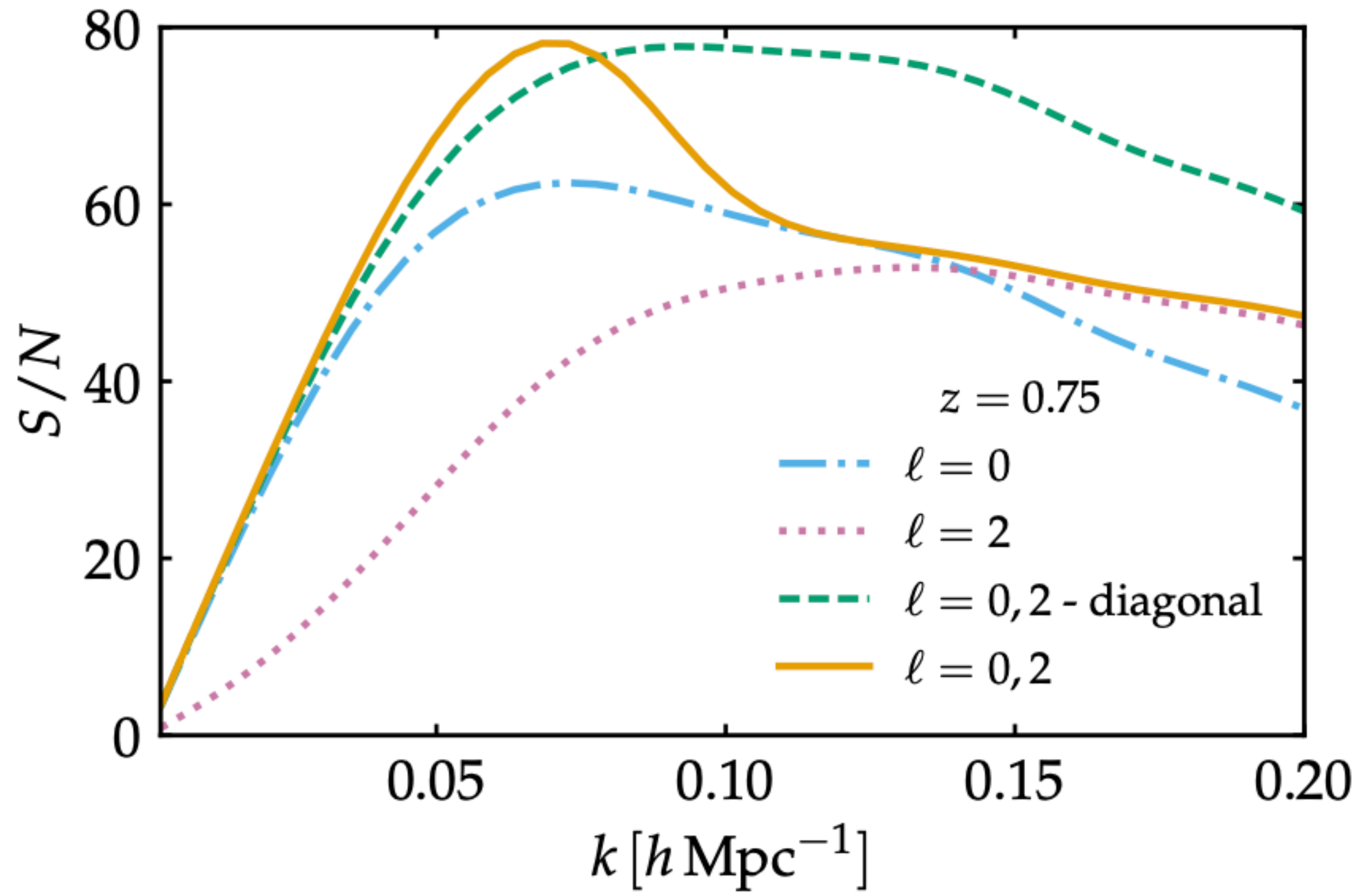
$$-\ln[\mathscr{L}] = \sum_{z_c} \frac{1}{2} \Delta\vec{\Theta}(z_c)^{\mathrm{T}} C^{-1}(z_c)\, \Delta\vec{\Theta}(z_c)$$

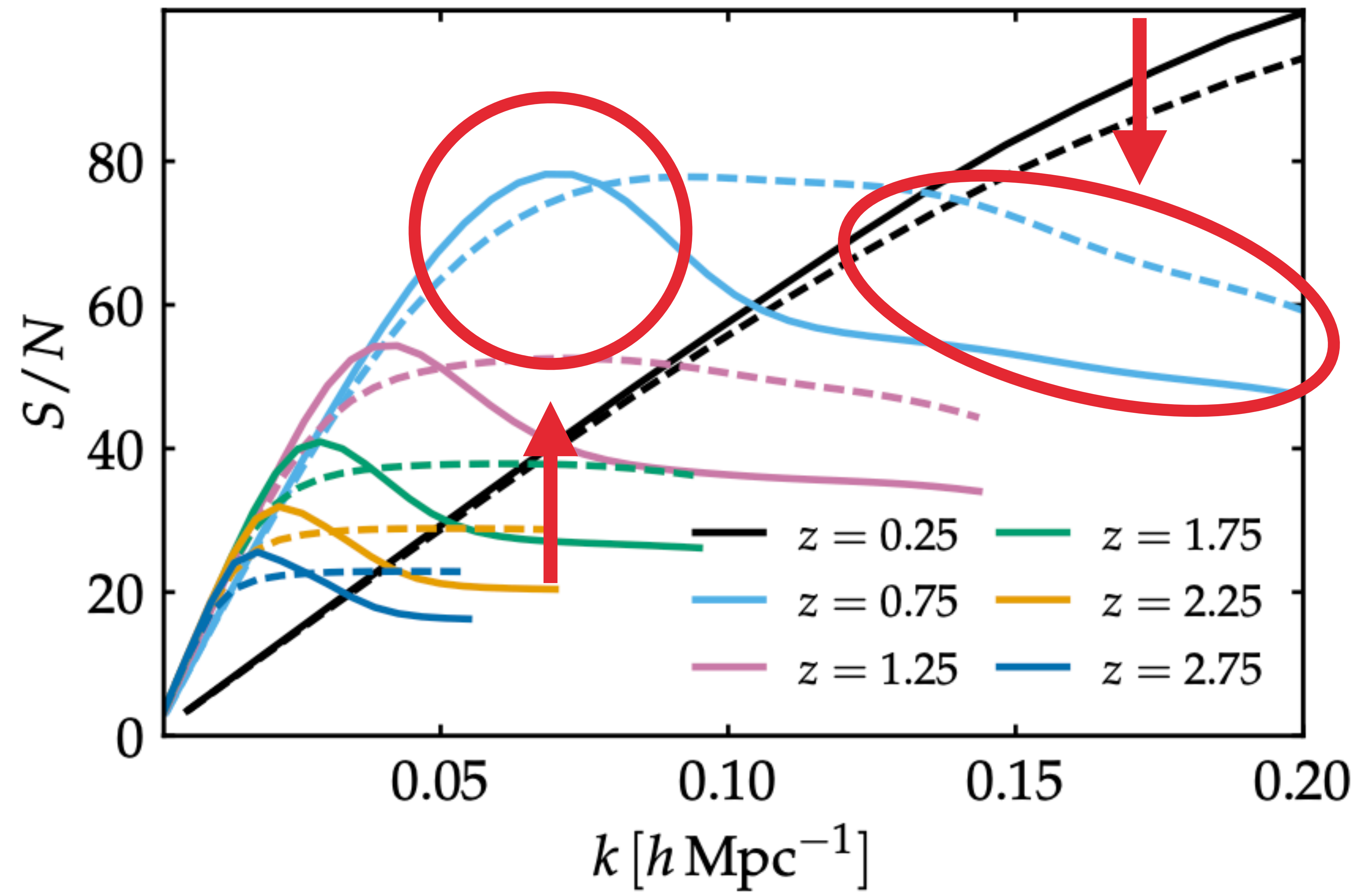
with $\Delta\vec{\Theta}(z_c) = \vec{\Theta}^{\mathrm{th}}(z_c) - \vec{\Theta}^{\mathrm{obs}}(z_c)$

NB redshift bins are independent!

MCMC analysis → modified version of **CosmoMC**

Lewis (2002)





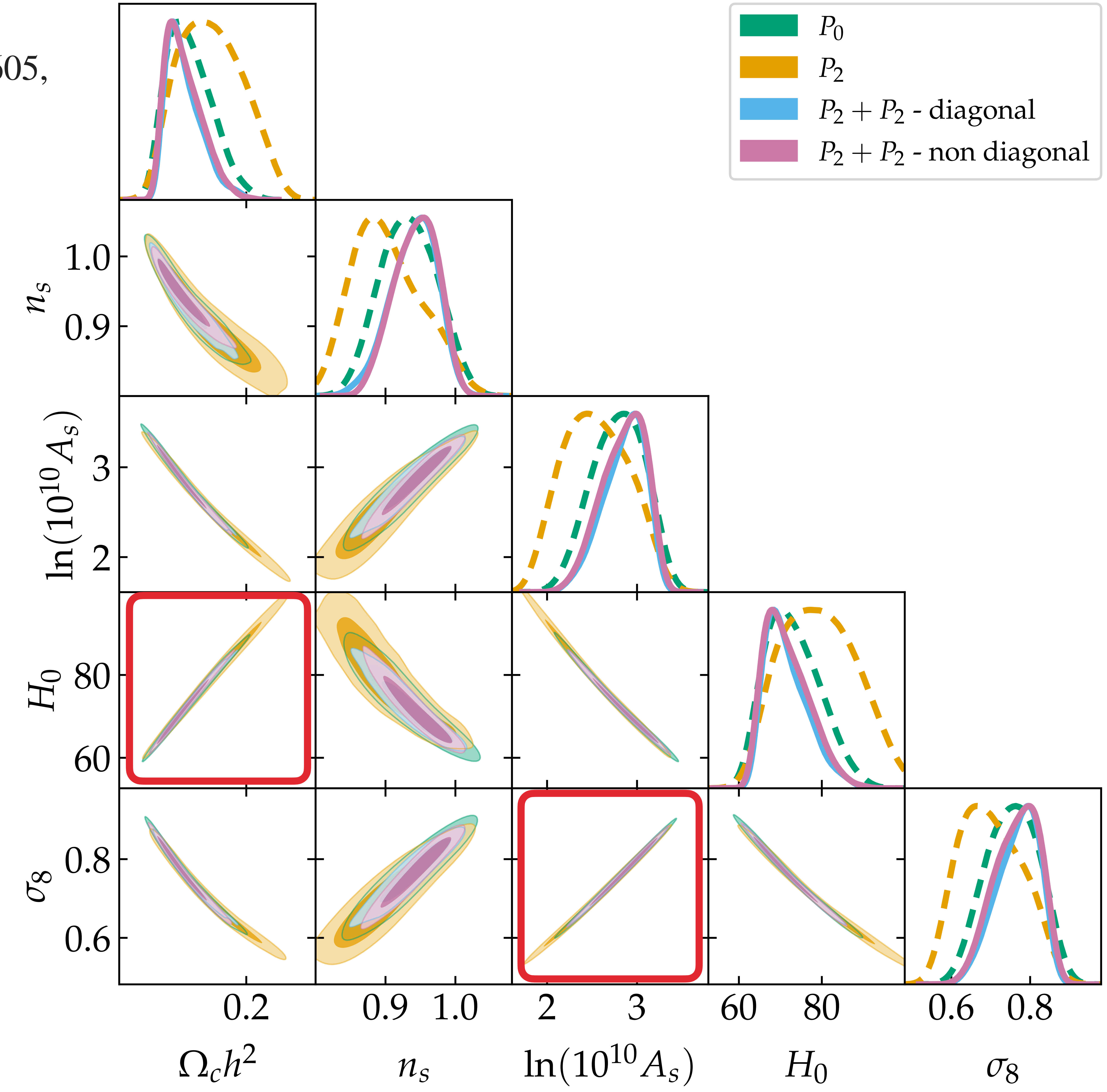
Results

Constraints From the Multipoles

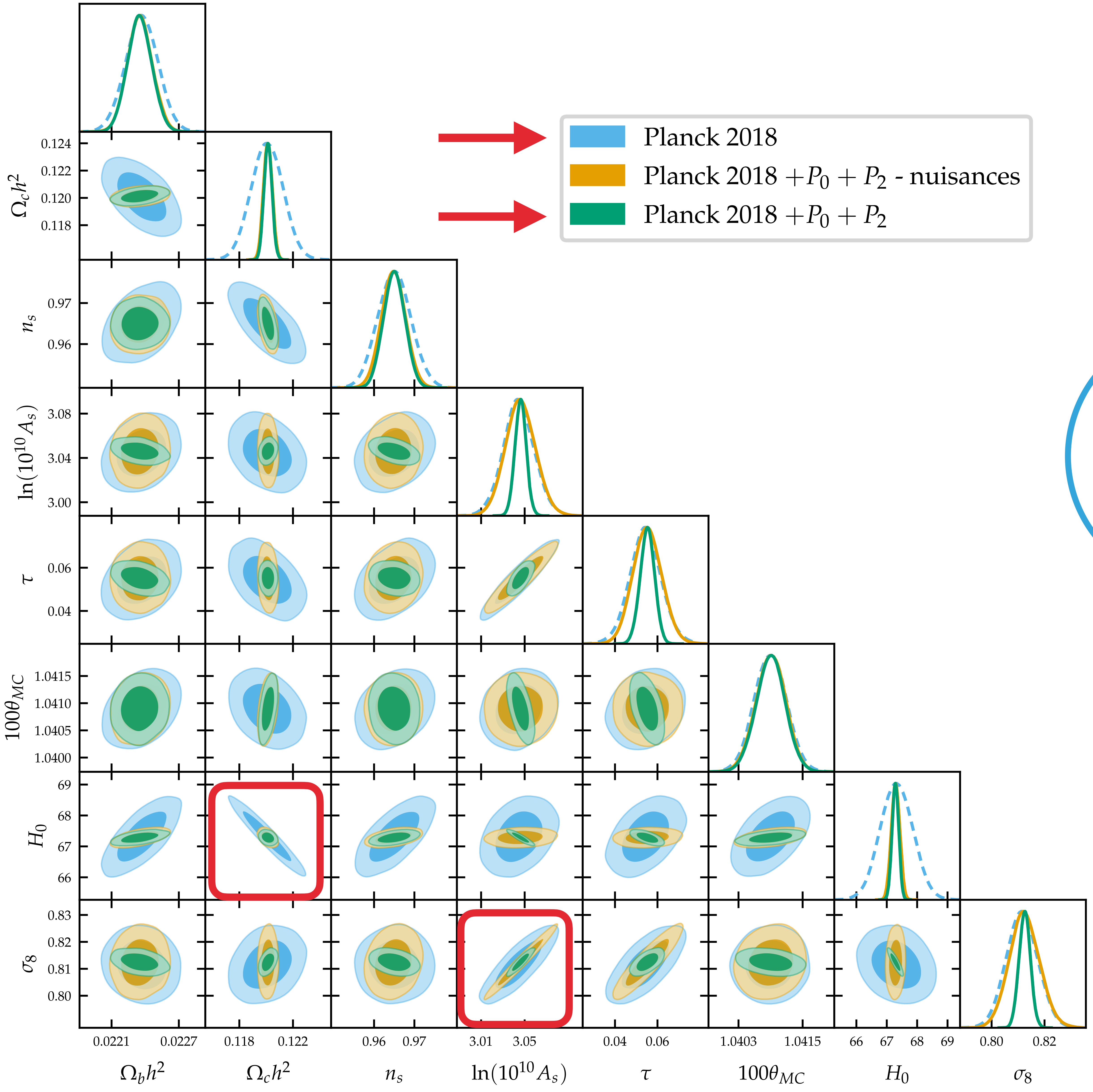
Fiducial values → $\{\Omega_b h^2 = 0.022383, \Omega_c h^2 = 0.12011, n_s = 0.96605,$
$\ln(10^{10} A_s) = 3.0448, \tau = 0.0543, H_0 = 67.32\}$

Parameter	P_0	P_2	$P_0 + P_2$ diagonal	$P_0 + P_2$ non diagonal
$\Omega_c h^2$	16.7%	21.6%	12.7%	13.4%
n_s	4.6%	5.6%	3.6%	3.4%
H_0	9.1%	12.0%	6.9%	7.4%
σ_8	9.6%	11.9%	7.1%	7.6%

- Tighter constraints using $P_0 + P_2$
- Using the full non diagonal matrix doesn't affect much the constraints
- 21cm signal alone from tomography has a good constraining power on cosmological parameters



Multipoles Combined With CMB Data



Planck TT, TE, EE + lowE + lensing

Parameter	Planck 2018	$+P_0 + P_2$
$\Omega_b h^2$	0.64%	0.49%
$\Omega_c h^2$	0.99%	0.25%
n_s	0.42%	0.27%
$\ln(10^{10} A_s)$	0.46%	0.17%
τ	13.44%	6.09%
$100\theta_{MC}$	0.03%	0.03%
H_0	0.79%	0.16%
σ_8	0.73%	0.26%

NB Always with non diagonal covariance!

Combining CMB and the 21cm multipoles

- Constraints are significantly improved
- Less marked degeneracies

Adding the Nuisance Parameters

One parameter per redshift bin → 12 new parameters

$$\{[T_b b_{\rm HI}\sigma_8]_1, [T_b b_{\rm HI}\sigma_8]_2, [T_b b_{\rm HI}\sigma_8]_3, [T_b b_{\rm HI}\sigma_8]_4, [T_b b_{\rm HI}\sigma_8]_5, [T_b b_{\rm HI}\sigma_8]_6\}$$

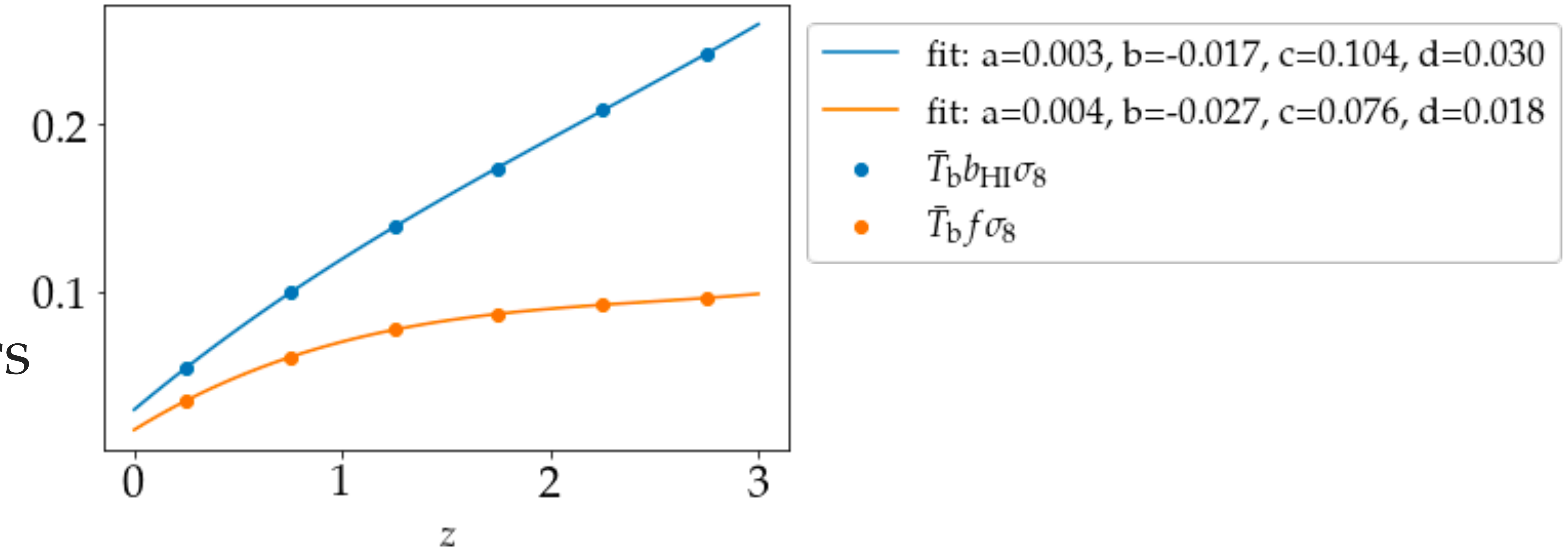
$$\{[T_b f\sigma_8]_1, [T_b f\sigma_8]_2, [T_b f\sigma_8]_3, [T_b f\sigma_8]_4, [T_b f\sigma_8]_5, [T_b f\sigma_8]_6\}$$

$$P_{21}(z,k,\mu) = \left[\bar{T}_{\rm b} b_{\rm HI}\sigma_8 + \bar{T}_{\rm b} f\sigma_8\,\mu^2\right]^2 P_{\rm m}/\sigma_8$$

Extrapolate redshift dependence → 8 parameters

$$\{[T_b b_{\rm HI}\sigma_8]_a, [T_b b_{\rm HI}\sigma_8]_b, [T_b b_{\rm HI}\sigma_8]_c, [T_b b_{\rm HI}\sigma_8]_d\}$$

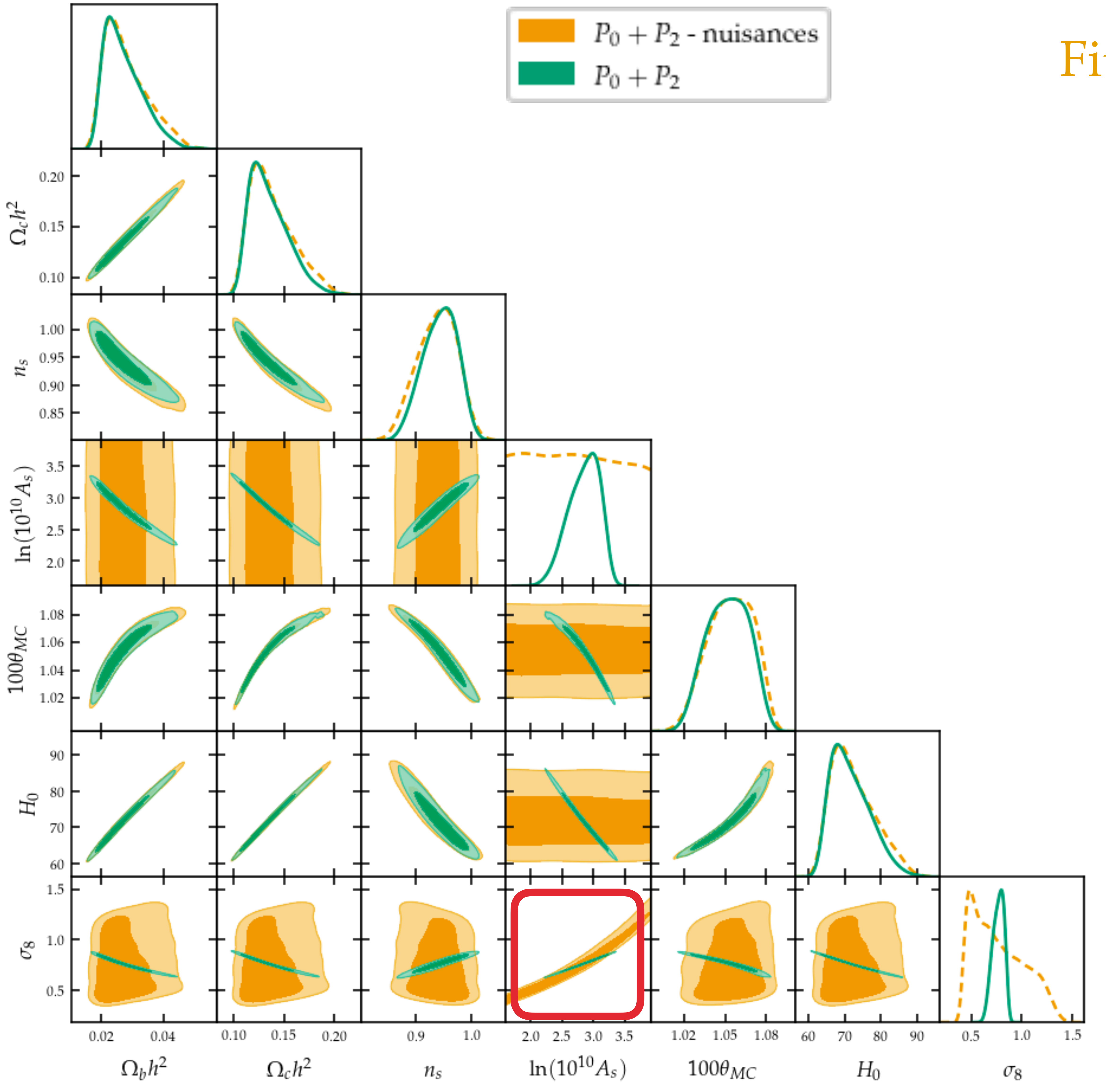
$$\{[T_b f\sigma_8]_a, [T_b f\sigma_8]_b, [T_b f\sigma_8]_c, [T_b f\sigma_8]_d\}$$



$$f(z) = az^3 + bz^2 + cz + d$$

for $f(z) = \bar{T}_{\rm b} b_{\rm HI}\sigma_8, \bar{T}_{\rm b} f\sigma_8$

Results Varying the Nuisance Parameters



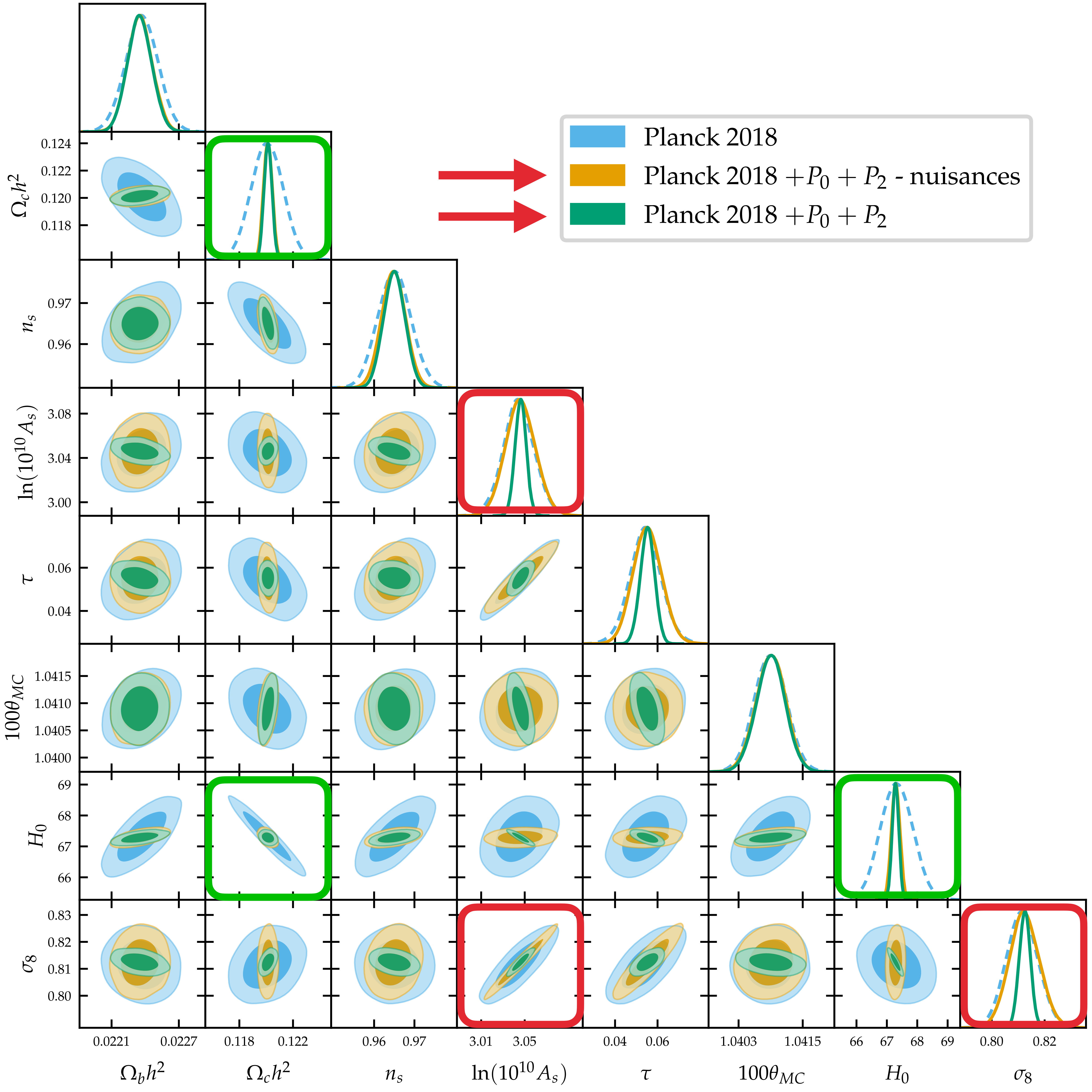
Fit redshift evolution

Parameter	$P_0 + P_2$	with nuisances
$\Omega_b h^2$	21.04%	22.81%
$\Omega_c h^2$	13.36%	14.66%
n_s	3.44%	3.94%
$\ln(10^{10} A_s)$	8.83%	—
$100\theta_{MC}$	1.53%	1.62%
H_0	7.39%	8.10%
σ_8	7.64%	32.48%

For the two nuisances models

- Same constraints on the cosmological parameters
- Faster and better convergence with 8 parameters (redshift fit)

Combined With CMB Data

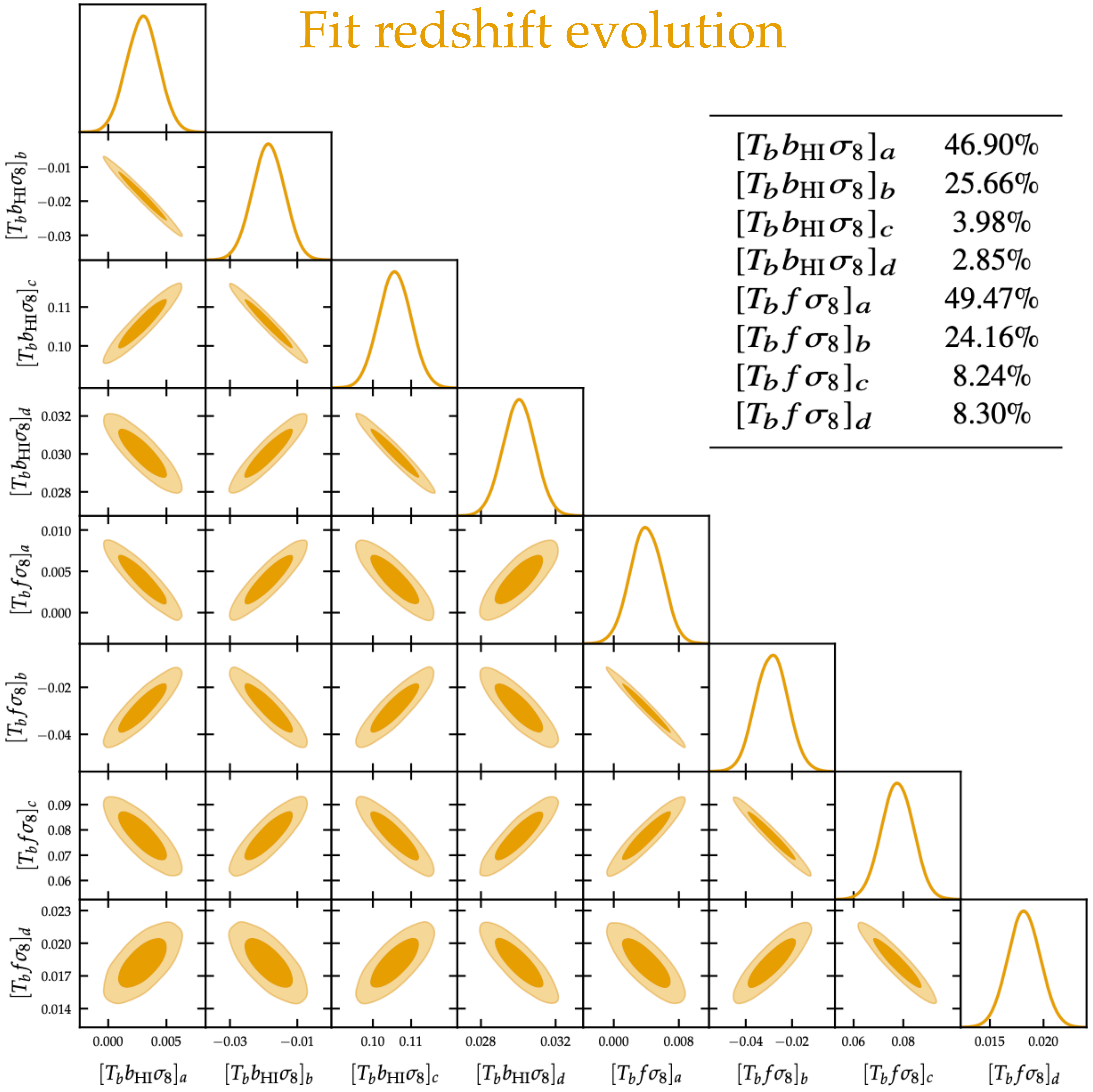


Fit redshift evolution

Parameter	Planck 2018	$+P_0+P_2$	+ with nuisances
$\Omega_b h^2$	0.64%	0.49%	0.49%
$\Omega_c h^2$	0.99%	0.25%	0.27%
n_s	0.42%	0.27%	0.31%
$\ln(10^{10}A_s)$	0.46%	0.17%	0.45%
τ	13.44%	6.09%	12.19%
$100\theta_{MC}$	0.03%	0.03%	0.03%
H_0	0.79%	0.16%	0.20%
σ_8	0.73%	0.26%	0.70%

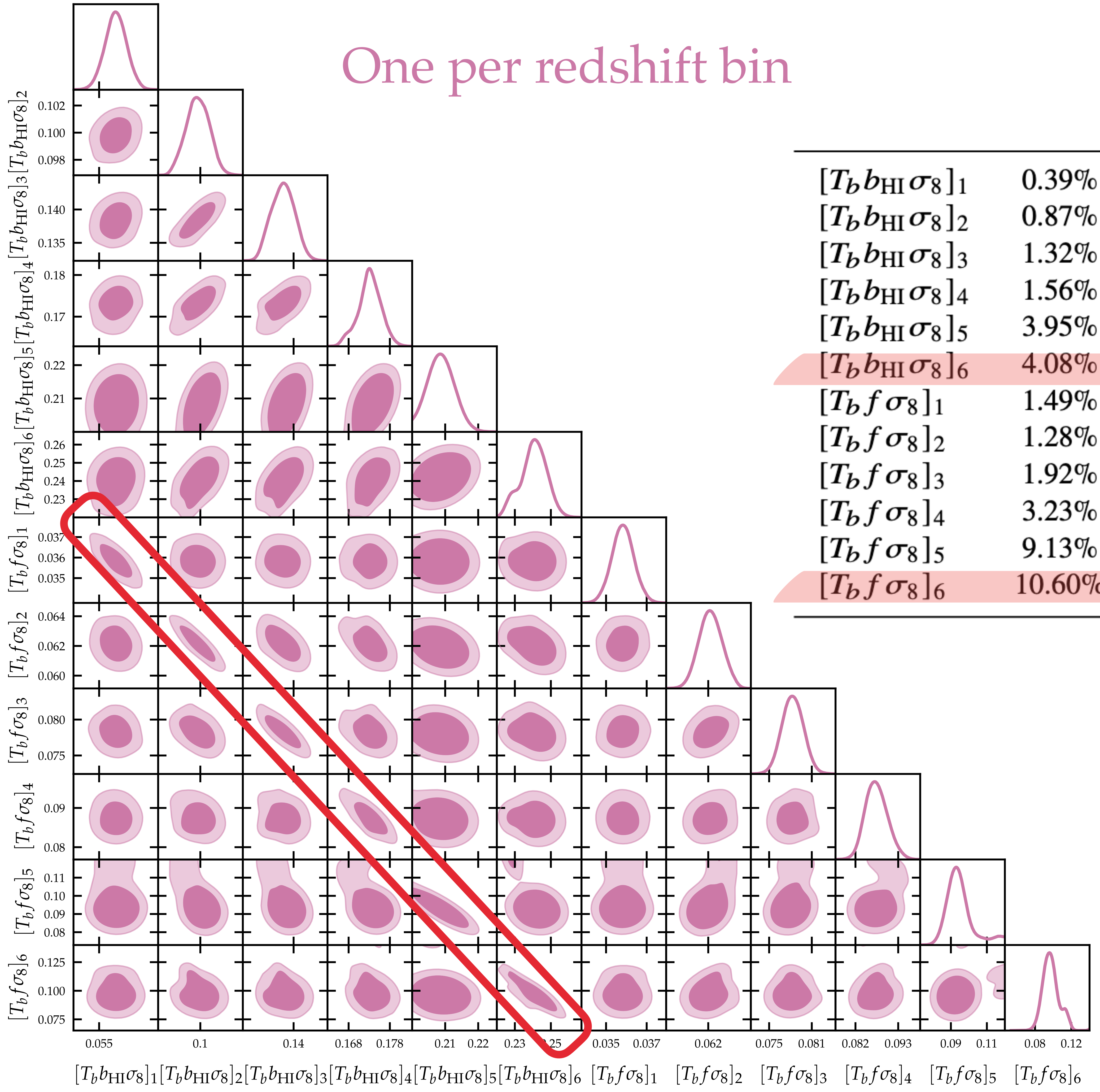
Constraints on the Nuisances

Fit redshift evolution



$[T_b b_{HI}\sigma_8]_a$	46.90%
$[T_b b_{HI}\sigma_8]_b$	25.66%
$[T_b b_{HI}\sigma_8]_c$	3.98%
$[T_b b_{HI}\sigma_8]_d$	2.85%
$[T_b f\sigma_8]_a$	49.47%
$[T_b f\sigma_8]_b$	24.16%
$[T_b f\sigma_8]_c$	8.24%
$[T_b f\sigma_8]_d$	8.30%

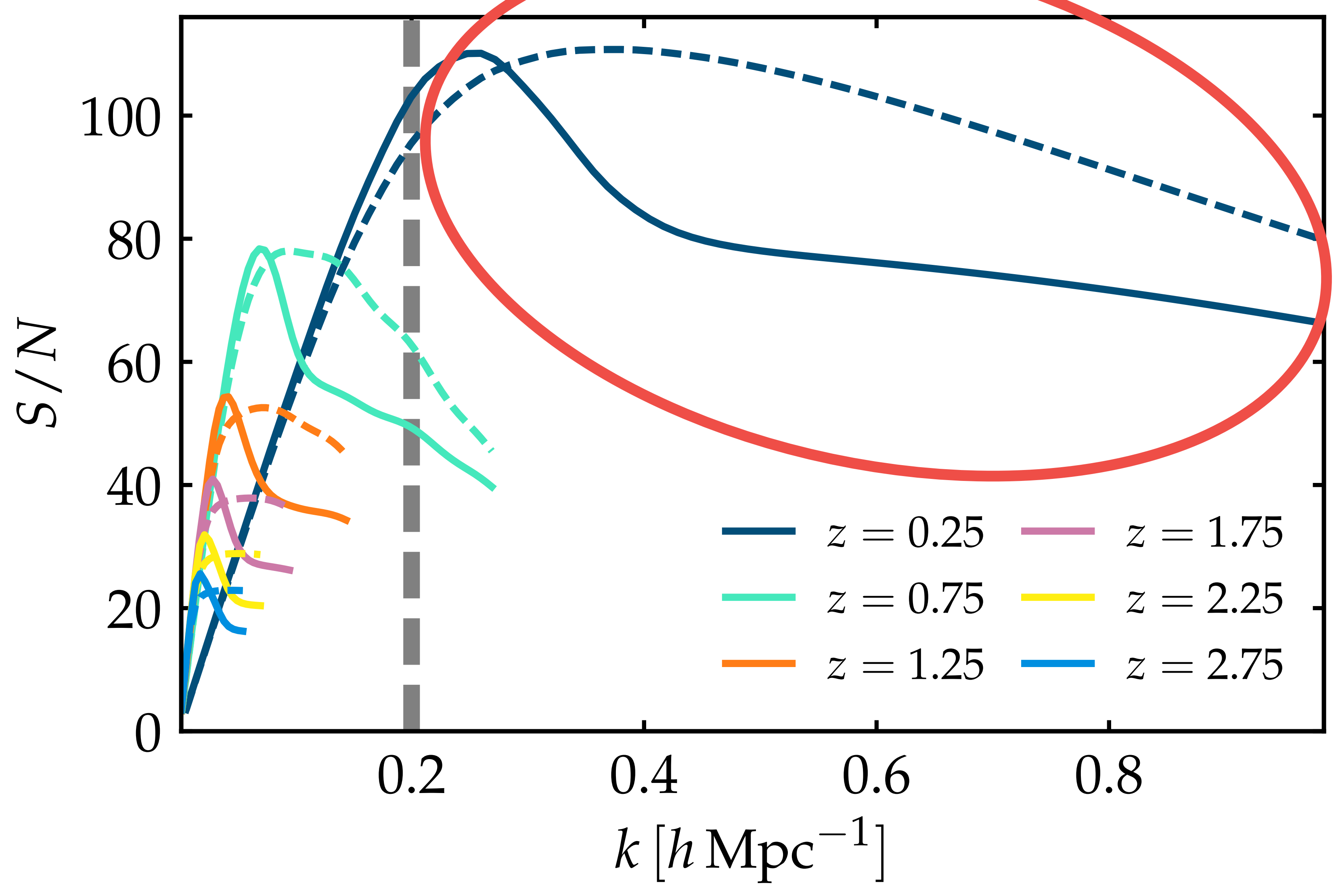
One per redshift bin



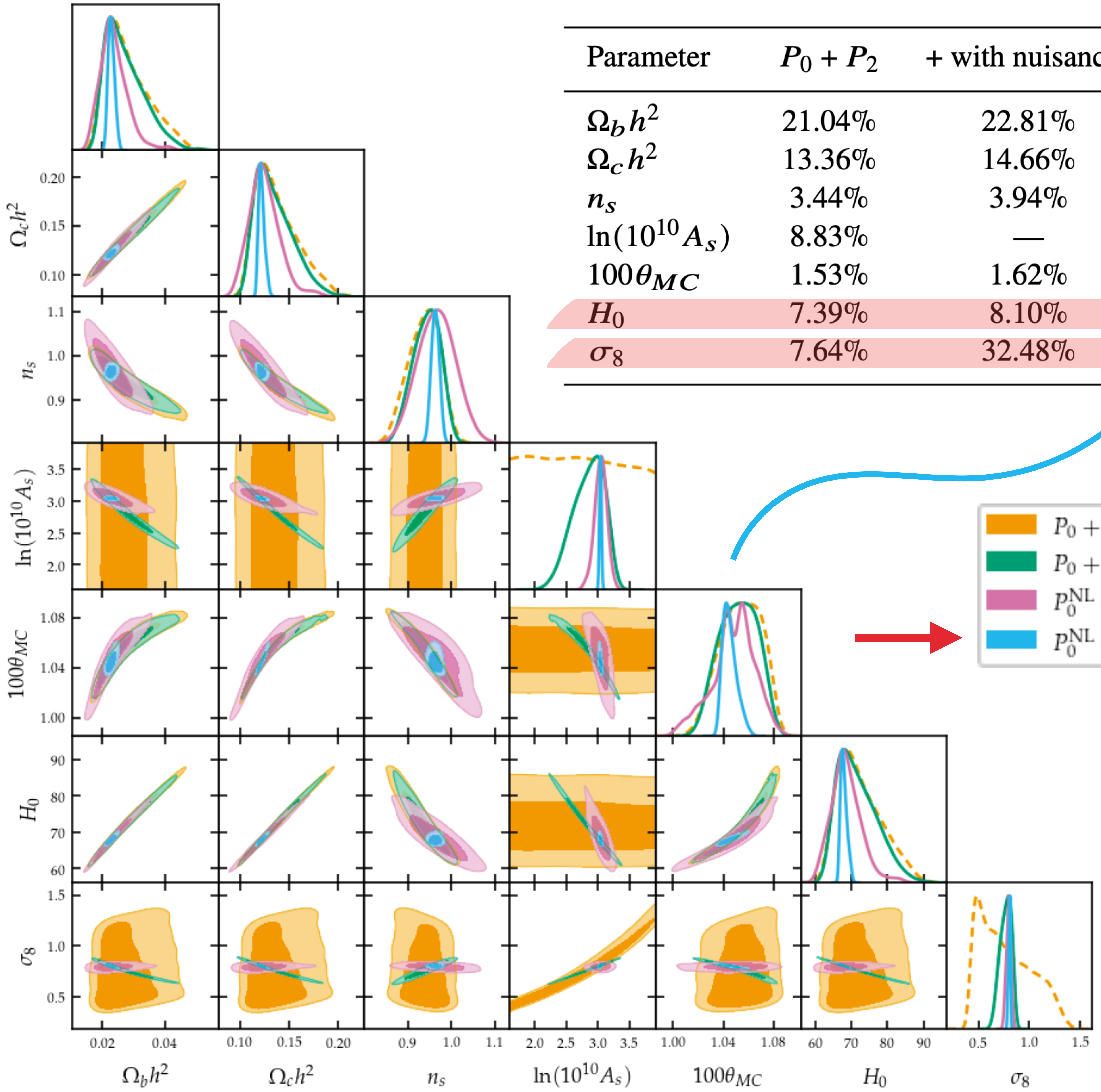
$[T_b b_{HI}\sigma_8]_1$	0.39%
$[T_b b_{HI}\sigma_8]_2$	0.87%
$[T_b b_{HI}\sigma_8]_3$	1.32%
$[T_b b_{HI}\sigma_8]_4$	1.56%
$[T_b b_{HI}\sigma_8]_5$	3.95%
$[T_b b_{HI}\sigma_8]_6$	4.08%
$[T_b f\sigma_8]_1$	1.49%
$[T_b f\sigma_8]_2$	1.28%
$[T_b f\sigma_8]_3$	1.92%
$[T_b f\sigma_8]_4$	3.23%
$[T_b f\sigma_8]_5$	9.13%
$[T_b f\sigma_8]_6$	10.60%

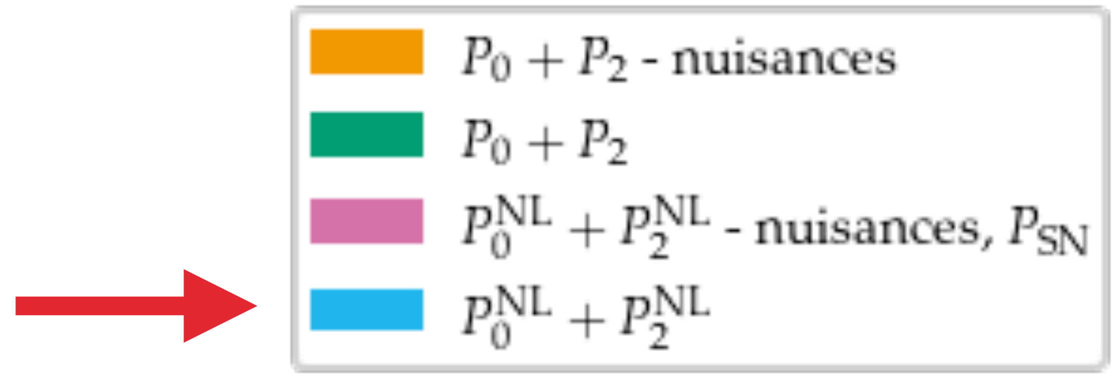
Extending to Non-Linear Scales

- Include non linear corrections
- Introduce $P_{\rm SN}$
- Minimum scale from beam size

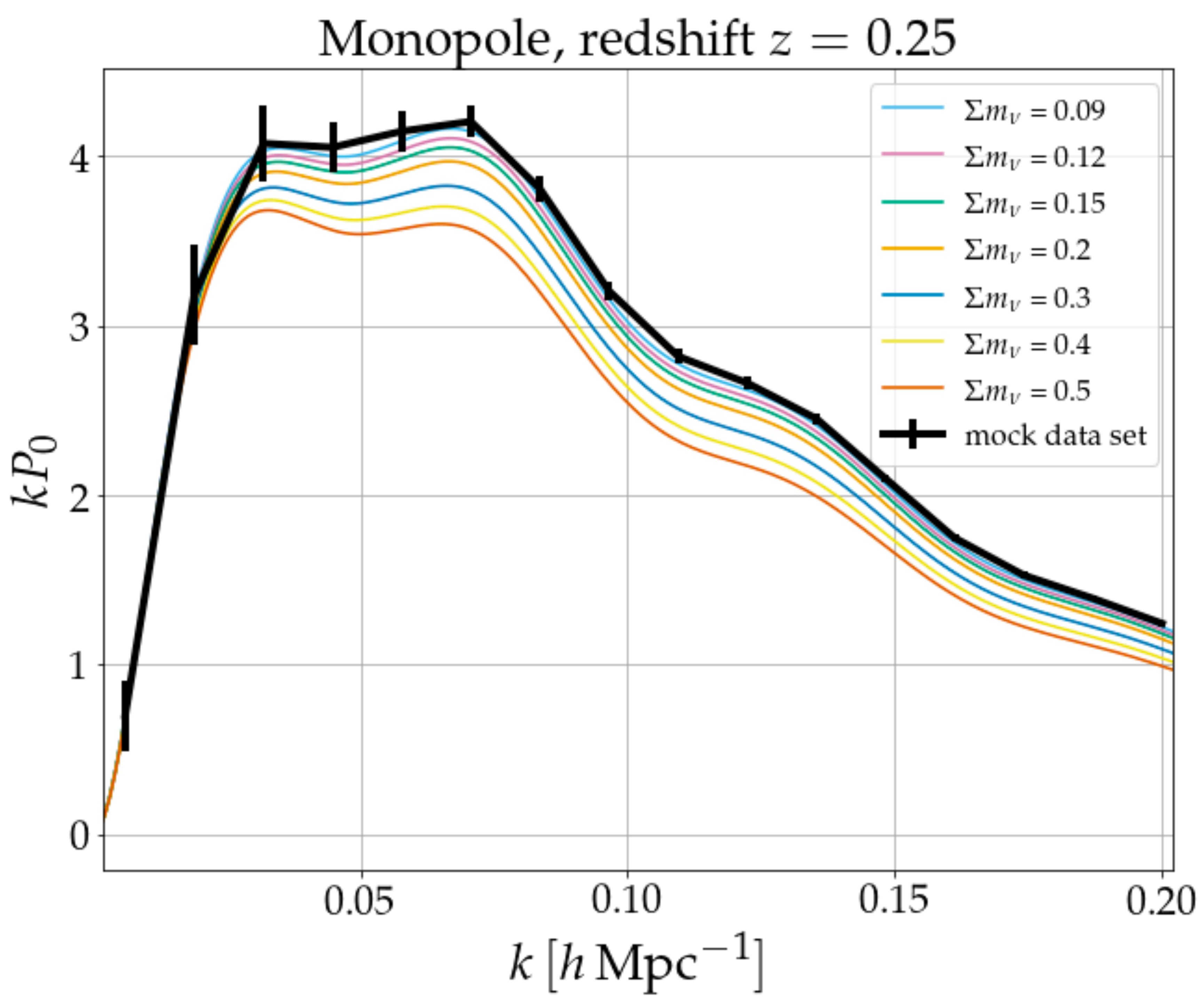


Parameter	$P_0 + P_2$	+ with nuisances	$P_0^{\rm NL} + P_2^{\rm NL}$
$\Omega_b h^2$	21.04%	22.81%	3.02%
$\Omega_c h^2$	13.36%	14.66%	1.16%
n_s	3.44%	3.94%	0.95%
$\ln(10^{10} A_s)$	8.83%	—	0.49%
$100\theta_{MC}$	1.53%	1.62%	0.18%
H_0	7.39%	8.10%	0.49%
σ_8	7.64%	32.48%	0.37%



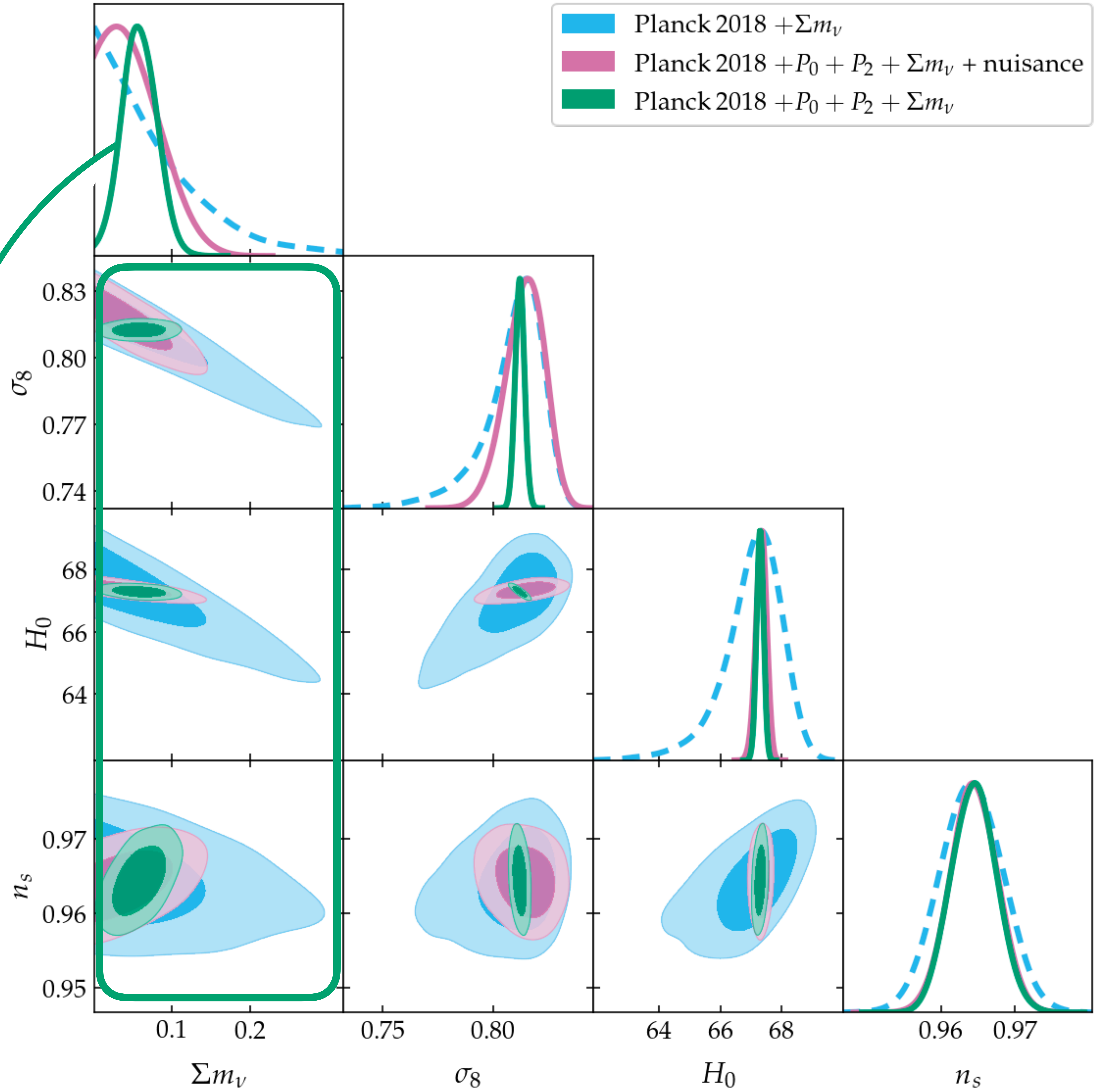


Constraints for the Neutrino Mass Σm_ν



Parameter	$P_0 + P_2 + \Sigma m_\nu$	+ with nuisance
Σm_ν	< 0.305	< 0.404

Parameter	Planck 2018 $+\Sigma m_\nu$	$+P_0 + P_2$	+ with nuisances
Σm_ν	< 0.219	$0.058^{+0.044}_{-0.043}$	< 0.118



Conclusions

Summary

WORK DONE

- Published work on Dark Energy constraints (no multipoles) arXiv:2109.03256
- Construct a tomographic data set forecasting SKAO observations within 6 redshift bin of the monopole and the quadrupole of the 21cm signal power spectrum
- Constrain the full set of cosmological parameters with a MCMC analysis, using the forecasted 21cm data set alone and combined with CMB data

RESULTS

- Adding observations of the 21cm signal to CMB significantly improved the constraints on the cosmological parameters
- When nuisances are taken into account, some of the constraining power is lost, e.g. on A_s (and σ_8), but results for $\Omega_c h^2/H_0$ are not spoiled
- Constraints on neutrinos to be further investigated

Analytical P_0, P_2

$$P_0 = \frac{\bar{T}_b^2 P_m}{2} e^{-A} \left[b_{HI}^2 \frac{\sqrt{\pi}\,\mathrm{erfi}(\sqrt{A})}{\sqrt{A}} + 2 b_{HI} f \left(\frac{e^A}{A} - \frac{\sqrt{\pi}\,\mathrm{erfi}(\sqrt{A})}{2A^{3/2}} \right) + \right.$$

$$\left. + f^2 \left(\frac{3\sqrt{\pi}\,\mathrm{erfi}(\sqrt{A})}{4A^{5/2}} + \frac{e^A(2A-3)}{2A^2} \right) \right]$$

$$P_2 = \frac{15 \bar{T}_b^2 P_m}{4} e^{-A} \left[b_{HI}^2 \left(\frac{e^A}{A} - \frac{\sqrt{\pi}\,\mathrm{erfi}(\sqrt{A})}{2A^{3/2}} \right) + 2 b_{HI} f \cdot \right.$$

$$\cdot \left(\frac{3\sqrt{\pi}\,\mathrm{erfi}(\sqrt{A})}{4A^{5/2}} + \frac{e^A(2A-3)}{2A^2} \right) + f^2 \left(- \frac{15\sqrt{\pi}\,\mathrm{erfi}(\sqrt{A})}{8A^{7/2}} + \right.$$

$$\left. \left. + \frac{e^A(15 - 10A + 4A^2)}{4A^3} \right) \right] - \frac{5}{2} P_0$$

$$A = k^2 R_{beam}$$